Phase space extensions at Null infinity for Gravity and Gauge theories

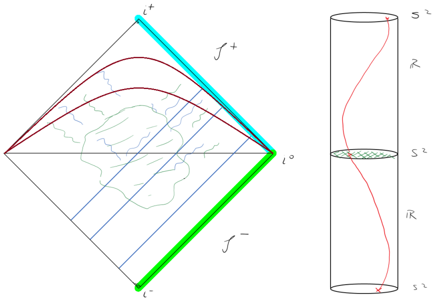
Javier Peraza

ENS, Lyon - July 21, 2022

Joint work with Miguel Campiglia: [arXiv:2002.06691],[arXiv:2111.00973]

General Idea

- Starting point: **Phase Space** at $\mathcal{I}^+$ with "radiative" boundary condition, Γ^{rad} + **Symmetries** $\mathcal{G}$, and its associated canonical charges, $Q_g, g \in \mathcal{G}$: Leading charges.
- Leading and Sub^n-leading **soft factorization theorems** imply new symmetries of the S matrix.
- Enlargement $\mathcal{G} \subset \hat{\mathcal{G}}$ implies relaxation of b.c. through more **Large Gauge Transformations**.
- Construct Γ^{ext} such that $\hat{\mathcal{G}}$ acts canonically. Introduction of **edge modes** for large gauge symmetries and new non-trivial conservation laws.



Extended phase space with radiative modes and edge mode fields: [Ciambelli, Donnay, Donnelly, Freidel, Geiller, Leigh, Livine, Pranzetti,...]

Gravity case

Bondi gauge

$$ds^2 = (v/r)e^{2\beta}du^2 - 2e^{2\beta}dudr + g_{ab}(dx^a - U^a du)(dx^b - U^b du),$$

$\beta = O(r^{-2}), \quad v = O(r), \quad U^a = O(r^{-2}), \quad g_{ab} = r^2 q_{ab}(\hat{x}) + r(C_{ab}(u,\hat{x}) + uT_{ab}(\hat{x})) + \ldots$

Gauge fixing: $\det(g_{ab}) = r^4 \det(q_{ab})$. Geroch Tensor: $D^b T_{ab} = -\frac{1}{2} D_a R(q_{ab})$

Symmetries: $\boxed{CKV(S^2) \ltimes C^\infty(S^2)}$ vs. $\boxed{Diff(S^2) \ltimes C^\infty(S^2)}$

1. $\mathcal{G}$:= Extended BMS: $\nabla_{(\mu}\xi_{\nu)} \overset{r\to+\infty}{\to} 0$. Fixes the metric q_{ab}. [Barnich, Troessaert '10]

   "Bondi frame" := $q_{ab} = q_{S^2}$, round metric on $S^2 \Leftrightarrow T_{ab} \equiv 0$

2. $\mathcal{G}$:=Generalized BMS: $\nabla_\mu \xi^\mu \overset{r\to+\infty}{\to} 0$. Relaxation of q_{ab}. (but $\sqrt{q_{ab}} = \sqrt{q_{S^2}}$) [Campiglia, Laddha '14]

$$\xi = V^a(\hat{x})\partial_a + (u\alpha(\hat{x}) + f(\hat{x}))\partial_u - r\alpha(\hat{x})\partial_r + O(r^{-1}), \quad \alpha = \frac{1}{2} D_c V^c$$

Variations

Diffeo algebra: $[\xi_f, \xi_{f'}] = 0, \quad [\xi_V, \xi_f] = \xi_{\mathcal{L}_V f - \alpha f}, \quad [\xi_V, \xi'_V] = \xi_{[V,V']}$.

General action:

$$\delta_f q_{ab} = 0, \quad \delta_f C_{ab} = f\partial_u C_{ab} - 2D_a D_b^{TF} f + fT_{ab}, \quad \delta_f T_{ab} = 0$$

$$\delta_V q_{ab} = \mathcal{L}_V q_{ab} - 2\alpha q_{ab}, \quad \delta_V C_{ab} = \mathcal{L}_V C_{ab} + \alpha u\partial_u C_{ab} - \alpha C_{ab}, \quad \delta_V T_{ab} = \mathcal{L}_V T_{ab} - 2D_a D_b \alpha^{TF}$$

Charge candidates [Campiglia,Laddha '14,'15],

$$P_f = \int_{\mathcal{I}} \delta_f C^{ab} \partial_u C_{ab} d\mu,$$

$$J_V = \underbrace{\int_{\mathcal{I}} \delta_V C_{ab} \partial_u C^{ab} d\mu}_{\text{Standard Ashtekar-Streubel terms}} + \underbrace{\int_{\mathcal{I}} u \partial_u C^{ab} \left(-4\bar{D}_a \bar{D}_b \bar{\alpha} + \bar{D}_a \bar{D}^c \delta_V q_{bc} - \frac{\bar{R}}{2} \delta_V q_{ab} \right) d\mu}_{\text{Acting with a } Diff(S^2)}$$

Problem:

$$\{P_f, J_V\} = \delta_V P_f \neq -\delta_f J_V$$

$$\delta_f J_V = P_{V(f)} - \delta_f \,(\text{Boundary Term}) + \text{magnetic}(f, V),$$

$$\text{Boundary Term} = \int_{\partial\mathcal{I}} \left(V^a C^{bc} \bar{D}_c C_{ab} + \frac{3}{2} \bar{\alpha} C^2 \right) \sqrt{q} d^2x =: J_V^{\partial\mathcal{I}}$$

$$\text{magnetic}(f, V) = -4 \int_{S^2} \left(\int_{\mathbb{R}} \partial_u C^{ab} du \right) \bar{D}_a \bar{D}^c \left(\bar{D}_{[b} f V_{c]} - \frac{1}{2} f \bar{D}_{[b} V_{c]} \right) \sqrt{q} d^2x.$$

magnetic term: vanishes by AF, $\lim_{u \to \pm\infty} \bar{D}_{[a} \bar{D}^c C_{b]c} = 0$. *Pure supertranslation.*

Resolution: add a boundary term.

$$
\begin{aligned}
P_f &= \int_{\mathcal{I}} \delta_f C^{ab} \partial_u C_{ab} d\mu, \\
J_V &= \underbrace{\int_{\mathcal{I}} \delta_V C_{ab} \partial_u C^{ab} d\mu}_{\text{Standard Bondi frame term}} + \underbrace{\int_{\mathcal{I}} u \partial_u C^{ab} \left(-4 \bar{D}_a \bar{D}_b \bar{\alpha} + \bar{D}_a \bar{D}^c \delta_V q_{bc} - \frac{\bar{R}}{2} \delta_V q_{ab} \right) d\mu}_{\text{Acting with a } Diff(S^2)} \\
&\quad + \underbrace{\int_{\partial\mathcal{I}} \left(V^a C^{bc} \bar{D}_c C_{ab} + \frac{3}{2} \bar{\alpha} C^2 \right) \sqrt{q} d^2 x}_{\text{Boundary term}}
\end{aligned}
$$

Phase Space:

$$
\Gamma := \bigcup_{q_{ab}: \sqrt{q} = \sqrt{q_{S^2}}} \{ C_{ab} : q^{ab} C_{ab} = 0, \partial_u C_{ab} \overset{u \to \pm\infty}{=} O(1/|u|^{2+\epsilon}), \lim_{u \to \pm\infty} \bar{D}_{[a} \bar{D}^c C_{b]c} = 0 \}_{q_{ab}}
$$

Simplectic Form:

$$
\boxed{\Omega = \int_{\mathcal{I}} \delta \partial_u C^{ab} \wedge \delta C_{ab} d\mu + \int_{S^2} (\delta p^{ab} \wedge \delta q_{ab} + \delta \Pi^{ab} \wedge \delta T_{ab}) \sqrt{q} d^2 x,}
$$

$$
p^{ab} = D^{(a} D_c \overset{1}{N}{}^{b)c} - \frac{R}{2} \overset{1}{N}{}^{ab} + (C^2)^{ab}|_{\partial\mathcal{I}}, \quad \Pi^{ab} = 2 \overset{1}{N}{}^{ab} + \frac{1}{2} C C^{ab}|_{\partial\mathcal{I}}, \quad \overset{1}{N}{}^{ab} = \int_{\mathbb{R}} u \partial_u C^{ab} du
$$

Yang-Mills case

Eom + Gauge symmetry :

$$\mathcal{D}^\mu \mathcal{F}_{\mu\nu} = \nabla^\mu \mathcal{F}_{\mu\nu} + [\mathcal{A}^\mu, \mathcal{F}_{\mu\nu}] = 0, \quad \delta_\Lambda \mathcal{A}_\mu = \mathcal{D}_\mu \Lambda = \partial_\mu \Lambda + [\mathcal{A}_\mu, \Lambda]$$

Harmonic gauge: $\nabla^\mu \mathcal{A}_\mu = 0 \quad \Rightarrow \quad [\delta_\Lambda, \delta_\Upsilon]\mathcal{A}_\mu = \delta_{[\Lambda,\Upsilon]^*}\mathcal{A}_\mu$.
Decays in (u, r, x^a): consistent with eom and tree level diagrams for "radiative" data, Miguel's talk!

$$\begin{aligned}
\mathcal{A}_r &= \tfrac{1}{r^2}(\ln r \overset{0,\ln}{A}_r + \overset{0}{A}_r) + \tfrac{1}{r^3}(\ln r \overset{1,\ln}{A}_r + \overset{1}{A}_r) + o(r^{-3}),\\
\mathcal{A}_u &= \tfrac{\ln r}{r} \overset{0,\ln}{A}_u + \tfrac{1}{r^2}(\ln^2 r \overset{1,\ln^2}{A}_u + \ln r \overset{1,\ln}{A}_u + \overset{1}{A}_u) + o(r^{-2}),\\
\mathcal{A}_a &= A_a + \tfrac{1}{r}(\ln r \overset{1,\ln}{A}_a + \overset{1}{A}_a) + o(r^{-1}),
\end{aligned}$$

$$\Gamma^{\text{rad}} = \{A_a : \partial_u A_a \overset{u\to\pm\infty}{=} O(1/|u|^\infty), \lim_{u\to+\infty} \mathcal{F}^{(0)}_{ru} = \lim_{u\to+\infty} \mathcal{F}^{(0)}_{ra} = 0\}$$

Subleading soft theorem are related to the $O(r)$ gauge transformation!

$$\begin{aligned}
\Lambda^0_\lambda(r,u,x) &= \lambda(x) + \frac{\ln r}{r}\overset{\ln}{\lambda}(u,x) + O(\ln^2 r/r^2),\\
\Lambda^1_\varepsilon(r,u,x) &= r\varepsilon(x) + \ln r \overset{0,\ln}{\varepsilon}(u,x) + \overset{0}{\varepsilon}(u,x) + O(\ln^3 r/r).
\end{aligned}$$

Problem: The set span$(\{\Lambda^0_\lambda, \Lambda^1_\epsilon\})$ contains $O(r^n)$, for any n.
Minimally extending the phase space, by linearizing at $O(r)$

Extension:

$$\Gamma^{\text{ext}} := \{\tilde{\mathcal{A}}_\mu = \mathcal{A}_\mu + \mathcal{D}_\mu \Lambda^1_\phi, \quad \mathcal{A}_\mu \in \Gamma^{\text{rad}}, \quad \phi \in C^\infty(S^2)\}$$

$$\delta^0_\lambda A_a = D_a \lambda, \quad \delta^0_\lambda \phi = [\phi, \lambda], \quad \delta^1_\epsilon A_a = 0, \quad \delta^1_\epsilon \phi = \epsilon$$

Remark: all identities are modulo $O(\phi^2)$, $O(\epsilon^2)$ and $O(\phi\epsilon)$.

Variations: $[\delta^0_\lambda, \delta^0_{\lambda'}] = \delta^0_{[\lambda,\lambda']}, \quad [\delta^1_\epsilon, \delta^0_\lambda] = \delta^1_{[\epsilon,\lambda]}, \quad [\delta^1_\epsilon, \delta^1_{\epsilon'}] = 0$

Imposing **Poisson algebra closure**

$$\{Q^0_\lambda, Q^0_{\lambda'}\} = Q^0_{[\lambda,\lambda']}, \quad \{Q^0_\lambda, {Q^1}_\epsilon\} = {Q^1}_{[\lambda,\epsilon]},$$

we have,

$$\boxed{Q^0_\lambda = \int_{\mathcal{I}} \text{tr}(\partial_u A^a D_a \lambda) d\mu + {Q^1}_{[\phi,\lambda]}} \boxed{{Q^1}_\epsilon = \int_{S^2} \text{Tr} d^2x(\epsilon\pi)} \quad \pi := -\frac{1}{2}\int_{\mathbb{R}} u\partial_u D^-_a (D^a F_{ru} + D_b F^{ba}) du$$

Symplectic Form

$$\boxed{\Omega^{\text{ext}} := \underbrace{\int_{\mathcal{I}} \text{Tr}(\delta\partial_u A^a \wedge \delta A_a) dud^2x}_{\Omega^{\mathcal{I}}} + \underbrace{\int_{S^2} \text{Tr}(\delta\pi \wedge \delta\phi) d^2x}_{\Omega^{S^2}}}$$

Abelian Case: Infinite tower of charges from first principles

Directions by different $O(r^n)$ extensions independent of each other. (No commutator! $\Rightarrow$ linearity and no field dependence)

Can be done directly from first principles (upon renormalization [Freidel, Hopfmüller, Riello, '19).

$$\Gamma^{\text{ext}} := \{\tilde{\mathcal{A}}_\mu = \mathcal{A}_\mu + \partial_\mu \sum_{n>0} \Lambda^n_{\phi_n}, \quad \mathcal{A}_\mu \in \Gamma^{\text{rad}}, \quad \phi_n \in C^\infty(S^2)\}$$

After renormalization

$$\Omega^{ext}_{ren} = \underbrace{\int_{\mathcal{I}} \omega_0(\delta,\delta) du dx^2}_{\Omega^{\mathcal{I}}} + \underbrace{\int_{S^2} \sum_{n=1}^{\infty} \delta F_{ru}^{(-2-n,0)} \wedge \delta \phi_n d^2x}_{\Omega^{S^2}}$$

with $F_{ru}^{(-2-i,0)}$ as in [Campiglia,Laddha '18]. **Symplectic Form before charges!**

Charges

$$\boxed{Q_{\{\epsilon_0,\ldots,\epsilon_n,\ldots\}} = \int_{S^2} \sqrt{q} \sum_{n=0}^{\infty} \epsilon_n F_{ru}^{-2-n,0} dx^2}$$

Ongoing work:

- Subleading charges in Einstein-Yang-Mills: consistency proof.
- Non-Abelian $O(r^n)$ extensions.

Discussion:

- Renormalization of Phase Space methods: Yang-Mills for $O(r^n)$ and Gravity?.
- Extended Double copy YM $\leftrightarrow$ Gravity ? ([Campiglia, Nagy '21], ongoing)
- Symmetry hierarchy in Gravity ? (Pranzetti's talk)

Thank you!