On the on-shell equivalence of general relativity and Holst theory

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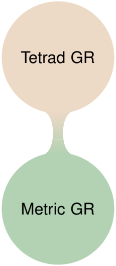
Part I: Introduction

Introduction

Metric GR

$$L^{(m)}_{\mathrm{GR}}(g) = g^{\alpha\beta}\mathring{\mathrm{Ric}}_{\alpha\beta}\mathrm{vol}_g$$

Introduction



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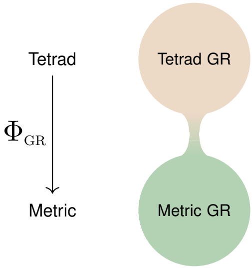

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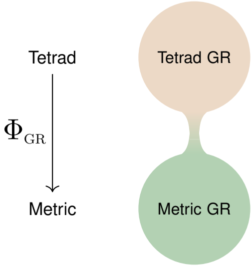
$$L^{(t)}_{\mathrm{GR}}(e) = \frac{1}{2}\epsilon_{IJKL}e^I \wedge e^J \wedge \mathring{F}^{KL}$$



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$$\overset{\circ}{F}{}^{KL} = \mathrm{d}\overset{\circ}{\omega}{}^{KL} + \overset{\circ}{\omega}{}^{K}{}_{M} \wedge \overset{\circ}{\omega}{}^{ML}$$

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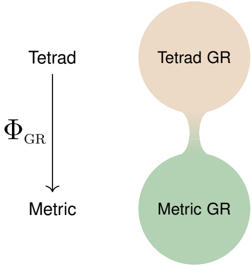

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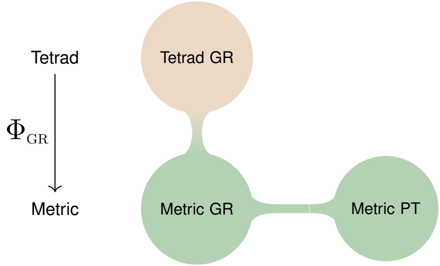

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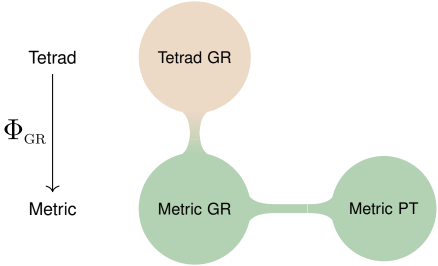

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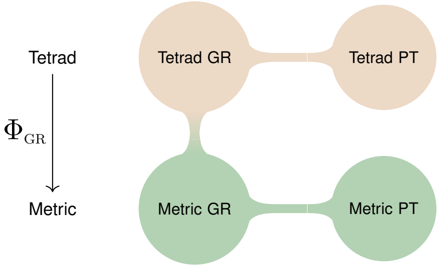

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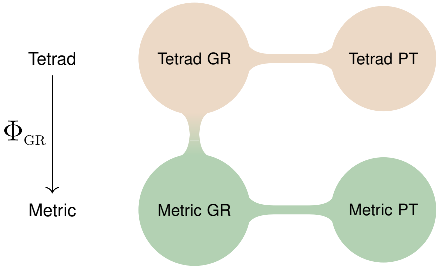

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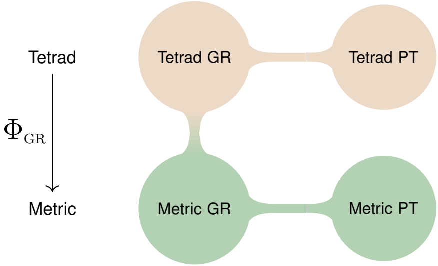

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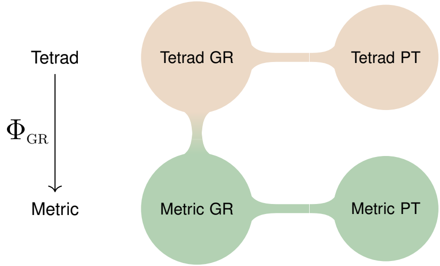

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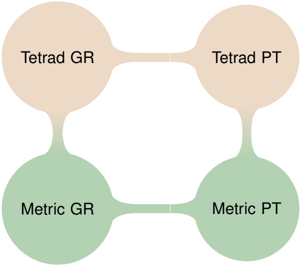
$$\text{Tetrad} \xrightarrow{\Phi_{\mathrm{GR}}} \text{Metric}$$



$$\text{(Tetrad,Spin conn.)} \xrightarrow{\Phi_{\mathrm{PT}}} \text{(Metric,Cov. deriv.)}$$

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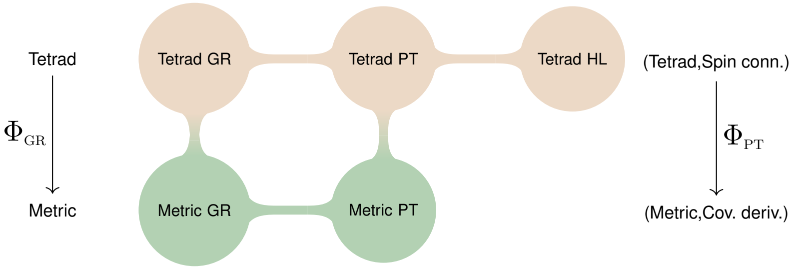

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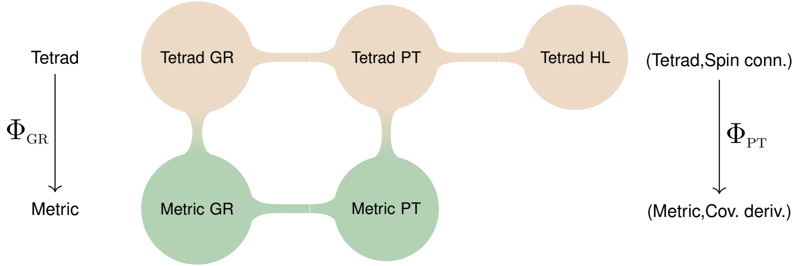

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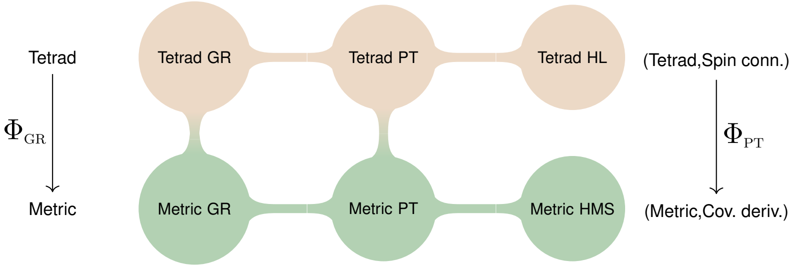

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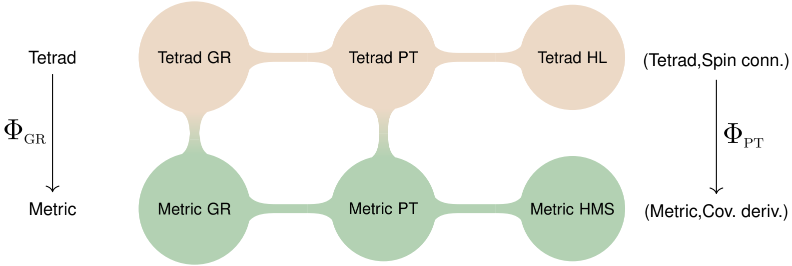

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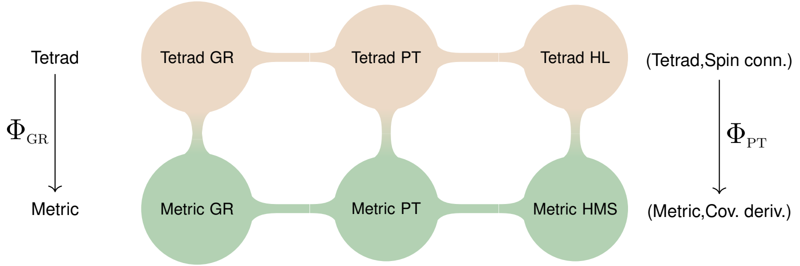

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Part II: Holst space of solutions

Space of solutions

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$$\mathcal{E}_{\mathrm{HL}} = 0$$

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$$\mathcal{E}_{\mathrm{HL}} = 0 \;\rightarrow\; Q^{\alpha}{}_{\beta\sigma} = U_{\beta}\delta^{\alpha}_{\sigma} \qquad E_{\mathrm{HL}}(g, U.\delta) = E_{\mathrm{GR}}(g) + 0 + 0$$

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$\mathrm{Sol}^{(m)}_{\mathrm{GR}} = \{g \ /$ satiesfies Einstein's eq.$\}$

Space of solutions

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$$\Omega = \int_{\Sigma} \mathbb{d}\Theta$$

- $\Omega_{\mathrm{HL}}^{(m)} = \Omega_{\mathrm{PT}}^{(m)} = \pi_1^{*}\Omega_{\mathrm{GR}}^{(m)}$

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$$(\mathrm{Sol}^{(m)}_{\mathrm{GR}}, \boldsymbol{\Omega}^{(m)}_{\mathrm{GR}}) \xleftarrow{\pi_1} (\mathrm{Sol}^{(m)}_{\mathrm{PT}}, \boldsymbol{\Omega}^{(m)}_{\mathrm{PT}}) \xleftarrow{\mathrm{Id}} (\mathrm{Sol}^{(m)}_{\mathrm{HL}}, \boldsymbol{\Omega}^{(m)}_{\mathrm{HL}})$$

Main result - Full equivalence

- CPS-symplectic equivalence (up to gauge).

$$
\begin{tikzcd}
\left(\boldsymbol{\Omega}^{(t)}_{\mathrm{GR}}, \mathrm{Sol}^{(t)}_{\mathrm{GR}}\right) \arrow[d, "\Phi_{\mathrm{GR}}"'] & & \left(\boldsymbol{\Omega}^{(t)}_{\mathrm{PT}}, \mathrm{Sol}^{(t)}_{\mathrm{PT}}\right) \arrow[ll, "\pi_1^{(t)}"'] \arrow[d, "\Phi_{\mathrm{PT}}"'] & & \left(\boldsymbol{\Omega}^{(t)}_{\mathrm{HL}}, \mathrm{Sol}^{(t)}_{\mathrm{HL}}\right) \arrow[ll, "\mathrm{Id}"'] \arrow[d, "\Phi_{\mathrm{HL}}"'] \\
\left(\boldsymbol{\Omega}^{(m)}_{\mathrm{GR}}, \mathrm{Sol}^{(m)}_{\mathrm{GR}}\right) & & \left(\boldsymbol{\Omega}^{(m)}_{\mathrm{PT}}, \mathrm{Sol}^{(m)}_{\mathrm{PT}}\right) \arrow[ll, "\pi_1^{(m)}"] & & \left(\boldsymbol{\Omega}^{(m)}_{\mathrm{HL}}, \mathrm{Sol}^{(m)}_{\mathrm{HL}}\right) \arrow[ll, "\mathrm{Id}"]
\end{tikzcd}
$$

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$$\mathrm{Sol}^{(t)}_{\mathrm{GR}} = \Phi^{-1}_{\mathrm{GR}} \mathrm{Sol}^{(m)}_{\mathrm{GR}}$$

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- Equivalence of charges on-shell (no dual charges).

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- Equivalence of charges on-shell (no dual charges).
- Some off-shell differences.

Main result with boundaries

- Lagrangian boundary terms are essential $\mathbb{S} = \int_M L - \int_{\partial M} \ell$.

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- Symplectic boundary terms are required $\Omega = \int_\Sigma \mathbb{d}\Theta - \int_{\partial\Sigma} \mathbb{d}\theta$.

Main result with boundaries

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$$L = \overbrace{\overbrace{\overbrace{L^{(m)}_{\mathrm{GR}}}^{\mathrm{EH}} + \hat{L}^{(m)}_{\mathrm{CPT}}}^{\mathrm{PT}} + \frac{1}{\gamma}\hat{L}^{(m)}_{\mathrm{CHL}}}^{\mathrm{HL}}$$

$$\ell = \overbrace{\overbrace{\overbrace{\ell^{(m)}_{\mathrm{GR}}}^{\mathrm{HGY}} + \hat{\ell}^{(m)}_{\mathrm{CPT}}}^{\mathrm{OB}} + \frac{1}{\gamma}\hat{\ell}^{(m)}_{\mathrm{CHL}}}^{\mathrm{new}}$$

- Symplectic boundary terms are required $\Omega = \int_\Sigma \mathbb{d}\Theta - \int_{\partial\Sigma} \mathbb{d}\theta$.
- The charges also require boundary contributions.

Main result with boundaries

- Lagrangian boundary terms are essential $\mathbb{S} = \int_M L - \int_{\partial M} \ell$.

$$L = \overbrace{\overbrace{\overbrace{L^{(m)}_{\mathrm{GR}}}^{\mathrm{EH}} + \hat{L}^{(m)}_{\mathrm{CPT}}}^{\mathrm{PT}} + \frac{1}{\gamma}\hat{L}^{(m)}_{\mathrm{CHL}}}^{\mathrm{HL}}$$

$$\ell = \overbrace{\overbrace{\overbrace{\ell^{(m)}_{\mathrm{GR}}}^{\mathrm{HGY}} + \hat{\ell}^{(m)}_{\mathrm{CPT}}}^{\mathrm{OB}} + \frac{1}{\gamma}\hat{\ell}^{(m)}_{\mathrm{CHL}}}^{\mathrm{new}}$$

- Symplectic boundary terms are required $\Omega = \int_\Sigma \mathbb{d}\Theta - \int_{\partial\Sigma} \mathbb{d}\theta$.
- The charges also require boundary contributions.
- The same results hold in the presence of boundaries.

Thanks for your attention.

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