Charges and fluxes on (perturbed) non-expanding horizons

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Uniwersytet Warszawski,
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A. Ashtekar, N. Khera, M. Kolanowski and J.L.,
Non-expanding horizons: multipoles and the symmetry group, **JHEP 01 (2022) 028 [2111.07873],**
Charges and Fluxes on (Perturbed) Non-expanding Horizons, **JHEP 02 (2022) 001 [2112.05608]**

Plan of the talk

BMS like structure of non-expanding horizons

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On the non-expanding horizons by appropriate gauge selection, we introduce a structure similar to the BMS structure for null scri.

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Application: gravitational radiation through settling down horizons

The settling down horizon is described by a perturbation of a non-expanding horizon. The generators of the very BMS like symmetry are used to define charges and fluxes.

Non-expanding horizons

The idea:

Killing horizons to the zeroth order, null surfaces that have relevant properties of the black hole / cosmological horizons.

Ashtekar, Beetle, Dreyer, Fairhurst, Krishnan, JL, Wiśniewski - 2000

Mechanics	*Ashtekar, Beetle, JL - 2001*	**4 dim**
	Korzyński, JL, Pawłowski - 2005	**n dim**
Geometry, DOF	*Ashtekar, Beetle, JL - 2002*	**4 dim**
	JL, Pawłowski - 2004	**n dim**

Non-expanding horizons equations

Extremal horizons

If non-expanding horizon is also Killing horizon to the first order and it is extremal, then it satisfies the constraint:

$$ {}^{(n)}\nabla_{(A}\omega_{B)} + \omega_A\omega_B - \frac{1}{2}{}^{(n)}R_{AB} + \frac{1}{n}\Lambda g_{AB} = 0 $$

Ashtekar, Beetle, JL - 2001
JL, Pawlowski - 2004

Exact solutions constructed from extremal horizons

Pawlowski, JL, Jezierski - 2005

Today the equation is called Near Horizon Geometry equation

Kunduri, J. Lucietti 2009

4d spacetime and NHG solutions for genus =0

$$\mathcal{S} = S_2$$

axial symmetry $\Rightarrow g_{AB},\ \omega_A = g_{AB}^{\rm extremal\ \ Kerr},\ \omega_A^{\rm extremal\ \ Kerr}$

$\Lambda = 0$

JL, Pawłowski 2002,

generalized to the Einstein-Maxwell case

uniqueness! no more solutions!

generalized to the Einstein-Yang-Mills case
and somehow to the $\Lambda \neq 0$ case *Kunduri, J. Lucietti 2009*
Buk, JL 2022

no axial symmetry $\Rightarrow$? only partial results known:

${}^{(n)}\nabla_{[A}\omega_{B]} = 0 \quad \Rightarrow K = \Lambda \geq 0,\ \omega_A = 0$ *Chruściel, Reall, Tod 2005*

(non-rotating)

the linearized equation about axisymmetric solution admits only axisymmetric solutions - partly numeric *Chruściel, Szybka, Tod 2017*

Applications to filing gaps in the BH uniqueness theorems Chruściel, Costa, Heusler 2012

NHG solutions for genus > 0

$\mathcal{S}\,,\, g_{AB}\,,\, \omega_A$ $\qquad {}^{(2)}\nabla_{(A}\omega_{B)} + \omega_A\omega_B + \frac{1}{2}(\Lambda - K)g_{AB} = 0$

K - the Gauss curvature $\qquad \Lambda$ - the cosmological constant

$\chi_E({}^2S) \leq 0 \quad \Rightarrow \quad K = \Lambda \leq 0, \qquad \omega_A = 0$

(genus > 0)

Dobkowski-Ryłko, Kamiński, JL, Szereszewski 2018

Embeddable in extremal cases $\Lambda = -\frac{1}{9M^2}$ of:

$$-(-1 - \frac{2M}{r} - r^2\frac{\Lambda}{3})dt^2 + \frac{dr^2}{-1 - \frac{2M}{r} - r^2\frac{\Lambda}{3}} + r^2\frac{2dzd\bar{z}}{(1 - \frac{1}{2}z\bar{z})^2}$$

this is really minus

compactified by suitable subgroup of isometries

Extremal Killing horizon to the 2nd order: Uniqueness of the extremal Kerr horizon

Suppose $\mathcal{S} = S_2$ and g_{AB}, ω_A, S_{AB} is axisymmetric and $\Lambda = 0$

Then, the solution of the first and the second equation is unique, modulo the obvious rescaling

$$g_{AB} \mapsto a\, g_{AB}, \qquad S_{AB} \mapsto b\, S_{AB}, \qquad a, b = \mathrm{const}$$

it corresponds to the horizon in the extremal Kerr spacetime

For every solution g_{AB}, ω_A, S_{AB} the horizon H, g_{ab}, ∇_a

is embeddable in the extremal Kerr spacetime of the corresponding horizon area.

Kolanowski, Lewandowski, Szereszewski 2019
Lewandowski, Pawłowski 2019 *Lucietti, Li 2016*

The Petrov type D equation

Non-extremal Killing horizon to the 2nd order satisfies

$$\Psi_2 = -\frac{1}{2}\left(K - \frac{\Lambda}{3} + i\Omega\right) \neq 0 \quad \textbf{and} \quad \bar{m}^A \bar{m}^B {}^{(2)}\nabla_A {}^{(2)}\nabla_B \left(K - \frac{\Lambda}{3} + i\Omega\right)^{-\frac{1}{3}} = 0$$

$$\Leftrightarrow$$

The spacetime Weyl tensor of the Petrov type D at the horizon

Therefore we call it: the Petrov type D equation.

Actually, this equation is is a generalization of the extremity (NHG) equation

This equation knows the secrets of the BH uniqueness theorems: the spherical topology, the rigidity, no-hair

JL, Pawłowski, 2002 Dobkowski-Ryłko, Pawłowski, JL 2018

Szereszewski JL 2018

NEH of the Hopf bundle structure

JL, Ossowski 2019, 2021, 2022

Embeddable in the NUT type spacetimes - due to them we have learned a lot about the global structure, the Misner extension.

Conclusion: investigating the non-expanding horizons we can learn a lot about about bh spacetimes and other exact solutions to Einsteins equations.
Today we will apply the NEHs to the radiation.

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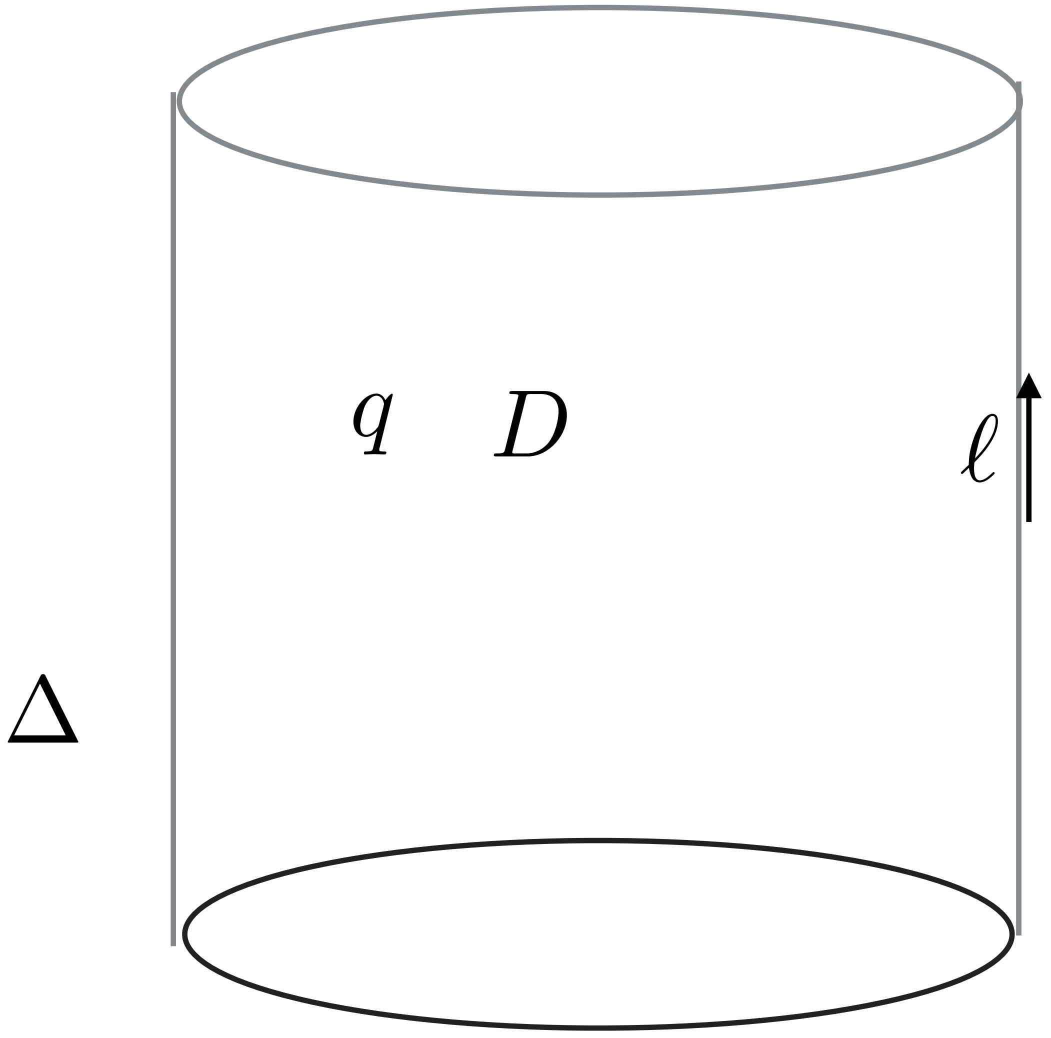
Rotation 1-form potential



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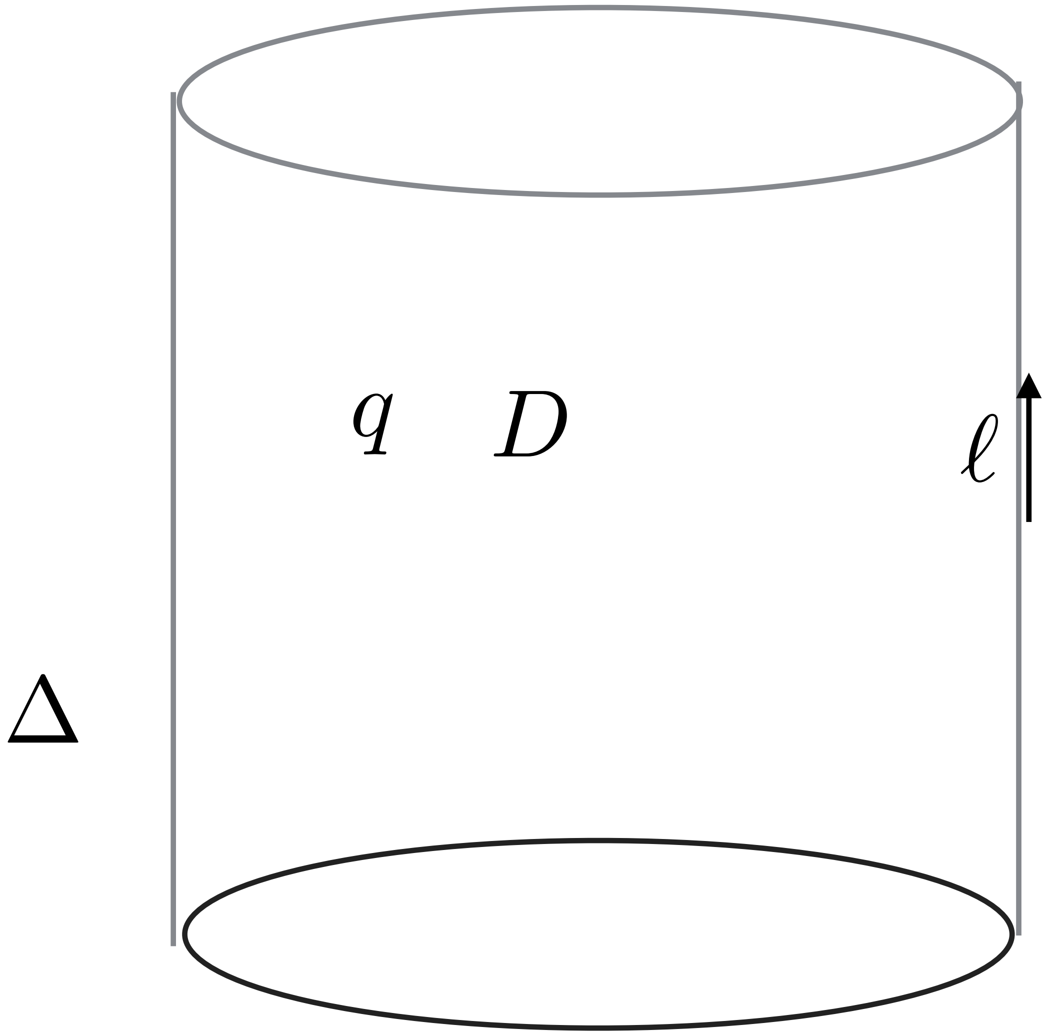

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$$D_a \ell^b = \omega_a \ell^b$$

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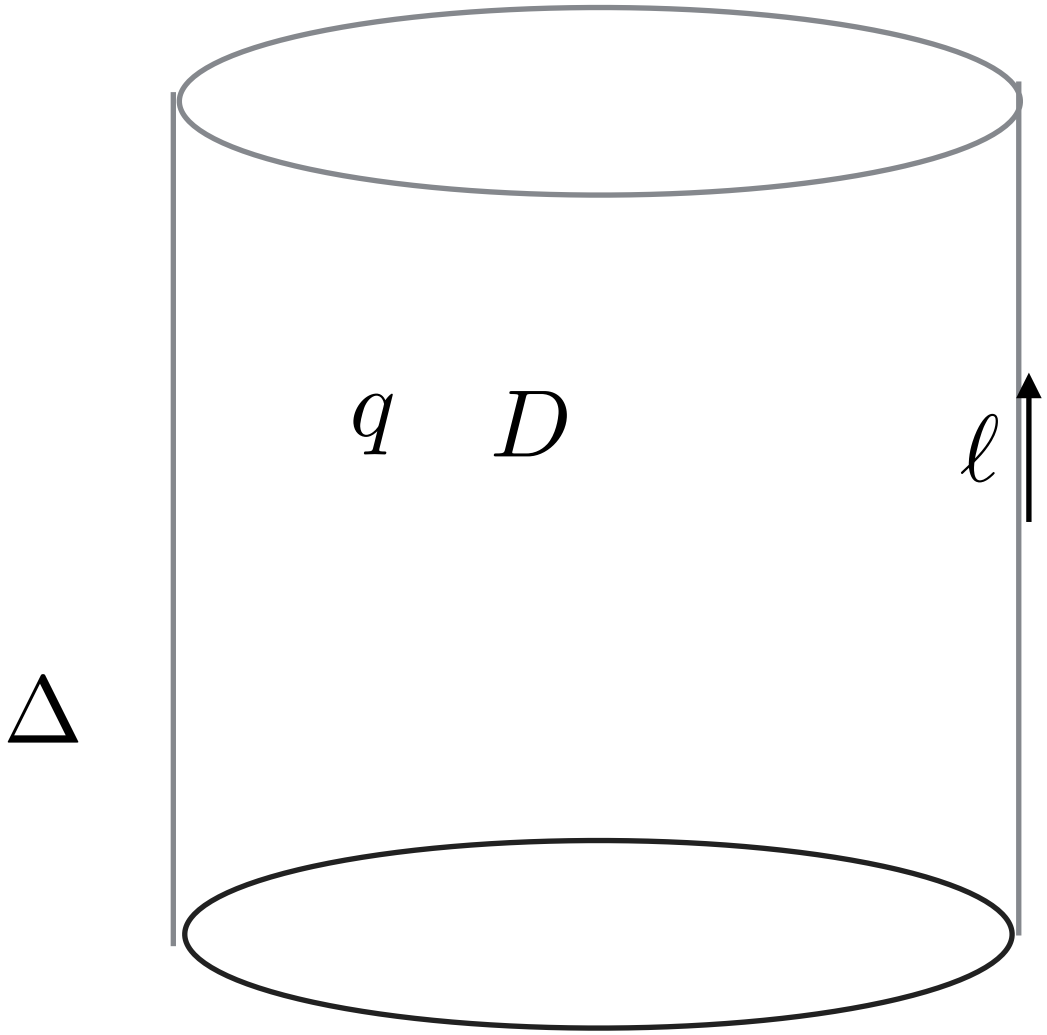

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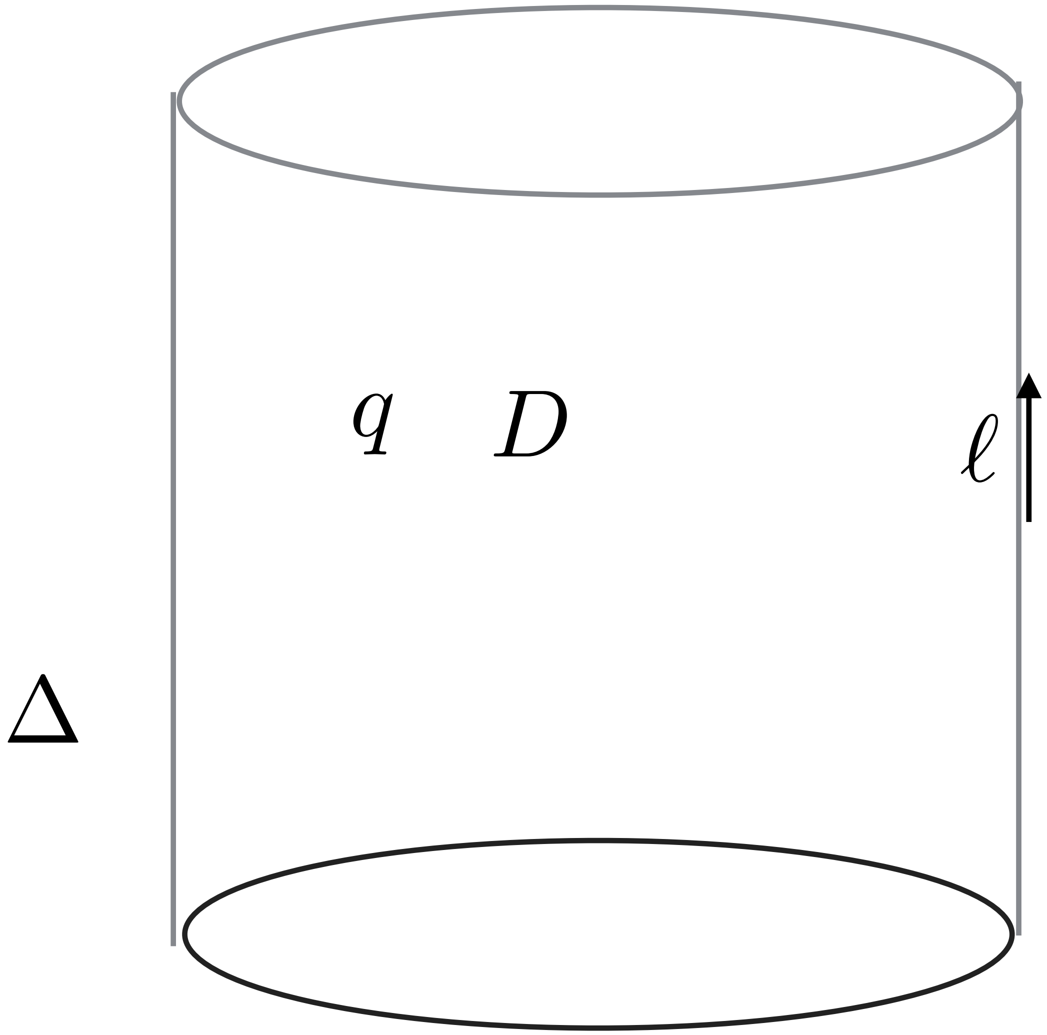

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$$\kappa = \ell^a \omega_a$$

$$\ell^a D_a \ell = \kappa \ell$$

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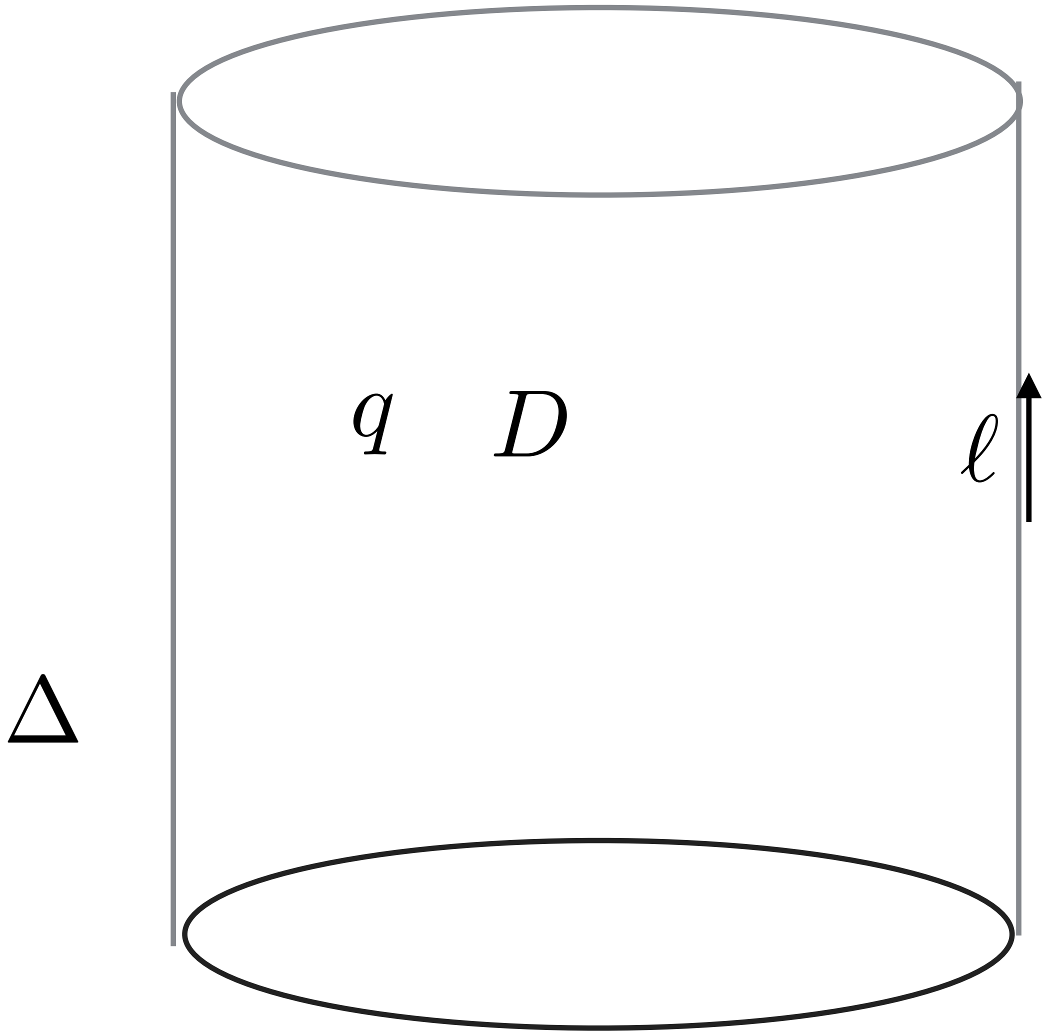

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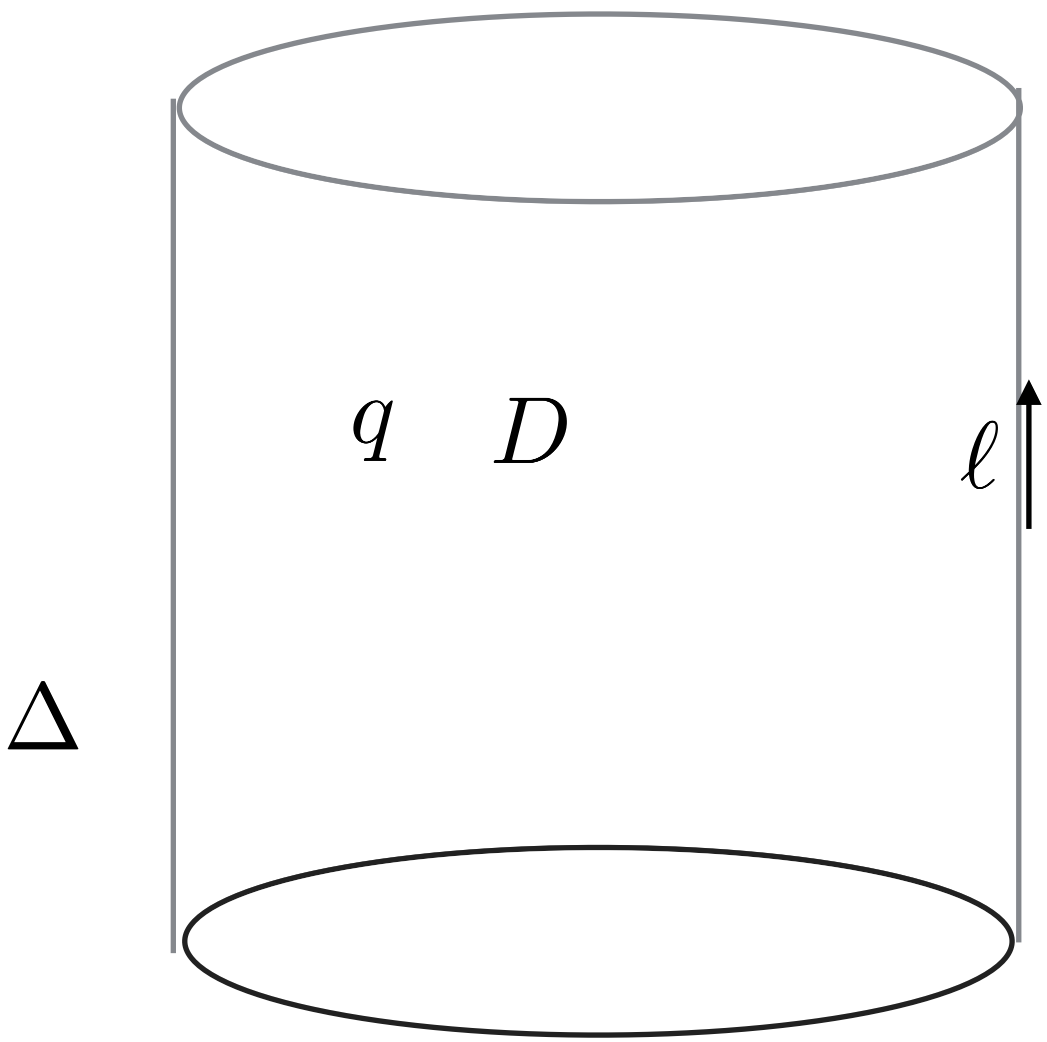

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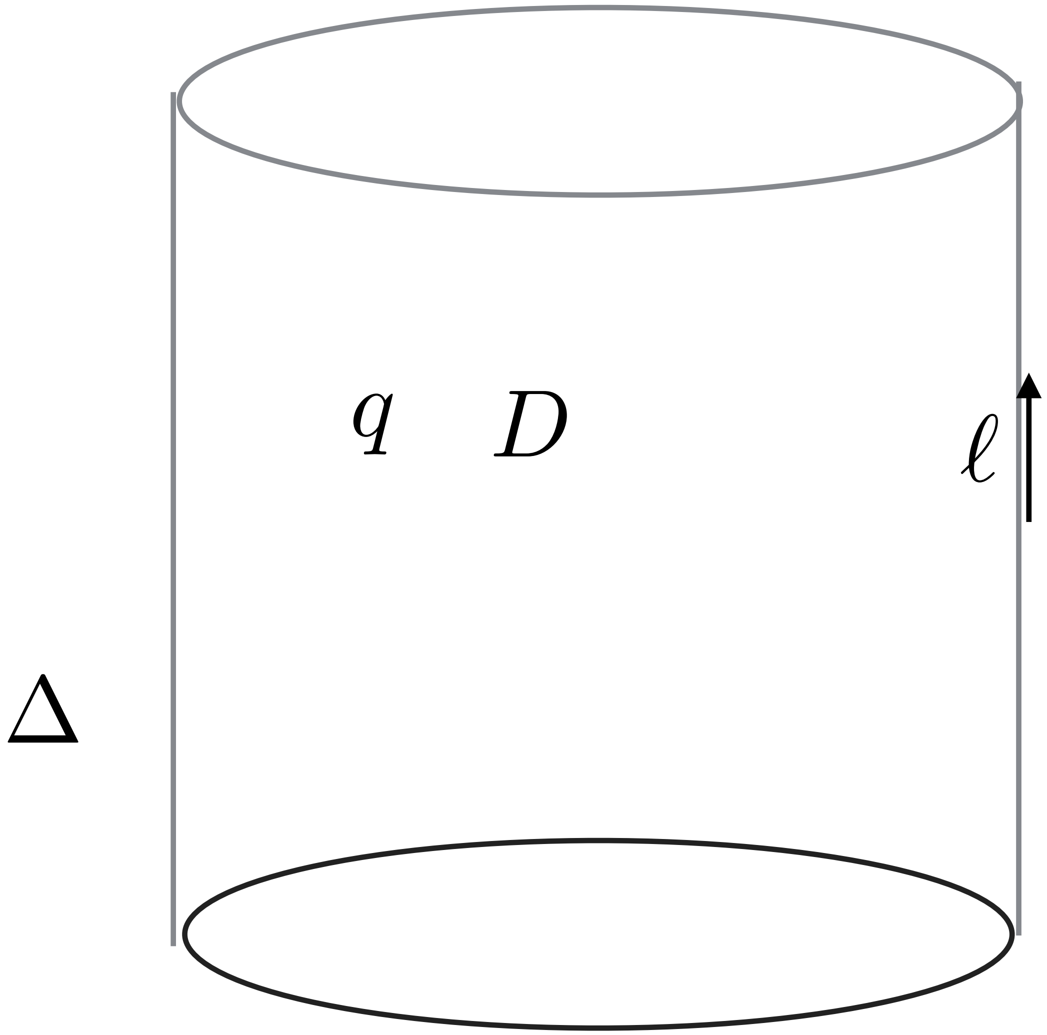
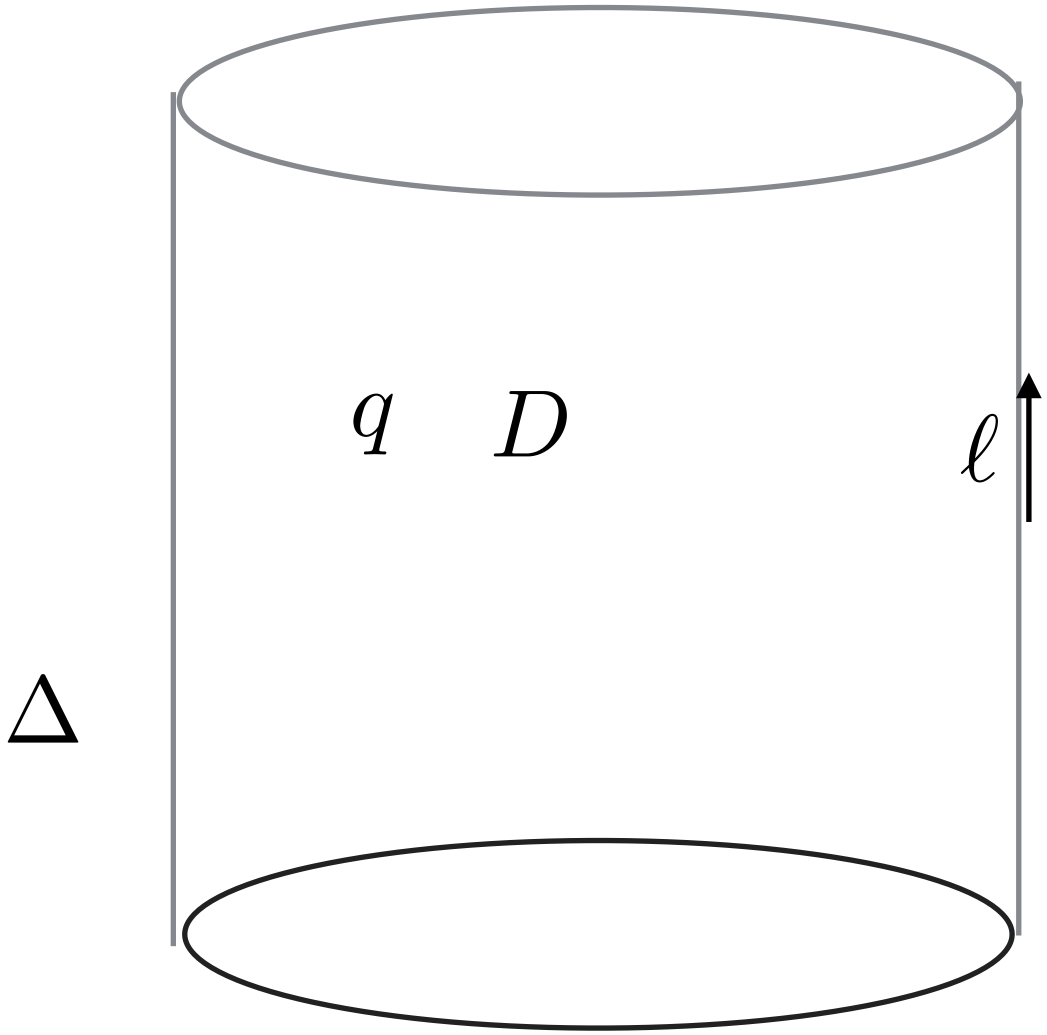
In this talk topology is trivial: $\Delta = \bar{\Delta} \times \mathbb{R}$

Embedded non-expanding horizon



Embedded non-expanding horizon

$\Delta \subset M$ **becomes a co-dim 1 null surface**



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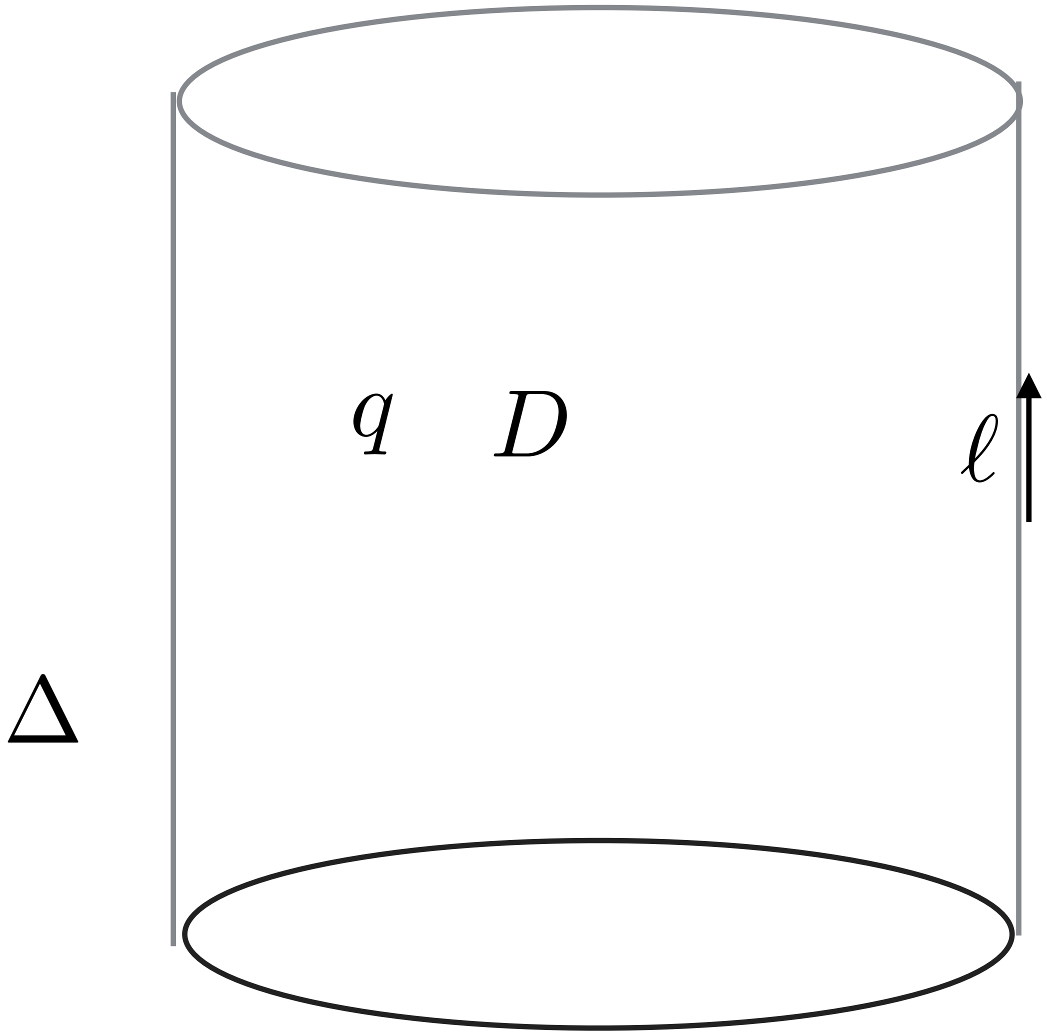
the spacetime Ricci

$$D_a \kappa = \mathcal{L}_\ell \omega_a + R_{ab} \ell^b$$



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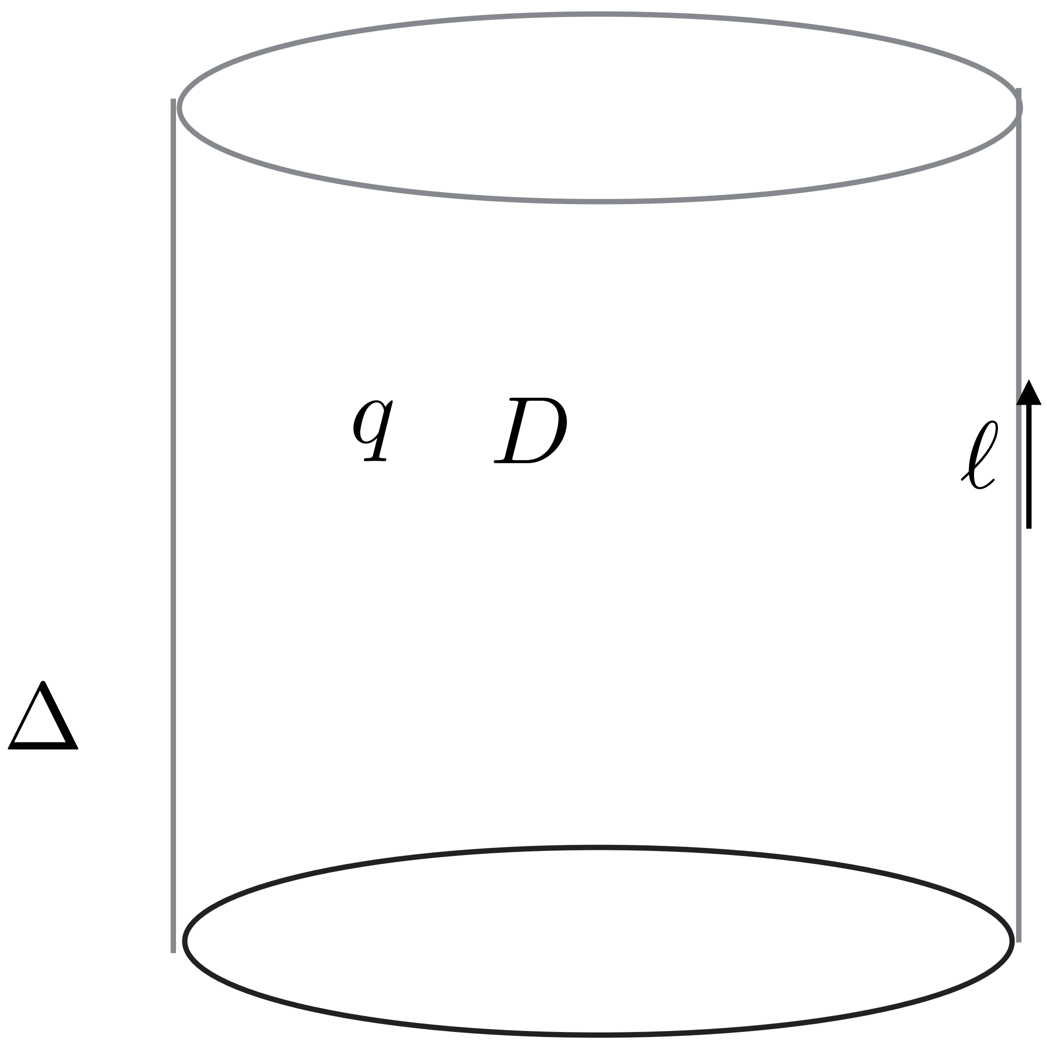

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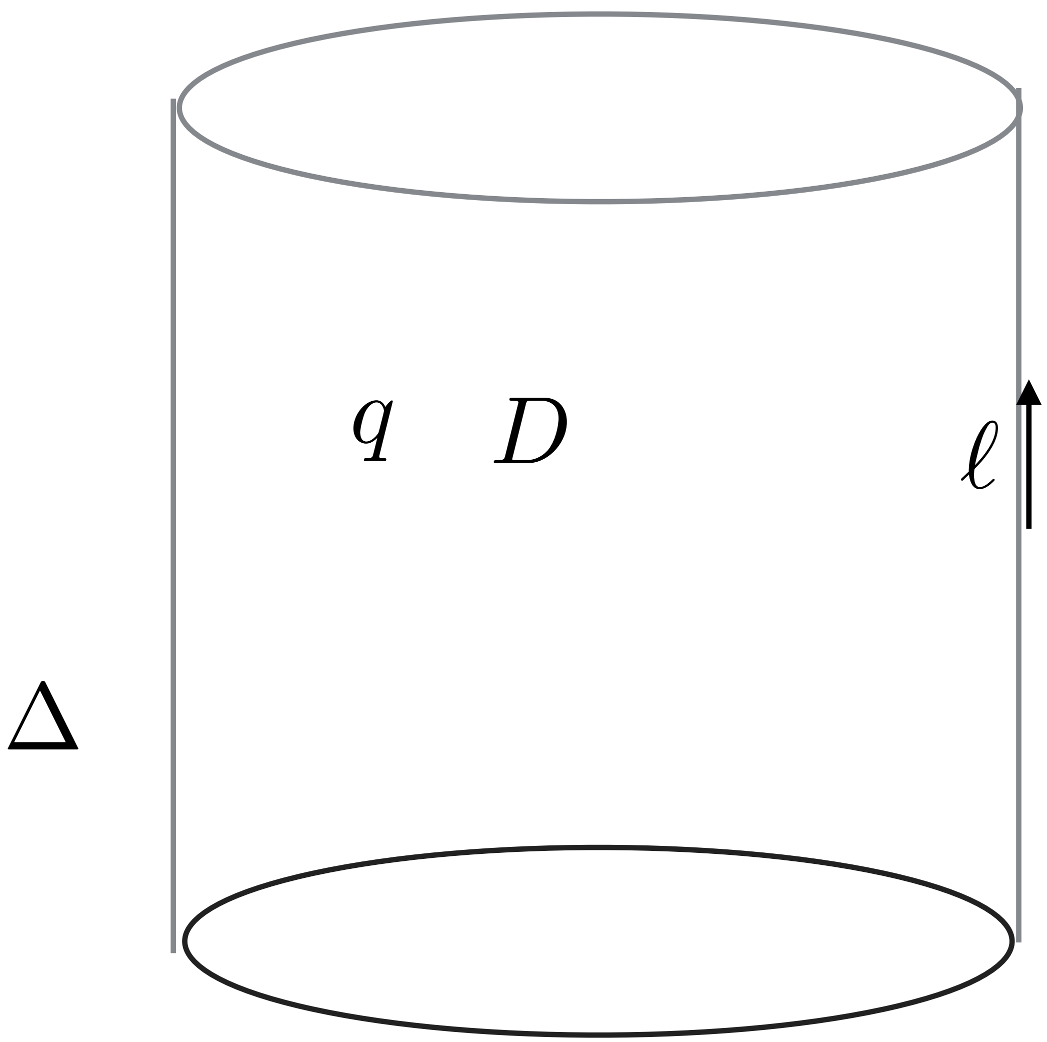

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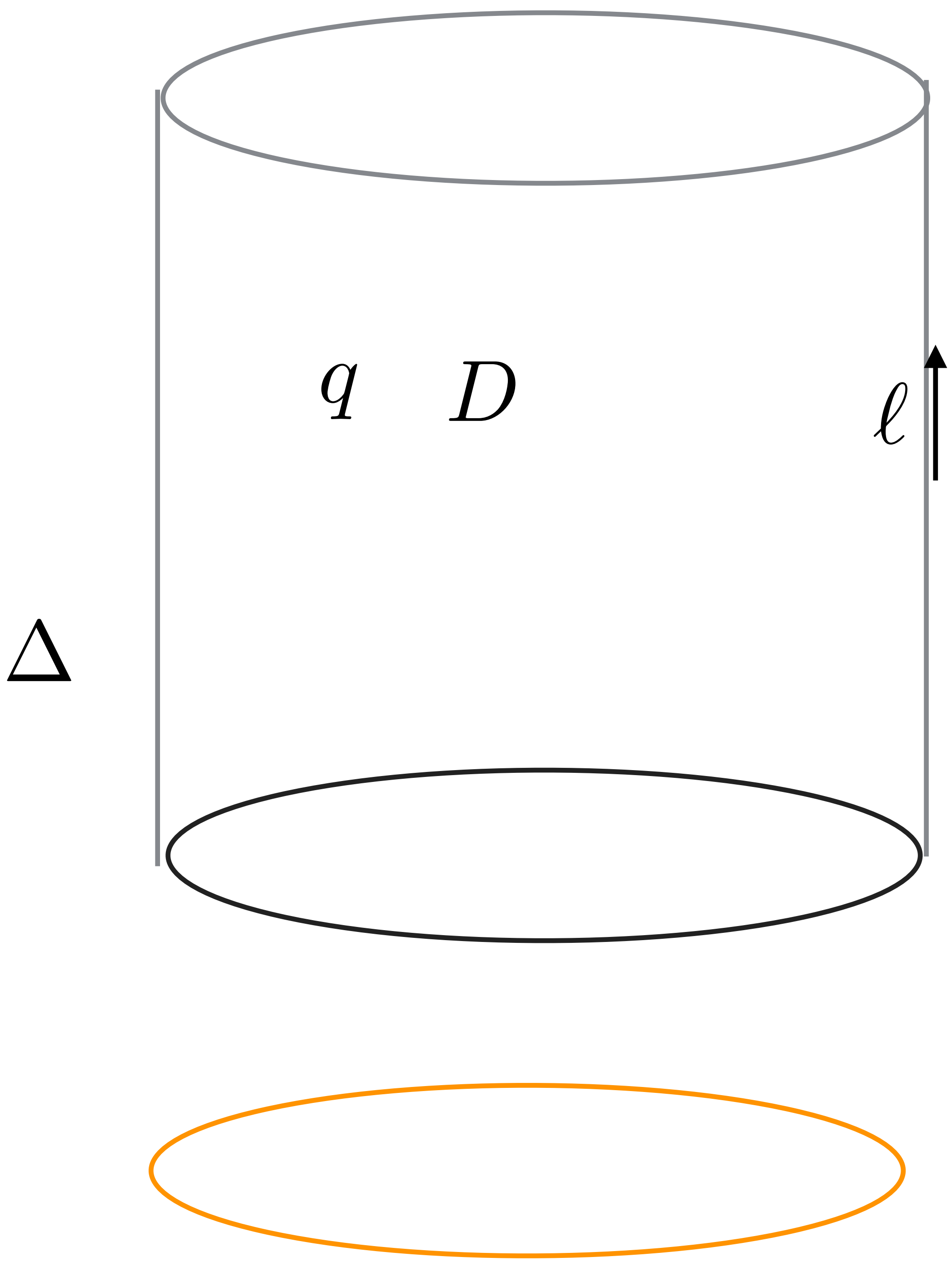

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$q_{AB}\,,\, \omega'_A$

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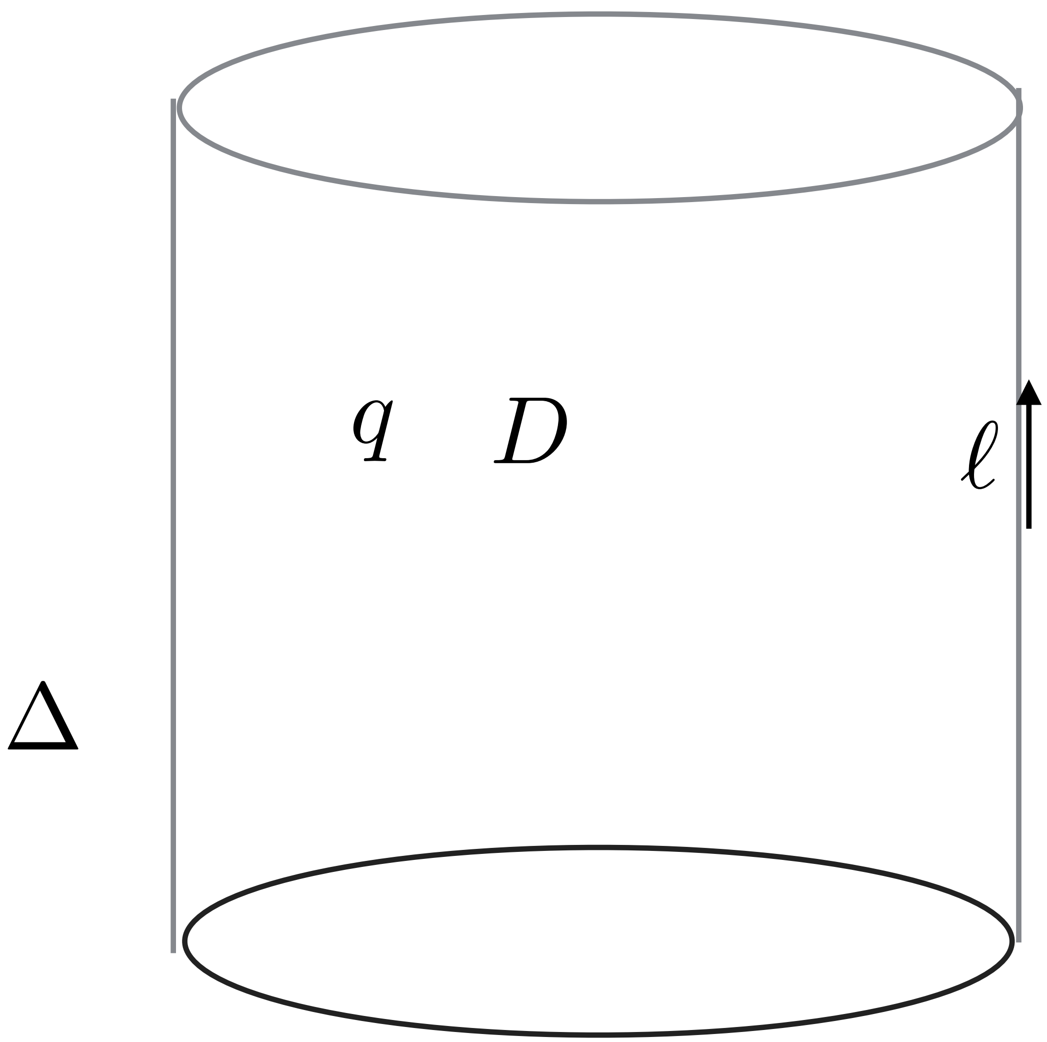

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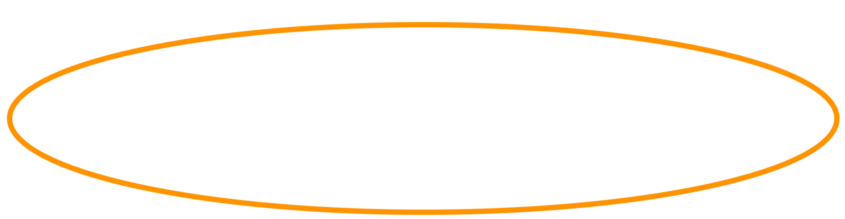

q_{AB} , ω'_A $\qquad$ $q^{AB} D_A \omega'_B = 0$

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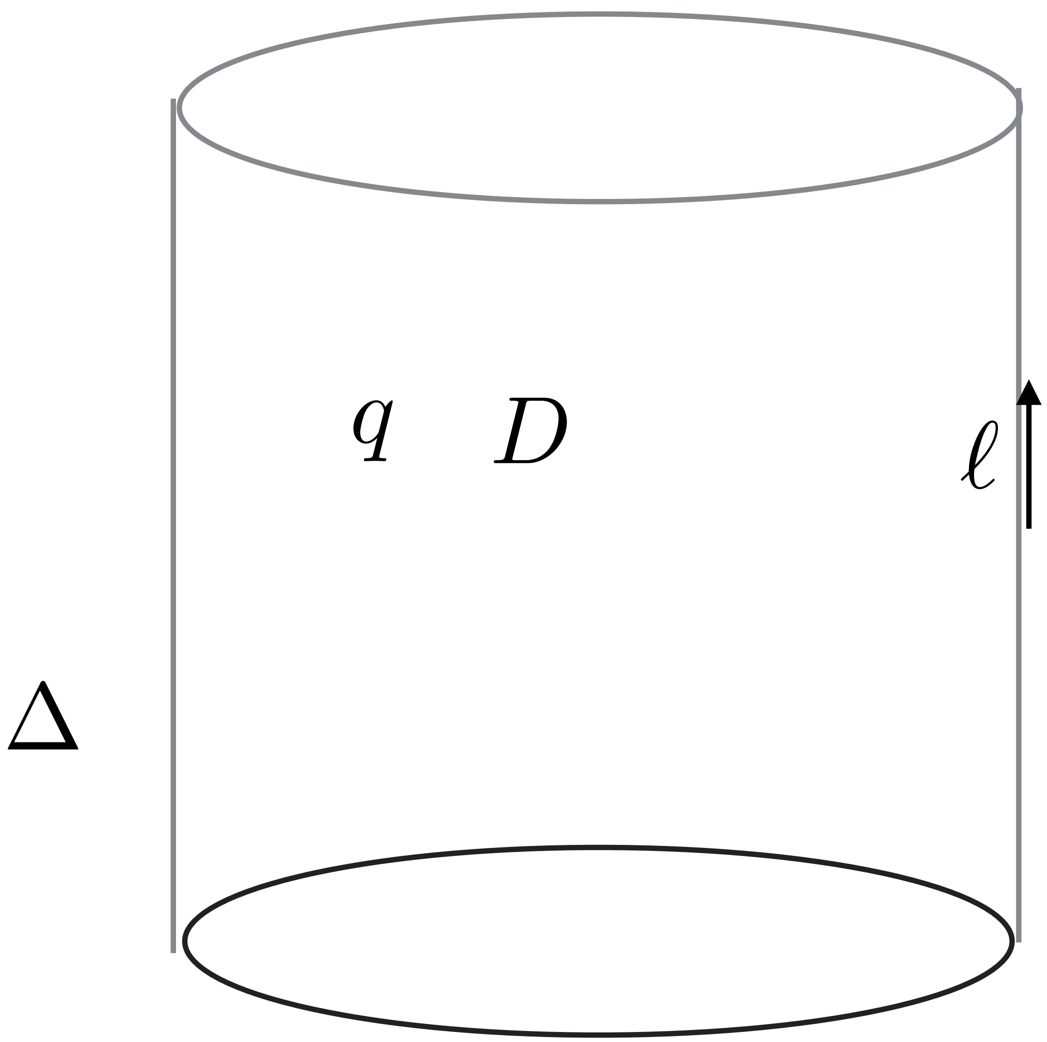

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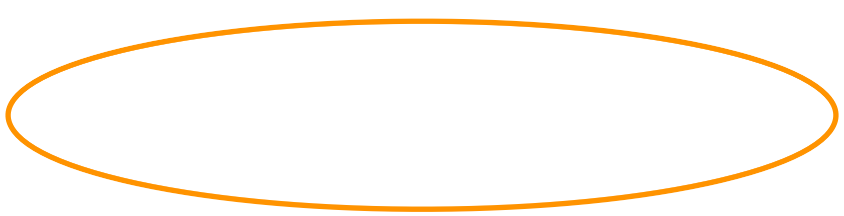
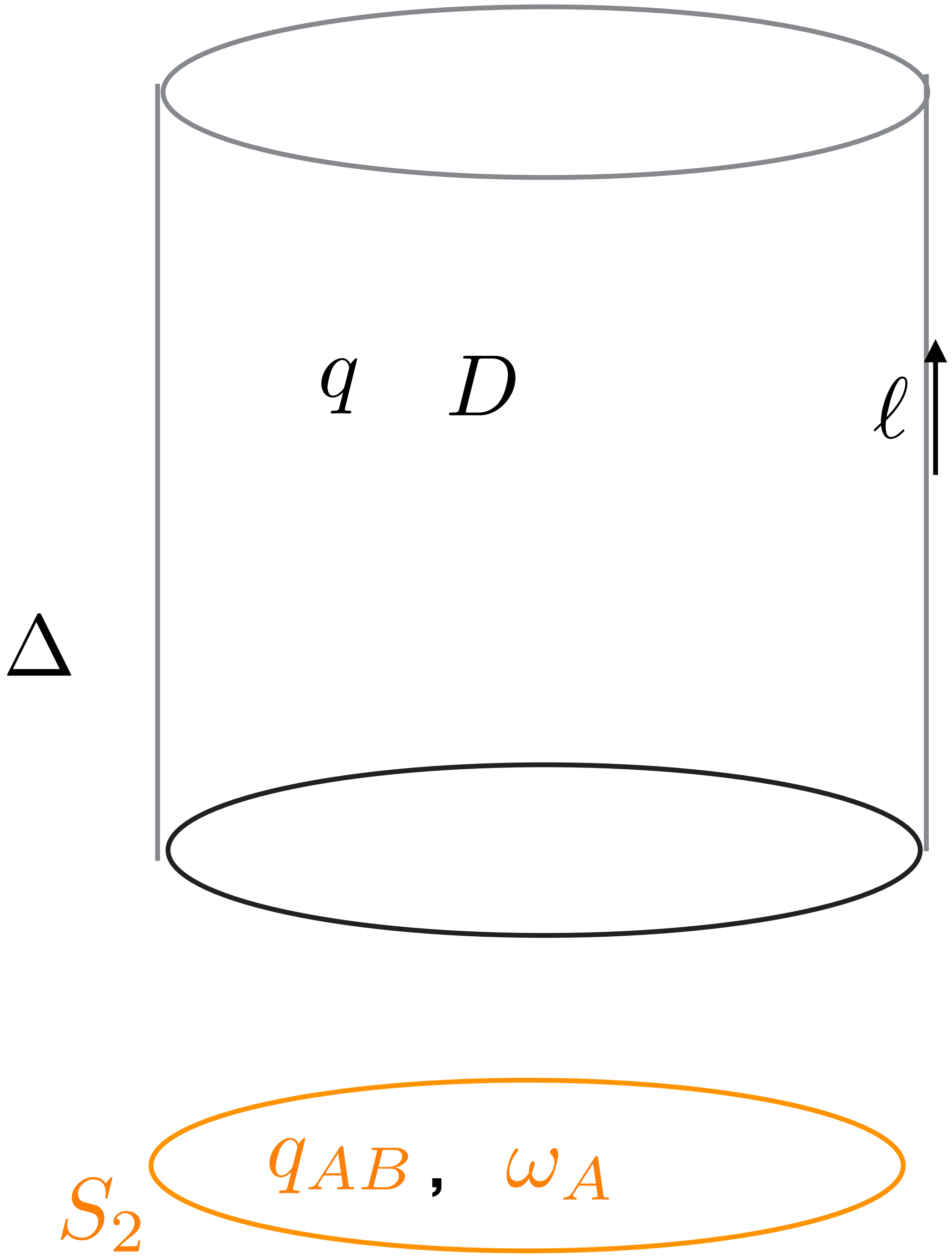


$q_{AB}\,,\,\omega'_A$ $\qquad q^{AB}D_A\omega'_B = 0$

$D_A q_{BC} = 0 \ \ D_A D_B f = D_B D_A f$

Embedded non-expanding horizons in 4d

$$\Delta = S_2 \times \mathbb{R}$$



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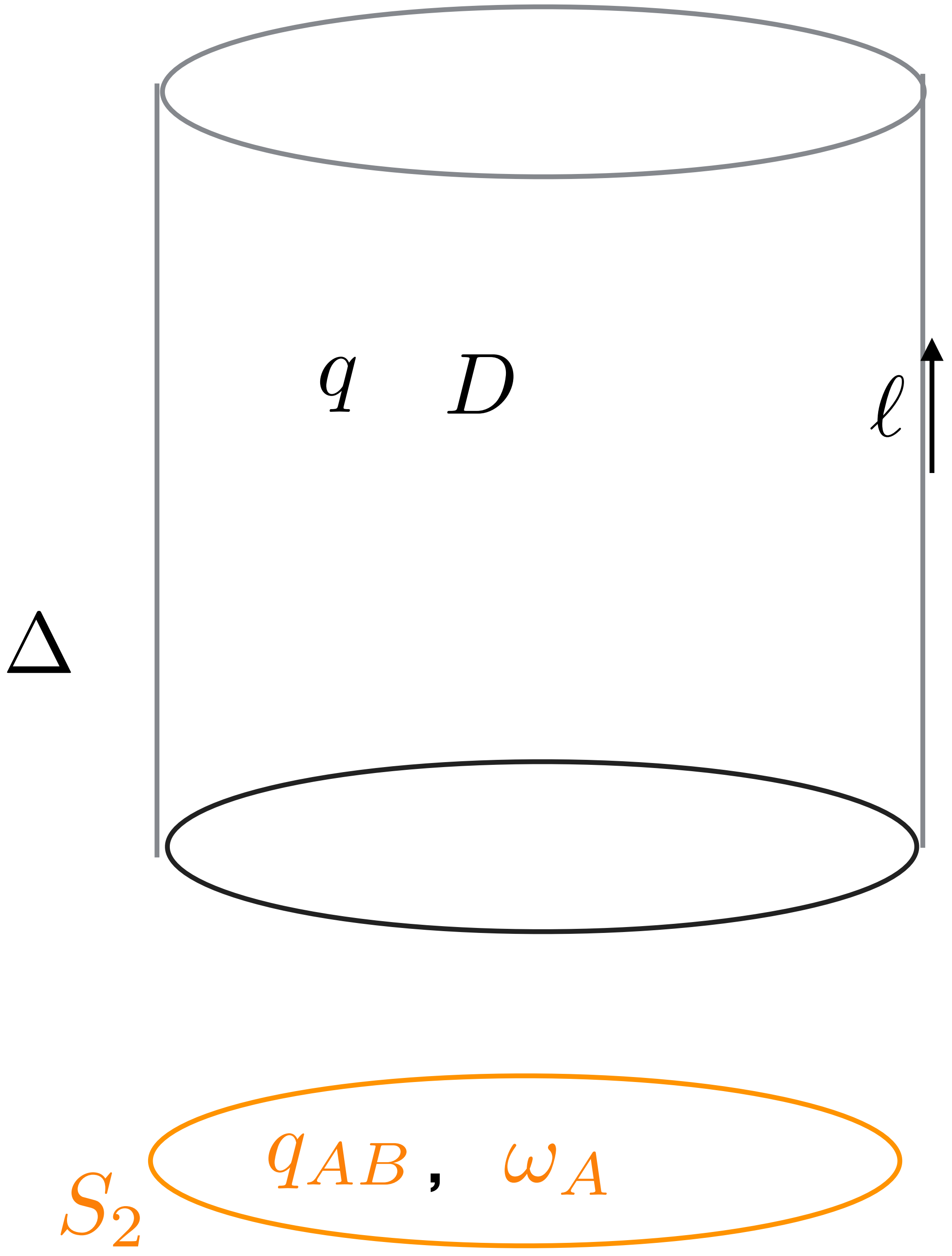

the condition:

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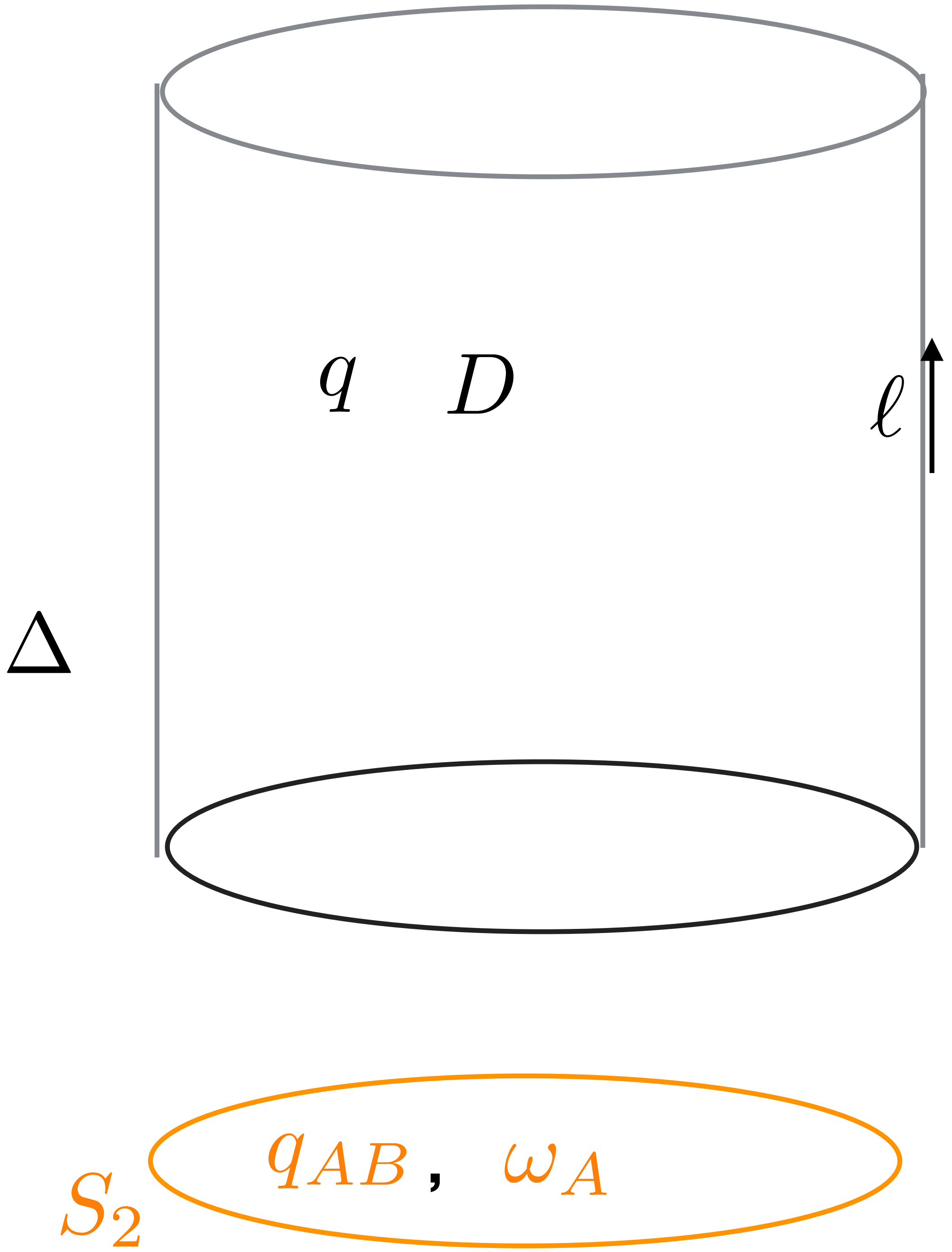

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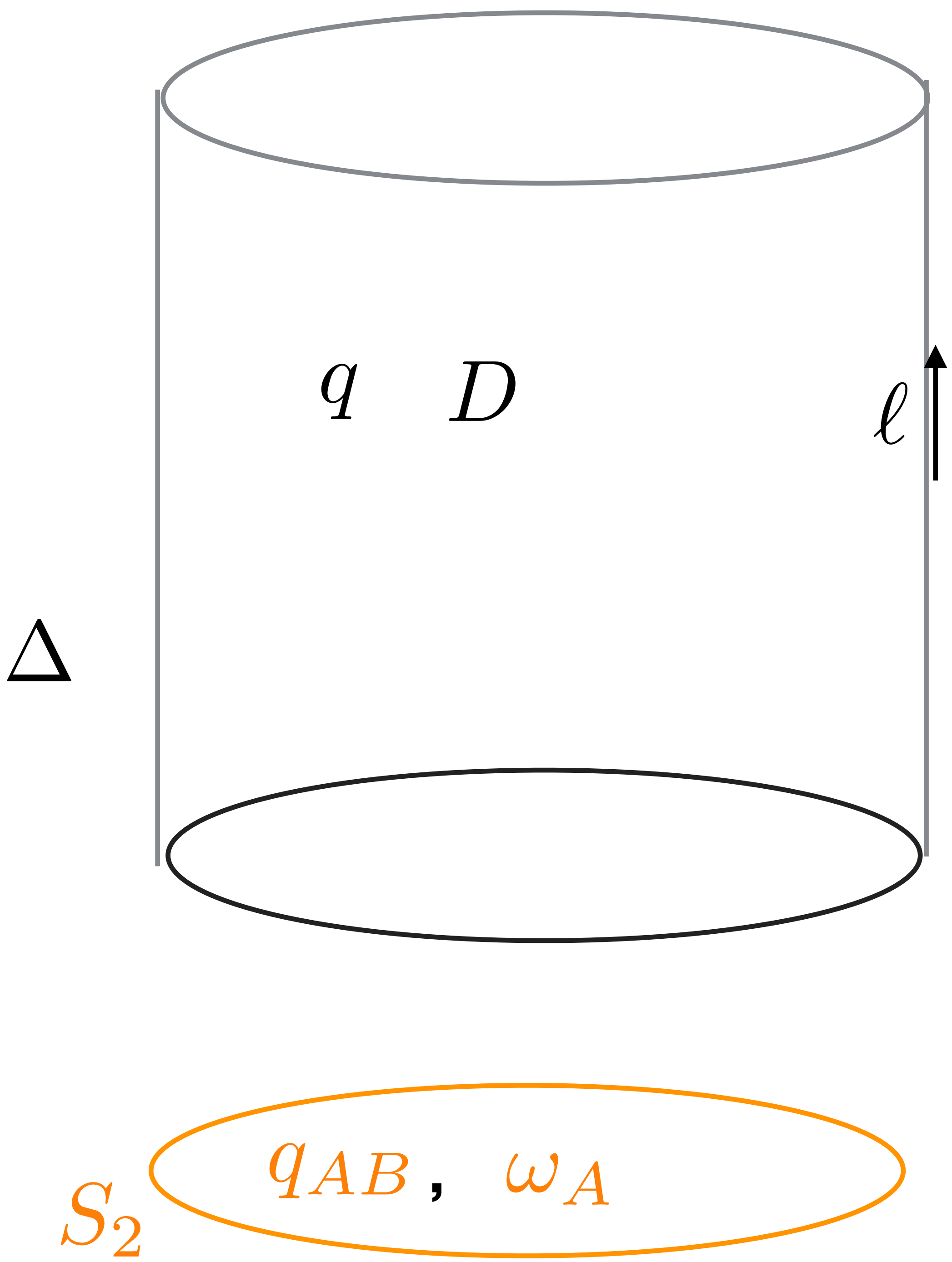

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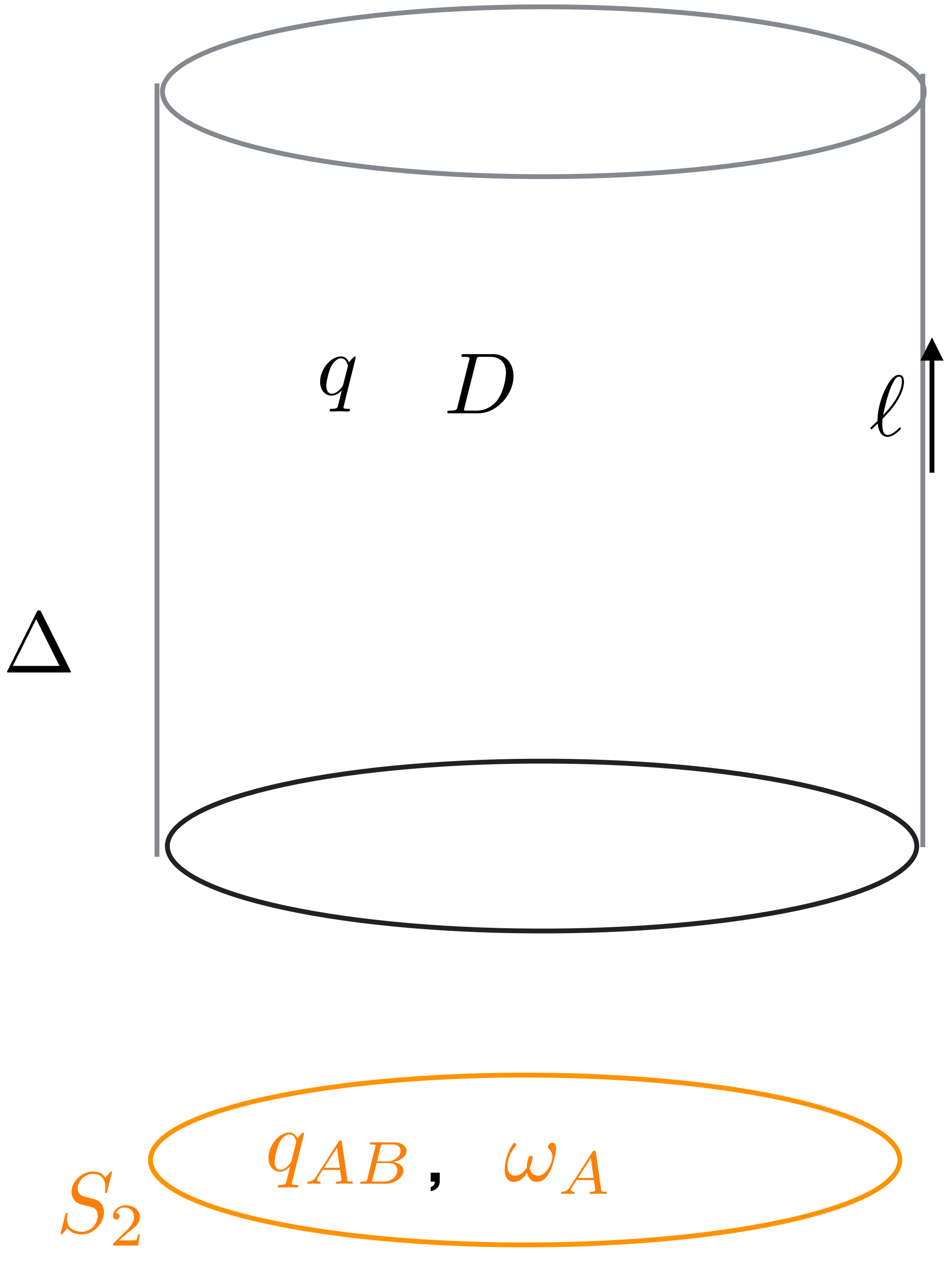

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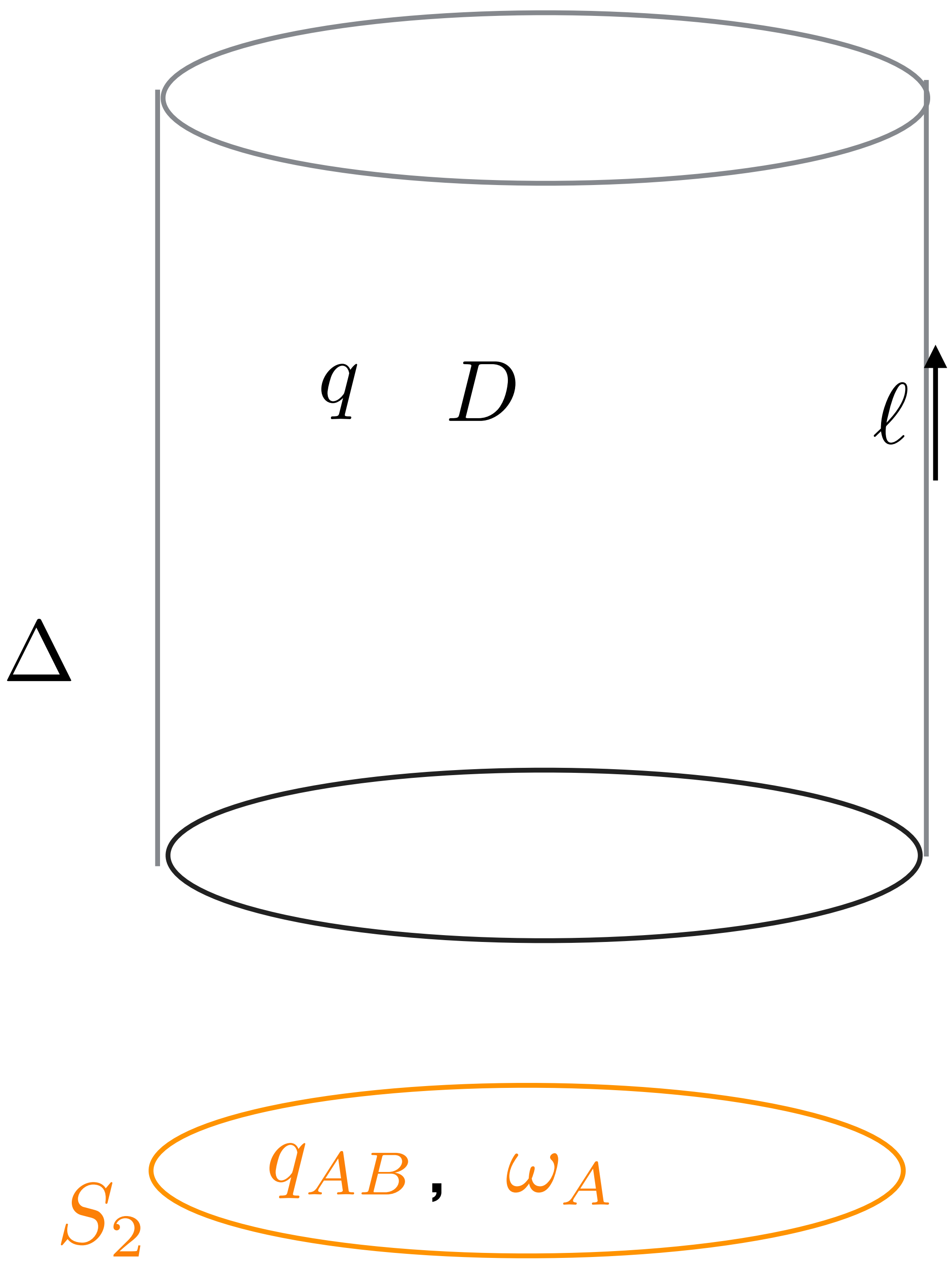

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This abiguity will imply the extension of the BMS group

A BMS like structure of NEH

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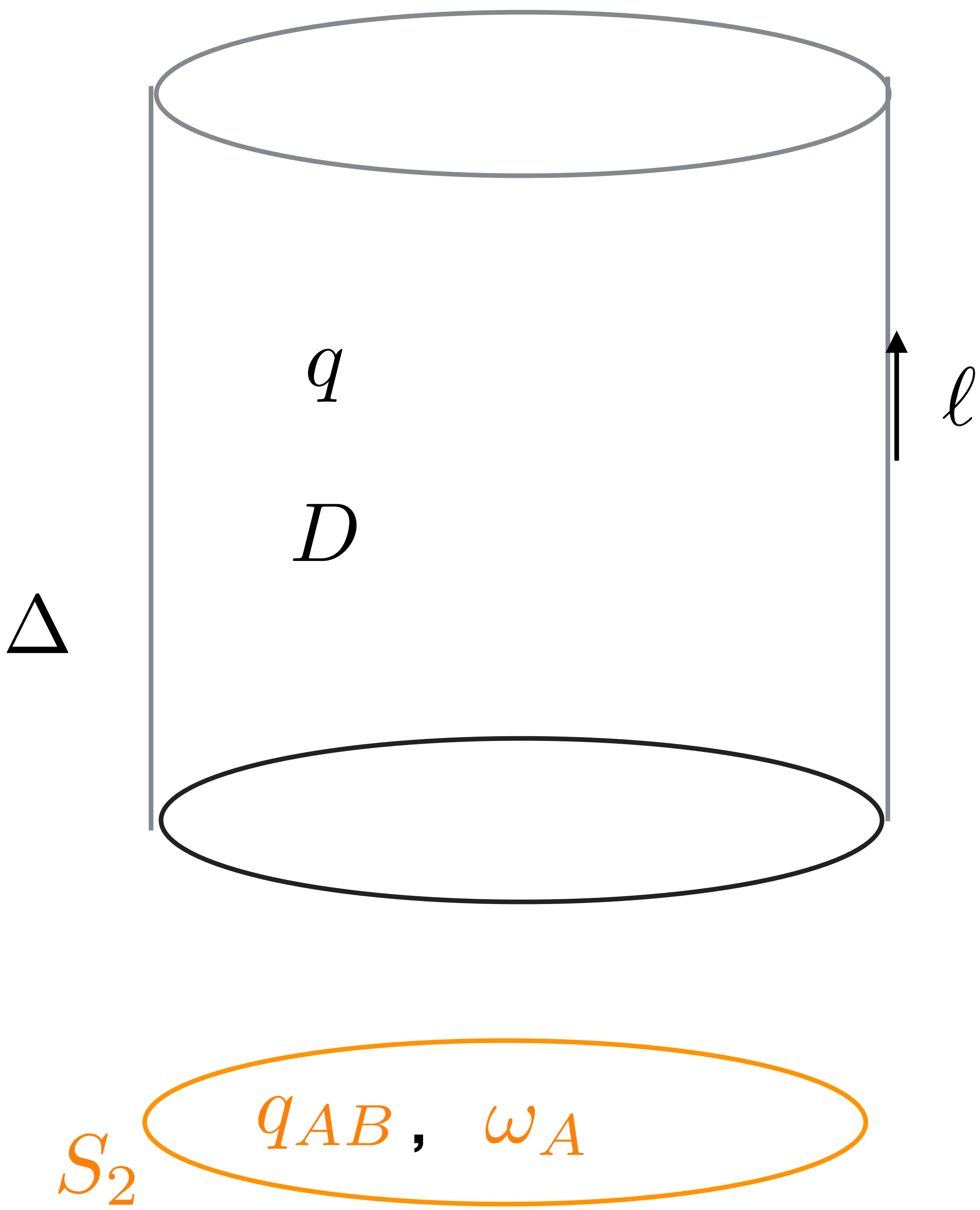

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A BMS like structure of NEH

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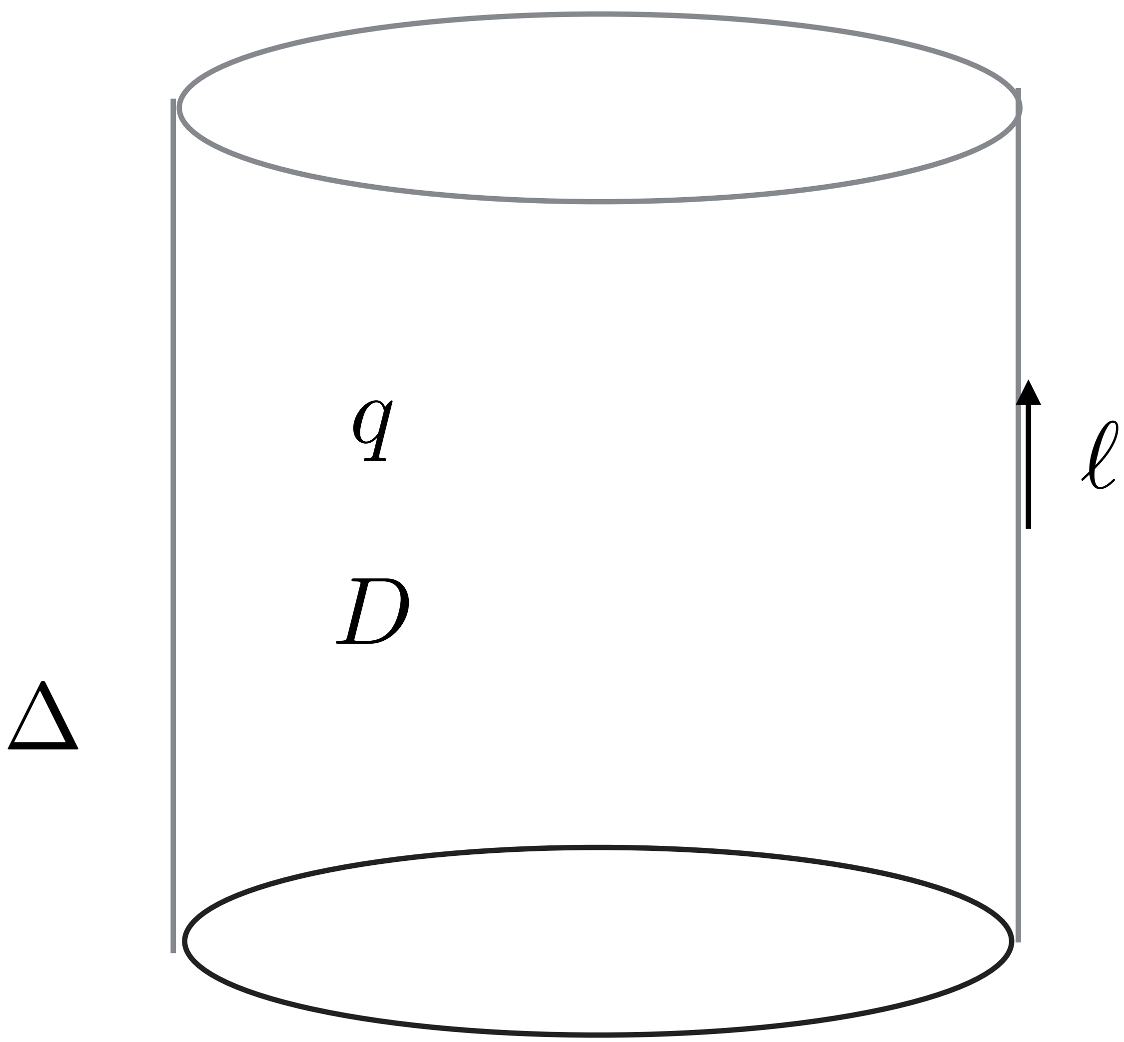
Universal structure $[\overset{\circ}{q}, \overset{\circ}{\ell}]$



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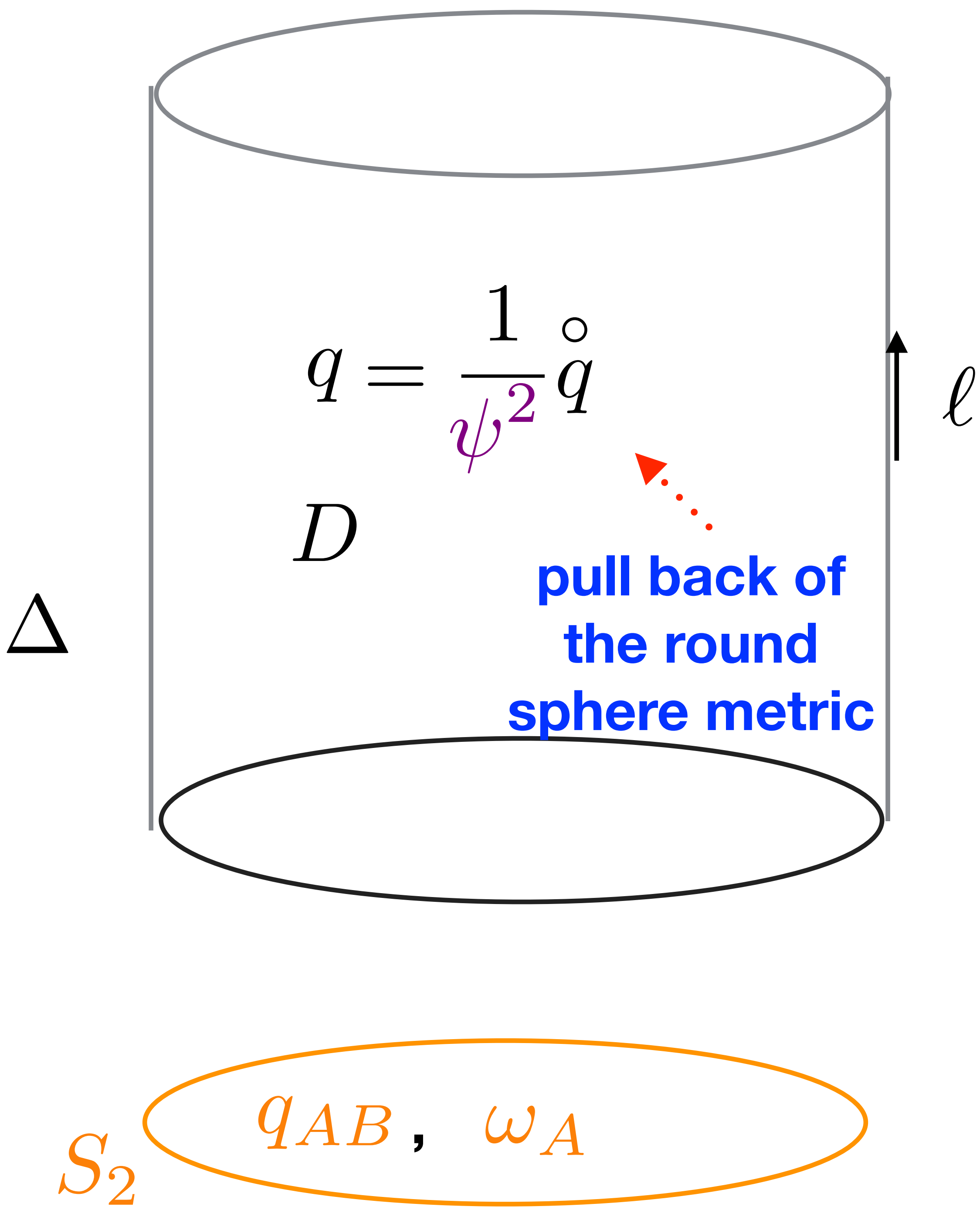

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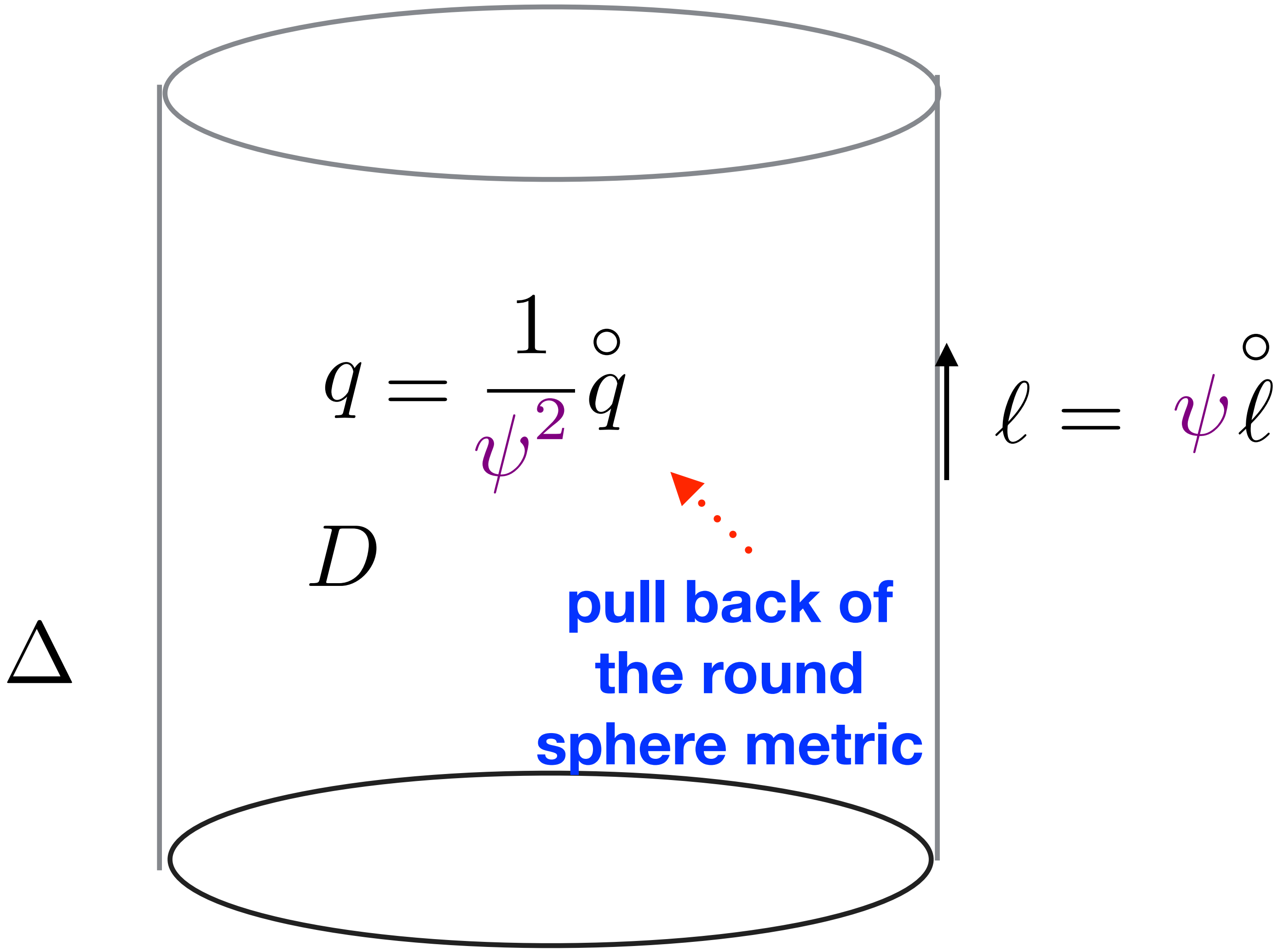

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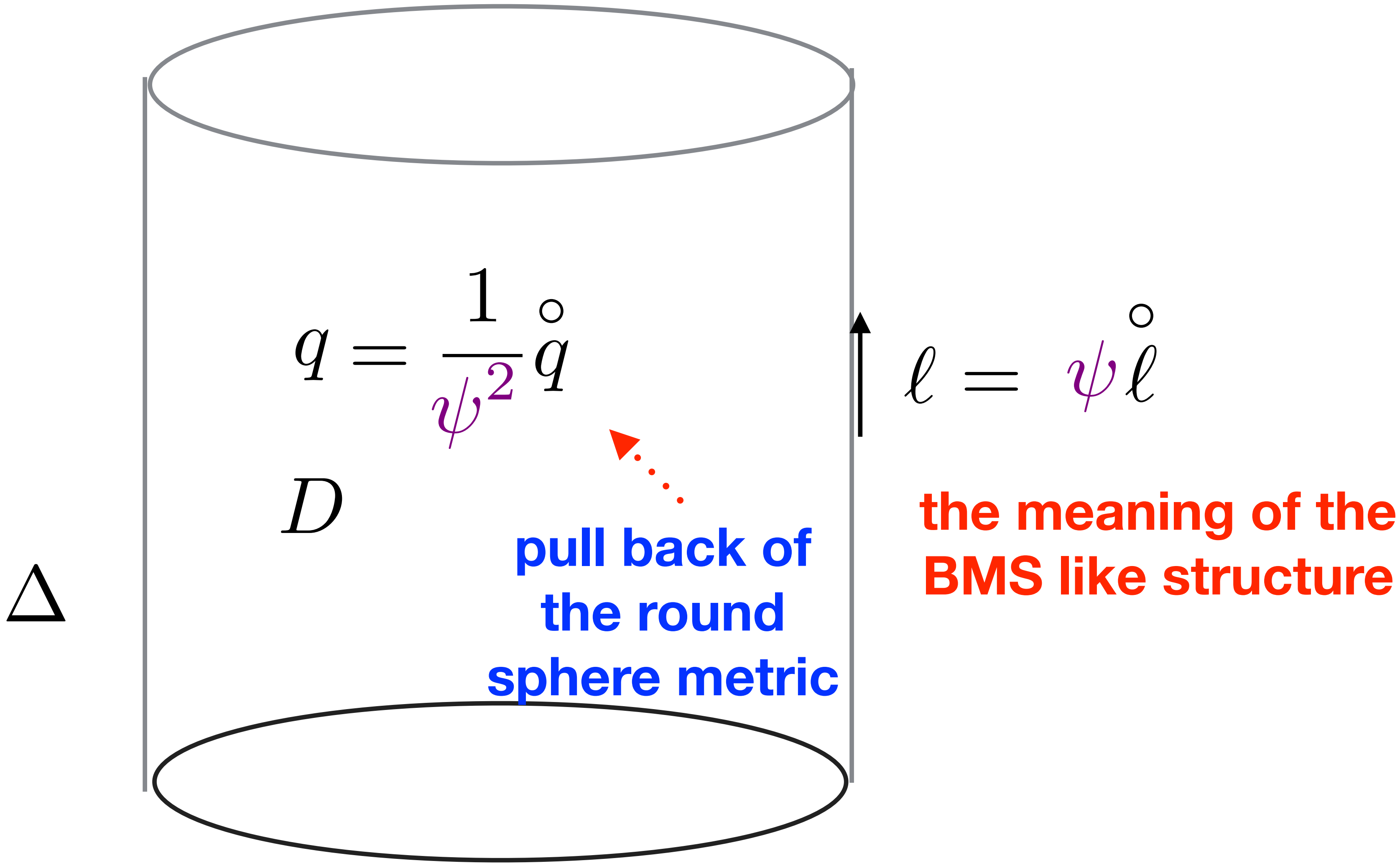

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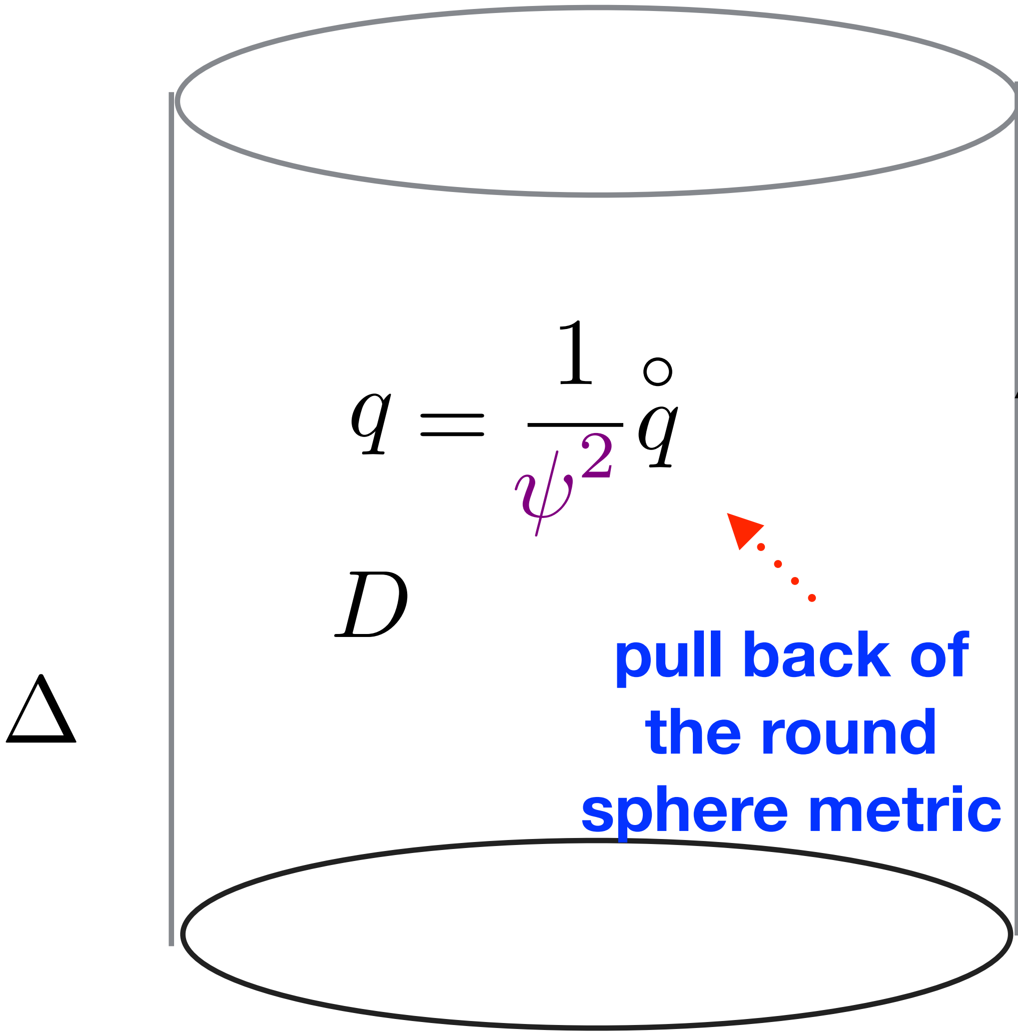

$$q = \frac{1}{\psi^2}\overset{\circ}{q}$$

$$\ell = \psi \overset{\circ}{\ell} \Leftrightarrow \left(1 - \overset{\circ}{\nabla}{}^A \overset{\circ}{\omega}_A\right)\overset{\circ}{\epsilon} = K\epsilon$$

$$D_a \overset{\circ}{\ell}{}^b =: \overset{\circ}{\omega}_a \overset{\circ}{\ell}{}^b$$

the 2-area elements

the meaning of the BMS like structure

S_2: q_{AB}, ω_A

$$q^{AB} D_A \omega_B = 0$$

A BMS like structure of NEH

$\Delta = S_2 \times \mathbb{R}$

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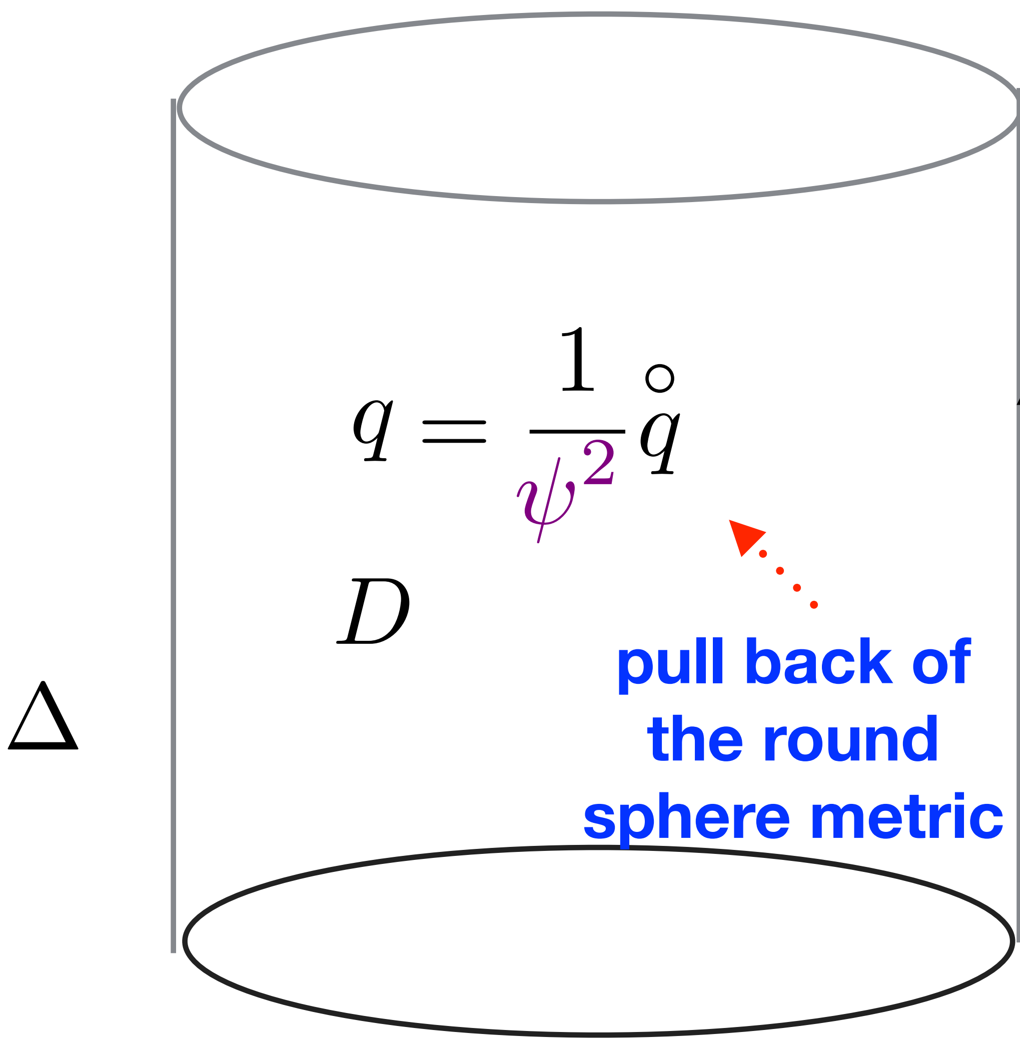

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pull back of the round sphere metric

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S_2: $q_{AB}, \; \omega_A$

$q^{AB} D_A \omega_B = 0$

ambiguity

$\mathring{q}' = \alpha^2 \mathring{q}$

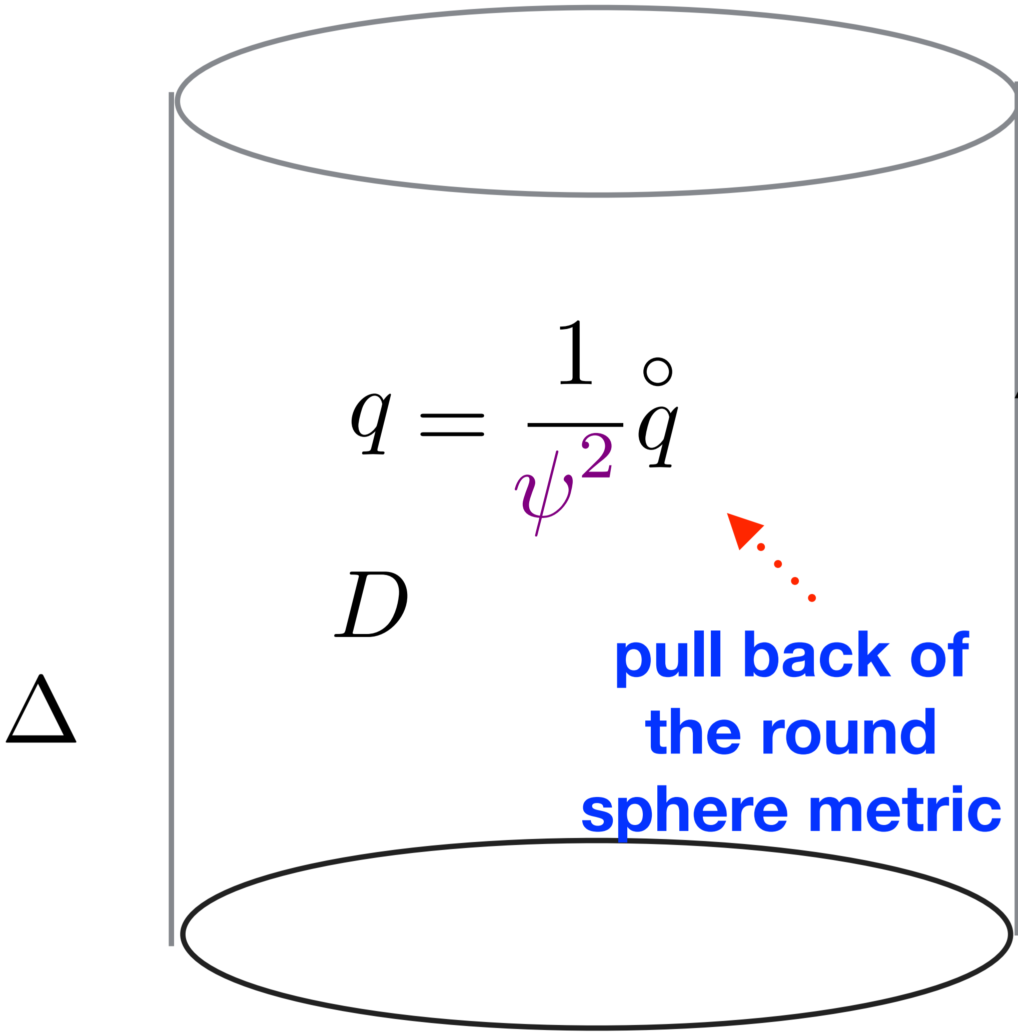
$\mathring{\ell}' = \frac{a}{\alpha}\mathring{\ell}$

a constant

A BMS like structure of NEH

$\Delta = S_2 \times \mathbb{R}$

Universal structure $[\mathring{q}, \mathring{\ell}]$



$\ell = \psi \mathring{\ell} \Leftrightarrow \left(1 - \mathring{\nabla}^A \mathring{\omega}_A\right)\mathring{\epsilon} = K\epsilon$

the 2-area elements

$D_a \mathring{\ell}^b =: \mathring{\omega}_a \mathring{\ell}^b$

the meaning of the BMS like structure

ambiguity

$\mathring{q}' = \alpha^2 \mathring{q}$

a constant

$\mathring{\ell}' = \frac{a}{\alpha}\mathring{\ell}$

S_2 $q_{AB}\,,\ \omega_A$

$q^{AB} D_A \omega_B = 0$

$[\mathring{q}, \mathring{\ell}]$ is the equivalence class

the BMS like structure

A BMS like structure of NEH

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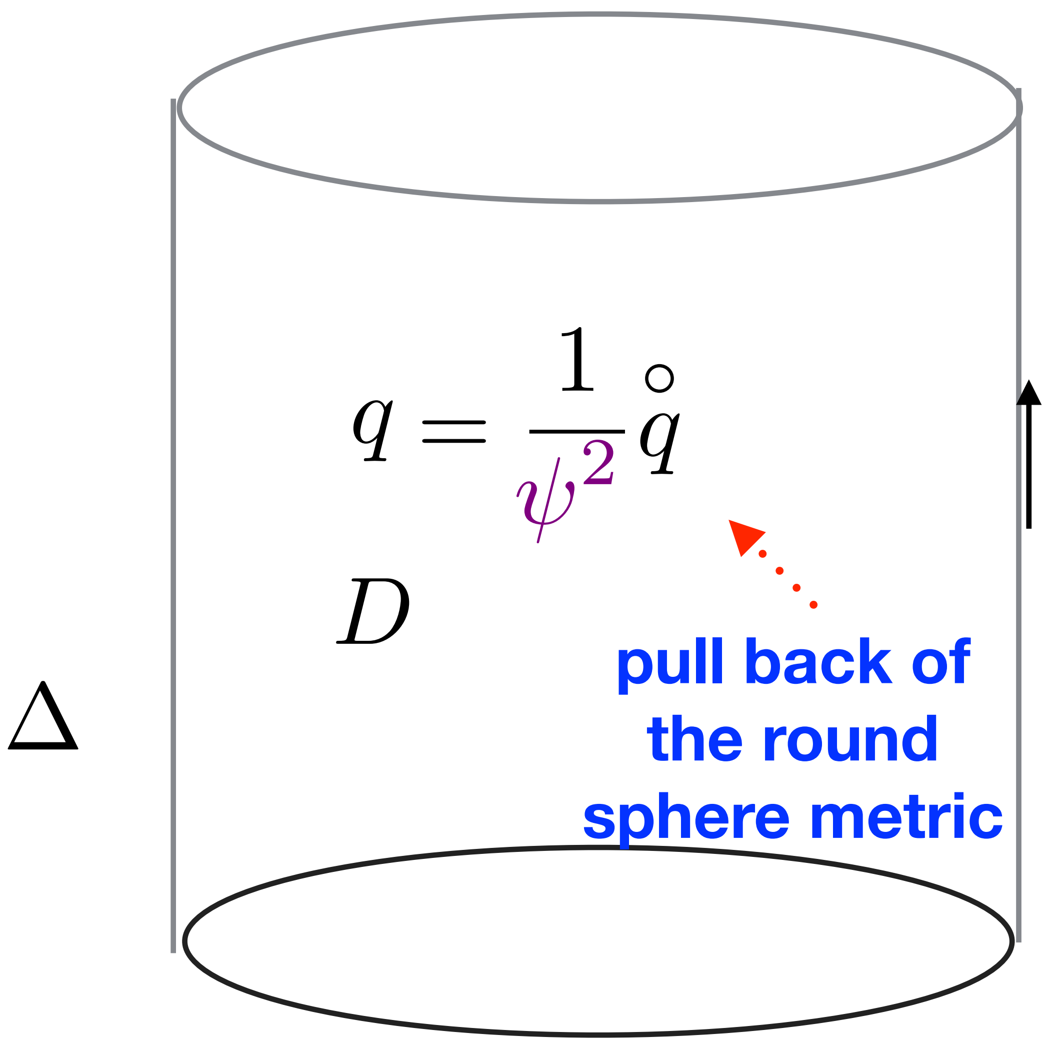

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a constant

$$\mathring{\ell}' = \frac{a}{\alpha}\mathring{\ell}$$

$[\mathring{q}, \mathring{\ell}]$ **is the equivalence class**

the BMS like structure

$$\underline{\alpha}^{-1} = \alpha_0 + \sum_{i=1}^{3} \alpha_i \, \hat{r}^i, \quad \text{for real constants } \alpha_0 \text{ and } \alpha_i, \text{ with}$$

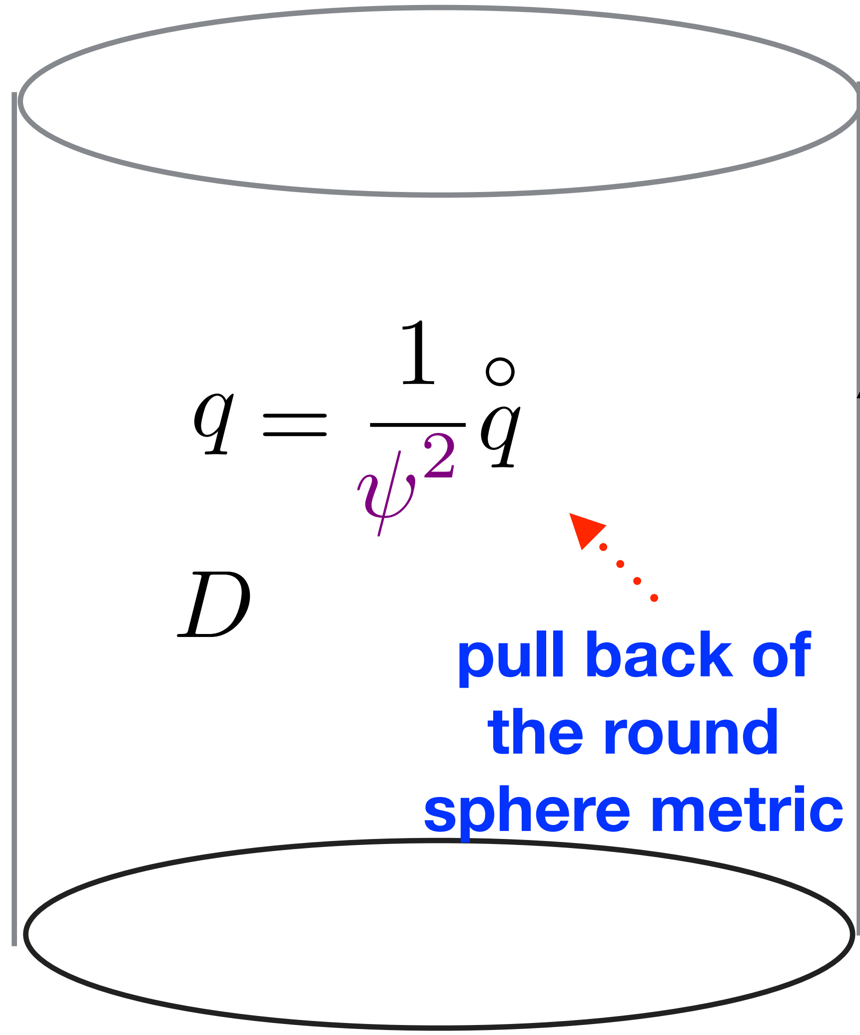
$$\hat{r}^i = (\sin\theta\cos\phi, \, \sin\theta\sin\phi, \, \cos\theta) \qquad \text{and} \qquad -\alpha_0^2 + \sum_{i=1}^{3}(\alpha_i)^2 = -1,$$

A BMS like structure of NEH

$\Delta = S_2 \times \mathbb{R}$

Universal structure $[\overset{\circ}{q}, \overset{\circ}{\ell}]$

Δ



$\ell = \psi \overset{\circ}{\ell}$

ambiguity

$\overset{\circ}{q}{}' = \alpha^2 \overset{\circ}{q}$

a constant

$\overset{\circ}{\ell}{}' = \frac{a}{\alpha} \overset{\circ}{\ell}$

$[\overset{\circ}{q}, \overset{\circ}{\ell}]$ is the equivalence class

the BMS like structure

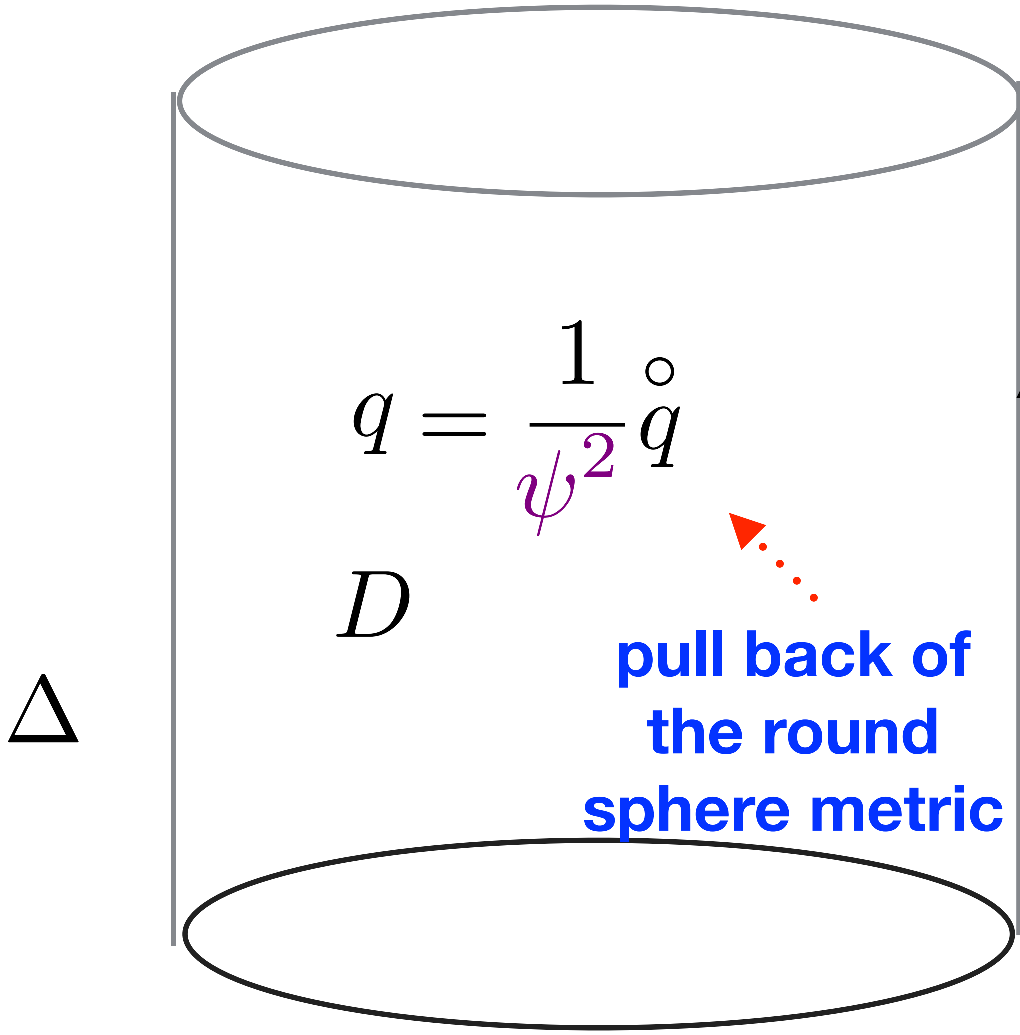
the SYMMETRIES are all the $\Delta \to \Delta$

that preserve $[\overset{\circ}{q}, \overset{\circ}{\ell}]$

A BMS like structure of NEH

$\Delta = S_2 \times \mathbb{R}$

Universal structure $[\mathring{q}, \mathring{\ell}]$



$\ell = \psi \mathring{\ell}$

symmetry generators

ξ

ambiguity $\mathring{q}' = \alpha^2 \mathring{q}$

a constant

$\mathring{\ell}' = \frac{a}{\alpha}\mathring{\ell}$

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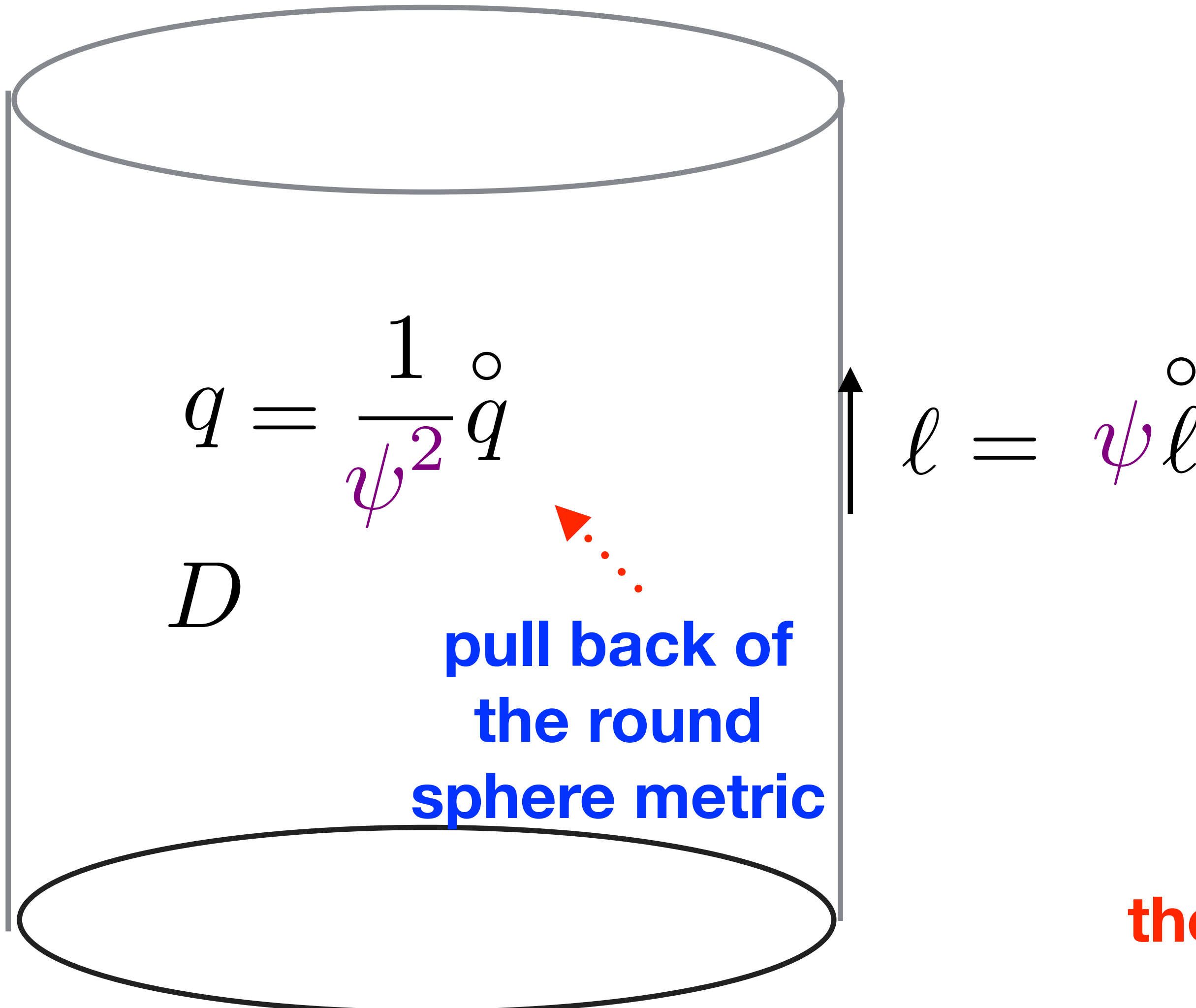
A BMS like structure of NEH

$\Delta = S_2 \times \mathbb{R}$

Universal structure $[\mathring{q}, \mathring{\ell}]$

Δ



symmetry generators

ξ

$$\mathcal{L}_\xi \, \mathring{q}_{ab} = 2\mathring{\phi} \, \mathring{q}_{ab} \qquad \mathcal{L}_\xi \, \mathring{\ell}^a = -(\mathring{\phi} + k) \, \mathring{\ell}^a$$

ambiguity

$$\mathring{q}' = \alpha^2 \mathring{q}$$

a constant

$$\mathring{\ell}' = \frac{a}{\alpha}\mathring{\ell}$$

$[\mathring{q}, \mathring{\ell}]$ is the equivalence class

the BMS like structure

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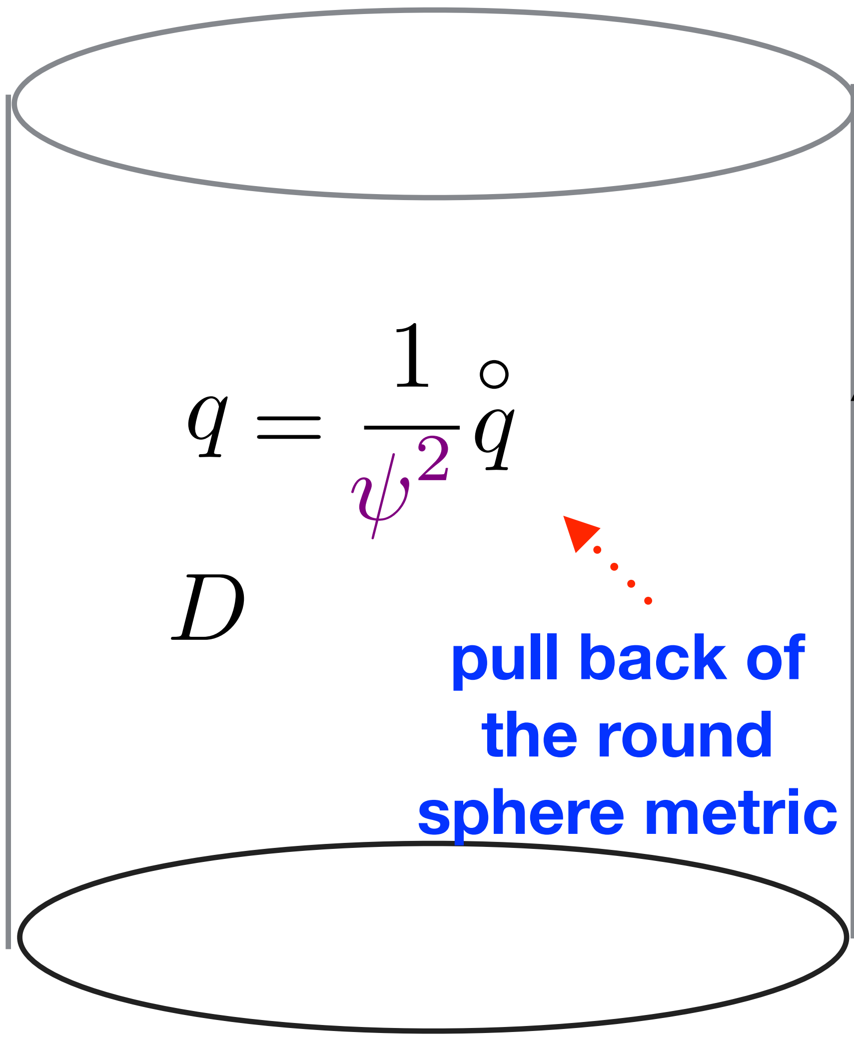
A BMS like structure of NEH

$\Delta = S_2 \times \mathbb{R}$

Universal structure $[\mathring{q}, \mathring{\ell}]$

Δ



$\ell = \psi \mathring{\ell}$

ambiguity → $\mathring{q}' = \alpha^2 \mathring{q}$

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$[\mathring{q}, \mathring{\ell}]$ is the equivalence class
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k is a constant

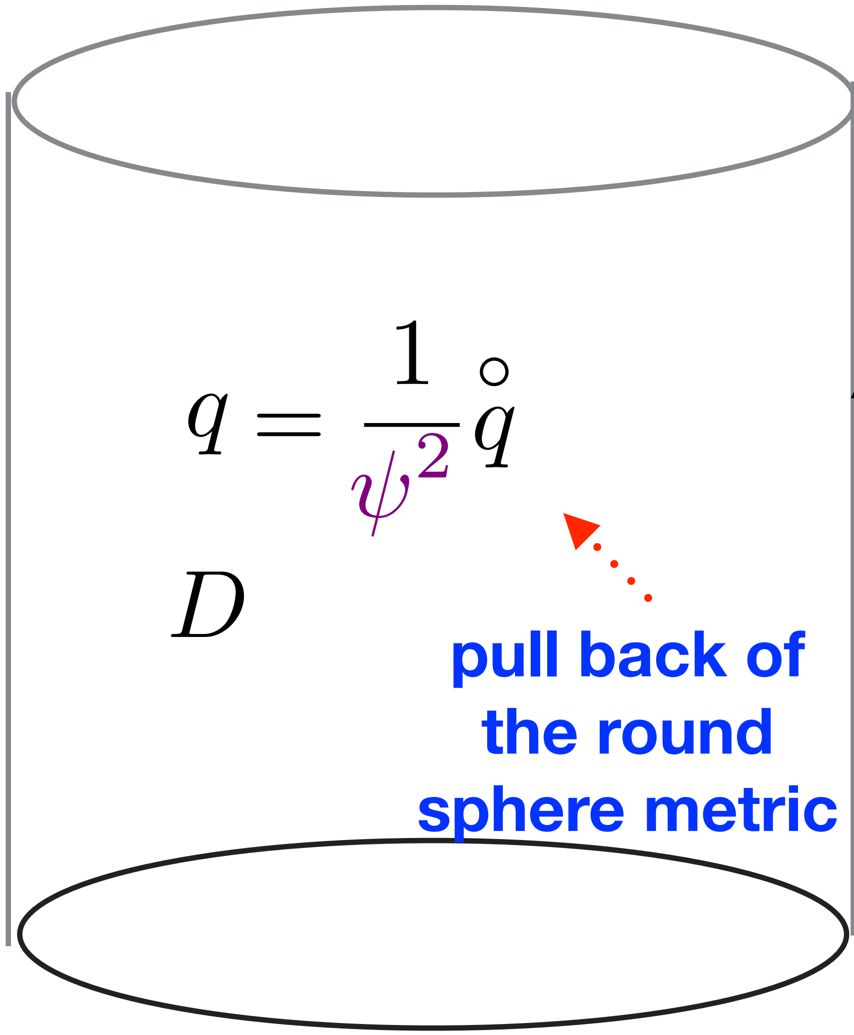
$\mathring{\phi} = \sum_m a_m Y_{1,m}$

A BMS like structure of NEH

$\Delta = S_2 \times \mathbb{R}$

Universal structure $[\mathring{q}, \mathring{\ell}]$

Δ



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symmetry generators

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k is a constant

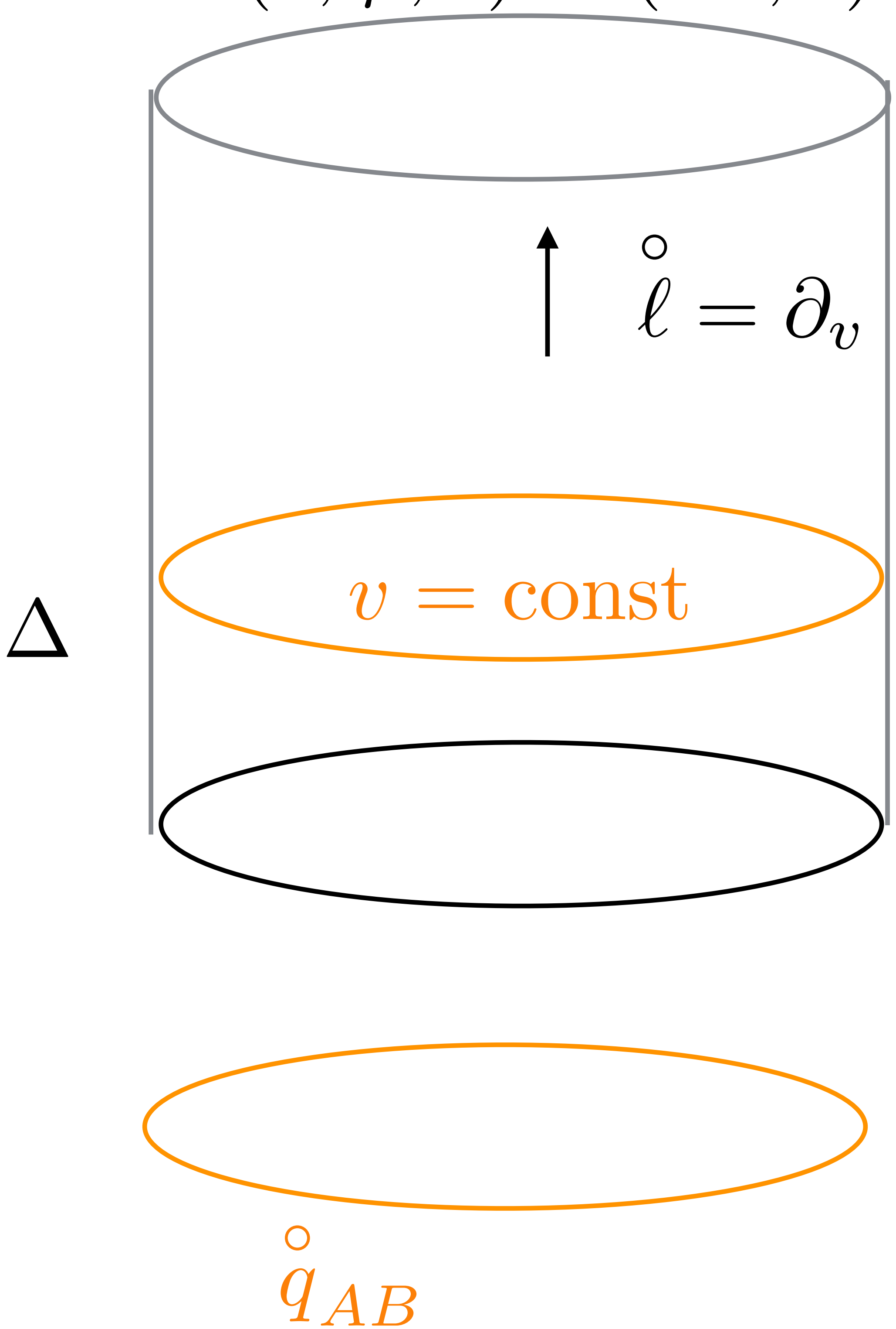
$\mathring{\phi} = \sum_m a_m Y_{1,m}$

$k = 0$ corresponds to the BMS

The symmetry generators

$$\Delta = S_2 \times \mathbb{R}$$

$$(\theta, \varphi, v) = (x^A, v)$$

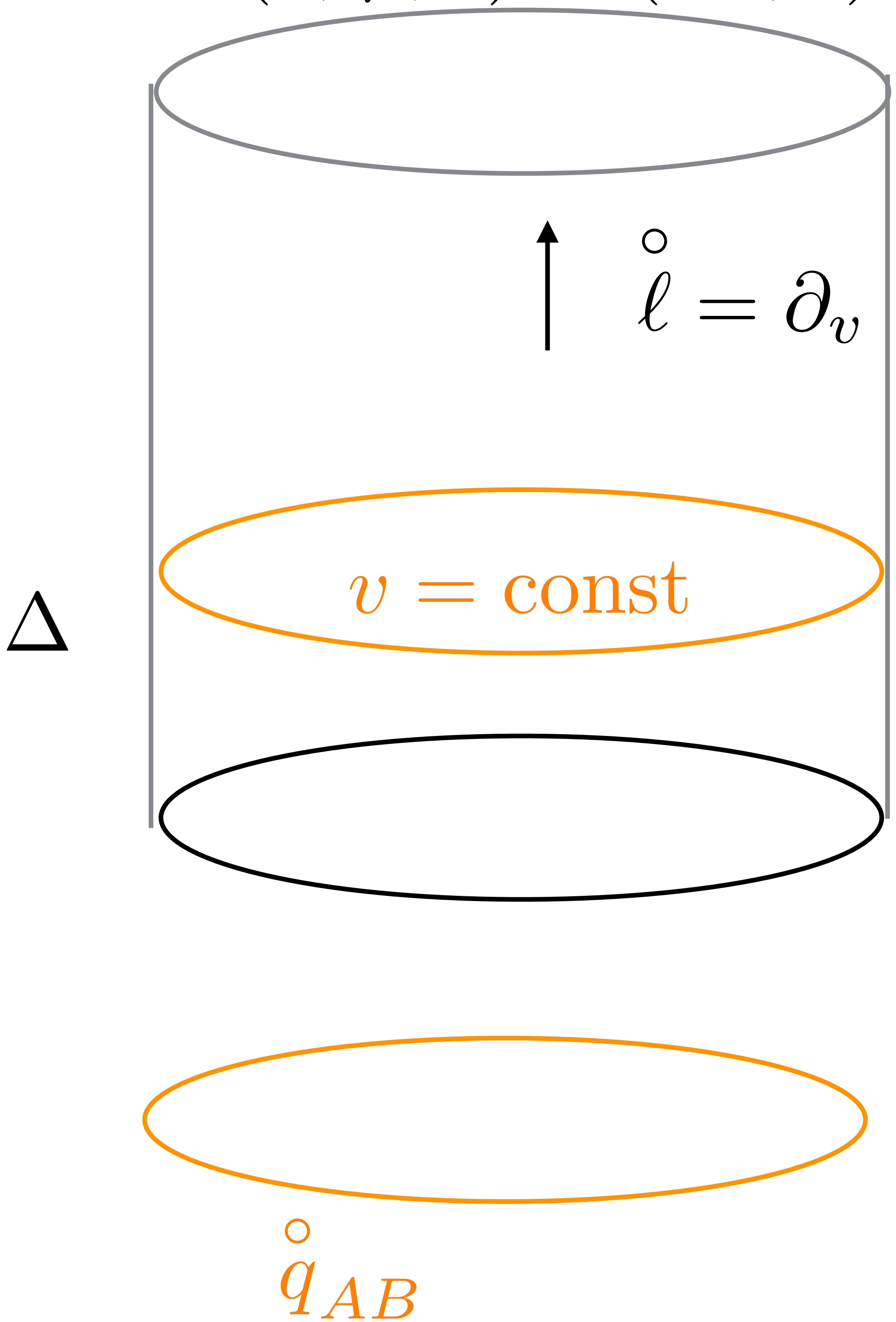


The symmetry generators

$$\Delta = S_2 \times \mathbb{R}$$

$$(\theta, \varphi, v) = (x^A, v)$$

dylations

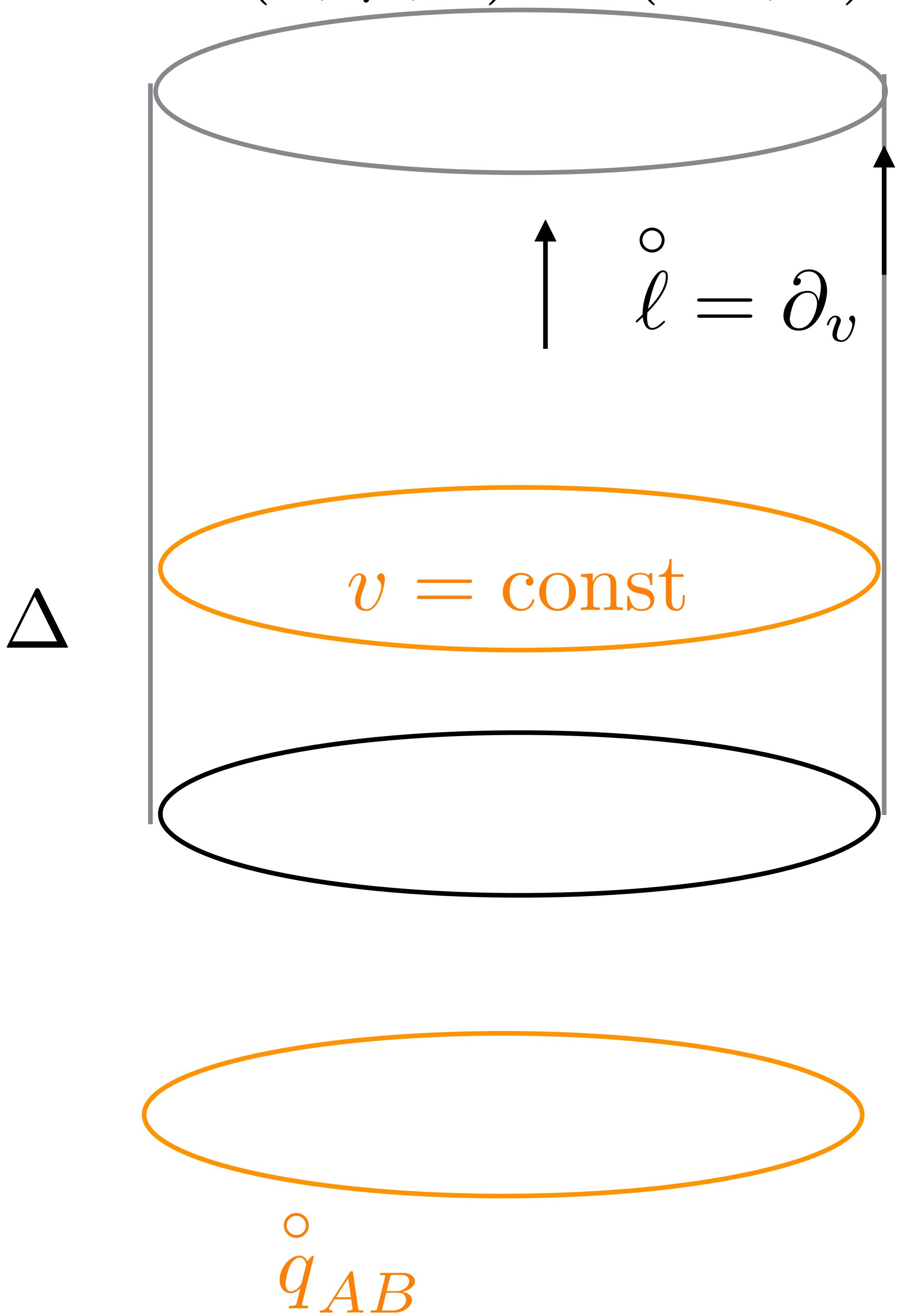


The symmetry generators

$$\Delta = S_2 \times \mathbb{R}$$

$$(\theta, \varphi, v) = (x^A, v)$$

$$d = kv\partial_v$$ **dylations**



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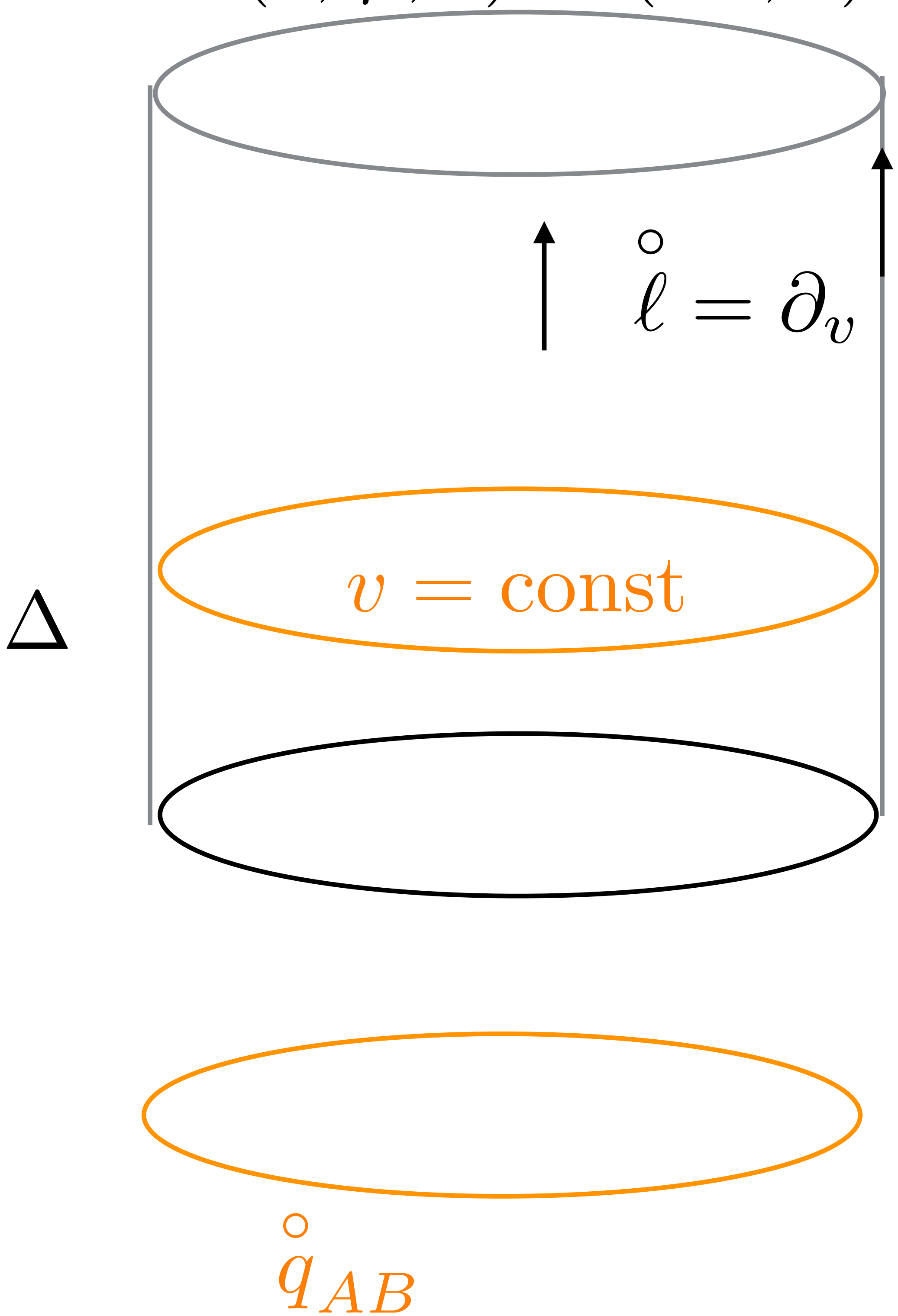


$$d = kv\partial_v$$

dylations

$k = \text{const}$

The symmetry generators

$$\Delta = S_2 \times \mathbb{R}$$

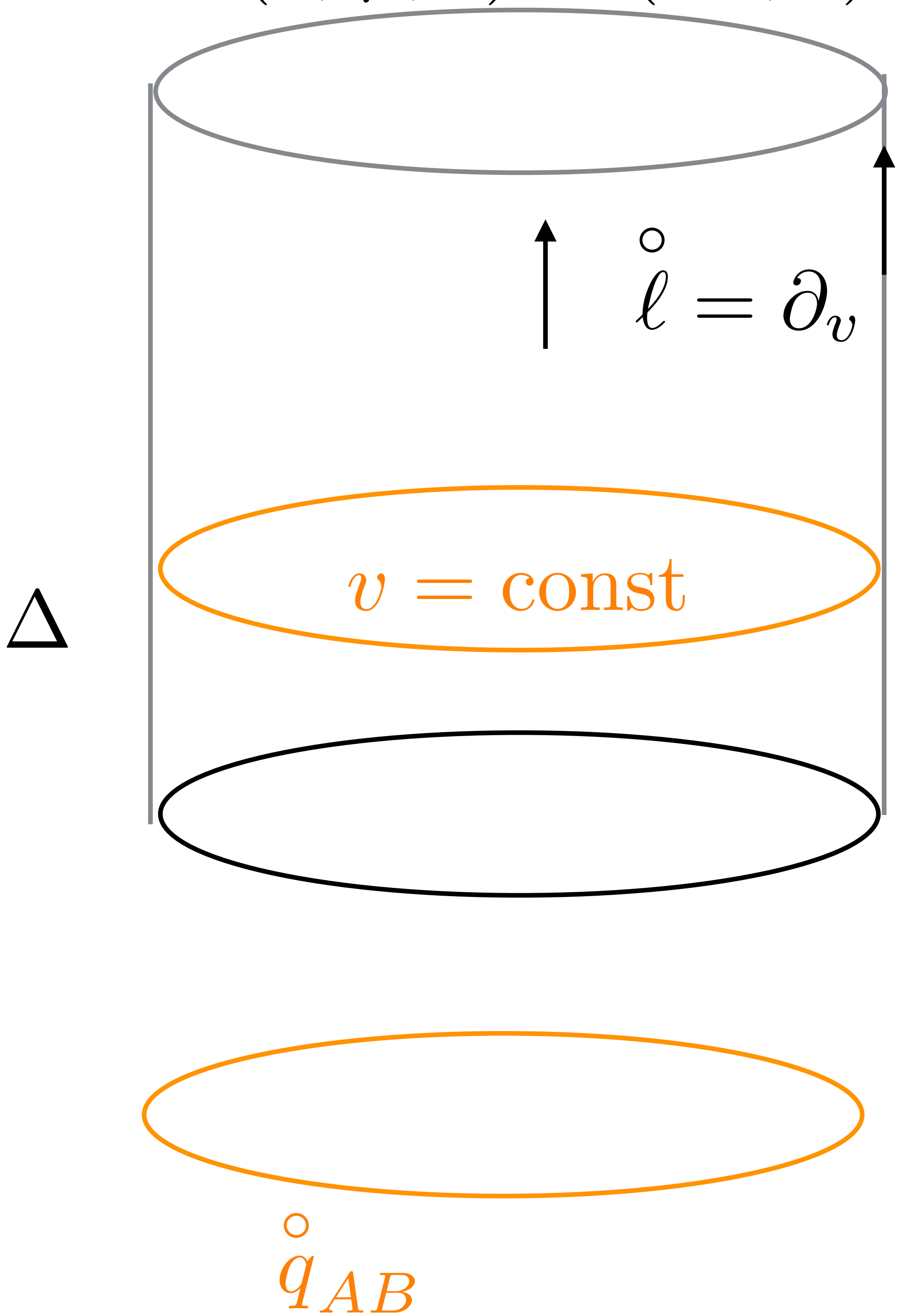
$$(\theta, \varphi, v) = (x^A, v)$$

$$d = kv\partial_v$$

dylations

$$k = \text{const}$$

super translations



The symmetry generators

$$\Delta = S_2 \times \mathbb{R}$$

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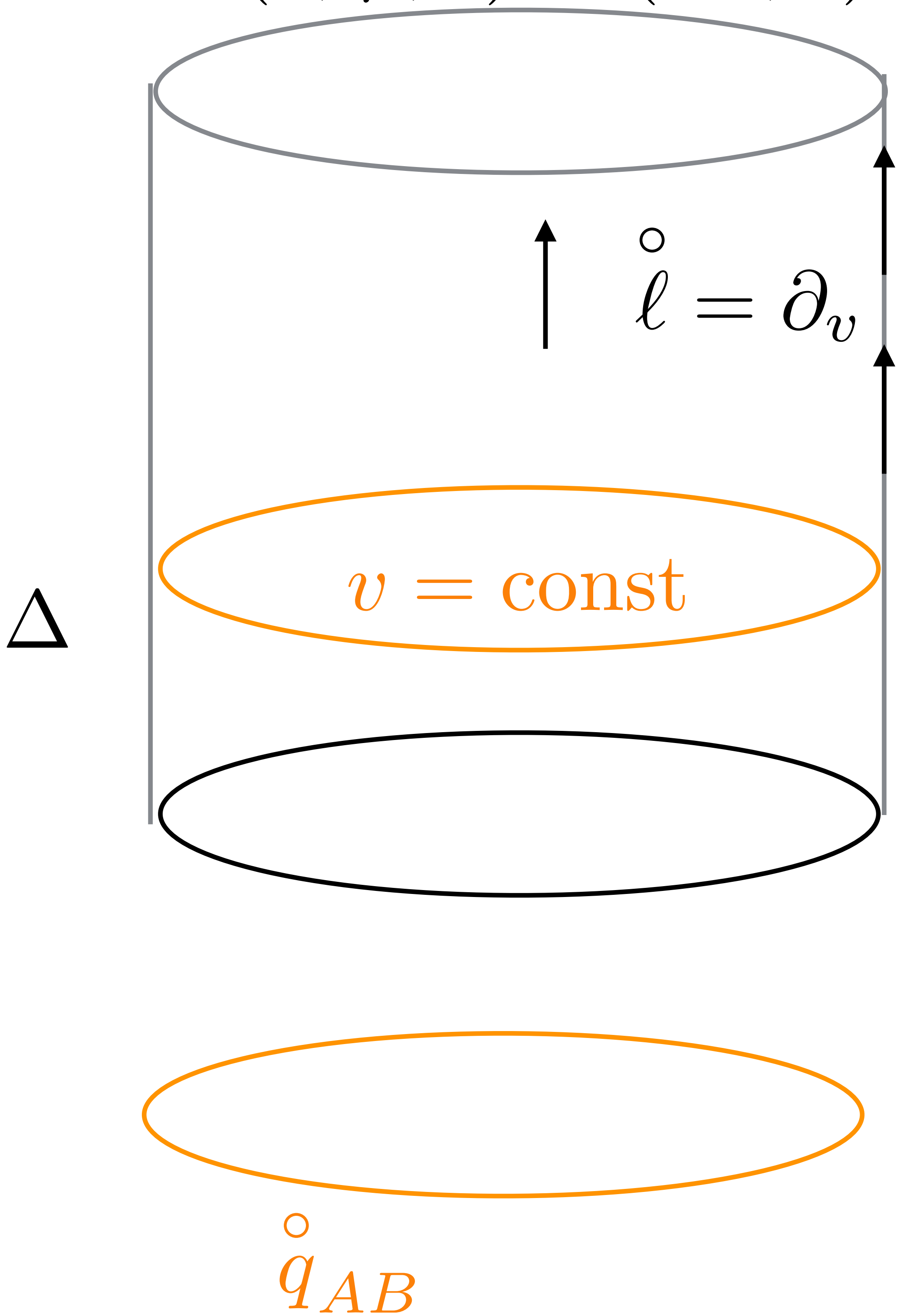


$$d = kv\partial_v$$ **dylations**

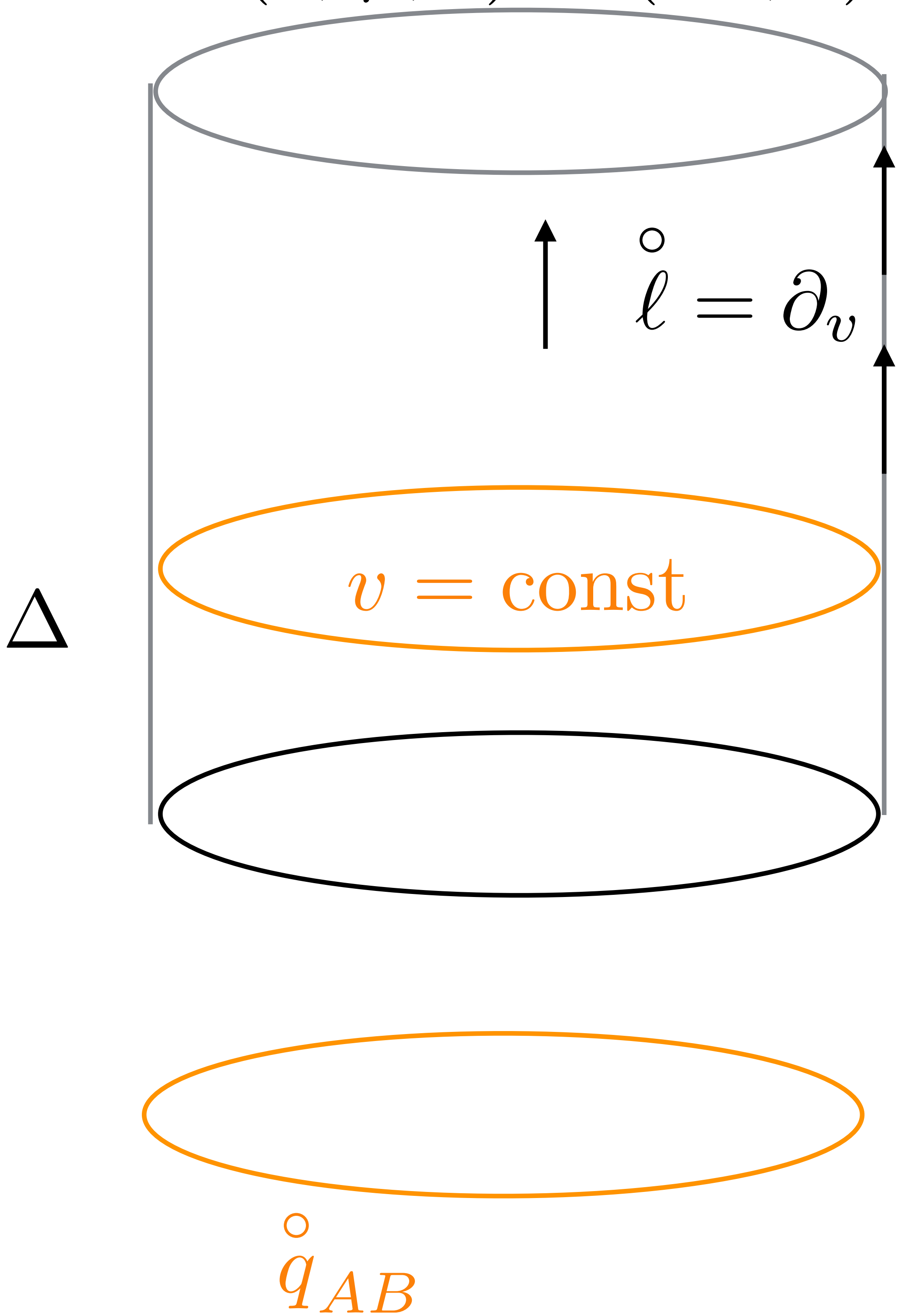
$k = \text{const}$

$$S = s\partial_v$$ **super translations**

The symmetry generators

$$\Delta = S_2 \times \mathbb{R}$$

$$(\theta, \varphi, v) = (x^A, v)$$



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dylations

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super translations

$$s = s(\theta, \varphi)$$

The symmetry generators

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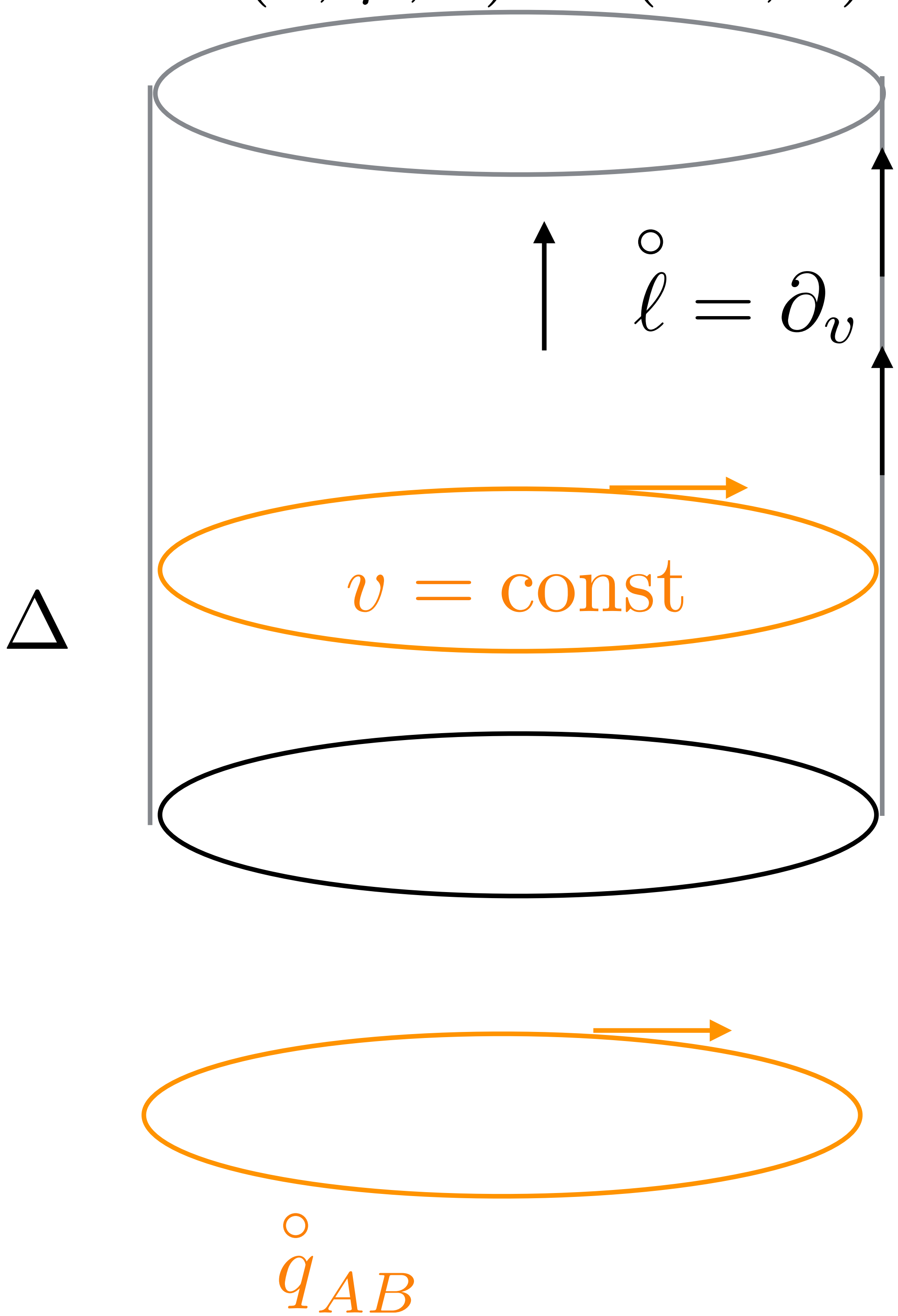

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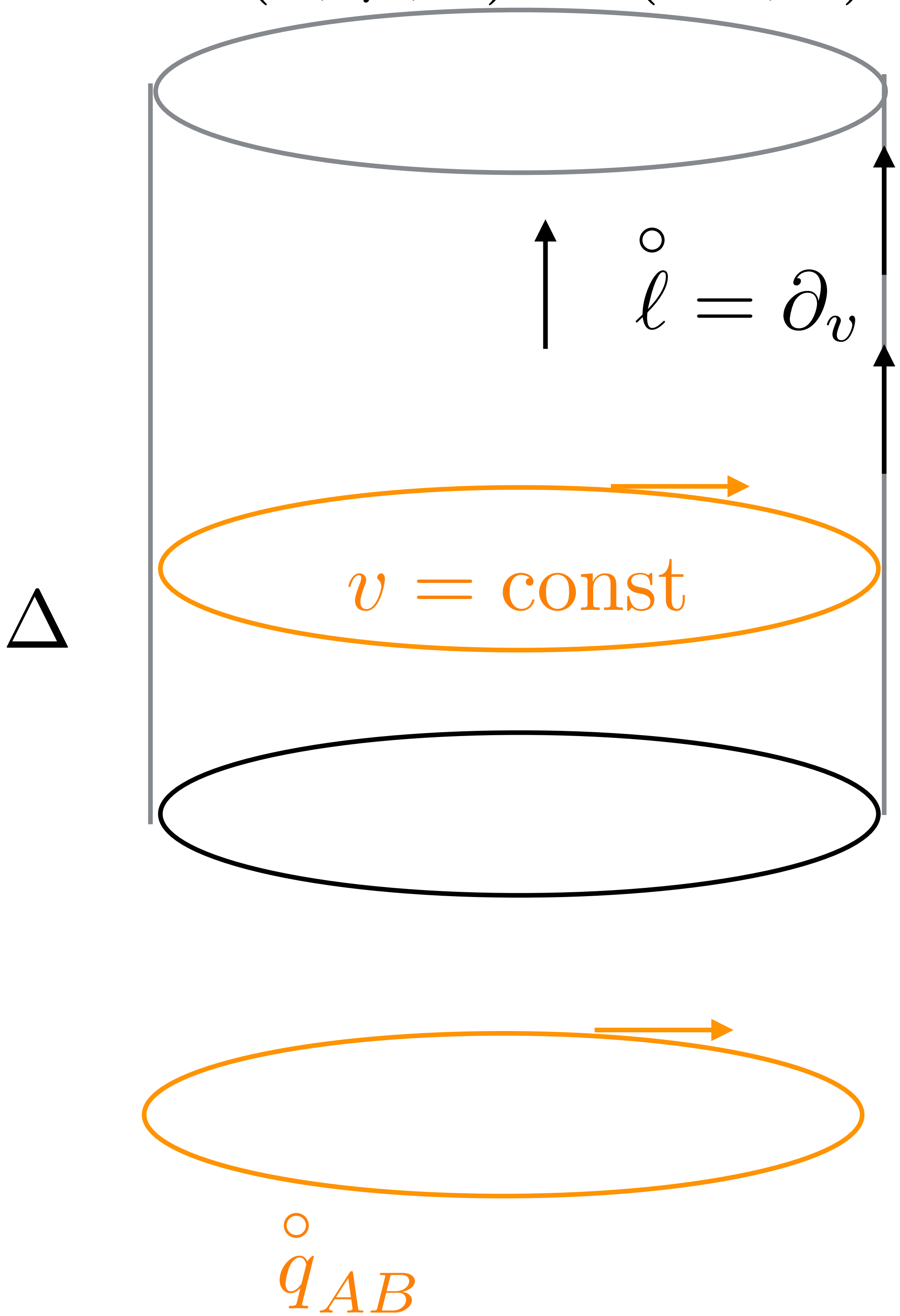

$$d = kv\partial_v$$

dylations

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super translations

$s = s(\theta, \varphi)$

$$R = \mathring{\epsilon}^{AB} \chi_{,B} \partial_A$$

rotations

The symmetry generators

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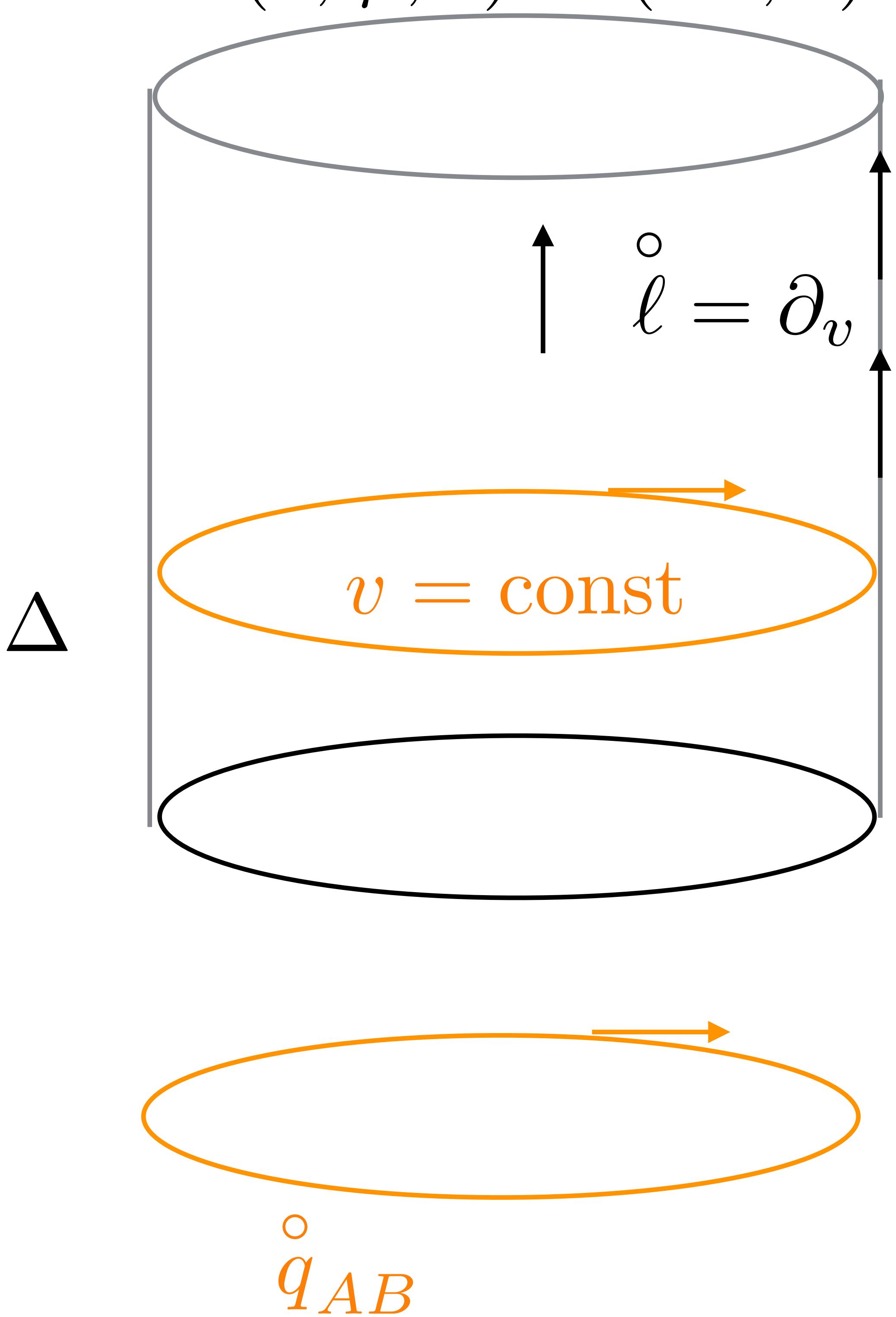

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dylations

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super translations

$$s = s(\theta, \varphi)$$

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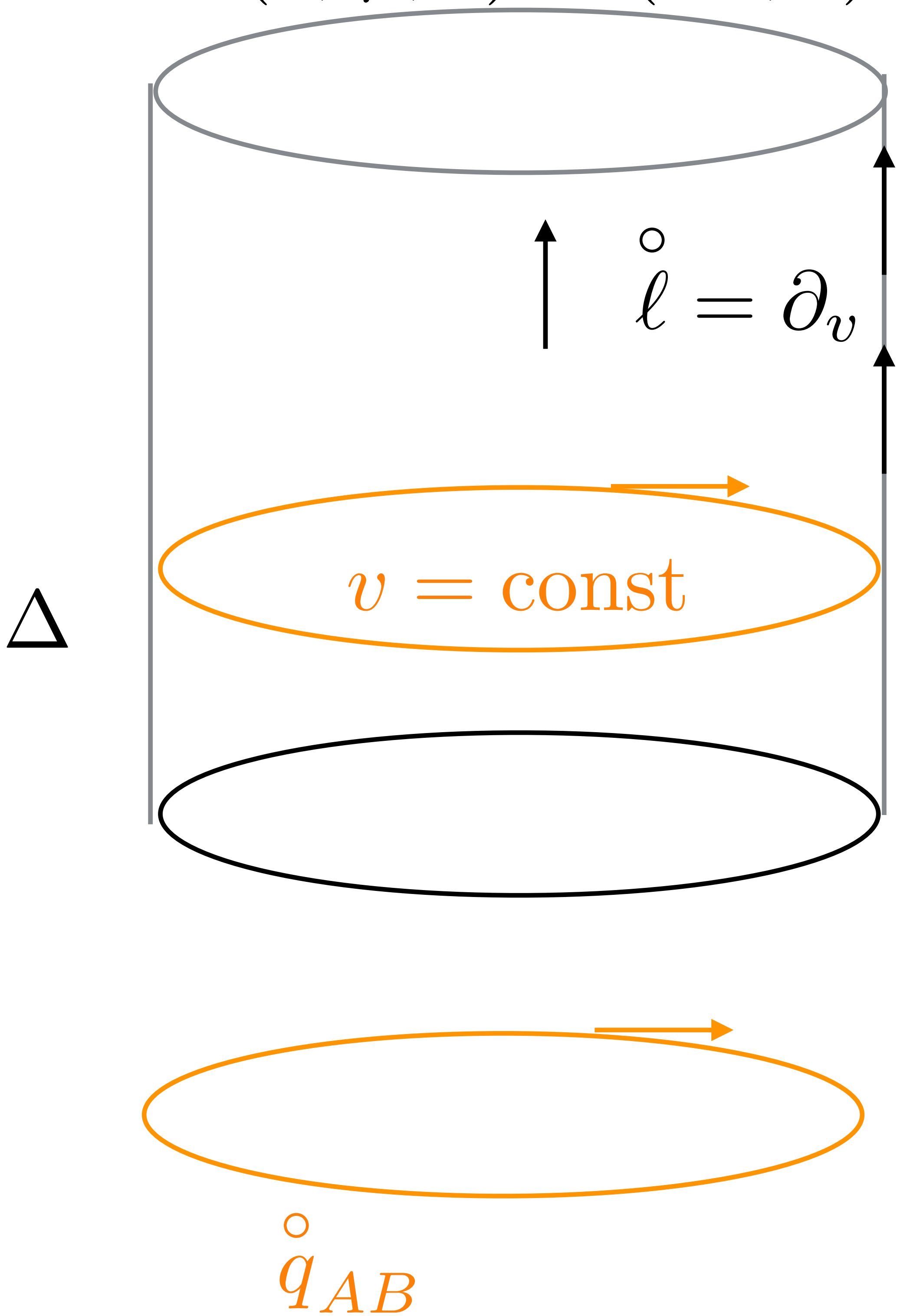
rotations

$$\chi = \chi(\theta, \varphi)$$

The symmetry generators

$$\Delta = S_2 \times \mathbb{R}$$

$$(\theta, \varphi, v) = (x^A, v)$$



$$d = kv\partial_v$$

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rotations

$$\chi = \chi(\theta, \varphi)$$

$$\mathring{\Delta}\chi = -2\chi$$

The symmetry generators

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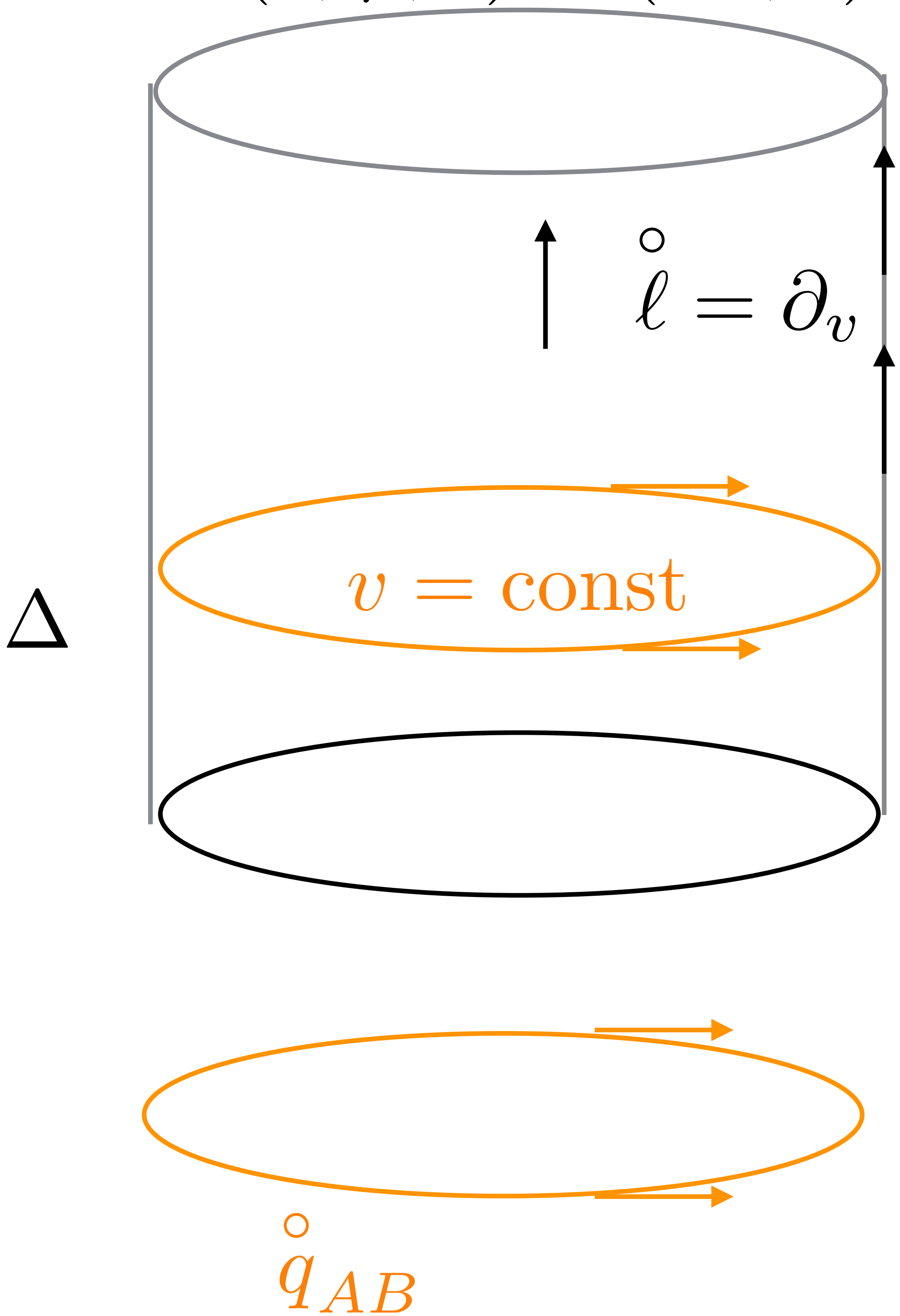

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boosts

The symmetry generators

$$\Delta = S_2 \times \mathbb{R}$$

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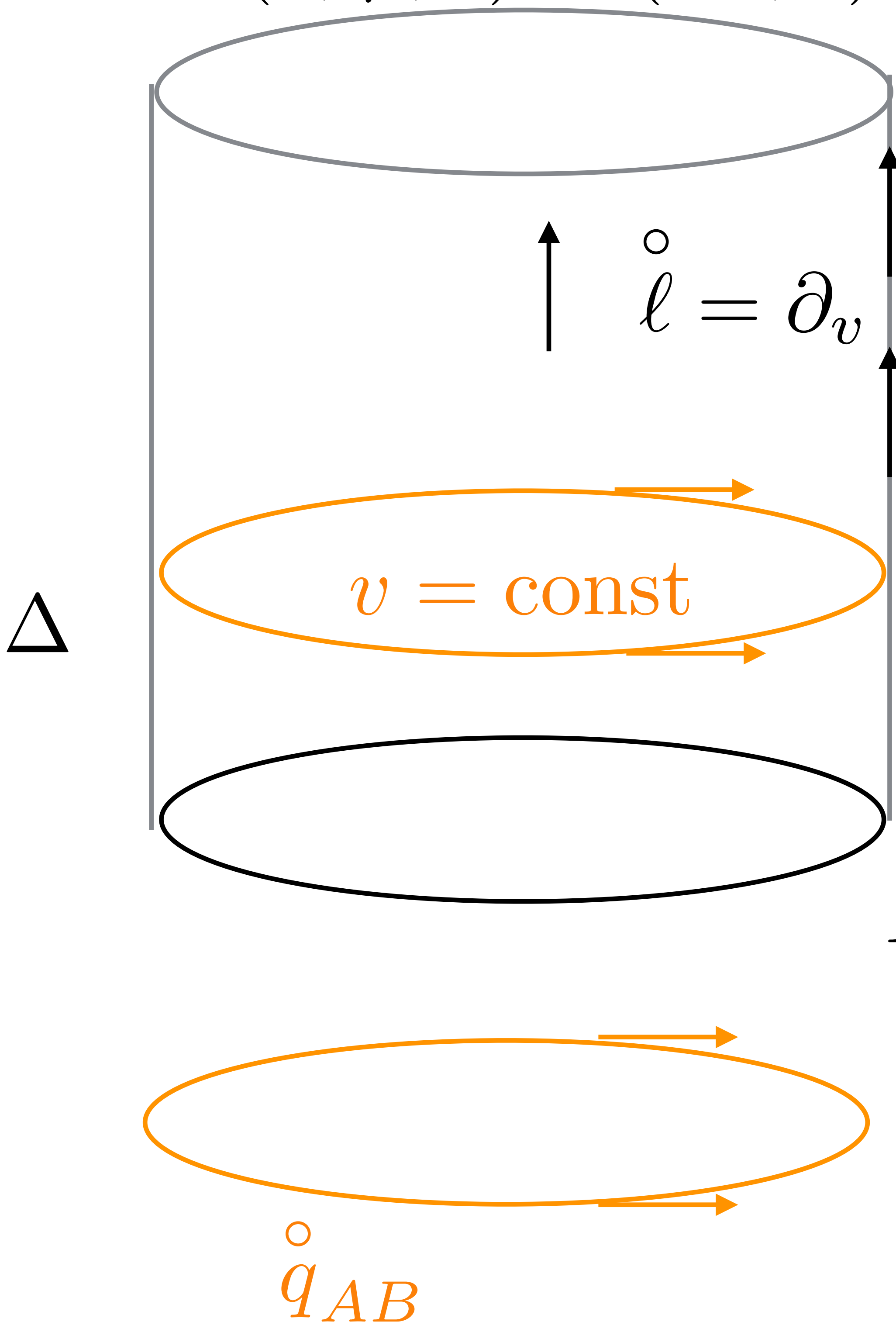


$$d = kv\partial_v$$

dylations

$k = \text{const}$

$$S = s\partial_v$$

super translations

$s = s(\theta, \varphi)$

$$R = \overset{\circ}{\epsilon}{}^{AB} \chi_{,B} \partial_A$$

rotations

$\chi = \chi(\theta, \varphi)$

$\overset{\circ}{\Delta}\chi = -2\chi$

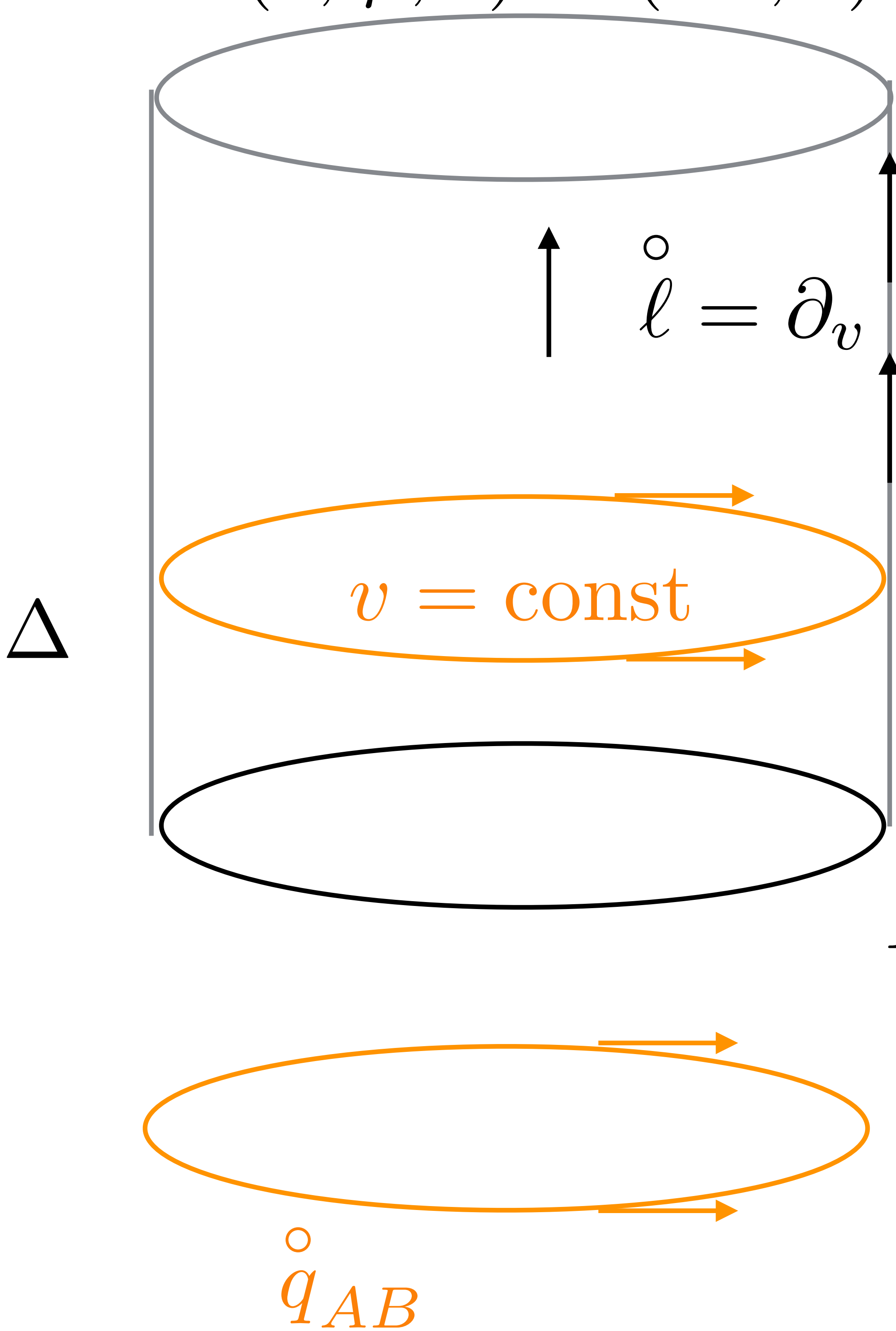
$$B = \overset{\circ}{q}{}^{AB} \overset{\circ}{\phi}_{,B} \partial_A$$

boosts

The symmetry generators

$$\Delta = S_2 \times \mathbb{R}$$

$$(\theta, \varphi, v) = (x^A, v)$$



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$$\chi = \chi(\theta, \varphi)$$

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boosts

The symmetry generators

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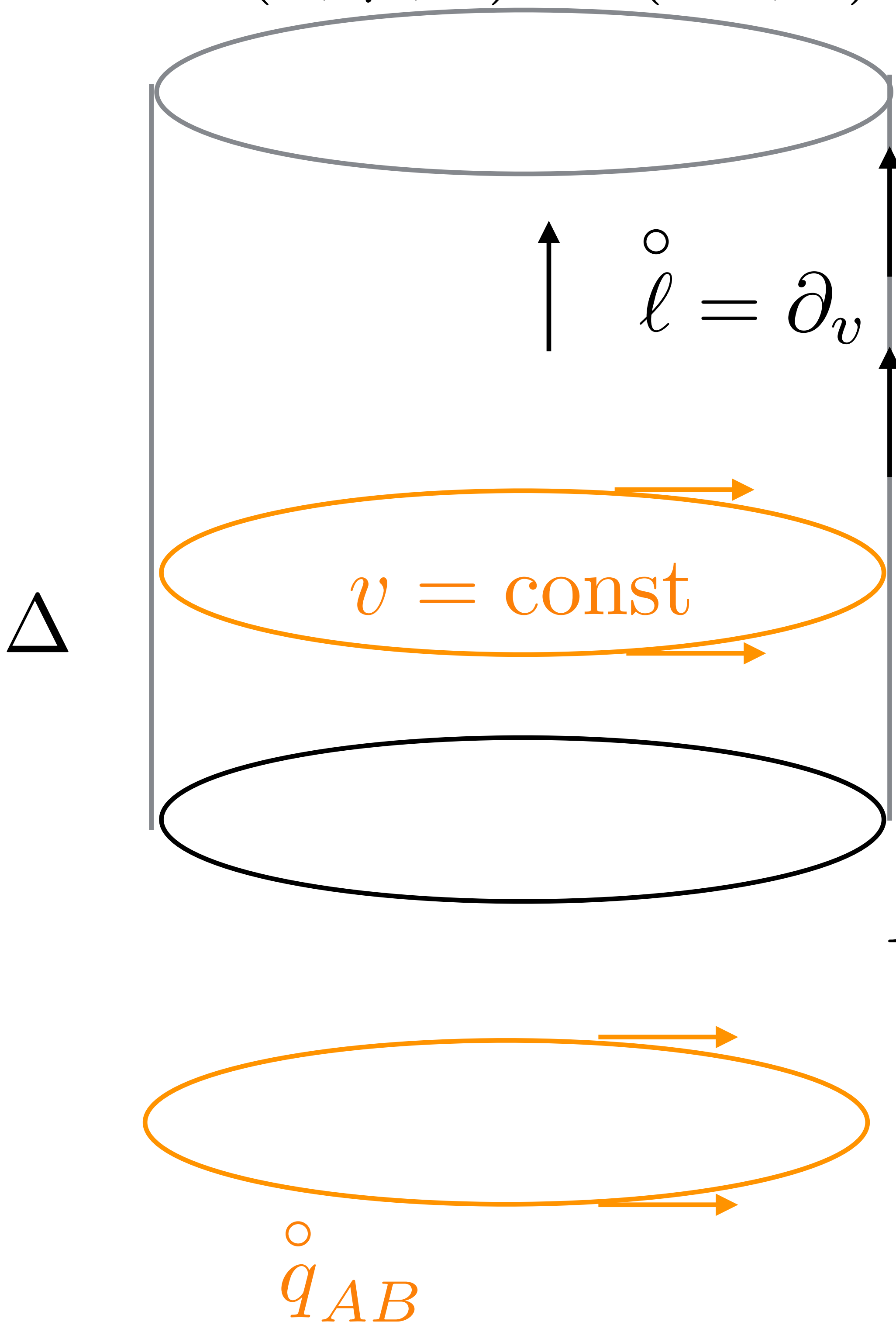

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dylations

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super translations

$$s = s(\theta, \varphi)$$

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$$B = \mathring{q}^{AB} \mathring{\phi}_{,B} \partial_A + v\mathring{\phi}\partial_v$$

boosts

$$\mathring{\phi} = \mathring{\phi}(\theta, \varphi)$$

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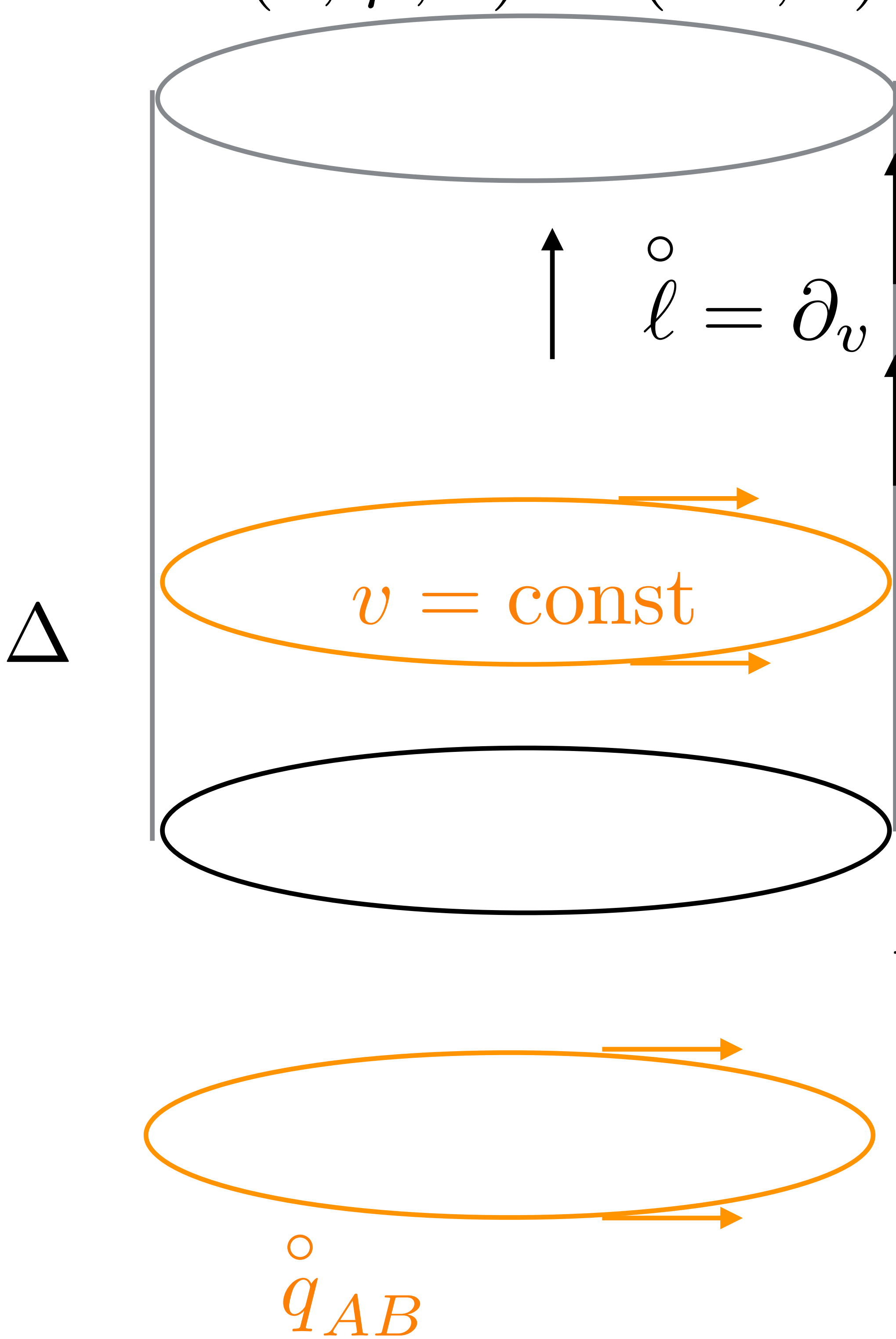

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super translations

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boosts

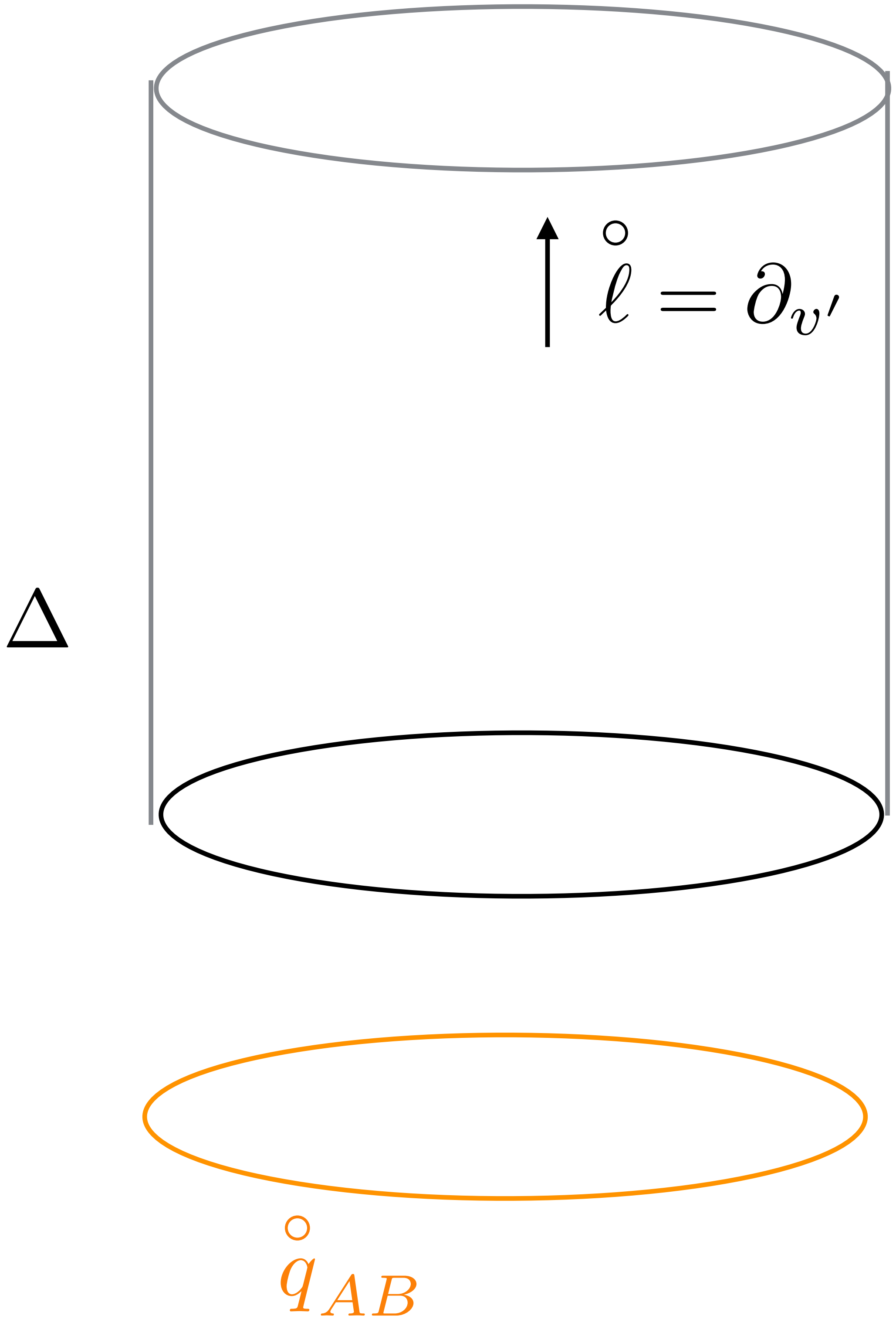
$$\mathring{\phi} = \mathring{\phi}(\theta, \varphi)$$

$$\mathring{\Delta}\mathring{\phi} = -2\mathring{\phi}$$

Ambiguities in the Lorenz transformations

$$\Delta = S_2 \times \mathbb{R}$$

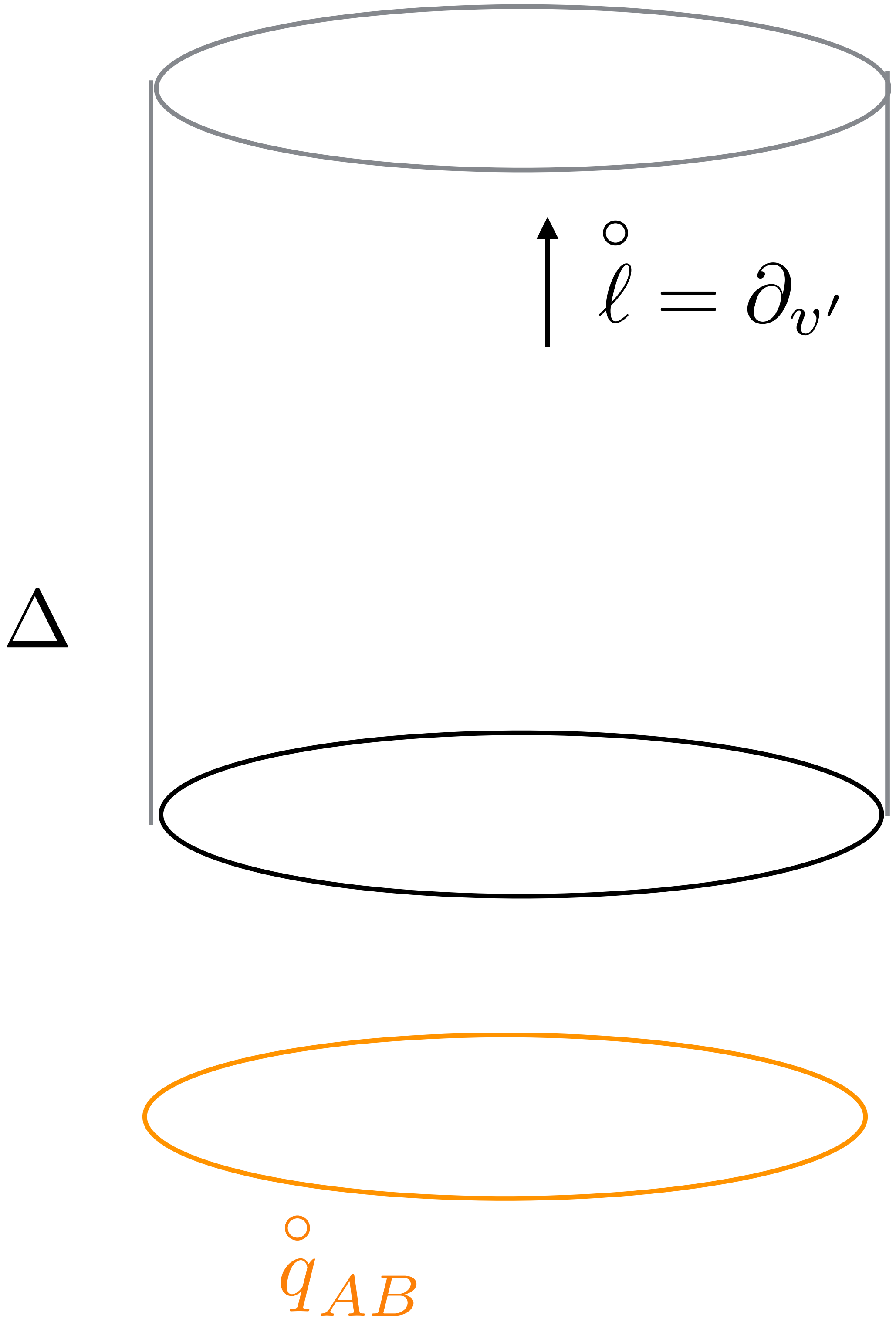
$$(\theta, \varphi, v' = v + f(\theta, \phi)) =: (x'^A, v')$$



Ambiguities in the Lorenz transformations

$$\Delta = S_2 \times \mathbb{R}$$

$$(\theta, \varphi, v' = v + f(\theta, \phi)) =: (x'^A, v') \qquad k v' \partial_{v'}$$



Ambiguities in the Lorenz transformations

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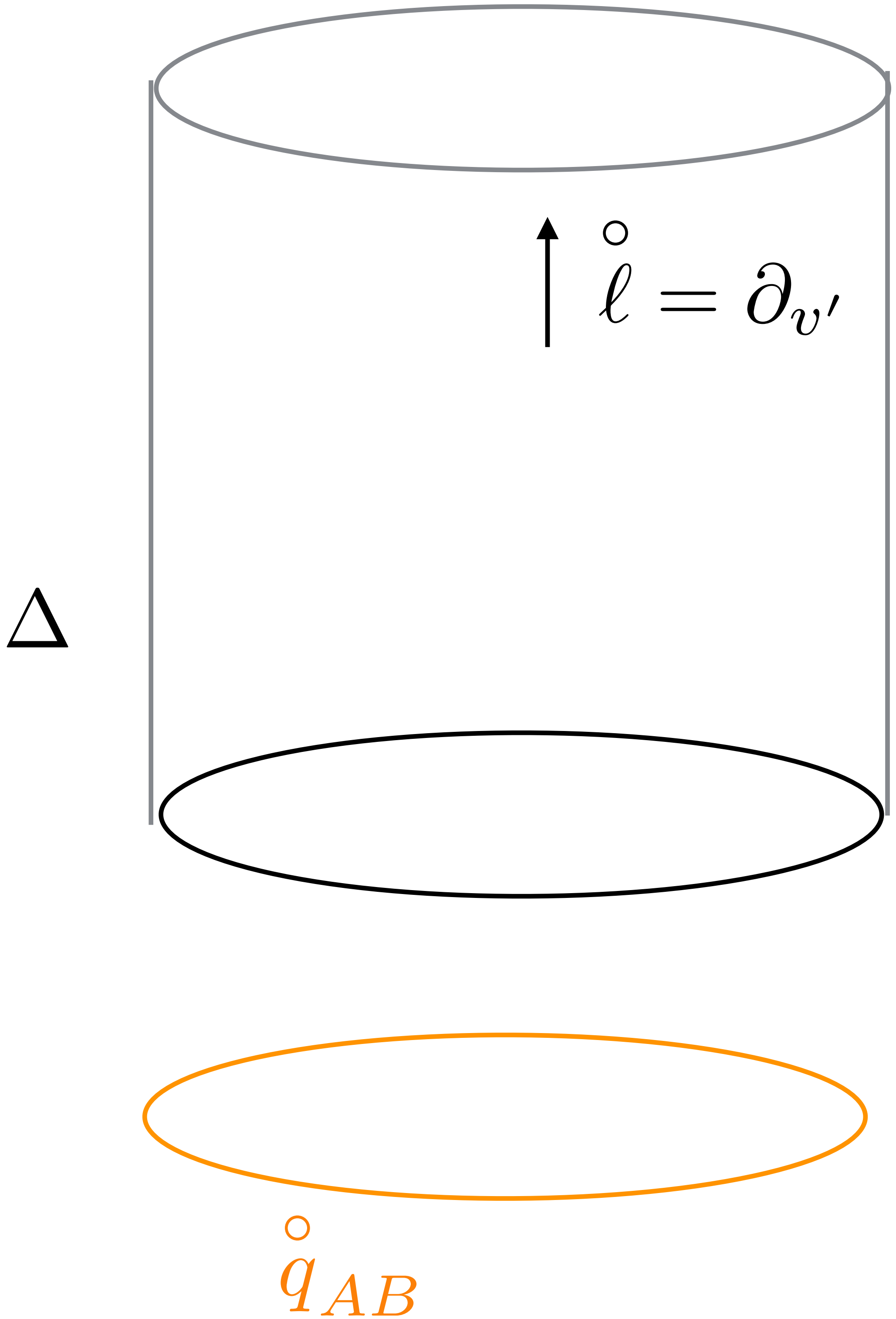
$$(\theta, \varphi, v' = v + f(\theta, \phi)) =: (x'^A, v') \qquad kv'\partial_{v'}$$



$$s\partial_{v'}$$

Ambiguities in the Lorenz transformations

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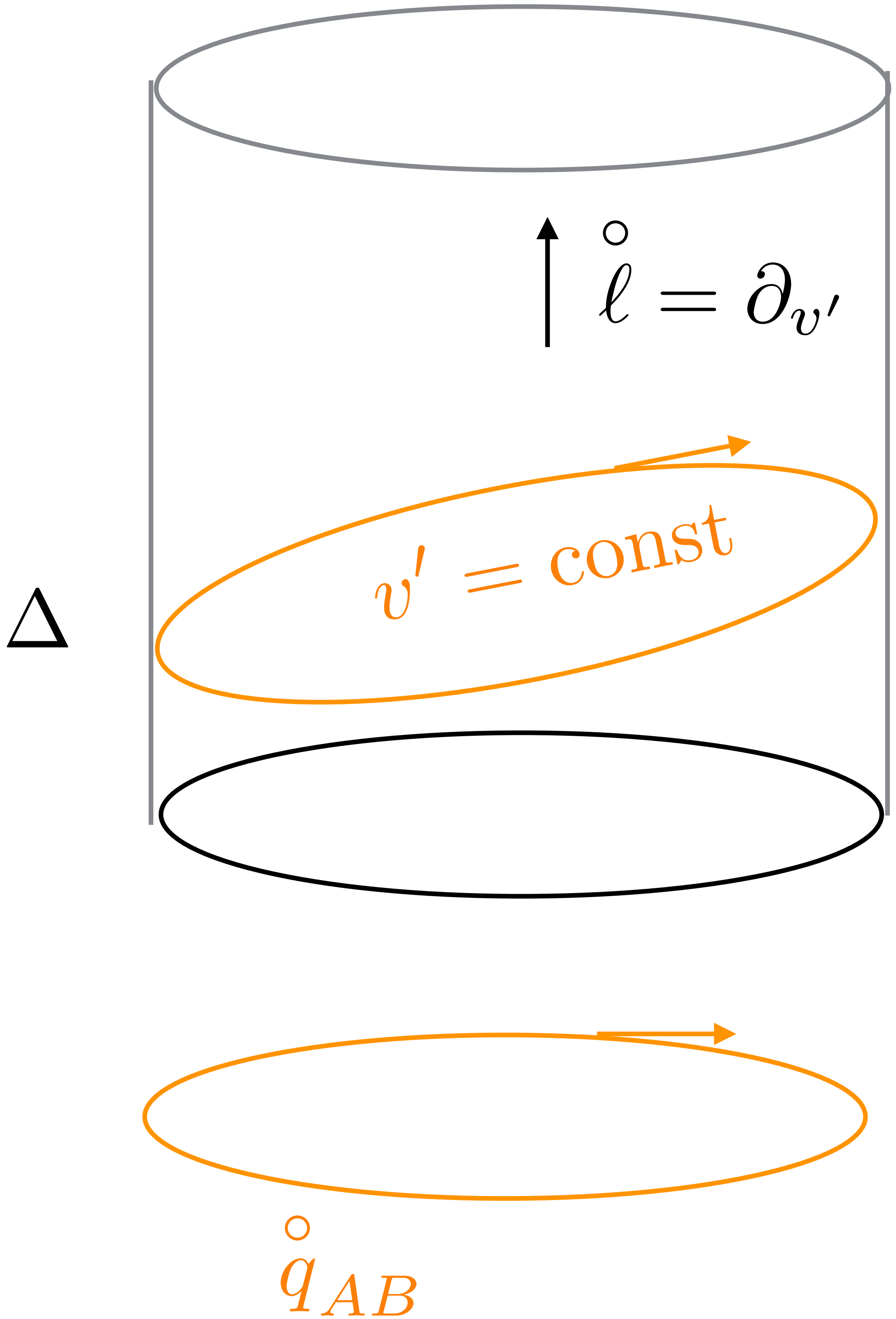
$$(\theta, \varphi, v' = v + f(\theta, \phi)) =: (x'^A, v') \qquad k v' \partial_{v'}$$



$$s\partial_{v'}$$

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Ambiguities in the Lorenz transformations

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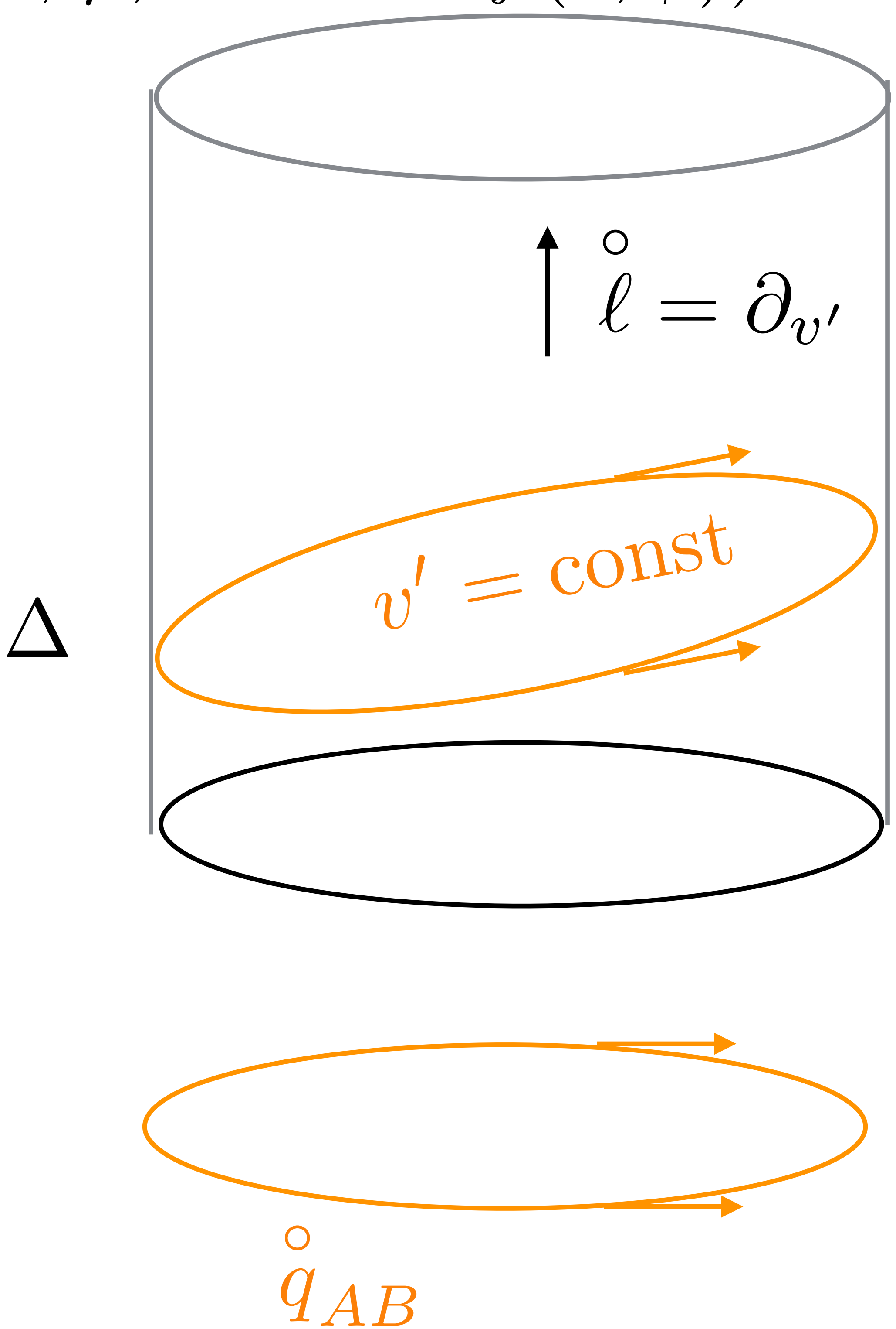

$$s\partial_{v'}$$

$$\mathring{\epsilon}^{AB}\chi_{,B}\partial'_A$$

$$\mathring{q}^{AB}\mathring{\phi}_{,B}\partial'_A + v'\phi\partial_{v'}$$

Comparison with BMS

$$\xi^a = d^a + S^a + R^a + B^a$$

dilation
additional generator
surface gravity

super translation

rotation

boost

BMS generators

Comparison with BMS

$$\xi^a = d^a + S^a + R^a + B^a$$

dilation

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BMS generators

Example: a horizon in the Kerr spacetime:

$$d = \partial_t + \Omega \partial_\phi$$

Comparison with BMS

$$\xi^a = d^a + S^a + R^a + B^a$$

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BMS generators

Example: a horizon in the Kerr spacetime:

$$d = \partial_t + \Omega \partial_\phi$$

That is why we accept that additional to BMS symmetry

Spacetime extension important for the charges

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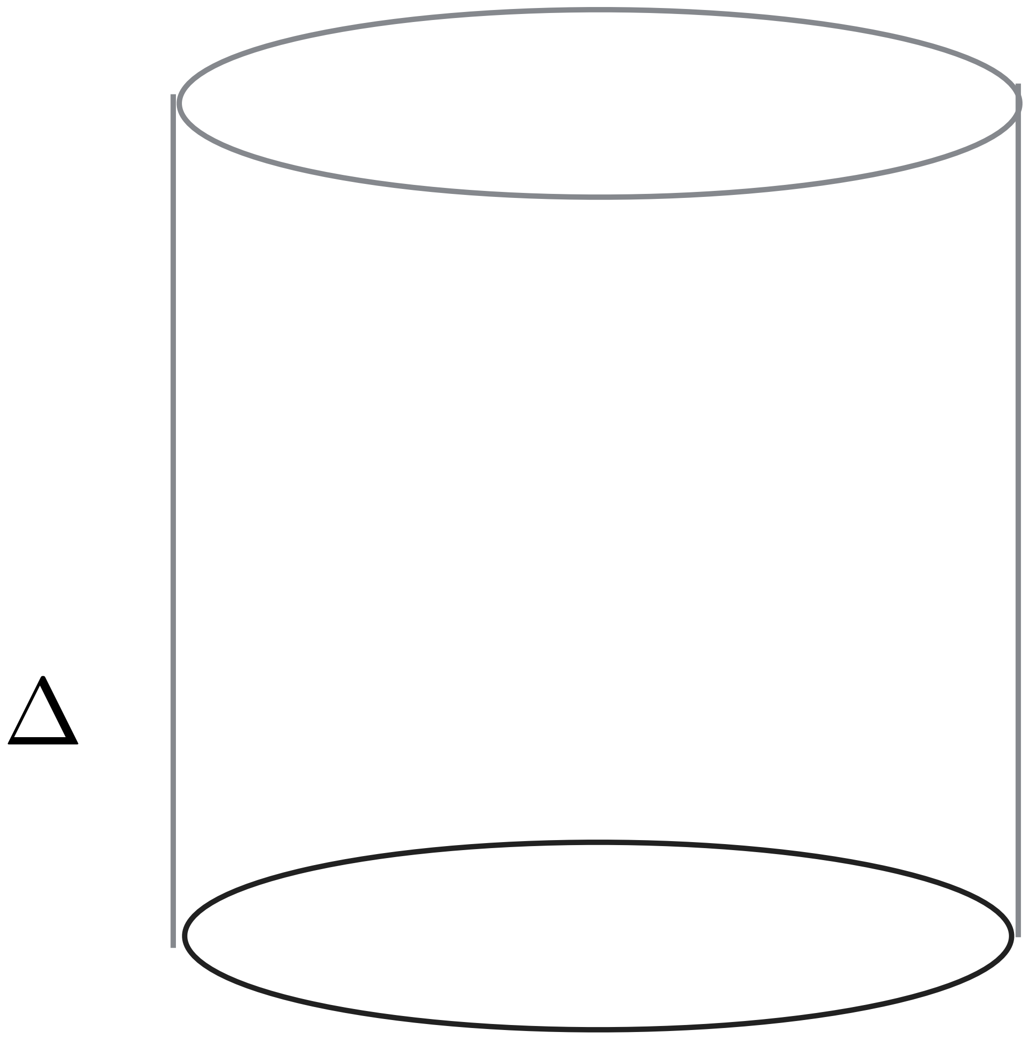
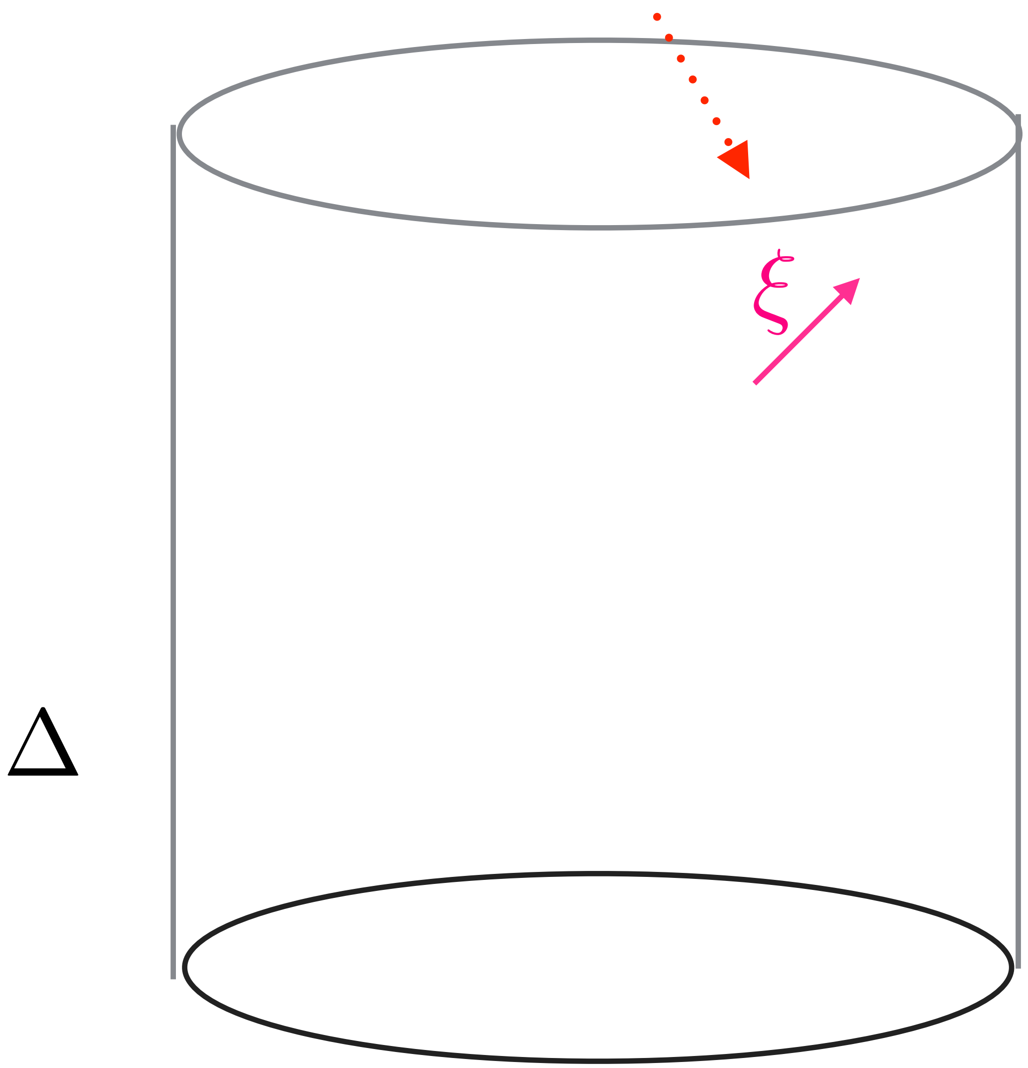
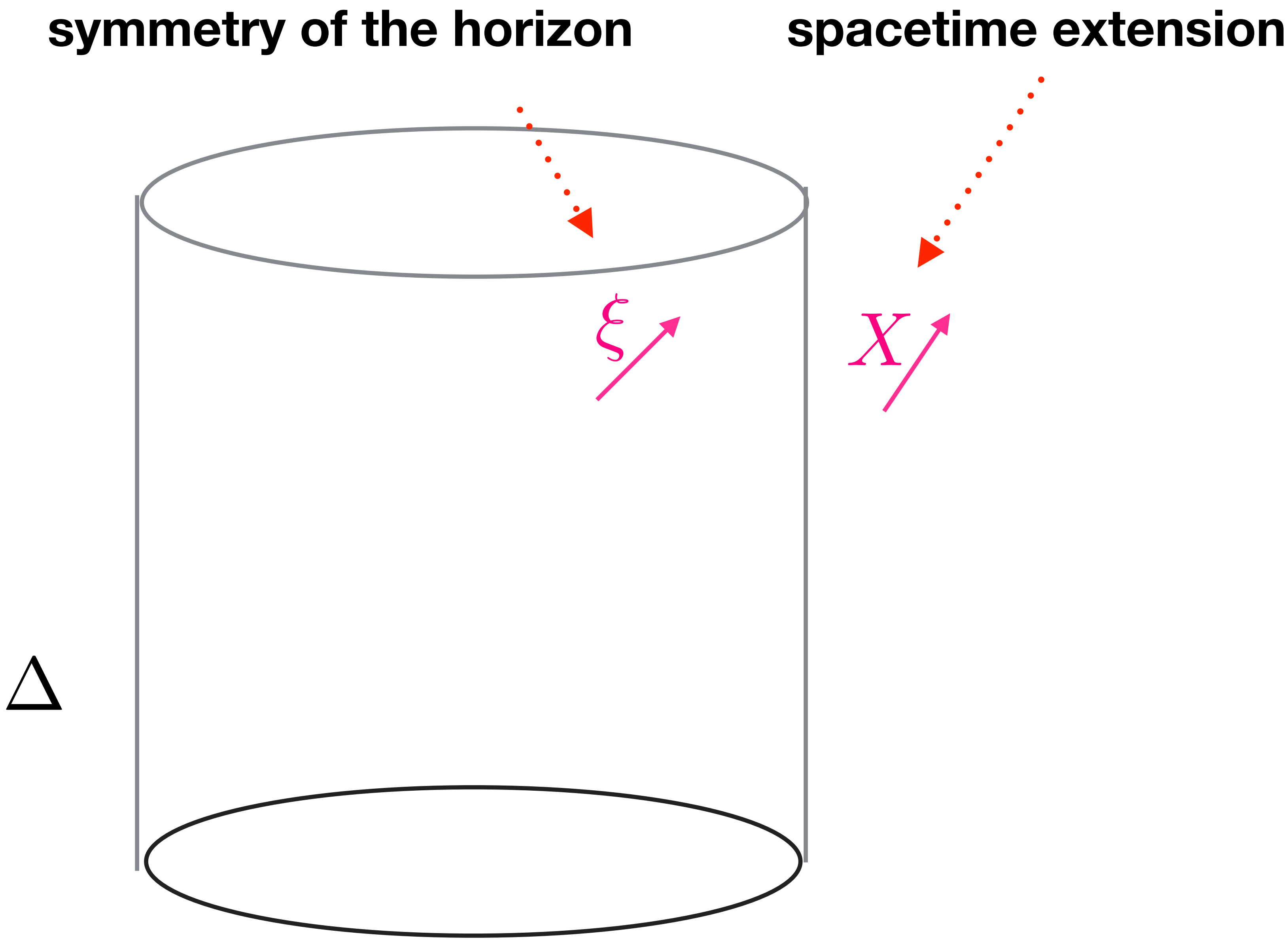


Spacetime extension important for the charges

symmetry of the horizon

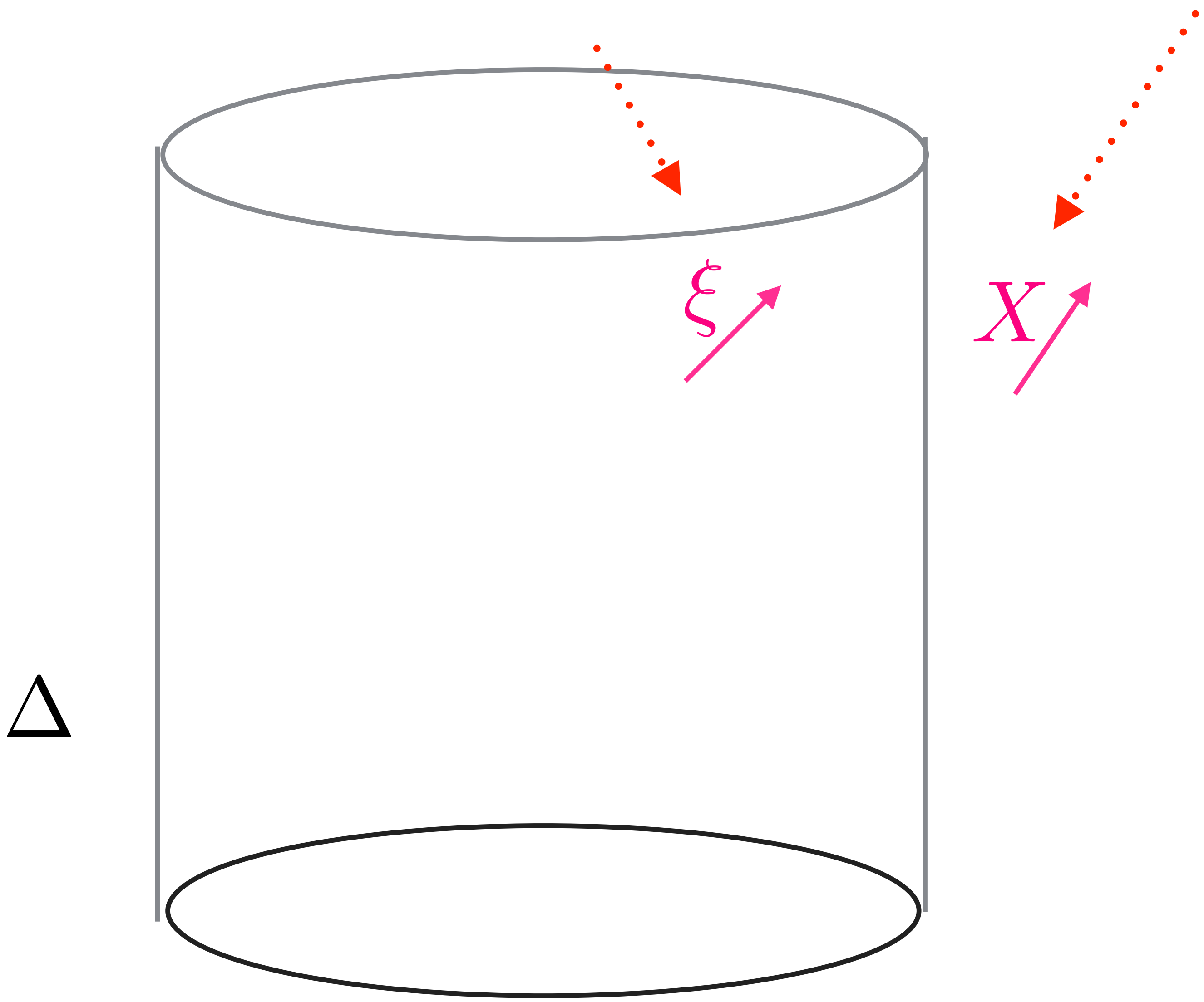


Spacetime extension important for the charges



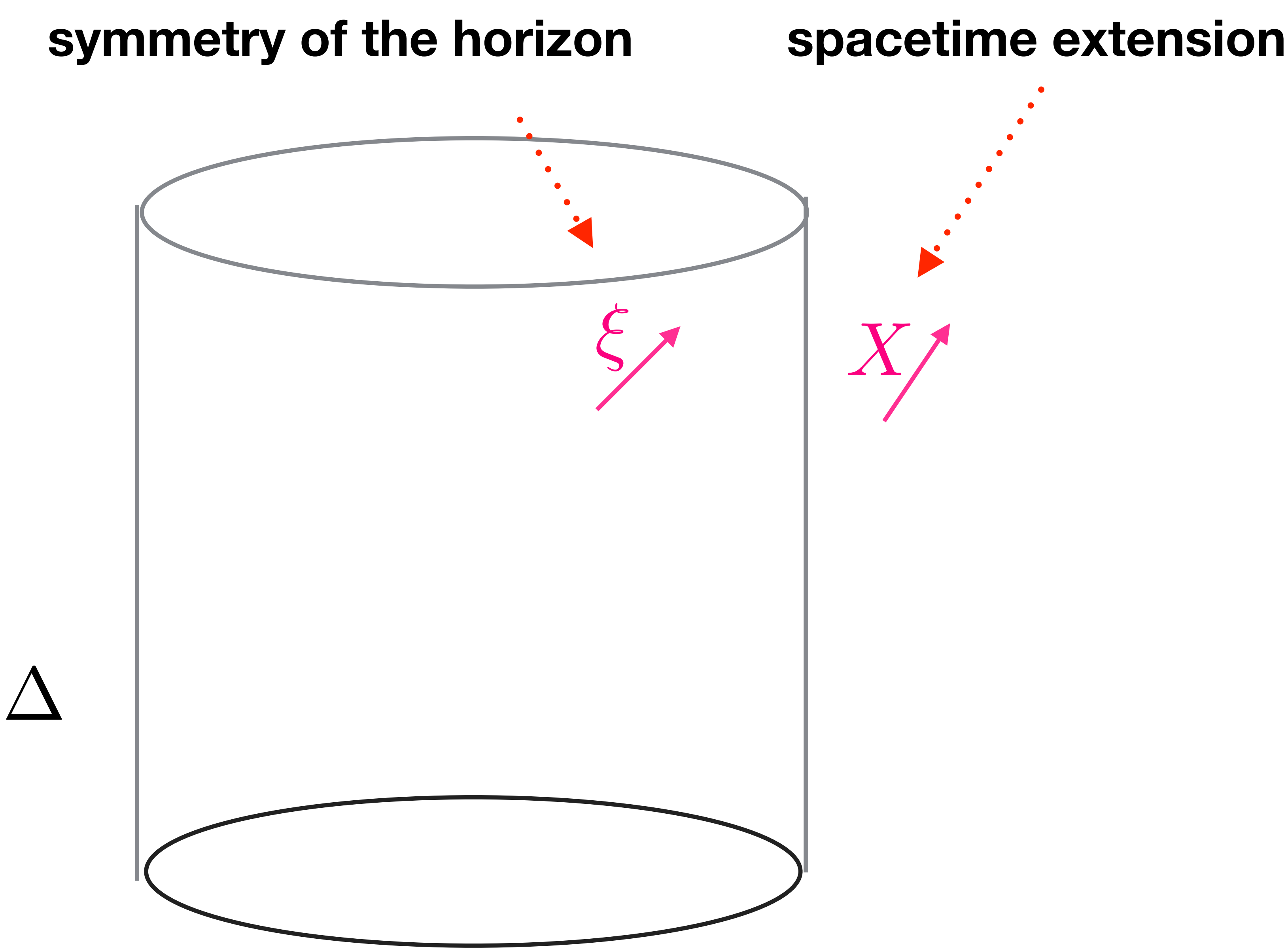
Spacetime extension important for the charges

symmetry of the horizon **spacetime extension**



$$X_{|\Delta} = \xi$$

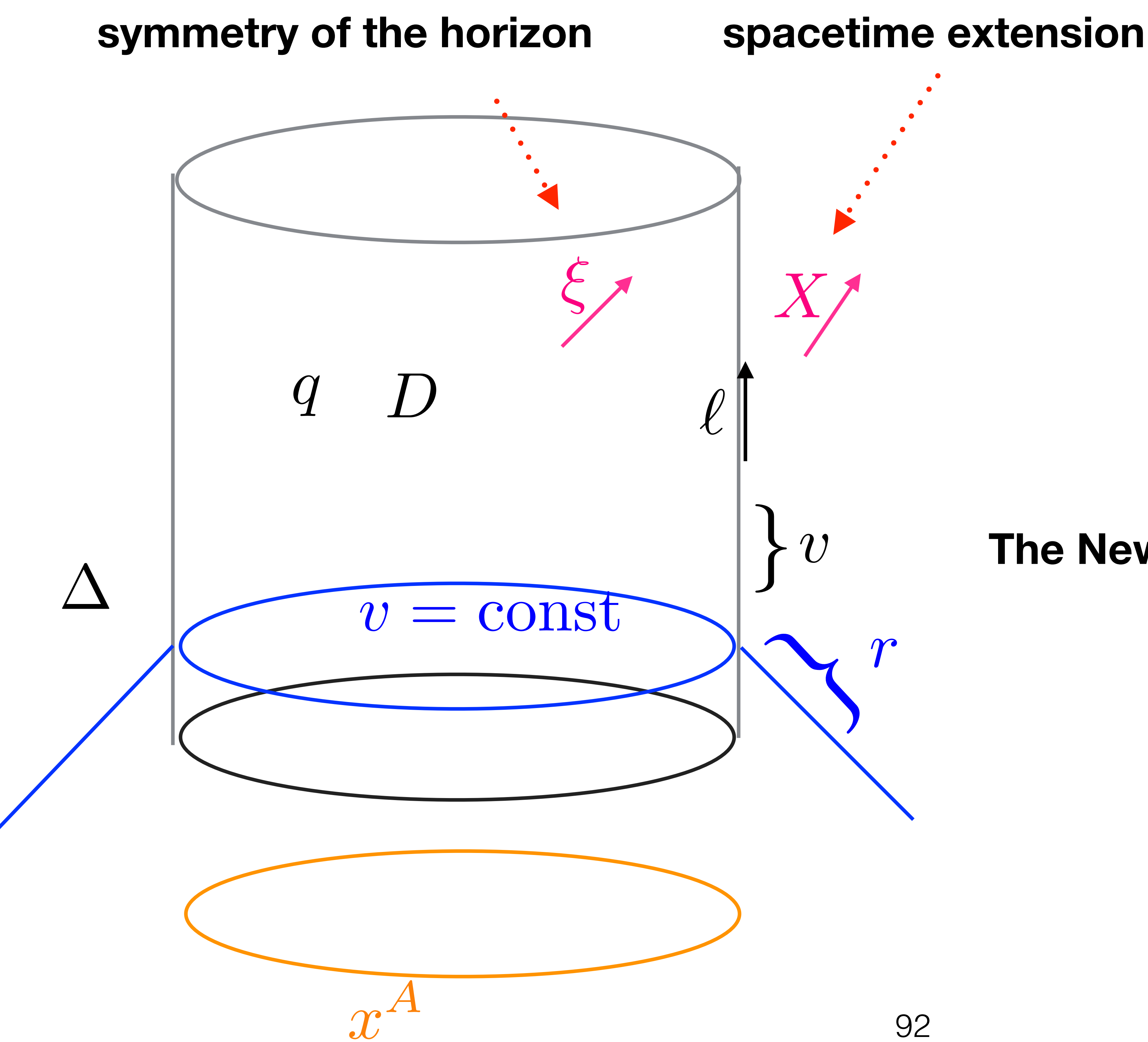
Spacetime extension important for the charges



$$X_{|\Delta} = \xi$$

$$(\mathcal{L}_X g_{\alpha a})\ell^a = 0$$

Spacetime extension important for the charges



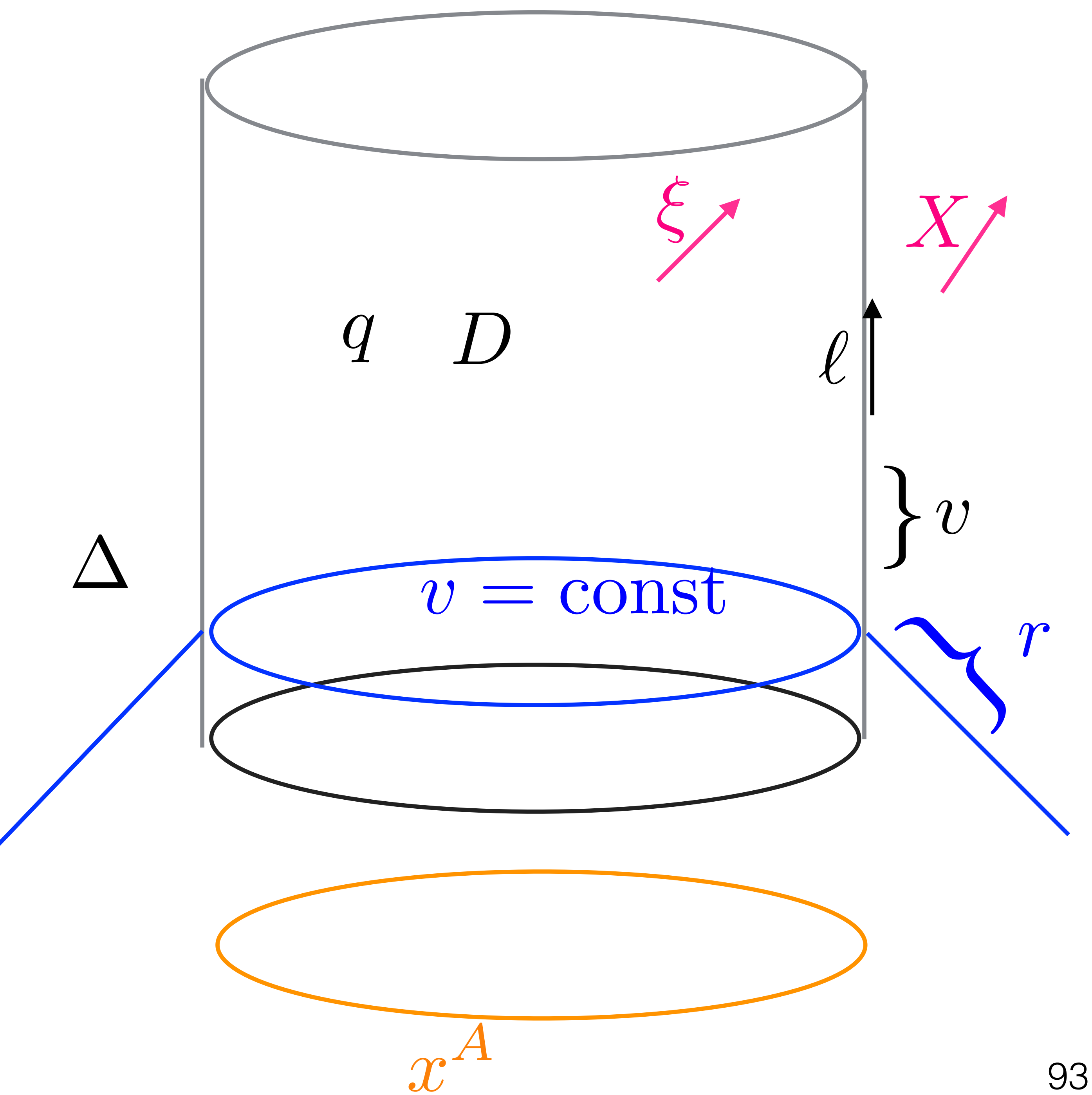
$$X_{|\Delta} = \xi$$

$$(\mathcal{L}_X g_{\alpha a})\ell^a = 0$$

The Newman-Unti coordinates

Spacetime extension important for the charges

$$X = (vf_1 + f_2)\partial_v + H^A\partial_A - rf_1\partial_r - r\tilde{X}^v\partial_v + r\tilde{X}^A\partial_A + r^2\tilde{\tilde{X}}^r\partial_r\,.$$



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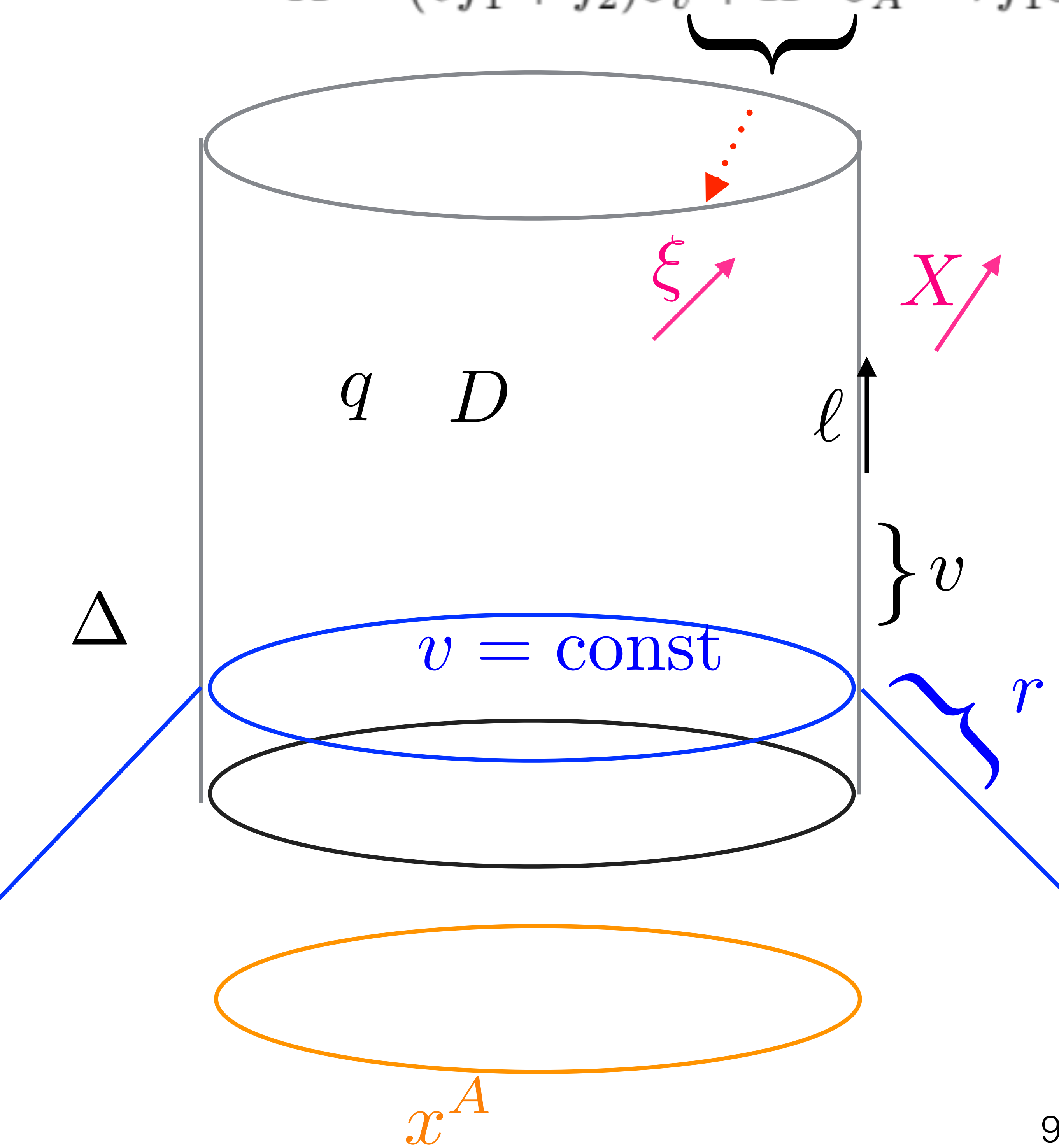
The Newman-Unti coordinates

Spacetime extension important for the charges

$$X = (vf_1 + f_2)\partial_v + H^A\partial_A - rf_1\partial_r - r\tilde{X}^v\partial_v + r\tilde{X}^A\partial_A + r^2\tilde{\tilde{X}}^r\partial_r \,.$$



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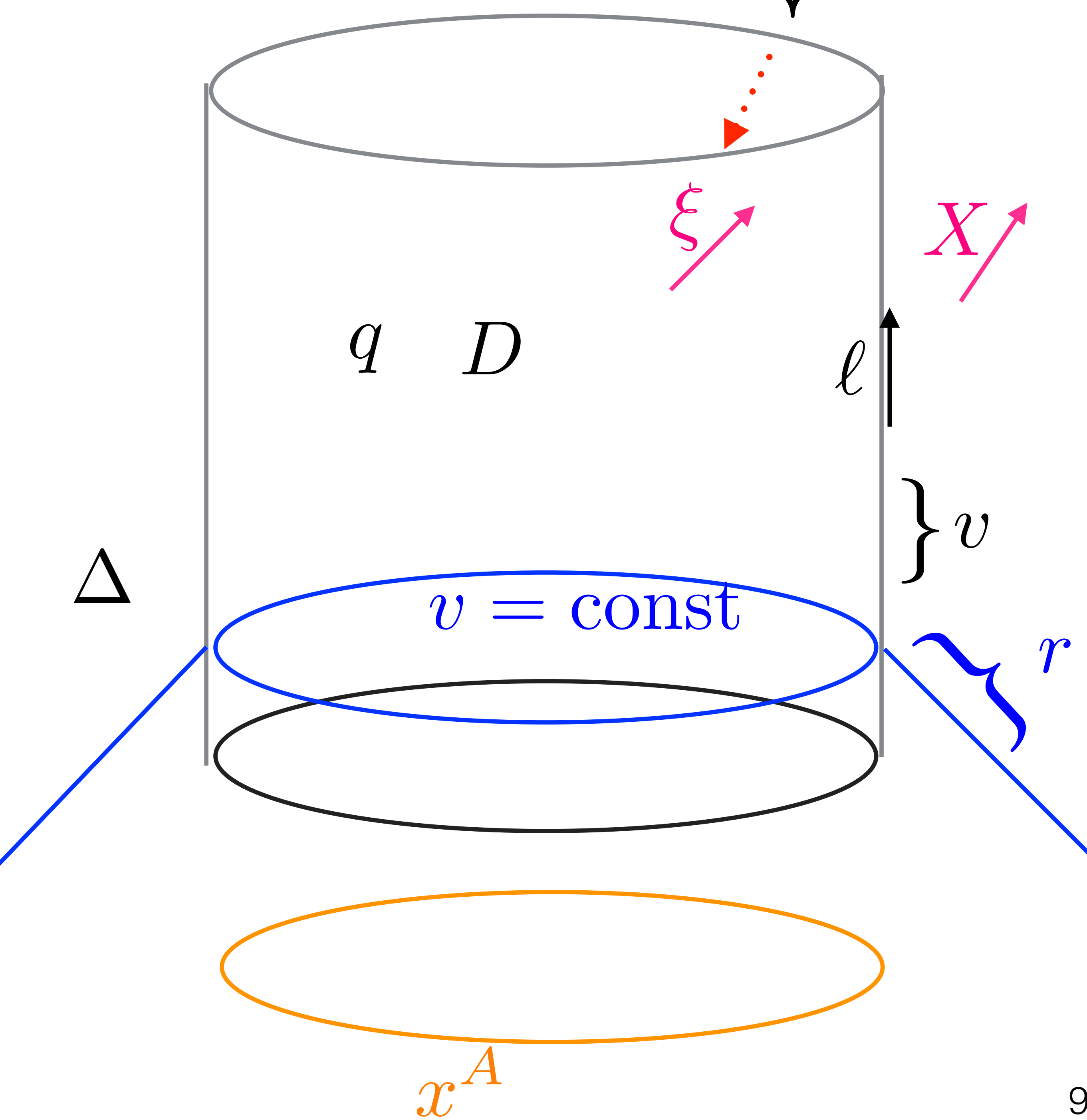

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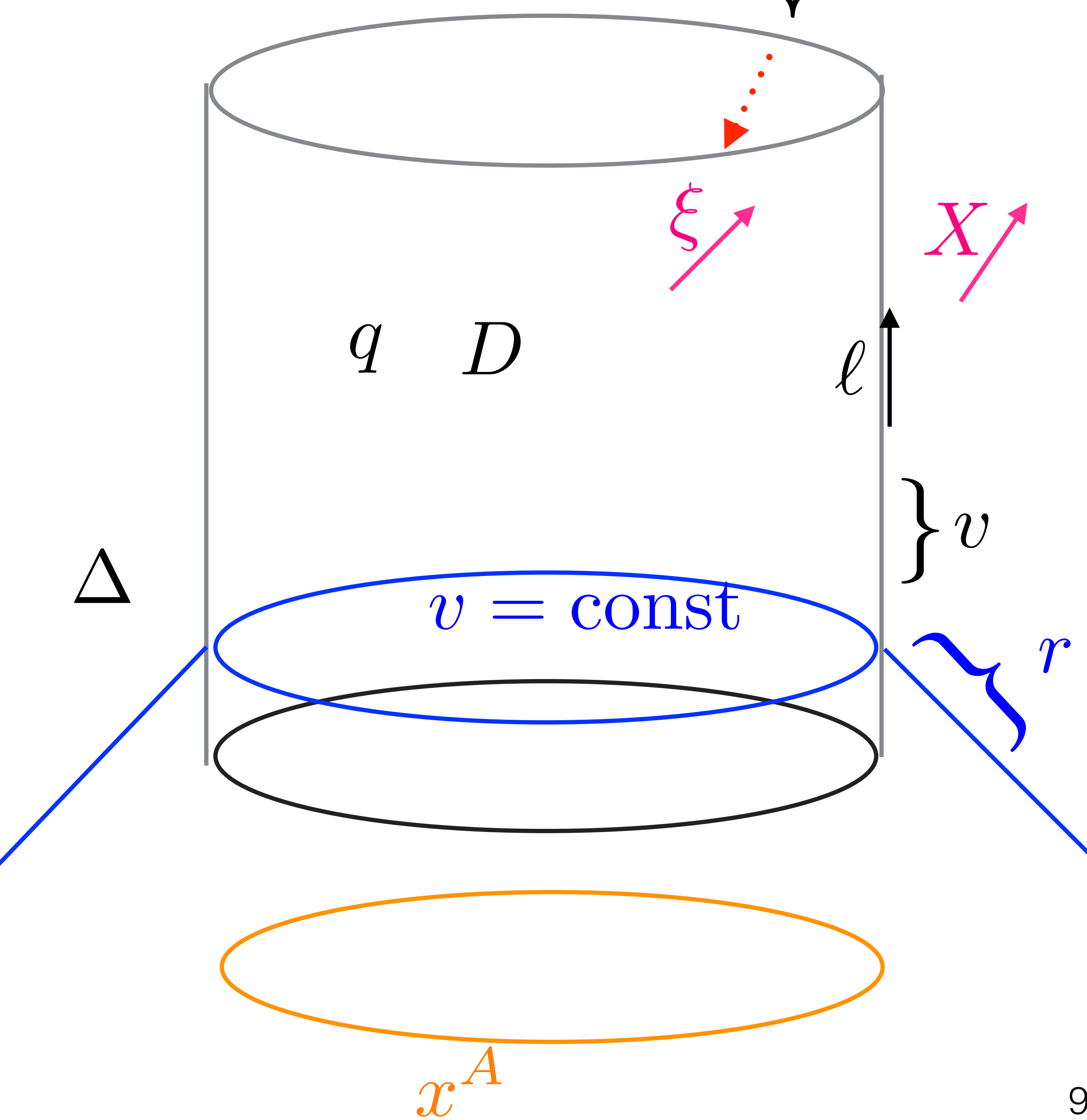

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The Newman-Unti coordinates

arbitrary

Settling down perturbed horizon

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consider 1-dimensional family of metric tensors:

Settling down perturbed horizon

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$$g_{ab}(\lambda) = {}^{\circ}g_{ab} + \lambda \frac{\mathrm{d}g_{ab}(\lambda)}{\mathrm{d}\lambda}|_{\lambda=0} + \frac{\lambda^2}{2} \frac{\mathrm{d}^2 g_{ab}(\lambda)}{\mathrm{d}\lambda^2}|_{\lambda=0} + \dots$$

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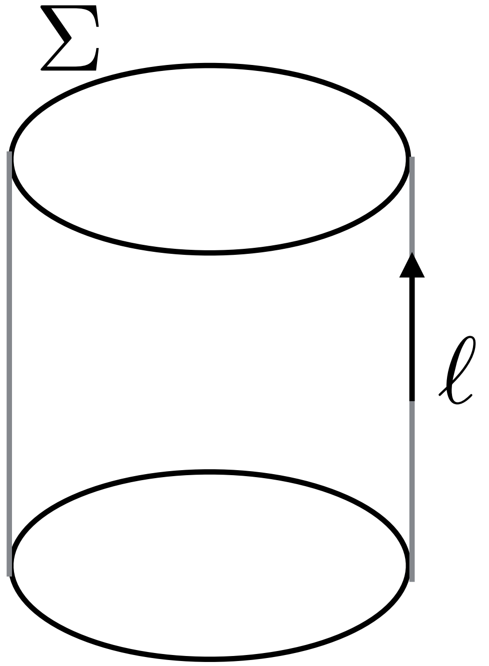

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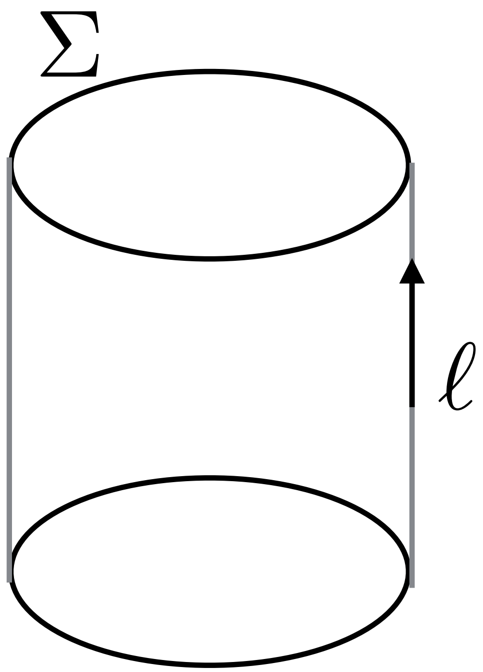

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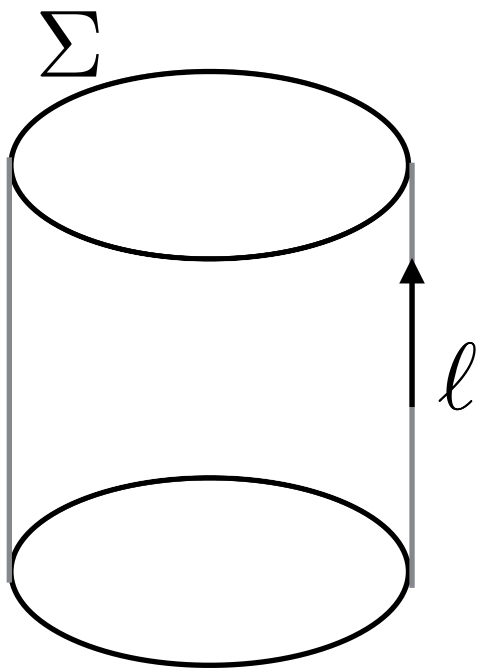
and a surface:



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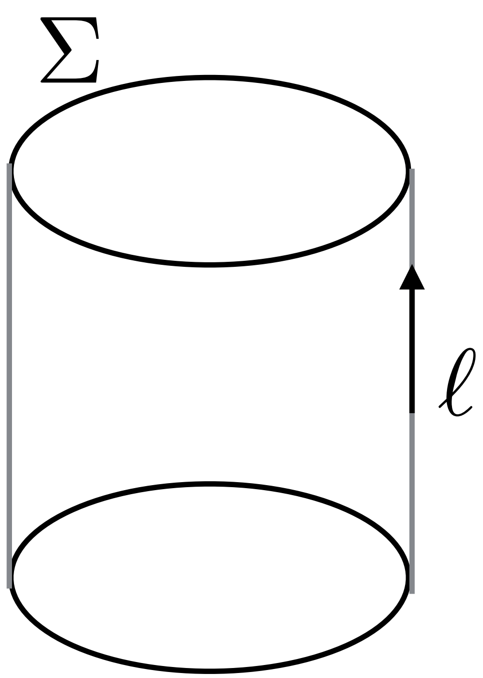

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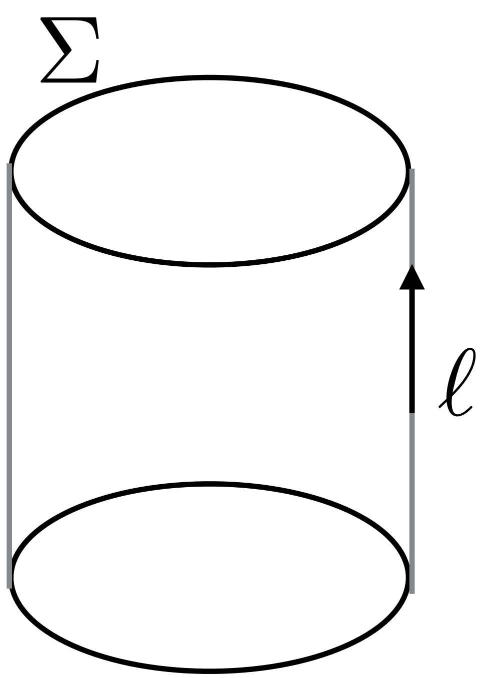

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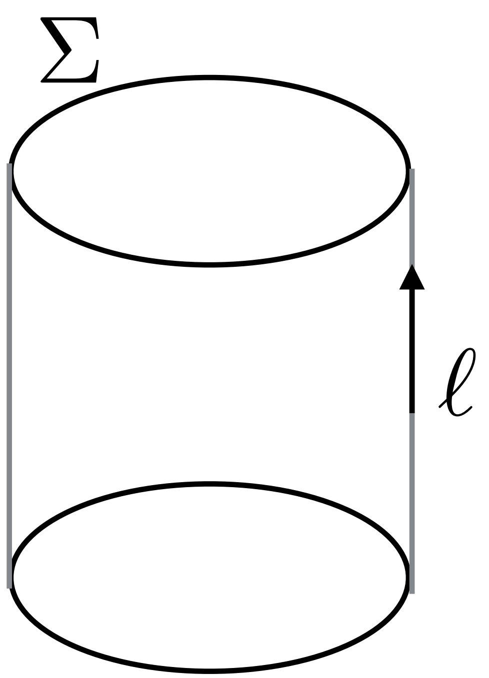

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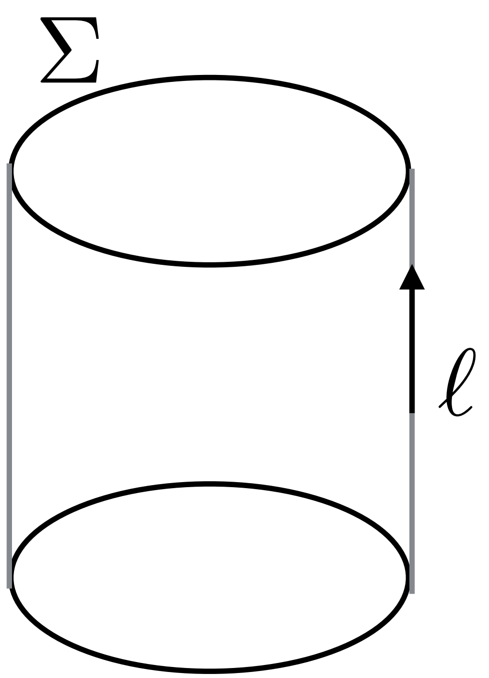

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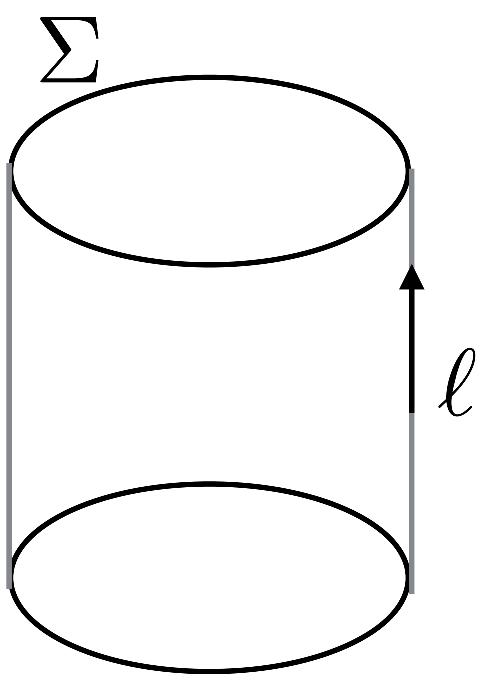

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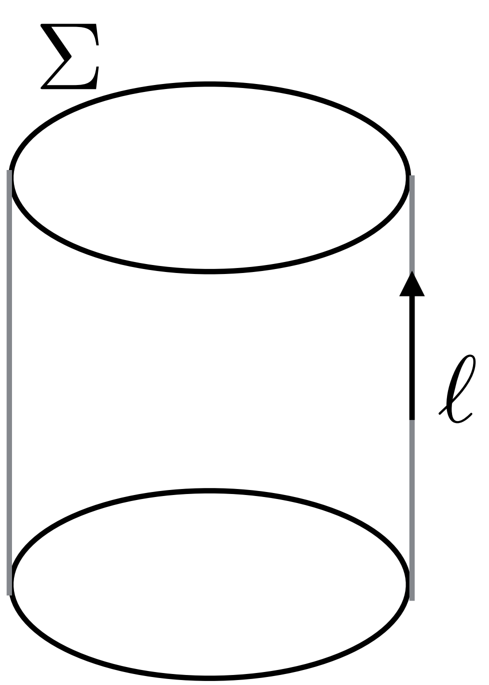

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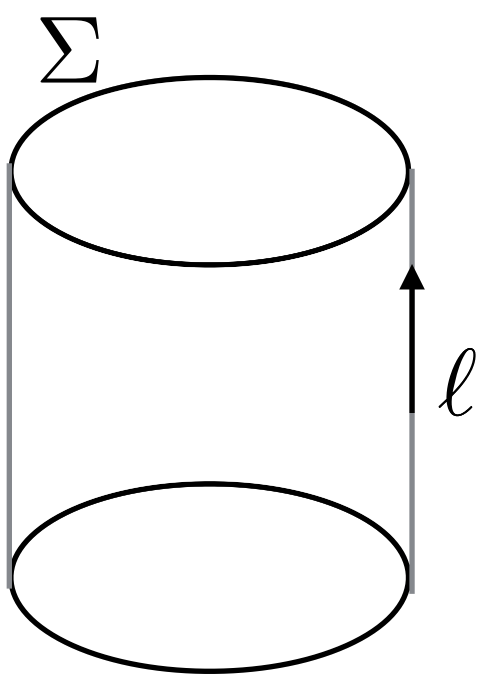

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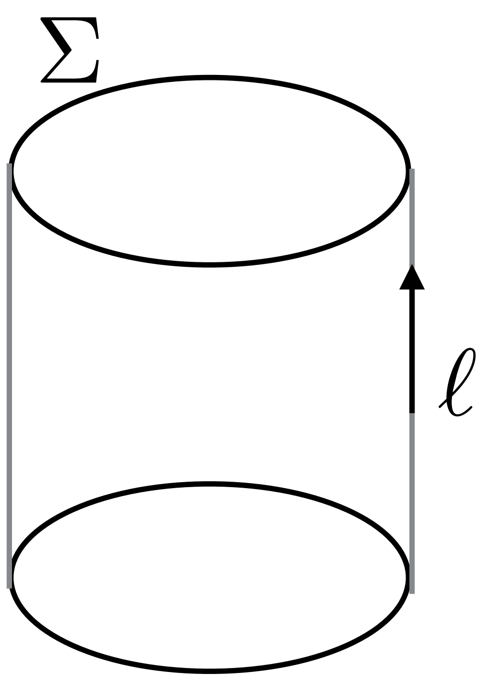

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We still use the symmetries of the background non-expanding horizon

Settling down perturbed horizon

$$g_{\mu\nu}(\lambda)dx^\mu dx^\nu = -r^2\,\gamma(\lambda)\,dv^2 + 2\,dvdr + 2r\,\beta_A(\lambda)\,dvdx^A + q_{AB}(\lambda)\,dx^A dx^B$$

$$D^A\beta_A(0) = 0 \qquad q^{AB}(0)\frac{d}{d\lambda}q_{AB,v}(0)_{|r=0} = 0 \qquad q_{AB,v}(0)_{|r=0} = 0$$

$$\xi = ((k+\phi(x))v + s(x))\,\partial_v + H^A(x)\partial_A,$$

$$X = \xi - r(k+\phi(x)\partial_r + r\tilde{X}^v\partial_v + r\tilde{X}^A\partial_A + r^2\tilde{\tilde{X}}^r\partial_r$$

Charges and fluxes - we use Wald-Zoupas and Chandrasekaran-Flanagan-Prambhu:

$$Q_\xi[C](\lambda) = \frac{1}{8\pi G}\oint_C\left(k+\phi\;-\;\frac{1}{2}\beta_A\,(\lambda)\;H^A -\;\theta\,(\lambda)\;((k+\phi)v+s)\right)\sqrt{\det q(\lambda)}\;dx^1\wedge dx^2$$

$$\mathcal{F}_\xi[\Sigma_{1,2}](\lambda) \;=\; \frac{1}{2}\lambda^2\frac{1}{16\pi G}\int_{H_{1,2}}\left(\mathcal{L}_\xi q'_{AB}\partial_v q'^{AB} + \phi\partial_v q"{}_A{}^A\right)\sqrt{\det q}dv\wedge\;dx^1\wedge dx^2 + O(\lambda^3)$$

$$\mathcal{F}[\Sigma_{1,2}](0) = \frac{d}{d\lambda}\mathcal{F}[\Sigma_{1,2}](0) = 0$$

$$\mathcal{F}_d[H_{1,2}](\lambda) = \frac{1}{2}\lambda^2\frac{1}{16\pi G}\int_{H_{1,2}} kv\partial_v q'_{AB}\partial_v q'^{AB}\sqrt{\det q}dv\wedge\;dx^1\wedge dx^2 + O(\lambda^3)$$

Summary

Fixing in a suitable way the gauge depending part of the 2-metric tensor and the rotation 1-form, respectively endowes NHE with a structure similar to that of the scri of asymptotically flat spacetime.

The symmetry group containes the BMS group plus one more generator: dilation.

Natural completeness and consistency conditions determine the extension to a neighborhood of the horizon.

Our framework is compatible with that of Wald-Zoupas-Chandrasekaran-Flanagan-Prambhu hence we can apply their charges and fluxes. One can also apply other charges, for exampe those of Barnich, Donnelly, Freidel, Spezialle

Thank you