Higher spin symmetry in gravity from asymptotic Einstein's equations

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Based on work in collaboration with L. Freidel, R. Oliveri, A-M. Raclariu, S. Speziale

- The Weyl BMS group and Einstein's equations, JHEP 07, 170 (2021), [hep-th/2104.05793];
- Gravity from symmetry: Duality and impulsive waves, JHEP 04, 125 (2022), [hep-th/2109.06342];
- Sub-subleading Soft Graviton Theorem from Asymptotic Einstein's Equations, JHEP 05, 186 (2022), [hep-th/2111.15607];
- Higher spin dynamics in gravity and $w_{1+\infty}$ celestial symmetries, [hep-th/2112.15573]

In our quest for quantum gravity, an essential task is to reach a proper understanding of the degrees of freedom and of the symmetries of gravity associated with local subregions

- In the bulk: spacetime diffeomorphisms = gauge symmetry

  -> Gauge redundancies with a vanishing charge which cannot be used to label physical states of QG

- On the boundary: a subset of transformations become physical symmetries

  -> Non-vanishing charges located on codimension-2 spheres (or corners S) with a non-trivial algebra

⭐ Goal: New description of quantum geometry by understanding the nature of the universal corner symmetry algebra $\mathfrak{g}_S \supset \mathfrak{su}(2)$ of any subregion of space

☞ Organizing principle for understanding quantum gravity [Laurent's talk]

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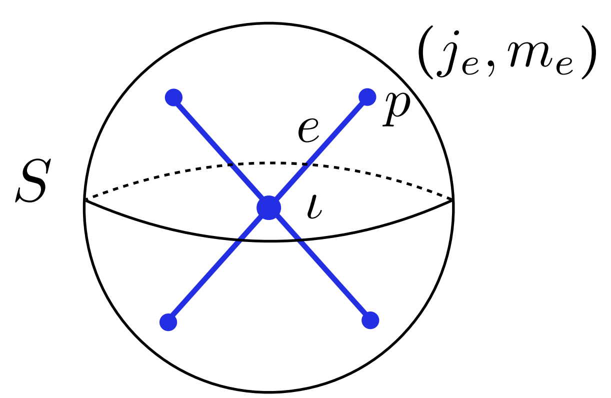
Loop quantum gravity:

☑ Local Hilbert space where the states of quantum geometry are labelled by boundary flux charges (SU(2) spin quantum numbers at the punctures) [Rovelli, Smolin 1995]; [Ashtekar, Lewandowski 1997]



$$\Sigma_i[S] = \int_S \epsilon_{abc} E_i^c \mathrm{d}x^a \wedge \mathrm{d}x^b$$

Discreteness of the area spectrum is the landmark of canonical LQG

- Decompose the bulk of spacetime into a collection of subregions and attach a symmetry algebra to the corner of each subregion;
- The corner Hilbert space forms an irreducible representation of the local corner symmetry algebra, and choices of states in this corner Hilbert space then encode quantum geometries;

Space = network of "bubbles" [Freidel, Livine 2019]

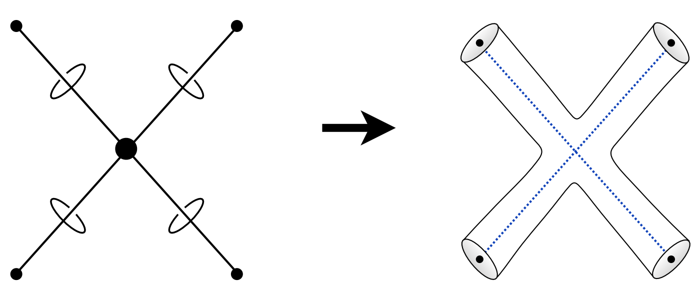
Corner symmetry charges = Coarse-grained information of geometrical DOF inside each region it encloses

Reconciliation between discrete spectrum for the area operator (derived from the continuum theory) and manifest Lorentz invariance [Freidel, Geiller, DP 2020] (see also [Wieland 2017])

Corner symmetry group: $G_S = \big(\mathrm{Diff}(S) \ltimes H^S\big) \ltimes (\mathbb{R}^2)^S$ with $H = \mathrm{SL}(2,\mathbb{R})_\perp \times \mathrm{SL}(2,\mathbb{R})_{||} \times \mathrm{SL}(2,\mathbb{C})$

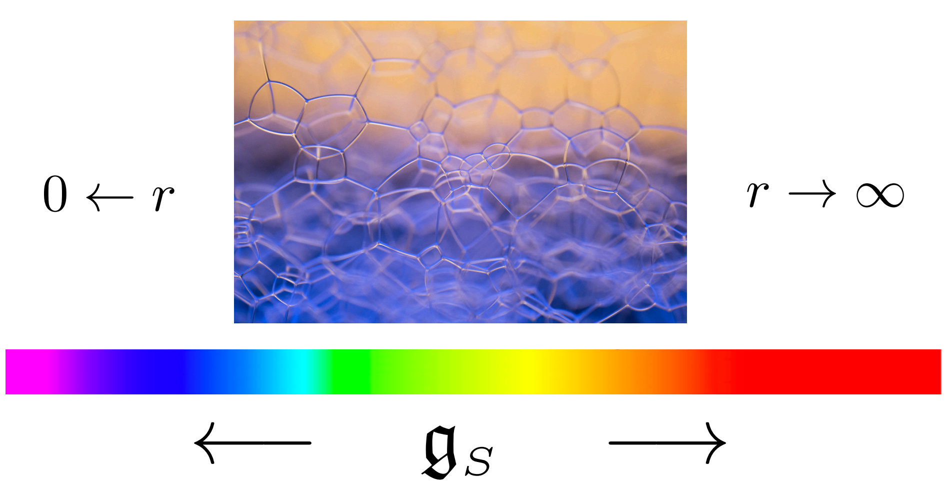
[Ciambelli, Donnelly, Freidel, Geiller, Leigh, Livine, Moosavian, Oliveri, Perez, DP, Speranza, Speziale]

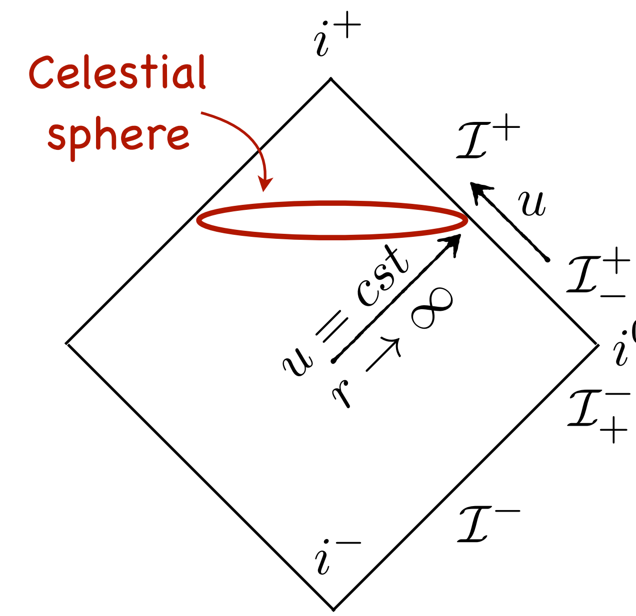
3D Poincaré networks → Kac-Moody modes



LQGs: SU(2) -> SL(2,C) -> Poincaré

[Freidel, Livine, DP 2019]





BMS group

Outline

1. Gravity from symmetry
    - Asymptotic Einstein equations as intertwiners
2. Soft graviton theorems
    - Conservation laws for charges of asymptotic symmetries
3. Higher spin dynamics
    - Canonical representation of $w_{1+\infty}$ loop algebra

Part 1: Gravity from symmetry

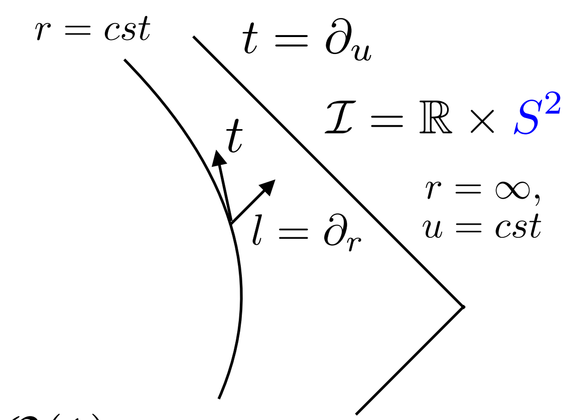
Weyl BMS group

Bondi-Sachs coordinates $x^\mu = (u, r, \sigma^A)$:

$$\mathrm{d}s^2 = -2e^{2\beta}\mathrm{d}u(\mathrm{d}r + \Phi \mathrm{d}u) + r^2\gamma_{AB}\left(\mathrm{d}\sigma^A - \frac{\Upsilon^A}{r^2}\mathrm{d}u\right)\left(\mathrm{d}\sigma^B - \frac{\Upsilon^B}{r^2}\mathrm{d}u\right)$$

The Bondi gauge conditions:

$$g_{rr} = 0\,, \quad g_{rA} = 0\,, \quad \partial_r\sqrt{\gamma} = 0 \qquad \text{(i)}$$



BMSW boundary conditions:

$$g_{ur} = -1 + \mathcal{O}(r^{-2}), \quad g_{uA} = \mathcal{O}(1), \quad g_{uu} = \mathcal{O}(1), \qquad q_{AB} = \mathcal{O}(1) \quad \text{(ii)}$$

Original BMS boundary conditions: $\quad g_{uu} = -1 + \mathcal{O}(r^{-1}), \quad q_{AB} = \mathring{q}_{AB} + \mathcal{O}(r^{-1})$

[Bondi, van der Burg, Metzner, Sachs 1962]

Metric coefficients:

$$\Phi = \frac{F}{4} - \frac{M}{r} + \mathcal{O}\left(r^{-2}\right)\,, \qquad \beta = \frac{b}{r^2} + \mathcal{O}\left(r^{-3}\right)\,,$$

$$\Upsilon^A = U^A - \frac{1}{r}\left(\frac{2}{3}\mathcal{P}^A + C^{AB}U_B + 2\partial^A b\right) + \mathcal{O}\left(r^{-2}\right)\,,$$

$$\gamma_{AB} = q_{AB} + \frac{1}{r}C_{AB} + \frac{1}{4r^2}q_{AB}\left(C_{CD}C^{CD}\right) + \frac{1}{r^3}\left(\frac{1}{3}\mathcal{T}_{AB} + \frac{1}{16}C_{AB}(C_{CD}C^{CD})\right) + \mathcal{O}\left(r^{-4}\right)$$

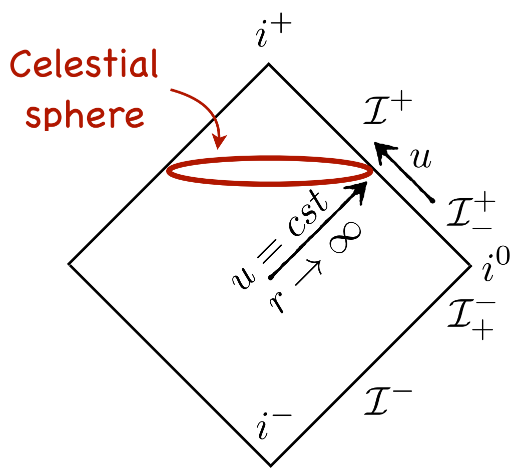
$$M, \mathcal{P}_A, C_{AB}, \mathcal{T}_{AB} \to 0\,, F \to \frac{1}{2} \quad \Rightarrow \quad \text{Minkowski spacetime}$$

Building on [Ashtekar, Streubel 1981], [Barnich, Troessaert 2010], [Campiglia, Laddha 2014]

- BMSW group = group of residual diffeos preserving the Bondi gauge-fixing (i) and the boundary conditions (ii):

2-sphere Conformal class and conformal scale allowed to vary [Freidel, Oliveri, DP, Speziale 2021]

(See [Geiller, Zwikel 2022] for further relaxation)



$$\xi_{(T,W,Y)} := Y^A(\sigma)\partial_A + W(\sigma)(u\partial_u - r\partial_r) + T(\sigma)\partial_u + \mathcal{O}(r^{-1})$$

$$\underbrace{\boxed{\mathrm{BMSW} = \left(\mathrm{Diff}(S) \ltimes \mathbb{R}^S_W\right) \ltimes \mathbb{R}^S_T \ \subset G_S}}_{\text{Kinematical subgroup}}$$

Preserving the null generator of $\mathcal{I}$

Transformation of the metric functionals: $\mathcal{L}_{\xi_{(T,Y,W)}} g_{\mu\nu}[\Phi^i] = \partial_\epsilon g_{\mu\nu}[\Phi^i + \epsilon\delta_{(T,Y,W)}\Phi^i]\big|_{\epsilon=0}$

$O_{(\Delta,s)} =$ Covariant observable for H_S if: $\delta_{(Y,W)} O_{(\Delta,s)} = (\mathcal{L}_Y + (\Delta - s)W) O_{(\Delta,s)}$

$\Delta =$ Conformal dimension, $s =$ Spin $\quad O_{(\Delta,s)} := O_{\Delta\langle A_1\cdots A_s\rangle}, \quad O_{(\Delta,-s)} := O_\Delta^{\langle A_1\cdots A_s\rangle}, \quad s \geq 0$

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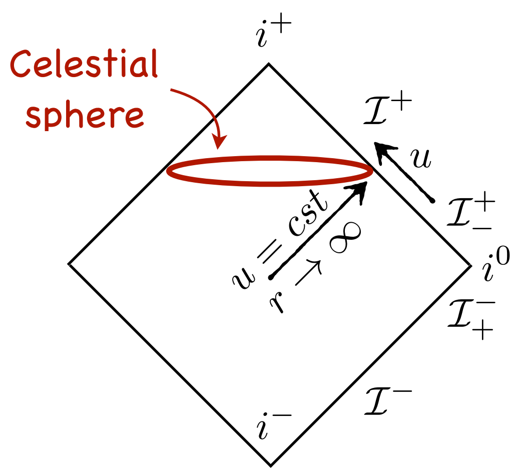

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Kinematical subgroup

Preserving the null generator of $\mathcal{I}$

Covariant observable	$\hat{N}_{AB}$	$\mathcal{N}^{AB}$	$\mathcal{J}^A$	$\mathcal{M}$	$\tilde{\mathcal{M}}$	$\mathcal{P}_A$	$\mathcal{T}_{AB}$
Dimension-Spin (Δ, s)	(2,2)	(3,-2)	(3,-1)	(3,0)	(3,0)	(3,1)	(3,2)

$$N_{AB} := \dot{C}_{AB}$$

Bondi news tensor

$$\hat{N}^{AB} := N^{AB} - \tau^{AB}$$

Shifted news Geroch tensor

$$\mathcal{J}^A := \tfrac{1}{2} D_B \hat{N}^{AB}$$

Covariant current

$$\mathcal{M} := M + \tfrac{1}{8} N^{AB} C_{AB}$$

Covariant mass

$$\tilde{\mathcal{M}} := \tfrac{1}{4}\epsilon^A{}_C \left(D_A D_B C^{CB} + \tfrac{1}{2} N^{CB} C_{AB}\right)$$

Dual covariant mass [Godazgar, Pope 2019]

Corner charges = Covariant observables for the kinematical subgroup

- Gravity from symmetry [Freidel, DP 2021]

Asymptotic Einstein equations can be reconstructed by identifying the combinations of covariant observables of the homogeneous subgroup and their derivatives that transform homogeneously under arbitrary BMSW

Holomorphic frame: $m = m^A \partial_A$ with normalization $m^A \bar{m}_A = 1$

Sphere metric: $q_{AB} = (m_A \bar{m}_B + m_B \bar{m}_A)\,,$ sphere volume form: $\epsilon_{AB} = -i(m_A \bar{m}_B - m_B \bar{m}_A)$

By assigning helicity +1 to m_A and -1 to $\bar{m}_A\,,$

we can convert spin-s tensors into scalars of a given helicity:

$$O_s = O_{A_1 \cdots A_s} m^{A_1} \cdots m^{A_s}, \qquad O_{-s} = O^{A_1 \cdots A_s} \bar{m}_{A_1} \cdots \bar{m}_{A_s}$$

with $O_{-s} = \bar{O}_s$ and

$$D = m^A D_A\,, \qquad \bar{D} = \bar{m}_A D^A$$

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Bondi mass loss formula

Asymptotic EEs* :

$$\boxed{\begin{aligned} \dot{\mathcal{J}} &= \tfrac{1}{2} D\mathcal{N}\,, \\ \dot{\mathcal{M}}_{\mathbb{C}} &= D\mathcal{J} + \tfrac{1}{4} C\mathcal{N}\,, \\ \dot{\mathcal{P}} &= D\mathcal{M}_{\mathbb{C}} + C\mathcal{J}\,, \\ \dot{\mathcal{T}} &= D\mathcal{P} + \tfrac{3}{2} C\mathcal{M}_{\mathbb{C}} \end{aligned}}$$

$\longleftrightarrow$ Leading soft th. $\mathcal{M}_{\mathbb{C}} = \mathcal{M} + i\tilde{\mathcal{M}}$

$\longleftrightarrow$ Subleading soft th.

$\longleftrightarrow$ Sub-subleading soft th.

* Equivalent to the Bianchi identities in the NP formalism [Newman, Penrose 1962]

Evolution Eq.s = Covariant observables for the full BMSW

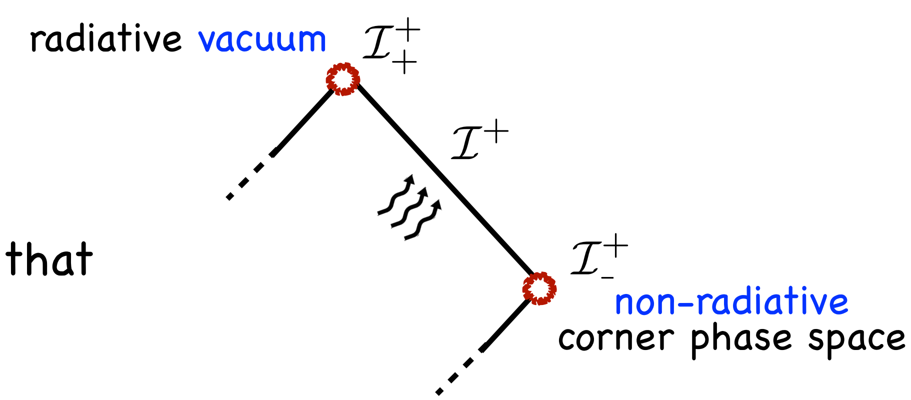
$\boxed{\text{Part 2: Soft graviton theorems}}$

Asymptotic conditions and renormalization

$$\mathcal{Q}_{-2} := \frac{\mathcal{N}}{2}, \quad \mathcal{Q}_{-1} := \mathcal{J}, \quad \mathcal{Q}_0 := \mathcal{M}_{\mathbb{C}}, \quad \mathcal{Q}_1 := \mathcal{P}, \quad \mathcal{Q}_2 := \mathcal{T}$$

EEs: $$\dot{\mathcal{Q}}_s = D\mathcal{Q}_{s-1} + \frac{(1+s)}{2} C\mathcal{Q}_{s-2}\,, \quad s = -1, 0, 1, 2$$

$$\hat{N} = \mathcal{O}(|u|^{-\alpha})\,, \quad \alpha > 3 \qquad \to \qquad \lim_{u\to+\infty} \mathcal{Q}_s(u,z) = 0$$



In order to integrate the recursion relation, we need to assume that

This allows us to define the charge aspects as integrals over their flux:

$$\mathcal{Q}_s(u,z) = \int_{+\infty}^{u} \mathrm{d}u' \dot{\mathcal{Q}}_s(u',z) \quad \to \quad \mathcal{Q}_s(u,z) = \underbrace{\mathcal{Q}^{\mathrm{S}}_s(u,z)}_{\text{Linear in } C,\bar{C} \text{ fields}} + \underbrace{\mathcal{Q}^{\mathrm{H}}_s(u,z)}_{\text{Higher orders}}$$

Renormalized charges: $$q_0 := \lim_{u\to-\infty} \mathcal{M}_{\mathbb{C}}\,, \quad q_1 := \lim_{u\to-\infty} \mathcal{P} - uD\mathcal{M}_{\mathbb{C}}\,, \quad q_2 := \lim_{u\to-\infty} \mathcal{T} + \cdots$$

Charge action

EEs

$$q_s \overset{\downarrow}{=} q_s[C,N] \;+$$ Radiative phase space [Ashtekar, Streubel 1981]: $\{N(u,z), C(u',z')\} = \frac{\kappa}{2}\delta(u-u')\delta(z,z')$

Use basic bracket of the radiative phase space at null infinity to compute the symmetry action on C:

$$\boxed{\delta_{\tau_s} C(u,z) = \{Q_s(\tau), C(u,z)\}}$$ where $$Q_s(\tau) := \int_S \tau_s(z) q_s(z) \qquad s = 0,1,2$$

- Supertranslation charge: $$Q(T) := \int_S T(z) q_0(z)$$

$$\delta_T C = \{Q(T), C\} = T\partial_u C - 2D^2 T \quad \rightarrow \quad \xi_T = T\partial_u$$

Generators of

$$\mathrm{GBMS} = \mathrm{BMSW}|_{W=\frac{1}{2}D_A Y^A}$$

[Campiglia, Laddha 2014];
[Miguel's talk]

- Super-Lorentz charge: $$Q(Y) := \int_S Y(z) q_1(z)$$

$$\delta_Y C = \{Q(Y), C\} = \frac{u}{2}(\delta_{T=DY} C) + YDC + \frac{3}{2} CDY \quad \rightarrow \quad \xi_Y = \frac{u}{2} DY\partial_u + YD$$

Charge action

$$q_s \overset{\text{EEs}}{=} q_s[C, N]$$ + Radiative phase space [Ashtekar, Streubel 1981]: $\{N(u,z), C(u',z')\} = \frac{\kappa}{2}\delta(u-u')\delta(z,z')$

Use basic bracket of the radiative phase space at null infinity to compute the symmetry action on C:

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where $Q_s(\tau) := \int_S \tau_s(z) q_s(z)$ $\qquad s = 0, 1, 2$

- Spin-2 charge: $Q(Z) := \int_S Z(z) q_2(z)$ [Freidel, DP, Raclariu 2021-I]

$$\delta_Z C = \{Q(Z), C\} = \cdots \xrightarrow{\text{Linear action}} \xi_Z = Z D^2 \partial_u^{-1} + \frac{2}{3} u DZD + \frac{u^2}{6} D^2 Z \partial_u$$

pseudo-vector field

New non-local (phase space) symmetry action

Universal behaviour of scattering amplitudes (at tree level) [Miguel's talk]

In the limit $\omega \to 0$: $\langle \mathrm{out}| a_\pm(\omega \hat{x}) \mathcal{S} |\mathrm{in}\rangle = \left(S^{(0)}_\pm + S^{(1)}_\pm + S^{(2)}_\pm \right) \langle \mathrm{out}|\mathcal{S}|\mathrm{in}\rangle + \mathcal{O}(\omega^2)$

Soft factors: Leading, Subleading, Sub-subleading

[Weinberg 1965] [Cachazo, Strominger 2014]

Large-r mode expansion of C near $\mathcal{I}^+$: $$C(u,\hat{x}) = \frac{i\kappa}{8\pi^2} \int_0^\infty d\omega \left[a_-^{\mathrm{out}\dagger}(\omega\hat{x}) e^{i\omega u} - a_+^{\mathrm{out}}(\omega\hat{x}) e^{-i\omega u} \right]$$

In order to have a well defined scattering problem in GR [Strominger 2013] :

$$q_s(z)|_{\mathcal{I}^+_-} = q_s(\epsilon(z))|_{\mathcal{I}^-_+} \qquad \longrightarrow \qquad \langle \mathrm{out}|\, q_s(z)|_{\mathcal{I}^+_-} \mathcal{S} - \mathcal{S}\, q_s(\epsilon(z))|_{\mathcal{I}^-_+} |\mathrm{in}\rangle = 0$$

antipodal match

Infinite number of Conservation laws = Symmetries of the S-matrix

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In the limit $\omega \to 0$: $\langle \text{out}| a_\pm(\omega\hat{x})\mathcal{S}|\text{in}\rangle = \left(\underset{\substack{\uparrow\\ \text{Leading}}}{S^{(0)}_\pm} + \underset{\substack{\uparrow\\ \text{Subleading}}}{S^{(1)}_\pm} + \underset{\substack{\uparrow\\ \text{Sub-subleading}}}{S^{(2)}_\pm}\right)\langle \text{out}|\mathcal{S}|\text{in}\rangle + \mathcal{O}(\omega^2)$

Soft factors: Leading Subleading Sub-subleading

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Infinite number of Conservation laws = Symmetries of the S-matrix

Truncated Ward identities:

$\langle \text{out}|[q^1_s,\mathcal{S}]|\text{in}\rangle = -\langle \text{out}|[q^2_s,\mathcal{S}]|\text{in}\rangle$

☞

Leading, subleading, sub-subleading soft th.s

$\underset{s=0}{\uparrow} \qquad \underset{s=1}{\uparrow} \qquad \underset{s=2}{\uparrow}$

Asymptotic Einstein's eq.s $\longleftrightarrow$ Ward identities

Asymptotic dynamics as charge conservation eq.s

[Strominger 2014]; [Kapec, Lysov, Pasterski, Strominger 2014]; [Campiglia, Laddha 2014]; [Freidel, DP, Raclariu 2021-I]

Recap of parts 1 & 2

- Symmetry principle as a powerful organizing tool: existence of a spin-2 charge

- Clear connection between the spin-2 conservation equation and the SSL soft theorem

- Unlike the spin-0 and -1 symmetries responsible for the leading and subleading soft theorems, the spin-2 symmetry is not simply an asymptotic diffeomorphism:

  Non-local transformation represented by a pseudo-vector field acting on scri

Classical GR at null infinity	$\longleftrightarrow$	Tree-level Feynman diagrams

Quantum asymptotic Einstein's eq.s from quantization of the symmetry charge bracket

➡ Application to non-perturbative derivation of scattering amplitudes [Abhay's talk]

- [ ] Spin-2 charge as one of the canonical generators for a bigger symmetry algebra that can be represented in the gravitational phase space?

- [ ] Are there physical observables associated to these new charges?

$\boxed{\text{Part 3: Higher spin dynamics}}$

Weyl scalars

Completing the null frame with the fields: $\ell = \partial_r,\qquad n = e^{-2\beta}(\partial_u - \Phi\partial_r + r^{-2}\Upsilon^A\partial_A)$

Weyl scalars: $\Psi_0 = -C_{\ell m \ell m}\,,\quad \Psi_1 = -C_{\ell n \ell m}\,,\quad \Psi_2 = -C_{\ell m \bar{m} n}\,,\quad \Psi_3 = -C_{n\bar{m}n\ell}\,,\quad \Psi_4 = -C_{n\bar{m}n\bar{m}}$

Asymptotic expansions around future null infinity:

$$\underset{\substack{\uparrow\\ \text{Incoming radiation}}}{\Psi_0} = \sum_{s=0}^{\infty}\frac{\Psi_0^{(s)}}{r^{5+s}}\,,\quad \Psi_1 = \frac{\Psi_1^0}{r^4} + \mathcal{O}(r^{-5})\,,\quad \Psi_2 = \frac{\Psi_2^0}{r^3} + \mathcal{O}(r^{-4})\,,\quad \Psi_3 = \frac{\Psi_3^0}{r^2} + \mathcal{O}(r^{-3})\,,\quad \underset{\substack{\uparrow\\ \text{Outgoing radiation at } \mathcal{I}^+}}{\Psi_4} = \frac{\Psi_4^0}{r} + \mathcal{O}(r^{-2})$$

- Spacetime interpretation: Covariant observables = Asymptotic Weyl scalars

$$\Psi_{2-s} = \frac{1}{r^{3+s}}\mathcal{Q}_s + \cdots \qquad \text{for} \qquad s = -2, -1, 0, 1, 2$$

$$\Psi_4^0 = \mathcal{Q}_{-2} = \frac{\mathcal{N}}{2}\,,\quad \Psi_3^0 = \mathcal{Q}_{-1} = \mathcal{J}\,,\quad \Psi_2^0 = \mathcal{Q}_0 = \mathcal{M}_{\mathbb{C}}\,,\quad \Psi_1^0 = \mathcal{Q}_1 = \mathcal{P}\,,\quad \Psi_0^{(0)} = \mathcal{Q}_2 = \mathcal{T}$$

☆ Higher spin charges [Freidel, DP, Raclariu 2021-II] :

NP charges [Newman, Penrose 1968]

$$\Psi_0 = \sum_{n=0}^{\infty} \frac{\Psi_0^{(n)}}{r^{5+n}}\,, \qquad \Psi_0^{(n)} = \Psi_{G0}^{(n)} + \Psi_{L0}^{(n)}\,, \qquad \Psi_{L0}^{(n)} = \frac{(-)^n}{n!}\bar{D}^n \mathcal{Q}_{n+2} + \cdots \quad \text{for} \quad n > 0$$

with $\ell = 2, \ldots, n+1$ for $\Psi_{G0}^{(n)}$ and $\ell \geq n+2$ for $\Psi_{L0}^{(n)}$.

such that: $$\dot{\mathcal{Q}}_s = D\mathcal{Q}_{s-1} + \frac{(1+s)}{2} C \mathcal{Q}_{s-2} \quad \text{for} \quad s \geq -1$$

The higher spin charges are associated to the remaining free data at $\mathcal{I}^+$ and encoded in the expansion modes of Ψ_0

$$\gamma_{AB} = q_{AB} + \frac{1}{r}C_{AB} + \frac{1}{4r^2}q_{AB}\left(C_{CD}C^{CD}\right) + \sum_{n\geq 0}\frac{1}{r^{n+3}}\left(\frac{2\,\bar{m}_A\bar{m}_B}{(n+3)(n+2)}\Psi_0^{(n)} + \text{c.c.} + \cdots\right)$$

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$$\Psi_0 = \sum_{n=0}^{\infty} \frac{\Psi_0^{(n)}}{r^{5+n}}\,, \qquad \Psi_0^{(n)} = \underset{\substack{\ell = 2,\dots,n+1 \\ \downarrow}}{\Psi_{G0}^{(n)}} + \Psi_{L0}^{(n)}\,, \qquad \underset{\substack{\ell \geq n+2 \\ \downarrow}}{\Psi_{L0}^{(n)}} = \frac{(-)^n}{n!}\bar{D}^n \mathcal{Q}_{n+2} + \cdots \quad \text{for} \quad n > 0$$

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Evidence:

i) Explicitly verified to all orders up to $s=3$

ii) Satisfied for all spin-s at linearized order from Bianchi identities of NP formalism [Newman, Penrose 1968]

iii) Satisfied for all spin-s in self-dual gravity [Ball, Narayanan, Salzer, Strominger 2021]; [Costello, Paquette 2022]

iv) Direct evidence that these charges form a canonical representation of a $w_{1+\infty}$ loop algebra on the gravitational phase space from the linearized contribution to the charge bracket

Higher spin charges

Renormalized higher spin generators with k oscillators:

$$\hat{q}_s^k(u,z) := \sum_{n=0}^{s} \frac{(-u)^{s-n}}{(s-n)!} D^{s-n} Q_n^k(u,z)\,, \qquad k = 1, 2, \dots, s+1$$

Higher spin charge aspects:

$$q_s^k(z) = \lim_{u\to-\infty} \hat{q}_s^k(u,z) \quad \to \quad q_s^1(z) = D^{s+2} N_s(z)\,, \qquad N_s(z) := \frac{1}{2}\frac{(-1)^{s+1}}{s!}\int_{-\infty}^{\infty} \mathrm{d}u\, u^s \hat{N}(u,z)$$

Neg. helicity (sub)^s-leading soft graviton operator
(or higher spin memory observables)

Action of the quadratic spin-s charge on the gravitational phase space: $\{q_s^2(z), C(u',z')\}$

Ward identities for higher spin charges q_s $\longrightarrow$ Tower of tree-level soft graviton theorems

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Linearized charge bracket $\qquad Q_s(\tau) := \frac{8}{\kappa}\int_S \mathrm{d}^2 z \sqrt{q}\, \tau_s(z) q_s(z)$

$$\{Q_s(\tau), Q_{s'}(\tau')\}^1 = \{Q^2_s(\tau), Q^1_{s'}(\tau')\} + \{Q^1_s(\tau), Q^2_{s'}(\tau')\} = Q^1_{s'+s-1}\left[(s'+1)\,\tau' D\tau - (s+1)\tau D\tau'\right]$$

$\boxed{w_{1+\infty} \text{ loop algebra}}$ [Freidel, DP, Raclariu 2021-II]

Higher spin memory observables

Gravitational wave (displacement) memory

[Zel'dovich, Polnarev 1974]; [Braginsky, Thorne 1987]; [Christodoulou 1991]:

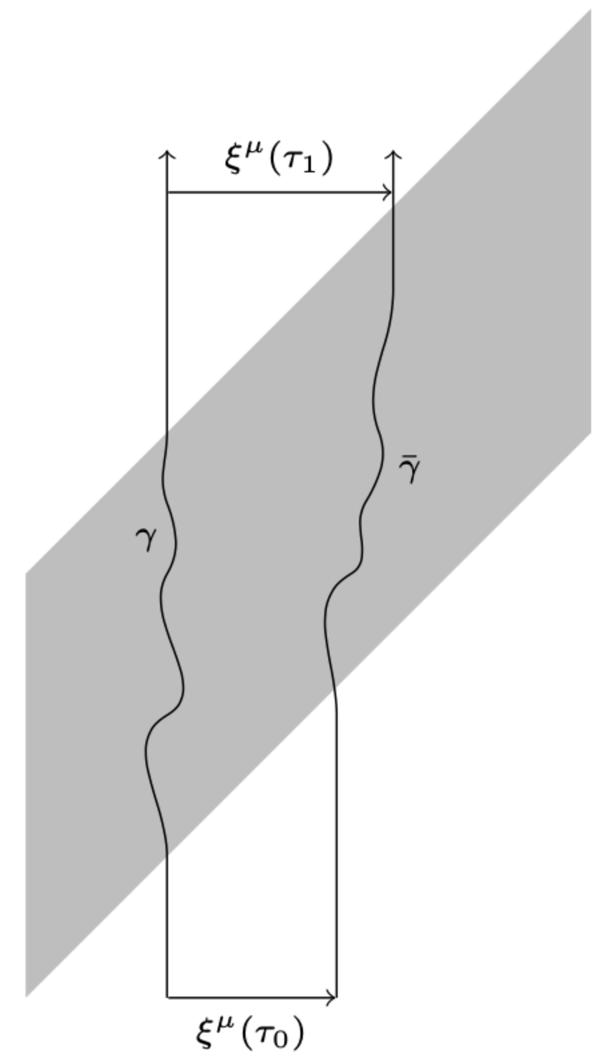
Change in displacement of initially comoving observers $\gamma, \bar{\gamma}$

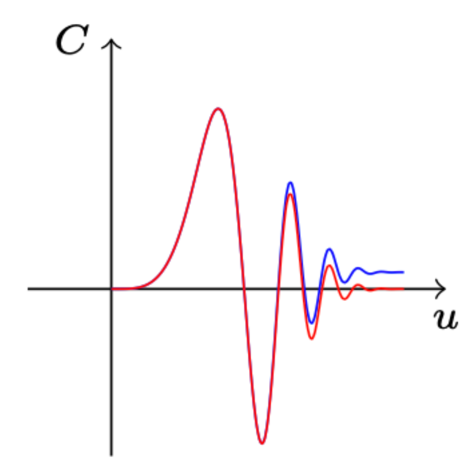
$$\Delta\xi^\alpha \sim \iint R^\alpha{}_{\mu\beta\nu}\dot{\gamma}^\mu\dot{\gamma}^\nu\xi^\beta$$

Displacement memory = Δ(source properties) $+ \int$ (flux of gravitational waves)

$$\underbrace{\frac{1}{2}\int_{u_0}^{u_1} D^2\hat{N}(u')}_{N_0=\text{spin}-0\text{ memory obs.}} = \Delta\mathcal{M} + \underbrace{\frac{1}{4}\int_{u_0}^{u_1} \mathrm{d}u' C(u')\mathcal{N}(u')}_{\text{integral of flux}}$$

For spin-1 memory see [Pasterski, Strominger, Zhiboedov 2016]; [Nichols 2018]





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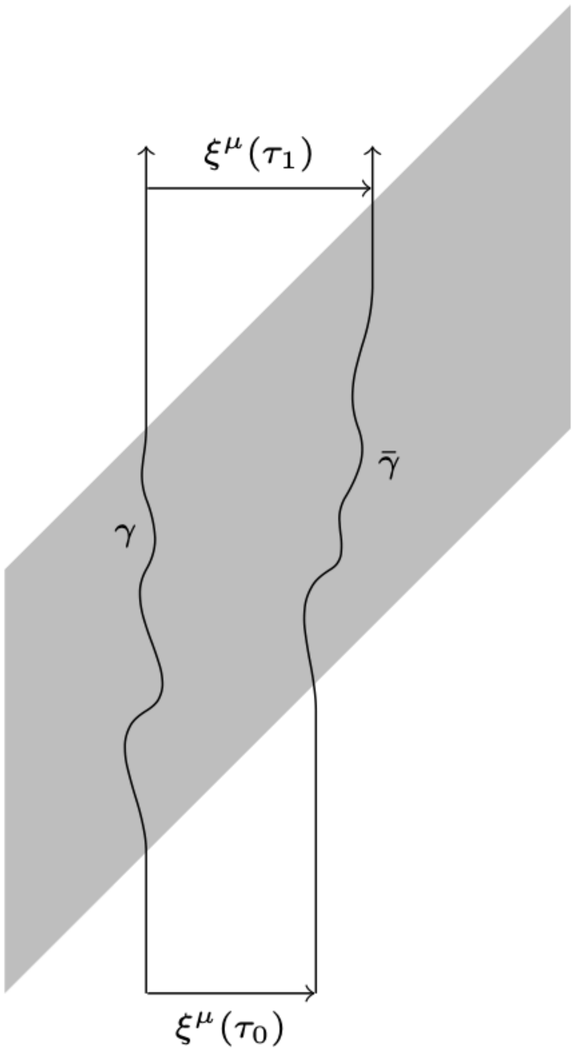
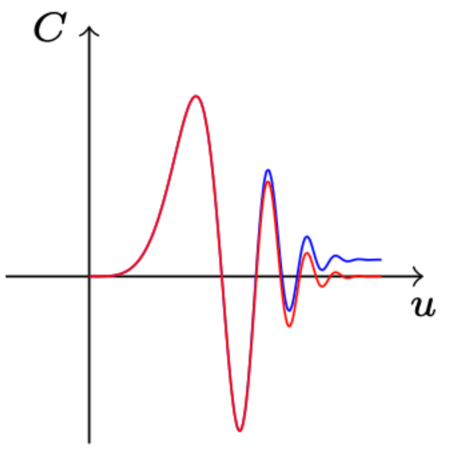


Allowing for initial relative velocity and acceleration(s): Higher spin-memory obs

Curve deviation (generalization of displacement memory) [Grant, Nichols 2019]

$$\Delta \xi(u_0, u_1) := \frac{1}{2r} \sum_{n \geq 0} \left[(n+1) N_n(u_0, u_1) - (u_1 - u_0) N_{n-1}(u_0, u_1) \right] \partial_u^n \xi|_{u=u_0}$$

where $\Delta \xi(u) = \xi(u) - \xi_{\text{flat}}(u)$



Gravitational memory effects are now being implemented in numerical relativity waveforms for binary black hole mergers [Khera, Krishnan, Ashtekar, De Lorenzo 2020]; [Mitman et al. 2020]

Outlook

- We have proposed a set of evolution equations for higher spin-s charges expressed as the recursion relations:

$$\dot{\mathcal{Q}}_s = D\mathcal{Q}_{s-1} + \frac{(1+s)}{2} C\mathcal{Q}_{s-2} \quad \text{for} \quad s \geq -1$$

New non-local symmetries associated to pseudo-vector fields

- To linear order the $w_{1+\infty}$ loop algebra has a canonical realisation in the gravitational phase space in terms of the Poisson bracket of the higher spin charges

- [ ] The relevance of the recursion relation in encoding the expression of the vacuum EEs at subleading orders in a large-r expansion needs to be firmly established in the case s>3

- [ ] Does the $w_{1+\infty}$ structure survives at quadratic order in the bracket and the inclusion of the the mixed helicity sector?

- [ ] How much of the radiation signal can be reconstructed from these higher spin charges?

Exciting recent results relating celestial charges to canonical multipole moments of the linearized gravitational field in the bulk parametrizing a generic post-Minkowskian metric without incoming radiation [Compere, Oliveri, Seraj 2022]

Universal behaviour of scattering amplitudes

$$\langle \text{out}|a_\pm(q)\mathcal{S}|\text{in}\rangle = \left(S_\pm^{(0)} + S_\pm^{(1)} + S_\pm^{(2)}\right)\langle \text{out}|\mathcal{S}|\text{in}\rangle + \mathcal{O}(q^2)$$

Soft factors:

$$S_\pm^{(0)} = \frac{\kappa}{2}\sum_{k=1}^{n}\frac{(p_k\cdot\varepsilon^\pm)^2}{p_k\cdot q}\,,$$

Leading

$$S_\pm^{(1)} = -\frac{i\kappa}{2}\sum_{k=1}^{n}\frac{(p_k\cdot\varepsilon^\pm)(q\cdot J_k\cdot\varepsilon^\pm)}{p_k\cdot q}\,,$$

Subleading

[Cachazo, Strominger 2014]

$$S_\pm^{(2)} = -\frac{\kappa}{4}\sum_{k=1}^{n}\frac{(\varepsilon^\pm\cdot J_k\cdot q)^2}{p_k\cdot q}$$

Sub-subleading

Large-r mode expansion of C near $\mathcal{I}^+$:

[He, Lysov, Mitra, Strominger 2014]

$$C(u,\hat{x}) = \frac{i\kappa}{8\pi^2}\int_0^\infty d\omega\left[a_-^{\text{out}\dagger}(\omega\hat{x})e^{i\omega u} - a_+^{\text{out}}(\omega\hat{x})e^{-i\omega u}\right]$$

Radiative data bracket -> Standard commutator for the modes : $[a_\pm(\omega\hat{x}), a_\pm^\dagger(\omega'\hat{x}')] = (2\pi)^3\frac{2}{\omega}\delta(\omega-\omega')\delta(z,z')$

$q = \omega\hat{x}$: (outgoing) graviton momentum

$\varepsilon_{\mu\nu}^{\pm\pm} = \varepsilon_\mu^\pm\varepsilon_\nu^\pm$: graviton polarization

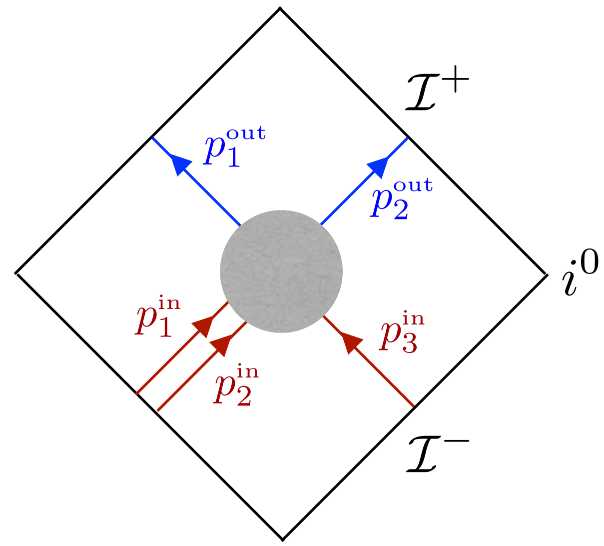
p_k, J_k : momenta and angular momenta of other (hard) particles

Flat retarded coordinates on the plane

$$q^\mu = \frac{\omega}{\sqrt{2}}(1+z\bar{z}, z+\bar{z}, -i(z-\bar{z}), 1-z\bar{z})$$

$$\varepsilon_\mu^+(q) = \bar{\varepsilon}_\mu^-(q) = \frac{1}{\sqrt{2}}(-\bar{z}, 1, -i, -\bar{z})$$

$$p_k^\mu = \frac{\epsilon_k\omega_k}{\sqrt{2}}(1+z_k\bar{z}_k, z_k+\bar{z}_k, -i(z_k-\bar{z}_k), 1-z_k\bar{z}_k)$$



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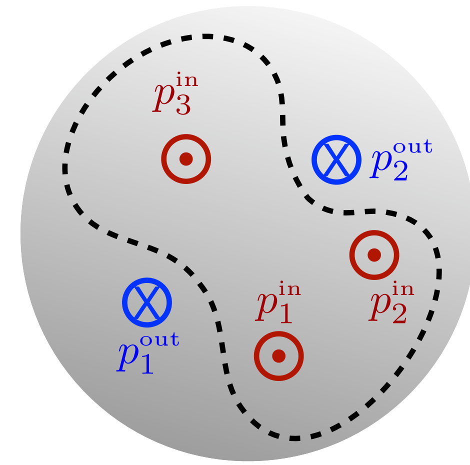

Celestial Holography
=
reformulation of the gravitational scattering problem
in a basis of asymptotic boost eigenstates
[Pasterski, Shao, Strominger 2017]

Conformal primary boost eigenstates

$$O^-_\Delta \propto -\frac{\Gamma(\Delta-1)}{2}\int_{-\infty}^{+\infty} \mathrm{d}u\, u^{-\Delta+1}\hat{N}(u,z)$$

Lorentzian CCFT with global conformal group $SL(2,\mathbb{R})_R \times SL(2,\mathbb{R})_L$

Celestial OPE of two gravitons in the antiholomorphic collinear limit $\bar{z}_{12} = \bar{z}_1 - \bar{z}_2 \to 0$

Euler beta function

$$O^-_{\Delta_1}(z_1)O^\pm_{\Delta_2}(z_2) \sim -\frac{\kappa}{2}\frac{1}{\bar{z}_{12}}\sum_{n=0}^{\infty} B(\Delta_1-1+n, \Delta_2 \pm 2+1)\frac{z_{12}^{n+1}}{n!}\partial^n O^\pm_{\Delta_1+\Delta_2}(z_2) + \mathcal{O}(\bar{z}^0_{12})$$

[Fan, Fotopoulos, Taylor 2019]; [Pate, Raclariu, Strominger, Yuan 2019]

☞ Infinite tower of soft theorems governed by a $w_{1+\infty}$ structure
[Guevara, Himwich, Pate, Strominger 2021]; [Strominger 2021]

Higher spin charges

Renormalized higher spin generators with k oscillators:

$$\hat{q}_s^k(u,z) := \sum_{n=0}^{s} \frac{(-u)^{s-n}}{(s-n)!} D^{s-n} \mathcal{Q}_n^k(u,z)\,, \qquad k = 1, 2, \ldots, s+1$$

Higher spin charge aspects:

$$q_s^k(z) = \lim_{u\to-\infty} \hat{q}_s^k(u,z) \qquad \rightarrow \qquad q_s^1(z) = D^{s+2} N_s(z)\,, \qquad N_s(z) := \frac{1}{2}\frac{(-1)^{s+1}}{s!}\int_{-\infty}^{\infty} \mathrm{d}u\, u^s \hat{N}(u,z)$$

Neg. helicity (sub)^s-leading soft graviton operator

Action of the quadratic spin-s charge on the gravitational phase space:

$$\{q_s^2(z), C(u',z')\} = \frac{\kappa^2}{8s!}\sum_{n=0}^{s}(-1)^{s+n}(n+1)(u'\partial_{u'}+3)_{s-n}(s)_n \partial_{u'}^{1-s} D_{z'}^n C(u',z') D_z^{s-n}\delta(z,z')$$

[Freidel, DP, Raclariu 2021-II]

Ward identities for higher spin charges q_s $\longrightarrow$ Tower of tree-level conformally soft graviton theorems

Celestial Holography $\subset$ $\dot{\mathcal{Q}}_s = D\mathcal{Q}_{s-1} + \frac{(1+s)}{2} C\mathcal{Q}_{s-2}$

$w_{1+\infty}$ structure

Canonical derivation on GR phase space:

$$Q_s(\tau) := \frac{8}{\kappa}\int_S \mathrm{d}^2 z \sqrt{q}\,\tau_s(z) q_s(z) \quad \text{with} \quad \tau_s(z,\bar{z}) \in V^{\mathrm{SL}(2,\mathbb{C})}_{\left(h=-\frac{s+1}{2},\,\bar{h}=\frac{s-1}{2}\right)}$$

Linearized charge bracket

$$\{Q_s(\tau), Q_{s'}(\tau')\}^1 = \{Q^2_s(\tau), Q^1_{s'}(\tau')\} + \{Q^1_s(\tau), Q^2_{s'}(\tau')\} = Q^1_{s'+s-1}\left[(s'+1)\,\tau' D\tau - (s+1)\tau D\tau'\right]$$

☞ $$[Q^s_{m,n}, Q^{s'}_{m',n'}] = i\,[m(1+s') - m'(1+s)]\,Q^{s+s'-1}_{m+m'-1,n+n'} \quad \text{with} \quad m,n,m',n' \geq 0$$

$\boxed{w_{1+\infty} \text{ loop algebra}}$ [Freidel, DP, Raclariu 2021-II]

[Guevara, Himwich, Pate, Strominger 2021]; [Strominger 2021]