causal structure in spin-foams

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take away:

The EPRL-FK spin-foam amplitude is summing over configurations that can describe:

      - different possible causal orderings

      - local flips of signature convention

      - non-causal (conjoined light-cones, acausal structure)

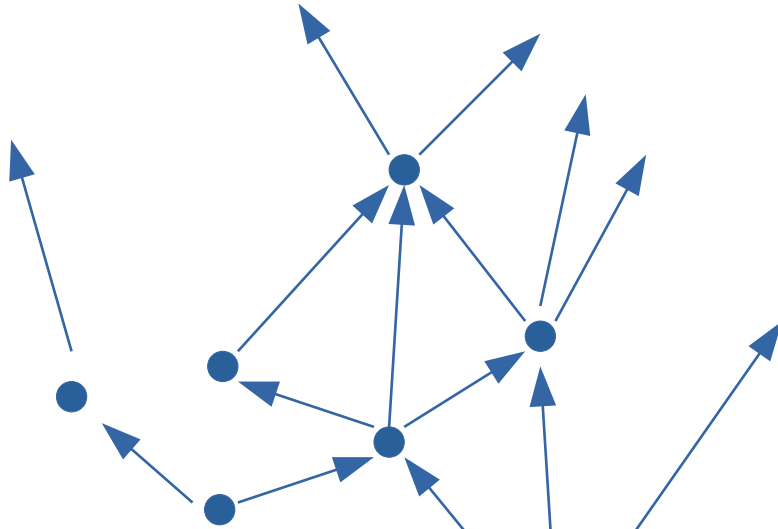Is causality a fundamental or an emergent property of space-time?

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Causal set theory

Metric = Causal structure + Conformal factor $\Omega(x)$
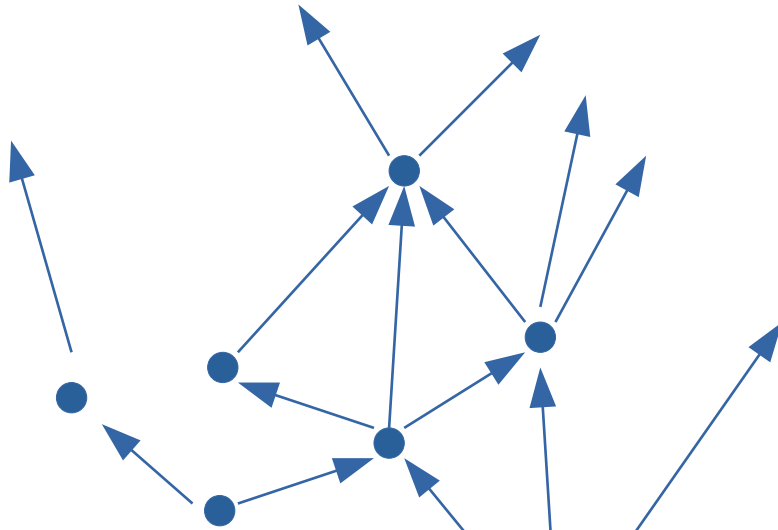[Hawking, King, McCarthy (1976) Malament (1977)]

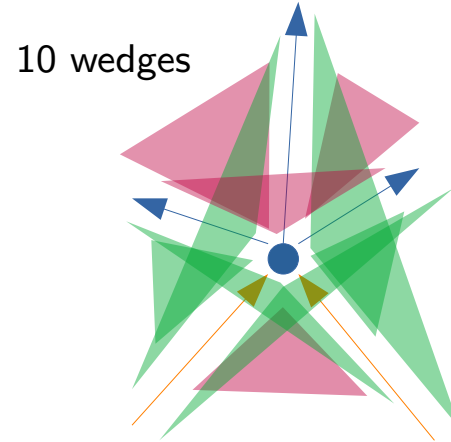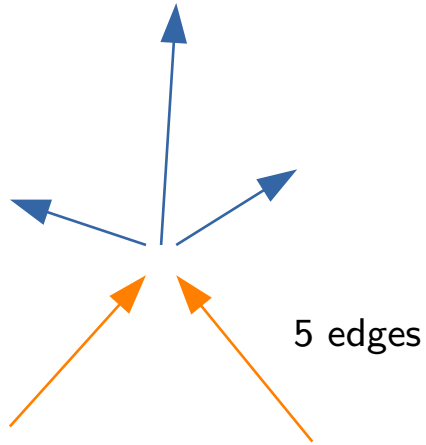Is causality a fundamental or an emergent property of space-time?

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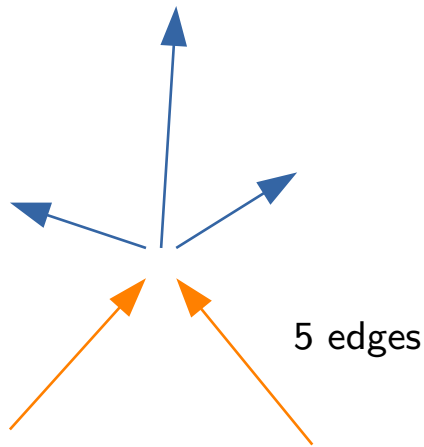[Hawking, King, McCarthy (1976) Malament (1977)]

Spinfoams ?

EPRL-FK based on a discretised version of Plebanski action (2-forms hence 2-surfaces objects)

5 edges

10 wedges

causality can be locally encoded on 2-surfaces (wedges)

$(\eta, -\eta, -\eta, -\eta)$

signature

10 wedges

5 edges

$$\varepsilon_w = \begin{cases} \eta & \text{co-chronal (time)} \\ -\eta & \text{anti-chronal (space)} \end{cases}$$
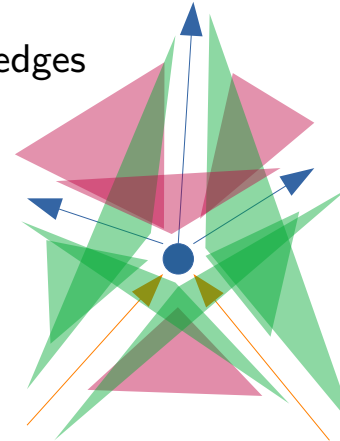
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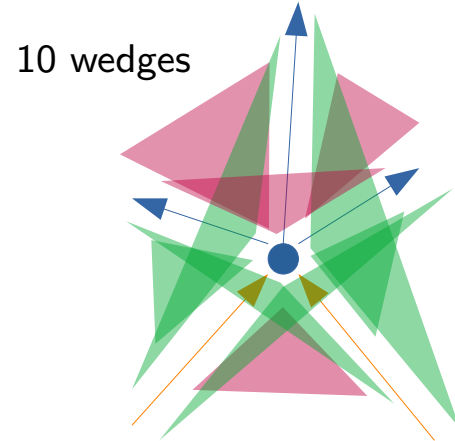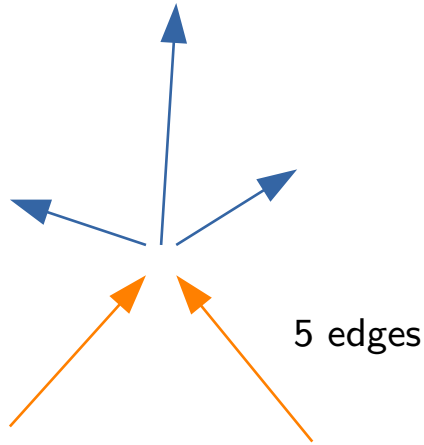**causality can be locally encoded on 2-surfaces (wedges)**

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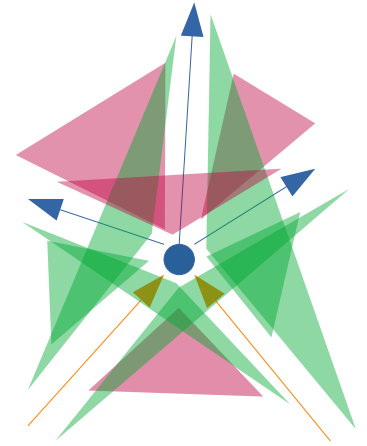wedge orientation $\varepsilon_w = \begin{cases} \eta & \text{co-chronal (time)} \\ -\eta & \text{anti-chronal (space)} \end{cases}$

causality can be locally encoded on 2-surfaces (wedges)

$$\prod_{w \in cycle} \varepsilon_w = \eta^{\#cycle}$$

Causal constraint

the wedge orientations form a space with
plenty of non-causal configurations

**the equations of motion select
standard light-cone configurations**

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plenty of non-causal configurations

$$\text{sign } \theta_w = \varepsilon_w$$

[Barrett 94']

1$^{\text{st}}$ order Regge action (lengths + 4D angles)

$$S[l_s, \theta_w, \mu_\sigma] = \sum_\sigma \sum_w A_t(l_s)\theta_w + \sum_\sigma \mu_\sigma \det \gamma_\sigma$$

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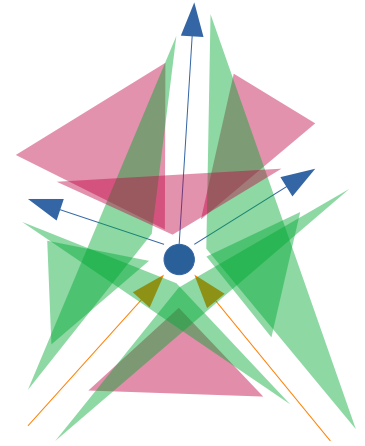
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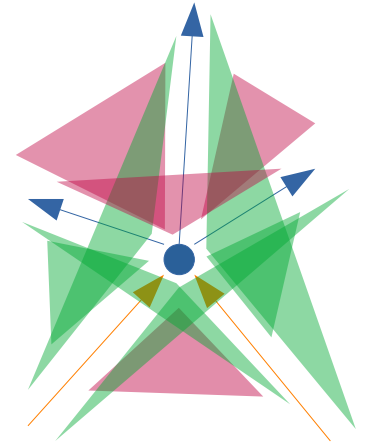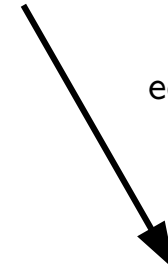
causal constraint

the 1ˢᵗ order Regge path-integral

admits causal and non-causal terms

$$\mathcal{W}_\Delta = \sum_{[\varepsilon_w]} \int [dl_s][dr_w] \prod_\sigma \delta(\det \gamma_\sigma) \, e^{\frac{i}{\hbar} \sum_w A_t \varepsilon_w r_w}$$

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$$\sum_{[\varepsilon_w]} = \underbrace{\sum_{[\varepsilon_w]}}_{\text{causal } \eta=1} + \underbrace{\sum_{[\varepsilon_w]}}_{\text{causal } \eta=-1} + \underbrace{\sum_{[\eta_v]}}_{\substack{\text{signature} \\ \text{changes}}} \underbrace{\sum_{[\varepsilon_w]}}_{\substack{\text{local} \\ \text{light-cones}}} + \underbrace{\sum_{[\varepsilon_w]}}_{\text{non-causal}}$$

$$\boxed{\prod_{w \in cycle} \varepsilon_w = \eta^{\#cycle}}$$

Causal constraint

Wedge amplitude

$$K(h, g) = \sum_{\varepsilon \in \{-1, 1\}} K^{\varepsilon}(h, g)$$

Spinfoam (EPRL-FK) sums can decomposed into sums of causal and non-causal terms.

$$K^{\varepsilon}(h, g) = \sum_{j} \frac{(2j + 1)^4}{\pi^3} \int [d\zeta][dz'][dz''] \langle z' | h^{\dagger} | z'' \rangle^{2j} \, \mathcal{B} \, \underline{\Theta(\varepsilon S_{\gamma})} \, e^{iS_{\gamma}}$$

Heaviside step-function

Total amplitude

$$W_{\mathcal{C}} = \sum_{[\varepsilon_w]} W_{\mathcal{C}}^{\varepsilon_w}$$

$$\sum_{[\varepsilon_{t\sigma}]} = \sum_{[\varepsilon_{t\sigma}]} + \sum_{[\varepsilon_{t\sigma}]} + \sum_{[\eta_\sigma]} \sum_{[\varepsilon_{t\sigma}]} + \sum_{[\varepsilon_{t\sigma}]}$$

causal $\eta = 1$ causal $\eta = -1$ signature   local   non-causal
changes   light-cones

14

$$\sum_{[\varepsilon_{t\sigma}]} = \sum_{[\varepsilon_{t\sigma}]} + \sum_{[\varepsilon_{t\sigma}]} + \sum_{[\eta_\sigma]} \sum_{[\varepsilon_{t\sigma}]} + \sum_{[\varepsilon_{t\sigma}]}$$

$$\text{causal } \eta=1 \quad \text{causal } \eta=-1 \quad \begin{matrix}\text{signature}\\\text{changes}\end{matrix} \begin{matrix}\text{local}\\\text{light-cones}\end{matrix} \quad \text{non-causal}$$

You could truncate this sum to keep only the causal terms
(analogue to the Feynman propagator).

Causal spinfoam: propagator of spin-networks (time-evolution)   [Teitelboim 82']

Full EPRL spinfoam: projector on the physical states (Hamiltonian constraint)   [Livine, Oriti 02']

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[Teitelboim 82']
[Livine, Oriti 02']

[Christodoulou, Längvik, Riello, Röken, Rovelli, 13']

Some non-causal terms are divergent.

[Engle, Zipfel 13']

"Proper vertex amplitude"

[Immirzi, Gupta, Smolin, Markopoulou, Cortes, Wieland...]

16

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