Categorified Phase Space and Spinning Geometries

LOOPS'22, Lyon

Hank Chen
with Florian Girelli and Matteo Laudoino

University of Waterloo

July 21, 2022

Recap

1. 3d BF theory (or equivalently Chern-Simons)[1]

$$S[A,B] = \int \underbrace{B}_{\in\mathfrak{g}^*} \wedge \underbrace{F}_{\in\mathfrak{g}}, \qquad F = dA + \frac{1}{2}[A \wedge A].$$

   - Symmetries form Drinfel'd double $\mathfrak{d} = \mathfrak{g}^* \bowtie \mathfrak{g}$; *cobracket* on $\mathfrak{g}^*$ and *action* $\mathfrak{g} \triangleright \mathfrak{g}^*$.
   - Cobracket integrates to Poisson bracket; $D = \exp \mathfrak{d}$ is Poisson-Lie.
   - Poisson-Lie symmetry of (B, A)-phase space; Heisenberg double.

2. What about 4D? B is now a *2-form*, and more symmetries can appear (eg. shifts, etc.)...
   - **How to understand dynamics of 4d theories?**

[1] Osei and Schroers, 2018.

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Overview

1. Categorfication of Drinfel'd double (DD) and phase space.

2. 2-Drinfel'd double (2DD) and 2-phase space structure from $IG \rtimes IG$ (here $IG = \mathfrak{g}^* \rtimes G$).

3. Application to spinning geometries with $G = SU(2)$.

4d BF-BB Gravity

- Consider 4d BF-BB gravity with (normalized) cosmological constant:

$$S[A,B] = \int B \wedge F - B \wedge B.$$

1-form shift symmetry $\delta_\alpha A = \alpha, \delta_\alpha B = d_A \alpha$; $B =$ "2-connection".

- 2-gauge theory based on **2-groups**[2] (2grp) $\mathfrak{G}$:

$$\underbrace{\mathfrak{h}(=\mathfrak{g})}_{\ni B} \xrightarrow{t=\mathrm{id}} \underbrace{\mathfrak{g}}_{\ni A}, \qquad \mathrm{EOMs}: \begin{cases} \text{"Fake-flatness"} & F = t(B) \\ \text{"2-flatness"} & d_A B = 0 \end{cases},$$

(subject to some algebraic conditions).

- **BF-BB encodes categorified DD/phase space structure.**

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Categorification of Drinfel'd Double

- Consider 4d *BFCG* theory (BF-BB: $t = \text{id}$),

$$S_{2BF} = \int \underbrace{B}_{\in \mathfrak{g}^*} \wedge \underbrace{(F - t(\Sigma))}_{\in \mathfrak{g}} + \underbrace{C}_{\in \mathfrak{h}^*} \wedge \underbrace{d_A \Sigma}_{\in \mathfrak{h}},$$

symmetries encode a **2-Drinfel'd double (2DD)**[3] $\mathfrak{D}$:

1. $\mathfrak{G}$ acting on $\mathfrak{G}^*$, where $\mathfrak{G}^* = \mathfrak{g}^* \xrightarrow{t^*} \mathfrak{h}^*$ (abelian dual 2grp),

$$\begin{cases} B \wedge [A, A] = -(A \rhd_0 B) \wedge A \\ C \wedge (A \rhd \Sigma) = -(A \rhd_{-1} C) \wedge \Sigma = \Delta_\Sigma(C) \wedge A \end{cases}$$

2. cobracket (δ_1, δ_0) on $\mathfrak{G}^*$,

$$\delta_1(B) \wedge (A \otimes A) = B \wedge [A, A], \qquad \delta_0(C) \wedge (A \otimes \Sigma) = C \wedge (A \rhd \Sigma),$$

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2-Drinfel'd Double Structure*

- What is a 2DD/2-Manin triple[4] in the most general sense?
  1. Action/back-action of Lie bialgebra 2grps $\mathfrak{G}, \mathfrak{G}^*$ on each other: $\mathfrak{G} \rhd \mathfrak{G}^*$ and $\mathfrak{G} \lhd \mathfrak{G}^*$.
  2. Components of $\rhd = (\rhd_0, \rhd_{-1}, \Delta)$ and $\lhd = (\lhd_0, \lhd_{-1}, \Delta^*)$ given by

$$\rhd_0 : \mathfrak{g} \to \text{End}\, \mathfrak{g}^*, \qquad \rhd_{-1} : \mathfrak{g} \to \text{End}\, \mathfrak{h}^*, \qquad \Delta : \mathfrak{h} \to \text{Hom}(\mathfrak{h}^*, \mathfrak{g}^*),$$
$$\lhd_0 : \mathfrak{h}^* \to \text{End}\, \mathfrak{h}, \qquad \lhd_{-1} : \mathfrak{h}^* \to \text{End}\, \mathfrak{g}, \qquad \Delta^* : \mathfrak{g}^* \to \text{Hom}(\mathfrak{g}, \mathfrak{h}).$$

     "categorified coadjoint representations".
  3. Coherence: *matched pair conditions* on $\mathfrak{G}, \mathfrak{G}^*$; gives consistent 2grp structure on $\mathfrak{D} = \mathfrak{G} \bowtie \mathfrak{G}^*$.
- What does this mean for gauge theory? Note 2-gauge transformations in $(\lambda, L) \in \mathfrak{G}$-sector (recall $\mathfrak{G}$ acts on the dual fields (C, B) via $\rhd$),

$$\delta_{(\lambda,L)} A = d_A \lambda + t(L), \qquad \delta_{(\lambda,L)} \Sigma = \lambda \rhd \Sigma + d_A L,$$
$$\delta_{(\lambda,L)} B = \lambda \rhd_0 B + \Delta_L(C), \qquad \delta_{(\lambda,L)} C = \lambda \rhd_{-1} C,$$

  and similarly for 2-gauge transformations in $(\tilde{\lambda}, \tilde{L}) \in \mathfrak{G}^*$-sector.

Theorem (H. Chen and Girelli, 2022)

2-gauge transformations are *unambiguous* — namely $\delta_{(\lambda+\tilde{\lambda}, L+\tilde{L})}$ is well-defined — iff $\mathfrak{G}$ and $\mathfrak{G}^*$ form a matched pair.

[4] Bai et al., 2013.

Categorified Dynamics in BF-BB gravity ($t = \mathrm{id}$)

- Here $\rhd_0 = \rhd_{-1} = -\Delta = [\cdot,\cdot]$; Lie brackets in 2DD $\mathfrak{D} = \mathfrak{d} \xrightarrow{\mathrm{id}} \mathfrak{d}$:

$$[A, A], \quad [A, \Sigma], \quad [\Sigma, \Sigma], \quad [A, B], \quad [A, C], \quad -[\Sigma, C],$$

six non-trivial brackets.

- Integrate (δ_1, δ_0) to *graded* Poisson bracket,

$$\{C^\infty(\mathfrak{h}^*), C^\infty(\mathfrak{h}^*)\}, \qquad C^\infty(\mathfrak{h}^*) \rhd_{\mathrm{fun}} C^\infty(\mathfrak{g}^*),$$

giving **Poisson 2-algebra** $C^\infty(\mathfrak{G}^*) = C^\infty(\mathfrak{g}^*) \xrightarrow{t^*=\mathrm{id}} C^\infty(\mathfrak{h}^*)$.

1. Poisson-Lie group $D = \exp \mathfrak{d} \Rightarrow$ Heisenberg double...
2. so *Poisson-Lie 2-group*[5] $\mathcal{D} = \exp \mathfrak{D} \Rightarrow$ **2-Heisenberg double**.

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2-Heisenberg double of $G = SU(2)$

- 2-phase space $M = T^*SU(2) \xrightarrow{\text{id}} T^*SU(2)$ of *higher*-holonomy-flux.
- Put on discretized 3d Σ — what does the 2grp picture buy us?
  1. *Organization:* Grading dictates dimension

     $$\text{Dim-2}: \ (X, y) \in T^*G, \qquad \text{Dim-1}: \ (J, h) \in T^*G.$$

     (X_p, y_f) on plaquettes/faces and (J_e, h_l) on edges/links.
  2. *Coherence:* t-map implements "Stokes's theorem",

     $$t(X_p) = \sum_{e \in \partial p} J_e, \qquad t(y_f) = \bigoplus_{l \in \partial f} h_l,$$

     through *fake-flatness* (non-Abelian sum $e^{\eta} e^{\eta'} = e^{\eta \oplus \eta'}$).
- Natural setting for spinning geometries[6].

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Structure of $SU(2)$ 2-Heisenberg double*

- Categorified-phase space structure of $M = T^*SU(2) \xrightarrow{t=\mathrm{id}} T^*SU(2)$,

	Dim-2	Dim-1
Momentum	$X \in \mathbb{R}^3$	$J \in \mathbb{R}^3$
Coordinate	$y \in SU(2)$	$h \in SU(2)$

with coherent Poisson brackets assigned on Σ:

$$\{X_p, X_{p'}\} = \epsilon X_{\mathbf{p}(\partial p \cap \partial p')}, \qquad \{X_p, y_f\} = \begin{cases} \delta_{J_{\tilde{e}}} y_f & ; f^* = \tilde{e} \in \partial p \\ 0 & ; \text{otherwise} \end{cases}, \qquad \{y_f, y_{f'}\} = 0,$$

$$J_e \rhd_{\text{fun}} X_p = \begin{cases} \epsilon X_{\mathbf{p}(e)} & ; e \in \partial p \\ 0 & ; \text{otherwise} \end{cases}, \qquad J_e \rhd_{\text{fun}} y_f = \delta_{e,f} \delta_{J_e} y_f, \qquad X_p \rhd_{\text{fun}} h_l = \delta_{p,l} \delta_{X_p} h_l$$

$$\{J_e, J_{e'}\} = \delta_{ee'} \epsilon J_e, \qquad \{J_e, h_l\} = \begin{cases} \delta_{J_e} h_l & ; e \perp l \\ 0 & ; \text{otherwise} \end{cases}, \qquad \{h_l, h_{l'}\} = 0,$$

- **Face-plaquette consistency condition**

$$\delta_{J_{\tilde{e}}} y_f = -h_{l_1} \cdots (\delta_{X_p} h_{\tilde{l}}) \cdots h_{l_n};$$

compute $\{X_p, y_f\}$ in two ways with Stokes's theorem.

1. *Well-defined*: $e \perp p \iff p \perp e$ (uniqueness of Poincaré duality).
   - Linked face-plaquette pairs: $\{X_p, y_f\} \neq 0$ iff $\partial p, \partial f$ are *linked*.
2. **Vestigial plaquettes** (a form of *whiskering*): for all e, there is a plaquette $p = \mathbf{p}(e)$ with $e = \partial p$.
   - Detects *intersections* of boundary edges.

- Tetrahedral τ phase space $\mathcal{P} = M^\tau // C$ (closure constraints C) $\Rightarrow$ 2-spin networks...

Summary

- *Categorification* of DD from gravity (useful also in condensed matter, cf. 3d/4d toric code, topological phases).
- Notion of "2-phase space" as dynamical content of BF-BB gravity (or more generally *BFCG* theory).
- Discretize ⇒ spinning geometries and 2-spin networks.
- Outlook:
  1. Kin. Hilbert space for 2-spin networks? Categorified Fourier transform.
  2. Gauss/diffeo & **simplicity** constraints? 2-spinfoams & 2-GFT[7]...
  3. Categorical interpretation of edge/corner modes?
  4. Relax Jacobi/assoc.: *weak* Poisson 2-alg. Relevance in QG?

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