3d Loop Quantum Gravity with a cosmological constant

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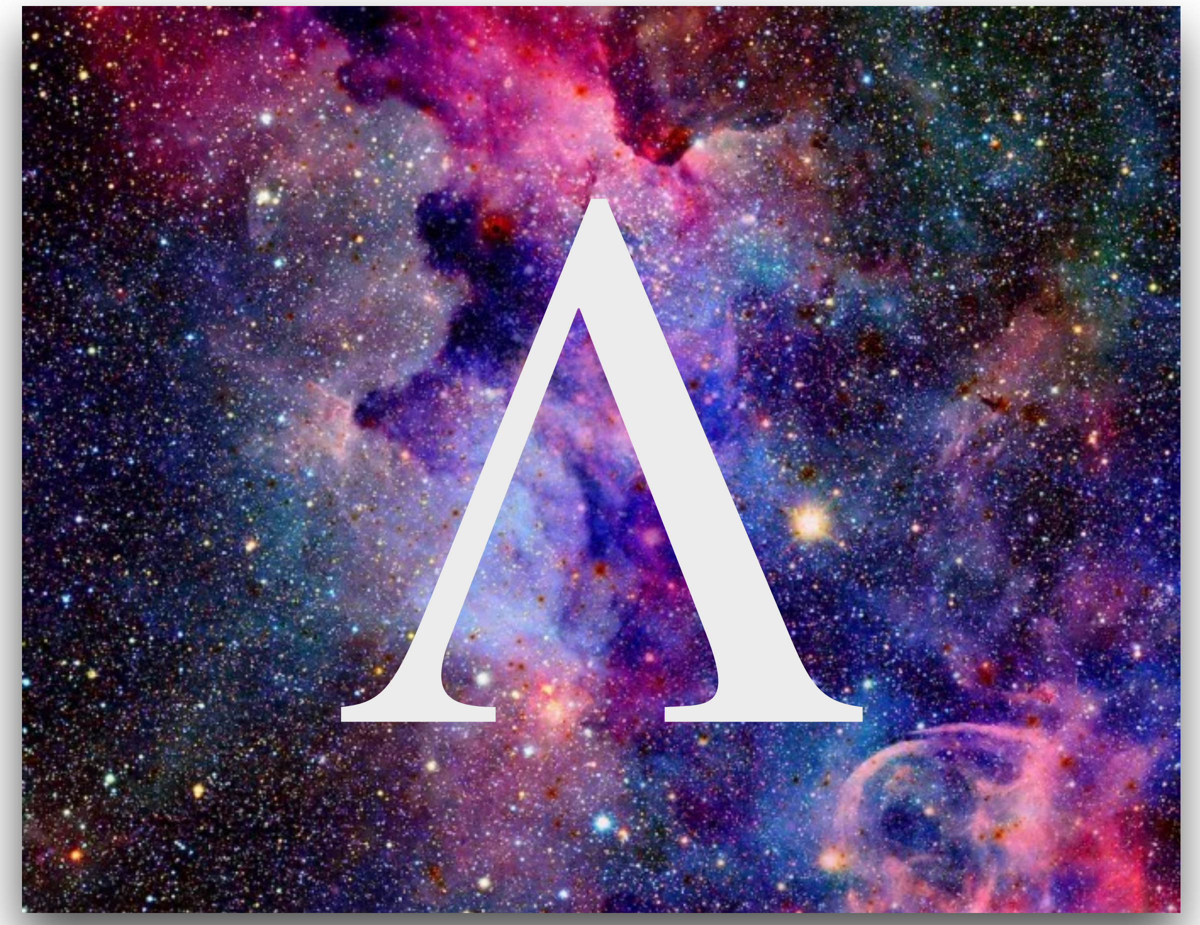
In collaboration with V. Bonzom, L. Freidel, F. Girelli, E. Livine, A. Osumanu, Q. Pan and J. Rennert

LOOPS'22

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

3d Loop Quantum Gravity with a cosmological constant

- Motivations
- The Program
- Latest results - see also Qiaoyin Pan's talk



Motivations

- Geometrically

- From other approaches

Motivations

- Geometrically
- From other approaches

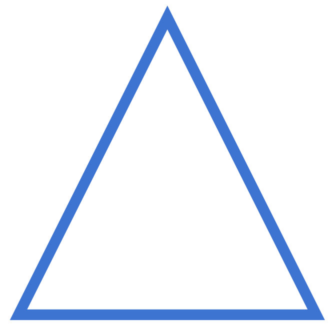
Regge calculus with curved simplices

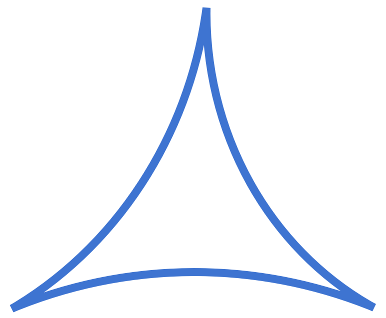
[B. Bahr, B. Dittrich '09]

Regge calculus when $\Lambda \neq 0$

Comparison between

Flat simplices & Simplices with homogeneous curvature





to approximate the (homogeneously) curved space

Better approximation of the continuum

(Perfect action - reflecting the dynamics and the symmetries of the continuum - can be computed)

Motivations

- Geometrically
- From other approaches

The cosmological constant in 3d quantum gravity

BF Action $\Longleftrightarrow$ Chern-Simons Action

$$S_{BF}[e,A] = \int_{\mathcal{M}} \mathrm{Tr}[e \wedge F(A) + \frac{\Lambda}{3} e \wedge e \wedge e]$$

$$S_{CS}[\mathbb{A}] = \frac{k}{4\pi} \int_{\mathcal{M}} \mathrm{Tr}[\mathbb{A} \wedge d\mathbb{A} + \frac{2}{3} \mathbb{A} \wedge \mathbb{A} \wedge \mathbb{A}]$$

canonical quantization → **Loop Quantum Gravity**

$SU(2)$ spin networks
Hamiltonian constraint?

path integral → **Spinfoam**

$\mathcal{U}_q(\mathfrak{su}(2))$ Turaev-Viro model [V. Turaev, O. Viro '92]

combinatorial quantization/ path integral → **Chern-Simons Amplitude**

[A. Achucarro, P. Townsend '86;
E. Witten '88
V. Fock, A. Rosly '99
A. Aleskeev, H. Grosse, V. Schomerus, '95
E. Buffenoir, Ph. Roche '95]

Reconciliation at the dynamical level
[K. Noui, A. Perez, D. Pranzetti '10-'11;
D. Pranzetti '14]

$Z_{TV} = |Z_{CS}|^2$ [J. Roberts '95]

Case $q = 1$ [L. Freidel, D. Louapre '04]

The Program

For Euclidean 3D gravity with a negative cosmological constant

- Continuous theory
- Classical discrete theory
- Quantum Theory

Seminal work on LQG with a quantum group using loop variables: S. Major, L. Smolin ’94.

The Program

$\Lambda = 0$

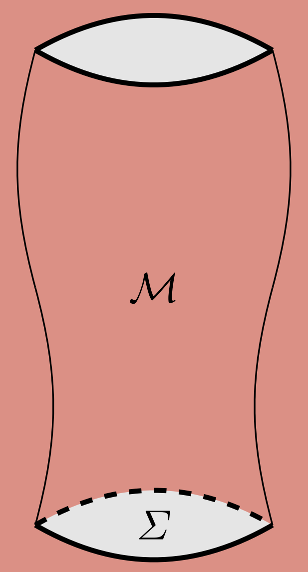
- Continuous theory ⟶ BF theory
- Classical discrete theory
- Quantum Theory

Example: Euclidean signature, $\Lambda = 0$

$$S_{BF}[e, A] = \int_{\mathcal{M}} \mathrm{Tr}[e \wedge F(A)]$$

EOM
- Torsion-free $d_A e = 0$
- Flatness $F(A) = 0$

Canonical analysis
$\mathcal{M} = \Sigma \times \mathbb{R}$



Phase space

$$\{A_a^j(x), E_k^b(y)\} = \delta_k^j \, \delta_a^b \, \delta(x, y)$$

$$E_j^a = \frac{\delta S}{\delta(\partial_0 A_a^j(x, t))} = \tilde{\epsilon}^{ab} \, e_{bj}$$

Constraints

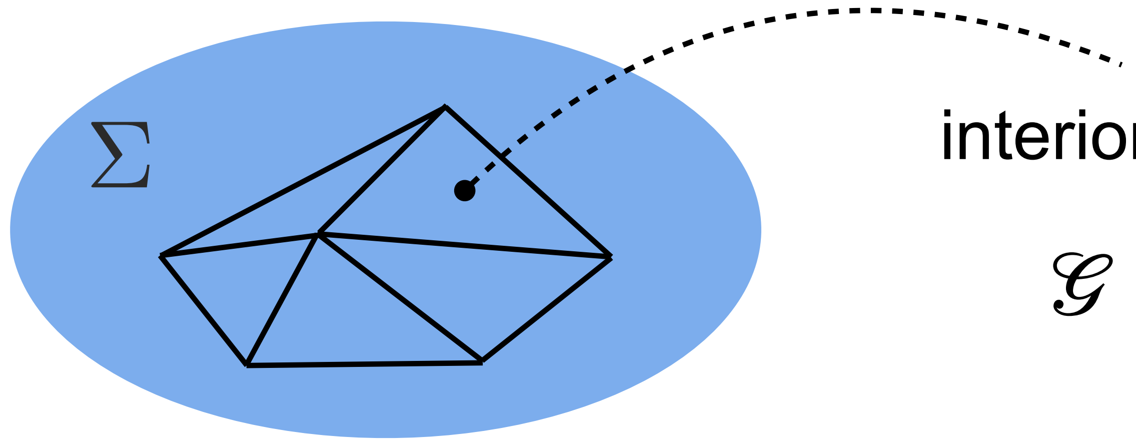
Gauss constraint and Flatness constraint

The Program

$\Lambda = 0$

- Continuous theory
- Classical discrete theory
- Quantum Theory

discretization
&
truncation



interior of the 2-cells:

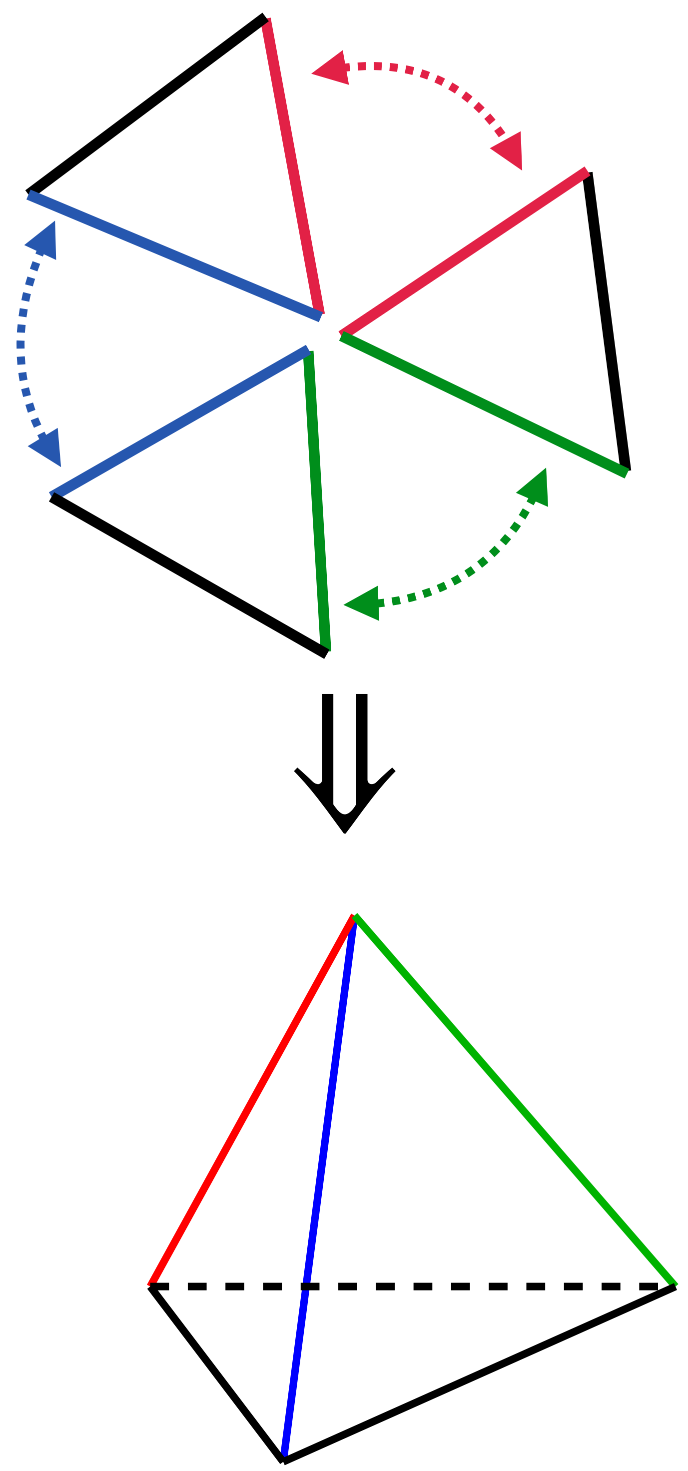
$$\mathscr{G} = \mathscr{F} = 0$$

L. Freidel, M. Geiller, J. Ziprick, ’13
MD, L.Freidel, F. Girelli, ‘17

The Program

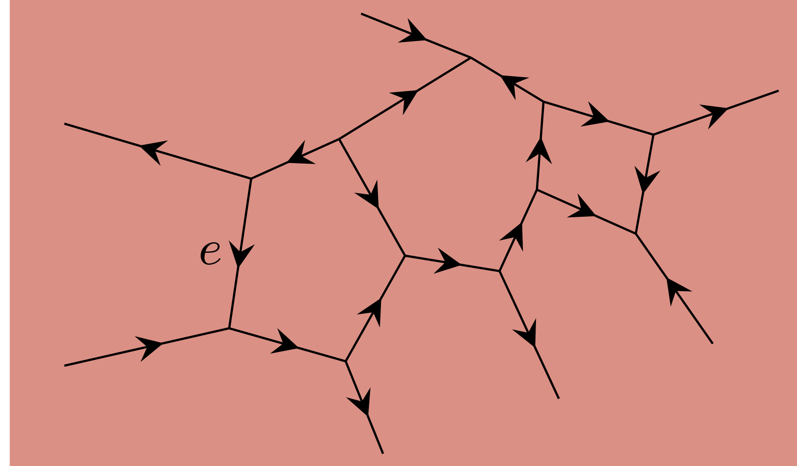
$\Lambda = 0$

- Continuous theory
- Classical discrete theory → Loop gravity
  Model of discrete geometries
- Quantum Theory



Example: Euclidean signature, $\Lambda = 0$

Discrete phase space $\times_e T^*\mathrm{SU}(2)_e$

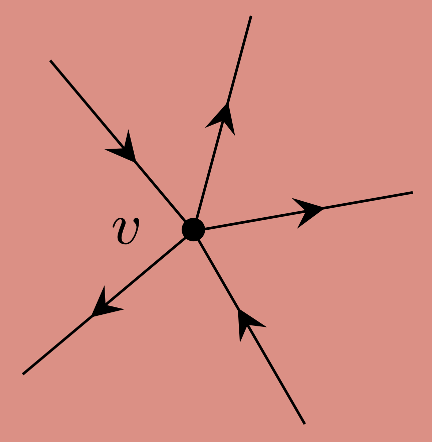


holonomy-flux variables
$(g_e, x_e) \in \mathrm{SU}(2) \times \mathfrak{su}(2)$

Discrete constraints

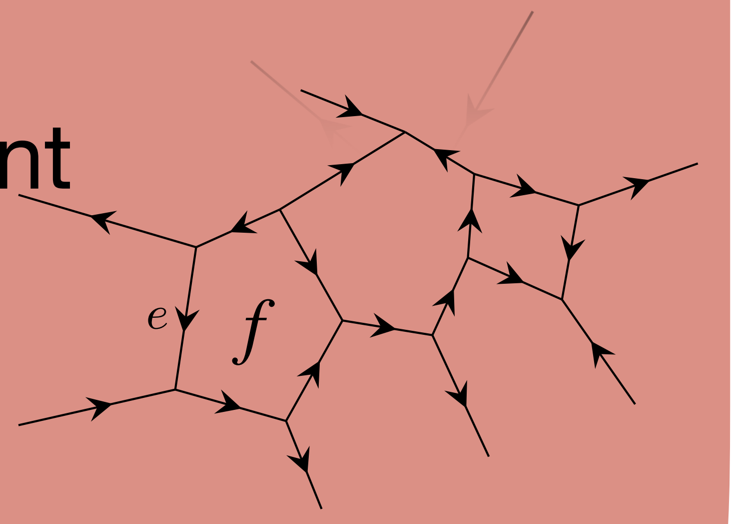
- Gauss constraint

$$\sum_{e|v=s(e)} x_e - \sum_{e|v=t(e)} \tilde{x}_e = 0$$



- Flatness constraint

$$\prod_{e\subset f} g_e = \mathbb{I}$$



Model of discrete flat geometries

V. Bonzom, L. Freidel, ’11

The Program

$\Lambda = 0$

- Continuous theory
- Classical discrete theory
- Quantum Theory

quantization - Dirac quantization

The Program

$\Lambda = 0$

- Continuous theory
- Classical discrete theory
- Quantum Theory $\longrightarrow$ Loop Quantum gravity

Example: Euclidean signature, $\Lambda = 0$

Kinematical Hilbert space

$$\mathscr{H}_\Gamma^{kin} := L^2(SU(2)^E, dg)$$

- Gauss constraint $\longrightarrow$ $\mathscr{H}_G^{kin}$

spinnetwork states

- The Flatness constraint $\longrightarrow$ $\mathscr{H}^{phys}$

Loop Quantum Gravity

$\Updownarrow$

Ponzano-Regge Spinfoam model

K. Noui, A. Perez, ‘04 V. Bonzom, L. Freidel, ‘11

The Program

- Continuous theory

discretization
&
truncation

- Classical discrete theory

quantization

- Quantum Theory

BF action with a non-zero cosmological constant

[Dupuis, Freidel, Girelli, Osumanu, Rennert '20]

$$S_{BF}[e,A] = \int_{\mathcal{M}} e^I \wedge \left(F_I[A] + \frac{\Lambda}{6}\epsilon_{IJK} e^J \wedge e^K \right)$$

EOM

$$d_A e = 0$$
$$F(A) + \frac{\Lambda}{2}[e \wedge e] = 0$$

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Here:
Euclidean signature
Negative cosmological constant

<u>*Change of variable*</u>

$$e^I P_I \in \mathbb{R}^3 \longrightarrow e^I \tau_I \in \mathfrak{sb}(2,\mathbb{C})$$

$$A_I J^I \longrightarrow \omega_I J^I = (A^I + (n \times e)^I) J_I$$

$$n^2 = -\Lambda$$

$$S[e,\omega] = \int_M e \cdot F(\omega) - \frac{1}{2}(e \times e) \cdot d_\omega n$$

EOM

$$\mathscr{T}' = d_\omega e + \frac{1}{2}[[e \wedge e], n] = 0$$

$$\mathscr{C}' = F[\omega] - [e \wedge d_\omega n] = 0$$

Phase space analysis

$$\{\omega^i_a(x), e^b_j(y) = \epsilon_i^{\ j}\delta^a_{\ b}\delta^2(x-y)$$

$$\{\omega^i_a(x), \omega^j_b(y)\} = \{e^a_i(x), e^b_j(y)\} = 0$$

The Program

- Continuous theory

↓ discretization & truncation

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↓ quantization

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$$n^2 = -\Lambda$$

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EOM

$$\mathcal{T}' = \mathrm{d}_A \mathbf{e} + \frac{1}{2}[[\mathbf{e} \wedge \mathbf{e}], \mathbf{n}] \simeq 0$$

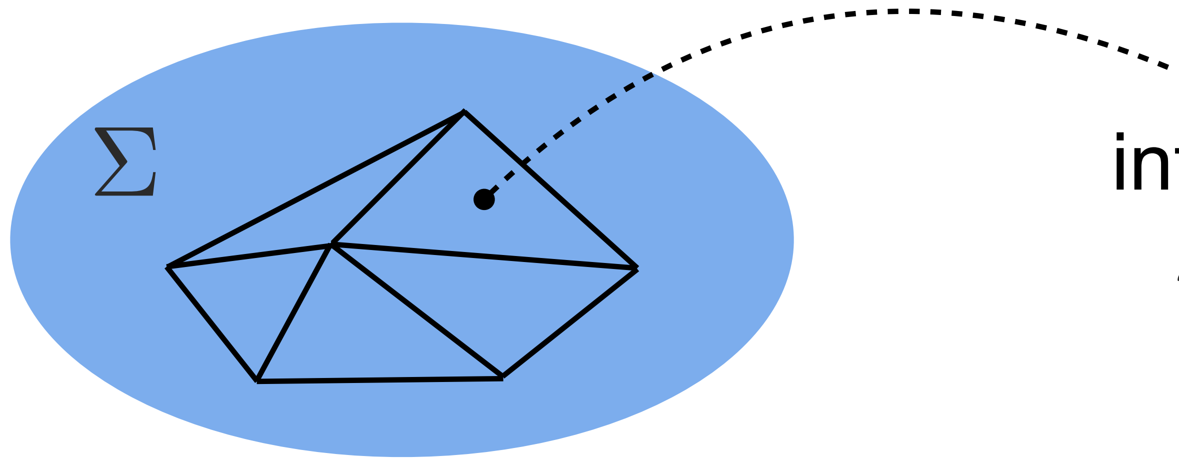
$$\mathcal{C}' = F[A] - [\mathbf{e} \wedge \mathrm{d}_A \mathbf{n}] \simeq 0$$

Phase space analysis

$$\{A^i_a(x), \mathbf{e}^b_j(y)\} = \epsilon_i{}^j \delta^a{}_b \delta^2(x-y)$$

$$\{A^i_a(x), A^j_b(y)\} = \{\mathbf{e}^a_i(x), \mathbf{e}^b_j(y)\} = 0$$

discretization & truncation



interior of the 2-cells:

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The Program

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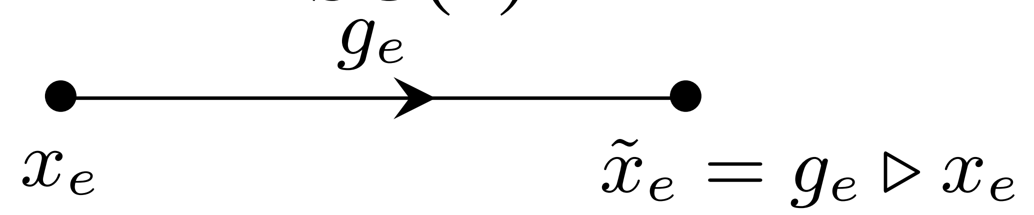
Euclidean signature
Negative cosmological constant

3D Loop Gravity phase space

Bonzom, MD, Girelli, Livine '14

Bonzon, MD, Girelli, Pan '22

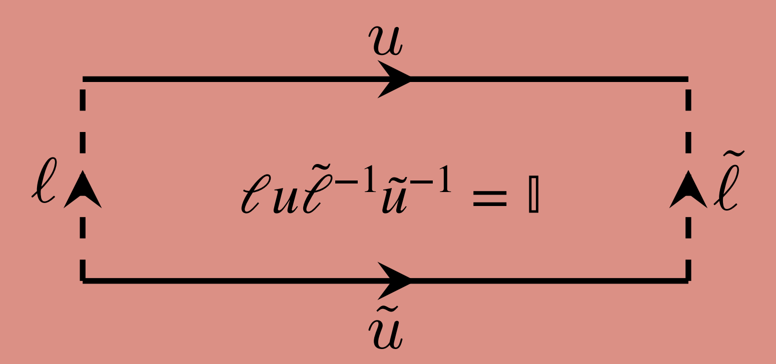
LQG $\Lambda = 0$	**LQG** $\Lambda < 0$
$T^*\mathrm{SU}(2)$	$\mathrm{SL}(2,\mathbb{C})$



$$(g_e, x_e) \in \mathrm{SU}(2) \times \mathfrak{su}(2)$$

$$\{x_e^a, g_e\} = \frac{i}{2}\sigma^a g_e\,, \{x_e^a, x_e^b\} = \epsilon^{abc} x_e^c\,, \{g_e, g_e\} = 0$$

$$\{x_e^a, \tilde{x}_e^b\} = 0\,, \{\tilde{x}_e^a, g_e\} = \frac{i}{2} g_e \sigma^a\,, \{\tilde{x}_e^a, \tilde{x}_e^b\} = -\epsilon^{abc} \tilde{x}_e^c$$



$$(\ell, u) \in \mathrm{SB}(2,\mathbb{C}) \times \mathrm{SU}(2)\,/\, D = \ell u \in \mathrm{SL}(2,\mathbb{C})$$

$$\{\ell_1, \ell_2\} = -[r, \ell_1 \ell_2]\,, \quad \{\ell_1, u_2\} = -\ell_1 r u_2\,,$$

$$\{u_1, \ell_2\} = \ell_2 r^t u_1\,, \quad \{u_1, u_2\} = -[r^t, u_1 u_2]$$

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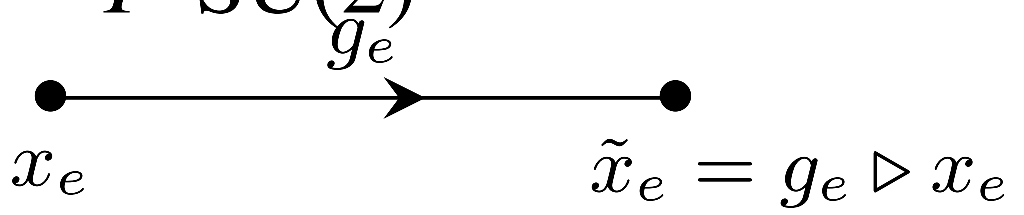

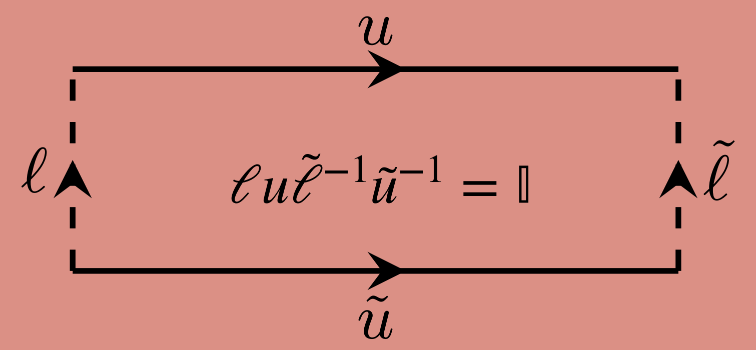

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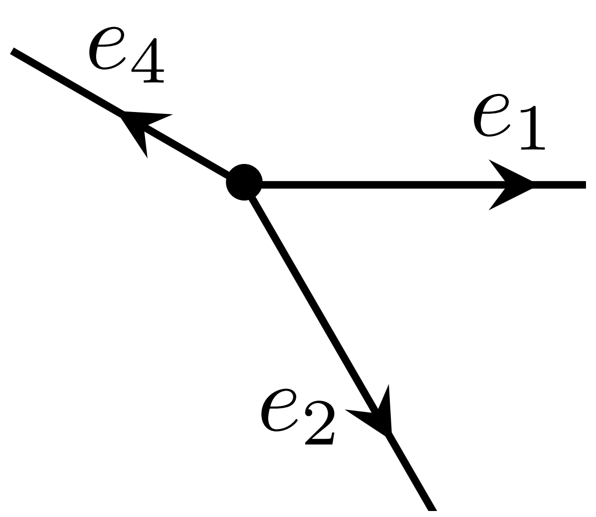
Gauss constraint



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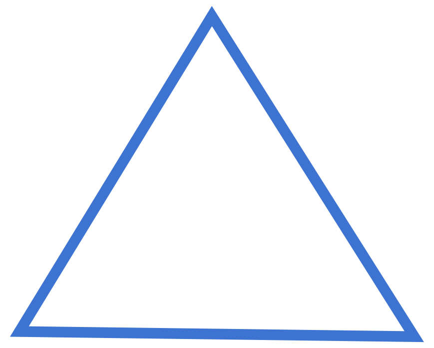

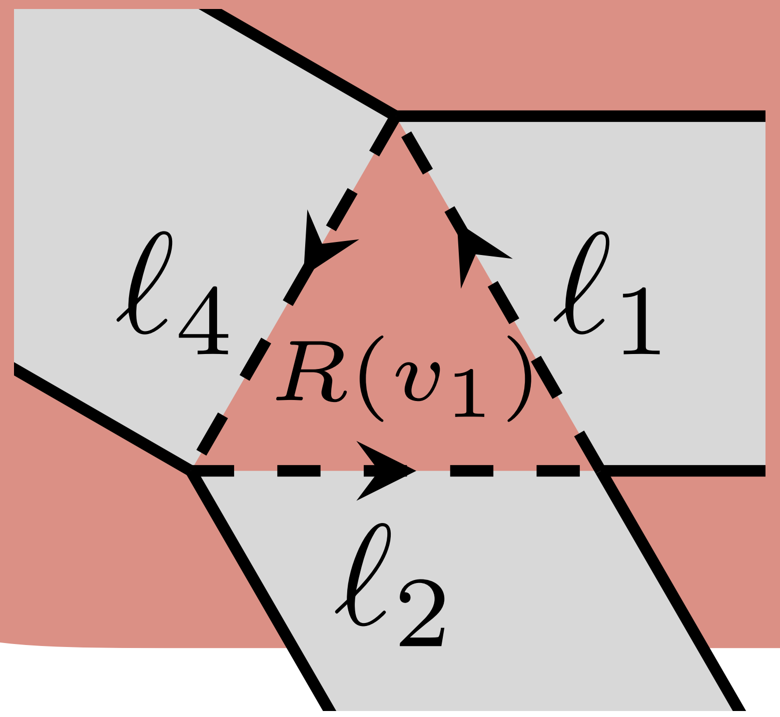

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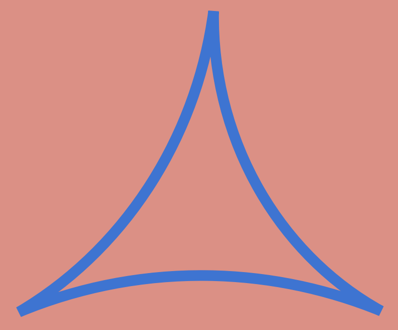

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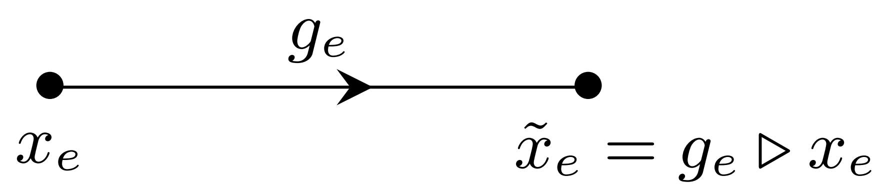
LQG $\Lambda = 0$

$T^*\mathrm{SU}(2)$



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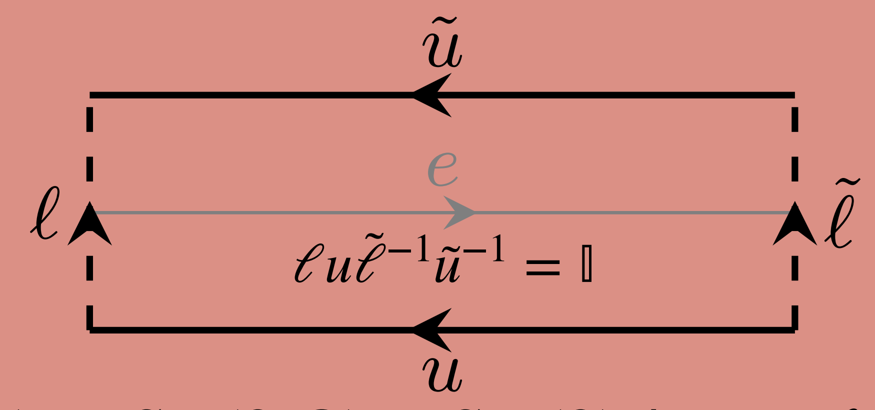
LQG $\Lambda \neq 0$

$\mathrm{SL}(2,\mathbb{C})$



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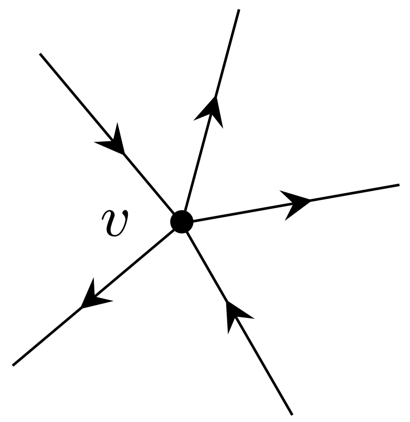
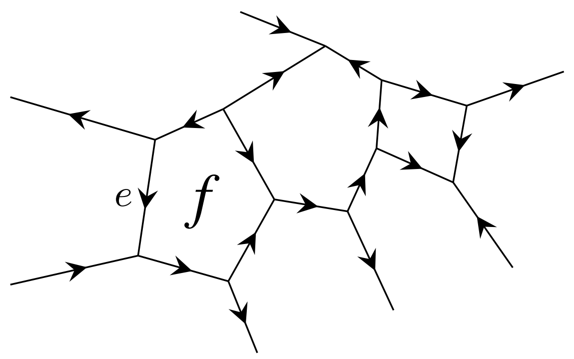
Constraints

Gauss constraint + Flatness constraint





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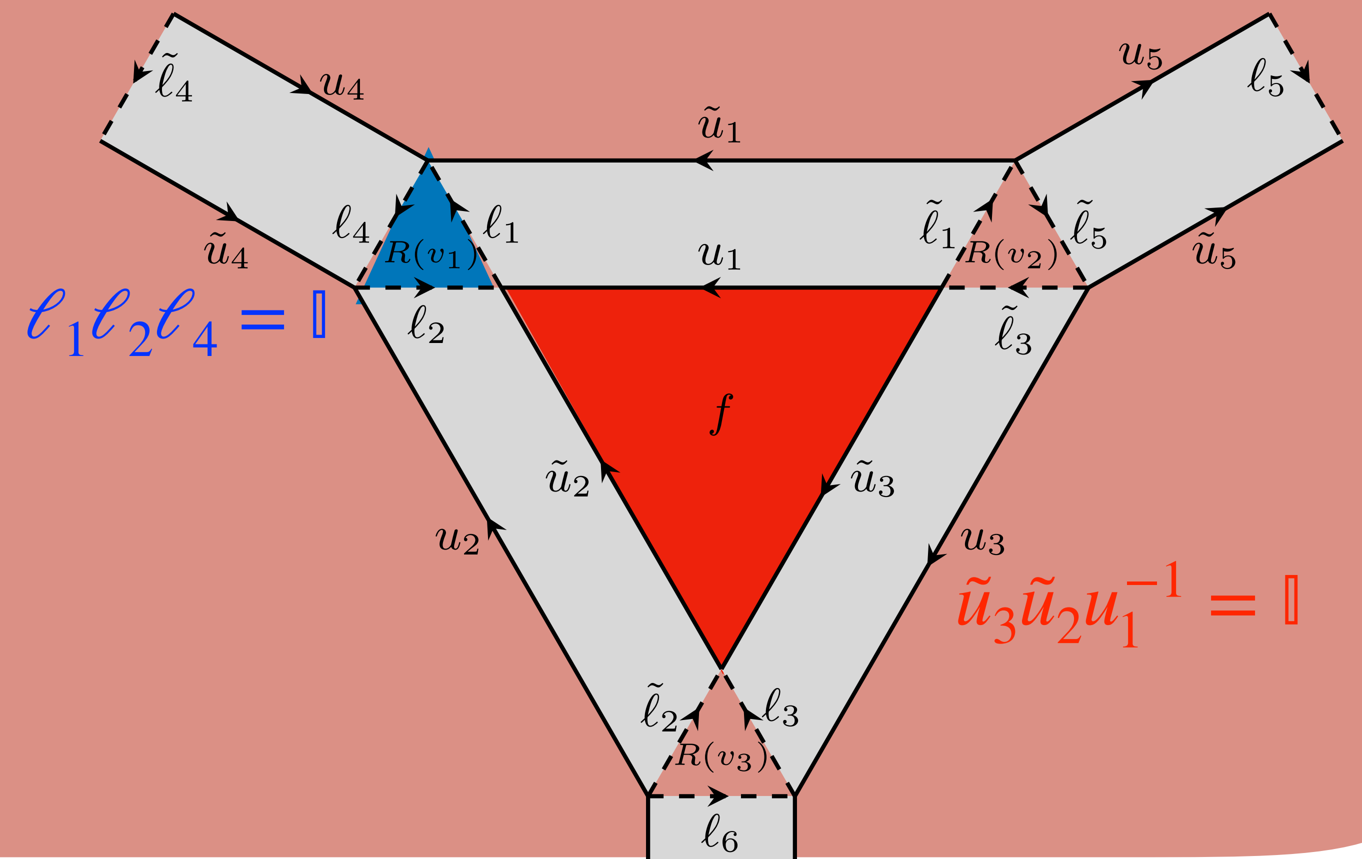

The Program

- Continuous theory

  ↓ discretization & truncation

- **Classical discrete theory**

  ↓ quantization

- Quantum Theory

Loop Gravity phase space
Geometrical interpretation

- Loop Quantum Gravity $\Lambda = 0 \overset{?}{\longrightarrow}$ Loop Quantum Gravity $\Lambda \neq 0$

  Deformation parameter in the Euclidean case with $\kappa = \dfrac{G\sqrt{|\Lambda|}}{c}$:

  Curvature introduced in the momentum space:

$$T^*\mathrm{SU}(2) \sim \mathrm{ISU}(2) = \mathrm{SU}(2) \ltimes \mathfrak{su}(2) \longrightarrow \mathrm{SL}(2,\mathbb{C}) = \mathrm{SU}(2) \bowtie \mathrm{SB}(2,\mathbb{C})$$

κ appears in the $\mathfrak{sb}(2,\mathbb{C})$ generators, the $\tau's$

Symplectic structure defined through the r-matrix, $r = \dfrac{1}{4}\sum_a \tau^a \otimes \sigma^a$

= Heisenberg Double $\big(D(G) = G \bowtie G^*, \pi_H\big)$

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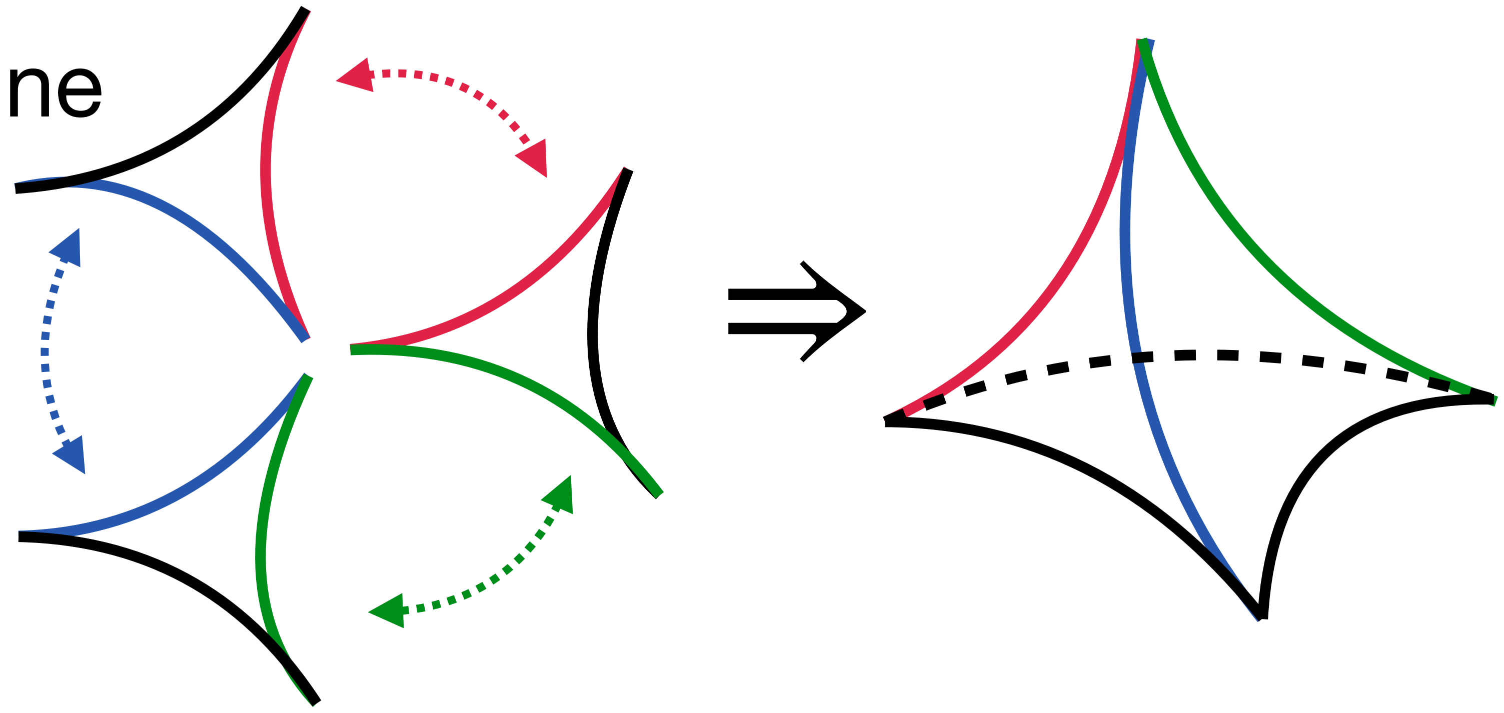
and the "deformed" Gauss and flatness constraints define

a model of discrete hyperbolic geometries.



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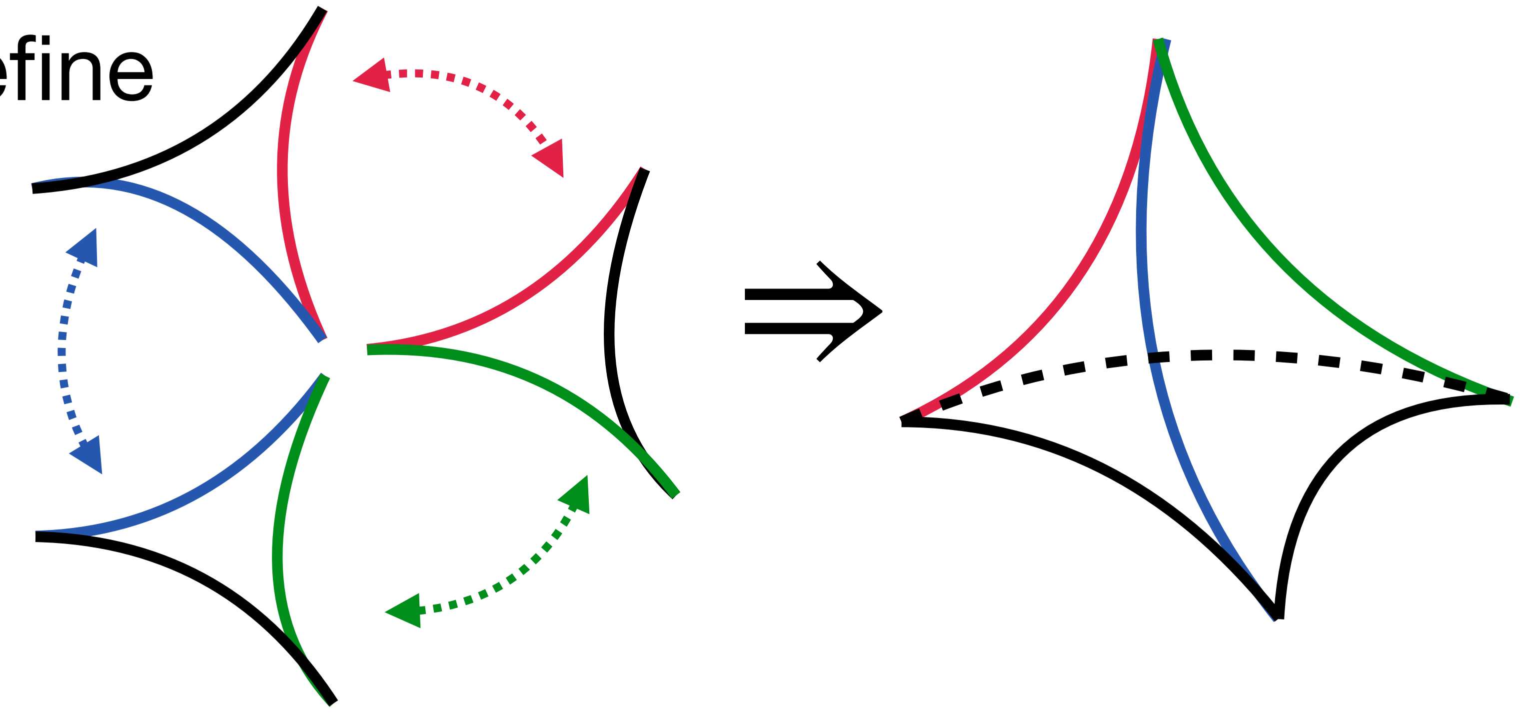

- $\kappa \to 0\,,\ \Lambda = 0$ Loop Quantum Gravity is recovered.

The Program

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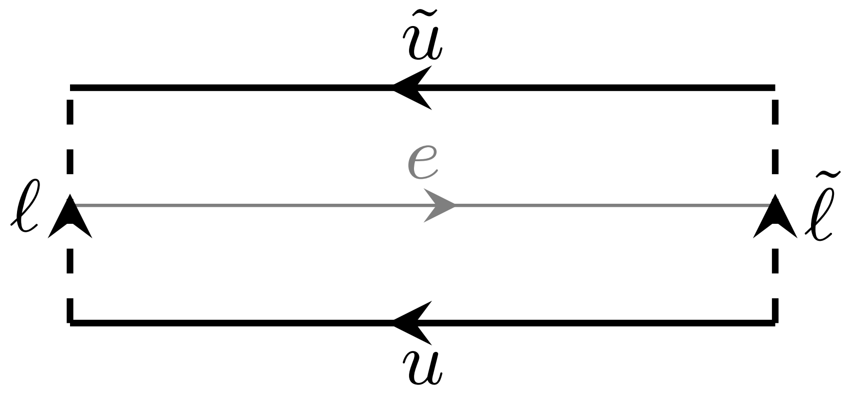
Quantization LQG $\Lambda \neq 0$

Bonzom, MD, Girelli, Pan, '22

Phase space



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Quantization

r-matrix $\quad r \longrightarrow \mathscr{R}$

$$R = \begin{pmatrix} q^{\frac{1}{4}} & 0 & 0 & 0 \\ 0 & q^{-\frac{1}{4}} & q^{-\frac{1}{4}}(q^{\frac{1}{2}} - q^{-\frac{1}{2}}) & 0 \\ 0 & 0 & q^{-\frac{1}{4}} & 0 \\ 0 & 0 & 0 & q^{\frac{1}{4}} \end{pmatrix} \approx \mathbf{1} \otimes \mathbf{1} + i\hbar r + O(\hbar^2)$$

Poisson brackets given by the r-matrix $\rightarrow$ **commutators** given by the $\mathcal{R}$-matrix

$$[\hat{A}, \hat{B}] = i\hbar \widehat{\{A, B\}}$$

The Program

• Continuous theory

discretization & truncation

• Classical discrete theory

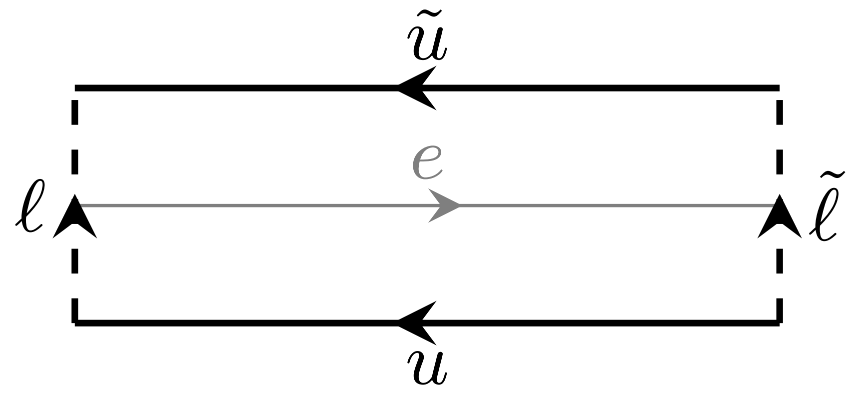
quantization

• Quantum Theory

Quantization LQG $\Lambda \neq 0$

Bonzom, MD, Girelli, Pan, '22

Phase space



$$(\ell, u) \in \mathrm{SB}(2,\mathbb{C}) \times \mathrm{SU}(2) \,/\, D = \ell u \in \mathrm{SL}(2,\mathbb{C})$$

$$\{\ell_1, \ell_2\} = -[r, \ell_1 \ell_2], \quad \{\ell_1, u_2\} = -\ell_1 r u_2,$$

$$\{u_1, \ell_2\} = \ell_2 r^t u_1, \quad \{u_1, u_2\} = -[r^t, u_1 u_2]$$

Quantization

• r-matrix $\quad r \longrightarrow \mathscr{R}$

$$R = \begin{pmatrix} q^{\frac{1}{4}} & 0 & 0 & 0 \\ 0 & q^{-\frac{1}{4}} & q^{-\frac{1}{4}}(q^{\frac{1}{2}} - q^{-\frac{1}{2}}) & 0 \\ 0 & 0 & q^{-\frac{1}{4}} & 0 \\ 0 & 0 & 0 & q^{\frac{1}{4}} \end{pmatrix} \approx \mathbf{1} \otimes \mathbf{1} + i\hbar r + O(\hbar^2)$$

• Phase space variables $\quad D = \ell u = \tilde{u}\tilde{\ell} \in SL(2,\mathbb{C})$

Quantization

$$\tilde{\ell} \in SB(2,\mathbb{C}) \longrightarrow \tilde{L} \in Fun_q(SB(2,\mathbb{C})) \cong \mathfrak{U}_q(\mathfrak{su}(2))$$

$$\tilde{u} \in SU(2) \longrightarrow \tilde{U} \in SU_q(2)$$

The Program

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Quantum Groups

Hopf algebra $\mathfrak{U}_q(\mathfrak{su}(2))$

• Generators $\mathbb{I}, J_\pm, K = q^{\frac{J_z}{2}}$

• Commutation relations $KJ_\pm K^{-1} = q^{\pm 1/2} J_\pm \,, \quad [J_+, J_-] = [2J_z]\,,$ **with** $[n] \equiv \dfrac{q^{\frac{n}{2}} - q^{-\frac{n}{2}}}{q^{\frac{1}{2}} - q^{-\frac{1}{2}}}$

• Hopf algebra with coproduct, antipode and counit

$$\Delta(J_\pm) := J_\pm \otimes K + K^{-1} \otimes J_\pm \,, \quad \Delta(K) := K \otimes K \,, \quad S(J_\pm) := -q^{\pm\frac{1}{2}} J_\pm \,, \quad S(K) := K^{-1} \,, \quad \epsilon\, K = 1 \,, \quad \epsilon\, J_\pm = 0$$

• Quasitriangular Hopf algebra with R-matrix

$$\mathscr{R} = q^{J_z \otimes J_z} \sum_{n=0}^{\infty} \frac{(1-q^{-1})^n}{[n]!} q^{\frac{n(n-1)}{4}} \left(q^{\frac{J_z}{2}} J_+\right)^n \otimes \left(q^{-\frac{J_z}{2}} J_-\right)^n$$

satisfying the quantum Yang-Baxter equation $\mathscr{R}_{12}\mathscr{R}_{13}\mathscr{R}_{23} = \mathscr{R}_{23}\mathscr{R}_{13}\mathscr{R}_{12}$

Hopf algebra $SU_q(2)$

$$T = \begin{pmatrix} \hat{a} & \hat{b} \\ \hat{c} & \hat{d} \end{pmatrix}$$

with $\det_q T := \hat{a}\hat{b} - q^{-\frac{1}{2}} \hat{c}\hat{d} = \mathbb{I}$

• Commutation relations

$$\hat{a}\hat{b} = q^{-\frac{1}{2}} \hat{b}\hat{a} \,, \quad \hat{a}\hat{c} = q^{-\frac{1}{2}} \hat{c}\hat{a} \,, \quad \hat{b}\hat{d} = q^{-\frac{1}{2}} \hat{d}\hat{b} \,,$$
$$\hat{c}\hat{d} = q^{-\frac{1}{2}} \hat{d}\hat{c} \,, \quad \hat{b}\hat{c} = \hat{c}\hat{b} \,, \quad [\hat{a}, \hat{d}] = -(q^{\frac{1}{2}} - q^{-\frac{1}{2}})\hat{b}\hat{c}$$

The Program

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- **Quantum Theory**

q-deformed LQG
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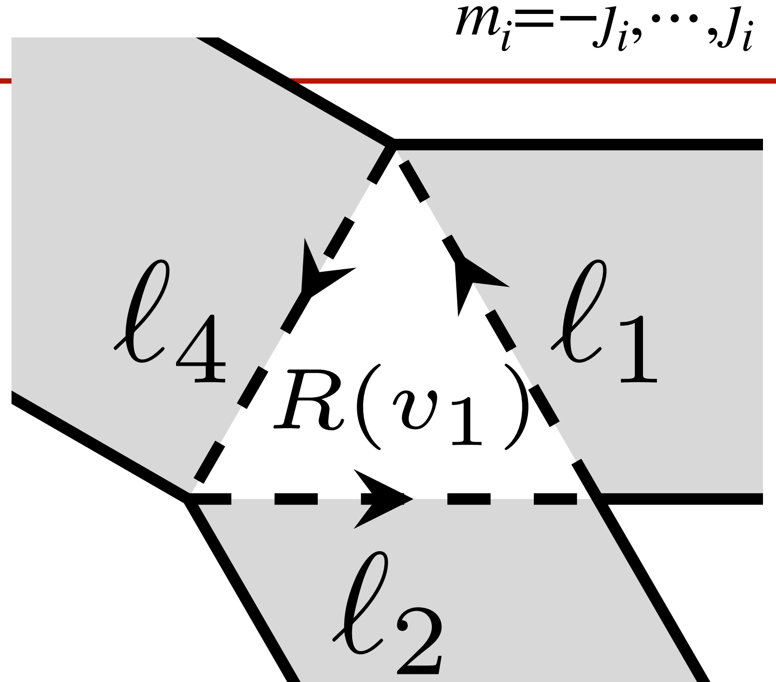
Variables

$$\ell \in SB(2,\mathbb{C}) \longrightarrow L \in Fun_{q^{-1}}(SB(2,\mathbb{C})) \cong \mathfrak{U}_{q^{-1}}(\mathfrak{su}(2))$$
$$u \in SU(2) \longrightarrow U \in SU_{q^{-1}}(2)$$
$$\tilde{\ell} \in SB(2,\mathbb{C}) \longrightarrow \tilde{L} \in Fun_{q}(SB(2,\mathbb{C})) \cong \mathfrak{U}_{q}(\mathfrak{su}(2))$$
$$\tilde{u} \in SU(2) \longrightarrow \tilde{U} \in SU_{q}(2)$$

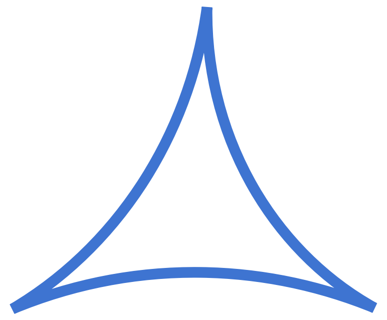
Gauss law at "vertex" v

$$\tilde{\ell}_{e_n v} \cdots \tilde{\ell}_{e_1 v} = \mathbb{I} \rightarrow \tilde{L}_{e_n v} \cdots \tilde{L}_{e_1 v}$$
$$\Delta \tilde{\ell}_{ij} = \sum_k \tilde{\ell}_{kj} \otimes \tilde{\ell}_{ik} \rightarrow \Delta \tilde{L} = \begin{pmatrix} \tilde{K} \otimes \tilde{K} & 0 \\ \alpha(q)(\tilde{J}_+ \otimes \tilde{K} + \tilde{K}^{-1} \otimes \tilde{J}_+) & \tilde{K}^{-1} \otimes \tilde{K}^{-1} \end{pmatrix}$$

$$\tilde{L}_3 \tilde{L}_2 \tilde{L}_1 \, i_{j_1 j_2 j_3} = i_{j_1 j_2 j_3} \Rightarrow i_{j_1 j_2 j_3} = \sum_{m_i = -j_i, \cdots, j_i} (-1)^{j_3 - m_3} q^{-\frac{m_3}{2}} C^{j_1 j_2 j_3}_{m_1 m_2 - m_3} |j_1, m_1\rangle \otimes |j_2, m_2\rangle \otimes |j_3, m_3\rangle$$



$\Leftrightarrow$



The Program

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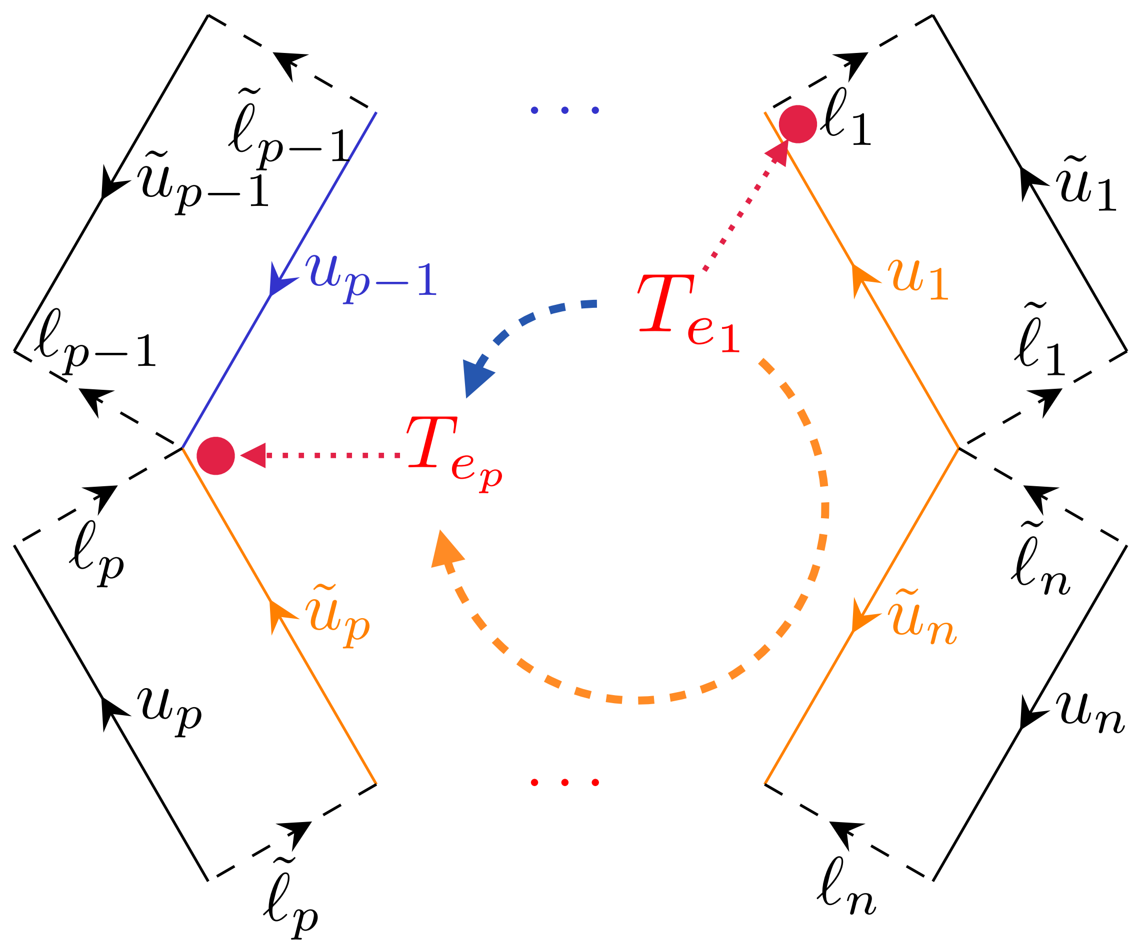
q-deformed LQG
$\Lambda \neq 0$

Bonzom, MD, Pan, '21

(Classical) Flatness constraint $\mathcal{F}_f = \overrightarrow{\prod_{e\in f}} u_e^{o_e} = \begin{cases} u_e & \text{if } o_e = + \\ \tilde{u}_e^{-1} & \text{if } o_e = - \end{cases}$



The Program

- Continuous theory

↓ discretization & truncation

- Classical discrete theory

↓ quantization

- **Quantum Theory**

q-deformed LQG
$\Lambda \neq 0$

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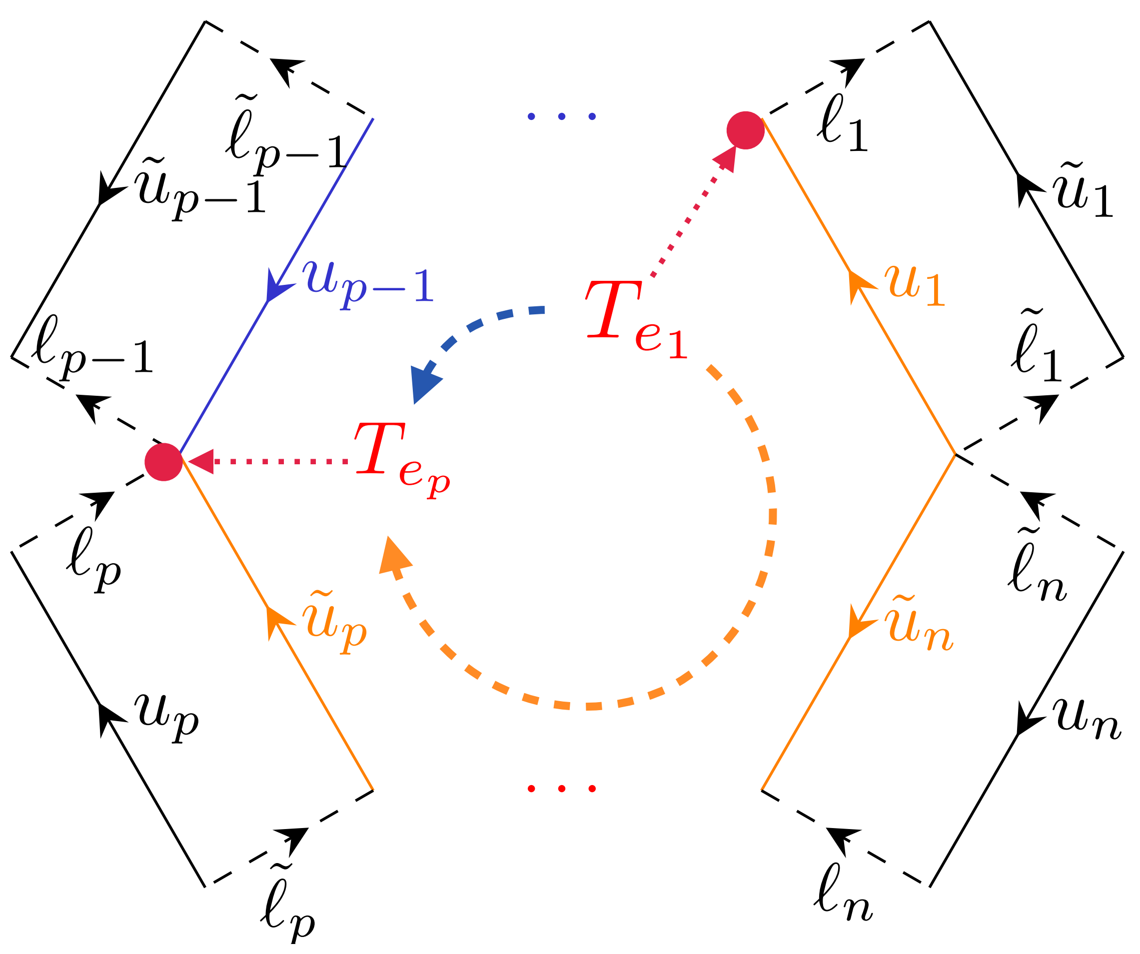
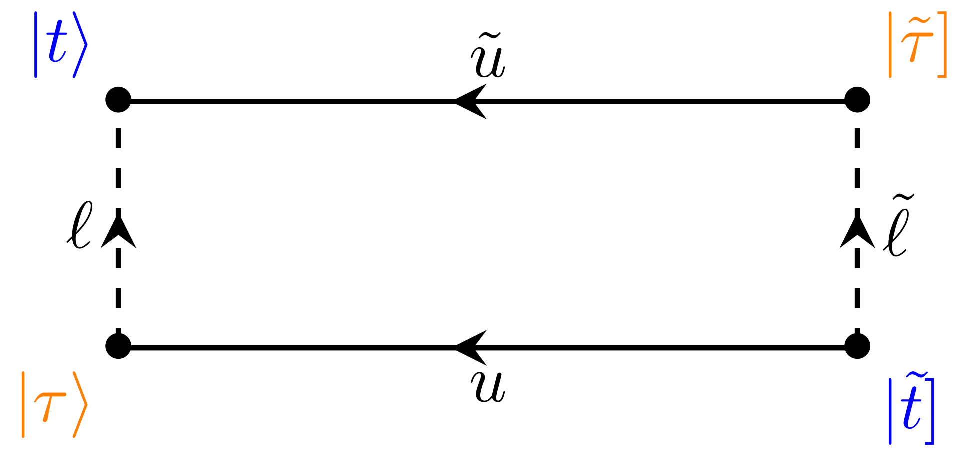
(Classical) Flatness constraint $\mathcal{F}_f = \overrightarrow{\prod_{e\in f}} u_e^{o_e} = \begin{cases} u_e & \text{if } o_e = + \\ \tilde{u}_e^{-1} & \text{if } o_e = - \end{cases}$



[Qiaoyin's talk]

Spinorial formalism for q-deformed loop gravity



Invariant Scalars $E_{e_1\to e_p} = \sum_{A,B} T_{e_p,-A} \left(u^{o_{p-1}}_{e_{p-1}} \cdots u^{o_2}_{e_2}\right)_{AB} T_{e_1,B} = \sum_{A,B} T_{e_p,-A} \left(u^{o_1}_{e_1} \cdots u^{o_p}_{e_p}\right)^{-1}_{AB} T_{e_1,B}$

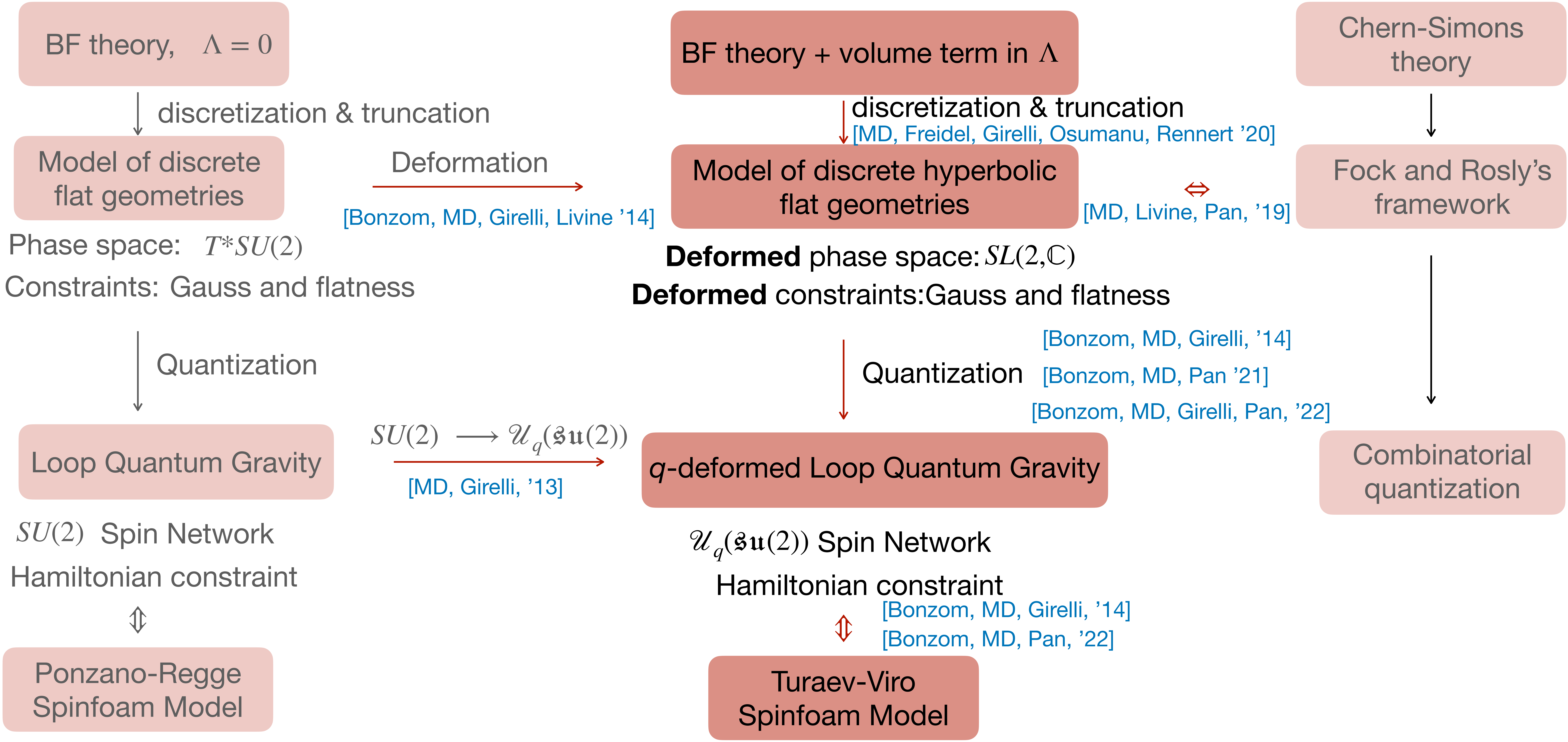
Hamiltonian for face *f* *and edges*

$$h^{\epsilon_1,\epsilon_p}_{f,e_1,e_p} = \sum_{\epsilon_2,\dots,\epsilon_{p-1}=\pm} \prod_{i=2}^{p} \frac{o_i\epsilon_i}{N_{e_i}} E^{\epsilon_i,\epsilon_{i-1}}_{e_i e_{i-1}} - (-1)^{d-p} o_1 o_p \epsilon_1 \epsilon_p \frac{N_{e_1}}{N_{e_p}} \sum_{\epsilon_{p+1},\dots,\epsilon_d=\pm} \prod_{i=p+1}^{d+1} \frac{o_i\epsilon_i}{N_{e_i}} E^{-\epsilon_i,-\epsilon_{i-1}}_{e_i e_{i-1}}$$

⟶ Quantum version: **quantum deformed Hamiltonian**
invariant under Pachner moves and related with the Turaev-Viro model

q-deformed Loop Quantum Gravity

A quantum theory for 3d Euclidean gravity with a negative cosmological constant



q-deformed Loop Quantum Gravity

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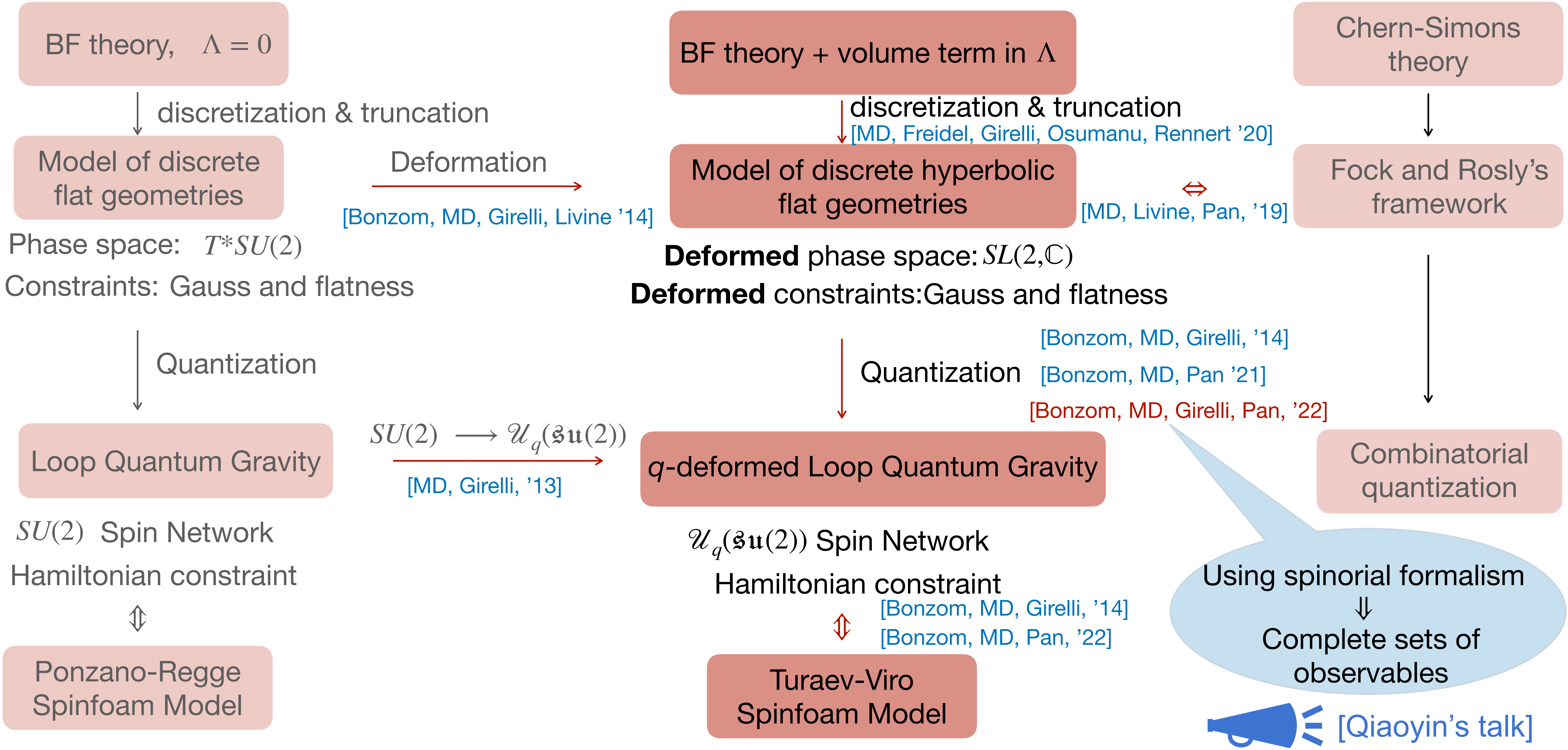

Some open questions

- Other cases of signatures, sign of cosmological constant
- Framework to study the BTZ black hole?
- What can we learn for the 4d case?
- Use of geometrical technics in other approaches?

Thank you!

Thank to Qiaoyin for the nice pictures!

Next: [Qiaoyin’s talk]