# Revisiting Loop Quantum Gravity with selfdual variables

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Joint work with Hanno Sahlmann, to be published soon

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# Setup and motivation

General relativity in Ashtekar variables: [Ashtekar 1986]

$A^i_a$ $\mathrm{SL}(2,\mathbb{C})$ connection: pullback of spacetime connection ${\omega_\mu}^I{}_J$ with

$$*\omega = i\omega$$

$E^a_i$ densitised triads: pullback of $e^I \wedge e^J$, with

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## Benefits & expectations:

- no ambiguous Immirzi parameter
- simple Hamilton constraint
- Bekenstein-Hawking entropy [Frodden, Geiller, Noui, and Perez 2014; Han 2014; Achour and Noui 2015]
- coupling of chiral fermions to selfdual connection
- Ashtekar's variables for supergravity [Eder and Sahlmann 2021a]

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In the literature (far from being exhaustive)

- ideas/fundamentals: [Ashtekar 1986; Ashtekar 1987; Jacobson and Smolin 1987; Samuel 1987; Jacobson and Smolin 1988; Ashtekar, Romano, and Tate 1989]
- model systems: [Thiemann and Kastrup 1993; Wilson-Ewing 2015; Eder and Sahlmann 2021b]
- Wick rotation/coherent state transform/complexifier: [Thiemann 1995; Ashtekar, Lewandowski, Marolf, Mourão, and Thiemann 1996; Thiemann 1996; Varadarajan 2019]
- Ashtekar variables for real LQG: [Wieland 2012]
- emergent space-time [Ashtekar and Varadarajan 2021] (Ashtekar's talk)
- spinfoams [Borissova and Dittrich 2022] (Dittrich's talk)

# Classical theory

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Classical theory:

- Hamilton formulation of selfdual part of complex Palatini action

$$S = \frac{1}{2\kappa}\int_{\mathcal{M}} \epsilon_{IJKL} e^I \wedge e^J \wedge F^{KL}[\omega]$$

- Poisson relation ("additional factor of i")

$$\left\{A_a^i(x), E_j^b(y)\right\} = \frac{i\kappa}{2}\delta_a^b\delta_j^i\delta(x,y)$$

- spatial metric: $qq^{ab} = \eta^{ij}E_i^aE_j^b$ with real internal metric $\eta_{ij} = \eta k_{ij}$
- Constraints (real $N$ and $N^a$)

$$NC = -N\frac{\sqrt{2\eta^3}s_e}{i\eta\sqrt{q}\kappa s_E}\mathring{\epsilon}_{kl}{}^{j}\eta^{li}E_i^aE_j^bF_{ab}^k \qquad \text{Hamilton}$$

$$N^aC_a = \frac{2}{i\kappa}N^aE_i^bF_{ab}^i \qquad \text{Diffeo}$$

$$(A\cdot t)^iG_i = -(A\cdot t)^i\frac{2}{i\kappa}D_a(E_i^a) \qquad \text{Gauß}$$

# Problems and solutions

Symplectic structure degenerate

$\rightarrow$ $\{\cdot\,,\,\cdot\}$ not defined on $\mathrm{Re}A$, $\mathrm{Im}A$, $\mathrm{Re}E, \mathrm{Im}E$

$\rightarrow$ holomorphic phase space description

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## Non-compactness of $\mathrm{SL}(2,\mathbb{C})$

$\longrightarrow$ choice of appropriate measure

## Reality conditions: [Ashtekar, Romano, and Tate 1989]

$\longrightarrow$ spatial metric is real: $q^{ab} \in \mathbb{R}$ (RCI)

$\longrightarrow$ and stays real: $\dot{q}^{ab} \in \mathbb{R}$ (RCII)

$\longrightarrow$ adjointness relations / conditions on measure

# Reality conditions

RCI: reality of the spatial metric

$$qq^{ab} = \eta^{ij} E_i^a E_j^b \in \mathbb{R} \text{ and pos. definite}$$

$\longrightarrow$ allows two different embeddings of the real theory

$\eta_{ij}$ positive definite $\longrightarrow$ $E_i^a \in \mathbb{R}$

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RCII: spatial metric stays real

$$(qq^{ab})^{\cdot} = 2\eta^{ij} E^{(a}_i \dot{E}^{b)}_j \in \mathbb{R} \quad \text{with derivative} \quad \{\,\cdot\,, C[N]\}$$

$\longrightarrow$ condition $A^i_a = \Gamma^i_a + iK^i_a$ with

$\Gamma^i_a(E)$ spin connection

$K^i_a = (A^i_a - \Gamma^i_a)/i$ real

$\longrightarrow$ RCI $\Rightarrow$ $\Gamma^i_a$ is rotation part of $A^i_a$

$\longrightarrow$ RCII: $K^i_a$ boost part of $A^i_a$

# Quantum theory

## Operators:

- $\mathrm{SL}(2,\mathbb{C})$ holonomies $h_e$
- fluxes $E_f(S)$: holomorphic invariant vector fields
- CCR: additional factor of „$i$“

$$L_i\Psi(g) = \frac{\mathrm{d}}{\mathrm{d}\epsilon}\Big|_{\epsilon=0}\Psi(ge^{\epsilon\tau_i}) \quad \& \quad R_i\Psi(g) = \frac{\mathrm{d}}{\mathrm{d}\epsilon}\Big|_{\epsilon=0}\Psi(e^{-\epsilon\tau_i}g)$$

$$[h_e, E_f(S)] = \frac{\kappa}{4}\kappa(e,S)\begin{cases} h_e f(p) & p \text{ source of } e \\ -f(p)h_e & p \text{ target of } e \end{cases}$$

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Holomorphic Hilbert space: $\mathcal{L}^2\left(\mathrm{SL}(2,\mathbb{C}), \mu(g,\overline{g})\mathrm{d}g_{\mathrm{H}}\right) \cap \mathcal{H}\left(\mathrm{SL}(2,\mathbb{C})\right)$

- inner product

$$\langle\phi,\psi\rangle := \int_{\mathrm{SL}(2,\mathbb{C})} \mathrm{d}g_{\mathrm{H}}\, \mu(g,\overline{g})\overline{\phi(g)}\psi(g)$$

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Observations:

- holom. derivatives in $\{\,\cdot\,,\,\cdot\,\}$ $\longrightarrow$ holom. invariant vector fields in $[\,\cdot\,,\,\cdot\,]$
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Idea: use freedom in $\mu\left(g,\overline{g}\right)$ for RC implementation

# Reality condition I

Measure-ansatz: $\mu\left(g,\overline{g}\right)=\mu\left(gg^{\dagger}\right)$, invariant under compact subgroup $\mathrm{SU}(2)$ & cyclic, e. g. $\mu(g,\overline{g})=\mu(\mathrm{tr}(gg^{\dagger}))$

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ONB for $\mathcal{H}\mathcal{L}^2(\mathrm{SL}(2,\mathbb{C}))$

- Weyl's unitary trick: holom. $\mathrm{SL}(2,\mathbb{C})$ irreps from $\mathrm{SU}(2)$ irreps $\Pi_j(g)_{mn}$, $g \in \mathrm{SL}(2,\mathbb{C})$
- RCI + $\mathrm{SU}(2)$ orthogonality relation:

$$\langle \Pi_{jmn}, \Pi_{j'm'n'} \rangle = \frac{\mu_j}{2j+1}\delta_{jj'}\delta_{mm'}\delta_{nn'} \quad \text{with} \quad \mu_j := \int \mathrm{d}|g|\, \mu(|g|^2)\mathrm{tr}(\Pi_j(|g|^2))$$

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Graph Hilbert space $\mathcal{H}_\gamma = \mathcal{L}^2(\overline{\mathcal{A}}_\gamma, \mathrm{d}\mu_\gamma(A))$ + product measure

$\longrightarrow$ $\mathrm{SL}(2,\mathbb{C})$ spin networks

$\longrightarrow$ for $\mu_0 = 1$ cylindrically cons. measure on $\mathcal{H} = \mathcal{L}^2\left(\overline{\mathcal{A}}, \mathrm{d}\Omega\right)$

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First reality condition $\Rightarrow$ new type of consistent measures on $\overline{\mathcal{A}}$, e. g. $\mu(g,\overline{g})\sim\exp\left\{-\mathrm{tr}\left(gg^{\dagger}\right)\right\}$

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Reality condition for holonomies

- short edges

$\rightarrow$ adjointness relation for holonomies

$\rightarrow$ $\Gamma(E)$ complicated function of $E$

$$\overline{A_a^j - \Gamma_a^j} = -(A_a^j - \Gamma_a^j) \quad \rightarrow \quad A_a^j + \overline{A_a^j} = 2\Gamma_a^j$$

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Consistency with CCR: $E_i{}^\dagger = \alpha E_i \longrightarrow 0 = \left[h_e(h_e{}^{-1})^\dagger, (E)^2\right] = (\alpha - 1)(\ldots)$ ? $\longrightarrow$ singles out $E_i{}^\dagger = E_i$

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Known problem! [Wilson-Ewing 2015]

- LQC with selfdual variables
- RCI inconsistency: selfadjoint $E$

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Questions / Where to go from here?

- how strong is this obstruction?
- incompatibility of RC's and quantum theory based on holonomies?
- different formulation of RCII?

# What has been done:

GR in selfdual variables & reality conditions

- degenerate symplectic structure, hence
  - $\longrightarrow$ holomorphic phase space
  - $\longrightarrow$ holomorphic quantisation
  - $\longrightarrow$ reality conditions are no constraints
- two embeddings of real theory

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## Holomorphic quantum theory

- new class of measures
  - $\rightarrow$ $\mathrm{SL}(2, \mathbb{C})$ holonomies
  - $\rightarrow$ cylindrical consistency
- tension between holonomies as operators
  & reality conditions

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Selfdual variables are still interesting to look at and the reality conditions might even be implementable!

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# THANK YOU!

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