Pushing the Loop Frontier in non-planar $\mathcal{N} = 4$ sYM

Alex Edison

6 Loops with JJ Carrasco & H. Johansson: 2112.05178

6 Points with S. He, H. Johansson, O. Schlotterer, F. Teng, Y. Zhang: 22xx.xxxxx

UPPSALA
UNIVERSITET

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Part 1: Non-Planar Six Loop Integrand

Motivations

Goal of program: UV behavior of 7 loop $\mathcal{N} = 8$ SUGRA
Why?

- SUSY arguments predict $L = 7$ counterterm in $D_c = 4$ (Bossard, Howe, Stelle; Green, Russo, Vanhove; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger; many more)
- Similar counterterms proven absent for $\mathcal{N} = 4, 5$ at $L = \mathcal{N} - 1$ (Bossard, Howe, Stelle, Vanhove; Bern, Davies, Dennen; Bern, Davies, Dennen, Huang)
- Improved behavior observed in $D = 4$ kinematics (AE, Hermann, Parra-Martinez, Trnka)

History of direct calculations:

- 1&2 loops '80 -'90s (Green, Schwarz, Brink; Bern, Dixon, Dunbar, Perelstein, Rozowsky)
- 3 loops '07-'10 (Bern, Carrasco, Dixon, Johansson, Kosower, Roiban)
- 4 loops '09-'12 (Bern, Carrasco, Dixon, Johansson, Roiban)
- 5 loops 2018 (Bern, Carrasco, Chen, AE, Johansson, Parra-Martinez, Roiban, Zeng)

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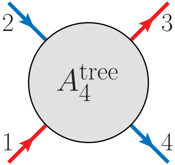
X Identity

Four-point ordered YM tree amplitudes only have s and t channel poles.

What if we try to "sit on the u pole" anyway, via $p_3 \to p_1, p_4 \to p_2$?



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$$A_4^{\text{tree, YM}} = \frac{t_8 F^4}{s\ t}$$

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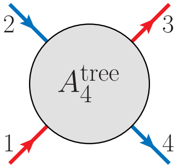

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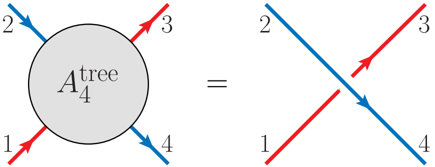
Dimensionless, must respect all symmetries: Can only get identity insertions! The diagram disconnects!



e.g.:
$$A_4^{\text{tree, YM}} = \frac{t_8 F^4}{s\ t} \xrightarrow{p_3=p_1} (\epsilon_1 \cdot \epsilon_3)(\epsilon_2 \cdot \epsilon_4)$$

Same for all other supersymmetric states

H Identity

Can we find a similar identity that maintains planarity?

- Consider $s_{13}A(1,3,2,4)|_{p_3=p_1}$: cut with zero momentum exchange
- Apply $s_{13}A(1,3,2,4) = -s_{12}A(1,2,3,4)$
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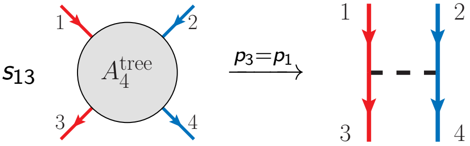

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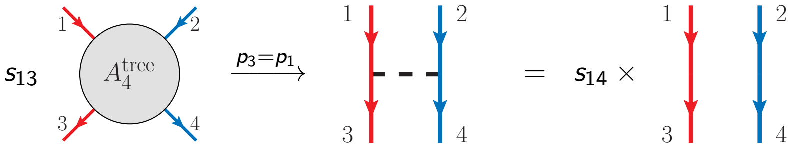

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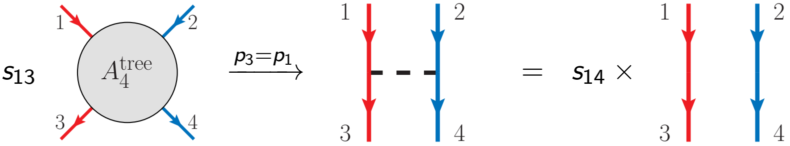
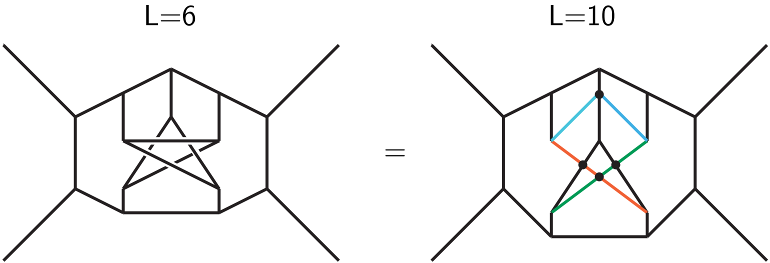


e.g.: $s_{13}A(1,3,2,4) = \dfrac{t_8 F^4}{s_{23}} \xrightarrow{p_3=p_1} s_{14}(\epsilon_1 \cdot \epsilon_3)(\epsilon_2 \cdot \epsilon_4)$

Same for all other supersymmetric states. Extends to (super)gravity.
N.B.: Physical identity (soft factorization), not heuristic rule.

Applications of X ID in MMC

X Identity: Evaluate $\mathcal{C}_{\gamma(k)}$ *directly* from limit of higher-loop cut



Challenges:

1. Edge crossing is NP-Hard[1]
2. Quickly outpace known planar cuts ($L \geq 11$ loops)
3. Only works for color-ordered cuts

[1] ex: There're better crossing schemes than in the diagram. Can you find one?

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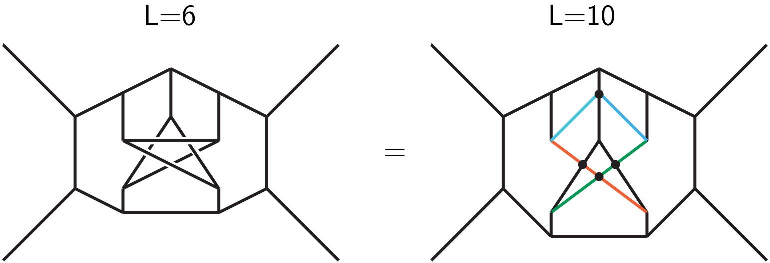

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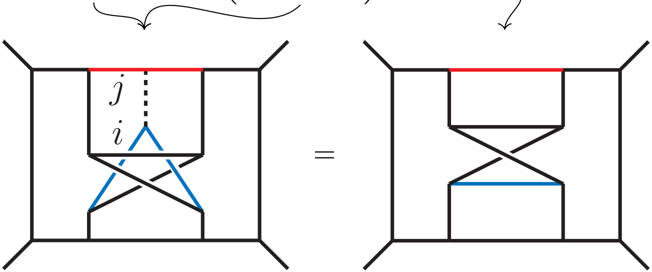
$$\lim_{\ell_m \to 0} \mathcal{P}_{\gamma^{(k)}} = -\left(\lim_{\ell_m \to 0} \mathcal{R}_{\gamma^{(k)}}\right) + (2p_i \cdot p_j)\mathcal{C}_{\gamma^{(k)} \setminus \ell_m}$$



Challenges:

1. Need to merge conditions
2. Many evaluations of lower-loop cuts

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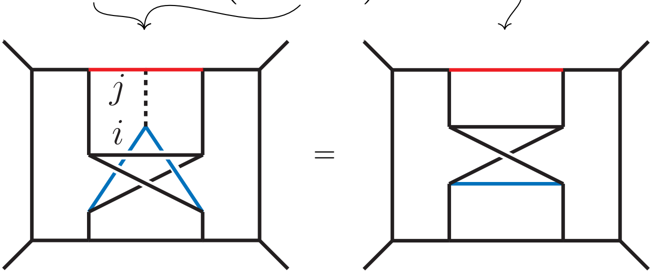

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Resolving the conditions

$\lim_{\ell_m \to 0}$ realized as projection on invariants: π_{ℓ_m}
Know that

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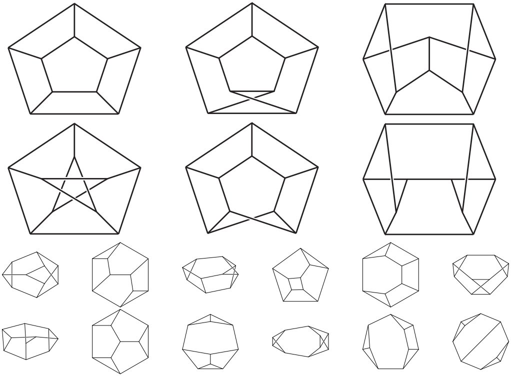
(Almost) always find that lowest degree generator of $\mathcal{I}_{\gamma^{(k)}}$ is unique and the correct degree to be $\mathcal{P}$. Significantly faster than solving ansatz.

Integrand Construction

DOI: 10.5281/zenodo.5765781



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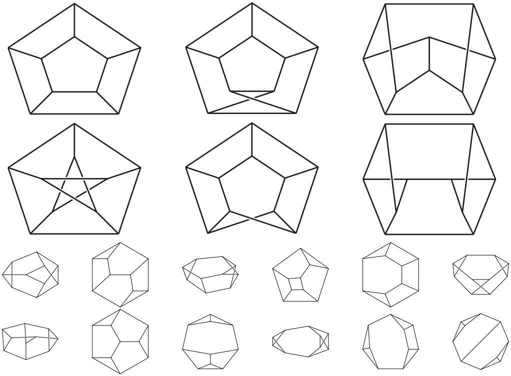

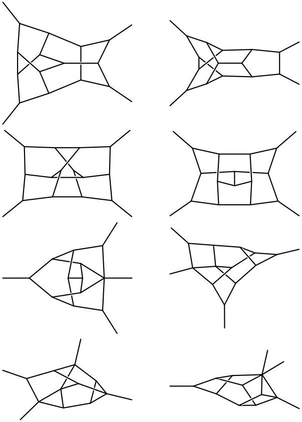

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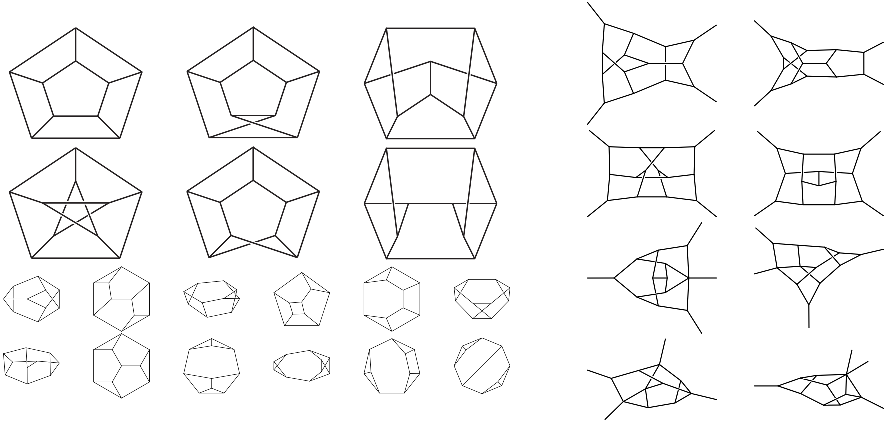

N^kM	0	1	2	3	4	$\sum$
cuts	5548	41649	156853	363963	576582	1,144,595
non-zero contacts	4420	16776	37653	56717	36087	151653

Integrand exitsts, but obviously too large to put on slides

Looking Forward

- Improve representation: diagram power counting, generalized double copy
- Prepping tools for SUGRA: gravity cuts, IBPs
- Improve efficiency for 7 loops: 6L MCs take seconds, 7L MCs take hours
  - Intersection of ideals is senstive to many superficial choices
  - Better diagram parameterization
  - Minimize number of limits to evaluate
- Other applications: higher points, QCD and lower SUSY, open string EFT

Part 2: Color-Kinematics Duality at $n = 6, L = 1$

Motivations

BCJ form at 6 points has long eluded multiple approaches:

- 4D: hexagon maximal cut impossible; Bjerrum-Bohr, Dennen, Monteiro, O'Connell '13: spurious poles in numerator...
- Brute force ansatz: degree-6 polynomial over $\varepsilon_i \cdot \varepsilon_j$, $\varepsilon_i \cdot p_j$, $p_i \cdot p_j$ – hundreds of thousands of terms
- String theoretic/forward limit approach – prescriptive, but...
  - Mafra & Schlotterer '14: via pure spinors
    Works well for five point. Six point numerators break BCJ
  - He, Schlotterer & Zhang '17: forward limit, only on linear propagators
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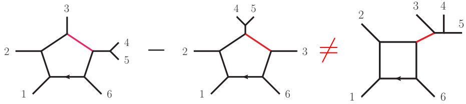

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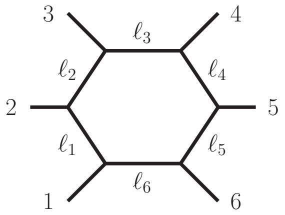
Mixing approaches

Use covariant structures that appear in forward limit/max cut: (AE, He, Schlotterer, Teng)

$$N_{\text{hex}} \supset \{(\varepsilon_1 \cdot \ell_1)(\varepsilon_2 \cdot \ell_2) t_8(f_3, f_4, f_5, f_6) + \text{perms}, \\ (\varepsilon_1 \cdot \ell_1) t_8(f_2, f_{[3,4]}, f_5, f_6) + \text{perms}, \\ t_8(f_1, f_{[2,3]}, f_{[4,5]}, f_6) + \text{perms}, \\ t_8(f_1, f_{[2,[3,4]]}, f_5, f_6) + \text{perms}, \\ t_{12}(f_1, f_2, f_3, f_4, f_5, f_6)\}$$



$$f_i^{\mu\nu} = k_i^\mu \varepsilon_i^\nu - k_i^\nu \varepsilon_i^\mu$$

$$f_{[i,j]} = f_i f_j - f_j f_i$$

$$t_8(f_w, f_x, f_y, f_z) = \text{tr}(f_w f_x f_y f_z) - \frac{1}{4}\text{tr}(f_w f_x)\,\text{tr}(f_y f_z) + \text{cyc}(x, y, z)$$

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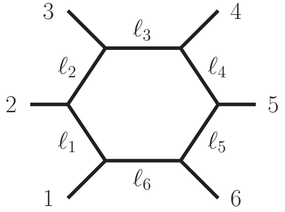

$$f_i^{\mu\nu} = k_i^\mu \varepsilon_i^\nu - k_i^\nu \varepsilon_i^\mu$$
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But allow contact freedom

$$N_{\text{hex}} \supset \{\varepsilon_1 \cdot \varepsilon_2 t_8(f_3, f_4, f_5, f_6)\{\ell_7^2, \ell_1^2, \ell_2^2\}, \quad \varepsilon_1 \cdot \varepsilon_3 t_8(f_2, f_4, f_5, f_6)\{\ell_7^2, \ell_1^2, \ell_2^2, \ell_3^2\}, \quad \varepsilon_1 \cdot \varepsilon_4 t_8(f_2, f_3, f_5, f_6)\{\ell_7^2, \ell_1^2, \ell_3^2, \ell_4^2\}\} + \text{cyc}$$

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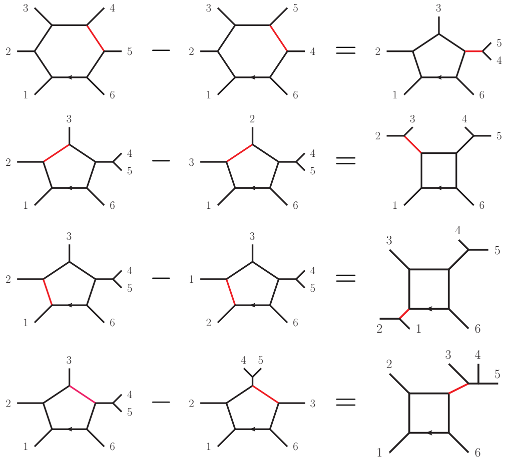
Color-kinematics duality relations

Define lower *n*-gons through Jacobis



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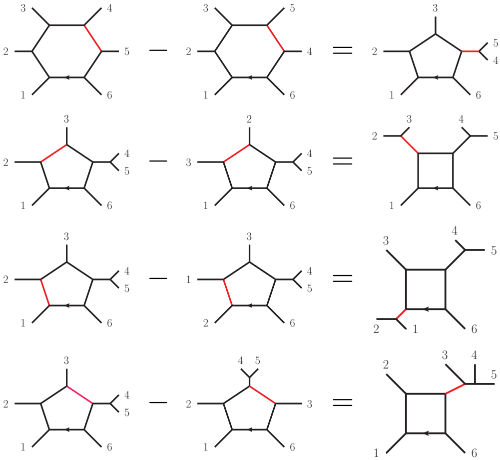
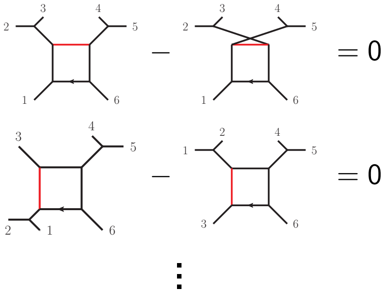


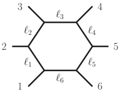
No-Triangle Jacobis



Relations lock contact terms to each other

The hexagon numerator

- Fixed by hexagon maximal cut
- Contact term corrections; related by no-triangle
- Related by gauge invariance of pentagon max cut

$N\Big[$  $\Big] =$

$$
\begin{aligned}
&(\varepsilon_1 \cdot \ell_1)(\varepsilon_2 \cdot \ell_2) t_8(f_3, f_4, f_5, f_6) - \frac{1}{2}(\varepsilon_1 \cdot \ell_1) t_8(f_2, f_{[3,4]}, f_5, f_6) \\
&+\frac{1}{4} t_8(f_1, f_{[2,3]}, f_{[4,5]}, f_6) + \frac{1}{6} t_8(f_1, f_{[2,[3,4]]}, f_5, f_6) + \text{perms} \\
&+ t_{12}(f_1, f_2, f_3, f_4, f_5, f_6) \\
&+ \frac{1}{40}\Big[\varepsilon_1 \cdot \varepsilon_2 (3\ell_6^2 - 10\ell_1^2 + 3\ell_2^2) t_8(f_3, f_4, f_5, f_6) \\
&\quad + \varepsilon_3 \cdot \varepsilon_3 (\ell_6^2 - 3\ell_1^2 - 3\ell_2^2 + \ell_3^2) t_8(f_2, f_4, f_5, f_6) \\
&\quad - \varepsilon_1 \cdot \varepsilon_4 (\ell_6^2 + \ell_1^2) t_8(f_2, f_3, f_5, f_6) + \text{cyc}(1,2,3,4,5,6)\Big]
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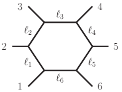
N [] =

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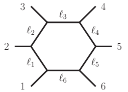
&+\frac{1}{4}t_8(f_1,f_{[2,3]},f_{[4,5]},f_6)+\frac{1}{6}t_8(f_1,f_{[2,[3,4]]},f_5,f_6)+\text{perms}\\
&+t_{12}(f_1,f_2,f_3,f_4,f_5,f_6)\\
&+\frac{1}{40}\Big[\varepsilon_1\cdot\varepsilon_2(3\ell_6^2-10\ell_1^2+3\ell_2^2)t_8(f_3,f_4,f_5,f_6)\\
&\quad+\varepsilon_3\cdot\varepsilon_3(\ell_6^2-3\ell_1^2-3\ell_2^2+\ell_3^2)t_8(f_2,f_4,f_5,f_6)\\
&\quad-\varepsilon_1\cdot\varepsilon_4(\ell_6^2+\ell_1^2)t_8(f_2,f_3,f_5,f_6)+\text{cyc}(1,2,3,4,5,6)\Big]
\end{aligned}
$$

The hexagon numerator

- Fixed by hexagon maximal cut
- Contact term corrections; related by no-triangle
- Related by gauge invariance of pentagon max cut

$N\left[\ \ \right] =$



$$
\begin{aligned}
&(\varepsilon_1\cdot\ell_1)(\varepsilon_2\cdot\ell_2)t_8(f_3,f_4,f_5,f_6) - \frac{1}{2}(\varepsilon_1\cdot\ell_1)t_8(f_2,f_{[3,4]},f_5,f_6)\\
&+\frac{1}{4}t_8(f_1,f_{[2,3]},f_{[4,5]},f_6) + \frac{1}{6}t_8(f_1,f_{[2,[3,4]]},f_5,f_6) + \text{perms}\\
&+t_{12}(f_1,f_2,f_3,f_4,f_5,f_6)\\
&+\frac{1}{40}\Big[\varepsilon_1\cdot\varepsilon_2(3\ell_6^2 - 10\ell_1^2 + 3\ell_2^2)t_8(f_3,f_4,f_5,f_6)\\
&\quad + \varepsilon_3\cdot\varepsilon_3(\ell_6^2 - 3\ell_1^2 - 3\ell_2^2 + \ell_3^2)t_8(f_2,f_4,f_5,f_6)\\
&\quad - \varepsilon_1\cdot\varepsilon_4(\ell_6^2 + \ell_1^2)t_8(f_2,f_3,f_5,f_6) + \text{cyc}(1,2,3,4,5,6)\Big]
\end{aligned}
$$

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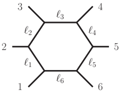
N [



] =

$$
\begin{aligned}
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&+ \frac{1}{4} t_8(f_1, f_{[2,3]}, f_{[4,5]}, f_6) + \frac{1}{6} t_8(f_1, f_{[2,[3,4]]}, f_5, f_6) + \text{perms} \\
&+ t_{12}(f_1, f_2, f_3, f_4, f_5, f_6) \\
&+ \frac{1}{40}\Big[\varepsilon_1 \cdot \varepsilon_2 (3\ell_6^2 - 10\ell_1^2 + 3\ell_2^2) t_8(f_3, f_4, f_5, f_6) \\
&\quad + \varepsilon_3 \cdot \varepsilon_3 (\ell_6^2 - 3\ell_1^2 - 3\ell_2^2 + \ell_3^2) t_8(f_2, f_4, f_5, f_6) \\
&\quad - \varepsilon_1 \cdot \varepsilon_4 (\ell_6^2 + \ell_1^2) t_8(f_2, f_3, f_5, f_6) + \text{cyc}(1,2,3,4,5,6)\Big]
\end{aligned}
$$

No free parameters remaining!

Going beyond

- Seven Points: Need new classes of contact terms

$$N_{\text{hept}} \supset \{(\varepsilon_1 \cdot \varepsilon_2)(\varepsilon_3 \cdot \ell_j)\,\ell_k^2\, t_8(f_4, f_5, f_6, f_7),\\ \varepsilon_1 \cdot \varepsilon_2\, \ell_k^2\, t_8(f_3, f_{[4,5]}, f_6, f_7), \ldots\}$$

  Heptagon cut + color-kinematics + gauge invariance leaves 30 free params; hexa-,penta-,tetra-gon cuts no extra info

- Eight Points: Will have double-contacts $\ell_i^2 \ell_j^2$. Interesting to see if n-gon+CK+GI is still everything.
- Two loops: MCs accessible via direct sewing $\rightarrow$ test limits of H-identity construction

Going beyond

- Seven Points: Need new classes of contact terms

$$N_{\text{hept}} \supset \{(\varepsilon_1 \cdot \varepsilon_2)(\varepsilon_3 \cdot \ell_j)\, \ell_k^2\, t_8(f_4, f_5, f_6, f_7), \\ \varepsilon_1 \cdot \varepsilon_2\, \ell_k^2\, t_8(f_3, f_{[4,5]}, f_6, f_7), \ldots\}$$

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Summary

1. New methods for dimension-agnostic cut recursion, applied to non-planar 6-loop sYM. Making progress towards 7-loop SUGRA
2. 11-parameter BCJ construction of 6 pt 1-loop sYM, solving decade-old problem
3. Opens door to studying non-planar higher points and higher loops in *d* dimensions

Thanks!