A CFT PERSPECTIVE ON AdS AMPLITUDES

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Based on papers w/ FARDELLI, GEORGOUDIS, MANENTI

GOAL: USE AdS/CFT CORRESPONDENCE
+
CONFORMAL BOOTSTRAP

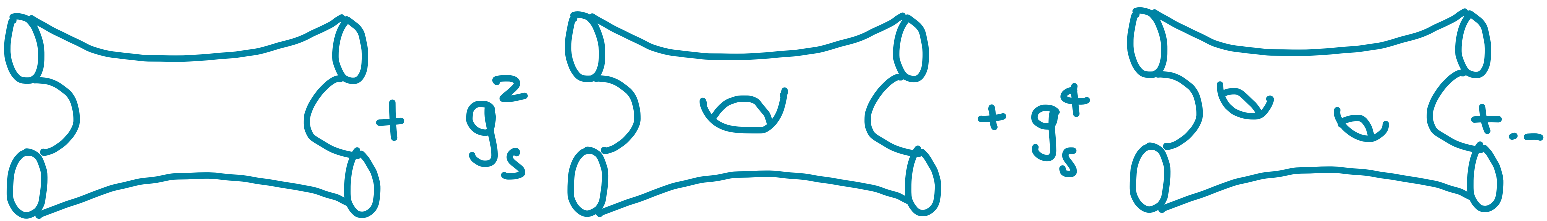
TO CONSTRUCT AMPLITUDES IN AdS.

PLAN:
- REVIEW THE METHOD
- STRUCTURE OF ALL LOOP AMPLITUDES
- HIGHER TRACE OPERATORS

- NEW DIRECTIONS & OPEN PROBLEMS

IN FLAT SPACE

SCATTERING AMPLITUDES IN STRING THEORY ORGANISE IN A GENUS EXPANSION :

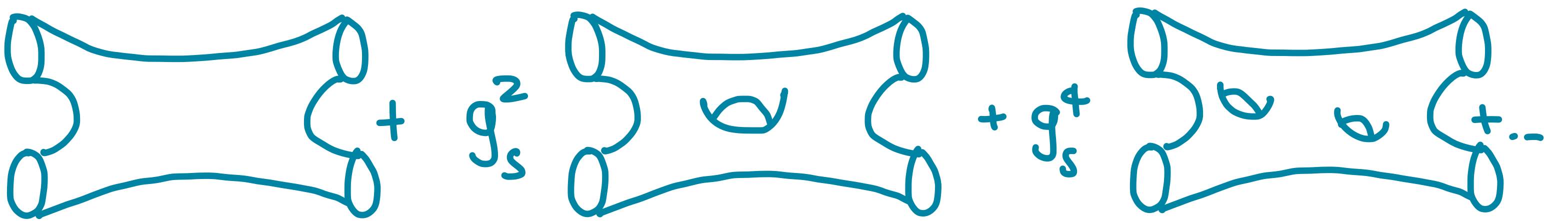


$$A^{(0)}(\alpha', \underline{s, t, u}) + g_s^2 \, A^{(1)}(\alpha', s, t, u) + g_s^4 \, A^{(2)}(\alpha', s, t, u) + \dots$$

MANDELSTAM VARIABLES

IN FLAT SPACE

SCATTERING AMPLITUDES IN STRING THEORY ORGANISE IN A GENUS EXPANSION :



$$A^{(0)}(\alpha', s, t, u) = \frac{\Gamma\left(-\frac{\alpha' s}{4}\right)\Gamma\left(-\frac{\alpha' t}{4}\right)\Gamma\left(-\frac{\alpha' u}{4}\right)}{\Gamma\left(1+\frac{\alpha' s}{4}\right)\Gamma\left(1+\frac{\alpha' t}{4}\right)\Gamma\left(1+\frac{\alpha' u}{2}\right)}$$

VIRASORO-SHAPIRO AMPLITUDE

IN FLAT SPACE

BY EXPANDING THE AMPLITUDE IN $\alpha' \to 0$, WE OBTAIN

$$A^{(0)}(\alpha', s, t, u) = \underbrace{\frac{1}{stu}}_{\text{SUPERGRAVITY}} + \underbrace{(\alpha')^3 + (\alpha')^5 (s^2 + t^2 + u^2)}_{\text{HIGHER DERIVATIVE CORRECTIONS}} + \dots$$

IT IS REMINISCENT OF THE LOW ENERGY EXPANSION OF THE ACTION.

FROM FLAT TO CURVED SPACE?

- WRITE DOWN THE ACTION, READ OFF THE VERTICES AND COMPUTE THE CORRESPONDING WITTEN DIAGRAMS.

  ↓

  THOUSANDS OF THEM AND NOT MANIFEST PRINCIPLE TO GROUP THEM

IDEA OF THIS TALK:

- USE THE DUAL CFT DESCRIPTION TO STUDY GRAVITON (AND GLUON) AMPLITUDES IN A g_s AND α' EXPANSION.

- SYMMETRIES (SUPERSYMMETRY + CONFORMAL SYM) ALLOW US TO COMPUTE / CONSTRAIN THE AMPLITUDE.

AdS / CFT CORRESPONDENCE

TYPE IIB SUPERSTRING THEORY ON $AdS_5 \times S^5$

- $\sqrt{\alpha'}$ STRING LENGTH
- g_s STRING COUPLING

4 DIMENSIONAL $\mathcal{N}=4$ SUPERYANG MILLS WITH $SU(N)$ AND $SU(4)_R$

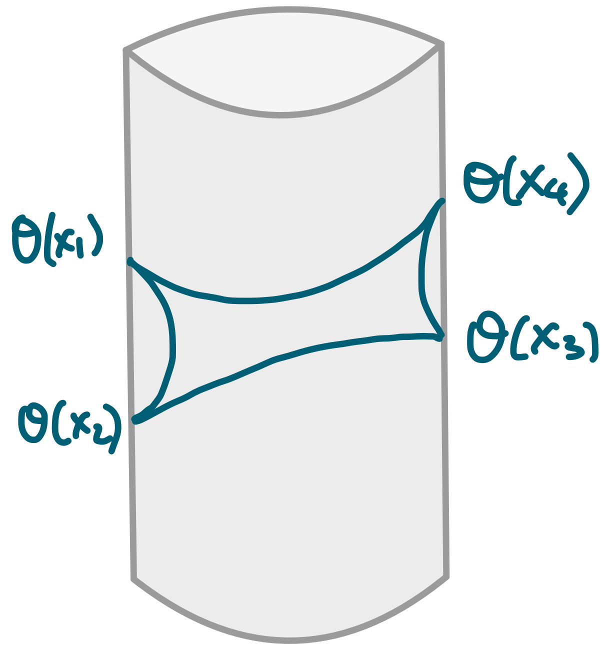
- N RANK OF THE GAUGE GROUP
- g_{YM} COUPLING

GENUS EXP. $$N \sim \frac{1}{g_s}$$

HIGHER DERIVATIVE EXP $$\lambda = g_{YM}^2 N = \frac{1}{(\alpha')^2}$$

IDEA

AMPLITUDES $\longleftrightarrow$ CORRELATORS



IDEA

AMPLITUDES $\longleftrightarrow$ CORRELATORS

SINGLE TRACE $\frac{1}{2}$BPS
OPERATORS

$$\mathcal{O}_p = \mathrm{Tr}\left(\phi_{i_1} \cdots \phi_{i_p}\right)$$

$$\Delta = p \qquad [0,p,0]_R$$

IDEA

AMPLITUDES $\longleftrightarrow$ CORRELATORS

GRAVITON — $p=2$ STRESS TENSOR θ_2

KALUZA KLEIN MODES — $p>2$

$$\langle \theta_2(x_1,y_1)\,\theta_2(x_2,y_2)\,\theta_2(x_3,y_3)\,\theta_2(x_4,y_4)\rangle$$

GRAVITON AMPLITUDES

$$\langle \mathcal{O}_2(x_1,y_1)\,\mathcal{O}_2(x_2,y_2)\,\mathcal{O}_2(x_3,y_2)\,\mathcal{O}_2(x_4,y_4)\rangle =$$

$$= \frac{(y_1\cdot y_2)^2(y_3\cdot y_4)^2}{x_{12}^4\,x_{34}^4}\,\mathcal{G}(u,v,\sigma,\tau)$$

GRAVITON AMPLITUDES

$$\langle \mathcal{O}_2(x_1,y_1)\,\mathcal{O}_2(x_2,y_2)\,\mathcal{O}_2(x_3,y_2)\,\mathcal{O}_2(x_4,y_4)\rangle =$$

$$= \frac{(y_1\cdot y_2)^2 (y_3\cdot y_4)^2}{x_{12}^4\, x_{34}^4}\, \mathcal{G}(u,v,\sigma,\tau)$$

CROSS RATIOS

$$\boxed{u} = \frac{x_{12}^2\, x_{34}^2}{x_{13}^2\, x_{24}^2} = z\bar{z} \qquad \boxed{v} = \frac{x_{14}^2\, x_{23}^2}{x_{13}^2\, x_{24}^2} = (1-z)(1-\bar{z})$$

$$\boxed{\sigma} = \frac{y_1\cdot y_3\;\, y_2\cdot y_4}{y_1\cdot y_2\;\, y_3\cdot y_4} = \alpha\bar{\alpha} \qquad \boxed{\tau} = \frac{y_1\cdot y_4\;\, y_2\cdot y_3}{y_1\cdot y_2\;\, y_3\cdot y_4} = (1-\alpha)(1-\bar{\alpha})$$

HARMONIC CROSS RATIOS

GRAVITON AMPLITUDES

$$\langle \mathcal{O}_2(x_1,y_1)\,\mathcal{O}_2(x_2,y_2)\,\mathcal{O}_2(x_3,y_2)\,\mathcal{O}_2(x_4,y_4)\rangle =$$

$$= \frac{(y_1\cdot y_2)^2(y_3\cdot y_4)^2}{x_{12}^4\,x_{34}^4}\,\mathcal{G}(u,v,\sigma,\tau)$$

polynomial in σ and τ.

- USE SUPERCONFORMAL WARD IDENTITIES TO ISOLATE THE CONTRIBUTION TO THE 4 POINT FUNCTION OF PROTECTED OPERATORS:

$$(z\,\partial_z - \alpha\,\partial_\alpha)\,\mathcal{G}(z,\bar{z},\alpha,\bar{\alpha})\Big|_{\alpha=\frac{1}{z}} = 0$$

DOLAN & OSBORN

GRAVITON AMPLITUDES

$$\langle \mathcal{O}_2(x_1,y_1)\,\mathcal{O}_2(x_2,y_2)\,\mathcal{O}_2(x_3,y_2)\,\mathcal{O}_2(x_4,y_4)\rangle =$$

$$= \frac{(y_1\cdot y_2)^2(y_3\cdot y_4)^2}{x_{12}^4\,x_{34}^4}\,\mathcal{G}(u,v,\sigma,\tau)$$

$$\mathcal{G}(u,v,\sigma,\tau) = \mathcal{G}^{\text{SHORT}}(u,v,\sigma,\tau) + \boxed{R(u,v,\sigma,\tau)\,\mathcal{H}^{\text{LONG}}(u,v)}$$

GRAVITON AMPLITUDES

$$\langle \mathcal{O}_2(x_1,y_1)\,\mathcal{O}_2(x_2,y_2)\,\mathcal{O}_2(x_3,y_2)\,\mathcal{O}_2(x_4,y_4)\rangle =$$

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$$\downarrow$$

$$\mathcal{H}(u,v) = \sum_{\substack{\Delta,\ell \\ \text{LONG}}} a_{\Delta,\ell}\, g^{s}_{\Delta,\ell}(u,v)$$

GRAVITON AMPLITUDES

$$\langle \mathcal{O}_2(x_1,y_1)\,\mathcal{O}_2(x_2,y_2)\,\mathcal{O}_2(x_3,y_3)\,\mathcal{O}_2(x_4,y_4)\rangle =$$

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$$\mathcal{G}(u,v,\sigma,\tau) = \mathcal{G}^{\text{SHORT}}(u,v,\sigma,\tau) + \boxed{R(u,v,\sigma,\tau)\, \mathcal{H}^{\text{LONG}}(u,v)}$$

$$\downarrow$$

$g^{S}_{\Delta,\ell}$: SUPERCONFORMAL BLOCKS.

$$\mathcal{H}(u,v) = \sum_{\substack{\Delta,\ell \\ \text{LONG}}} a_{\Delta,\ell}\, g^{S}_{\Delta,\ell}(u,v)$$

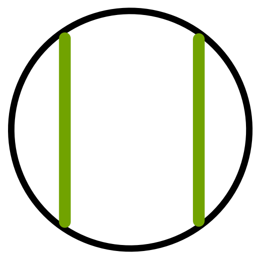
SUPERGRAVITY APPROXIMATION

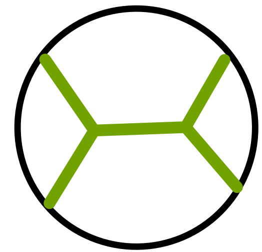
$$\mathcal{H}(u,v) = \mathcal{H}^{(0)}(u,v) + \frac{1}{N^2}\mathcal{H}^{(1)}(u,v) + \frac{1}{N^4}\mathcal{H}^{(2)}(u,v) + \dots$$

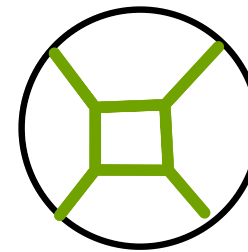
- THE IDEA IS TO USE CROSSING SYMMETRY (THUS ALSO THE KNOWLEDGE OF $\mathcal{G}^{\text{SHORT}}$) AND THE TECHNOLOGY OF THE ANALYTIC CONFORMAL BOOTSTRAP TO CONSTRUCT $\mathcal{H}^{(k)}$ STARTING WITH $\mathcal{H}^{(0)}$.

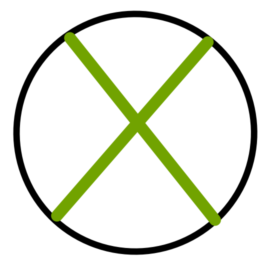
SUPERGRAVITY APPROXIMATION

$$\mathcal{H}(u,v) = \mathcal{H}^{(0)}(u,v) + \frac{1}{N^2}\mathcal{H}^{(1)}(u,v) + \frac{1}{N^4}\mathcal{H}^{(2)}(u,v) + \dots$$









- INTERMEDIATE OPERATORS ARE DOUBLE TRACES

COMMENTS ON THE METHOD

- IT IS POSSIBLE TO RECONSTRUCT THE CORRELATOR BY KNOWING ITS SINGULARITIES / DOUBLE DISCONTINUITY:

$$c_{\Delta,\ell} \sim \int_0^1 dz\, d\bar{z}\; \mu(z,\bar{z})\; \mathrm{dDisc}\left[\mathcal{G}(z,\bar{z})\right]$$

$$\mathrm{dDisc}\left[\mathcal{G}(z,\bar{z})\right] = \mathcal{G}_{\text{Eucl}}(z,\bar{z}) - \underbrace{\frac{1}{2}\mathcal{G}^{\circlearrowleft}(z,\bar{z}) - \frac{1}{2}\mathcal{G}^{\circlearrowright}(z,\bar{z})}_{\text{ANALYTIC CONT. AROUND } \bar{z}=1}$$

CARON-HUOT

COMMENTS ON THE METHOD

- IT IS POSSIBLE TO RECONSTRUCT THE CORRELATOR BY KNOWING ITS SINGULARITIES / DOUBLE DISCONTINUITY:

$$c_{\Delta,\ell} \sim \int_0^1 dz\, d\bar{z}\; \mu(z,\bar{z})\; \mathrm{dDisc}\left[\mathcal{G}(z,\bar{z})\right]$$

$$\boxed{c_{\Delta,\ell}} \xrightarrow[\Delta \to \Delta_k]{} \frac{a_{\Delta_k,\ell}}{\Delta - \Delta_k}$$

- HAS POLES AT THE DIMENSION OF THE EXCHANGED OPS WITH RESIDUE THE 3pt FUNCT

BACK TO OUR PROBLEM

$$\mathcal{H}(u,v) = \mathcal{H}^{(0)}(u,v) + \frac{1}{N^2}\mathcal{H}^{(1)}(u,v) + \frac{1}{N^4}\mathcal{H}^{(2)}(u,v) + \dots$$

$$\Delta = 4 + 2n + \ell + \frac{1}{N^2}\gamma^{(1)}_{n,\ell} + \frac{1}{N^4}\gamma^{(2)}_{n,\ell} + \dots \rightsquigarrow \mathcal{O}_2 \Box^n \partial_\ell \mathcal{O}_2$$

$$a_{n,\ell} = a^{(0)}_{n,\ell} + \frac{1}{N^2} a^{(1)}_{n,\ell} + \frac{1}{N^4} a^{(2)}_{n,\ell} + \dots$$

$$\downarrow$$

PLUG INTO THE SUPERCONFORMAL BLOCK DECOMPOSITION

$$\sum_{n,\ell}\left(a^{(0)}_{n,\ell} + \frac{1}{N^2} a^{(1)}_{n,\ell} + \dots\right) u^{2+n+\frac{\gamma^{(1)}}{N^2}+\dots} g_{n,\ell}(u,v)$$

UP TO ORDER N^{-4}:

1) AT N^0 AND N^{-2}, THE ONLY CONTRIBUTION TO THE dDisc COMES FROM THE PROTECTED PART OF THE CORRELATOR:

$$\mathrm{dDisc}\left(\log(1-z)(1-\bar{z})\right) = 0 \qquad \mathrm{dDisc}\left[\left(\frac{1-\bar{z}}{\bar{z}}\right)^p\right] \neq 0$$

IN A DISTRIBUTIONAL SENSE

$\downarrow$

IT IS POSSIBLE TO UNAMBIGUOUSLY DETERMINE $\mathcal{H}^{(0)}(u,v)$ AND $\mathcal{H}^{(1)}(u,v)$

UP TO ORDER N^{-4}:

1) AT N^0 AND N^{-2}, THE ONLY CONTRIBUTION TO THE dDisc COMES FROM THE PROTECTED PART OF THE CORRELATOR.

2) AT ORDER N^{-4}:

- PROTECTED OPERATORS STOP CONTRIBUTING
- THERE IS A TERM $\sim \log^2 v \sum a_0 (\gamma^{(1)})^2$

$\downarrow$

RECONSTRUCTED FROM N^0 AND N^{-2}

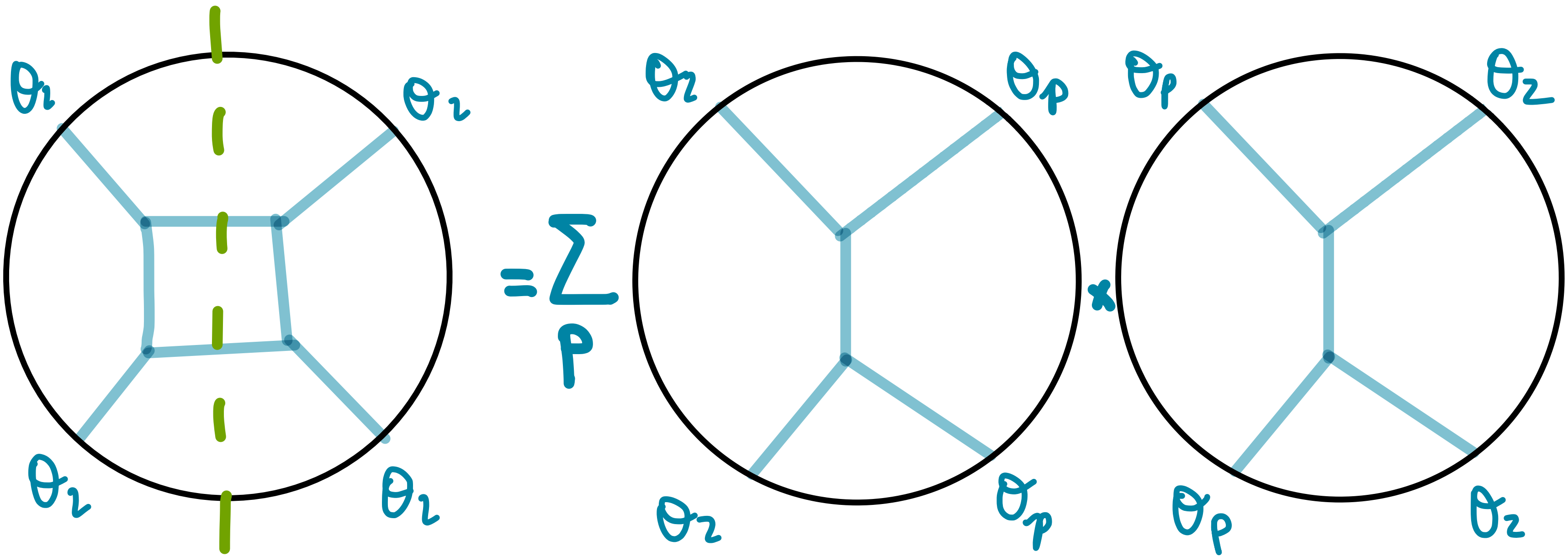
UP TO ORDER N^{-4}:

1) AT N^0 AND N^{-2}, THE ONLY CONTRIBUTION TO THE dDisc COMES FROM THE PROTECTED PART OF THE CORRELATOR.

2) AT ORDER N^{-4}: (NEED TO SOLVE MIXING PROBLEM)

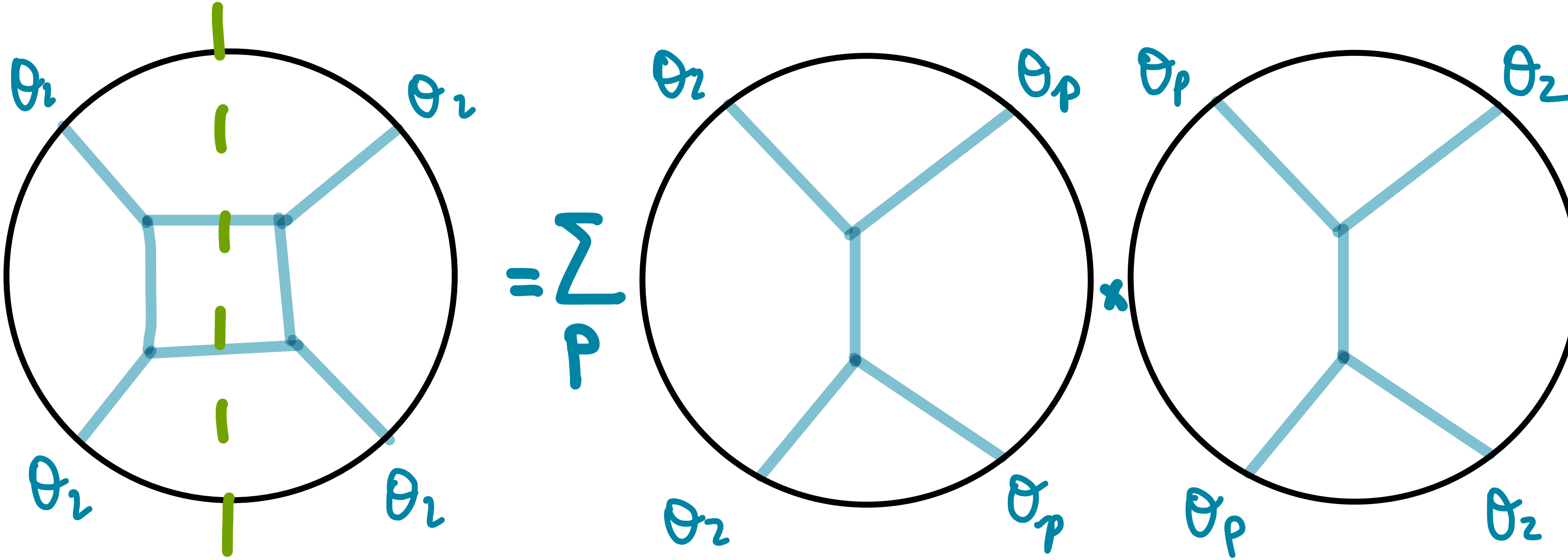
$\downarrow$ $\sim \langle \mathcal{O}_p \mathcal{O}_p \mathcal{O}_2 \mathcal{O}_2 \rangle \quad \forall p$

IT IS POSSIBLE TO FIND THE FULL AMPLITUDE



ALDAY, AB

SERIES OF PAPERS APRILE, DRUMMOND, HESLOP, PAUL



- EFFECT OF THE MIXING
- THE ONLY POSSIBLE CUT IS A DOUBLE TRACE CUT.

ALL LOOP ? (WITH FARDELLI & GEORGOUDIS)

- PUSHING THIS REASONING TO HIGHER LOOPS HAS A FEW OBSTACLES:

  1) HIGHER TRACES OPERATORS START CONTRIBUTING TO THE dDisc

  2) MIXING PROBLEM TO BE SOLVED AT HIGHER LOOPS.

(SOME RECENT DEVELOPMENTS AT TWO LOOPS BY DRUMMOND & PAUL, HUANG & YUAN)

- DESPITE THESE PROBLEMS, THERE IS A TERM WHICH IS COMPLETELY DETERMINED AND PRESENT AT ANY LOOP ORDER:

$$\mathcal{H}^{(k)}(u,v) \subset \log^k u \sum_{\substack{n,\ell \\ I}} \frac{u^{n+2}}{2^k\, k!} \underbrace{a^{(0)}_{n,\ell,I} \left(\gamma^{(1)}_{n,\ell,I}\right)^k}_{\text{KNOWN!}} g_{n,\ell}(u,v)$$

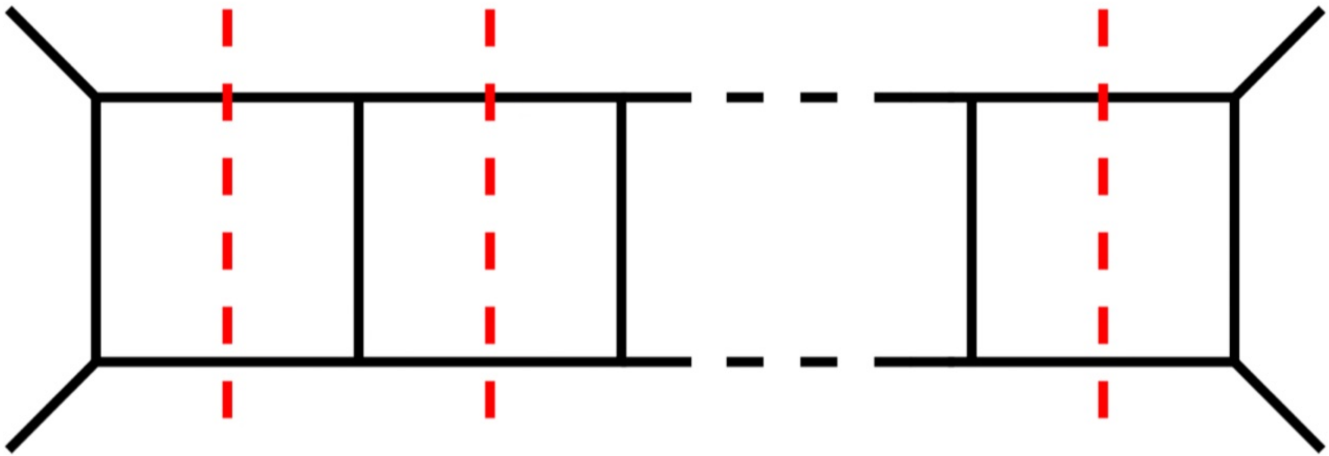
- THE QUESTION IS TO WHICH PART OF THE FULL ANSWER THIS TERM CORRESPOND TO?

TO UNDERSTAND THIS:

- COMPUTE THE SUM $\longrightarrow$ $\log^k u \, \underbrace{f(u,v)}$
- TAKE A SPECIFIC LIMIT (BULK POINT LIMIT) THAT CONNECTS WITH THE FLAT SPACE AMPLITUDE
- COMPUTE THE AMPLITUDE IN FLAT SPACE AND COMPARE.

ALONG THE LINES OF ALDAY & CARON-HUOT

RESULT



CONSECUTIVE S-CHANNEL CUTS → DOUBLE TRACES CUTS.

HIGHER TRACE OPERATORS (WITH FARDELLI & MANENTI)

- IN ORDER TO PROCEED TO HIGHER LOOPS, WE NEED TO UNDERSTAND HOW TO DEAL WITH MULTITRACE OPERATORS.

$\swarrow$ HIGHER POINT FUNCTIONS

GONCALVES, PEREIRA, ZHOU

$\searrow$ MULTITRACE EXTERNAL OPERATORS

1/4 BPS OPERATORS

- THE MOST CONTROLLED SETUP IS TO STUDY 4 POINT FUNCTIONS OF DOUBLE TRACE PROTECTED OPERATORS

- $\frac{1}{4}$ BPS OPERATORS : MULTITRACE $[q,p,q]_R$

PROTECTED DIMENSION $\Delta = 2q + p$

$\ell = 0$

$$\mathcal{O}_{pq} = \mathrm{Tr}\left(\varphi^{M_1} \dots \varphi\right) \mathrm{Tr}\left(\varphi \dots \varphi^{M_\Delta}\right) C_{M_1 \dots M_\Delta} + \frac{1}{N}\left(\text{SINGLE TRACE}\right)$$

SETUP

- THERE IS A PROLIFERATION OF POLARIZATION TO TAKE INTO ACCOUNT
- DISENTANGLE THE CONTRIBUTION OF PROTECTED OPERATORS IN THE OPE (USE THE CHIRAL ALGEBRA)

$\downarrow$

AS IN THE 1/2 BPS CASE, VIA THE dDisc IT IS POSSIBLE TO FIX THE UNPROTECTED PART (AT LEADING ORDER IN N).

SPINNING CORRELATORS (w/ FARDELLI MANENTI & ZHOU) TO APPEAR

- SPINNING CORRELATORS IN $\mathcal{N}=2$ SCFT IN 4D → GLUON

↓

SUPERSPACE EXPRESSION FOR THE FULL SUPERMULTIPLET

- CONNECTION WITH THE 4 POINT CORRELATOR OF THE STRESS TENSOR SUPERMULTIPLET IN $\mathcal{N}=4$.

FOR INSTANCE

$$\langle JJJJ \rangle \propto \mathbb{D}_1 \mathbb{D}_2 \mathbb{D}_3 \mathbb{D}_4 \; \mathcal{Y} H^{\text{GLUON}}$$

$$\langle TTTT \rangle \propto \mathbb{D}_1^2 \mathbb{D}_2^2 \mathbb{D}_3^2 \mathbb{D}_4^2 \; \mathcal{Y}^2 H^{\text{GRAVITON}}$$

- HINTING TO A DOUBLE COPY RELATION.

CONCLUSIONS

- STUDY AMPLITUDES IN CURVED SPACES USING THE CFT COUNTERPART
- THIS METHOD RELYS ONLY ON THE SYMMETRIES, SO IT CAN BE APPLIED VASTLY.
- FOR THE $\mathcal{N}=4$ SYM CASE, THIS HAS BEEN COMPLEMENTED WITH INTEGRABILITY, VERY PROMISING.

(ALDAY, HANSEN, SILVA – CARON HUOT, CORONADO, TRINH, ZAHRAEE – CAVAGLIA, GROMOV, JULIUS, PRETI)