Line Operators in Chern-Simons-Matter Theories and Bosonization in Three Dimensions

Amit Sever, Tel-Aviv University

With **Barak Gabai** and **De-liang Zhong**

short	[2204.05262]
perturbative	[22xx.xxxxx]
bootstrap	[22xx.xxxxx]

Motivation

Chern-Simons theory posses a well established **non-perturbative** level-rank duality

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extensive evidence, especially at large N

- correlation function of local operators
- spectrum of monopole/baryon operators
- thermal free-energies
- S-matrices
- relating non-susy dualities to well-established susy ones

[Witten, Minwalla, Prakash, Trivedi, Wadia, Yin, Aharony, Gur-Ari, Yacoby, Maldacena, Zhiboedov, Giombi, Gaiotto, Kapustin, Hsin, Seiberg, Naculich, Schnitzer, Mlawer, Naculich, Riggs, Schnitzer, Camperi, Levstein, Zemba, Bedhotiya, Prakash, Gurucharan, Kirilin, Prakash, Skvortsov, Radivcevic, Jain, Yokoyama, Sharma, Takimi, Mandlik, Inbasekar, Mazumdar, Giveon, Kutasov, Benini, Closset, Cremonesi, …]

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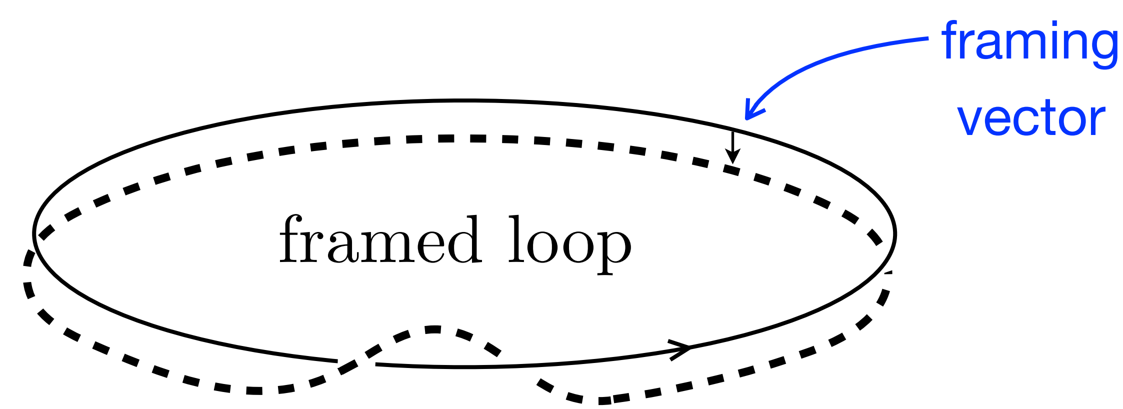

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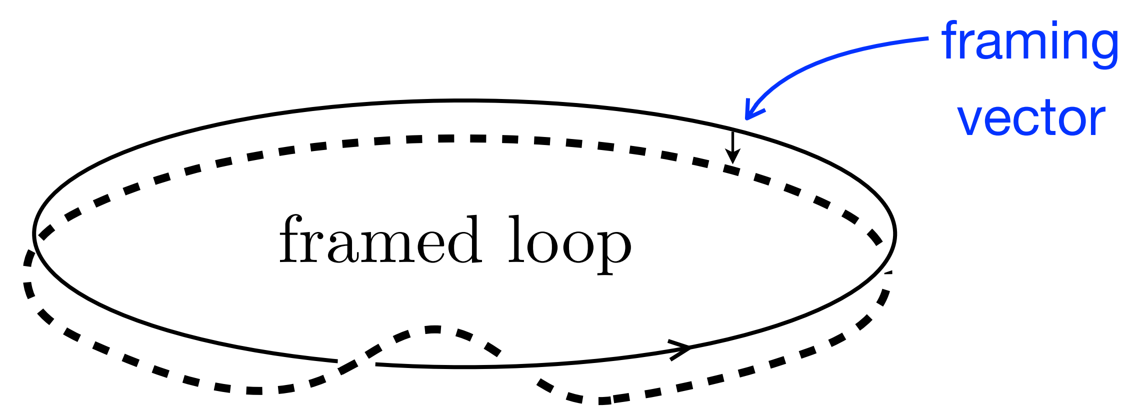
$$\left\langle W^{\mathfrak{f}}_{\text{unknot}} \right\rangle = e^{i\pi\lambda\mathfrak{f}} \times k\,\frac{\sin(\pi\lambda)}{\pi}$$



$SU(N)$ or $U(N)$ gauge group at level k

Only consider the $N \to \infty$ limit with $\lambda \equiv \dfrac{N}{k} \in [-1, 1]$ fixed

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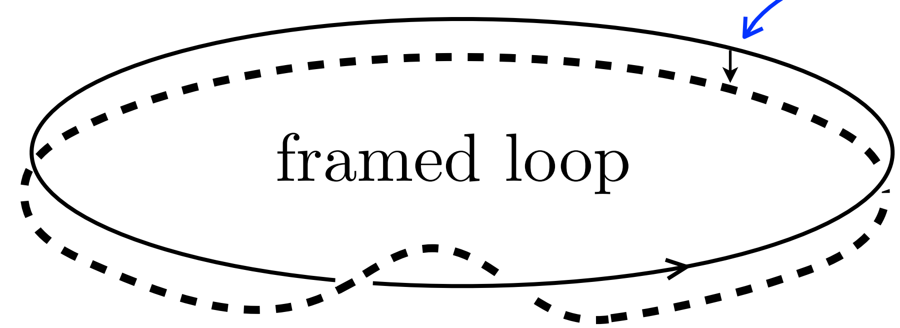
framing vector



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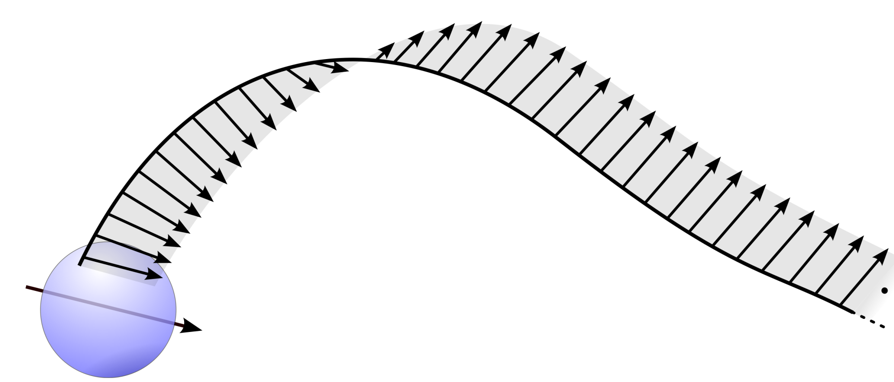
Only consider the $N \to \infty$ limit with $\lambda \equiv \dfrac{N}{k} \in [-1,1]$ fixed

$\Rightarrow$ Expect that the same dependance on the framing vector would lead to fractional statistics, ranging between boson (fermion) at $\lambda = 0$ to fermion (boson) at $|\lambda| = 1$

(Will be shortly proven)



level-rank $(k,\lambda) \quad \leftrightarrow \quad (-k, \lambda - \mathrm{sign}(k))$

Plan

Consider CFTs in 3d

= fixed points of CS theory + fundamental bosons/fermions

$$S_E^{\text{bos}} = S_{CS} + \int d^3x\, (D_\mu \phi)^\dagger \cdot D^\mu \phi + \frac{\lambda_6}{N^2} (\phi \cdot \phi^\dagger)^3$$ "quasi-boson"

$$S_E^{\text{fer}} = S_{CS} + \int d^3x\, \bar{\psi} \cdot \gamma^\mu D_\mu \psi$$ "quasi-fermion"

$$S_{CS} = \frac{ik}{4\pi} \int d^3x\, \epsilon^{\mu\nu\rho}\, \mathrm{tr}(A_\mu \partial_\nu A_\rho - \tfrac{2i}{3} A_\mu A_\nu A_\rho)$$

or their double trace deformation with respect to $\left(J^{(0)}\right)^2$

scalar current - $J^{(0)}_{\text{bos}} = \phi^\dagger \cdot \phi, \quad J^{(0)}_{\text{fer}} = \bar{\psi} \cdot \psi$

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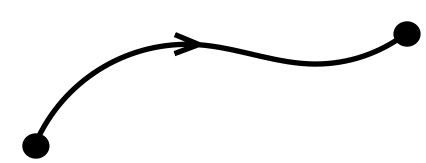
Study the most fundamental operators in CS-matter theory

"mesonic line operator"



= line operators along an arbitrary smooth path between a fundamental and an anti-fundamental boundary operators

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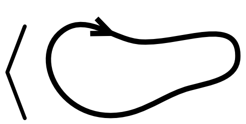
<u>At large N</u>

The matter contributes at $\mathcal{O}(1/N)$ to $\langle$  $\rangle$

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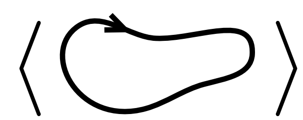
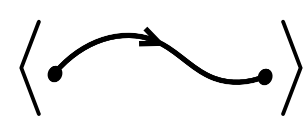
The matter contributes at $\mathcal{O}(1/N)$ to 

$\mathcal{O}(1)$ to 

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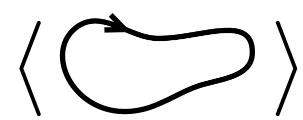
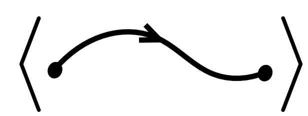
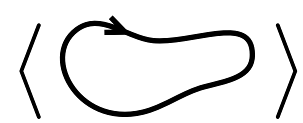
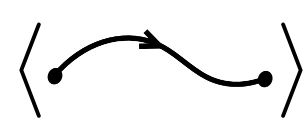
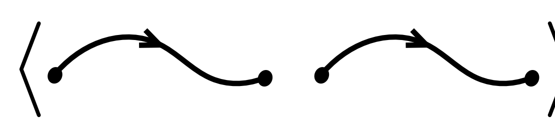
The matter contributes at $\mathcal{O}(1/N)$ to 

$\mathcal{O}(1)$ to 

The double trace deformation contributes at $\mathcal{O}(1/N^2)$ to 

$\mathcal{O}(1/N)$ to 

$\mathcal{O}(1)$ to  connected

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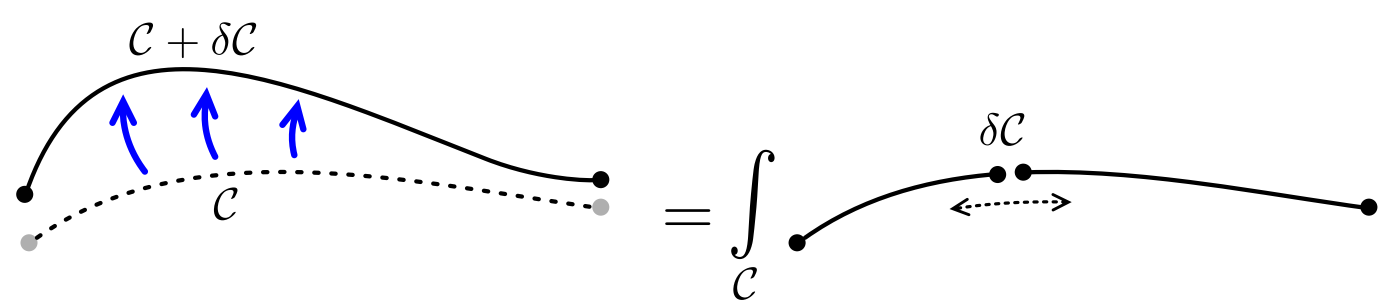

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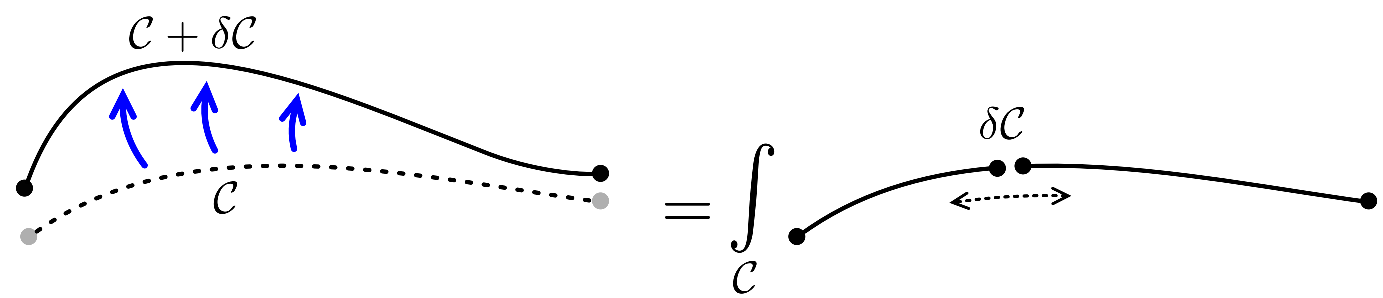

- Line bootstrap — [spectrum of boundary operators] + [evolution equation] uniquely determine the expectation values of the mesonic line operators, as well as the 1/N correction to closed loops expectation values.

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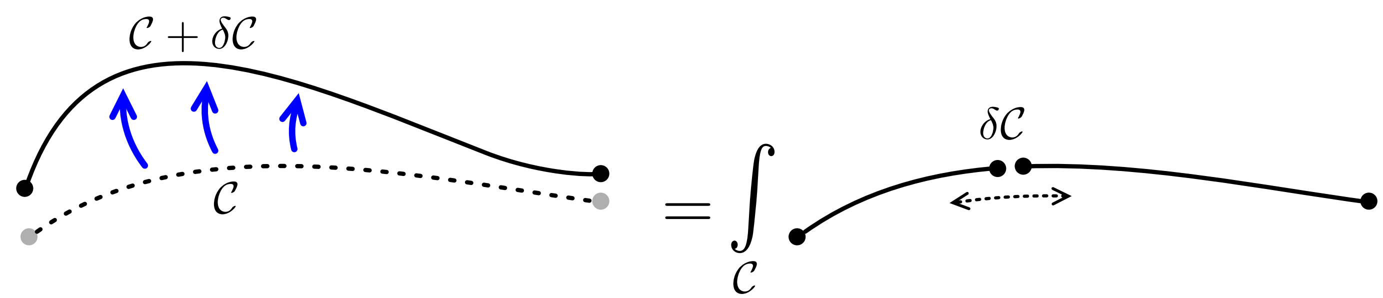

- Line bootstrap — [spectrum of boundary operators] + [evolution equation] uniquely determine the expectation values of the mesonic line operators, as well as the 1/N correction to closed loops expectation values.

- The conformal line operators of the bosonic and fermionic theories satisfy the same evolution equation and that their spectra of boundary operators are related to each other through the level-rank duality map $(k,\lambda) \leftrightarrow (-k, \lambda - \mathrm{sign}(k)) \;\Rightarrow\; \lambda_f = \lambda_b - \mathrm{sign}(k_b)$

Results

Demonstrate this by bootstrapping and computing explicitly the two-point function of the displacement operator

displacement operator

$$\frac{\langle \bullet \rightarrow \mathbb{D}_\perp(x_t) \longrightarrow \mathbb{D}_\perp(x_s) \rightarrow \bullet \rangle}{\langle \underset{x_1}{\bullet} \longrightarrow \underset{x_0}{\bullet} \rangle} = \frac{\Lambda(\Delta)}{x_{st}^4}\left(\frac{x_{10}x_{st}}{x_{1s}x_{t0}}\right)^{2\Delta}$$

boundary operators of minimal dimension Δ and opposite transverse spin

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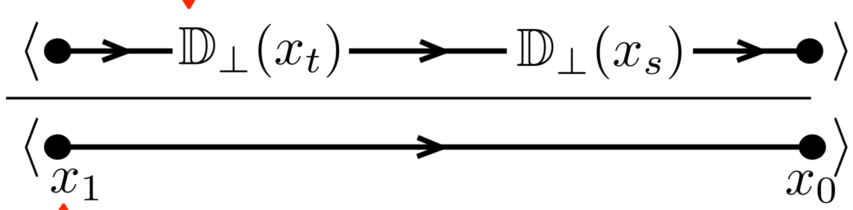

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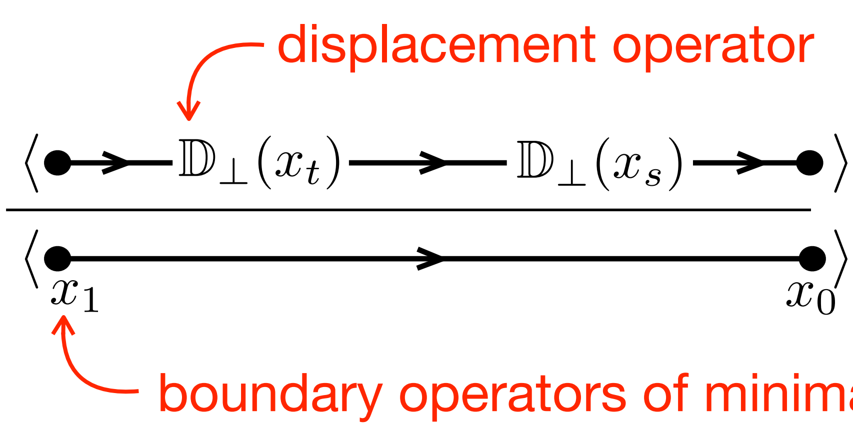

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$$\Lambda(\Delta) = -\frac{(2\Delta-1)(2\Delta-2)(2\Delta-3)\sin(2\pi\Delta)}{2\pi}$$

The dependance of Δ on λ depends on the line operator and whether we use λ_b or $\lambda_f = \lambda_b - \mathrm{sign}(k_b)$.

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Bosonic theory $\longrightarrow$ Fermionic theory $\longrightarrow$ Bootstrap

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Without loss of generality, we assume $\mathrm{sign}(k_b) > 0 \quad \Rightarrow \quad \begin{array}{l} \lambda_b \in [0,1] \\ \lambda_f \in [-1,0] \end{array}$

(parity $k \leftrightarrow -k \,, \quad \lambda \leftrightarrow -\lambda$.)

Warning — no detailed derivations

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path

framing vector

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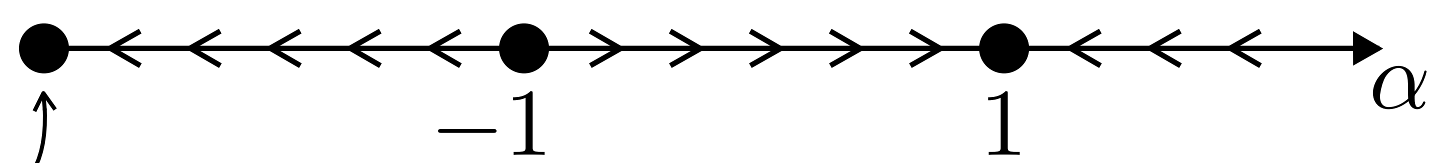

(almost) trivial line

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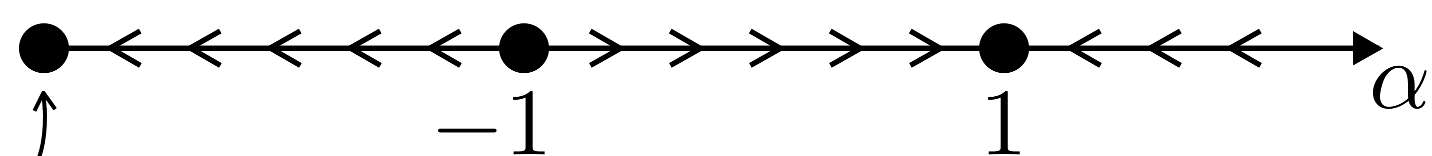

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There are also other conformal line operators — operators with d.o.f. on the line and non-unitary ones, that we will not describe in this talk.

Mesonic Line Operators

anti-fundamental boundary operator

fundamental boundary operator

Mesonic line operator $M = \mathcal{O}_L \bullet\!\xrightarrow{\mathcal{W}}\!\bullet\, \mathcal{O}_R$

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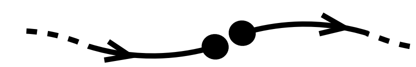
Adjoint operators on the line $\mathcal{O}_{\text{inner}} = \mathcal{O}_R \times \mathcal{O}_L$ 

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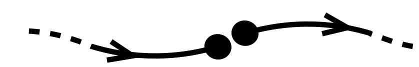
Adjoint operators on the line $\mathcal{O}_{\text{inner}} = \mathcal{O}_R \times \mathcal{O}_L$ 

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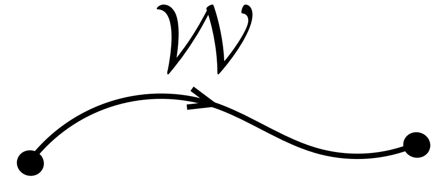
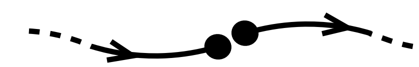
Mesonic line operator $M = \mathcal{O}_L$  $\mathcal{O}_R$

Adjoint operators on the line $\mathcal{O}_{\text{inner}} = \mathcal{O}_R \times \mathcal{O}_L$ 

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At tree level
line in the x^3- direction

$$\mathcal{O}^{(n,s)}_{R,\,\text{tree}} = \frac{1}{\sqrt{N}} \times \begin{cases} \partial^n_{x^3_R} \partial^s_{x^+_R} \phi & s \geq 1 \\ \partial^n_{x^3_R} \partial^{-s}_{x^-_R} \phi & s \leq 0 \end{cases}$$

tree level spin

$$\Delta^{(n,s)}_{\text{tree}} = 1/2 + n + |s|$$

$ds^2 = 2dx^+ dx^- + (dx^3)^2$

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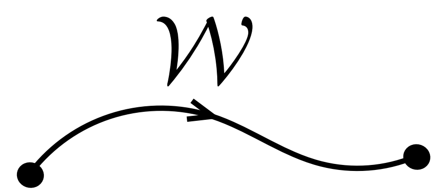
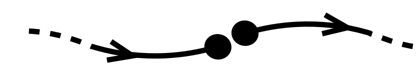
Mesonic line operator $M = \mathcal{O}_L$  $\mathcal{O}_R$

Adjoint operators on the line $\mathcal{O}_{\text{inner}} = \mathcal{O}_R \times \mathcal{O}_L$ 

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An infinite straight line preserve an $SL(2,\mathbb{R}) \times U(1)$ conformal symmetry

The operators with $n=0$ are $SL(2,\mathbb{R})$ primaries

Mesonic $\alpha = 1$ Line Operators

tree level spin

Spectrum -

$$\mathfrak{s}_L = s_L + \lambda/2$$
$$\mathfrak{s}_R = s_R - \lambda/2$$

exact transverse spin

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exact transverse spin

Operators with the same anomalous dimension are related by path derivatives (keeping framing perpendicular and fixed)

i.e. $$\mathcal{O}_L^{(0,s+1)} = \delta_+ \mathcal{O}_L^{(0,s)}\,, \qquad \mathcal{O}^{(n+1,s)} = \delta_3 \mathcal{O}_L^{(n,s)}$$

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At the bottom of these four towers we have the operators

$$\{\mathcal{O}_L^{(0,0)}, \mathcal{O}_L^{(0,-1)}\} \quad \text{and} \quad \{\mathcal{O}_R^{(0,0)}, \mathcal{O}_R^{(0,1)}\}$$

opposite anomalous dimension

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The relation between the dimension and spin can be shown to follow from supersymmetry

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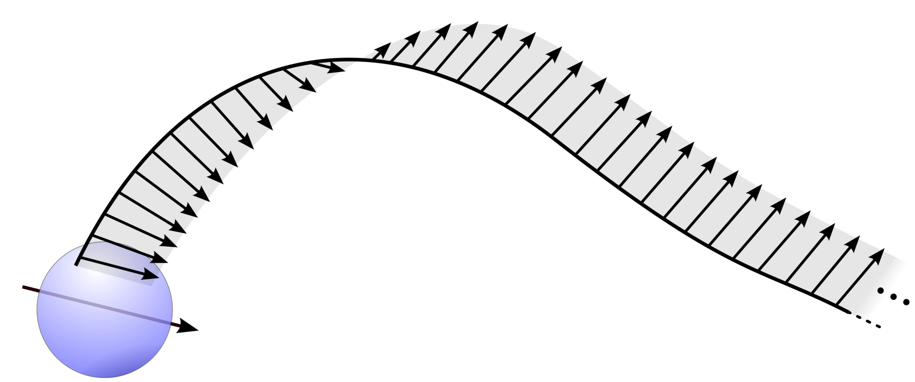
tree level spin

Spectrum - $$\mathfrak{s}_L = s_L + \lambda/2$$ $$\mathfrak{s}_R = s_R - \lambda/2$$ $$\Delta^{(n,s)} = 1/2 + n + |\mathfrak{s}|$$

exact transverse spin

This confirms the expectation that the dependance on the framing vector leads to fractional statistics, ranging between boson at $\lambda = 0$ to fermion at $\lambda = 1$



Mesonic $\alpha = 1$ Line Operators

tree level spin

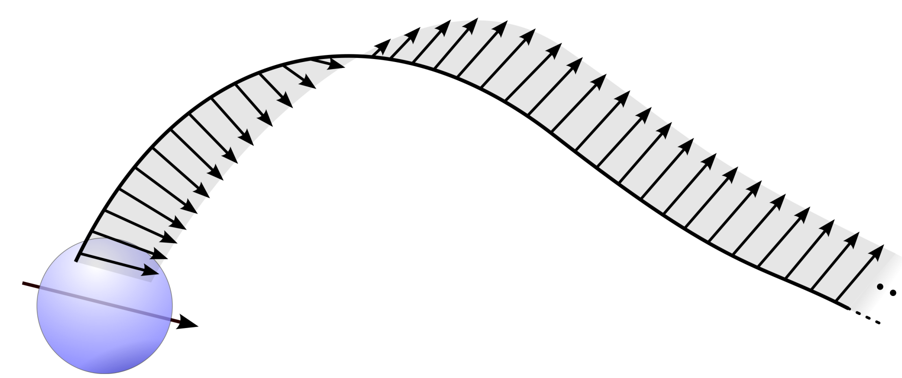
Spectrum -

$$\mathfrak{s}_L = s_L + \lambda/2$$
$$\mathfrak{s}_R = s_R - \lambda/2$$

exact transverse spin

$$\Delta^{(n,s)} = 1/2 + n + |\mathfrak{s}|$$

This confirms the expectation that the dependance on the framing vector leads to fractional statistics, ranging between boson at $\lambda = 0$ to fermion at $\lambda = 1$



The operator **on the line** with minimal dimension is the bi-scalar adjoint

$\mathcal{O}_R^{(0,0)} \times \mathcal{O}_L^{(0,0)}$ with $\Delta_{\min} = \Delta_L^{(0,0)} + \Delta_R^{(0,0)} = 1 + \lambda > 1$

$\Rightarrow$ The $\alpha = 1$ line is a stable fix-point

The evolution equation ($\alpha = 1$)

Expectation values take the form

transverse spin

conformal invariant functional of the path

$$\left\langle \mathcal{O}_L^{(0,0)} \mathcal{W} \mathcal{O}_R^{(0,0)} \right\rangle = \frac{\left(n_L^+ n_R^-\right)^{\lambda/2}}{|x_L - x_R|^{1+\lambda}} \times F^{(0,0)}[x(\cdot)]$$

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conformal dimension

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$$\delta\mathcal{W} = \int ds\, |\dot{x}(s)|\, v^\mu(s)\, \mathcal{P}\left[\mathbb{D}_\mu(s)\mathcal{W}\right]$$

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$$\Rightarrow \qquad \Delta(\mathbb{D}_\pm) = 2\,, \quad \mathfrak{s}(\mathbb{D}_\pm) = \pm 1$$

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Displacement operator

$$\mathbb{D}_- = -4\pi\lambda\, \mathcal{O}_R^{(0,0)}\, \mathcal{O}_L^{(0,-1)} \qquad \mathbb{D}_+ = -4\pi\lambda\, \mathcal{O}_R^{(0,1)}\, \mathcal{O}_L^{(0,0)}$$

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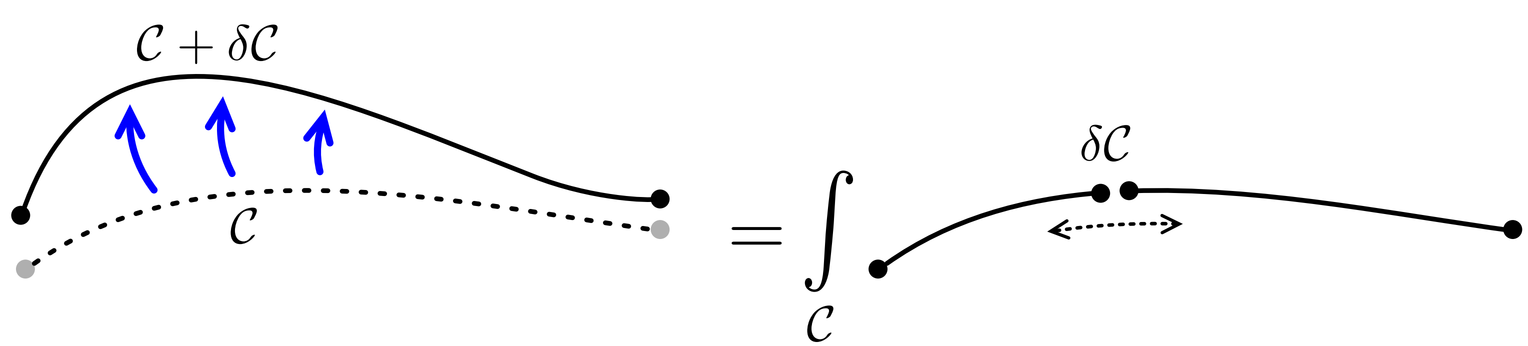
First order chiral Schwinger-Dyson equation



Boundary equation ($\alpha = 1$)

The boundary operators also satisfy an operator equation.

It relates $SL(2,\mathbb{R})$ primaries from the same tower as

part of a smooth deformation of the line

$$\delta_- \mathcal{O}_L^{(0,s+1)} = \bar{\beta}\, \mathcal{O}_L^{(2,s)}\,, \qquad s \geq 0$$

$$\delta_+ \mathcal{O}_L^{(0,-s-1)} = \beta\, \mathcal{O}_L^{(2,-s)}\,, \qquad s \geq 1$$

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Bootstrap or explicate computation $\quad \Rightarrow \quad \beta = \bar\beta = -\dfrac{1}{2}$

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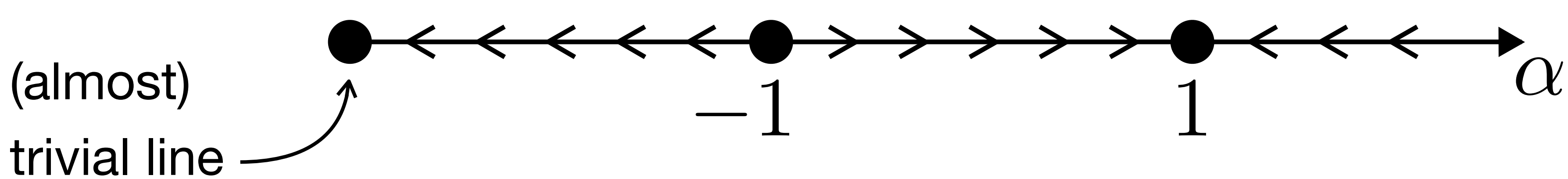

Plan

Bosonic theory ✓ $\longrightarrow$ Fermionic theory $\longrightarrow$ Bootstrap

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A Wilson line is a good conformal operator $W[x(\cdot)] = \mathcal{P}e^{i\int A_\mu dx^\mu}$

We have repeated the same steps as in the bosonic theory.
We find a perfect match with the $\alpha = 1$ mesonic line operators for both, the **spectrum** and the form of the **evolution equation**.

Fermionic	Tree	Bosonic	Tree	$\mathfrak{s}$	Δ
$\mathcal{O}_R^{(0,-\frac{1}{2})}$	ψ_-	$\mathcal{O}_R^{(0,0)}$	ϕ	$-\frac{\lambda_b}{2}$	$\frac{1+\lambda_b}{2}$
$\mathcal{O}_R^{(0,\frac{1}{2})}$	ψ_+	$\mathcal{O}_R^{(0,1)}$	$\partial_+\phi$	$\frac{2-\lambda_b}{2}$	$\frac{3-\lambda_b}{2}$
$\mathcal{O}_L^{(0,\frac{1}{2})}$	$\bar{\psi}_+$	$\mathcal{O}_L^{(0,0)}$	$\phi^\dagger$	$\frac{\lambda_b}{2}$	$\frac{1+\lambda_b}{2}$
$\mathcal{O}_L^{(0,-\frac{1}{2})}$	$\bar{\psi}_-$	$\mathcal{O}_L^{(0,-1)}$	$\partial_-\phi^\dagger$	$\frac{\lambda_b-2}{2}$	$\frac{3-\lambda_b}{2}$

$\lambda_f = \lambda_b - 1$

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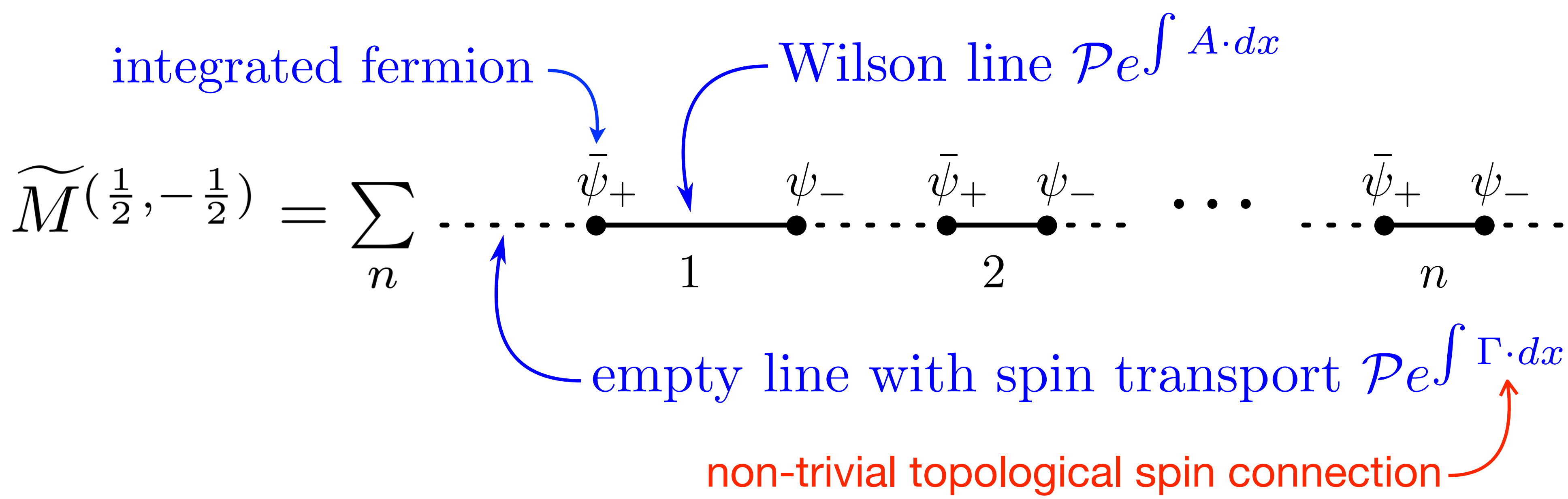

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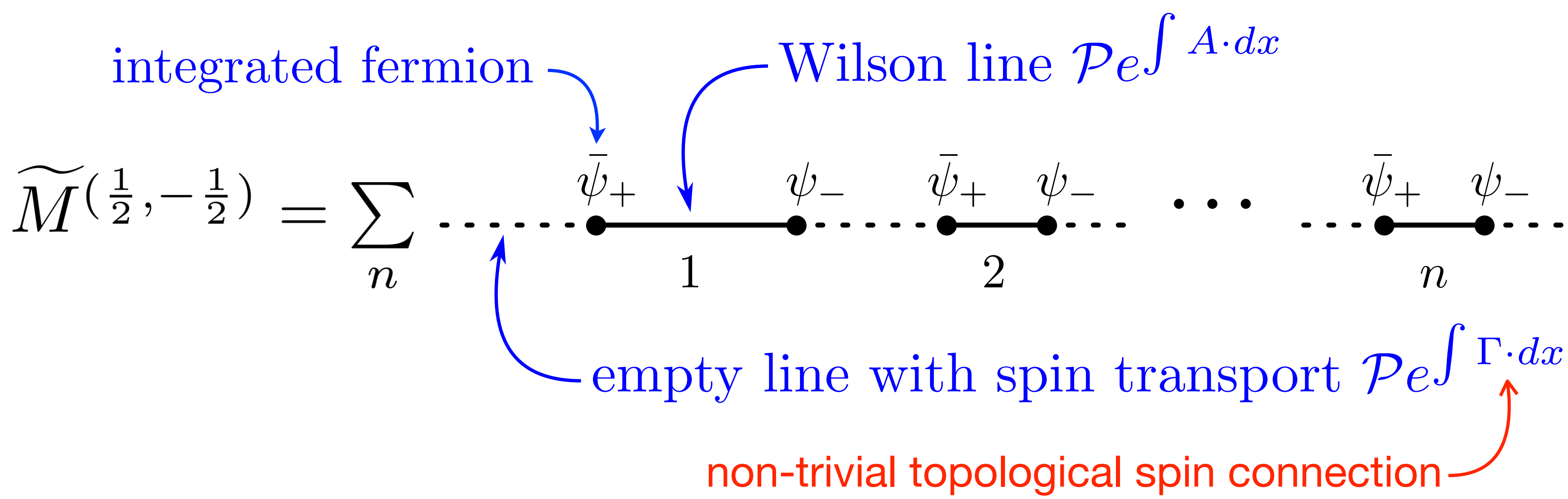

Fermionic	Tree	Bosonic	Tree	$\mathfrak{s}$	Δ
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$\lambda_f = \lambda_b - 1$

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Main point — [spectrum of boundary operators] + [evolution equation] uniquely determine the expectation values of the mesonic line operators

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- Pick a regularization scheme and fix all coefficients by demanding
    - Conformal symmetry with the appropriate spectrum of boundary operators.
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We find that these conditions are sufficient to systematically fix all the coefficients.

Bootstrap

For example, at second order we have the two-point function of the displacement operator

$$\frac{\langle \bullet \!\rightarrow\! \mathbb{D}_\perp(x_t) \!\rightarrow\! \mathbb{D}_\perp(x_s) \!\rightarrow\! \bullet \rangle}{\langle \underset{x_1}{\bullet} \!\longrightarrow\! \underset{x_0}{\bullet} \rangle} = \frac{\Lambda(\Delta)}{x_{st}^4} \left(\frac{x_{10} x_{st}}{x_{1s} x_{t0}} \right)^{2\Delta}$$

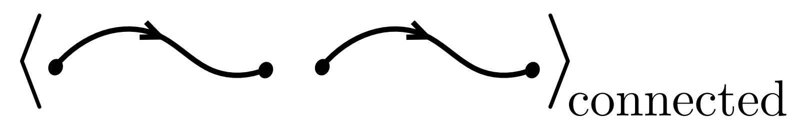
with $$\Lambda(\Delta) = \Lambda(2-\Delta) = -\frac{(2\Delta-1)(2\Delta-2)(2\Delta-3)\sin(2\pi\Delta)}{2\pi}$$

Here, Δ is the dimension of any of the four bottom operators.
For example, for the $\alpha = 1$ line operator it is $\Delta = (1+\lambda_b)/2$

or $\Delta = (3-\lambda_b)/2$

Future directions

- To complete the derivation of the duality at the planar level, one should also match the connected piece of the correlation functions between mesonic line operators, as well as the local single trace operators.



We expect that known [spectrum of single trace local operators]
+[spectrum of boundary operators]
+[evolution equation]

would be sufficient.

- Find an explicit solution for the expectation values.

- Derive the holographic dual — parity breaking versions of Vasiliev's higher-spin theory (is our original motivation)

Thank you

Mesonic Line Operators in the fermonic theory

$$\widetilde{M}^{(\frac{1}{2},-\frac{1}{2})}[x(\cdot)] = \left[\mathcal{P}e^{\mathrm{i}\int \widetilde{\mathcal{A}}_\mu \dot{x}^\mu \mathrm{d}s}\right]_{2\,2}$$

$$\widetilde{\mathcal{A}}_\mu \equiv \begin{pmatrix} A_\mu & \mathrm{i}\, P_\mu^- \psi \\ -\mathrm{i}\frac{4\pi}{k}\bar{\psi} P_\mu^- & \frac{4\pi}{k}\frac{1}{\epsilon}\gamma_\mu + \Gamma_\mu \end{pmatrix}$$

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$$\widetilde{\mathcal{O}}_L^{(0,\frac{3}{2})} = D_+(\bar{\psi}P_\nu^+ e_L^\nu)/\sqrt{N} \qquad \widetilde{\mathcal{O}}_R^{(0,-\frac{3}{2})} = D_-(e_R^\rho P_\rho^- \psi)/\sqrt{N}$$

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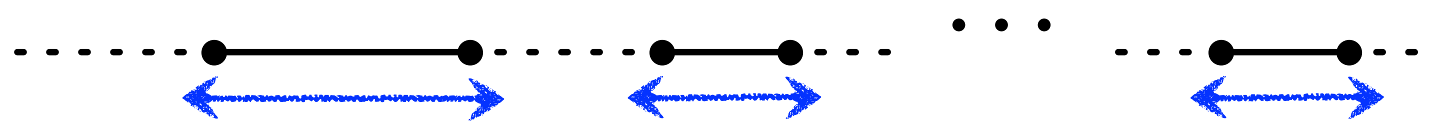
Mesonic Line Operators in the fermonic theory

Mesonic Line Operators in the fermonic theory

Deforming by $\widetilde{\mathcal{O}}_R^{(0,-\frac{1}{2})} \times \widetilde{\mathcal{O}}_L^{(0,\frac{1}{2})}$ with the appropriate sign generates a flow to the normal Wilson line.



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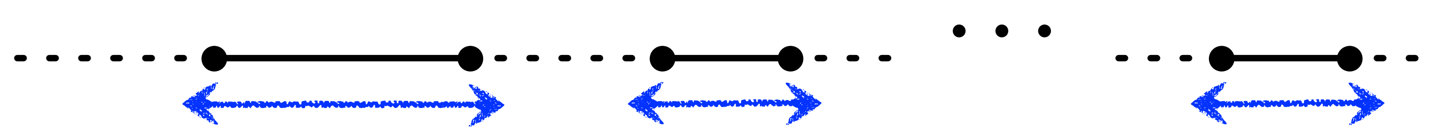

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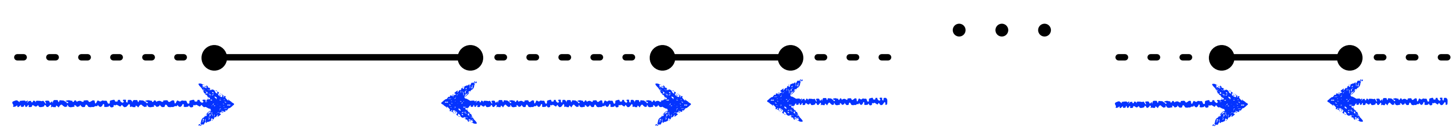

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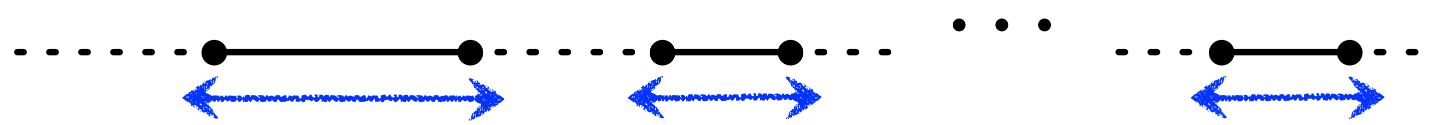

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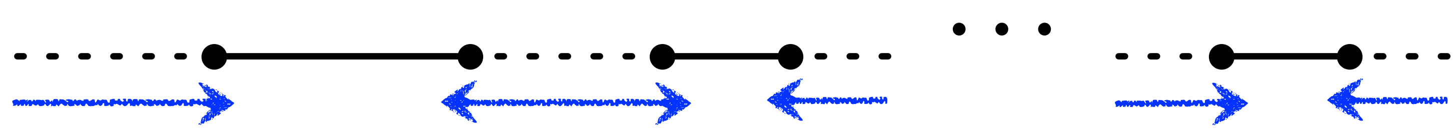

The condensed fermion operator itself can also be understood as the fix-point of a different RG flow on the line.

Mesonic Line Operators in the fermonic theory

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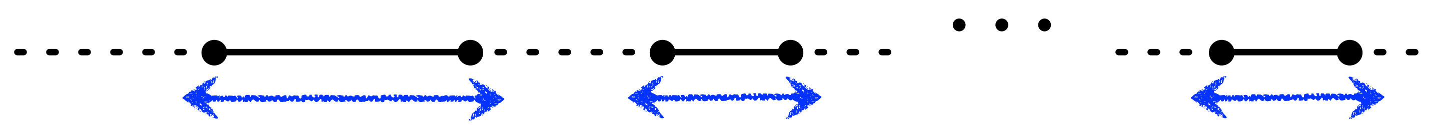
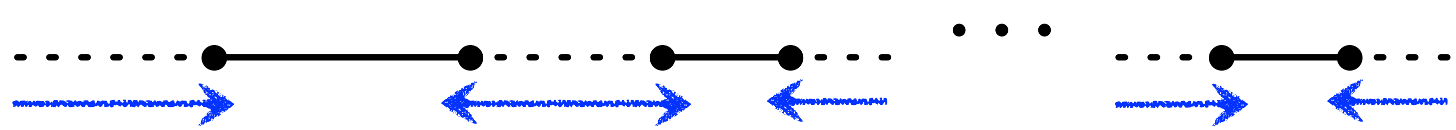


Deforming by $\widetilde{\mathcal{O}}_R^{(0,-\frac{1}{2})} \times \widetilde{\mathcal{O}}_L^{(0,\frac{1}{2})}$ with the opposite sign generates a flow to an empty line with a topological spin $(1+\lambda_f)/2$ transport.



The condensed fermion operator itself can also be understood as the fix-point of a different RG flow on the line.

Consider a normal Wilson line times a trivial line

$$a_\mu = \begin{pmatrix} A_\mu & \\ & (1+\lambda_f)\Gamma_\mu \end{pmatrix} \quad \text{and deform by} \quad \delta a_\mu = \epsilon \begin{pmatrix} & \mathcal{O}_R^{(0,-\frac{1}{2})} \\ -\mathcal{O}_L^{(0,\frac{1}{2})} & \end{pmatrix}$$

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<u>Hint</u>

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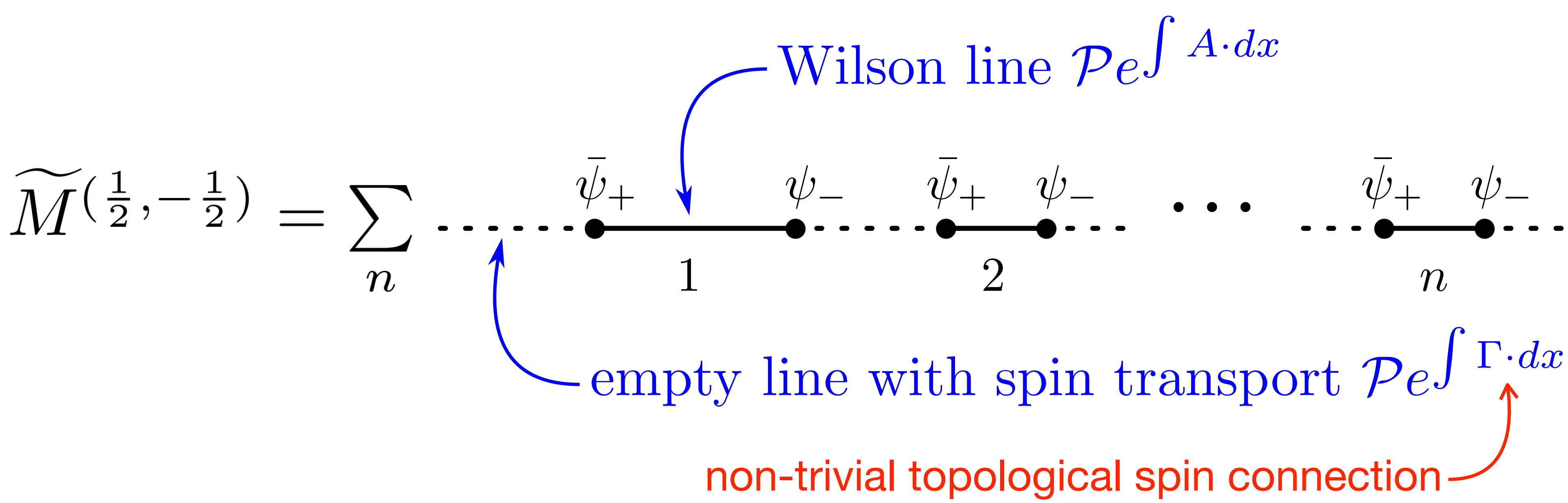
Answer - the dual has fundamental fermion condensed in the exponent



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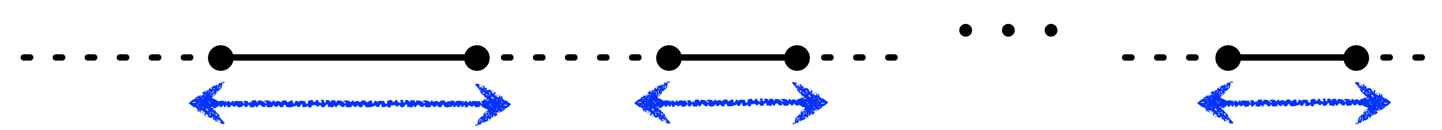

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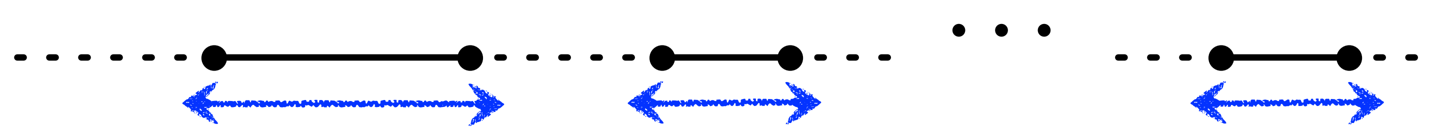

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