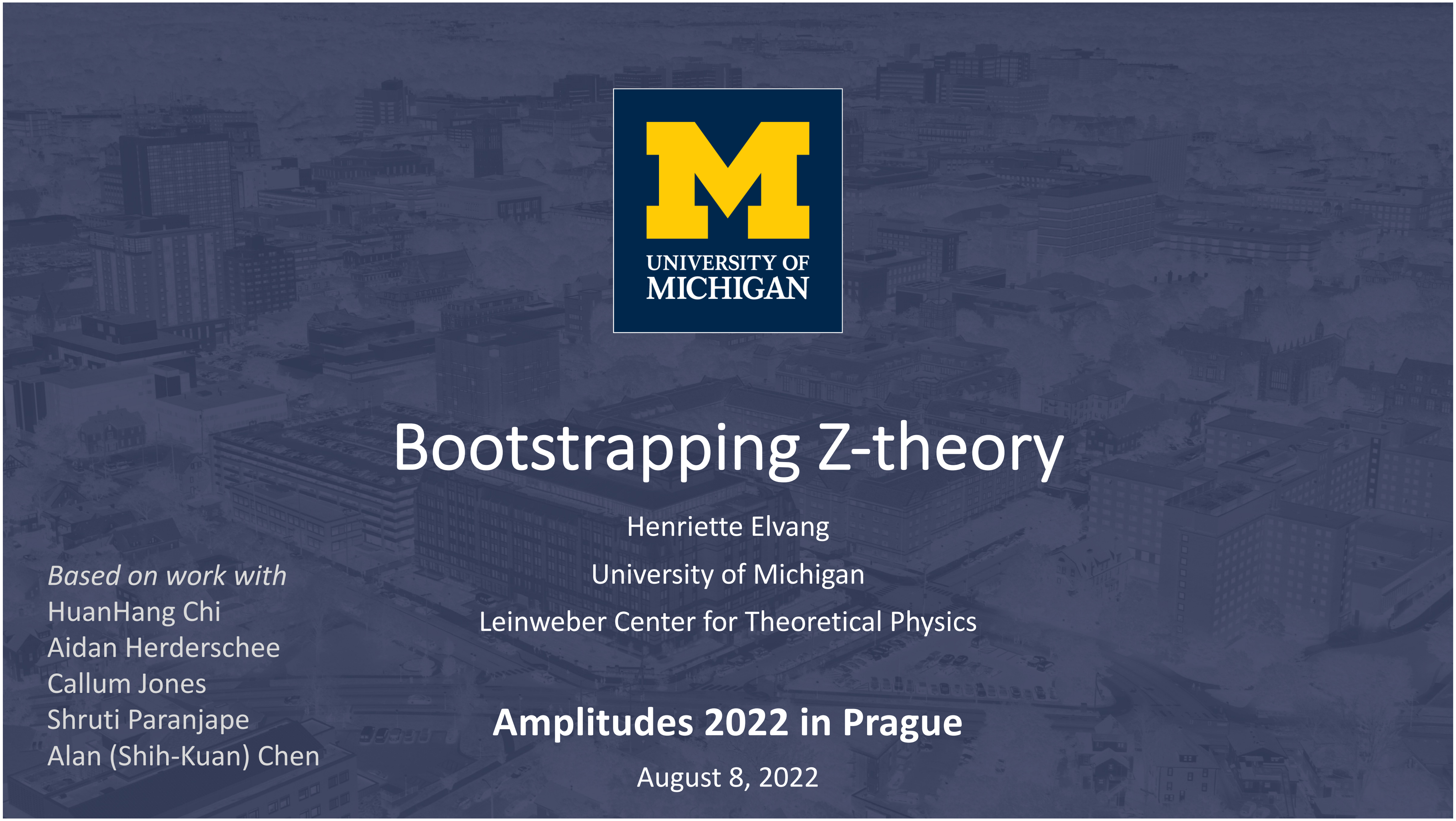

UNIVERSITY OF MICHIGAN

Bootstrapping Z-theory

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Based on work with
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Aidan Herderschee
Callum Jones
Shruti Paranjape
Alan (Shih-Kuan) Chen

KLT Double-Copy Bootstrap

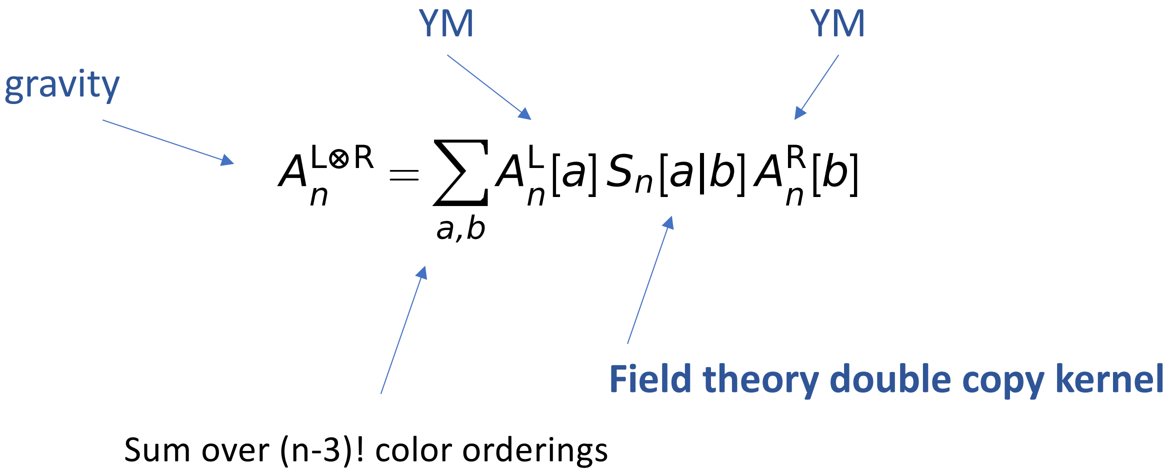
- Review ideas of the KLT Double-Copy Bootstrap.
- New results at 6-point.
- Bootstrapping Z-theory.
- Where is the string theory kernel?

This talk:
Tree Level

KLT Double-Copy Bootstrap

We have been investigating the fundamental structure of the KLT double-copy and how it may generalize, in particular in the context of EFTs.

Chi, HE, Herderschee, Jones, Paranjape (2021)



YM

YM

gravity

$$A_n^{\mathrm{L}\otimes\mathrm{R}} = \sum_{a,b} A_n^{\mathrm{L}}[a]\, S_n[a|b]\, A_n^{\mathrm{R}}[b]$$

Field theory double copy kernel

Sum over (n-3)! color orderings

KLT Double-Copy Bootstrap

We have been investigating the fundamental structure of the KLT double-copy and how it may generalize, in particular in the context of EFTs.

Chi, HE, Herderschee, Jones, Paranjape (2021)

Open string

Open string

Closed string

$$A_n^{\mathrm{L}\otimes\mathrm{R}} = \sum_{a,b} A_n^{\mathrm{L}}[a]\, S_n[a|b]\, A_n^{\mathrm{R}}[b]$$

String theory double copy kernel

Sum over (n-3)! color orderings

KLT Double-Copy Bootstrap

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Chi, HE, Herderschee, Jones, Paranjape (2021)

YM + h.d.

YM + h.d.

GR + h.d.

$$A_n^{\mathrm{L}\otimes\mathrm{R}} = \sum_{a,b} A_n^{\mathrm{L}}[a]\, S_n[a|b]\, A_n^{\mathrm{R}}[b]$$

What is the most general kernel?

Sum over (n-3)! color orderings

KLT kernels

Field theory KLT kernel is inverse of any (n-3)! x (n-3)! submatrix of the tree amplitudes of the cubic Bi-Adjoint Scalar model (BAS).

Cachazo, He, Yuan

$$\mathcal{L}_{\text{BAS}} = -\frac{1}{2}\left(\partial_\mu \phi^{aa'}\right)^2 - g\, f^{abc}\, \tilde{f}^{a'b'c'}\, \phi^{aa'}\, \phi^{bb'}\, \phi^{cc'}$$

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String theory KLT kernel is inverse of any (n-3)! x (n-3)! submatrix of the tree amplitudes of the cubic Bi-Adjoint Scalar model (BAS) + *specific* higher derivative terms determined by the alpha' expansion.

Mizera

$$\mathcal{L}_{\alpha'} = \mathcal{L}_{\text{BAS}} + \alpha'\, \partial^2 \phi^4 + \alpha'^3\, \partial^6 \phi^4 + \ldots$$

KLT kernels

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Mizera

$$\mathcal{L}_{\alpha'} = \mathcal{L}_{\mathrm{BAS}} + \alpha'\, \partial^2 \phi^4 + \alpha'^3\, \partial^6 \phi^4 + \ldots$$

Generalized EFT KLT kernel is inverse of any (n-3)! x (n-3)! submatrix of the amplitudes of the cubic Bi-Adjoint Scalar model (BAS) + *general* higher derivative terms.

Chi, HE, Herderschee, Jones, Paranjape (2021)

..but what are the rules for adding higher-derivative terms without wrecking the desired kernel properties?

The Rules: The KLT algebra.

"Standard" field theory double-copy has an **identity element**

FT $\otimes$ FT	YM	$\mathcal{N}=4$ SYM	χPT	BAS
YM	gravity+	$\mathcal{N}=4$ SG	BI	YM
$\mathcal{N}=4$ SYM	$\mathcal{N}=4$ SG	$\mathcal{N}=8$ SG	$\mathcal{N}=4$ sDBI	$\mathcal{N}=4$ SYM
χPT	BI	$\mathcal{N}=4$ sDBI	sGalileon	χPT
BAS	YM	$\mathcal{N}=4$ SYM	χPT	BAS

Cubic Bi-Adjoint Scalar model (BAS)

$$\mathcal{L}_{\mathrm{BAS}} = -\frac{1}{2}\left(\partial_\mu \phi^{aa'}\right)^2 - g f^{abc}\tilde{f}^{a'b'c'}\phi^{aa'}\phi^{bb'}\phi^{cc'}$$

$$\mathrm{L} = \mathrm{L}\otimes\mathbf{1}\,,\qquad \mathrm{R} = \mathbf{1}\otimes\mathrm{R}\,,\qquad \underbrace{\mathbf{1} = \mathbf{1}\otimes\mathbf{1}\,.}\qquad\qquad \mathbf{1} = \mathrm{BAS\ model}$$

This links identity element to the kernel => inverse submatrix relation

Chi, HE, Herderschee, Jones, Paranjape (2021)

Proposal

Proposal: take the KLT algebra as the fundamental property

$$\underbrace{\mathrm{L} = \mathrm{L} \otimes \mathbf{1}\,, \qquad \mathrm{R} = \mathbf{1} \otimes \mathrm{R}\,,}_{\text{KKBCJ / monodromy relations}} \qquad \underbrace{\mathbf{1} = \mathbf{1} \otimes \mathbf{1}\,.}_{\textbf{KLT Bootstrap Equation}}$$

The double-copy kernel is the inverse of a suitable submatrix of the tree amplitudes in the identity model.

To generalize the double-copy kernel, we generalize the identity element

“Minimal rank condition”

$\mathbf{1} = \mathbf{1} \otimes \mathbf{1}$ ⇔ (n-1)! x (n-1)! matrix of tree amplitudes of the identity model has rank (n-3)!

Chi, HE, Herderschee, Jones, Paranjape (2021)

4-point bootstrap

4-point: (n-1)! = 6 independent single-trace color-orderings: 1234, 1243, 1324, 1342, 1423, 1432

=> 6 x 6 matrix of doubly-ordered tree amplitudes m[a|b] for the identity model

Cyclic symmetry and momentum relabeling: can express in terms of just three functions f_1, f_2, f_6

$$\mathbf{m}_4 = \begin{pmatrix} \boxed{m_4[1234|1234]} & \boxed{m_4[1234|1243]} & m_4[1234|1324] & \cdots & \boxed{m_4[1234|1432]} \\ m_4[1243|1234] & m_4[1243|1243] & m_4[1243|1324] & \cdots & m_4[1243|1432] \\ \vdots & \vdots & \vdots & & \vdots \\ m_4[1432|1234] & m_4[1432|1243] & m_4[1432|1324] & \cdots & m_4[1432|1432] \end{pmatrix} = \begin{pmatrix} \boxed{f_1(s,t)} & \boxed{f_2(s,t)} & f_2(u,t) & f_2(s,t) & f_2(u,t) & \boxed{f_6(s,t)} \\ f_2(s,u) & f_1(s,u) & f_2(t,u) & f_6(s,u) & f_2(t,u) & f_2(s,u) \\ f_2(u,s) & f_2(t,s) & f_1(t,s) & f_2(t,s) & f_6(t,s) & f_2(u,s) \\ f_2(s,u) & f_6(t,u) & f_2(t,u) & f_1(t,u) & f_2(t,u) & f_2(s,u) \\ f_2(u,s) & f_2(t,s) & f_6(u,s) & f_2(t,s) & f_1(u,s) & f_2(u,s) \\ f_6(u,t) & f_2(s,t) & f_2(u,t) & f_2(s,t) & f_6(u,t) & f_2(u,t) \end{pmatrix}$$

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$\mathbf{1} = \mathbf{1} \otimes \mathbf{1}$ ⇔ All 2x2 minors vanish ⇔ 6x6 matrix has rank 1

$$\boxed{f_1(s,t) = \frac{f_2(s,t) f_2(-s-t,s)}{f_2(t,s)}\,,} \quad \boxed{f_6(s,t) = f_1(s,t)\,.}$$

$$\boxed{f_2(s,t) f_2(-s-t,s) f_2(t,-s-t) = f_2(t,s) f_2(-s-t,t) f_2(s,-s-t)\,.}$$

Let’s look more at the bootstrap results at 4-point

Ansatz $f_2(s,t) = -\frac{g^2}{s} + a_{0,0} + a_{1,0}\,t + a_{1,1}\,s + a_{2,0}\,t^2 + a_{2,1}\,st + a_{2,2}\,s^2 + \dots$

Let’s look more at the bootstrap results at 4-point

Ansatz

$$f_2(s,t) = -\frac{g^2}{s} + \cancel{a_{0,0}} + a_{1,0}\, t + a_{1,1}\, s + a_{2,0} \underbrace{t^2 + a_{2,1}}_{\text{same}} st + \cancel{a_{2,2}}\, s^2 + \ldots$$

4-point bootstrap + locality

$$f_2(s,t) = -\frac{g^2}{s} + a_{1,0}\, t + a_{1,1}\, s + a_{2,0}\, t(s+t)$$

$$+\, a_{3,0}\, t^3 + a_{3,1}\, st^2 + a_{3,2}\, s^2 t + a_{3,3}\, s^3$$

$$+\, a_{4,0}\, t^4 + a_{4,1}\, st^3 + a_{4,2}\, s^2 t^2 + \underbrace{\big(a_{4,0} + a_{4,1} + a_{4,2}\big)}_{a_{4,3} \text{ fixed to this}} s^3 t$$

$$+ \ldots$$

All $a_{2k,2k} = 0$
by locality of f_1

Chi, HE, Herderschee, Jones, Paranjape (2021)

Compare with the inverse string kernel

$$f_2^{\mathrm{string}}(s,t) = -\frac{1}{\sin(\pi\alpha' s)} = -\frac{1}{\pi\alpha' s} - \frac{1}{6}\pi\alpha' s - \frac{7}{360}(\pi\alpha' s)^3 - \frac{31}{15120}(\pi\alpha' s)^5 + \dots$$

$$\mathcal{L}_{\alpha'} = \mathcal{L}_{\mathrm{BAS}} + \alpha'\,\partial^2\phi^4 + \alpha'^3\,\partial^6\phi^4 + \dots$$

Compare with the inverse string kernel

$$f_2^{\text{string}}(s,t) = -\frac{1}{\sin(\pi\alpha' s)} = -\frac{1}{\pi\alpha' s} - \frac{1}{6}\pi\alpha' s - \frac{7}{360}(\pi\alpha' s)^3 - \frac{31}{15120}(\pi\alpha' s)^5 + \dots$$

4-point bootstrap + locality

$$g^2 = \frac{1}{\pi\alpha'} \qquad -\frac{1}{6}\pi\alpha' \qquad -\frac{7}{360}\pi^3\alpha'^3$$

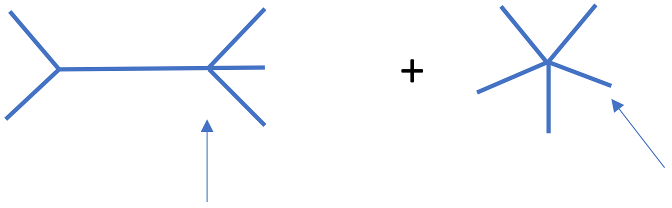
$$\begin{aligned} f_2(s,t) = \boxed{-\frac{g^2}{s}} &+ \cancel{a_{1,0}}\, t + \boxed{a_{1,1}\, s} + \cancel{a_{2,0}}\, t(s+t) \\ &+ \cancel{a_{3,0}}\, t^3 + \cancel{a_{3,1}}\, st^2 + \cancel{a_{3,2}}\, s^2 t + \boxed{a_{3,3}\, s^3} \\ &+ \cancel{a_{4,0}}\, t^4 + \cancel{a_{4,1}}\, st^3 + \cancel{a_{4,2}}\, s^2 t^2 + \cancel{\left(a_{4,0} + a_{4,1} + a_{4,2}\right)}\, s^3 t \\ &+ \dots \end{aligned}$$

So the result of the 4-pt KLT bootstrap is *much more general than the strings kernel*.

5-point bootstrap

$\mathbf{1} = \mathbf{1} \otimes \mathbf{1}$ ⇔ (n-1)! x (n-1)! matrix has rank (n-3)!

$m_5 \supset$



Boostrap: 24 x 24 matrix of tree amplitudes must have rank 2

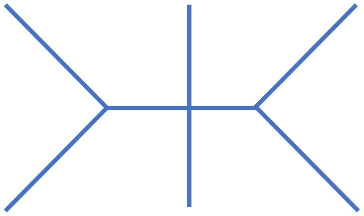
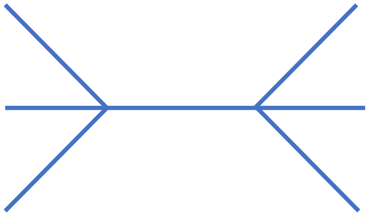
5-point bootstrap gives *no constraints on 4-point coefficients.*
There is freedom in local coefficients starting at order (Mandelstam)^3.

Chi, HE, Herderschee, Jones, Paranjape (2021)

6-point bootstrap

$\mathbf{1} = \mathbf{1} \otimes \mathbf{1}$ ⇔ (n-1)! x (n-1)! matrix has rank (n-3)!

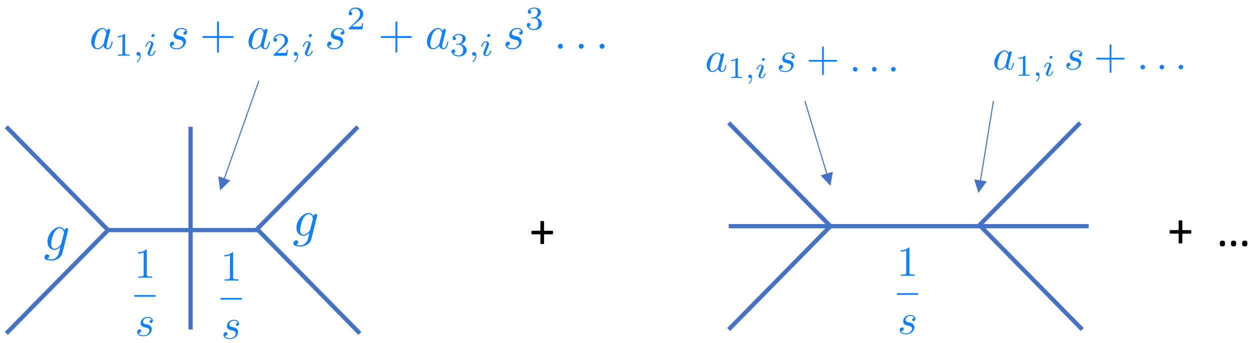
At 6-points, things start to get interesting because factorization to lower-point plays a crucial role

$m_6 \supset$  $+$  $+ \ldots$

Alan (Shih-Kuan) Chen & HE (to appear)

6-point bootstrap

At 6-points, things start to get interesting because factorization to lower-point plays a crucial role



s denotes
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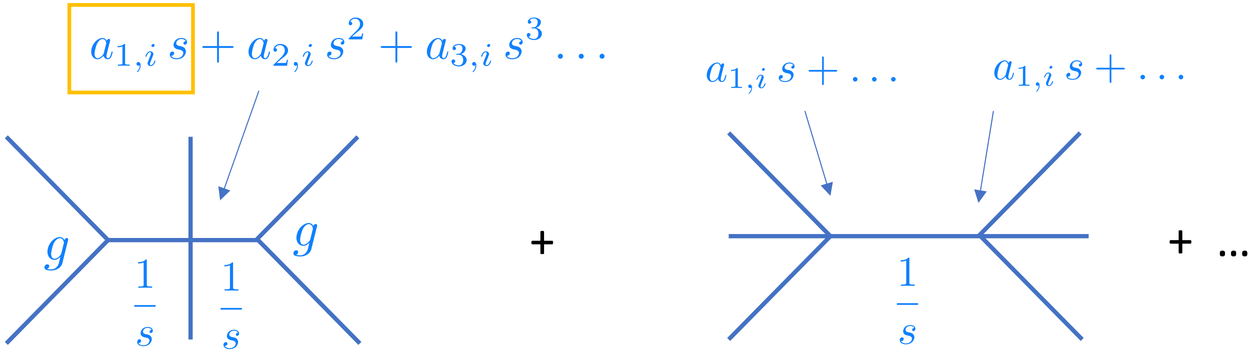
Alan (Shih-Kuan) Chen & HE (to appear)

Factorization plays a key role at 6-point

At 6-points, things start to get interesting because factorization to lower-point plays a crucial role



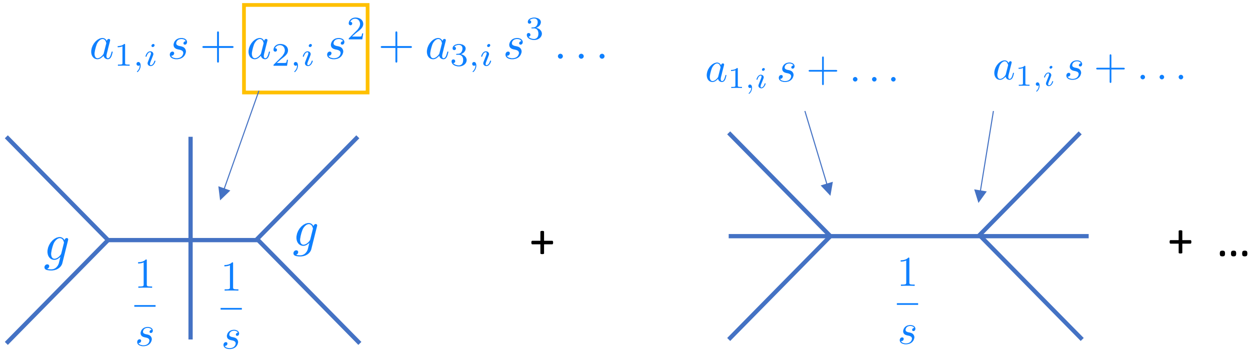
s denotes generic Mandelstam

O(1/s) Non-trivial that minimal rank conditions holds, no adjustments possible from local 6pt terms. **It does work.**

Alan (Shih-Kuan) Chen & HE (to appear)

Factorization plays a key role at 6-point

At 6-points, things start to get interesting because factorization to lower-point plays a crucial role



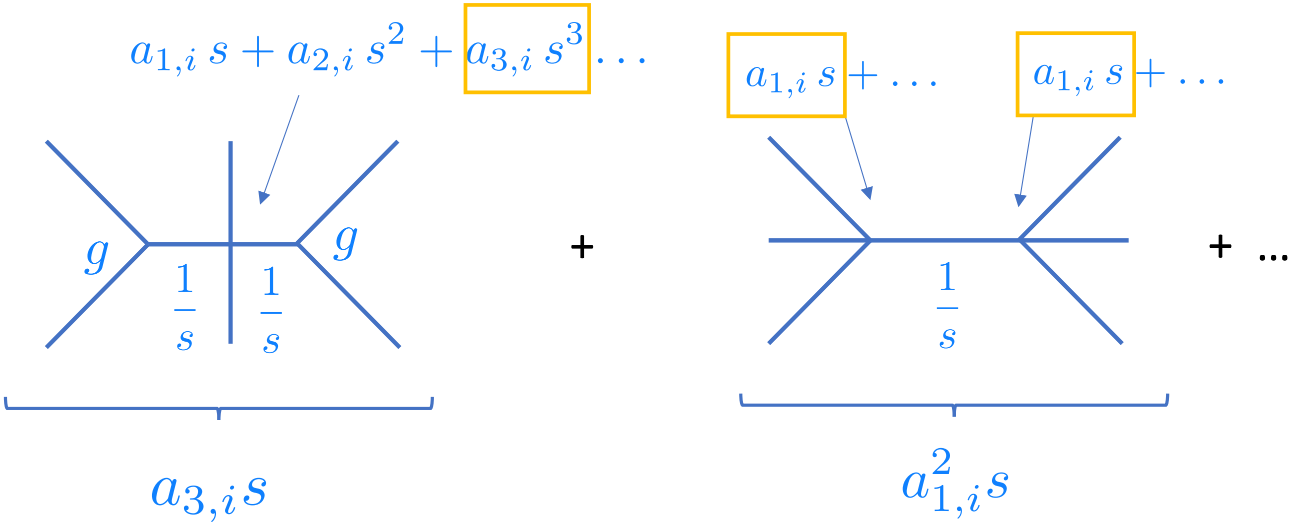
s denotes generic Mandelstam

$O(s^0)$ No constraints on $a_{2,0}$

Alan (Shih-Kuan) Chen & HE (to appear)

Factorization plays a key role at 6-point

At 6-points, things start to get interesting because factorization to lower-point plays a crucial role



s denotes generic Mandelstam

O(s^1)

Rank condition imposes relations like $a_{3,j} \propto a_{1,i}^2$ **All four $a_{3,i}$ are completely fixed!**

Alan (Shih-Kuan) Chen & HE (to appear)

New constraints at 4- and 5-point follow from the 6-point bootstrap

$$a_{3,0} = \frac{2}{5g^2} a_{1,0} \left(a_{1,0} - 2a_{1,1}\right)$$

$$a_{3,1} = \frac{1}{10g^2} a_{1,0} \left(a_{1,0} - 12a_{1,1}\right)$$

$$a_{3,2} = \frac{1}{5g^2} a_{1,0} \left(2a_{1,0} - 9a_{1,1}\right)$$

$$a_{3,3} = -\frac{7}{10g^2} a_{1,1}^2$$

Alan (Shih-Kuan) Chen & HE (to appear)

New constraints at 4- and 5-point follow from the 6-point bootstrap

$$g^2 = \frac{1}{\pi\alpha'}$$

$$\left.\begin{array}{lcl}
a_{3,0} = \dfrac{2}{5g^2}a_{1,0}\left(a_{1,0} - 2a_{1,1}\right) & \rightarrow & 0 \\
a_{3,1} = \dfrac{1}{10g^2}a_{1,0}\left(a_{1,0} - 12a_{1,1}\right) & \rightarrow & 0 \\
a_{3,2} = \dfrac{1}{5g^2}a_{1,0}\left(2a_{1,0} - 9a_{1,1}\right) & \rightarrow & 0 \\
a_{3,3} = -\dfrac{7}{10g^2}a_{1,1}^2 & \rightarrow & -\dfrac{7}{360}\pi^3\alpha'^3
\end{array}\right\}$$

Recall that string theory has

$$a_{1,0} = 0 \qquad a_{1,1} = -\frac{1}{6}\pi\alpha'$$

$$a_{2,0} = 0$$

Exactly the string kernel values

Alan (Shih-Kuan) Chen & HE (to appear)

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$$g^2 = \frac{1}{\pi\alpha'}$$

$$\left.\begin{aligned}
a_{3,0} &= \frac{2}{5g^2} a_{1,0}\left(a_{1,0} - 2a_{1,1}\right) &&\to \quad 0 \\
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\end{aligned}\right\}$$

Recall that string theory has

$$a_{1,0} = 0 \qquad a_{1,1} = -\frac{1}{6}\pi\alpha'$$

$$a_{2,0} = a_{4,0} = a_{6,0} = 0$$

6-pt bootstrap: all $a_{4,i}$ fixed, except for one.

all $a_{5,i}$ fixed.

all $a_{6,i}$ fixed, except for one.

…

Alan (Shih-Kuan) Chen & HE (to appear)

Can we bootstrap the string theory kernel?

Not quite, there must be free parameters.
For a good reason: our results must in fact be more general than the string kernel.

To see why, we need to take a little detour…

… to another corner of the KLT double-copy relation world….

Z-theory

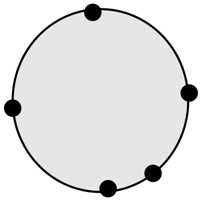
Open string with its alpha’ corrections

Z-theory $\otimes_{\mathrm{SS}}$ YM = open string

Result of string disk integrals

Standard BAS kernel

YM *without* higher derivative corrections



$$Z[\sigma|\rho] = Z_\sigma(\rho(1,2,\ldots,n)) = (2\alpha')^{n-3}\int_{\sigma\{-\infty\leq z_1\leq z_2\leq\ldots\leq z_n\leq\infty\}} \frac{dz_1\ \ldots\ dz_n}{\mathrm{vol}(\mathrm{SL}(2,\mathbb{R}))}\ \frac{\prod_{i<j}^n |z_{ij}|^{\alpha' s_{ij}}}{\rho\left\{z_{12}z_{23}\cdots z_{n-1,n}z_{n,1}\right\}}$$

Mafra, Schlotterer, Stieberger (2011)+(2013)

Z-theory

Open string with its alpha’ corrections

Z-theory $\otimes_{\mathrm{SS}}$ YM = open string

Result of string disk integrals

Standard BAS kernel

YM *without* higher derivative corrections

$$\sum_{a,b} Z_n[c|a]S_n[a|b]A^{\mathrm{YM}}[b] = A^{\text{open str}}[c]$$

Sum over (n-3)! color-orderings

Mafra, Schlotterer, Stiewerger (2011)+(2013)

Z-theory

$$\sum_{a,b} Z_n[c|a] S_n[a|b] A^{\text{YM}}[b] = A^{\text{open str}}[c]$$

Sum over (n-3)! color-orderings

Independence of the choice of (n-3)! color-orderings in the KLT sum requires that the Z-theory amplitudes must obey the L-sector standard field theory KKBCJ relations:

Specifically, the Z-theory amplitudes must obey $\mathbf{L} \otimes \mathbf{1} = \mathbf{L}$ (but not necessarily $\mathbf{1} \otimes \mathbf{R} = \mathbf{R}$)

Z-theory

$$\sum_{a,b} Z_n[c|a]S_n[a|b]A^{\mathrm{YM}}[b] = A^{\mathrm{open\ str}}[c]$$

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Specifically, the Z-theory amplitudes must obey $\mathbf{L} \otimes \mathbf{1} = \mathbf{L}$ (but not necessarily $\mathbf{1} \otimes \mathbf{R} = \mathbf{R}$)

This can be imposed as $\sum_a Z[c|a]\, n_i[a] = 0$ where $n_i[a] = 0$ are the (n-1)! – (n-3)! null vectors of the matrix of BAS amplitudes

Sum over (n-1)! color-orderings

That means the matrix of Z-amplitudes itself has the (n-1)! – (n-3)! null vectors under R-multiplication.

So it too has rank (n-3)! and therefore *it solves the KLT bootstrap equation*.

Z-theory

Z-theory then defines its own kernel: $\mathbf{1}_Z = \mathbf{1}_Z \otimes_Z \mathbf{1}_Z$

This means that **Z-theory must be a special case of our generalized KLT bootstrap**!!

The most general solution to the KLT bootstrap equations must include both

1) the inverse string kernel
2) Z-theory

Alan (Shih-Kuan) Chen & HE (to appear)

What do Z-theory amplitudes look like?

$$f_1^{\mathrm{Z}}(s,t) = m_4^{\mathrm{Z}}[1234|1234] = \frac{1}{\pi}\frac{\Gamma(\alpha' s)\Gamma(\alpha' u)}{\Gamma(\alpha' s + \alpha' u)}$$

Beta function

Use the KKBCJ relation $A_4[1\underline{24}3] = \frac{u}{t}A[1\underline{23}4]$

$$f_2^{\mathrm{Z}}(s,t) = m_4^{\mathrm{Z}}[1234|1\underline{24}3] = \frac{u}{t}m_4^{\mathrm{Z}}[1234|1\underline{23}4] = \frac{u}{t}f_1^{\mathrm{Z}}(s,t) = -\frac{1}{\pi\, s}\frac{\Gamma(1+\alpha' s)\Gamma(1+\alpha' u)}{\Gamma(1+\alpha' s + \alpha' u)}$$

What do Z-theory amplitudes look like?

Low-energy expansion

$$f_2^{\mathrm{Z}}(s,t) = -\frac{1}{\pi s}\frac{\Gamma(1+\alpha' s)\Gamma(1+\alpha' u)}{\Gamma(1+\alpha' s+\alpha' u)}$$

$$= -\frac{1}{\pi\alpha' s} \underbrace{- \frac{1}{6}\pi\alpha'\,(s+t) - \frac{\zeta(3)}{\pi}\alpha'^2\, t(s+t) - \frac{1}{360}\pi^3\alpha'^3(s+t)(7s^2+7st++4t^2)+\ldots}$$

BAS with $g^2 = \frac{1}{\pi\alpha'}$

Z-theory is BAS plus specific higher-derivative operators!

Z-theory comparison

Low-energy expansion

$$f_2^{Z}(s,t) = -\frac{1}{\pi s}\frac{\Gamma(1+\alpha' s)\Gamma(1+\alpha' u)}{\Gamma(1+\alpha' s+\alpha' u)}$$

$$= -\frac{1}{\pi\alpha' s} \underbrace{-\frac{1}{6}\pi\alpha'\,(s+t)}_{a_{1,1}=a_{1,0}=-\frac{1}{6}\pi\alpha'} \underbrace{-\frac{\zeta(3)}{\pi}\alpha'^2\,t(s+t)}_{a_{2,0}=-\frac{\zeta(3)}{\pi}\alpha'^2} \underbrace{-\frac{1}{360}\pi^3\alpha'^3(s+t)(7s^2+7st++4t^2)}_{\substack{a_{3,0}=-\frac{1}{90}\pi^3\alpha'^3 \quad a_{3,1}=-\frac{11}{360}\pi^3\alpha'^3 \\ a_{3,2}=-\frac{7}{180}\pi^3\alpha'^3 \quad a_{3,3}=-\frac{7}{360}\pi^3\alpha'^3}} + \dots$$

BAS with $g^2 = \frac{1}{\pi\alpha'}$

$$g^2 = \frac{1}{\pi\alpha'}$$

6-pt bootstrap:

all $a_{3,i}$ fixed
all $a_{4,i}$ fixed, except for $a_{4,0}$
all $a_{5,i}$ fixed.
all $a_{6,i}$ fixed, except for $a_{6,0}$
…

Inverse string theory kernel

$$a_{1,0} = 0 \qquad a_{1,1} = -\frac{1}{6}\pi\alpha'$$

$$a_{2,0} = a_{4,0} = a_{6,0} = 0$$ etc

Z-theory

$$a_{1,1} = a_{1,0} = -\frac{1}{6}\pi\alpha' \qquad a_{4,0} = -\frac{\zeta(5)}{\pi}\alpha'^4$$

$$a_{2,0} = -\frac{\zeta(3)}{\pi}\alpha'^2 \qquad a_{6,0} = -\frac{\zeta(7)}{\pi}\alpha'^6$$ etc

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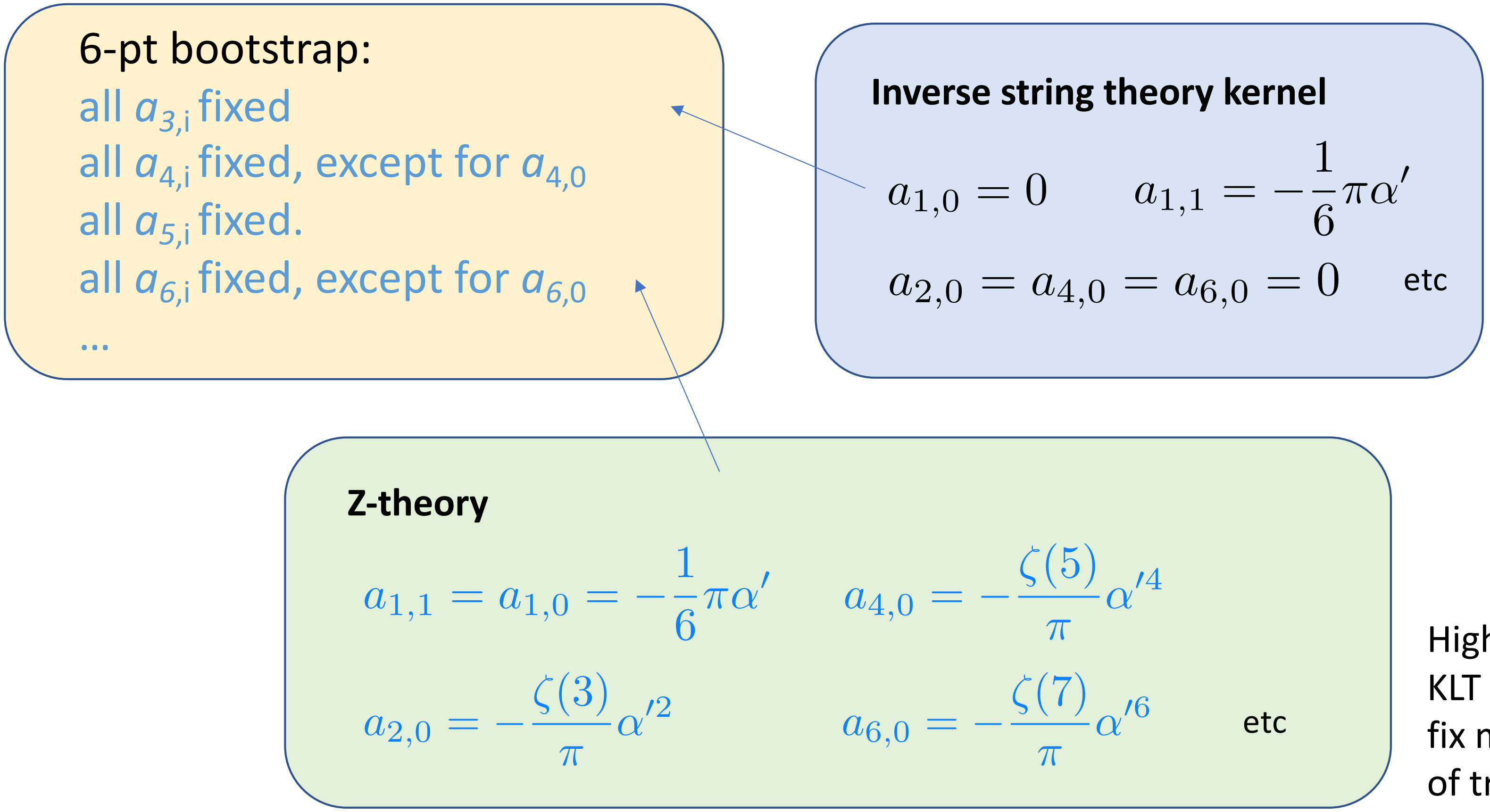
$$g^2 = \frac{1}{\pi\alpha'}$$



6-pt bootstrap:

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Z-theory

$$a_{1,1} = a_{1,0} = -\frac{1}{6}\pi\alpha' \qquad a_{4,0} = -\frac{\zeta(5)}{\pi}\alpha'^4$$

$$a_{2,0} = -\frac{\zeta(3)}{\pi}\alpha'^2 \qquad a_{6,0} = -\frac{\zeta(7)}{\pi}\alpha'^6$$ etc

Higher-point
KLT bootstrap *cannot*
fix more at 4pt b/c
of transcendentality

Alan (Shih-Kuan) Chen & HE (to appear)

Observations

- Z-theory obeys $\mathbf{Z} \otimes_{\mathrm{SS}} \mathbf{1}_{\mathrm{SS}}$ Standard BAS KKBCJ relations from the right
- Impose BAS KKBCJ from the right on most general ansatz GS model:
- Find at 4-point: GS = Z V for *stu*-symmetric local function V.

This means: KLT bootstrap + fixing $a_{1,0}$ & $a_{1,1}$ => Z-theory

Where is the string theory kernel?

string monodromy BAS KKBCJ

Z-theory obeys string monodromy relations on the left: $Z_n[a|b]$

Let Z^T be the model whose matrix of tree amplitudes are the transpose of the (n-1)! x (n-1)! tree amplitudes of Z-theory amplitudes. Will obey BAS KKBCJ relations from the *left* and *string monodromy on the right.*

So: The result of $Z \otimes_{\text{SS}} Z^T$ will obey string monodromy on both L and R.

Is it the inverse string kernel??

Almost! $Z \otimes_{\text{SS}} Z^T = m^{\alpha'\alpha'}\, U$, where $U = -\dfrac{\Gamma[\alpha' s]\Gamma[\alpha' t]\Gamma[\alpha' u]}{\Gamma[-\alpha' s]\Gamma[-\alpha' t]\Gamma[-\alpha' u]}$

Where is the string theory kernel?

Hybrid decomposition conjecture:

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General minimal rank model amplitudes obey $\mathrm{GG} = \mathrm{GS} \otimes_{\mathrm{SS}} \mathrm{SG}$

where the “hybrid models” GS and SG obey BAS KKBCJ from the R / L, respectively

4-point: $\mathrm{GS} \sim Z$ after fixing $a_{1,0}=a_{1,1}$ gives

Differ by *stu*-symmetric function

$$\mathrm{GG} \sim \mathrm{GS} \otimes_{\mathrm{SS}} \mathrm{SG} \sim \mathrm{GS} \otimes_{\mathrm{SS}} (\mathrm{GS})^T \sim Z \otimes_{\mathrm{SS}} Z^T \sim \mathrm{inv\ str\ kernel}$$

Closing in on bootstrapping the most general 4-point kernel to be in the *same* equivalence class as the string kernel!

Collaborators

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Greetings from Michigan