Double copy with topological masses

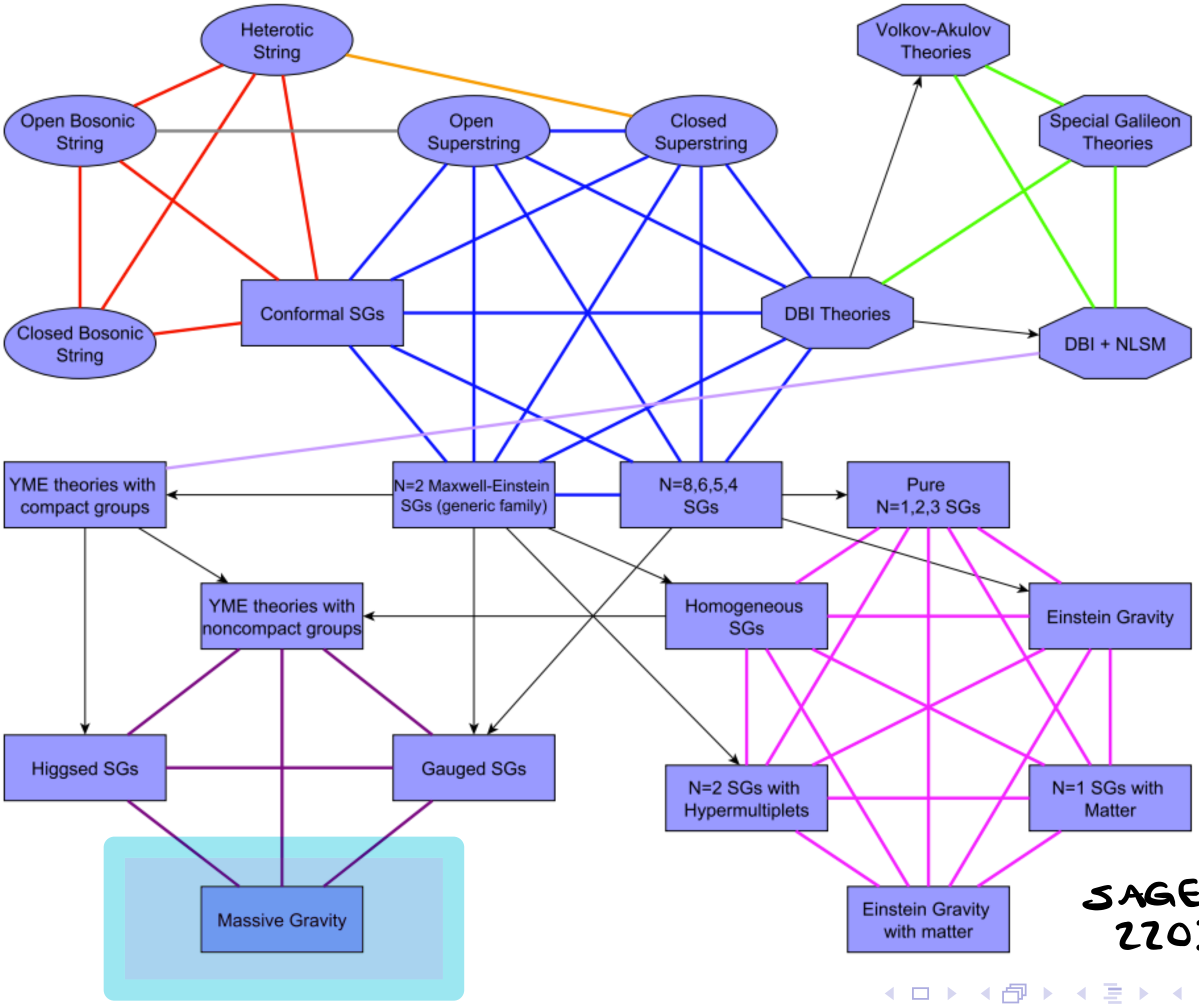
Mariana Carrillo González

Amplitudes 2022

Imperial College London

Based on work with A. Momeni and J. Rumbutis: 2107.00611, 2112.08401, 2202.10476 and work in progress with W. Edmond , N. Moynihan, J. Rumbutis, C. D. White

Web of double copy constructible theories



$= (YM)^2$

SAGEX REVIEW
2203.13013

Imperial College London

BCJ Double Copy

$$\mathcal{A}_{YM} = \sum_i \frac{c_i n_i}{s_i + m^2} = c^{\mathsf{T}} D^{-1} n \quad \xleftrightarrow[\text{dual}]{\text{CK}} \quad \mathcal{M}_G = \sum_i \frac{n_i n_i}{s_i + m^2} = n^{\mathsf{T}} D^{-1} n$$

Color factors satisfy Jacobi relations: $Mc = 0$

Color-kinematics duality $\Rightarrow Mn = 0$

For $Mn \neq 0$ use generalized gauge transformations:

$$A^{YM} \to A^{YM}$$

$n \to n + \Delta n$ s.t. $c^T D^{-1} \Delta n = 0$ $\quad Mn \to Mn + MDM^T v = 0$

$$\to \Delta n = DM^T v$$

Imperial College London

BCJ Double Copy

$$\mathcal{A}_{YM} = \sum_i \frac{c_i n_i}{s_i + m^2} = c^T D^{-1} n \quad \overset{\text{CK dual}}{\longleftrightarrow} \quad \mathcal{M}_G = \sum_i \frac{\tilde{n}_i \tilde{n}_i}{s_i + m^2} = \tilde{n}^T D^{-1} \tilde{n}$$

Color factors satisfy Jacobi relations: $Mc = 0$

Color-Kinematics duality $\Rightarrow$ $M\tilde{n} = Mn + MDM^T v = 0$

Invertible

$$\mathcal{M}_G = n^T D^{-1} n + (Mn)^T (MDM^T)^{-1} (Mn)$$

Unphysical poles!

$\Rightarrow$ CK duality for any theory!

Previously observed in:
1701.02519 Generalized D.C. Bern, Carrasco, Chen, Johansson, Roiban

Massive case:
Johnson-Engelbrecht, Jones, Paranjape
Momeni, Rombouts, Tolley

Avoiding unphysical poles

$m=0 \xrightarrow{SSB} m\neq 0$

MDM^T has minimal rank

$\Rightarrow$ 4 5pt BCJ relations

- SSB in Supergravities
  Chiodaroli, Gunaydin, Johansson, Roiban
- Kaluza-Klein theories
  Johnson-Engelbrecht, Jones, Paranjape
  Momeni, Rumbutis, Tolley
- $(\phi^a)^3 \quad U(N)\times G$
  MCG, Liang, Trodden

- Mass-deformed minimal $(DF)^2$
  Johansson, Mogull, Teng; Menezes

3D Kinematics

$\det(MDM^T) \propto \det(p_i \cdot p_j), \quad i,j<5$

$\Rightarrow \det(MDM^T)^{3d} = 0$

$\boxed{\text{Only 1 5pt ``BCJ'' relation}}$

No spurious poles + correct factorization

Topologically massive theories

MCG, Momeni, Rumbutis
Burger, Emond, Moynihan
Hang, He, Shen

Topologically massive theories

Imperial College London

2107.00611
MCG, Momeni, Rumbutis

DC @
3, 4, 5 pt

PARITY BROKEN

Topologically Massive Gravity 1 dof

$$S_{TMG} = \frac{1}{\kappa^2} \int d^3x \sqrt{-g} \left(-R - \frac{1}{2m} \varepsilon^{\mu\nu\rho} \left(\Gamma^{\alpha}_{\mu\sigma} \partial_\nu \Gamma^{\sigma}_{\alpha\rho} + \frac{2}{3} \Gamma^{\alpha}_{\mu\sigma} \Gamma^{\sigma}_{\nu\beta} \Gamma^{\beta}_{\rho\alpha} \right) \right)$$

Topologically Massive Yang-Mills 1 dof

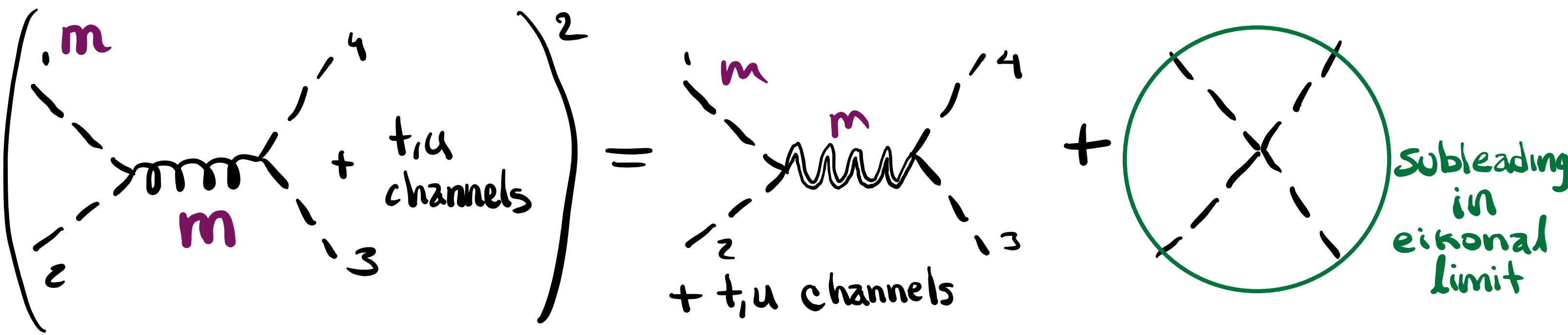
$$S_{TMYM} = \int d^3x \left(-\frac{1}{4} F^{a\mu\nu} F_{a\mu\nu} + \varepsilon_{\mu\nu\rho} \frac{m}{12} \left(6 A^{a\mu} \partial^\nu A^\rho_a + g\sqrt{2} f_{abc} A^{a\mu} A^{b\nu} A^{c\rho} \right) \right)$$

$1 \otimes 1 = 1$ ✓

Including matter

$$(T_{MYM})^2 = T_{MG} + \cancel{?}$$

for matter with $T_\mu{}^\mu = 0$



2112.08401

MCG, Momeni, Rumbotis

DC in eikonal limit

Also observed in:

Burger, Emond, Moynihan

Kerr-Schild metric $(g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu})$

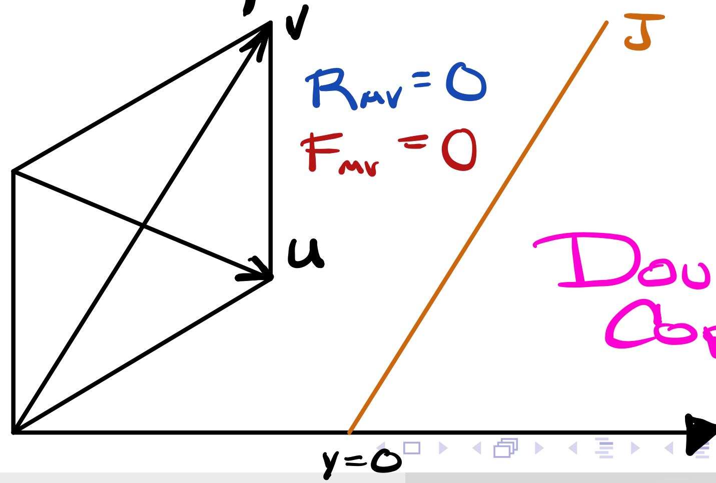
★ Special choice of b.c.

$$h_{\mu\nu} = k_\mu k_\nu \phi \qquad k_\mu dx^\mu = du$$

$$\phi = \frac{\kappa}{2} \frac{2E}{m} \left(e^{-my} \Theta(y) + (1 - my) \Theta(-y) \right)$$

$$A^a_\mu = c^a k_\mu \phi$$

$$\phi = g \frac{Q}{m} \left(e^{-my} \Theta(y) + \Theta(-y) \right)$$



$R_{\mu\nu} = 0$

$F_{\mu\nu} = 0$

$\phi \to 0$

$\phi \to 0$

$y \to \infty$

Double Copy

Cotton Double Copy

Imperial College London

Shockwaves satisfy: $C_{uu} = \frac{m}{2} \frac{{}^*F_u \, {}^*F_u}{\phi}$ Why?

Look at linearized e.o.m plane waves

$$\varepsilon_{\mu\alpha\beta}\nabla^{\alpha}\left(R_{\nu}{}^{\beta} - \tfrac{1}{4} g_{\nu}{}^{\beta} R\right) \overset{\text{lin. eom}}{=} \frac{m}{2}\varepsilon_{\zeta\nu\alpha\beta}F^{\alpha\beta}$$

$$\parallel$$

$$C^{lin}_{\mu\nu} \propto \frac{\overbrace{\nabla^{\lambda} F^{lin}_{\lambda(\nu}}\, \varepsilon_{\mu)\rho\gamma} F^{lin\,\rho\gamma}}{e^{ip\cdot x}}$$

$\mathcal{O}(p^3)$ $\mathcal{O}(p^2)$ $\mathcal{O}(p)$

Imperial College London

Weyl D.C.

$$SO^+(1,3) \cong SL(2,\mathbb{C})/\mathbb{Z}_2$$

$$\psi_{ABCD} = \frac{f_{(AB}f_{CD)}}{S}$$

$$\nabla^{A\dot{A}}\psi_{ABCD} = 0$$

$$\nabla^{A\dot{A}}f_{AB} = 0$$

$$(\Box - R/6)S = 0$$

Cotton D.C.

$$SO^+(1,2) \cong SL(2,\mathbb{R})/\mathbb{Z}_2$$

$$C_{ABCD} = \frac{f_{(AB}f_{CD)}}{S}$$

$$\nabla^{AE}C_{ABCD} = m\,C^{E}{}_{BCD}$$

$$\nabla^{AE}f_{AB} = m\,f^{E}{}_{B}$$

Dimensional reduction?

$$(\Box - R/6 - m^2)S = 0$$

Imperial College London

Weyl D.C.	Cotton D.C.
$V^{\mu} = \sigma^{\mu}_{A\dot{B}} V^{A\dot{B}}$	$V^{\mu} = \sigma^{\mu}_{AB} V^{AB}$
$\Psi_{ABCD} = \dfrac{f_{(AB} f_{CD)}}{S}$	$C_{ABCD} = \dfrac{f_{(AB} f_{CD)}}{S}$
$\nabla^{A\dot{A}} \Psi_{ABCD} = 0$	$\nabla^{AE} C_{ABCD} = m\, C^{E}{}_{BCD}$
$\nabla^{A\dot{A}} f_{AB} = 0$	$\nabla^{AE} f_{AB} = m\, f^{E}{}_{B}$
$(\Box - R/6) S = 0$	$(\Box - R/6 - m^2) S = 0$

Dimensional reduction?

Cotton DC for waves (Type N)

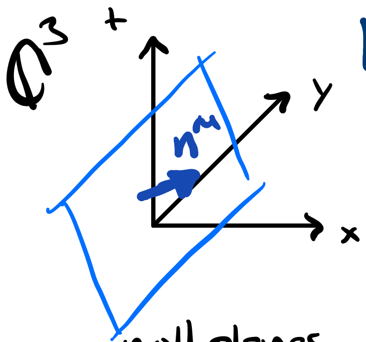
$$C_{ABCD} = \Psi_4\, o_A o_B o_C o_D$$

2202.10476 — MCG, Momeni, Rumbutis — Cotton DC

$$f_{AB} = \Phi_2\, o_A o_B$$

$$\Psi_4 = \frac{m}{2}\frac{\Phi_2^2}{\phi}$$

	Ψ_4	Φ_2	ϕ
Shockwaves	$E m^2 e^{-my}$	$Q e^{-my}$	$\lambda\, e^{-my}$
Gyratons	$(E+\sigma m) m^2 e^{-my}$	$(Q+mQ') e^{-my}$	$\lambda+\lambda' m\, e^{-my}$
AdS-Shockwaves	$E m^2 \left(\frac{z}{z_0}\right)^{-1-Lm}$	$Q \left(\frac{z}{z_0}\right)^{-Lm}$	$\lambda \left(\frac{z}{z_0}\right)^{1-Lm}$



null planes

$\downarrow$

Minitwistor space

Work in progress with Emond, Moynihan, Rombotis, White

$$\mathbb{MT} = \{ Z^\alpha = (u, \lambda_A),\; u = x^{AB}\lambda_A\lambda_B \}$$

$$(u, \lambda_A) \sim (r^2 u, r\lambda_A)$$

$$u = n_\mu x^\mu,\quad n_{AB} = n_\mu \sigma^\mu_{AB} = \lambda_A\lambda_B$$

$$\left(\mathbb{MT} = \frac{ESL(2,\mathbb{C})}{Q} \right)$$

$\uparrow$

isometries null planes

From Dim. Red. $x^{4d}_{A\dot{A}} T_B{}^{\dot{A}} = x^{3d}_{AB} + iz\,\epsilon_{AB}$

$$(x^{AB}\lambda_A\lambda_B, \lambda_A) \sim \left(r^2 (x^{AB}\lambda_A + iz\lambda^B)\lambda_B,\; r\lambda_A \right)$$

spin n field

$$\underbrace{\phi_{AB\cdots D}}_{2n} = \frac{1}{2\pi i}\oint_{\Gamma} \langle \lambda d\lambda \rangle \lambda_A \cdots \lambda_D \, \rho_x[f(z)]$$

complex rep. of Q

$$\text{where } f(r^2(x^{AB}\lambda_A + iz\lambda^B))\lambda_B, r\lambda_A) = r^{-2n-2} e^{-izm} f(x^{AB}\lambda_A\lambda_B, \lambda_A)$$

$$\Rightarrow f(z) = e^{-m\frac{\langle a|x|\lambda\rangle}{\langle a\lambda\rangle}} g(u, \lambda_\alpha) \in H(M\mathbb{T}, \mathcal{O}(-2n-2, m))$$

$$\boxed{\nabla^H{}_A \phi_{HB\cdots D} = m\,\phi_{AB\cdots D}}$$

Imperial College London

Choose twistor representatives for type N (waves)

$$\boxed{f_2 = f_1^2 / f_0}$$

DOUBLE COPY

$$f_n = \frac{e^{-m\frac{\langle a|x|\lambda\rangle}{\langle a\lambda\rangle}}}{(A_\alpha Z^\alpha)} g^{-2n-1}(z) \qquad \lambda_\alpha = (1, z)$$

$$\phi_{\underbrace{AB\cdots D}_{2n}} = \frac{1}{2\pi i}\oint_{f_0} dz\, \lambda_A \cdots \lambda_D \frac{e^{-m\,q(x;z)}}{(z - z_0)} g^{-2n-1}(x;z)$$

$$= \lambda^0_A \cdots \lambda^0_D\, g^{-2n-1}(x;z_0)\, e^{-m\,q(x;z_0)}$$

$$\Rightarrow \text{Type N Cotton DC}$$

$$g^{-5} = \frac{(g^{-3})^2}{g^{-1}}$$

Imperial College London

Choose twistor representatives for type D (field around isolated objects)

$$f_n = \frac{e^{-m\frac{\langle a|x|\lambda\rangle}{\langle a\lambda\rangle}}}{Q_{\alpha\beta} Z^\alpha Z^\beta}$$

$\lambda_\alpha = (1, z)$

Related to Amplitudes

$$\boxed{f_2 = f_1^2 / f_0}$$

DOUBLE COPY

$$\phi_{\underbrace{AB\cdots D}_{2n}} = \frac{1}{2\pi i}\oint_{\Gamma_0} dz\, \lambda_A \cdots \lambda_D \frac{e^{-mq(x;z)}}{(z-z_0)^{1+2n}(z-z_1)^{1+2n}}$$

$$\propto \lim_{z\to z_0} \frac{d^{2n}}{dz^{2n}}\left(\frac{z^r e^{-mq(x;z)}}{(z-z_1)^{1+2n}}\right)$$

! No squaring relation in position space

$$(\text{TMYM})^2 = \text{TMG}$$

Robust evidence: Amplitudes + Classical

Open Questions:

- Coupling to non-eikonalized matter? (Anyons)
- Type D? (Squashed AdS)
- Origin via dimensional reduction?