~~ON~~ OFF - SHELL ~~BAD~~ GOOD

or Why String Field Theory is Needed to Determine the Effects of D-instantons

based on 2205.00609 (section 6.4 in particular)

w/ N. Agmon, B. Balthazar, M. Cho, V. Rodriguez

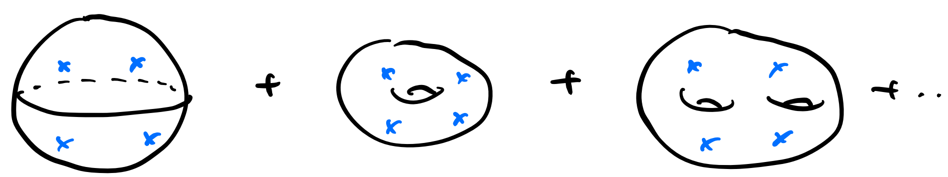
see also earlier and closely related works by

Balthazar, Rodriguez, Yin, Sen, Alexandrov, Stefanski, ...

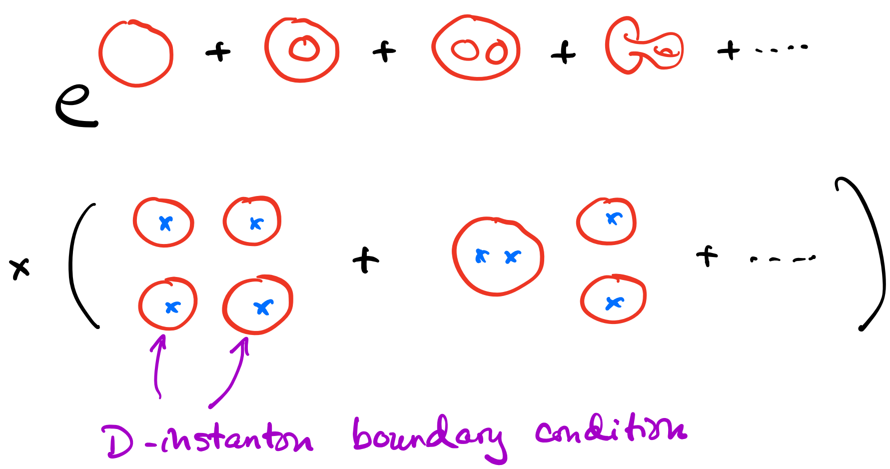
• The worldsheet formulation of string theory captures more than just the perturbative expansion in g_s

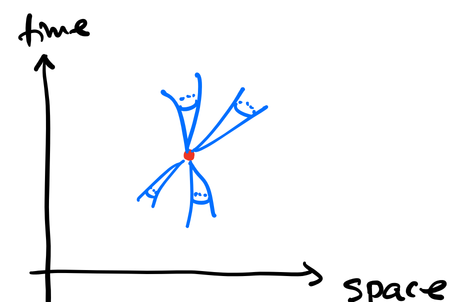
Polchinski '94 :

In addition to



Also ;

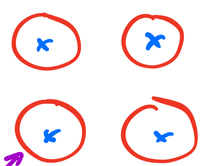
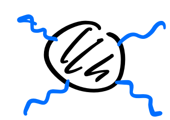




* Deficiencies of the on-shell approach to D-instanton effects

1. normalization (of measure on $\mathcal{M}_{D\text{-inst}}$)

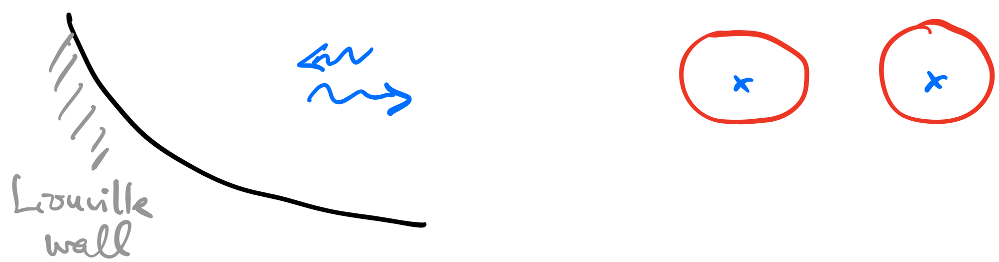
- Green, Gutperle '97:

computed  contribution to  in type IIB string theory

1-D-instanton

up to normalization, undetermined from 1st principle but fixed indirectly via S-duality

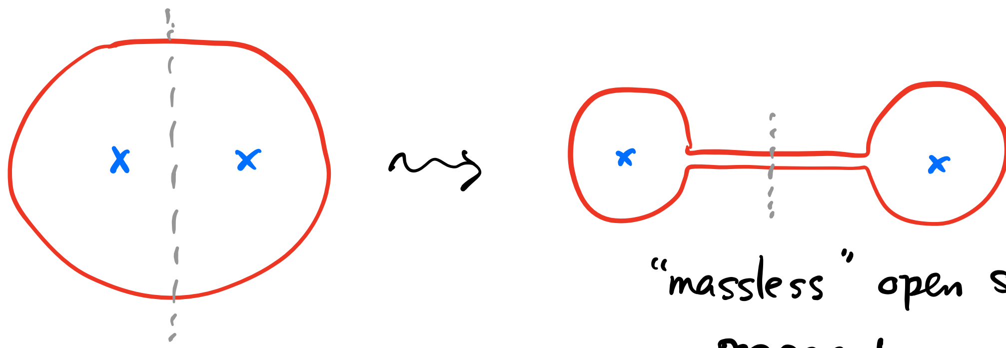
- Balthazar, Rodriguez, Yin '19, '22



D-instanton effects in 2D c=1 (bosonic) string and type 0B (nonsupersymmetric NSR) string

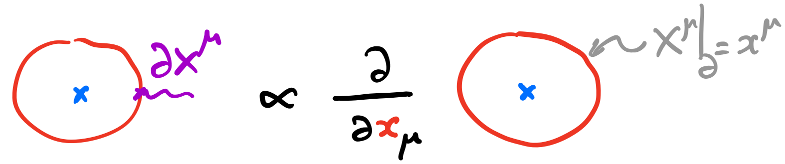
- normalization fixed by matching w/ conjectured non-perturbative dual MQM

② Open string divergences



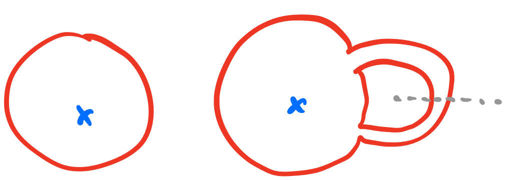
"massless" open string propagator $\sim \frac{1}{0}$.

e.g.



Polchinski '94

divergence cancels against



- Finite ambiguity?

  Need a consistent cutoff near boundary of moduli space of WS surfaces of different topologies

  - Noted by Green, Gutperle '97 but unresolved.

  - Balthazar, Rodriguez, XY '19 fixed ambiguity in 2D $c=1$ string by comparison with proposed dual MQM.

  - Sen '20 resolved the ambiguity in 2D $c=1$ string using open + closed SFT, and proposed general scheme also applicable to superstrings.

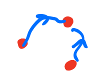
  - In this talk, we will discuss an example in which this ambiguity is resolved in type IIB string in 10D via O+C SFT [2205.00609]

(3) $\int_{\mathcal{M}_{D\text{-inst}}}$ may be singular

e.g. for multiple D-instantons in type IIB string

In principle, this problem is also present in the standard treatment of instantons in QFT.

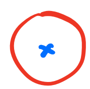
However, OSFT is smart!



non-Abelian open string fields

On-shell

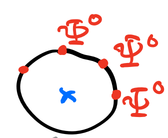
$$\int_{\mathcal{M}_{D\text{-inst}}} \cdots$$



deformed BCFT

off-shell

$$\int D\Psi^0 \cdots$$



fixed BCFT

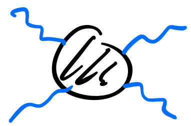
- leading N-D-instanton contribution to R^4 eff. coupling in IIB string theory captured by IKKT matrix integral

cannot be evaluated by expanding perturbatively around classical vacua

Green, Gutperle '97

Sen '21

Now we discuss a particular observable of interest, in type IIB string in 10D



$2 \to 2$ Supergraviton amplitude

$$\mathcal{A}_{2\to 2} = \mathcal{A}^{\text{sugra}}_{2\to 2} \times \mathcal{M}(s,t \,|\, \tau, \bar{\tau})$$

$\underline{s} \equiv \frac{s}{m_{pl}^2}$.

$$\mathcal{M} = 1 + \underline{s}\,\underline{t}\,\underline{u}\left[\underbrace{f_0(\tau,\bar{\tau})}_{\text{“}R^4\text{”}} + \cdots\right]$$

momentum expansion

$$\overset{?}{=} \sum_{h=0}^{\infty} \tau_2^{-2h} M_h(\alpha' s, \alpha' t)$$

$(\tau_2^{-1} = g_s)$

$$\text{“}+\text{”} \sum_{(n,m)} e^{2\pi i (n\tau - m\bar{\tau})} \mathcal{M}^{(n,m)}(\alpha' s, \alpha' t; \tau_2)$$

↑ D-instanton expansion.

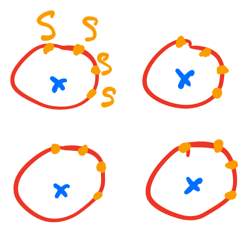
$$+ \cdots$$

Focus on 1-D-instanton sector $\quad (n, m) = (1, 0)$

$$\mathcal{M}^{(1,0)} = \tau_2^{-\frac{11}{2}} \sum_{L \geq 0} \tau_2^{-L} \mathcal{M}_L^{(1,0)}$$

open string "loop" order

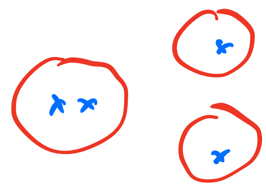
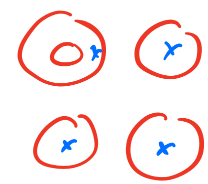
LO:

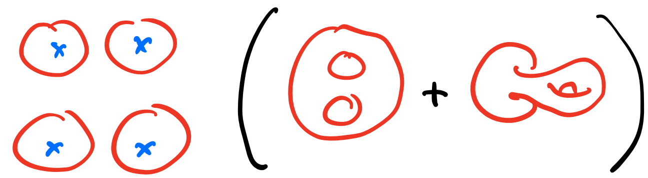


Green-Gutperle '97
Sen '20

$$\frac{1}{stu} \mathcal{M}_0^{(1,0)} = \frac{\pi}{16} .$$

$e^{2\pi i \tau}$ coeff. in $f_0(\tau, \bar{\tau}) R^4$.

NLO :  also 



$$\frac{1}{stu}\mathcal{M}_1^{(1,0)} = C_1 + \sum_{n\geq 2}\frac{\zeta(n)}{2^{2n+3}}\left(s^n+t^n+u^n\right)$$

$e^{2\pi i\tau}\tau_2^{-1}R^4$ $\qquad$ $e^{2\pi i\tau}\tau_2^{-1}D^{2n}R^4$

ABCRY '22

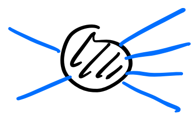
susy + soft thm : $C_1 = \frac{3}{256}$

Green, Gutperle '97
Green, Sethi '98
Wang, XY '15

However, naively on-shell computation gets C_1 wrong !

• We will analyze "a part of C_1":

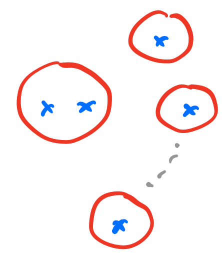
consider generalization to N-pt MRV amplitude



$2 \to N-2$

$$D^{2n}(\delta\tau)^{N-4}R^4 + \cdots$$

1-D-instanton contribution at NLO: e.g.



$$e^{2\pi i \tau}\,\tau_2^{-1}\,\delta^{16}(Q) \times \Big(C_1^{(N)} + \sum_{\substack{i<j \\ n\geq 2}} \#\, s_{ij}^{n} \Big)$$

✓ "easy"

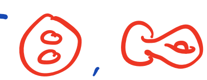
$$C_1^{(N)} = \frac{3}{256} - \frac{(N-4)(N-5)}{64}$$

(from susy + soft thm)

$$\equiv A_0 \binom{N}{2} + A_1 N + A_2$$

easiest in SFT
focus on this!

involves subtleties of open string field redefinition.



In SFT approach to D-instanton effects Sen '20, '21

the D-instanton contribution to closed string (field) effective action is captured by the functional integral over open string fields (on the D-instanton)

schematically,

$$e^{-\Gamma[\Psi_c]}\Big|_{D\text{-inst}} = \int D\Phi_o\Big|_{\mathcal{L}} \, e^{-S_{oc}[\Psi_o, \Psi_c]}$$

$\Psi_o = (\Phi_o, \Phi_o^{\ddagger})$

BV anti-field fixed by gauge cond. $\mathcal{L}$

$$S_{oc} \supset \underbrace{\tfrac{1}{2}\langle \Psi_0 | Q_B | \Psi_0 \rangle}_{\text{kinetic term}} + \cdots$$

In the conventionally adopted Siegel gauge $b_0 | \Psi_0 \rangle = 0$,

can write kinetic term as

$$\tfrac{1}{2}\langle \Psi_0 | c_0 \underbrace{b_0\, Q_B}_{\Downarrow} | \Psi_0 \rangle$$

$$\{b_0, Q_B\} = L_0$$

zero-weight ($L_0 = 0$) states ("massless") have no kinetic term ?!

Two types of massless open string modes:

1. collective modes

$$c\, e^{-\phi} \psi^{\mu} \quad \text{and} \quad c\, e^{-\frac{\phi}{2}} S^{\alpha}$$

roughly $\longleftrightarrow$ bosonic and fermionic moduli of D-instanton

2. "ghost zero modes"

finite propagator (not in Siegel gauge)

Grassmann even

$$\boxed{\kappa^{1}\, \beta_{-\frac{1}{2}}\, c_0\, c_1 \overbrace{|-1\rangle}^{e^{-\phi}}}$$

obey Sen gauge (good)

$$+ \boxed{\zeta^{2}\, \beta_{-\frac{1}{2}}\, c_1\, |-1\rangle}$$

FP ghost

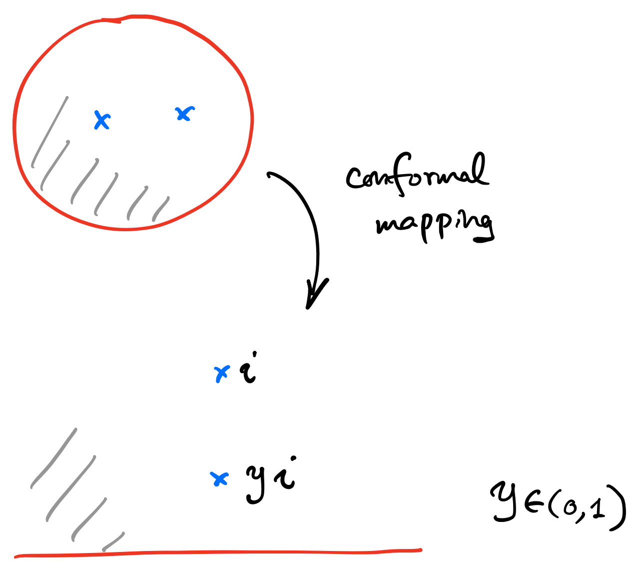
odd

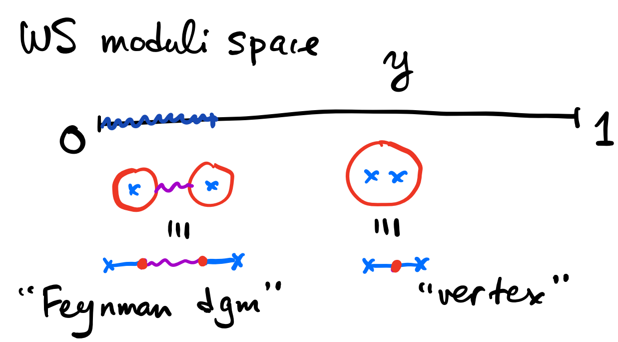
obey Siegel gauge (bad)

$$+ \zeta_{1}\, \gamma_{-\frac{1}{2}}\, c_1\, |-1\rangle$$

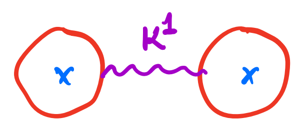
$$+ \kappa_{2}\, \gamma_{-\frac{1}{2}}\, c_0\, c_1\, |-1\rangle$$

even





In particular, working in Sen gauge, one must include the Feynman diagram



which has no analog in the naive on-shell approach

(K^1 absent in Siegel gauge)

This contribution from K^1-propagator is crucial in obtaining the correct value $A_0 = -\frac{1}{32}$.

[2205.00609 section 6.4]

Recap: 1-D-instanton contribution at NLO:

$$e^{2\pi i \tau}\,\tau_2^{-1}\,\delta^{16}(Q)\times\Big(C_1^{(N)}+\sum_{\substack{i<j\\ n\geq 2}}\#\,S_{ij}^{n}\Big)$$

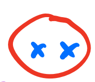
determined from susy + soft thm.

first-principle computation requires SFT

can compute on-shell, agrees with SFT

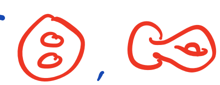
$$C_1^{(N)}=\frac{3}{256}-\frac{(N-4)(N-5)}{64}$$

$$\equiv A_0\binom{N}{2}+\boxed{A_1 N+A_2}$$



a nontrivial test of non-pert. susy of IIB vacua!

involve further subtleties having to do with open SF redefinition ...



For the near future

- important tests of the SFT formalism!