The Geometry of Perturbative Unitarity

Paolo Benincasa

Max Planck Institut für Physik, München

09 August 2022 – Amplitudes 2022, Prague

based on:

S. Albayrak, **P.B.**, C. Duaso Pueyo – *to appear*

other relevant works:

N. Arkani-Hamed, **P.B.** – 1811.01125

P.B. – 1909.02517

P.B., A. J. McLeod, C. Vergu – 2009.03047

P.B., W. J. Torres Bobadilla – 2112.09028

Physical Processes & Fundamental Principles

Physical processes are highly constrained by fundamental principles (Locality, Causality, Unitarity)

Flat space

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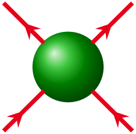
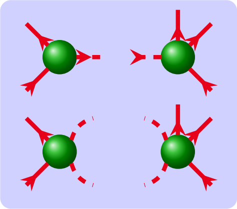
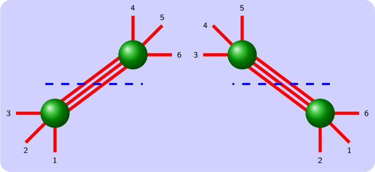


Poles

Branch cuts



[Cutkosky, 60]



[Stapp 71; Cahill, Stapp 73,75; Lassalle, 74]
[Bourjaily, Hannesdottir, McLeod, Schwartz, Vergu, 20]
[**P.B.**, McLeod, Vergu, 20; **P.B.**. W. Torres Bobadilla, 21]

- Computational apprachés

- Constraints on the interactions

[Weinberg, 64; **P. B.**, Cachazo, 07; **P. B.**, Conde, 11; McGady, Rodina 13; Arkani-Hamed, Huang, Huang, 17]

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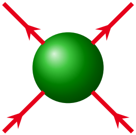

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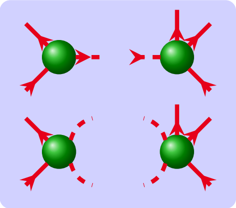

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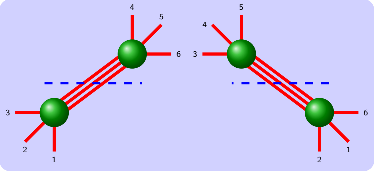

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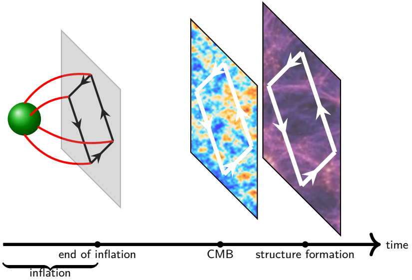

$$\langle \prod_{j=1}^{n} \Phi(\vec{p}_j) \rangle = \int \mathcal{D}\Phi \, \tilde{\rho}(\Phi) \prod_{j=1}^{n} \Phi(\vec{p}_j)$$

$\langle \Phi\Phi \cdots \rangle$ $\langle \delta T \delta T \cdots \rangle$ $\langle \delta\rho_g \delta\rho_g \cdots \rangle$

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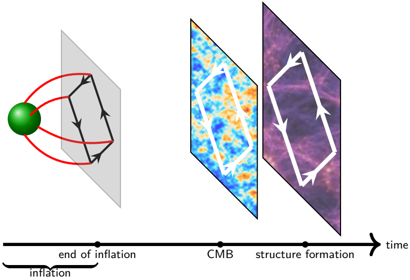

$\langle \Phi\Phi\cdots\rangle$ $\langle \delta T \delta T \cdots\rangle$ $\langle \delta\rho_g \delta\rho_g \cdots\rangle$

$$\langle \prod_{j=1}^{n} \Phi(\vec{p}_j)\rangle = \int \mathcal{D}\Phi\, \tilde{\rho}(\Phi) \prod_{j=1}^{n} \Phi(\vec{p}_j)$$

$$\tilde{\rho}(\Phi) \equiv |\Psi(\Phi)|^2$$

probability distribution

$$\Psi[\Phi] = \langle \Phi | \hat{U}(0,-\infty)|0\rangle = \mathcal{N} \int_{\phi(-\infty)=0}^{\phi(0)=\Phi} \mathcal{D}\phi\, e^{iS[\phi]}$$

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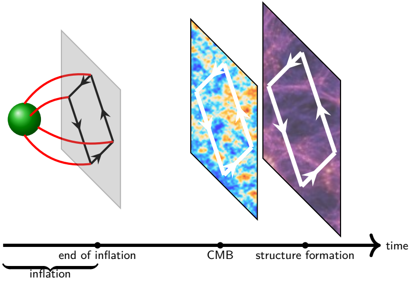

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Wavefunction of the universe

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Bunch-Davies vacuum

$$\phi \xrightarrow{\eta\longrightarrow-\infty} \sim e^{iE\eta}$$

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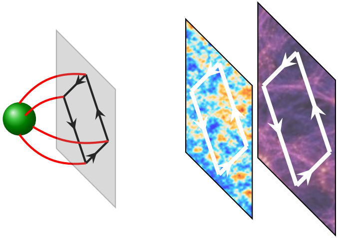


time

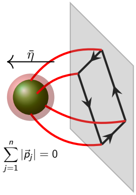
end of inflation — CMB — structure formation

inflation

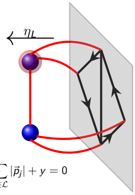
$\langle \Phi\Phi\cdots\rangle$ $\quad$ $\langle \delta T \delta T\cdots\rangle$ $\quad$ $\langle \delta\rho_g \delta\rho_g\cdots\rangle$

$$\Psi[\Phi] = \Psi_{\text{free}}[\Phi]\left[1 + \sum_{n\geq 2}\int \prod_{j=1}^{n}\left[\frac{d^d p_j}{(2\pi)^d}\,\Phi(\vec{p}_j)\right]\sum_{L\geq 0}\psi_n^{(L)}(\vec{p}_1,\ldots,\vec{p}_n)\right]$$

Singularities & Factorisation



Scattering Amplitude $\mathcal{A}^{\text{h.e.}}$



$\mathcal{A}_{\mathcal{L}} \times \tilde{\Psi}_{\mathcal{R}}$

Constraints on higher codim singularities

$$\text{Disc}_{E_{\mathfrak{g}_1}}\cdots\text{Disc}_{E_{\mathfrak{g}_k}}\psi_n^{(L)} = 0$$

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Unitarity & The $i\epsilon$–prescription

$$\Psi[\Phi] = \Psi_{\text{free}}[\Phi] \underbrace{\left[1 + \sum_{n\geq 2} \int \prod_{j=1}^{n} \left[\frac{d^d p_j}{(2\pi)^d} \Phi(\vec{p}_j)\right] \sum_{L\geq 0} \psi_n^{(L)}(\vec{p}_1, \dots, \vec{p}_n)\right]}_{\langle \Phi | \hat{U}_{\text{int}} | 0 \rangle}$$

$$\hat{U}_{\text{int}} := \mathcal{T}\left\{\exp\left[-i \int_{-\infty}^{0} d\eta\, \hat{H}_{\text{int}}(\eta)\right]\right\}$$

It is unitary: $\hat{U}_{\text{int}}^{\dagger} = \hat{U}_{\text{int}}^{-1}$

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No time reversal invariance

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In order to make any statement about unitarity
$i\epsilon$-prescription needs to preserve unitarity

e.g.: $\hat{U}_{\epsilon} = \mathcal{T}\left\{\exp\left[-i \int_{-\infty}^{0} d\eta\, e^{\epsilon\eta} \hat{H}_{\text{int}}(\eta)\right]\right\}$ ← **It is not the only choice**

[Baumgart, Sundrum, 20]

Unitarity & The wavefunction of the universe

$$\Psi[\Phi] = \Psi_{\text{free}}[\Phi]\left[1 + \sum_{n\geq 2}\int \prod_{j=1}^{n}\left[\frac{d^d p_j}{(2\pi)^d}\Phi(\vec{p}_j)\right]\sum_{L\geq 0}\psi_n^{(L)}(\vec{p}_1,\ldots,\vec{p}_n)\right]$$

$$\hat{U}_\epsilon \hat{U}_\epsilon^\dagger = \hat{\mathbb{1}}$$

$$\hat{U}_\epsilon = \hat{1} + \hat{V}_\epsilon$$

$$\longrightarrow \quad \hat{V}_\epsilon + \hat{V}_\epsilon^\dagger = -\hat{V}_\epsilon \hat{V}_\epsilon^\dagger$$

$$\psi_n^{(L)}(\vec{p}_j;\epsilon) + \psi_n^{(L)\dagger}(-\vec{p}_j;\epsilon) = -\sum_{\text{cuts}}\psi_n^{(L)}$$

Not Unique

[Goodhew, Jazayeri, Pajer, 20], [Melville, Pajer, 21], [Goodhew, Jazayeri, Lee, Pajer, 21], [Baumann, Chen, Duaso Pueyo, Joyce, Lee, Pimentel, 21] [Albayrak, **P.B.**, Duaso Pueyo, *to appear*]

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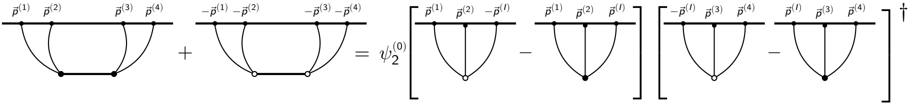

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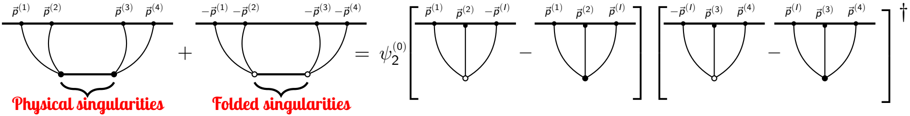

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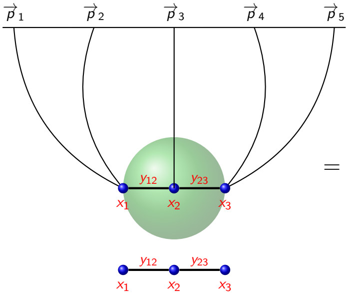
A Combinatorial Description of the Wavefunction

[N. Arkani-Hamed, P.B., A. Postnikov, 17]; [P.B., 19]

$$S[\phi] = \int_{-\infty}^{0} d\eta \int d^d x \left[\frac{1}{2}(\partial\phi)^2 - \sum_{k\geq 3}\frac{\lambda(\eta)}{k!}\phi^k\right], \qquad \lambda(\eta) = \int_{-\infty}^{+\infty} d\varepsilon\, e^{i\varepsilon\eta}\tilde{\lambda}_k(\varepsilon)$$



$$= \prod_{v\in\mathcal{V}}\left[\int_{X_v}^{+\infty} dx_v \tilde{\lambda}(x_v)\right] \underbrace{\int_{-\infty}^{0}\prod_{v\in\mathcal{V}}\left[d\eta_v\, e^{ix_v\eta_v}\right]\prod_{e\in\mathcal{E}} G(y_e, \eta_{v_e}, \eta_{v'_e})}_{\psi_{\mathcal{G}}(x_v,\, y_e)}$$

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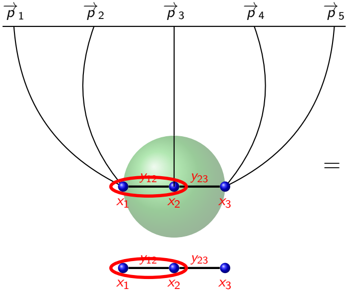

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Poles $\Longleftrightarrow$ Subgraphs

$E_{\mathfrak{g}} = x_1 + x_2 + y_{23}$

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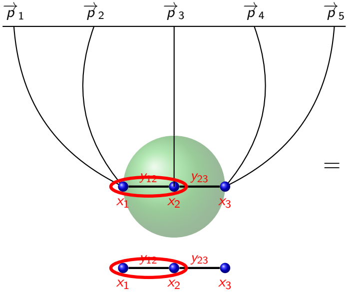

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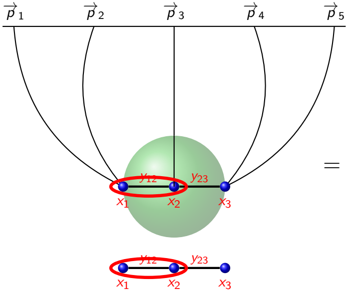
- Conformally-coupled ϕ, ϕ^k-interactions in FRW
  [N. Arkani-Hamed, **P.B.**, A. Postnikov, 17]
- More general scalars via operators on conformally-coupled ϕ
  [**P.B.**, 19]; [D. Baumann, C. Duaso Pueyo, A. Joyce, G. Pimentel, 19]
- Spinning propagating states
  [D. Baumann, C. Duaso Pueyo, A. Joyce, G. Pimentel, 19]
- Scalar integrals appearing in wavefunctions for states with a flat-space counter-part.



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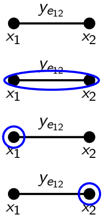
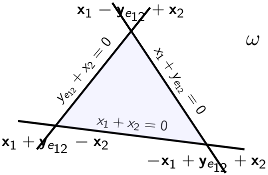


$\mathcal{Y} = (x_1, y_{e_{12}}, x_2)$

$x_1 + x_2 = 0$

$x_1 + y_{e_{12}} = 0$

$y_{e_{12}} + x_2 = 0$

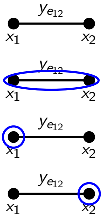


$$\omega = \frac{\langle 123\rangle^2 \langle \mathcal{Y} d^2 \mathcal{Y}\rangle}{\langle \mathcal{Y}12\rangle\langle \mathcal{Y}23\rangle\langle \mathcal{Y}31\rangle} = \frac{dx_1\, dy_{e_{12}}\, dx_2 / \mathrm{Vol}\{GL(1)\}}{(y_{e_{12}} + x_2)(x_1 + x_2)(x_1 + y_{e_{12}})}$$

$$\{Z_1^{(e_{12})}, Z_2^{(e_{12})}, Z_3^{(e_{12})}\} = \{\mathbf{x}_1 - \mathbf{y}_{e_{12}} + \mathbf{x}_2, \mathbf{x}_1 + \mathbf{y}_{e_{12}} - \mathbf{x}_2, -\mathbf{x}_1 + \mathbf{y}_{e_{12}} + \mathbf{x}_2\}$$

A Combinatorial Description of the Wavefunction

[Arkani-Hamed, P.B., Postnikov, 17; Arkani-Hamed, P.B.,18; P.B., 18; P.B., 19]



$\mathcal{Y} = (x_1, y_{e_{12}}, x_2)$

$x_1 + x_2 = 0$

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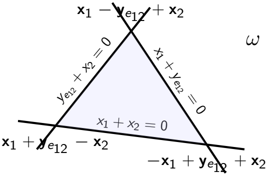
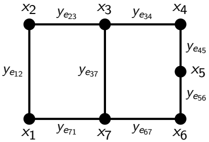


$$\omega = \frac{\langle 123\rangle^2 \langle \mathcal{Y} d^2 \mathcal{Y}\rangle}{\langle \mathcal{Y}12\rangle\langle \mathcal{Y}23\rangle\langle \mathcal{Y}31\rangle} = \frac{dx_1\, dy_{e_{12}}\, dx_2 / \text{Vol}\{GL(1)\}}{(y_{e_{12}} + x_2)(x_1 + x_2)(x_1 + y_{e_{12}})}$$

$$\{Z_1^{(e_{12})}, Z_2^{(e_{12})}, Z_3^{(e_{12})}\} = \{\mathbf{x}_1 - \mathbf{y}_{e_{12}} + \mathbf{x}_2, \mathbf{x}_1 + \mathbf{y}_{e_{12}} - \mathbf{x}_2, -\mathbf{x}_1 + \mathbf{y}_{e_{12}} + \mathbf{x}_2\}$$

locus of the intersections of the facets outside $\mathcal{P}_{\mathcal{G}}$



Cosmological polytopes $\mathcal{P}_{\mathcal{G}}$ as convex hull

$$\{\mathbf{x}_i - \mathbf{y}_{e_{ij}} + \mathbf{x}_j, \mathbf{x}_i + \mathbf{y}_{e_{ij}} - \mathbf{x}_j, -\mathbf{x}_i + \mathbf{y}_{e_{ij}} + \mathbf{x}_j\}_{e_{ij} \in \mathcal{E}}$$

$$\omega = \frac{\mathfrak{n}_\delta(\mathcal{Y}) \langle \mathcal{Y} d^{n_s + n_e - 1} \mathcal{Y}\rangle}{\prod_{\mathfrak{g} \subseteq \mathcal{G}} q_{\mathfrak{g}}(\mathcal{Y})}$$

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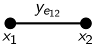

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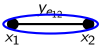
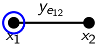
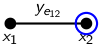


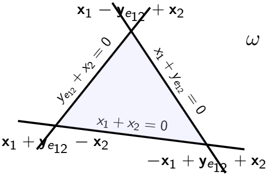
$x_1 + x_2 = 0$



$x_1 + y_{e_{12}} = 0$



$y_{e_{12}} + x_2 = 0$



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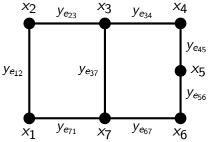

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Wavefunction $\psi_{\mathcal{G}}$

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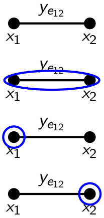

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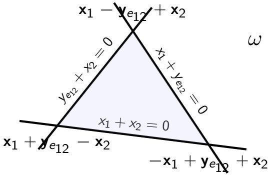

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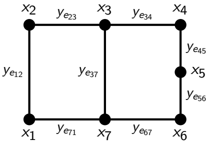

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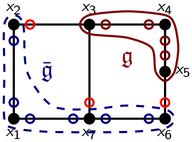


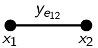
Flat-space cutting rules!

$$\omega = \left(\prod_{e\in\mathcal{E}} \frac{1}{2y_e}\right) \mathcal{A}[\mathfrak{g}] \times \mathcal{A}[\bar{\mathfrak{g}}] \langle Z_\mathcal{G} Z_\mathfrak{g} \mathcal{Y} d^{n_s+n_e-3} \mathcal{Y}\rangle$$

Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)

Unitarity from Non-Convexity

[Albayrak, P.B., Duaso Pueyo, *to appear*]



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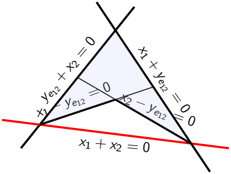

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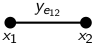

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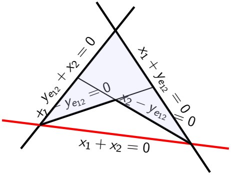
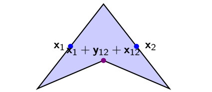


- Optical theorem as a consequence of the equivalence of triangulations



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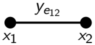

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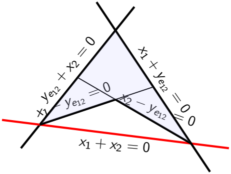

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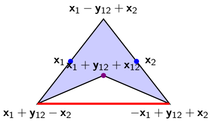

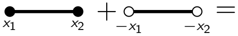

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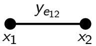

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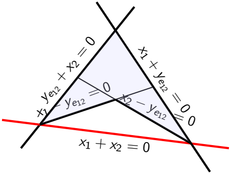

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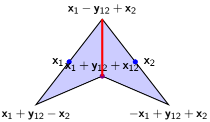

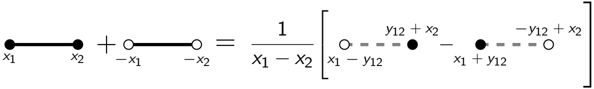

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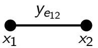

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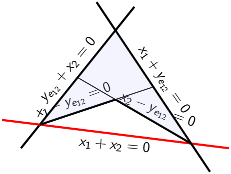

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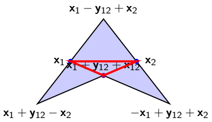

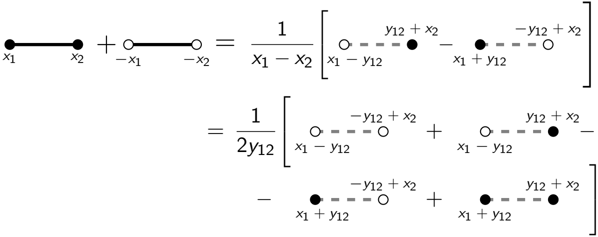

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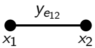

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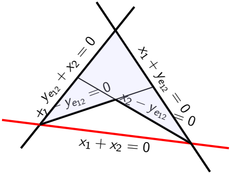
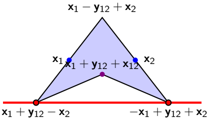


- **Flat-space cutting rules as sum of disconnected codimension-2 boundaries**



$$\delta(x_1 + x_2)\left[\frac{1}{2y_{e_{12}}}\delta(x_1 - y_{e_{12}}) + \frac{1}{2y_{e_{12}}}\delta(x_1 + y_{e_{12}}) + \frac{1}{2y_{e_{12}}}\delta(x_2 - y_{e_{12}}) + \frac{1}{2y_{e_{12}}}\delta(x_2 + y_{e_{12}})\right]$$

More generally

$$\delta(E_{\mathcal{G}}) \sum_{\mathfrak{g}\subset\mathcal{G}} \left[\Omega(\mathcal{P}'_{\mathcal{G}} \cap \mathcal{W}^{(\mathcal{G})} \cap \mathcal{W}^{\mathfrak{g}})\delta(E_{\mathfrak{g}})\right]$$

Conclusion

- Fundamental questions:

1. What's the imprint of basic principles such as unitarity and causality in the wavefunction of the
2. How is it connect to the one in flat-space scattering amplitudes?

- Unitarity & the $i\epsilon$-prescription

1. So far $i\epsilon$ just introduced for convergence reasons;
2. Not all the choices preserve the unitarity of the evolution operator;
3. They correspond to kinematic $i\epsilon$;
4. In the flat-space limit they can redue to the Feynman $i\epsilon$;
5. A careful analysis of the $i\epsilon$-contour needed. What about causality?

- The geometry of perturbative unitarity

1. Cosmological polytopes encode the cosmological optical theorem in a non-convex part;
2. The total energy hyperplane intersects such a non-convex polytope just on codimension-2 boundaries, providing the flat-space cutting rules;
3. All the possible *cutting rules* are triangulations of such non-convex polytopes.
4. The geometrical picture can prescind from the explicit $i\epsilon$-analysis which is encoded in its boundaries;
5. Th $i\epsilon$ can be made explicit via the contour integral representation of the canonical form;
6. The factorisation properties of ψ do not appear to come directly from the cosmological optical theorem. What is their origin?