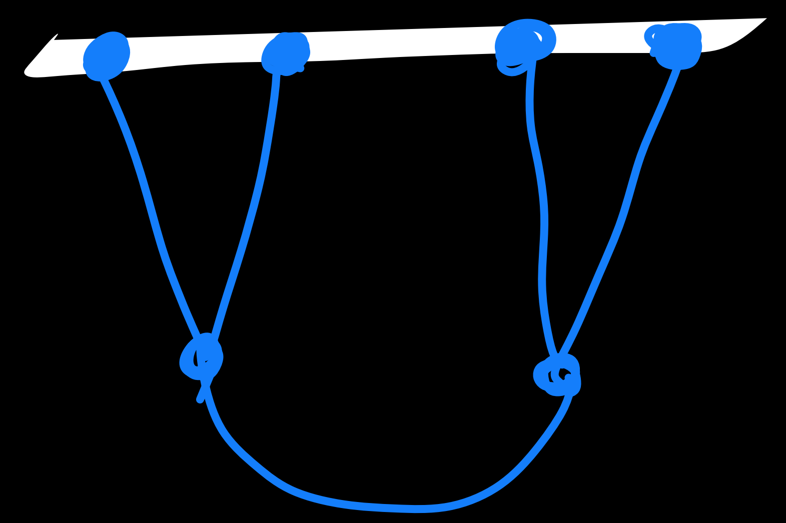
Differential Equations for Cosmological Correlators



Guilherme L. Pimentel

SCUOLA NORMALE SUPERIORE

Based On Work With

Nima Arkani-Hamed, Daniel Baumann, Hayden Lee; Aaron Hillman; Austin Joyce; Wei-Ming Chen; Scott Melville; Carlos Duaso Pueyo; Dong-Gang Wang

Related Work & Discussions With

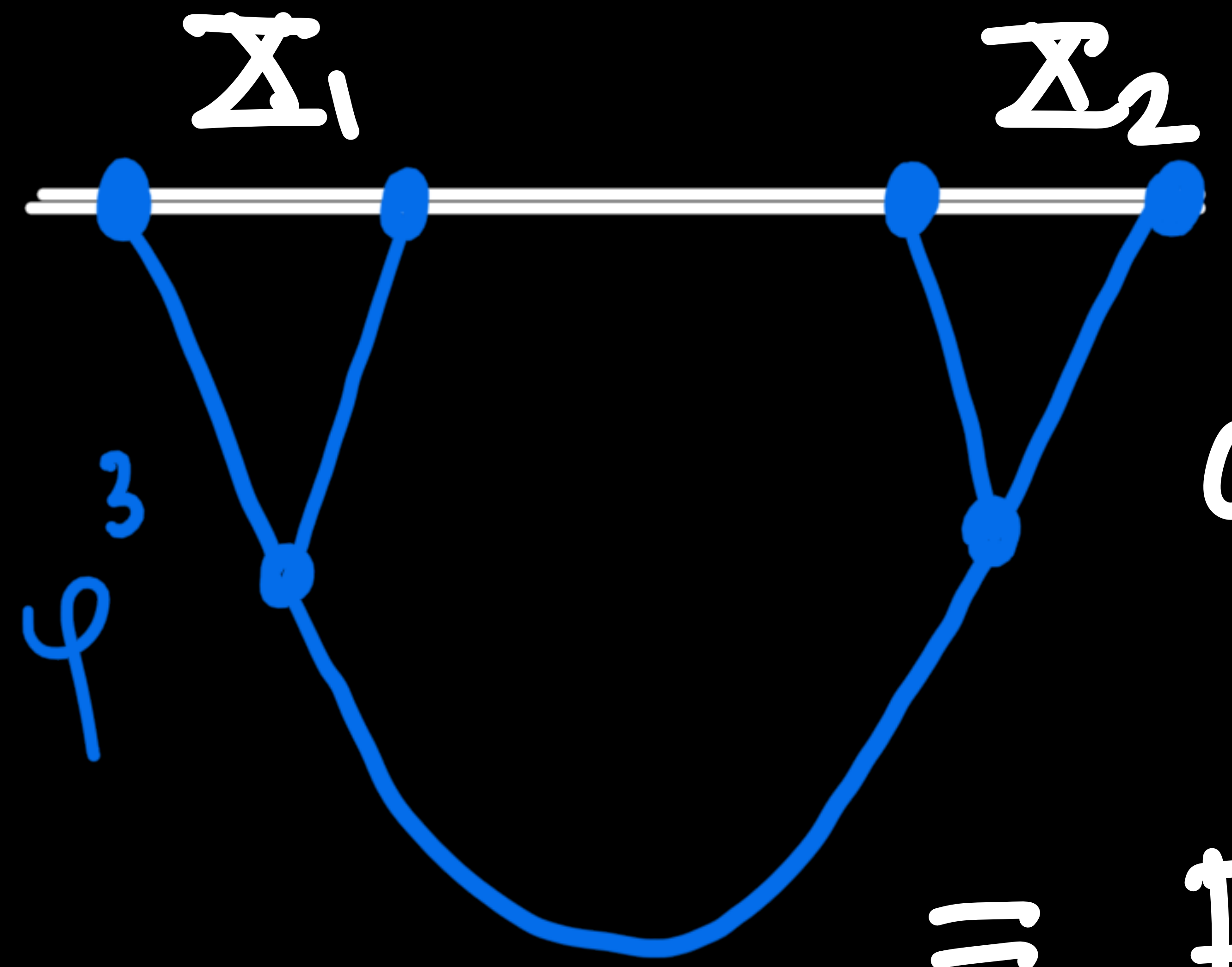
A. Achúcarro, E. Pajer, J. Maldacena, M. Zaldarriaga, C. Sleight, M. Taronna, X. Chen, Y. Wang, L. Senatore, A. Kalaja, D. Meerburg, W. Coulton, D. Green, S. Jazayeri, S. Renaux-Petel, D. Stefanyszyn, T. Anous, D. Anninos, P. Benincasa, M. Loparco, F. Rost, S. Albayrak, ...

DISCLAIMER

Very few (no?) references!

Snowmass: { Inflation: Theory & Observations
The Cosmological Bootstrap

The Main Result



$$a(\eta) = \eta^{-(1+\varepsilon)}$$

$$\equiv F(X_1, X_2)$$

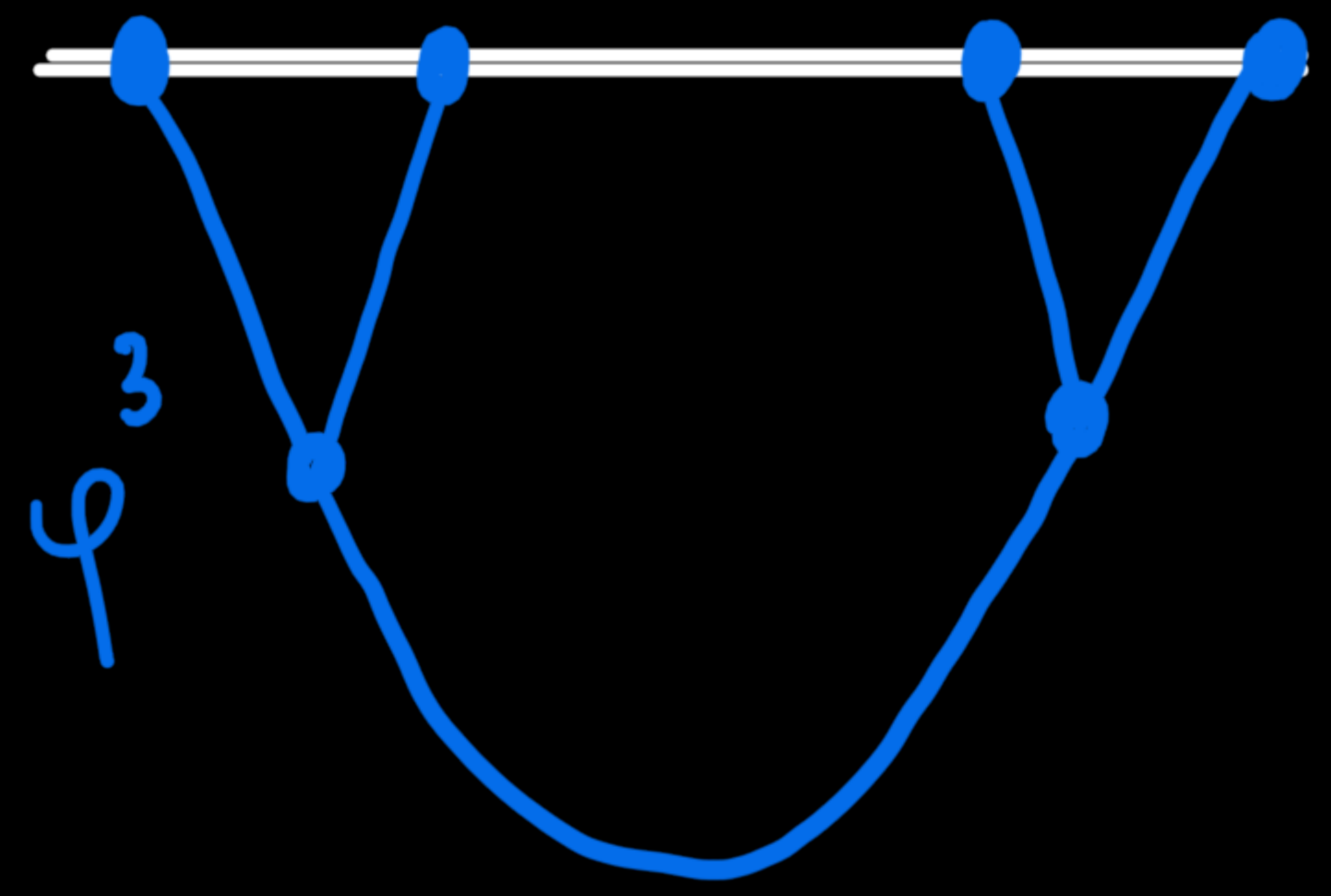
A toy model for structure formation $\rightsquigarrow$ complicated time integrals $\rightsquigarrow$ ...

$$\rightsquigarrow (\mathbb{d} + \varepsilon \mathbb{A})\Psi = 0$$

CANONICAL FORMS

COSMOLOGICAL POLYTOPE

FLAT CONNECTIONS



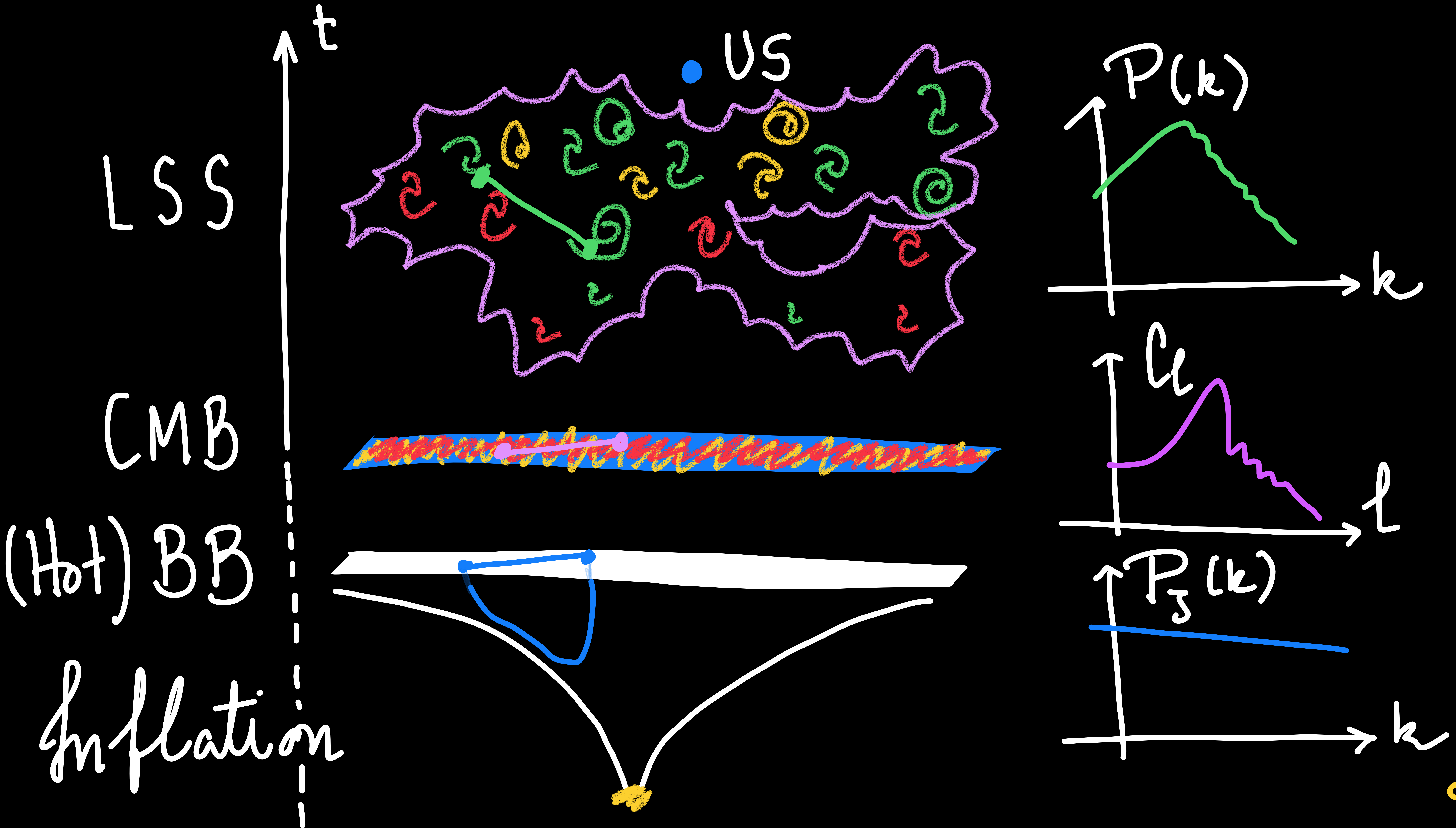
AMPLITUDES

TWISTED COHOMOLOGY

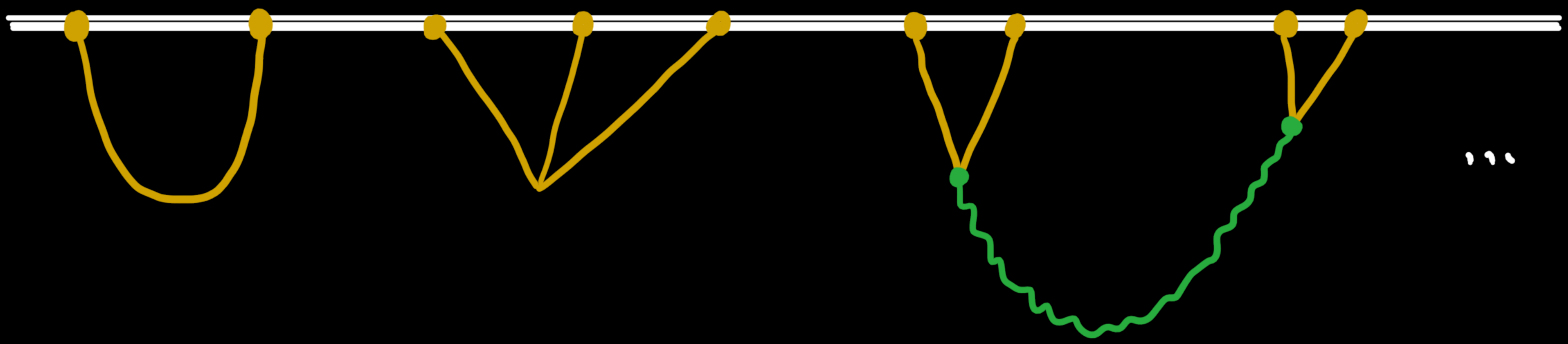
${}_2F_1$

Li_2

Motivation


t
US
LSS
CMB
(Hot) BB
Inflation
P(k)
k
C_ℓ
ℓ
$P_\zeta(k)$
k

Cosmological Bootstrap



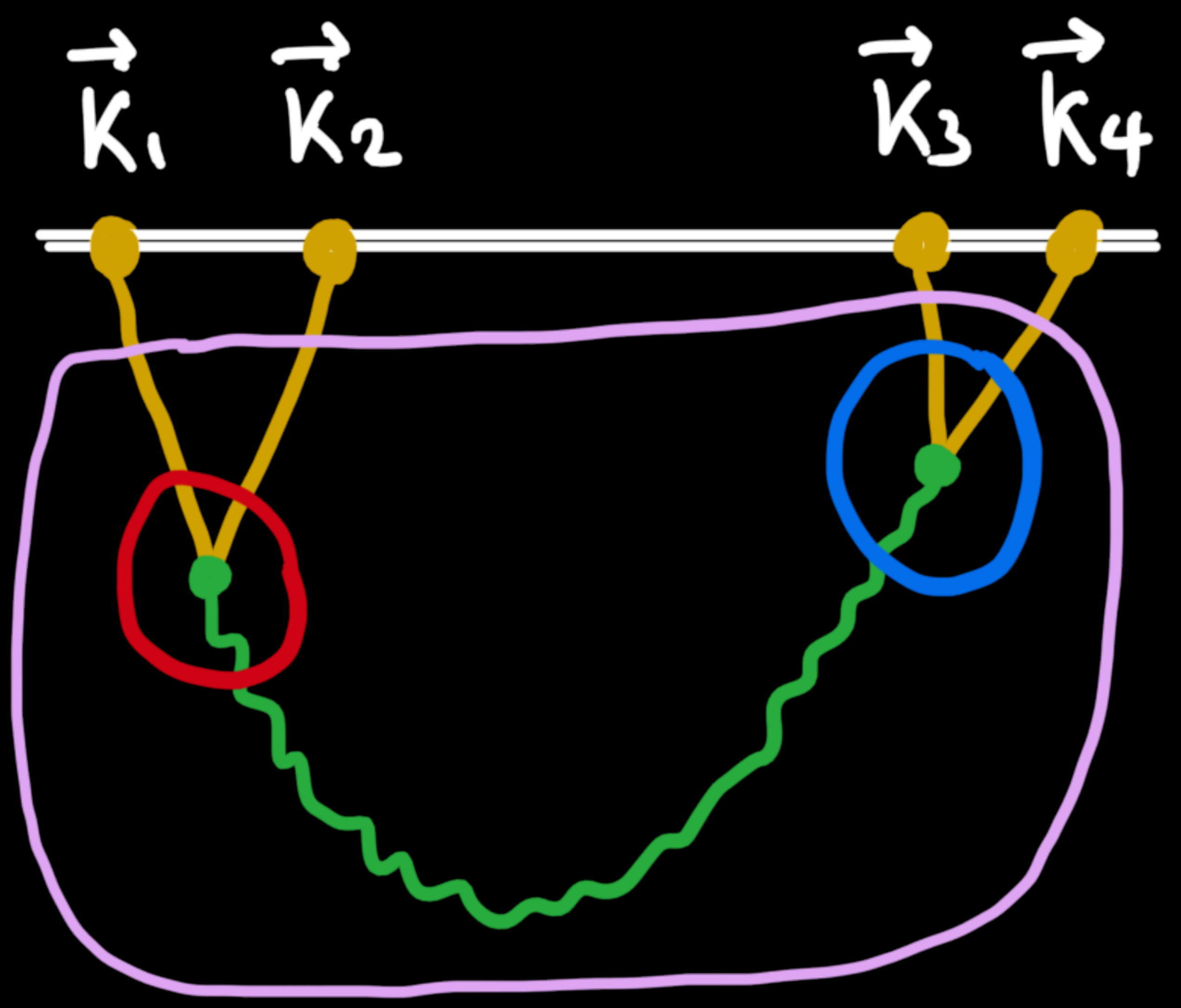
In dS_4 (inflation), with & without boost symmetry

Locality, Unitarity, perturbative & non-perturbative

...

Time Evolution

Differential Equations + Singularities



- $X_1 + 1 \to 0$
- $X_2 + 1 \to 0$

} Amplitude ⊗ Correlator Limit

- $X_1 + X_2 \to 0$ Flat Space Limit

- $X_1 - 1 \to 0$
- $X_2 - 1 \to 0$

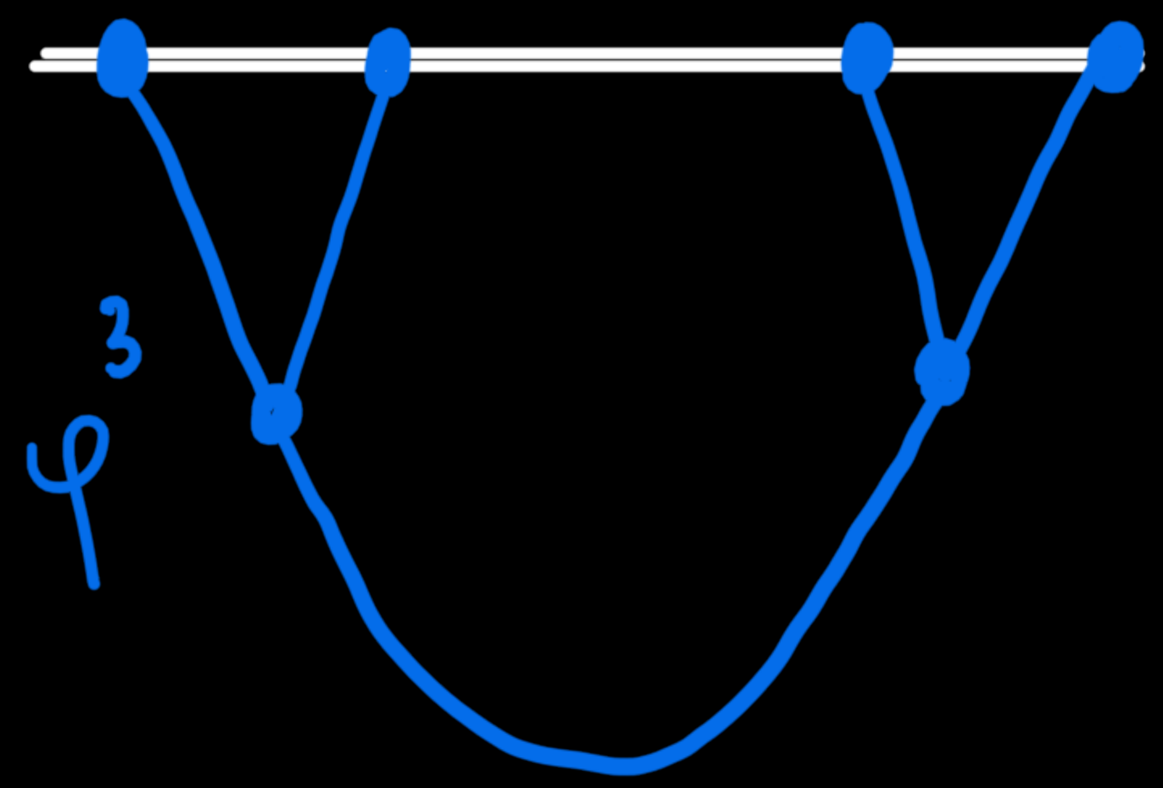
} Forbidden Singularities (vacuum choice)

Where do the differential equations come from?

Can we derive them (+ solve them) systematically?

Model

Prototype: Conformal Scalars (φ) in Power Law (ε) Cosmologies



$$F(X_1, X_2) = \int_0^\infty \frac{(x_1 x_2)^{\varepsilon}\, dx_1\, dx_2}{(x_1 + X_1 + 1)(x_2 + X_2 + 1)(x_1 + x_2 + X_1 + X_2)}$$

$$ds^2 = \eta^{2\varepsilon}\left[\frac{-d\eta^2 + d\vec{x}^2}{\eta^2}\right]$$

$\varepsilon = 0$ dS $_0$

$\varepsilon = 1$ Flat

$\varepsilon = 2$ Radiation

$\varepsilon = 3$ Matter

How to find D.E. for F?

How to solve it efficiently?

Key Point

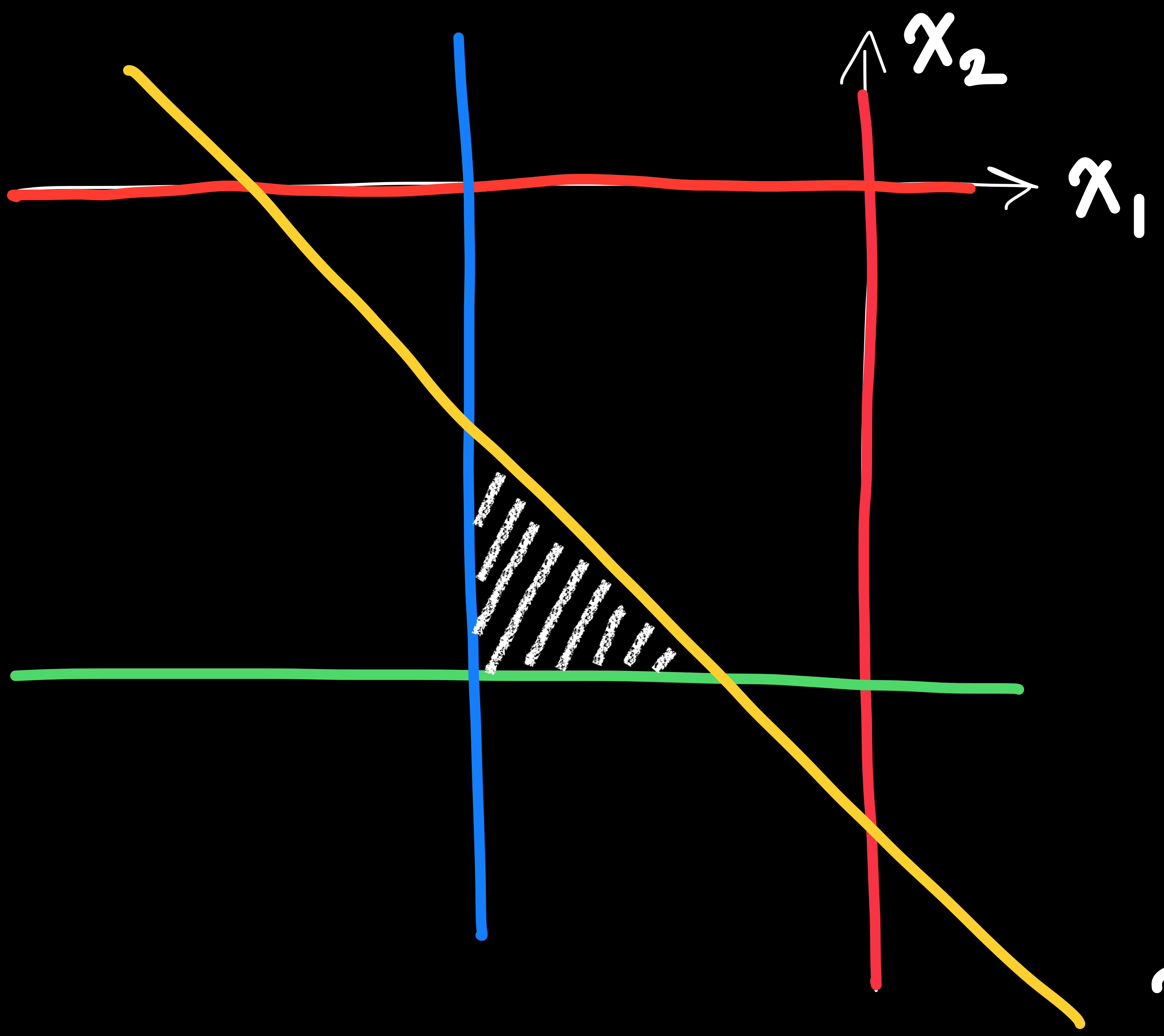
"twist"

$$F = \int (x_1 x_2)^{\varepsilon}\, \Omega$$

Canonical Form for bounded region in (x_1, x_2) plane (Cosmological Polytope)

Strategy: Exploit (twisted) cohomology of integrands

Consider $F_{n_1,\dots,n_5} = c_{n_1\dots n_5} \int_0^\infty \frac{dx_1 dx_2 (x_1 x_2)^\varepsilon}{L_1^{n_1} L_2^{n_2} L_3^{n_3} L_4^{n_4} L_5^{n_5}}$



$L_1 : x_1$

$L_2 : x_2$

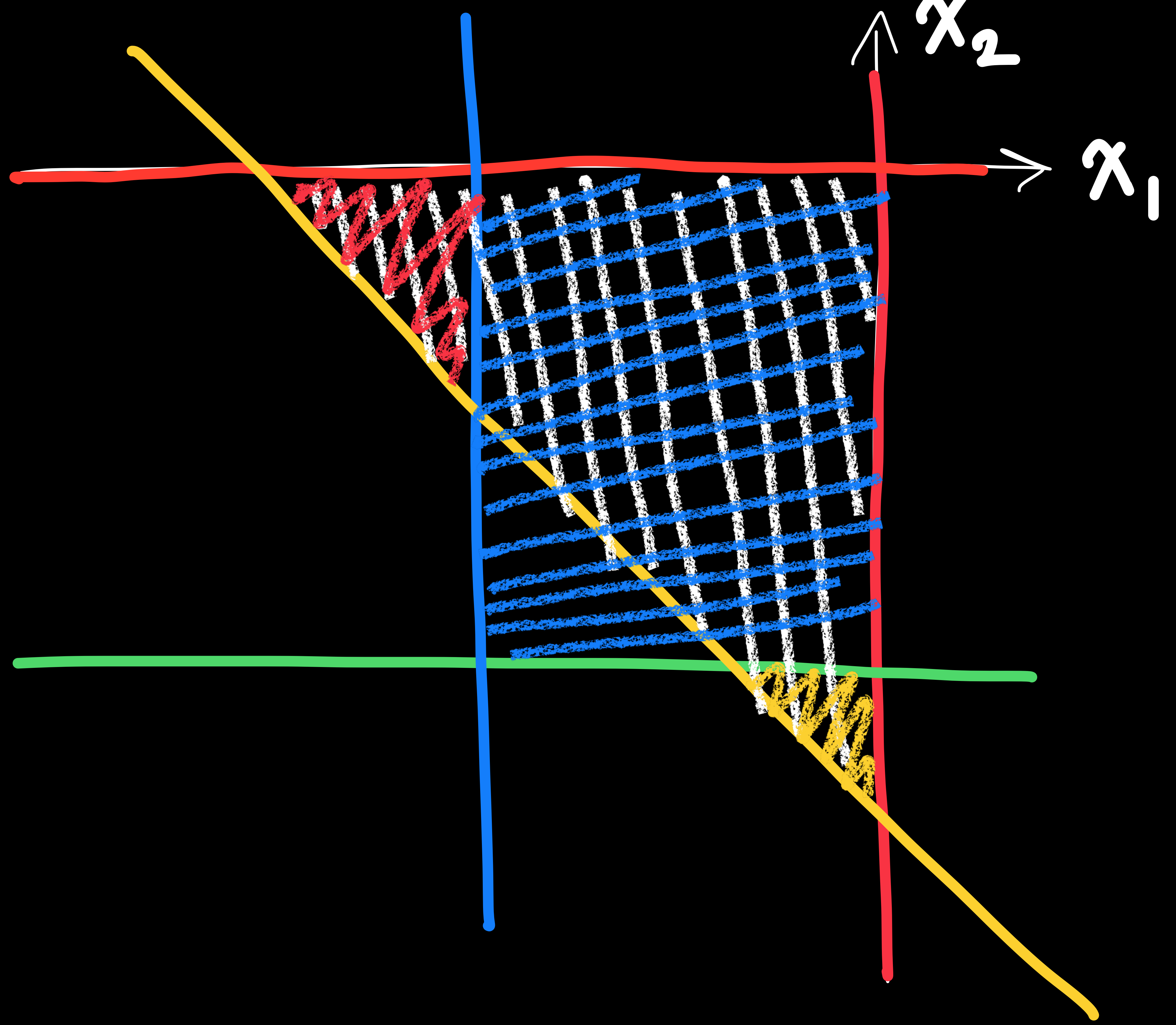
$L_3 : x_1 + X_1 + 1$

$L_4 : x_2 + X_2 + 1$

$L_5 : x_1 + x_2 + X_1 + X_2$

Basis size: # of bounded regions

In the "good" basis



$$\Psi = \int_0^\infty (x_1 x_2)^{\varepsilon} \begin{bmatrix} \Omega \\ \Omega \\ \Omega \\ \Omega \end{bmatrix}$$

"good": hug twisted lines as much as possible!

If $\Psi = \int_0^\infty (x_1 x_2)^{\varepsilon} \begin{bmatrix} \Omega \\ \Omega \\ \Omega \\ \Omega \end{bmatrix}$

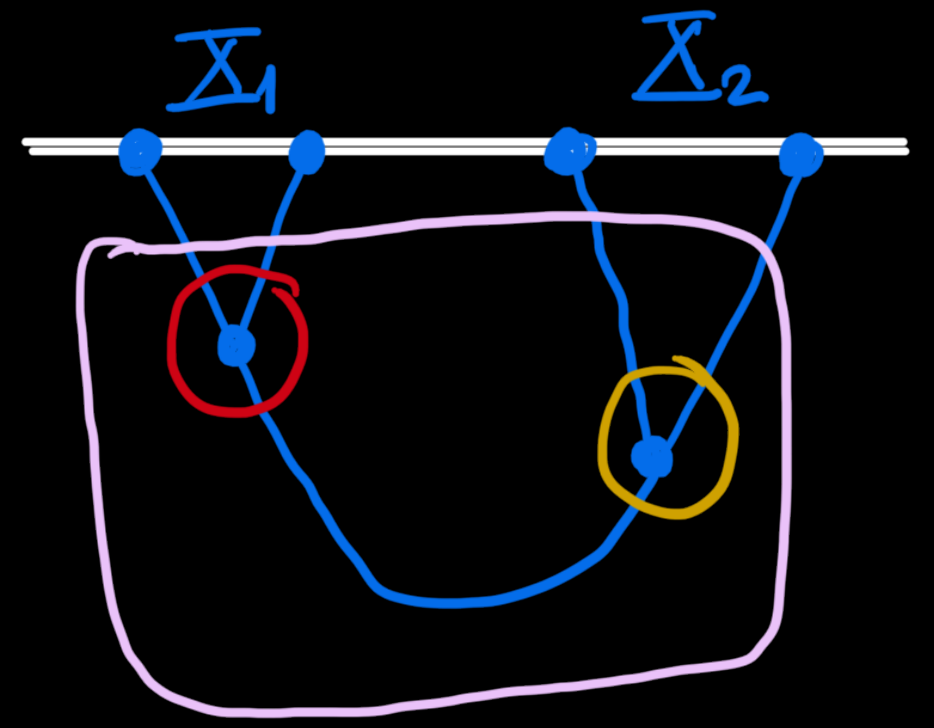
then $(\mathrm{d} + \varepsilon \mathbb{A})\Psi = 0$

$\mathbb{A}$ MAGIC

$$d\mathbb{A} = 0 \quad \& \quad \mathbb{A} \wedge \mathbb{A} = 0$$

Solution: $$\Psi = \exp\left[-\varepsilon \int_{(\mathbb{X}_1^0, \mathbb{X}_2^0)}^{(\mathbb{X}_1, \mathbb{X}_2)} \mathbb{A}\, d\mathbb{X}\right] \Psi_0$$

The real magic... $\Downarrow$



- $X_1 + 1$
- $X_2 + 1$
- $X_1 + X_2$
- $X_1 - 1$
- $X_2 - 1$

$$
A = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} d\log(X_1+1) + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} d\log(X_2+1) + \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} d\log(X_1+X_2) +
$$

$$
+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} d\log(X_1-1) + \begin{bmatrix} -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} d\log(X_2-1)
$$

$\mathbb{A}$ is triangular! Solve for each line individually (Lower lines have upper entries as "sources")

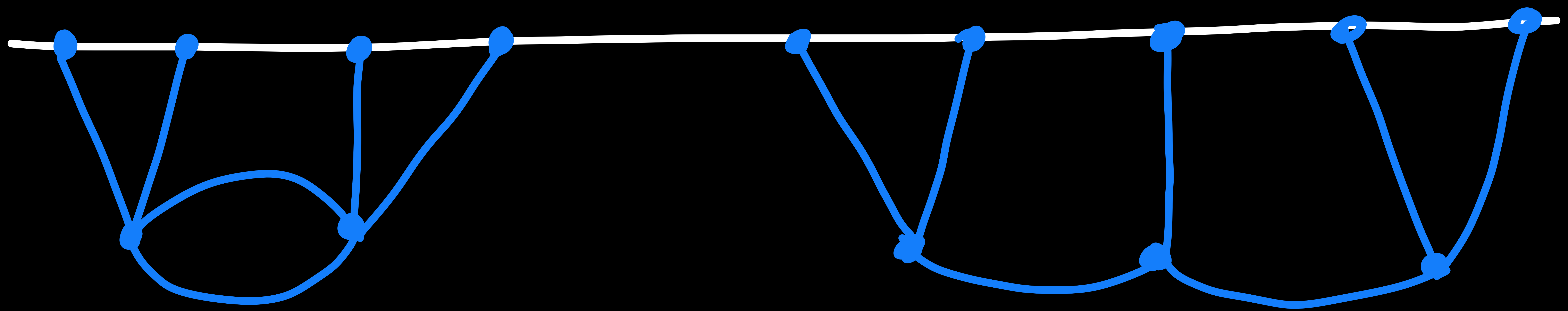
<u>Explicit Solution:</u> ${}_2F_1$'s etc. etc. $\longrightarrow$

$$\longrightarrow \begin{cases} Li_2 + \log\text{'s} & \epsilon \to 0 \\ \text{rational} & \epsilon \to 1, 2, 3 \end{cases}$$

"Dimension Shift" relations $\longleftrightarrow$ Change the Cosmology!

Outlook

Works for more sophisticated examples also!



10 dim' ${}_2F_1$'s etc.

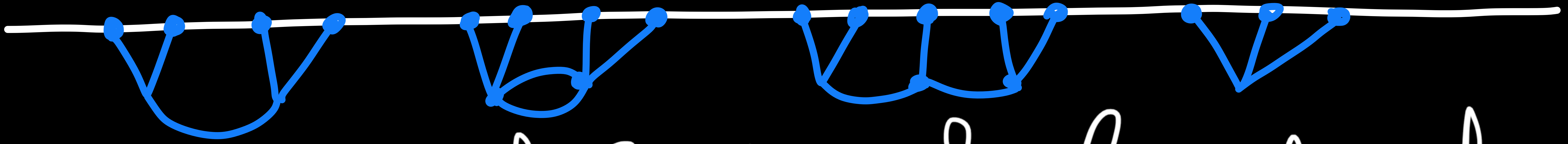
25 dim. Fancier than ${}_2F_1$'s

Why so simple? $\underline{\text{LOCALITY}}$

$$\tilde{\Psi} = \int (x_1 x_2)^{\varepsilon} (x_1 + x_2 + X_1 + X_2)^{\varepsilon} \left[\Omega\right]$$

4^{th} order diff. equations, much harder...

Avatar: Kinematic-Independent Eigenvectors of $\mathbb{A}$



- Differential Equations for Cosmological Correlators seem to exist for many examples
- Practically useful. How to find them in general?
- Conceptually interesting. Time evolution is traded by evolution in kinematic space
- Loops in Cosmology? Spin?