Symmetries & Celestial Amplitudes

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2201.06805 + 2205.10901

What has Celestial Holography taught us about Amplitudes?

Not an Apology!

An Apologia for Celestial Amplitudes

Celestial Holography

Flat + boost basis

A rose by any other name would smell as sweet.

Tenents of Celestial Holography

1. We don't like LSZ

2. We take r large first

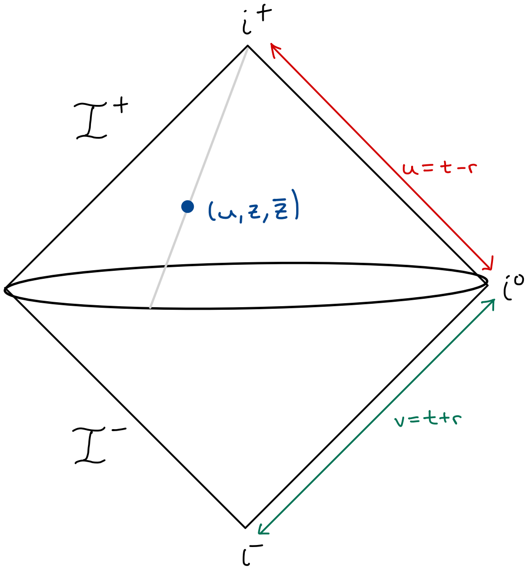
. . .

pert qft

$$\langle \phi(x_1^\mu) \ldots \phi(x_n^\nu) \rangle \xrightarrow[\text{holo.}]{\text{turn on grav}} \langle \phi(\mathbf{x}_1^a) \ldots \phi(\mathbf{x}_n^b) \rangle$$

$\uparrow$ $x_1 \in \mathbb{R}^{1,3}$

$\uparrow$ $\partial \mathcal{M}$



p^μ $\qquad\qquad$ x^μ

co-dim 1 from on-shell $\longleftrightarrow$ co-dim 1 from $x \to \partial M$

What does this perspective help with?

- Symmetries
- Causality @ bndy
- eom near bndy

how the onshell data evolve

⟷ constraint equations

first celestial result to celebrate

ASG = soft thm = Co-dim 2 current

Later!

New
- subleading soft graviton theorem
- spin memory
- Lorentz → $\mathrm{Vir} \times \overline{\mathrm{Vir}}$

A simple extrapolate dictionary

$r \to \infty$ $u = t - r$ fixed

$$e^{ip\cdot X} = e^{-ip^0 u - ip^0 r(1-\cos\theta)} \longrightarrow e^{-ip^0 u} \times \frac{i}{p^0 r}\, \frac{\delta(\theta)}{\sin\theta}$$

implies

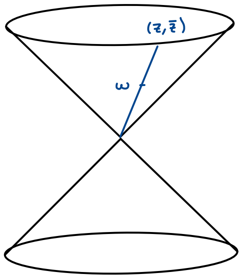
$$h_{\mu\nu} = \sum_{\alpha=\pm} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2p^0} \left[\varepsilon^{\alpha *}_{\mu\nu}\, a_\alpha e^{ip\cdot x} + \varepsilon^{\alpha}_{\mu\nu}\, a^\dagger_\alpha e^{-ip\cdot x} \right]$$

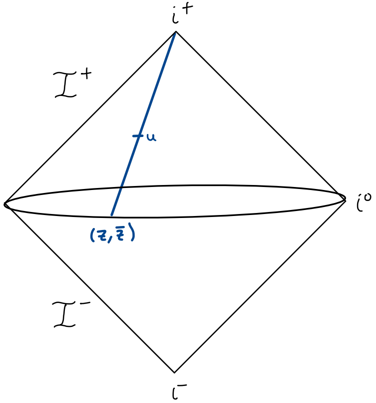
$$\downarrow$$

$$h_{zz} \propto r \int_0^\infty d\omega\, a_+(\omega \hat{x}) e^{-i\omega u} + \text{h.c.}$$

p^μ $\quad\quad\quad\quad$ x^μ

co-dim 1 from on-shell $\longleftrightarrow$ co-dim 1 from $x \to \partial M$





ASG

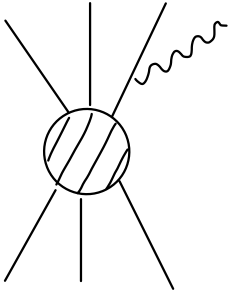
phase space $\{m_B^0(z,\bar z), N_z^0(z,\bar z), C_{zz}(u,z,\bar z)\}$

ASG $\xi = f(z,\bar z)\partial_u + \dots + Y(z)\partial_z + \dots$

$\mathcal{L}_\xi$ phase space $\circlearrowleft$

Amplitudes

$$\langle out + 1 | S | in\rangle = S^{(0)\pm}\langle out | S | in\rangle + \dots$$



$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar z}dzd\bar z \longleftarrow \text{flat}$$

$\frac{1}{r^\#}$ corrections

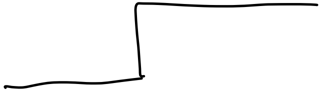
$$+\frac{2m_B}{r}du^2 + rC_{zz}dz^2 + rC_{\bar z\bar z}d\bar z^2 + D^z C_{zz}dudz + D^{\bar z}C_{\bar z\bar z}dud\bar z + \dots$$

ASG

$\hat{p} = \hat{x}$

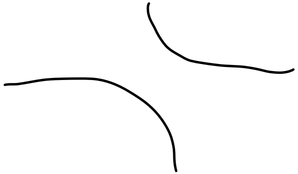
Amplitudes

$\Theta(u)$



$$\xleftrightarrow{\text{1D F.T.}}$$

$\frac{1}{\omega}$



"Memory = Soft Operator"

[Zhiboedov, Strominger '14]

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This spacetime perspective continues to teach us about the appropriate sym & reps.

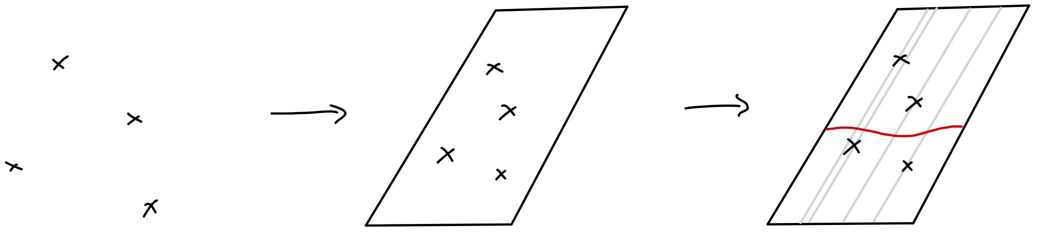
1. View CCFT as dim reduction of $\mathcal{I}^{\pm}$
2. Canonical charges → BMS fluxes
3. Spacetime pov fixes ambiguity in identifying loop corrections to T^{CCFT}

The Boost basis is a particular smearing along $\mathcal{I}^{\pm}$ so that the operators transform as conformal primaries under the Lorentz subgroup of Poincaré.

$$\phi_{\Delta}(z,\bar{z}) = \int_{-\infty}^{\infty} du \, (u - i\varepsilon)^{-\Delta} \, \phi(u,z,\bar{z})$$

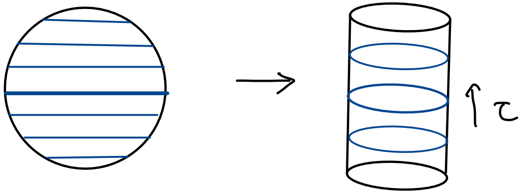
We trade u or ω for a continuous spectrum of Δ.

$$\begin{tikzcd} u \arrow[r, leftrightarrow, "\text{F.T.}"] & \omega \arrow[r, leftrightarrow, "\text{Mellin}"] & \Delta \end{tikzcd}$$

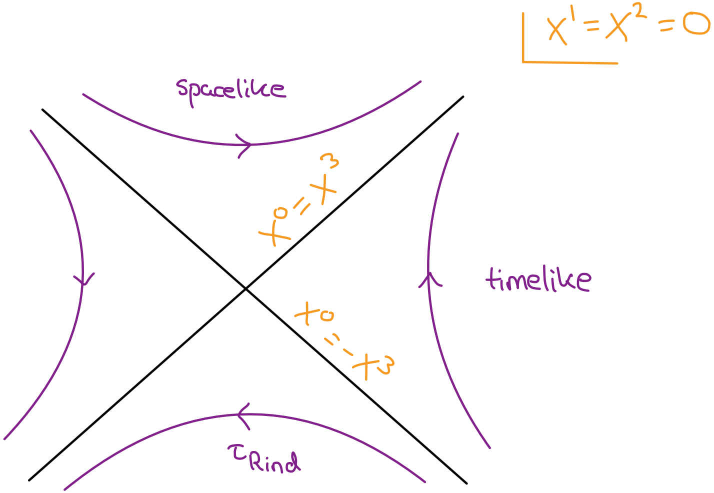


If we want to treat this as a 2D CFT, we should have

radial evolution $\Longleftrightarrow$ Rindler evolution

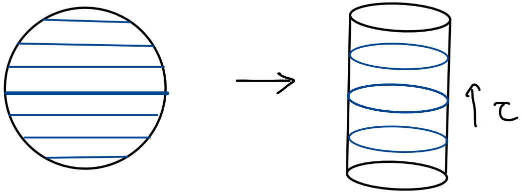


$z = e^{-\tau + i\phi}$



To match the bulk & boundary perspectives, let's start with the locus $X^3 = 0$

$$X^\mu = \left(u+r,\ r\,\frac{z+\bar{z}}{1+z\bar{z}},\ ir\,\frac{\bar{z}-z}{1+z\bar{z}},\ r\,\frac{1-z\bar{z}}{1+z\bar{z}}\right) \quad \rightarrow \quad |z|^2 = 1$$



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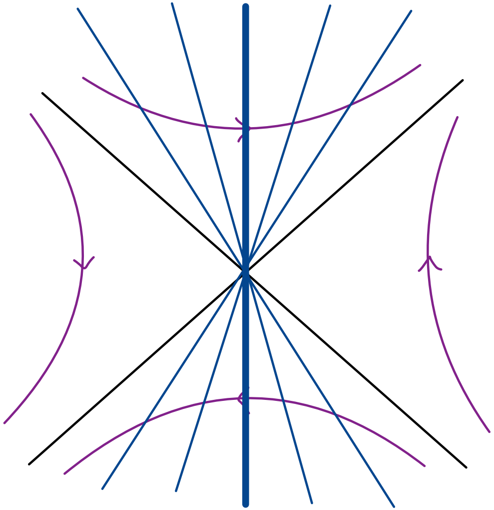
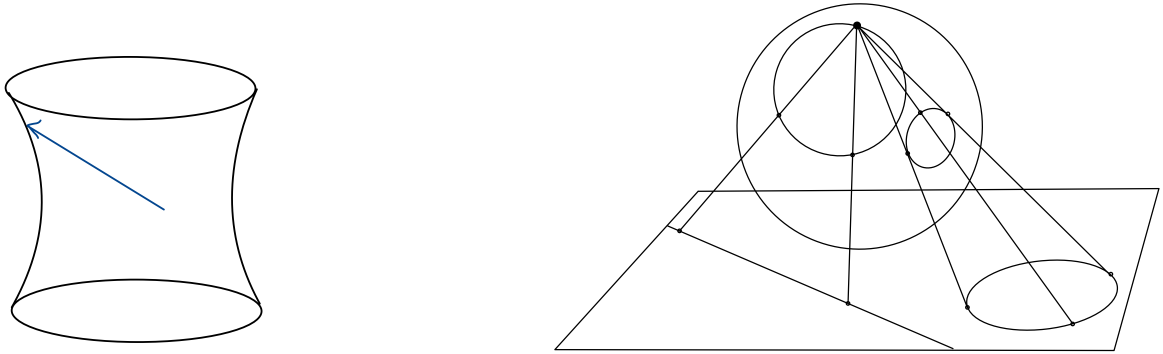


This hyperplane splits the celestial sphere along its equator, while boosts in the X^3 direction sweep a foliation.

Boosting in an arbitrary direction sweeps out hyperplanes with spacelike normal through the origin in the bulk...



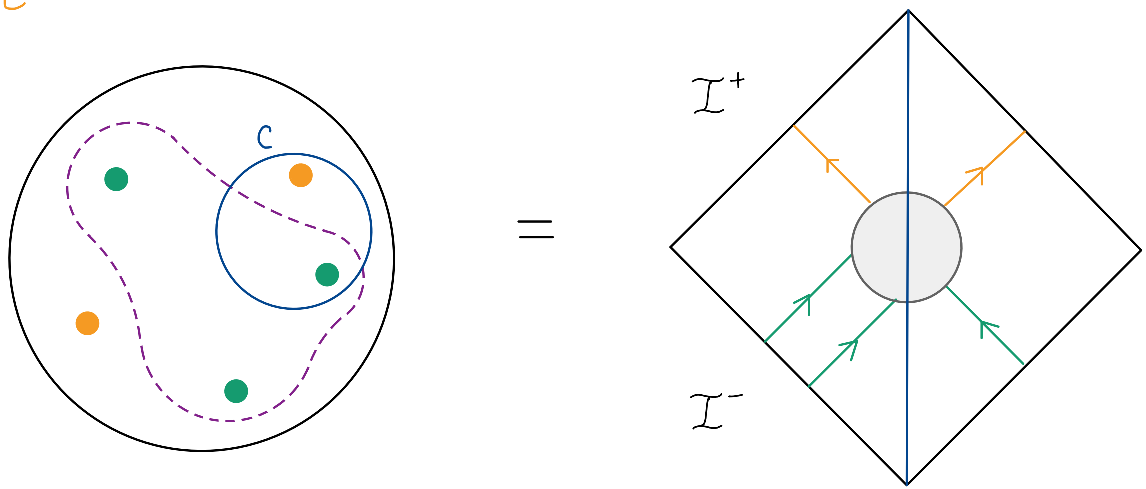
.. and maps us to different circles on the celestial sphere.

The charges in gauge theory are co-dimension 2. If I lift a path C on the celestial sphere to a co-dimension 1 hypersurface Σ_C of the bulk that runs along the generators of $\mathscr{I}$ I have

$$\mathcal{J}_C(\lambda) = \int_{\partial\Sigma_C} \star\kappa(\lambda)$$

charge generating transformation on celestial state defined on contour C

bulk 'charge' evaluated on hypersurface Σ_C



Let us use the U(1) case as an example. The gauge transformation

$$A_\mu \to A_\mu + \partial_\mu \lambda$$

is generated by the canonical charge with 2-form

$$\kappa = \lambda F \qquad F = \frac{1}{2} F_{\mu\nu}\, dx^\mu \wedge dx^\nu$$

For an ordinary Cauchy slice $\partial\Sigma = S^2 @ i^0$

$$Q^+(\lambda) = \lim_{r\to\infty} \int_{S^2} d^2z \sqrt{\gamma}\, \lambda\, (r^2 F_{ru})$$

By contrast if we pull back this form to the bndy Σ_C we have

$$Q^C(\lambda) = -i \int du \oint_C dx^A \lambda\, \epsilon_{AC} \gamma^{CB} F_{Bu} + \mathscr{I}^- \text{ contrib.}$$

can evaluate with the soft thm. directly

The radial = Rindler picture let's us jump directly to the statement that soft operators in the bulk generate symmetries on the celestial sphere.

We also see that our co-dimension 2 hologram is consistent with AdS/CFT expectations for how bulk and boundary symmetries are related.

Can we further see why this basis is a better way to present the flat space hologram?

$$\left.\underset{\substack{\uparrow \\ \text{built from spin-s radiative}}}{Q_s} + \underset{\substack{\uparrow \\ \text{built from matter} + \omega \neq 0}}{Q_H} = 0 \quad\right]$$

Decompose into reps of:

$SL(2, \mathbb{C})$
Virasoro
BMS
...

Demanding the soft & hard operators transform under the full ASG as conformal fields with the expected weights provides higher order corrections to the Celestial stress tensor

$$T = \oint \frac{dz}{2\pi i} J_{soft}(z,\bar{z}) \qquad J_{soft} = -\bar{\mathcal{D}}^3 \underset{\substack{\uparrow \\ \text{subleading soft graviton}}}{\mathcal{N}^{(1)}} - \bar{\mathcal{D}}^3 C \overset{\substack{\text{leading soft graviton} \\ \downarrow}}{\mathcal{N}^{(0)}} - 3\bar{\mathcal{D}}^2 \underset{\substack{\uparrow \\ \text{Supertranslation Goldstone}}}{C} \bar{\mathcal{D}} \mathcal{N}^{(0)}$$

where

$$\mathcal{D}\phi_{h,\bar{h}} = [D_z - h\partial\Phi]\phi_{h,\bar{h}} \qquad \overset{\substack{\text{Superrotation Goldstone} \\ \downarrow}}{\Theta_{zz}} = \frac{1}{2} D_z\Phi D_z\Phi - D_z^2\Phi$$

[Donnay, Ruzziconi '21]

Now we commonly considered scattering around the trivial superrotation vacuum

$$\Theta_{zz} = 0 \implies \mathcal{D} \longrightarrow D_z$$

The remaining terms almost look like the proposed loop correction

$$\Delta \bar{T} \propto \underset{\underset{\text{IR div.}}{\uparrow}}{\frac{1}{\epsilon}} \int d^2w \underset{\underset{\text{contour}}{\uparrow}}{\frac{1}{z-w}} \left(\bar{\mathcal{D}} \underset{\underset{\text{memory vs. Goldstone}}{\uparrow}}{\bar{N}^{(0)}} N^{(0)} + 3\, \bar{N}^{(0)} \bar{\mathcal{D}} N^{(0)} \right)$$

[He, Kapec, Raclariu, Strominger '17]

Trust the symmetries!

That derivation predated our understanding of the supertranslation vertex operators.

$$\mathcal{O}_k = W_k \tilde{\mathcal{O}}_k \qquad W_k = e^{i\eta_k \omega_k C(z_k,\bar{z}_k)}$$

This leading soft sector exhibits a Kac-Moody like symmetry

$$P = \frac{1}{4G}\bar{\mathcal{D}} N^{(0)}, \quad \tilde{P} = i\mathcal{D}C \; : \quad P_z P_w \sim 0 \quad P_z \tilde{P}_w \sim \frac{1}{(z-w)^2} \quad \tilde{P}_z \tilde{P}_w \sim -\frac{1}{\varepsilon}\frac{G}{\pi}\frac{\bar{z}-\bar{w}}{z-w}$$

[Himwich, Narayanan, Pate, Paul, Strominger '20]

The off diagonal level structure implies an ambiguity when there are no other soft insertions

$$\mathcal{D}^2 C \Longleftrightarrow -\frac{i}{2\pi\varepsilon} N^{(0)}$$

The proposal to allow superrotations

prompted Cachao and Strominger to look for a subleading soft graviton thm

and led to the identification of a new memory effect.

Meanwhile it's ward Identity promotes the Lorentz group to a Virasoro symmetry

with this soft graviton mode providing a candidate 2D stress tensor

in the boost basis

Canonical phase space methods also predict the form of the loop corrections

The celestial basis actually points to a full tower of soft modes whose collinear limits encode a $Lw_{1+\infty}$ symmetry!

Themes

It is beneficial to examine both the asymptotic spacetime and amplitudes manifestations of a given structure.

CCFT further advocates for a particular dimensional reduction because it elucidates a larger symmetry algebra.

Ongoing Questions

- Role of IR regulators
  - ⟶ non-zero levels
  - ⟶ deforming symmetry algebra
- Analytic Continuations
  - ⟶ crossing between in and out
  - ⟶ crossing between 2D channels
- Toy Models
  - ⟶ identifying necessary ingredients
  - ⟶ SSB governed soft sector

Thank You!