Advances in Celestial Holography

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What is celestial holography?

What evidence supports the proposal?

What have we learned?

Introduction & Motivation

- *Celestial Holography* is the proposal:

Quantum gravity in 4D asymptotically flat spacetimes ⟷ (*Holographic dual*) Conformal field theory in 2D

- Basic observation underlying the proposal:

$$\underbrace{\mathrm{SO}(3,1)}_{\text{4D Lorentz}} \cong \underbrace{\mathrm{SL}(2,\mathbb{C})}_{\text{2D global conformal}}$$

⇒ Scattering amplitudes for 4D particles in highest weight reps of $\mathrm{SL}(2,\mathbb{C})$ ⟷ (*Transform under Lorentz*) Correlation functions of primary operators in a 2D CFT

Primary Operators

- To provide intuition for particles in highest weight representations, first recall labelling of primary operators:

$$\underbrace{\mathcal{O}_{h,\bar{h}}}_{\text{left \& right conformal weights}}(z, \bar{z})$$

left & right
conformal weights

Point on 2D plane

- Under SL(2,ℂ)

$$z \to \frac{az+b}{cz+d}, \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{C})$$

Primary Operators

- Primary operators *simultaneously diagonalize* the pair of conformal transformations that preserve $(z, \bar{z})$:

Dilation about $(z, \bar{z})$: $\Delta = h + \bar{h}$

Rotation about $(z, \bar{z})$: $s = h - \bar{h}$

- Corresponds to a pair of *mutually commuting* Lorentz transformations.

Dilation about $(z, \bar{z})$ $\Leftrightarrow$ Boost towards a fixed direction

Rotation about $(z, \bar{z})$ $\Leftrightarrow$ Rotation about a fixed direction

$\Rightarrow$ *Points on 2D plane are naturally identified with spatial directions in 4D.*

$\Rightarrow$ *Particles in highest-weight reps simultaneously diagonalize a pair of Lorentz generators.*

- There exists a simple construction for massless particles.

Massless Particles

- Typically study *momentum & helicity eigenstates*.
  $\longrightarrow$ Diagonalize rotations about the direction of the null momentum.
- *Spinor helicity variables* $\longrightarrow$ useful labels for such states

$$p_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$$

  - Transform non-trivially (but are eigenvectors of) rotations about $\vec{p}$

$$\lambda \to e^{i\phi}\lambda, \qquad \tilde{\lambda} \to e^{-i\phi}\tilde{\lambda}.$$

  - Ratio of components is invariant $\Rightarrow$ specifies direction of null momentum!

$$\frac{\lambda_1}{\lambda_2} \sim \text{direction of } \vec{p}$$

Massless Particles

- General Lorentz:

$$\lambda \to M\lambda, \qquad M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{C})$$

⇒ *Ratio* transforms like coordinate on 2D plane:

$$z \equiv \frac{\lambda_1}{\lambda_2} \sim \text{direction of } \vec{p}, \qquad z \to \frac{az+b}{cz+d}$$

Explicit example of identification between points on 2D plane and spatial directions in 4D.

Massless Particles

⇒ Motivates further parametrization of p by points on 2D plane:

$$p_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}} = \omega \begin{pmatrix} z \\ 1 \end{pmatrix} \begin{pmatrix} \bar{z} & 1 \end{pmatrix}$$

ω parametrizes overall scale

- *Upshot*: label massless particles in momentum and helicity eigenstates

$$|\omega, s, z, \bar{z}\rangle$$

- $(z, \bar{z})$: point on 2D plane
- s: eigenvalue under rotation about $(z, \bar{z})$
- ω: scale of momentum (~ energy)

Only mismatch!

Massless Particles

- Transformation under $\mathrm{SL}(2,\mathbb{C})$ confirms mismatch:

$$p_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}} = \omega \begin{pmatrix} z \\ 1 \end{pmatrix} \begin{pmatrix} \bar{z} & 1 \end{pmatrix} \qquad \lambda \to M\lambda, \quad M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{C})$$

$$\omega \to |cz+d|^2 \omega \qquad \Rightarrow \qquad \omega \neq \Delta$$

- Unsurprising because Δ is eigenvalue of dilation in 2D → boost in 4D
  - Null momenta are *not* invariant under boosts

  ⇒ *Impossible* to work with momentum eigenstates that are also in highest weight reps

  ⇒ *Need to change basis.*

Massless Particles in Boost Eigenstates

Already diagonalized rotations about $\vec{p}$

→ Easiest to diagonalize

Boost along $\vec{p}$

$$p_{\alpha\dot{\alpha}} \to \xi p_{\alpha\dot{\alpha}} \quad \Rightarrow \quad \omega \to \xi\omega$$

- Diagonalized by the *Mellin transform*:

$$\int_0^\infty \frac{d\omega}{\omega}\omega^\Delta |\omega, s, z, \bar{z}\rangle \equiv |\Delta, s, z, \bar{z}\rangle$$

Celestial Amplitudes

- Mellin transform momentum space amplitudes (of massless particles) with respect to each ω_i

$\Rightarrow$ Obtain objects that transform under Lorentz like *correlation functions of primary operators* in a 2D CFT

$$\Big(\prod_{i=1}^{n}\int_0^\infty \frac{d\omega_i}{\omega_i}\omega_i^{\Delta_i}\Big)\mathbf{A}(p_1,\cdots,p_n) \equiv \langle \mathcal{O}_{h_1,\bar{h}_1}(z_1,\bar{z}_1)\cdots\mathcal{O}_{h_n,\bar{h}_n}(z_n,\bar{z}_n)\rangle$$

[Kapec, Mitra, Raclariu & Strominger, 1609.00282
Cheung, de la Fuente & Sundrum, 1609.00732;
Pasterski & Shao, 1705.01027;
Pasterski, Shao & Strominger, 1706.03917]

Celestial Amplitudes

- Construction was not systematic
  - *Not the unique way to obtain highest weight reps from momentum space*
  - *However, will be the focus for today*
- Key property :

Points on the 2D plane ⟷ *Actually correspond to* Points on spatial cross sections of null infinity

Follows from $z \equiv {}^{\lambda_1}/_{\lambda_2}$ labelling direction of null momentum

Outline

✓ *What is celestial holography?*

1. Review of celestial holography & construction of celestial amplitudes

- *What evidence supports the proposal?*

1. Locality in 2D: OPEs $\leftrightarrow$ Collinear limits
2. OPEs from Poincaré Symmetry

- *What have we learned?*

Operator Product Expansions & Collinear Limits

- *Immediate Implication*:

$$\lim_{z_1 \to z_2} \mathcal{O}_1(z_1)\mathcal{O}_2(z_2) \quad \underset{\text{dual to}}{\longleftrightarrow} \quad p_1 || p_2$$

$$p_1 \cdot p_2 \sim [12]\langle 12 \rangle \sim \omega_1 \omega_2 z_{12} \bar{z}_{12}$$

- Standard CFT: limit is governed by the *operator product expansion*

  ⇒ To interpret boost weight states as *local operators*, need collinear limits to admit OPE-like structure.

- Encouragingly, collinear limits of *tree-level massless scattering* amplitudes *do* appear to provide compatible structure.

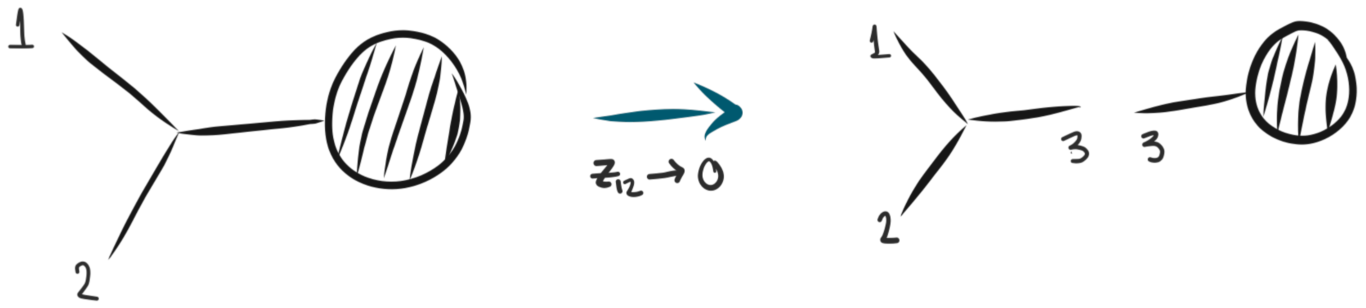
Collinear Limits Review

- The behavior of tree-level amplitudes in the collinear limit is dominated by factorization:

$$\mathbf{A}_n(p_1, p_2, \cdots) \to \mathcal{V}_3 \frac{1}{(p_1+p_2)^2} \mathbf{A}_{n-1}(p_1+p_2, \cdots)$$

Collinear splitting factor

Written to elucidate origin of factorization in Feynman diagrams



Key Ingredients from Collinear Limits

1. Collinear splitting factor can supply singularities in z_{12}

$$\frac{1}{(p_1+p_2)^2} \sim \frac{1}{\omega_1 \omega_2 z_{12} \bar{z}_{12}}$$

2. Bulk (4D) dimension d_V of 3-point interaction determines overall scaling with energy

$$\mathcal{V}_3 \sim \omega^{d_V - 3} \quad \rightarrow \quad \underbrace{\mathcal{V}_3 \frac{1}{(p_1+p_2)^2}}_{\text{Collinear splitting factor}} \sim \omega^{d_V - 5}$$

OPE from Collinear Factorization

- Collinear factorization:

$$\mathbf{A}_n(p_1, p_2, \cdots) \overset{z_{12}\to 0}{\sim} \omega^{d_V - 5} \frac{\bar{z}_{12}^p}{z_{12}} \mathbf{A}_{n-1}(p_1 + p_2, \cdots)$$

Treat z & $\bar{z}$ as independent

- Implies OPE limit for celestial amplitudes

$$\langle \mathcal{O}_{\Delta_1, s_1}(z_1, \bar{z}_1) \mathcal{O}_{\Delta_2, s_2}(z_2, \bar{z}_2) \cdots \rangle \overset{z_{12}\to 0}{\propto} \frac{\bar{z}_{12}^p}{z_{12}} \langle \mathcal{O}_{\Delta_1 + \Delta_2 + d_V - 5, s}(z_2, \bar{z}_2) \cdots \rangle$$

Determined by ω-scaling:

- Δ_1, Δ_2 from Mellin integrals for $\mathcal{O}_1$, $\mathcal{O}_2$
- $d_V - 5$ from splitting factor

1. Singularity in z_{12} ⇒ boost weight states behave like local operators in 2D.
2. Energy scaling fixed by d_V ⇒ fusion rule for celestial operators

MP, Raclariu, Strominger & Yuan [1901.07424]

OPEs from Collinear Limits

- Interpret factorization as arising from an OPE:

$$\mathcal{O}_{\Delta_1,s_1}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,s_2}(z_2,\bar{z}_2) \sim C_{123}\frac{\bar{z}_{12}^{p}}{z_{12}}\mathcal{O}_{\underbrace{\Delta_1+\Delta_2+d_V-5,s}_{\equiv\Delta_3,s_3}}(z_2,\bar{z}_2)$$

- Deduce p by comparing transformation of each side under 2D conformal symmetry

$$\begin{aligned} p &= d_V - 4 && \text{(match net } \Delta\text{-weight)} \\ &= s_1 + s_2 - s_3 - 1 && \text{(match net } s\text{-weight)} \end{aligned}$$

MP, Raclariu, Strominger & Yuan [1910.07424]

- Can explicitly compute OPE coefficient with more careful treatment of this argument.
  - Originally done in Yang-Mills by Fan, Fotopoulos and Taylor [1903.01676]

OPE Coefficients from Symmetry

- *Instead today*:

Provide *holographic first principled* derivation of

OPE coefficients from symmetry

- *Previously,* carried out similar type of analysis in 1901.07424 by **MP**, Raclariu, Strominger & Yuan.
  - *Used more exotic symmetries associated to subleading soft theorems to derive OPE coefficients in EYM.*

- *Today,* we'll use Poincaré symmetry to determine leading OPE coefficients in generic theories with massless particles.

Himwich, **MP** & Singh [2108.07763]

Poincaré Constraints on OPE Coeff.

- Why is Poincaré sufficient?

Irrep of 4D Poincaré { Massless particle = *Family* of operators of fixed spin s & varying boost weight Δ

- OPE involves primaries of *fixed* boost weight

  ⇒ Grouping of boost weights into single 4D-particles is captured by non-trivial dependence of OPE coefficients on boost weight Δ

- Goal: Determine Δ_i dependence
- Logic: 4D translations relate different conformal families

  ⇒Thereby impose further constraints on OPE coefficients of the primaries

- Payoff: learn how how 4D particles (i.e. irreps of Poincaré) emerge from 2D CFT data

Symmetry Constraints on OPE Coefficients

Basic Logic:

Ansatz:

$$\mathcal{O}_1(z,\bar{z})\mathcal{O}_2(0,0) = \sum_k A_{12k}(z,\bar{z})\mathcal{O}_k(0,0)$$

Symmetry implies:

$$\underbrace{[Q, \mathcal{O}_1(z,\bar{z})\mathcal{O}_2(0,0)]}_{[Q,\mathcal{O}_1(z,\bar{z})]\,\mathcal{O}_2(0,0) + \mathcal{O}_1(z,\bar{z})\,[Q,\mathcal{O}_2(0,0)]} = \sum_k A_{12k}(z,\bar{z})\,[Q, \mathcal{O}_k(0,0)]$$

Poincaré for Celestial Amplitudes

- Reference: Stieberger & Taylor [1812.01080]
- *Lorentz Transformations* = standard global conformal transformations

$$\left[\bar{L}_m, \mathcal{O}_{h,\bar{h}}(z,\bar{z})\right] = \bar{z}^m \left((m+1)\bar{h} + \bar{z}\partial_{\bar{z}}\right) \mathcal{O}_{h,\bar{h}}(z,\bar{z})$$

$$m = -1, 0, 1$$

$$\left[\bar{L}_m, \bar{L}_n\right] = (m-n)\bar{L}_{m+n}$$

Poincaré for Celestial Amplitudes

- Reference: Stieberger & Taylor [1812.01080]; Donnay, Puhm, Strominger [1810.05219]
- *Translations*:

$$p_{\alpha\dot{\alpha}} \sim \omega \begin{pmatrix} z \\ 1 \end{pmatrix} \begin{pmatrix} \bar{z} & 1 \end{pmatrix}$$

$$|\Delta\rangle \sim \int_0^\infty \frac{d\omega}{\omega} \omega^\Delta |\omega\rangle$$

Shifts $\Delta \rightarrow \Delta + 1$

(Otherwise multiples by $z, \bar{z}$)

$$\left[P_{m,n}, \mathcal{O}_{h,\bar{h}}(z,\bar{z})\right] = \frac{1}{2} z^{m+\frac{1}{2}} \bar{z}^{n+\frac{1}{2}} \mathcal{O}_{h+\frac{1}{2},\bar{h}+\frac{1}{2}}(z,\bar{z})$$

$m, n = \pm\frac{1}{2}$ and label mode number under global conformal transformations (analogue of subscript on $\bar{L}_n$'s)

Ansatz

From collinear limits:

$$\mathcal{O}_{h_1,\bar{h}_1}(z,\bar{z})\mathcal{O}_{h_2,\bar{h}_2}(0,0) \sim C_{123}\frac{\bar{z}^p}{z}\mathcal{O}_{\underbrace{h_1+h_2-1,\bar{h}_1+\bar{h}_2+p}_{\equiv h_3,\bar{h}_3}}(0,0)$$

$$p = d_V - 4$$

Translations mix primaries & descendants

⇒ Need to include these too

$$\mathcal{O}_{h_1,\bar{h}_1}(z,\bar{z})\mathcal{O}_{h_2,\bar{h}_2}(0,0) \sim \frac{\bar{z}^p}{z}\sum_{m=0}^{\infty} C_{123}^{(m)}\,\bar{z}^m\bar{\partial}^m\mathcal{O}_{h_1+h_2-1,\bar{h}_1+\bar{h}_2+p}(0,0)$$

(R-moving is sufficient)

Summary of Poincaré Constraints

- $\bar{L}_1$: $$(2\bar{h}_1 + p + m)C_p^{(m)}(\bar{h}_1, \bar{h}_2) = (m+1)(2\bar{h}_1 + 2\bar{h}_2 + 2p + m)C_p^{(m+1)}(\bar{h}_1, \bar{h}_2)$$ (recursion in m)

- $P_{-\frac{1}{2},-\frac{1}{2}}$: $$C_p^{(m)}(\bar{h}_1 + \tfrac{1}{2}, \bar{h}_2) + C_p^{(m)}(\bar{h}_1, \bar{h}_2 + \tfrac{1}{2}) = C_p^{(m)}(\bar{h}_1, \bar{h}_2)$$ (recursion in $\bar{h}_1$)

- $P_{-\frac{1}{2},+\frac{1}{2}}$: $$C_p^{(m)}(\bar{h}_1 + \tfrac{1}{2}, \bar{h}_2) = (m+1)C_p^{(m+1)}(\bar{h}_1, \bar{h}_2)$$ (recursion in $\bar{h}_1$ and m)

$$(2\bar{h}_1 + p + m)C_p^{(m)}(\bar{h}_1, \bar{h}_2) = (2\bar{h}_1 + 2\bar{h}_2 + 2p + m)C_p^{(m)}(\bar{h}_1 + \tfrac{1}{2}, \bar{h}_2)$$

(recursion in $\bar{h}_1$ from combining $\bar{L}_1$ and $P_{-\frac{1}{2},+\frac{1}{2}}$)

Summary of Poincaré Constraints

- $\bar{L}_1$: $$(2\bar{h}_1 + p + m)C_p^{(m)}(\bar{h}_1, \bar{h}_2) = (m+1)(2\bar{h}_1 + 2\bar{h}_2 + 2p + m)C_p^{(m+1)}(\bar{h}_1, \bar{h}_2)$$

- $P_{-\frac{1}{2},-\frac{1}{2}}$: $$C_p^{(m)}(\bar{h}_1 + \tfrac{1}{2}, \bar{h}_2) + C_p^{(m)}(\bar{h}_1, \bar{h}_2 + \tfrac{1}{2}) = C_p^{(m)}(\bar{h}_1, \bar{h}_2)$$

- $P_{-\frac{1}{2},+\frac{1}{2}}$: $$C_p^{(m)}(\bar{h}_1 + \tfrac{1}{2}, \bar{h}_2) = (m+1)C_p^{(m+1)}(\bar{h}_1, \bar{h}_2)$$

$$(2\bar{h}_1 + p + m)C_p^{(m)}(\bar{h}_1, \bar{h}_2) = (2\bar{h}_1 + 2\bar{h}_2 + 2p + m)C_p^{(m)}(\bar{h}_1 + \tfrac{1}{2}, \bar{h}_2)$$

Two fixed m constraints are recursion relations for the Euler beta function

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

OPE Coefficients from Poincaré

- Solution to fixed m constraints:

$$C_p^{(m)}(\bar{h}_1, \bar{h}_2) \propto B(2\bar{h}_1 + p + m, 2\bar{h}_2 + p)$$

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

- $\bar{L}_1$ constraint further fixes relative coefficient between different m:

$$C_p^{(m)}(\bar{h}_1, \bar{h}_2) = \gamma_p^{s_1,s_2} \frac{1}{m!} B(2\bar{h}_1 + p + m, 2\bar{h}_2 + p)$$

Himwich, MP & Singh [2108.07763]

  - $\gamma_p^{s_1,s_2}$ is undetermined spin-dependent coefficient.
  - Can verify formula by Mellin-transforming collinear splitting factor.

→ $\gamma_p^{s_1,s_2}$ is just the coupling constant for the 3-point interaction between 4D particles of spin s_1, s_2, and $p + 1 - s_1 - s_2$.

Outline

✓ *What is celestial holography?*

✓ *What evidence supports the proposal?*

1. Locality in 2D: OPEs ↔ Collinear limits
2. OPEs from Poincaré Symmetry

- *What have we learned?*

1. Conformally Soft Gravitons & $\mathrm{w}_{1+\infty}$
2. $\mathrm{w}_{1+\infty}$ for Hard Massless Particles

Outline

✓ *What is celestial holography?*

2D hologram of 4D scattering from Lorentz *symmetry*

✓ *What evidence supports the proposal?*

1. Locality in 2D: OPEs ↔ Collinear limits
2. OPEs from Poincaré Symmetry

Interactions (OPE coefficients) from Poincaré *symmetry*

- *What have we learned?*

1. Conformally Soft Gravitons & $\mathrm{w}_{1+\infty}$
2. $\mathrm{w}_{1+\infty}$ for Hard Massless Particles

New symmetries from *interactions* (OPE coefficients)

Poles in the OPE coefficients

- Notice that the OPE coefficients have poles in $\bar{h}_i$

$$C_p^{(m)}(\bar{h}_1, \bar{h}_2) = \frac{\gamma_p^{s_1,s_2}}{m!} B(2\bar{h}_1 + p + m, 2\bar{h}_2 + p)$$

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

⇒ Poles @ $2\bar{h}_1 + p + m = \Delta_1 - s_1 + p + m \in \mathbb{Z}_{\leq 0}$

Physical Significance of Poles

- *To determine physical significance, consider Mellin transform of function with a Laurent expansion about $\omega = 0$:*

$$\widetilde{f}(\Delta) = \int_0^\infty \frac{d\omega}{\omega} \omega^\Delta f(\omega) \subset \int_0 \frac{d\omega}{\omega} \omega^\Delta \sum_n \omega^n f_n \sim \sum_n \frac{f_n}{\Delta + n}$$

⇒ Powers of ω turn into simple poles in Δ at integer values.

✶ Residues give Laurent expansion coefficients.

⇒ *Infinite tower of poles* in OPE coefficients captures a *series expansion in energy*.

⇒ For scattering amplitudes, series admits universal behavior, characterized by *soft theorems*.

✶ Soft particles behave like currents generating ∞-dimensional symmetries whose Ward identities are soft theorems.

Soft Graviton Symmetries from Graviton OPE

- Consider OPE capturing the minimal coupling of a positive-helicity graviton to matter:

$$G^{+}_{\Delta_1}(z,\bar{z})\mathcal{O}_{h_2,\bar{h}_2}(0,0) \sim -\frac{\kappa}{2}\frac{\bar{z}}{z}\sum_{m=0}^{\infty}\frac{1}{m!}B(\Delta_1-1+m,2\bar{h}_2+1)\bar{z}^m\bar{\partial}^m\mathcal{O}_{h_2+\frac{\Delta_1}{2},\bar{h}_2+\frac{\Delta_1}{2}}(0,0)$$

$(\Delta = \Delta_1, s=2)$

- Leading pole @ $\Delta_1 = 1$:

Residue @ $\Delta_1 = 1$ $\Leftrightarrow$ Coefficient of $^1/_\omega$ } Content of the *leading soft graviton theorem*

- Poles @ $\Delta_1 = 0, -1, -2, \ldots$:

$\Rightarrow$ Suggests that a *universal symmetry action* associated to *subleading* soft theorems persists *to all orders*!

Holographic Symmetry Algebra

- *What is the algebra generated by these soft graviton symmetry actions?*
  - Determine from current-current OPE.
  - Currents are "conformally soft gravitons".

$$H^k(z,\bar{z}) \equiv \lim_{\varepsilon \to 0} \varepsilon G^{+}_{k+\varepsilon}(z,\bar{z}), \qquad k = 2, 1, 0, \cdots$$

Gravitons in highest weight states,
normalized to extract residue of pole in Δ

Holographic Symmetry Algebra

- Use $G^+_{\Delta_1} G^+_{\Delta_2} \sim G^+_{\Delta_1+\Delta_2}$ OPE to determine OPE for conformally soft gravitons

$$H^k(z,\bar{z})H^\ell(0,0) \sim -\frac{\kappa}{2}\frac{\bar{z}}{z}\sum_{m=0}^{1-k}\frac{1}{m!}\binom{2-k-\ell-m}{1-\ell}\bar{z}^m\bar{\partial}^m H^{k+\ell}(0,0)$$

- Mode expand, compute commutator & relabel

$$H^k(z,\bar{z}) = \sum_{n=\frac{k-2}{2}}^{-\frac{k-2}{2}}\frac{H^k_n(z)}{\bar{z}^{n+\bar{h}}}, \qquad [A,B](z) \equiv \oint_z \frac{dw}{2\pi i}A(w)B(z), \qquad w^p_n = \frac{1}{\kappa}(p-n-1)!(p+n-1)!H^{4-2p}_n,$$

$$[w^p_m, w^q_n] = (m(q-1)-n(p-1))\, w^{p+q-2}_{m+n}$$

$w_{1+\infty}$ symmetry algebra

Guevara, Himwich, **MP** & Strominger [2103.03961]
Strominger [2105.14346]

$w_{1+\infty}$ Symmetry Algebra

$$[w_m^p, w_n^q] = (m(q-1) - n(p-1))\, w_{m+n}^{p+q-2}$$

- Here $p = 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots$ & $1 - p \leq m \leq p - 1$ ("wedge subalgebra")
- w_m^2 generates a $\mathbf{SL(2,}\mathbb{R})$ **action** under which $\boldsymbol{w_n^q}$ transforms as $\boldsymbol{n}$**th mode of primary of weight** $\boldsymbol{q}$
- $w_m^{p\leq 2}$ form closed subalgebra
- $w_m^{5/2}$ generates infinite tower (corresponds to subsubleading soft graviton).

Guevara, Himwich, **MP** & Strominger [2103.03961]
Strominger [2105.14346]

$w_{1+\infty}$ Action on Massless Particles

- *(How) does $w_{1+\infty}$ act on generic massless particles (i.e. not just soft gravitons)?*
- Determine from current-matter OPE
  - Use minimal coupling OPE ($G^+\mathcal{O}\sim\mathcal{O}$) to determine:

$$H^k(z,\bar{z})\mathcal{O}_{h,\bar{h}}(0,0) \sim \frac{1}{z}\sum_{m=0}^{1-k}\lim_{\varepsilon\to 0}\varepsilon B(k+\varepsilon-1+m, 2\bar{h}+1)\ \bar{z}^m\bar{\partial}^m\mathcal{O}_{h+\frac{k}{2},\bar{h}+\frac{k}{2}}(0,0)$$

  - Mode expand, compute commutator & relabel (or equivalently perform light transform)

$$\left[\widehat{\mathrm{w}}^q_n, \mathcal{O}_{h,\bar{h}}(z,\bar{z})\right] = \frac{1}{2}\sum_{\ell=0}^{2q-3}\binom{q+n-1}{\ell}\frac{(2q-2-\ell)\Gamma(2\bar{h}+1)}{\Gamma(2\bar{h}+1-\ell)}\bar{z}^{q+n-1}\partial_{\bar{z}}^{2q-3-\ell}\mathcal{O}_{h+2-q,\bar{h}+2-q}(z,\bar{z})$$

Himwich, **MP** & Singh [2108.07763]; Jiang [2108.08799]

$w_{1+\infty}$ Action on Massless Particles

- Can show

$$\left[\widehat{w}^p_m, \left[\widehat{w}^q_n, \mathcal{O}_{h,\bar{h}}(z,\bar{z})\right]\right] - \left[\widehat{w}^q_n, \left[\widehat{w}^p_m, \mathcal{O}_{h,\bar{h}}(z,\bar{z})\right]\right] = \left[\left[\widehat{w}^p_m, \widehat{w}^q_n\right], \mathcal{O}_{h,\bar{h}}(z,\bar{z})\right]$$

where

$$\left[\widehat{w}^p_m, \widehat{w}^q_n\right] = \left(m(q-1) - n(p-1)\right)\widehat{w}^{p+q-2}_{m+n}$$

$\Rightarrow$ Massless particles transform in (non-trivial) representations of $w_{1+\infty}$

Himwich, MP & Singh [2108.07763]

Summary

✓ *What is celestial holography?*

Symmetry: $SO(3,1) \cong SL(2,\mathbb{C})$ ⇒ Quantum gravity in 4D asymptotically flat spacetimes is holographically dual to a 2D CFT

✓ *What evidence supports this approach?*

- Massless particles in highest weight states behave like *local operators*, admitting operator product expansions.

Poincaré Symmetry ⇒ Fixes leading OPE coefficients

✓ *What have we learned?*

Leading OPE coefficients (in graviton OPE) ⇒ Algebra of soft symmetries organizes into $\mathrm{w}_{1+\infty}$

$\mathrm{w}_{1+\infty}$ symmetry ⇒ Additional constraints on OPE?

* Leading coefficients are consistent with, but not further constrained by $\mathrm{w}_{1+\infty}$

Thank you!