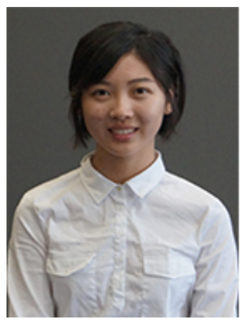
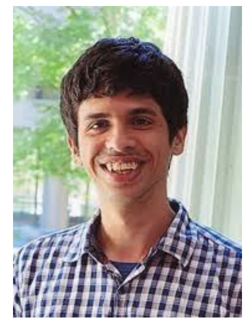
Effective Field Theories with Celestial Duals

M. Spradlin, Brown University, Amplitudes 2022






Rishabh Bhardwaj

Shounak De

Yangrui Hu ⇒ Perimeter Institute

Luke Lippstreu

Lecheng Ren

Akshay Yelleshpur Srikant

Anastasia Volovich

Outline of the Talk

1. Effective Field Theories with Celestial Duals

   2206.08322 Ren, MS, Yelleshpur Srikant, Volovich

2. All-loop Celestial OPEs

   soon Bhardwaj, Lippstreu, Ren, MS, Yelleshpur Srikant, Volovich

The OPE for conformally soft graviton modes

One property you might want to have is an associative OPE (operator product expansion).

Let's quickly review the story for + helicity gravitons which have an OPE

$$B(a,b) = \frac{\Gamma(a)\,\Gamma(b)}{\Gamma(a+b)}$$

$$G^{+}_{\Delta_1}(z,\bar{z})\,G^{+}_{\Delta_2}(0,0) \sim -\frac{1}{2}\,\frac{\kappa_{2,2,-2}}{z}\sum_{n=0}^{\infty} B(\Delta_1-1+n,\,\Delta_2-1)\,\frac{\bar{z}^{\,n+1}}{n!}\,\bar{\partial}^{n} G^{+}_{\Delta_1+\Delta_2}$$

$\sqrt{G_N}$

[Fan, Fotopoulos, Taylor, Pate, Raclariu, Strominger, Yuan]

Noting that the OPE coefficient B has poles one defines the conformally soft graviton operators

$$H^k(z,\bar z) = \lim_{\Delta\to k} (\Delta-k)\, G^+_\Delta(z,\bar z)$$

and their mode expansion $$H^k(z,\bar z) = \sum_n \frac{H^k_n(z)}{\bar z^{\,n+\frac{k-2}{2}}}$$

Guevara, Himwich, Pate, Strominger worked out their commutation relations from the OPE and Strominger showed that they are equivalent to

$$[w^p_m, w^q_n] = (m(q-1)-n(p-1))\, w^{p+q-2}_{m+n} \qquad q\in\{1,\tfrac{3}{2},2,\tfrac{5}{2},\dots\}$$
$$1-q \le n \le q-1$$

"$W_{1+\infty}$" [Penrose 1976, Bakas 1989]

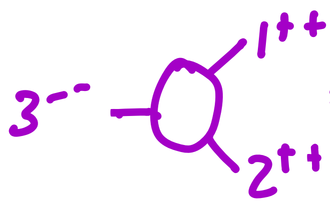
(There is an analogous story for gluons.)

Deformed $w_{1+\infty}$ Algebra

In 2108.07763 Himwich, Pate, Singh considered corrections to the OPE from non-minimal couplings.

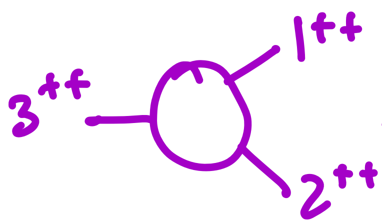
In 2111.11356 Mago, Ren, Yelleshpur Srikant, Volovich worked out the corresponding corrections to the algebra.

Notation: K_{h_1,h_2,h_3} is the 3-point coupling for helicities h_1, h_2, h_3, e.g.



$$= K_{2,2,-2} \frac{[12]^6}{[23]^2[31]^2}$$

Einstein term



$$= K_{2,2,2} [12]^2[23]^2[13]^2$$

R^3 interaction

Deformed $w_{1+\infty}$ Algebra

[2111.11356 Mago, Ren, Yelleshpur Srikant, Volovich]

They found that the algebra of conformally soft modes satisfies the Jacobi identity only if the Wilson coefficients satisfy certain constraints, in particular

Einstein

R^3

$$K_{0,2,2}^2 = \frac{3}{10} K_{-2,2,2} K_{2,2,2}$$

massless scalar

$$K_{0,1,1}^2 = 2 \; K_{-1,1,1} \; K_{1,1,1}$$

F^3

Yang-Mills

$$K_{-1,1,2} K_{1,1,1} = 3 \; K_{1,1,2} K_{-1,1,1}$$

On EFTs with Celestial Duals

In 2206.08322 Ren, MS, Yelleshpur Srikant, Volovich reformulated the condition for OPE associativity directly at the level of momentum space scattering amplitudes, finding the same constraints as above, and worked out the implications of these constraints for amplitudes.

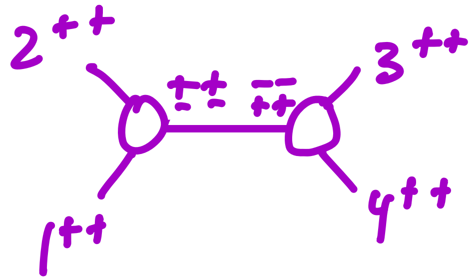
i.e. Is there anything special about the amplitudes in an EFT whose celestial dual has an associative OPE.

Yes! The 3/10, for example, is "famous" (in certain circles).

Four-Point Amplitudes

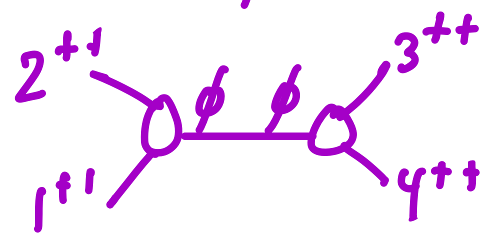
Consider the all-plus four-graviton amplitude

$$A(1^{++}, 2^{++}, 3^{++}, 4^{++}) =$$



graviton exchange

$+$



scalar exchange

$+$

permutations

$$\propto \left(K_{-2,2,2} K_{2,2,2} - \frac{3}{10} K_{0,2,2}^2 \right) = 0$$

if the celestial dual has an associative OPE

We showed that all (all-line shift constructible) four-point amplitudes involving external scalars, gluons & gravitons vanish on the support of the constraints.

The caveat is important; not all four-point amplitudes vanish, e.g.

$$A(1^{++},2^{++},3^{++},4^{--}) = K_{2,2,2}\, K_{-2,-2,2}$$

$$\times \frac{\langle 14\rangle^2 [13]^2 \langle 34\rangle^2 [12][23][31]}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}$$

[Broedel, Dixon]

For higher point amplitudes that are all-line shift constructible, the general statement is that all contributions that involve purely holomorphic $(h_1+h_2+h_3<0)$ or purely anti-holomorphic $(h_1+h_2+h_3>0)$ vertices vanish on support of the constraints.

The vanishing of the four-positive helicity graviton amplitude $A(1^{++},2^{++},3^{++},4^{++})$ is not, by itself, enough to guarantee associativity of the corresponding celestial OPE, but it serves as a simple and powerful check.

Aside: in 2204.05301 Costello & Paquette showed that the celestial dual of self-dual YM theory does not have an associative chiral algebra unless one adds an "axion" with a specific coupling constant.

Rampant Speculation

It seems (?) that "most" 4d EFTs do not have celestial duals with an associative OPE.

For a person interested only in 4d amplitudes, what is the value of the question "does the celestial dual have an associative OPE?" What does that tell you? What does it mean?

Maybe it indicates that the 4d theory is very special and possesses some type of "integrability". Costello

Maybe its like a swampland constraint that only "good" theories of gravity satisfy. (The all-plus 4 graviton amplitude vanishes in heterotic but not bosonic string). Strominger

All-loop Celestial OPEs

[to appear, Bharduaj, Lippstreu, Ren, Yelleshpur Srikant, MS, Volovich]

A motivation for our work is to investigate the celestial avatar of an "exact" amplitude that actually exists.

In the case of $\mathcal{N}=4$ super-Yang-Mills theory, this means going onto the Coulomb branch where the scalar fields ϕ_i are given a VEV.

The Coulomb branch is vastly underexplored but there is corner which suits us well:

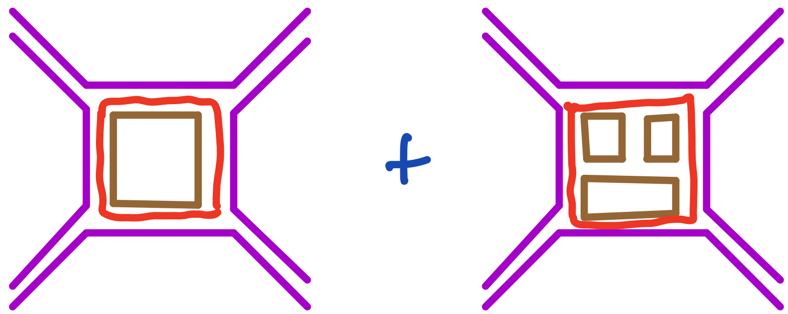
$$U(N) \longrightarrow U(N-M) \times U(M) \quad \text{with} \quad N \gg M \gg 1.$$

giving the off-diagonal gluons mass m.

In this limit the dominant contributions to the scattering of the massless $U(M)$ gluons come from planar diagrams framed by a single external loop of massive gluons

$U(N-M)$

$U(M)$



etc.

— = massive loop

Alday, Henn, Plefka and Schuster suggested that the exact amplitude has an exponential BDS-like form with m instead of ε playing the role of IR regulator:

$$A_4(+,+,-,-) = \frac{\langle 34\rangle^3}{\langle 12\rangle\langle 23\rangle\langle 41\rangle} \cdot \exp\left[-\frac{\gamma}{8}\cdot\left(\log^2(s/m) + \log^2(t/m) + \log^2(s/t)\right) - G\cdot\left(\log(s/m^2) + \log(t/m^2)\right) + C + \mathcal{O}(m^2)\right]$$

where γ, G, C are functions of the coupling.

Advertisement: one nice feature is the absence of cross-order contamination when you expand the exponential

$$\lim_{M\to 0} M^2 \cdot \log(M^2) = 0$$

(unlike for the BDS ansatz in dim reg).

Now we can compute the celestial avatar of this amplitude by taking the Mellin transform — calculationally, identical to work of Gonzalez, Puhm, Rojas — who did it for the BDS ansatz — and from this result we can read off the all-loop OPE:

$$\boxed{\mathcal{O}^a_{\Delta_1,+}(z,\bar z)\,\mathcal{O}^b_{\Delta_2,+}(0,0) \sim \frac{if^{abc}}{z}\cdot \exp\left[-\frac{\pi^2}{48}\gamma - \frac{\gamma}{4}\left(\mathcal{D}^2 - \partial_{\Delta_1}\partial_{\Delta_2}\right) + G\cdot\mathcal{D}\right] B(\Delta_1-1,\Delta_2-1)\,\mathcal{O}^c_{\Delta,+}(0)\Big|_{\Delta=\Delta_1+\Delta_2-1}}$$

where $\mathcal{D} = \partial_{\Delta_1} + \partial_{\Delta_2} + \partial_\Delta + \frac{1}{2}\log(-z\bar z)$

(the part in blue is just the tree-level OPE)

We've also shown that this is the OPE of "hard" gluon operators — using hard/soft factorization.

[see also Magnea; Nastase, Rojas, Rubio; Gonzalez, Rojas].

Conclusion

We've looked at the implications for amplitudes in EFTs whose celestial duals have an associative OPE.

We've looked at an example of an "all-loop" OPE, which exhibits some unusual features.

Many natural questions would benefit from studying simpler examples (self-dual gravity, YM, $\mathcal{N}=2$ strings, ...)