AMPLITUDES 2023

CERN

Conference: August 7 - 11

School: July 31 - August 4

Local Organizers:

Samuel Abreu, Andrew McLeod,
Ben Page, Lorenzo Tancredi

Novel Analytic Constraints on Feynman Integrals

Andrew McLeod

Amplitudes 2022
August 11

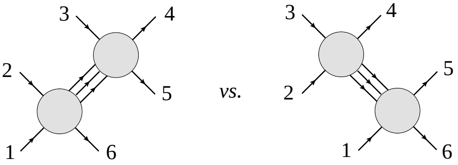
arXiv:2109.09744 [hep-th] and ongoing work

with H. Hannesdottir, M. Schwartz, and C. Vergu

Motivation

In recent years, a number of surprising **empirical properties** have been observed in the **analytic structure** of Feynman integrals

- The **locations of branch cuts** in large classes of Feynman integrals exhibit intriguing connections to cluster algebras and related algebraic structures
  [Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, Trnka (2012)] [Golden, Goncharov, Spradlin, Vergu, Volovich (2013)] $\cdots$
- Moreover, the **sequential discontinuities** of many Feynman integrals obey generalized versions of the Steinmann relations
  [Drummond, Foster, Gürdoğan (2017)] [Caron-Huot, Dixon, von Hippel, AJM, Papathanasiou (2018)]



[Steinmann (1960)] (see also Dixon's talk)

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- Moreover, the **sequential discontinuities** of many Feynman integrals obey generalized versions of the Steinmann relations (also observed in the nonplanar sector!)
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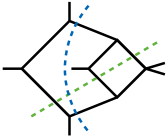
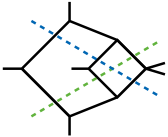


extended Steinmann satsifed



extended Steinmann not satisfied

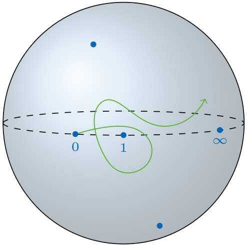
[Abreu, Ita, Page, Tschernow (2021)]

Constraints from Landau Analysis

Can we derive these types of properties of Feynman integrals directly from Landau analysis?

We also bring to this analysis detailed knowledge of the types of **iterated integrals** that are known to appear in Feynman integrals

- The first class of iterated integrals that naturally arise are **multiple polylogarithms**
  (see also talks by Dixon, He, Henn, Schwartz, Wilhelm, Zoia)



Multiple Polylogarithms

- **Multiple polylogarithms** are functions F that have the property that
$$dF = \sum_i F^{s_i} d\log s_i$$
where the s_i are algebraic functions, and each F^{s_i} is also a multiple polylogarithm

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- It is useful to define the **symbol** of a multiple polylogarithm by upgrading this total differential to a tensor product [Goncharov, Spradlin, Vergu, Volovich (2010)]

$$\mathcal{S}(F) = \sum_i \mathcal{S}(F^{s_i}) \otimes s_i$$

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Examples of such functions include $\log(z)$ and $\mathrm{Li}_m(z)$:

$$\mathrm{Li}_1(z) = -\log(1-z), \quad \mathrm{Li}_m(z) = \int_0^z \frac{\mathrm{Li}_{m-1}(t)}{t} dt$$

$$\mathcal{S}\big(\mathrm{Li}_m(z)\big) = -(1-z) \otimes z \otimes \cdots \otimes z$$

Multiple Polylogarithms

- The symbol map can thus be used to transparently expose the analytic structure of polylogarithmic Feynman integrals $\mathcal{I}(p)$:

$$\mathcal{S}\big(\mathcal{I}(p)\big) = \sum \overbrace{a_1(p)}^{\text{first discontinuity}} \otimes \overbrace{a_2(p)}^{\text{second discontinuity}} \otimes \overbrace{a_3(p)}^{\cdots} \otimes \cdots \otimes \underbrace{a_{w-2}(p)}_{\cdots} \otimes \underbrace{a_{w-1}(p)}_{\text{second derivative}} \otimes \underbrace{a_w(p)}_{\text{first derivative}}$$

Constraints from Landau Analysis

Two broad strategies for constraining the symbol of Feynman integrals:

- From the front — restrict what **sequences of discontinuities** are allowed in Feynman integrals by studying where singularities can appear in these integrals [Hannesdottir, AJM, Schwartz, Vergu (forthcoming)]

- From the back — restrict the **derivatives** of Feynman integrals by studying their behavior when expanded near branch points [Hannesdottir, AJM, Schwartz, Vergu (2021) and (forthcoming)]

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  Hannesdottir's talk

- From the back — restrict the **derivatives** of Feynman integrals by studying their behavior when expanded near branch points [Hannesdottir, AJM, Schwartz, Vergu (2021) and (forthcoming)]

  this talk

Landau Analysis

- The locations where Feynman integrals can become singular and develop branch cuts are described by solutions to the **Landau Equations** [Landau (1959)]

$$\alpha_e(q_e^2 - m_e^2) = 0 \qquad\qquad \sum_{e \in \text{loop}} \alpha_e q_e^\mu = 0$$

- Near a branch points that is approached as some kinematic variable $\varphi \to 0$, the **leading non-analytic behavior** of a Feynman integral is expected to take the form

$$\mathcal{I}(p, \varphi \to 0) \sim C(p)\, \varphi^\gamma \log^\nu \varphi + \ldots$$

All-Mass Example

Consider the class of Feynman integrals **with generic masses in D dimensions**

- Near a branch point that corresponds to an **ℓ-loop Landau diagrams with E internal propagators**, these integrals are expected to behave as [Landau (1959)]

$$\mathcal{I}(p, \varphi \to 0) \sim \begin{cases} C(p)\varphi^{\gamma} \log \varphi & \text{if } \gamma \in \mathbb{Z}, \gamma \geq 0 \\ C(p)\varphi^{\gamma} & \text{otherwise} \end{cases} \qquad \gamma = \frac{\ell D - E - 1}{2}$$

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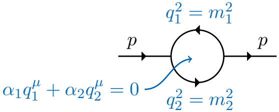
For example, two-particle thresholds and pseudothresholds are associated with the bubble Landau diagram



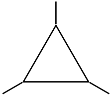
$$\Rightarrow \quad p^2 = (m_1 \pm m_2)^2$$

$$\gamma = (D-3)/2$$

All-Mass Example

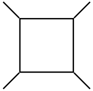
- The branch cuts that develop near the two-particle thresholds of all-mass Feynman integrals in different dimensions thus behave as

$D = 3$



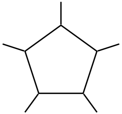
$\sim \log \varphi$

$D = 4$



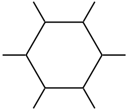
$\sim \varphi^{\frac{1}{2}}$

$D = 5$



$\sim \varphi \log \varphi$

$D = 6$



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$\vdots$

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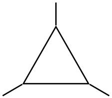
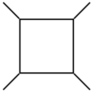
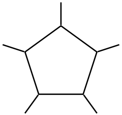
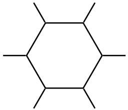
$D = 3$  $\sim \log \varphi$

$D = 4$  $\sim \varphi^{\frac{1}{2}}$

$D = 5$  $\sim \varphi \log \varphi$

$D = 6$  $\sim \varphi^{\frac{3}{2}}$

$\vdots$ $\qquad$ $\vdots$

If we can predict the leading-order behavior of Feynman integrals near a given branch point, what constraints does this put on the symbol of this integral?

General Strategy

Study the order at which non-analytic behavior appears when polylogarithms are expanded around the branch points in their symbol

$$\lim_{\varphi \to 0} (a_1 \otimes \cdots \otimes \varphi \otimes \cdots \otimes a_n) \sim \varphi^p \log^q \varphi$$

Approximate the value of Feynman integrals near their branch points

$$\mathcal{I}(\varphi \to 0) \sim \varphi^\gamma \log^\nu \varphi$$

Compare these expansions to put new constraints on the positions of branch points in the symbols of Feynman integrals

Logarithmic Singularities of Symbols

- We first study symbol terms in which a single letter becomes singular as $\varphi \to 0$:

$$a_1(p) \otimes \cdots \otimes a_{m-1}(p) \otimes \varphi \otimes a_{m+1}(p) \otimes \cdots \otimes a_n(p)$$

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$$\Downarrow$$

$$\int_{0 \le t_1 \le \cdots \le t_n \le 1} \sigma^*\big(d \log a_1(p)\big)(t_1) \cdots \sigma^*\big(d \log \varphi\big)(t_m) \cdots \sigma^*\big(d \log a_n(p)\big)(t_n)$$

where $\sigma_i(t)$ pramaterizes the integration contour in the space of external kinematics

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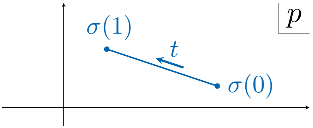
- We can take this integration contour to be

$$\sigma_i(t) = (1-t)p_i^\bullet + tp_i$$

  where $p_i^\bullet$ is an arbitrary integration base point



Logarithmic Singularities of Symbols

- By changing the order we do the integrations, this iterated integral can be written as

$$\int_0^1 U(t)\,\sigma^*(d\log\varphi)(t)\,V(t)$$

where

$$U(t) = \int_0^t \sigma^*(d\log a_1)(t_1)\cdots\int_{t_{m-2}}^t \sigma^*(d\log a_{m-1})(t_{m-1})$$

$$V(t) = \int_t^1 \sigma^*(d\log a_{m+1})(t_{m+1})\cdots\int_{t_{n-1}}^1 \sigma^*(d\log a_n)(t_n)$$

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- If we choose the integration base point $\varphi^\bullet = 1$, we have $\sigma(t) = (1-t)+t\varphi$ and thus

$$\sigma^*(d\log\varphi)(t) = \frac{\varphi-1}{(1-t)+t\varphi}dt$$

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- If we choose the integration base point $\varphi^\bullet = 1$, we have $\sigma(t) = (1-t) + t\varphi$ and thus

$$\sigma^*(d\log\varphi)(t) = \frac{\varphi - 1}{(1-t) + t\varphi}dt$$

Logarithmic Singularities of Symbols

- To find the leading non-analytic behavior of this integral, we expand $U(t)$ and $V(t)$ around $t \to 1$:

$$\int_0^1 U(t)\,\sigma^*(d\log\varphi)(t)\,V(t) \quad \sim \quad U(1)\int_0^1 dt\,\frac{\varphi-1}{(1-t)+t\varphi}\;\overbrace{\frac{(t-1)^{n-m}}{(n-m)!}\left(\frac{d^{n-m}V}{dt^{n-m}}(1)\right)}^{\text{leading-order contribution to } V(t\to 1)} \quad + \quad \dots$$

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- Evaluating this integral and dropping all rational terms, we find

$$\boxed{\frac{U(1)}{(n-m)!}\left(\frac{d^{n-m}V}{dt^{n-m}}(1)\right)\varphi^{n-m}\log\varphi \;+\;\ldots}$$

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Non-analytic contributions are **power-suppressed by the number of letters after φ**:

$$a_1\otimes\cdots\otimes a_{m-1}\otimes\varphi\otimes\underbrace{a_{m+1}\otimes\cdots\otimes a_n}_{n-m}$$

New Constraints on Symbol Letters

We conclude that any polylogarithmic integral with leading behavior

$$\mathcal{I}(p, \varphi \to 0) \;\sim\; \varphi^{\gamma} \log \varphi$$

(i) cannot involve symbol letters that vanish as $\varphi \to 0$ in the last γ entries:

$$\mathcal{S}\big(\mathcal{I}(p,\varphi)\big) = \sum a_1 \otimes \cdots \otimes a_{n-\gamma} \otimes \underbrace{a_{n-\gamma+1} \otimes \cdots \otimes a_n}_{\text{no logarithmic branch points at } \varphi = 0}$$

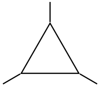
(ii) must have at least one term in which a logarithmic branch point at $\varphi = 0$ appears in the $n-\gamma$ entry (and nowhere else):

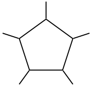
$$\mathcal{S}\big(\mathcal{I}(p,\varphi)\big) = a_1 \otimes \cdots \otimes a_{n-\gamma-1} \otimes \varphi \otimes a_{n-\gamma+1} \otimes \cdots \otimes a_n \; + \; \ldots$$

[Hannesdottir, AJM, Schwartz, Vergu (2021)]

All-Mass Example

We recall that the logarithmic branch cuts in odd-dimensional all-mass Feynman integrals were suppressed by $\varphi^{\frac{D-3}{2}}$ near two-particle thresholds:

$D = 3$  $\sim \log \varphi$

$D = 5$  $\sim \varphi \log \varphi$

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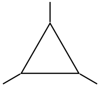
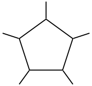
$D = 3$  $\sim \log \varphi$ $\Rightarrow$ φ appears in **last entry** of the symbol

$D = 5$  $\sim \varphi \log \varphi$ $\Rightarrow$ φ appears in **second-to-last entry** of the symbol

$\vdots$ $\qquad$ $\vdots$

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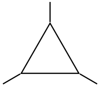
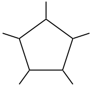
$D = 3$  $\sim \log\varphi$ $\Rightarrow$ φ appears in **last entry** of the symbol

$D = 5$  $\sim \varphi\log\varphi$ $\Rightarrow$ φ appears in **second-to-last entry** of the symbol

$\vdots$ $\vdots$

- The one-loop n-gon symbols in n dimensions are known at one loop for all n [Schläfli (1860)] [Aomoto (1977)] [Davydychev, Delbourgo (1998)]
- We can thus check that our analysis correctly predicts the position of **all logarithmic branch points** that appear in these one-loop symbols

Algebraic Singularities of Symbols

We also saw that all-mass integrals can develop algebraic branch points near two-particle thresholds; can we also constrain these algebraic branch points?

Algebraic Singularities of Symbols

We also saw that all-mass integrals can develop algebraic branch points near two-particle thresholds; can we also constrain these algebraic branch points?

A similar analysis allows us to bound where algebraic branch points at $\sqrt{\varphi}$ can appear

$$a_1(p) \otimes \cdots \otimes a_{m-1}(p) \otimes \left(\frac{b(p) + \sqrt{\varphi}}{b(p) - \sqrt{\varphi}} \right) \otimes a_{m+1}(p) \otimes \cdots \otimes a_n(p)$$

- In this case, the total differential of the singular letter is given by

$$d \log \left(\frac{b(p) + \sqrt{\varphi}}{b(p) - \sqrt{\varphi}} \right) \quad = \quad \frac{b(p)}{b(p)^2 - \varphi} \frac{d\varphi}{\sqrt{\varphi}} - \frac{2\sqrt{\varphi}}{b(p)^2 - \varphi} db(p) \quad \xrightarrow{\varphi \to 0} \quad \frac{dt}{b(p)\sqrt{\varphi}}$$

- Expanding $U(t)$ and $V(t)$ around $t \to 1$ and using $\sigma(t) = (1-t) + t\varphi$, one finds

$$\int_0^1 dt\, U(t) \, \frac{1}{b(p)\sqrt{(1-t) + t\varphi}} V(t) \quad \sim \quad \varphi^{m-n+\frac{1}{2}}$$

New Constraints on Symbol Letters

Similar to before, we conclude that any polylogarithmic integral with leading behavior

$$\mathcal{I}(p, \varphi \to 0) \ \sim \ \varphi^{\gamma}, \qquad \gamma \in \mathbb{Z} + \tfrac{1}{2}$$

(i) cannot involve symbol letters that depend on $\sqrt{\varphi}$ in the last $\gamma - \frac{1}{2}$ entries:

$$\mathcal{S}\big(\mathcal{I}(p,\varphi)\big) = \sum a_1 \otimes \cdots \otimes a_{n-\gamma+\frac{1}{2}} \otimes \underbrace{a_{n-\gamma+\frac{3}{2}} \otimes \cdots \otimes a_n}_{\substack{\text{no algebraic branch} \\ \text{points at } \varphi = 0}}$$

(ii) must have at least one term in which $\sqrt{\varphi}$ appears in the $n - \gamma + \frac{1}{2}$ entry:

$$\mathcal{S}\big(\mathcal{I}(p,\varphi)\big) = a_1 \otimes \cdots \otimes a_{n-\gamma-\frac{1}{2}} \otimes \left(\frac{b+\sqrt{\varphi}}{b-\sqrt{\varphi}}\right) \otimes a_{n-\gamma+\frac{3}{2}} \otimes \cdots \otimes a_n \ + \ \ldots$$

- These predictions are exactly borne out in the one-loop n-gons

[Hannesdottir, AJM, Schwartz, Vergu (forthcoming)]

Singularities of Symbols

More generally, we can analyze symbol terms in which logarithmic or algebraic branch points at $\varphi \to 0$ occur in repeated letters:

$$a_1 \otimes a_2 \otimes \cdots \otimes a_m \otimes \cdots \otimes a_{n-1} \otimes a_n$$

Location of Branch Points	**Leading Non-Analytic Behavior**
$a_m = \varphi$	$\sim \varphi^{n-m} \log \varphi$
$a_{m-r+1} = \cdots = a_m = \varphi$	$\sim \varphi^{n-m} \log^r \varphi$
$a_m = \frac{a+\sqrt{\varphi}}{a-\sqrt{\varphi}}$	$\sim \varphi^{n-m+\frac{1}{2}}$
$a_{m-r+1} = \cdots = a_m = \frac{a+\sqrt{\varphi}}{a-\sqrt{\varphi}}$	$\sim \varphi^{n-m+\frac{1}{2}}$

- This provides a dictionary between the leading behavior of Feynman integrals near their branch points and where these branch points can appear in the symbol

Beyond All-Mass Integrals

While the behavior of multiple polylogarithms near branch points is under good control, predicting the leading behavior of general Feynman integrals near branch points can be subtle

Two general strategies that can be pursued in more general examples:

- Expand around kinematic branch points in dimensional regularization, keeping dependence on ϵ exact [Polkinghorne, Screaton (1960)]
- Perhaps the tropical analysis recently used to understand the leading behavior of UV/IR divergences can be extended to kinematic singularities [Arkani-Hamed, Hillman, Mizera (2022)] (see also Mizera's talk)

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Note, however, that the results already presented go beyond all-mass integrals, and constrain any Feynman integral that can be contracted into an all-mass Landau diagram

Conclusions and Future Directions

In this talk, we presented a new method for deriving constraints on where logarithmic and algebraic branch points can appear in the symbols of polylogarithmic Feynman integrals

Future directions:

- Develop general methods for estimating the leading non-analytic behavior of Feynman integrals near their branch points
- Generalize the analysis of singularities in the symbol to **elliptic polylogarithms**
- Combine these new constraints with constraints on the sequential discontinuities of Feynman integrals to **bootstrap** Feynman integrals directly

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Thanks!

All-Mass Example

As a corollary to our new bounds, we can also put an upper bound on the **transcendental weight** of all-mass Feynman integrals:

- The number of letters that can appear after a given logarithmic branch point is γ
- For fixed D and ℓ, we can maximize γ by making E as small as possible while still requiring $\gamma \in \mathbb{Z}$ and that $E > 0$

$$\gamma \;=\; \frac{\ell D - E - 1}{2} \;\leq\; \left\lfloor \frac{\ell D}{2} \right\rfloor - 1$$

- It follows that the number of symbol letters that can appear in each term is bounded from above by $\left\lfloor \frac{\ell D}{2} \right\rfloor$

This matches the expected maximum transcendental weight