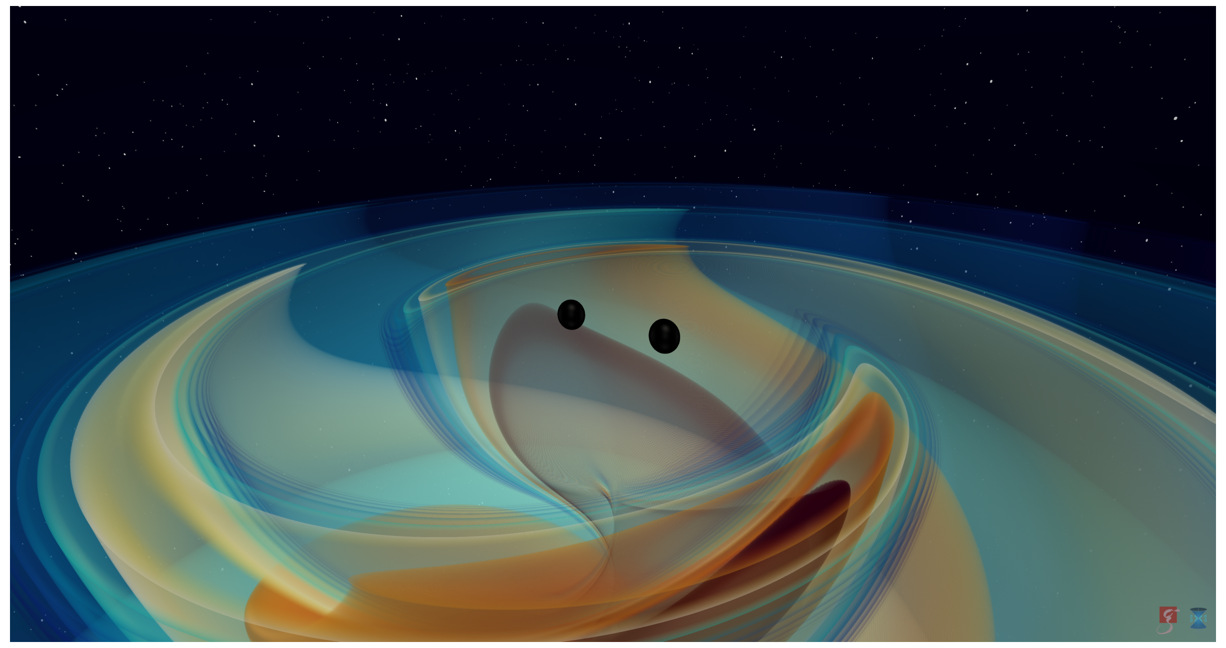

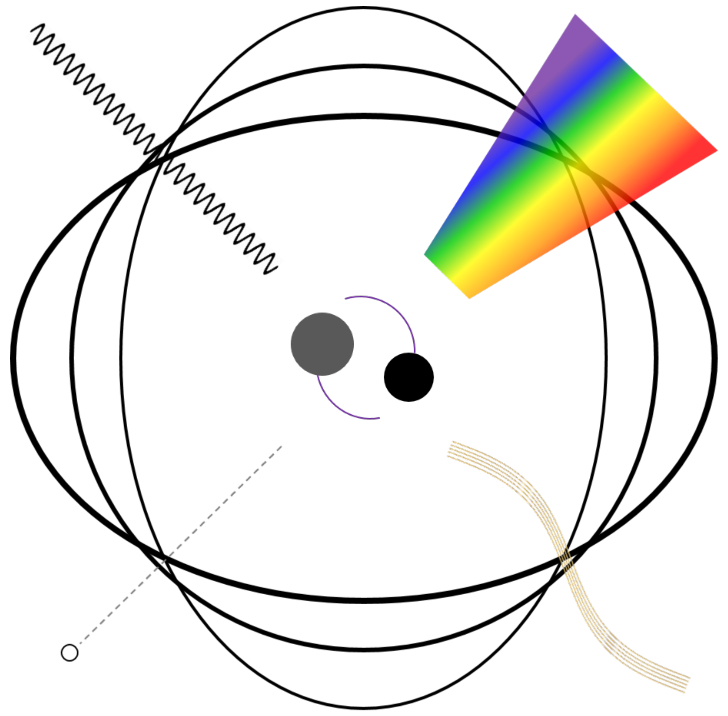
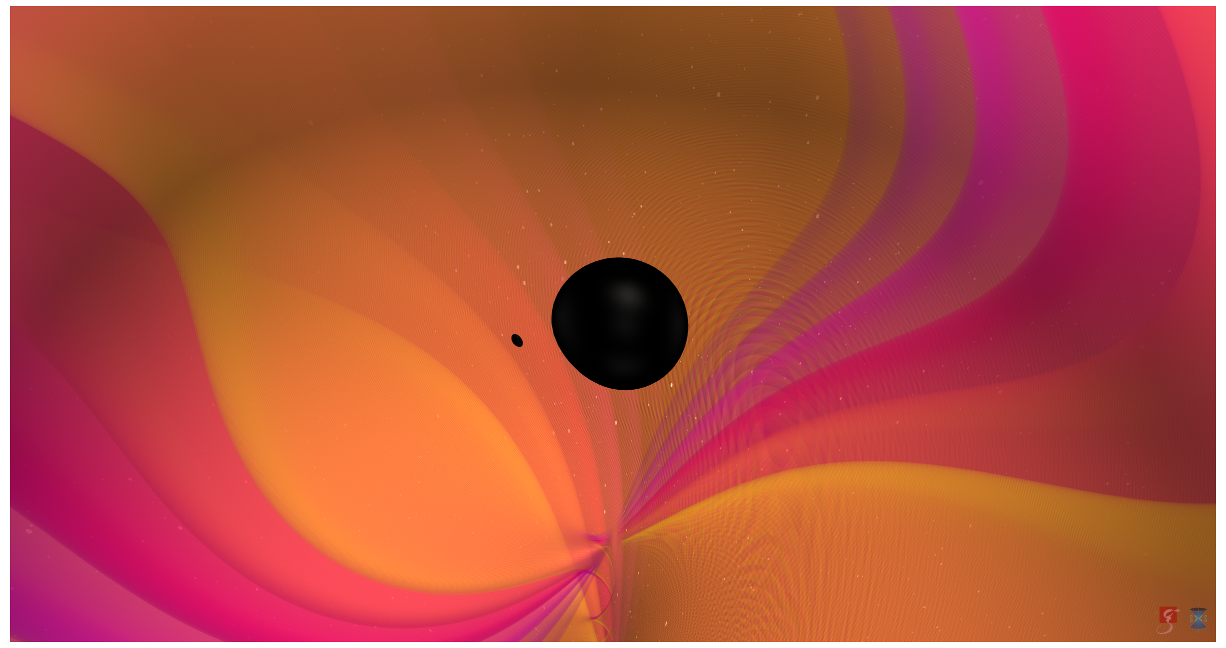

From Scattering Amplitudes to Gravitational-Wave Observations

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DFG Deutsche Forschungsgemeinschaft German Research Foundation

“Amplitudes 2022”, Prague

August 12, 2022

Motivations/Outline

- **Gravitational waves** are the **new tool to explore** the Universe.

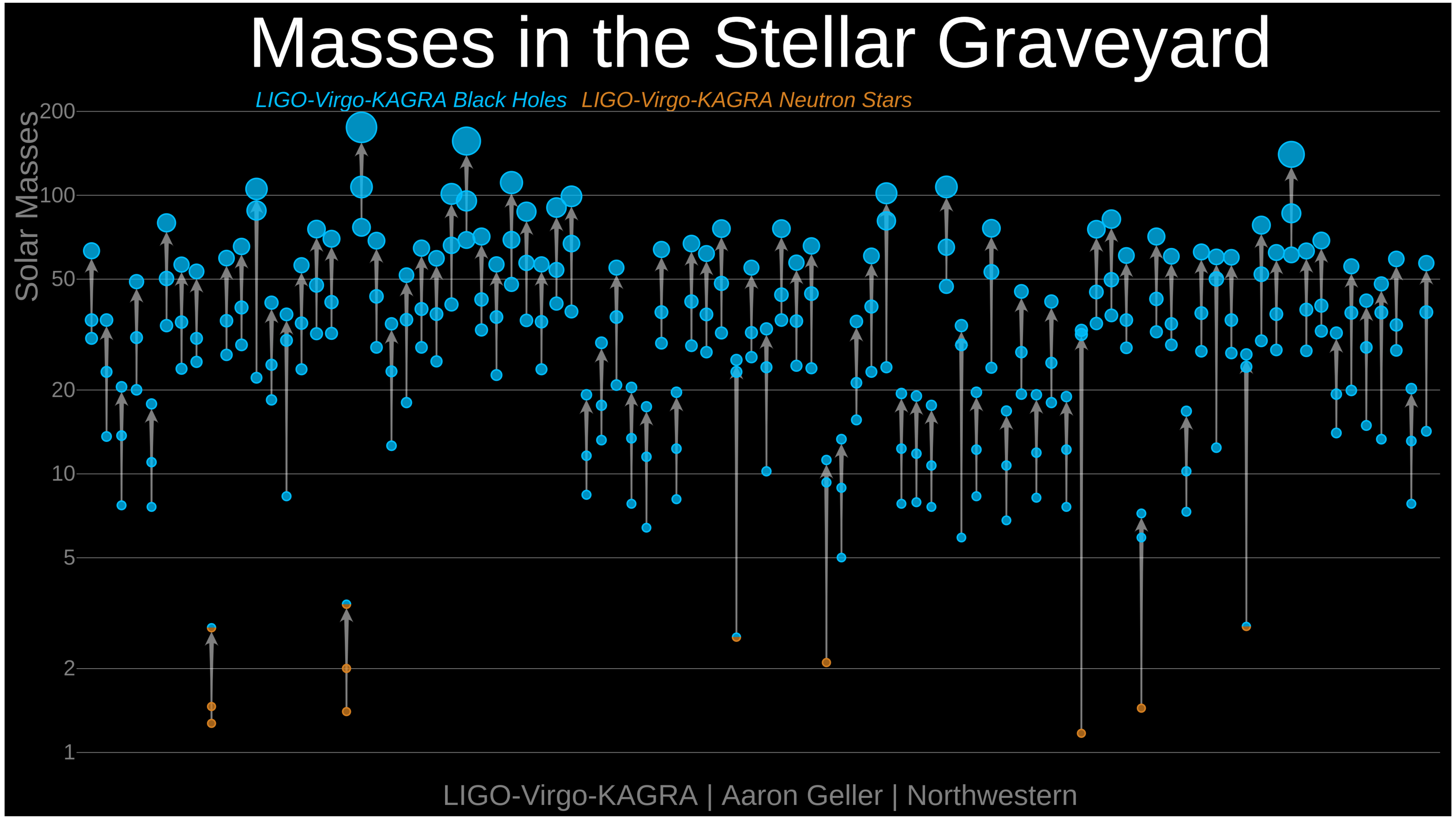
- Inferring **astrophysical and cosmological information** from GW observations, detecting **possible deviations from GR** and **discriminating** them **from astrophysical environmental effects, rely on accurate predictions** of **two-body dynamics** and **gravitational radiation.**

- **Upcoming runs with LIGO-Virgo-KAGRA and future detectors** in space and on the ground, **require ever more accurate and precise** waveform models, which **include *all* physical effects** (spins, tides, eccentricity, beyond-GR effects, non-vacuum GR's effects, etc.).

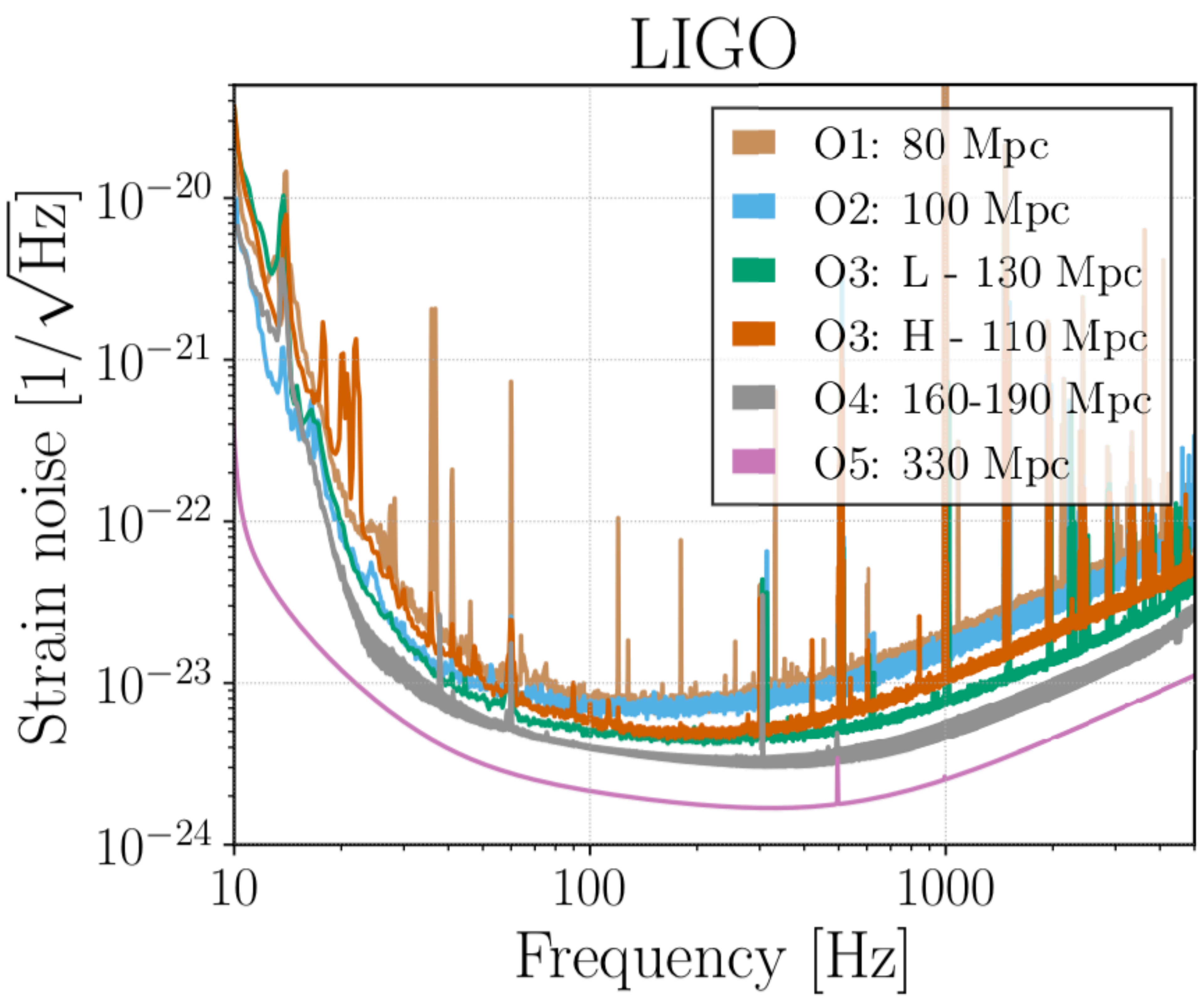
- What does it take to **build faithful waveform models** for the entire coalescence **combining the different analytical** methods with **numerical** relativity, and **how perturbative results from scattering-amplitude** calculations could **be employed to improve waveforms?**

Gravitational Waves Ushered in New Era of AstroPhysics

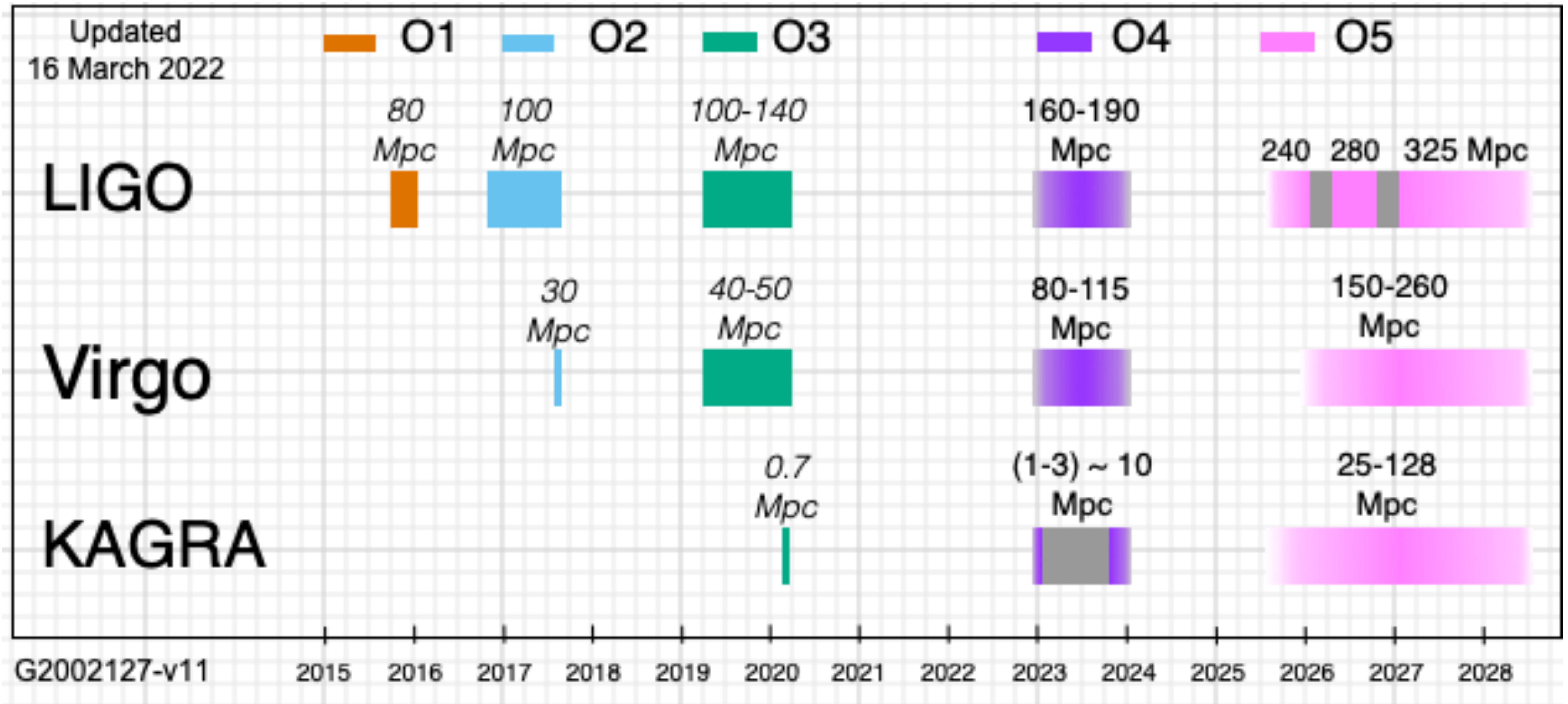
- **Since the discovery** in 2015, LIGO-Virgo have observed, **90 GW events**; the majority are **binary black holes (BBH)**, but also **2 binary neutron stars (BNS) and mixed NSBHs**.



Gravitational-Wave Landscape until ~2030



(update of Aasi et al. Living Rev. Rel. 21, 2020)



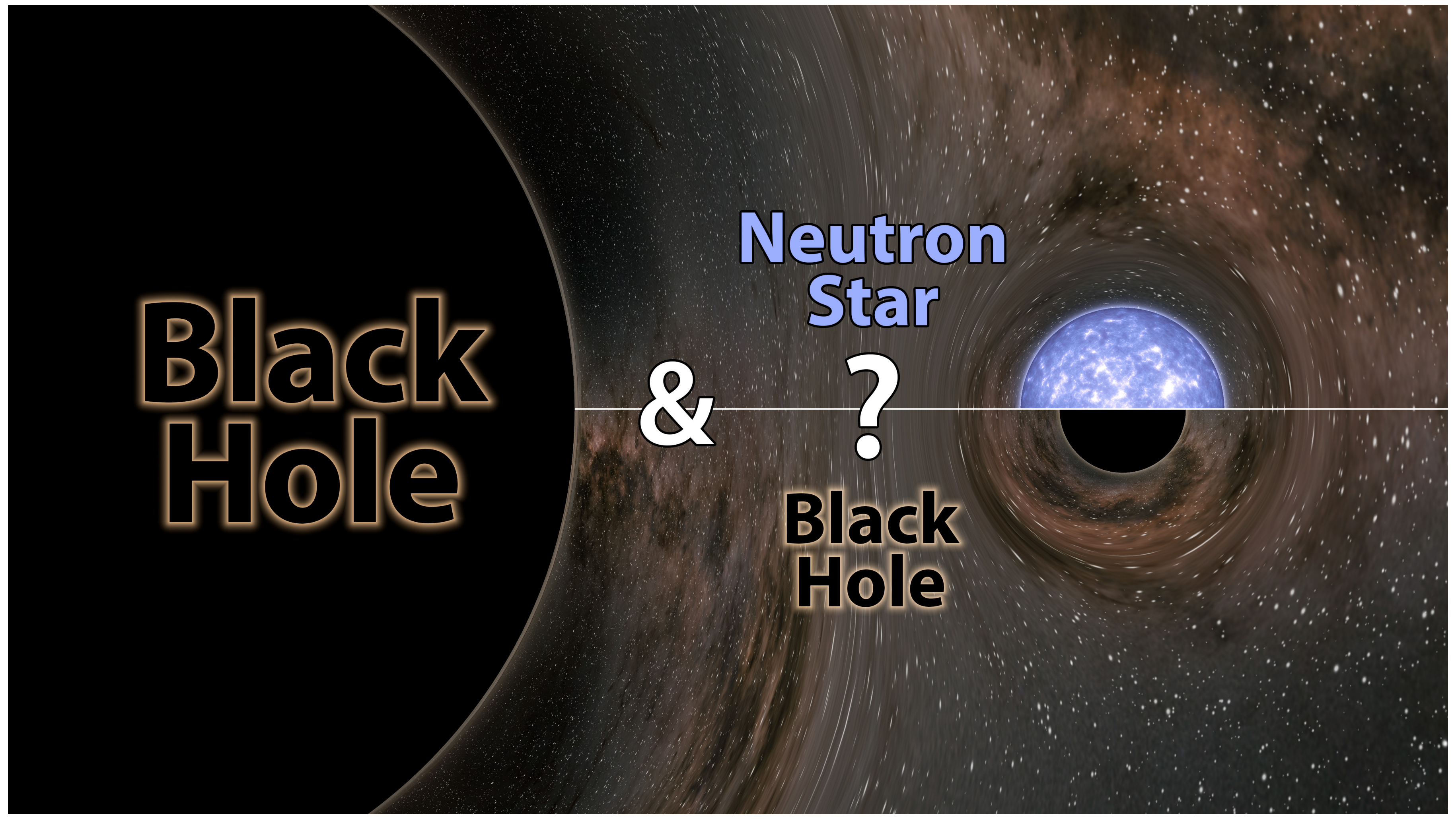
- From **several tens (O3) to hundreds (O4-O5)** of compact binary detections per year.
- Inference of **astrophysical properties** of BBHs, NSBHs and BNSs **still in our local Universe $z \lesssim 1$.**

Some highlights on the science of the last observing run (O3).

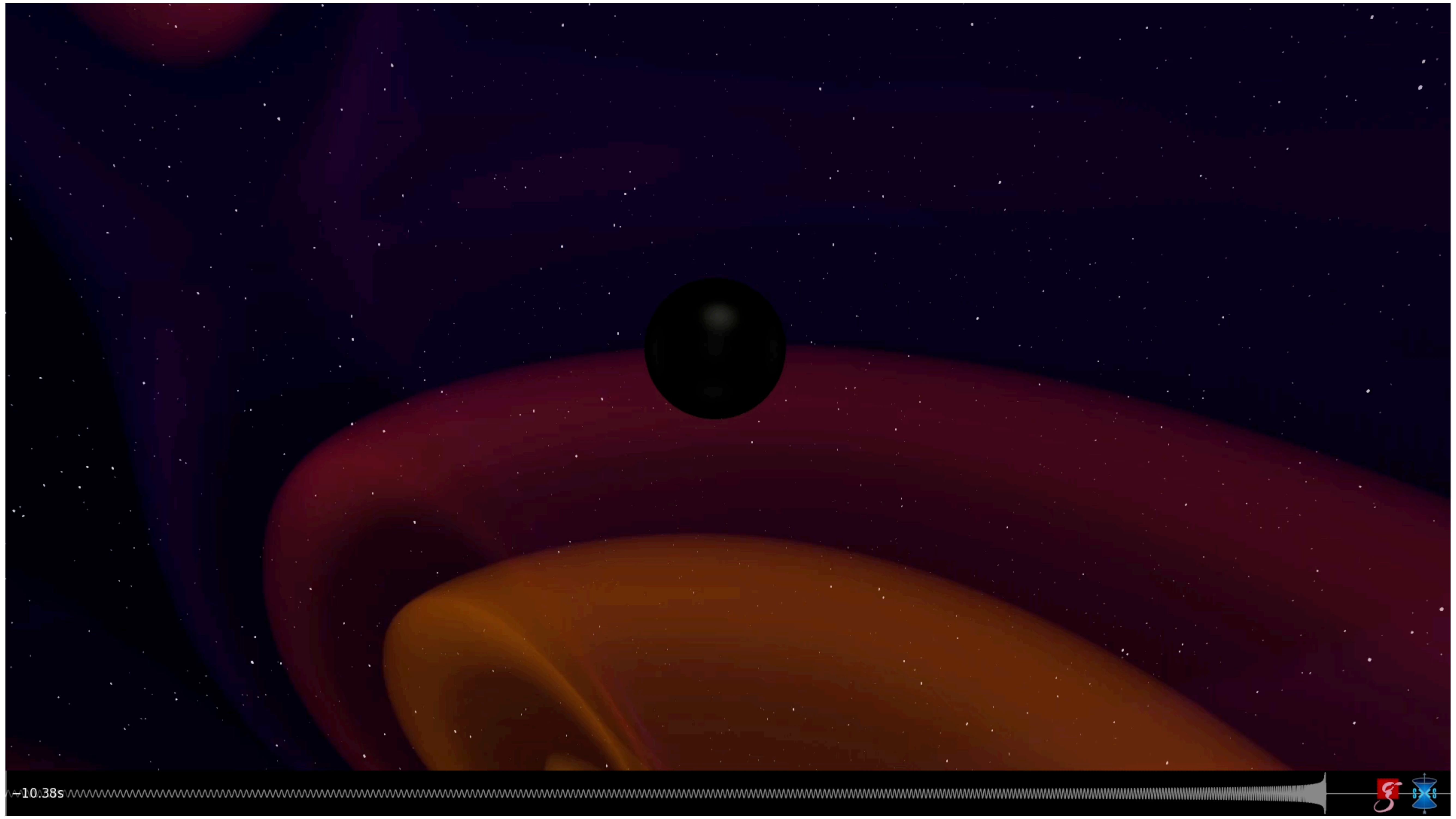
GW190814: a Binary with a Puzzling Companion

GW190814: a binary with a puzzling companion

- A black hole 23 times the mass of our Sun merging with an *object* just 2.6 times the mass of the Sun.



- The more substructure and complexity the binary has (e.g., masses or spins of black holes are different) the richer is the spectrum of radiation emitted: higher harmonics.

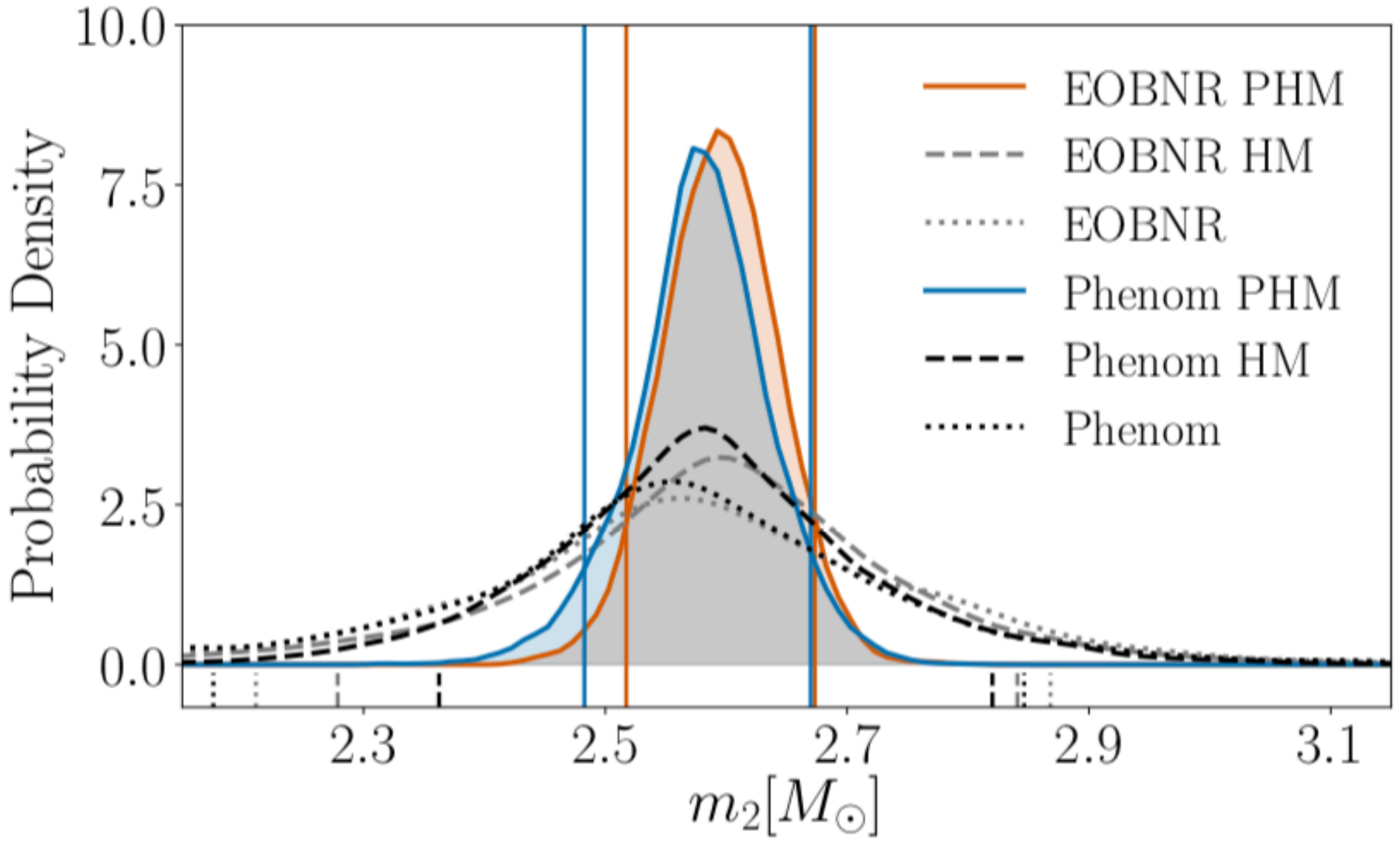


(credit: Fischer, Pfeiffer, Ossokine & AB; SXS project)

GW190814: a Binary with a Puzzling Companion (contd.)

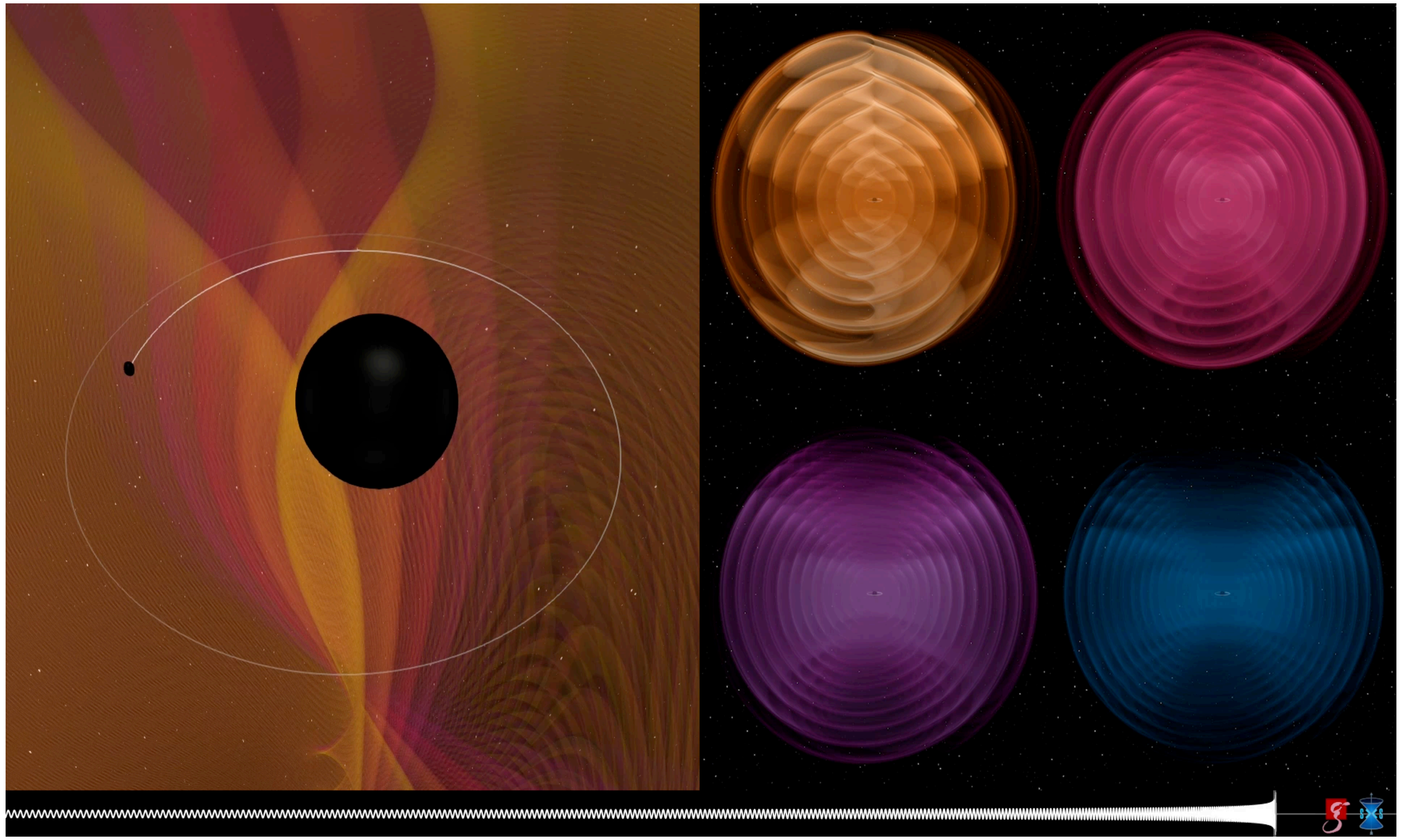
- Either the **largest neutron star** or the **smallest black hole.**

$$m_1 = 23.2^{+1.1}_{-1.0}\, M_\odot \qquad m_2 = 2.59^{+0.08}_{-0.09}\, M_\odot$$



- Using waveform models with **higher-modes and spin-precession constrains more tightly** the **secondary mass.**

- The **more substructure and complexity** the binary has (e.g., masses or spins of black holes are different) **the richer is the spectrum of radiation** emitted: **higher harmonics**.



(credit: Fischer, Pfeiffer, Ossokine & AB; SXS project)

GW190521: a Signal Produced by the Largest BHs

(*Abbott et al. PRL 125 (2020) 10, ApJ Lett 900 (2020) L13*)

- Likely, BHs **too massive** to have been formed **from a collapsed star, because of Pair-Instability SN (high mass gap).**

$$m_1 = 91.4^{+29.3}_{-17.5}\,M_\odot \qquad m_2 = 66.8^{+20.7}_{-20.7}\,M_\odot$$

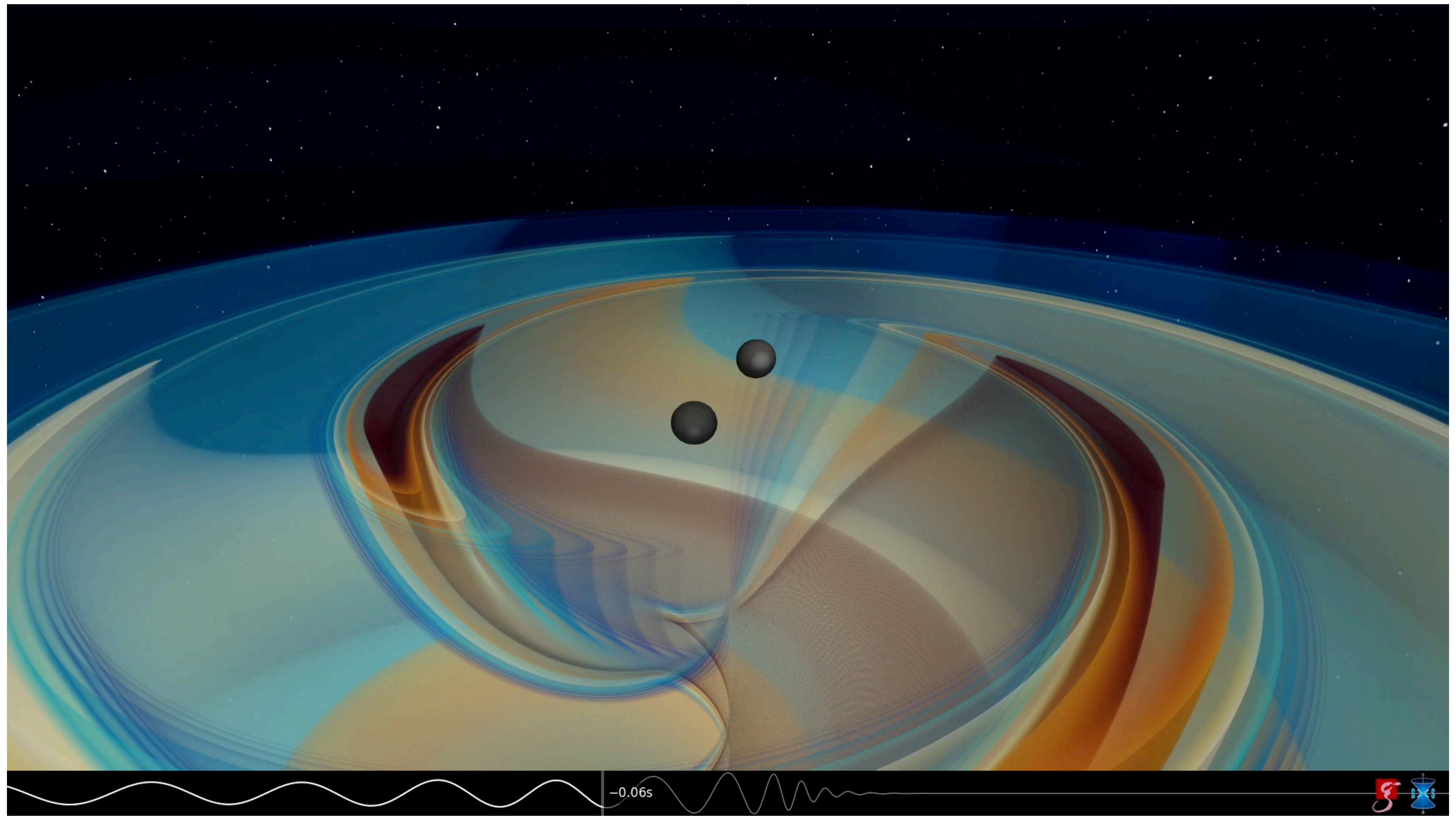
$$q = m_1/m_2$$

$$\chi_1 = S_1/m_1^2$$

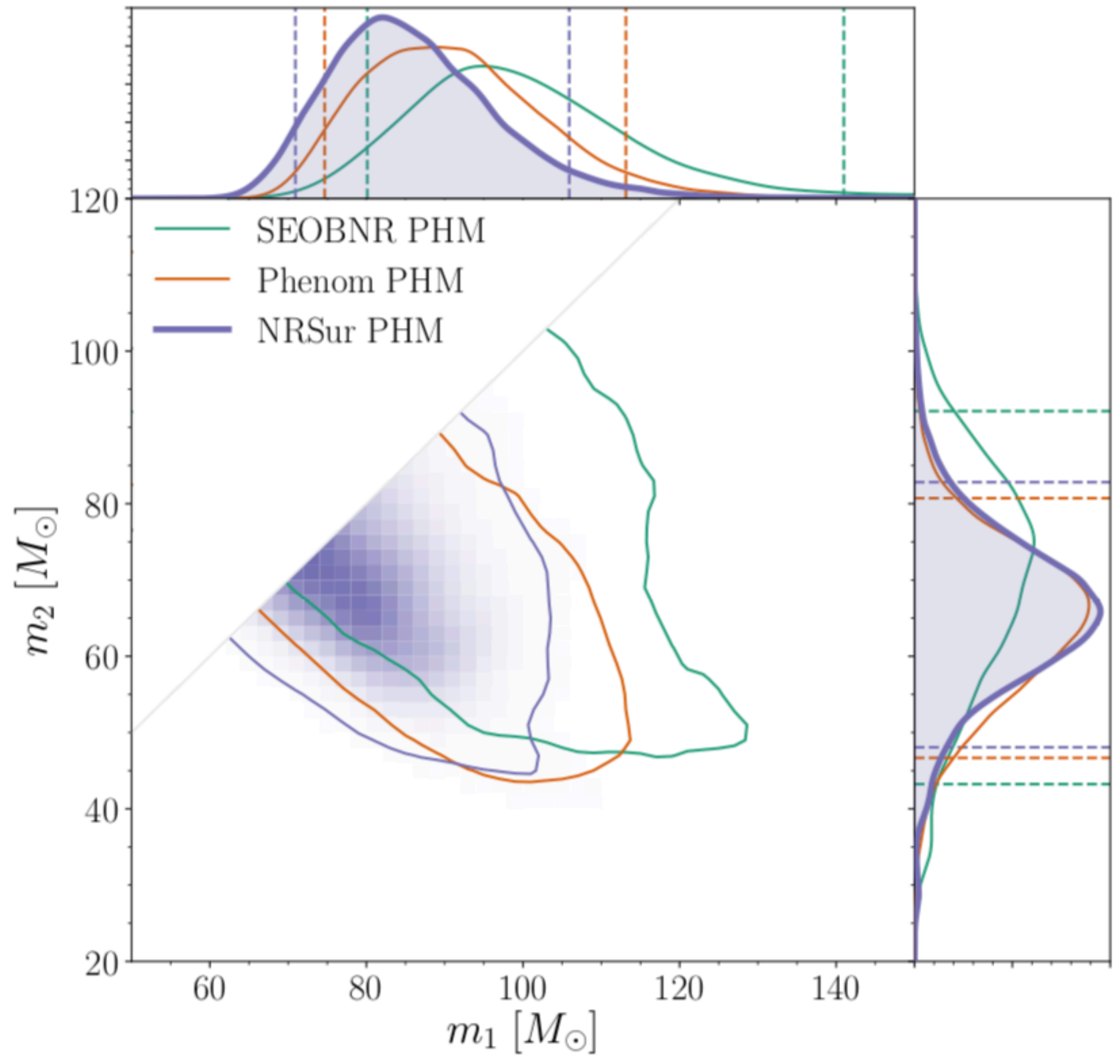
$$\chi_2 = S_2/m_2^2$$

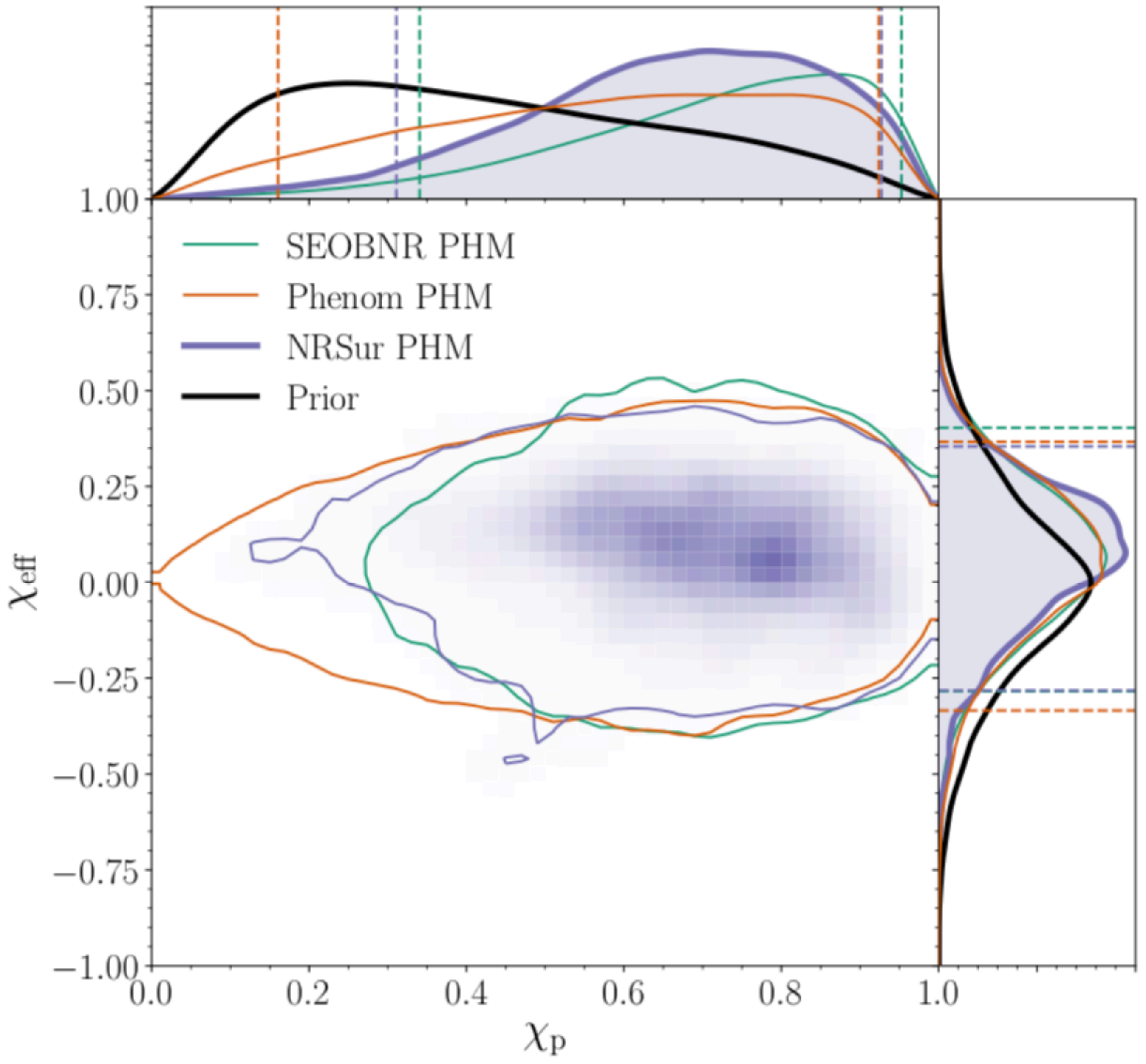
$$\chi_{\rm eff} = \left(\frac{m_1}{M}\chi_1 + \frac{m_2}{M}\chi_2\right)\cdot \hat{\mathbf{L}}$$

χ_p measures the spin components on the orbital plane



(*credit: Fischer, Pfeiffer & AB; SXS Collaboration*)

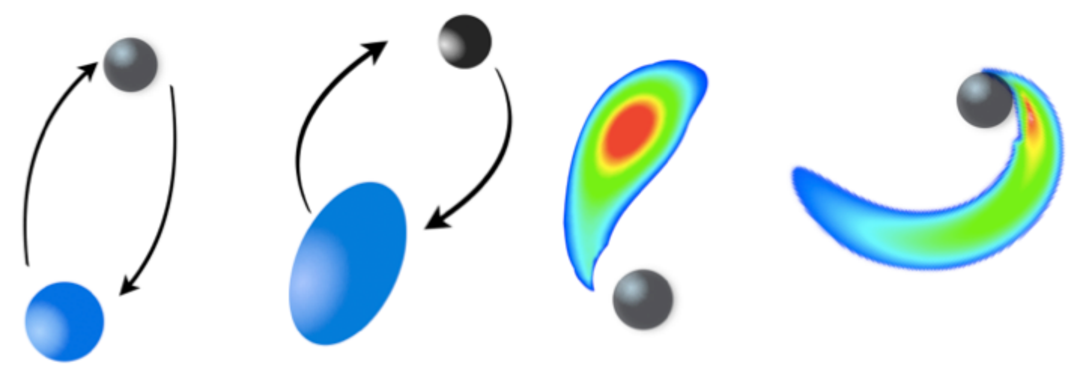




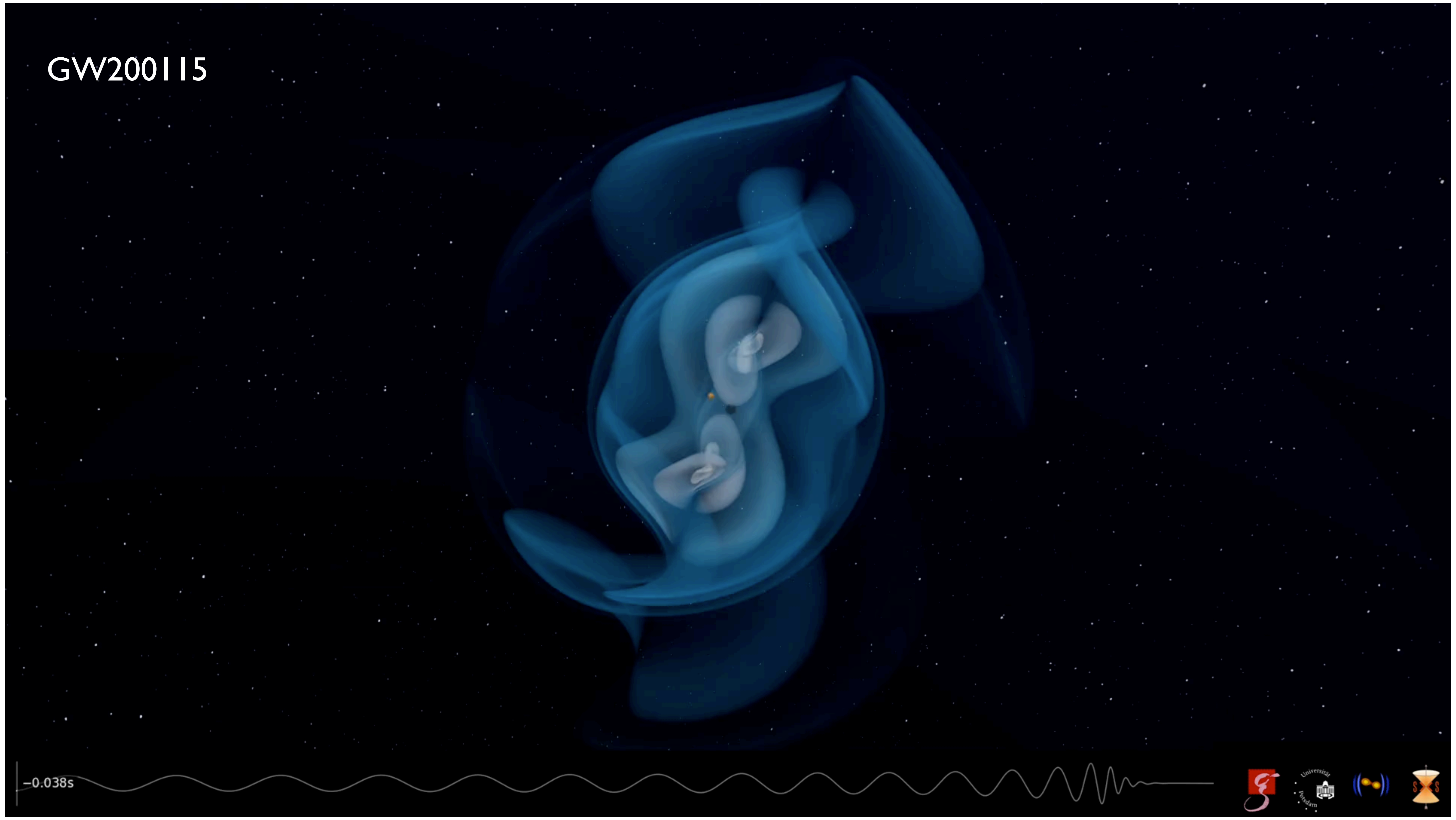
- **Systematics** due to waveform modeling **are not negligible when spin precession** and **higher modes are relevant,** but they are still **subdominant with respect to statistical uncertainty.**

GW200115: a BH swallowing the NS whole

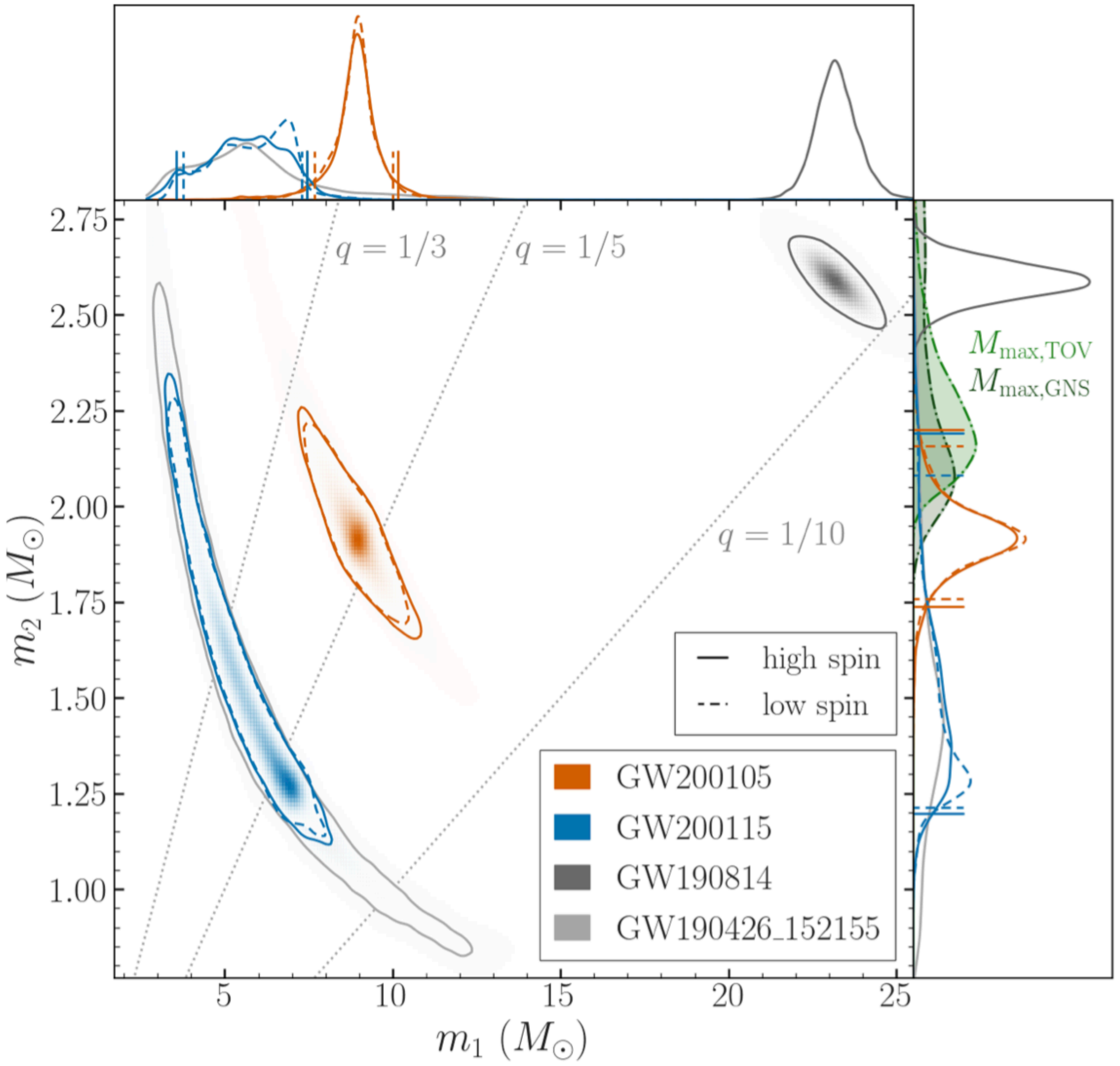
- First **robust detection** of a mixed binary.



(Abbott et al. ApJ 915 (2021))



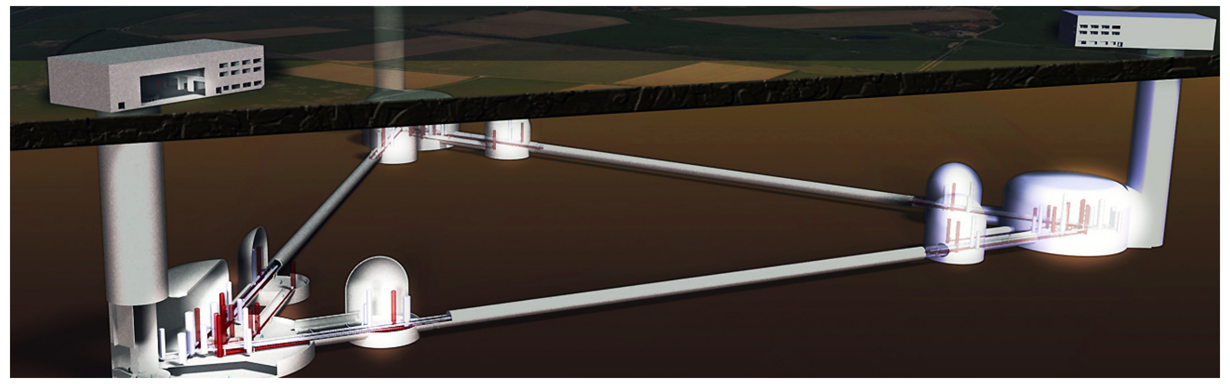
(credit: Chaurasia, Dietrich, Fischer, Ossokine & Pfeiffer)



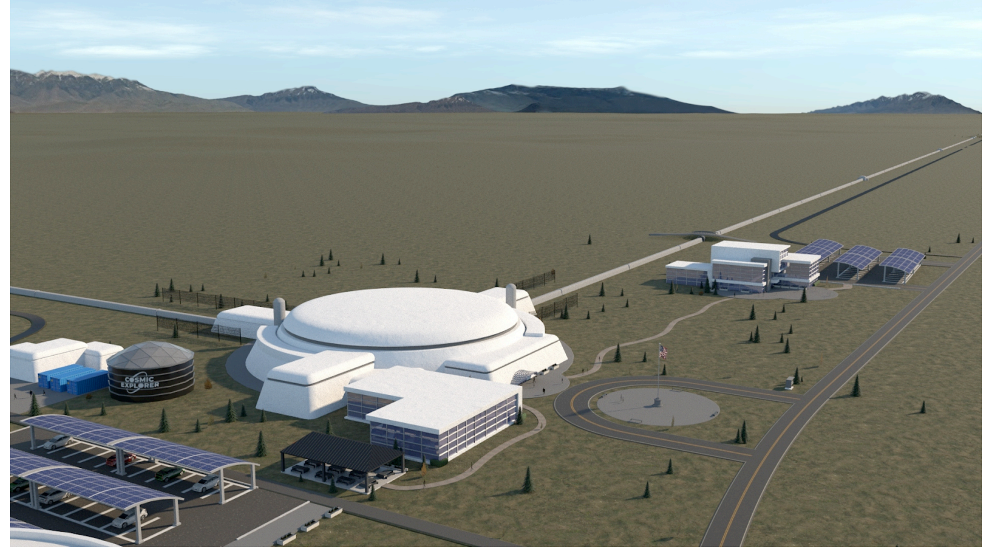
Ever more sensitive detectors in the next decade.

Gravitational-Wave Landscape in late 2030 on the Ground

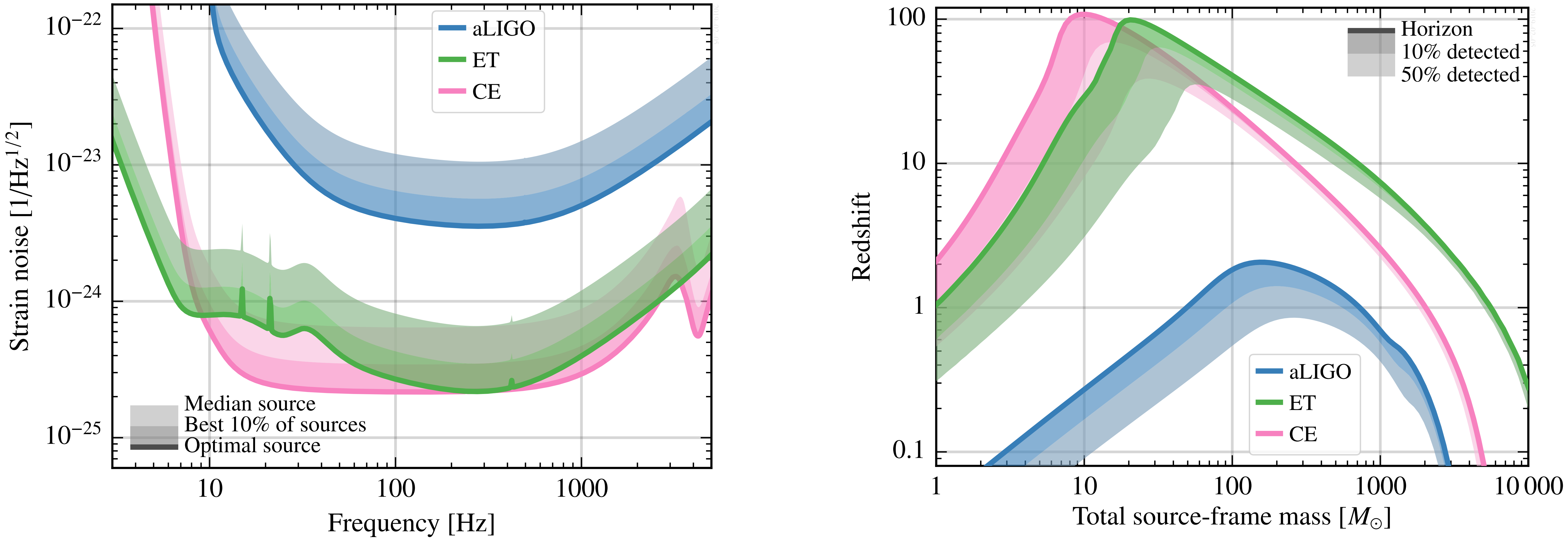
Einstein Telescope



Cosmic Explorer



(Third-Generation Science-Case Report 21)

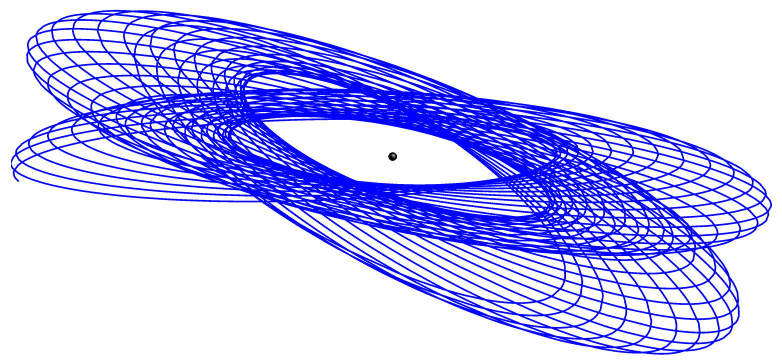


High-frequency detector: NEMO in Australia

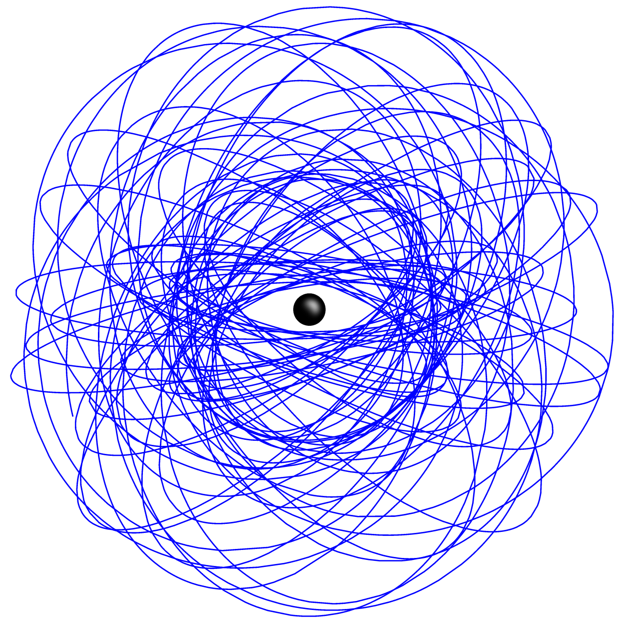
- **Stellar-mass binaries:**
  - Observe each year ~ 30 **BBH signals**, which last for up to 10 minutes, with **SNRs** > 1000 (and $20{,}000$ BBHs with SNRs > 100).
  - Observe each year ~ 10 **BNS signals**, which last several hours, with **SNRs** > 500 (and 780 BNSs with SNRs > 100).

(Borhanian & Sathyaprakash 22)

- **Intermediate Mass-Ratio Inspirals (IMRIs),** with mass ratio 10^3



at GW frequency ~1Hz

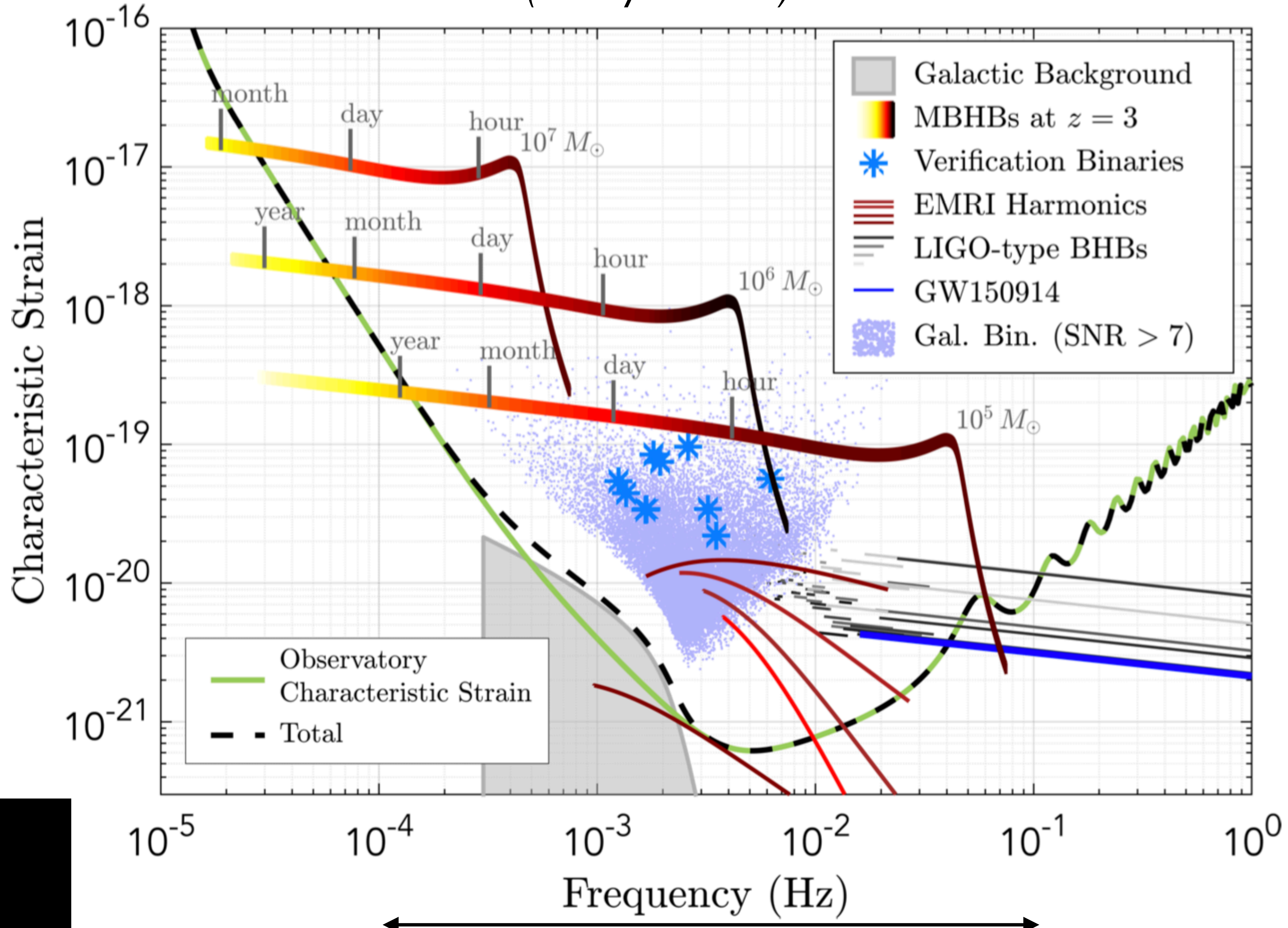


at GW frequency ~10 Hz

(credit: van de Meent)

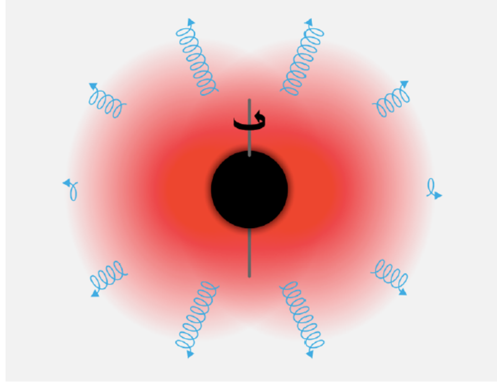
Gravitational-Wave Landscape in late 2030 in Space

(Audley et al. 17)



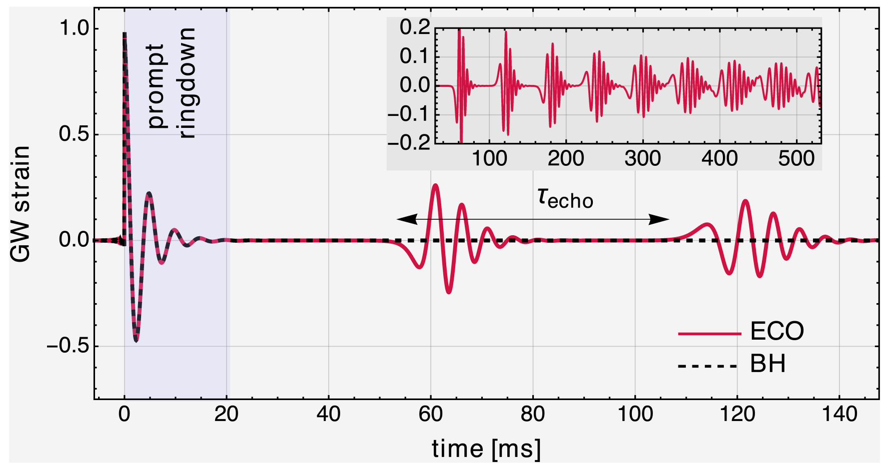
opening **three decades** of GW spectrum

- Are there **new fundamental particles** (axions, ultra-light bosons)?

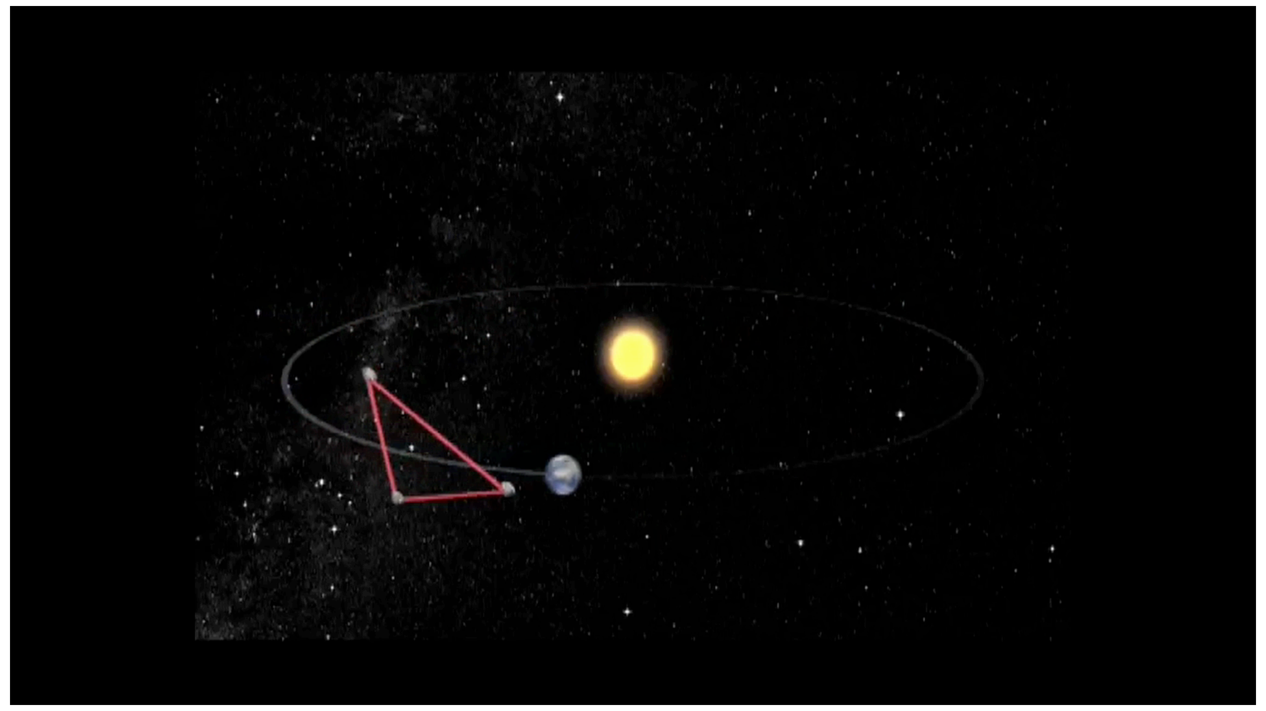


BH with boson cloud

- Do black holes **have an horizon**?



LISA in 2035

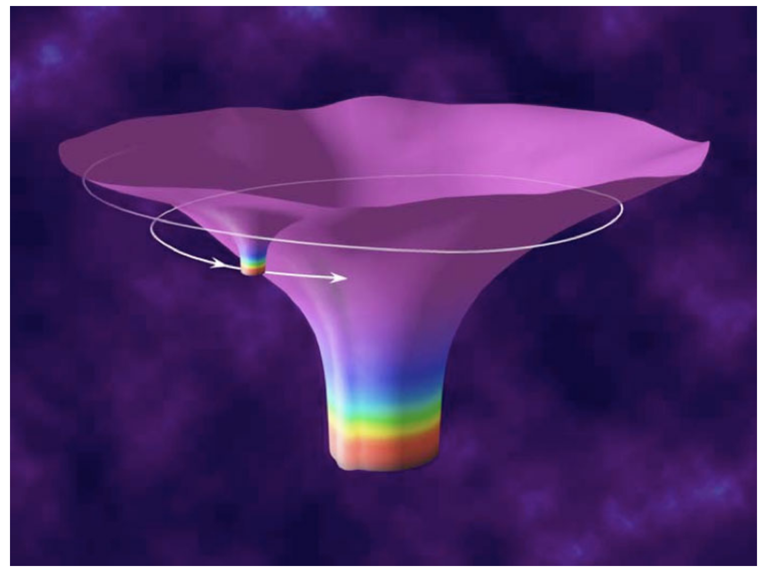


(credit: AEI/Milde Marketing)

ESA leading mission with NASA junior partner

Tianquin (2035+), China

- **New** GW **sources**:
  - extreme mass-ratio inspirals (**EMRIs**)
  - massive BHs (**MBHs**)
  - White-Dwarf binaries in our galaxy

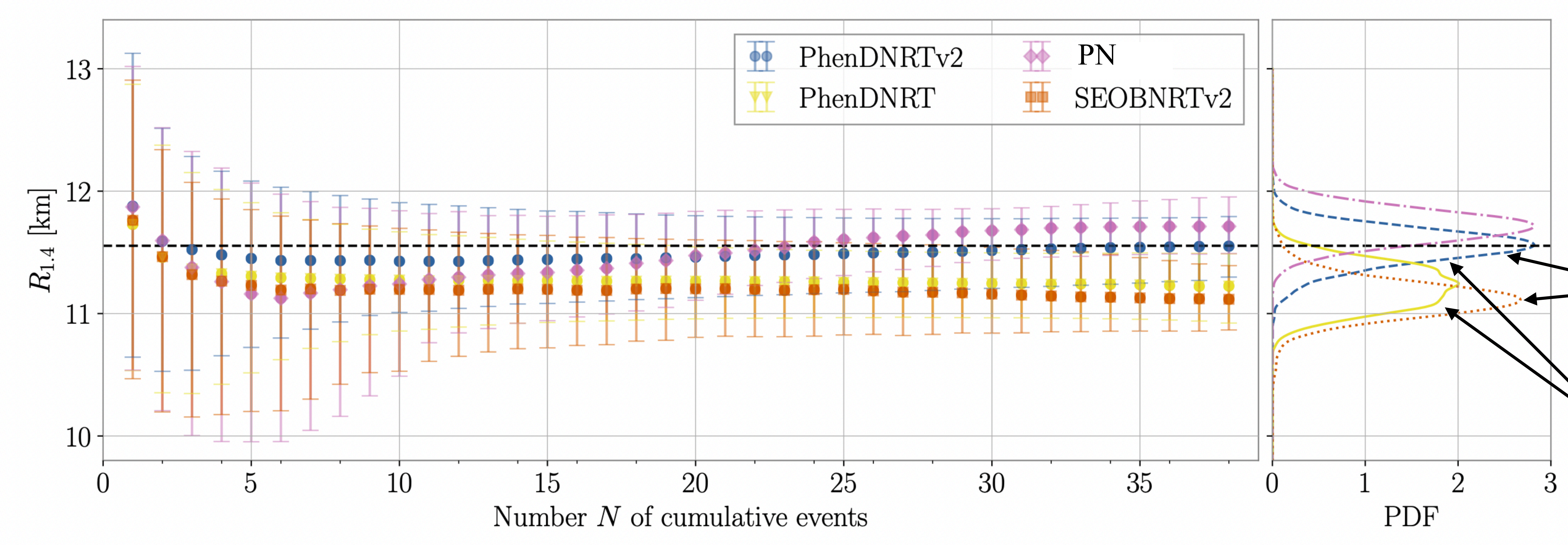


EMRI

To take full advantage of discovery potential with ever more sensitive GW detectors, we need ever more accurate waveform models.

Systematics in Waveform Models with Future Detectors

- "Stacking" events reduces statistical errors, but systematic biases can show up. *(Kunert, Pang, Tews, Coughlin & Dietrich 22)*



LIGO-Virgo O5 run (~2025)

Synthetic signal injected is PhenDNRTv2.

Differences in models only due to BBH baseline model, i.e., tidal terms are identical.

Differences in models only due to tidal corrections.

- With 38 NS detections, statistical uncertainties in NS radius decrease to $\pm 250\,\mathrm{m}$ ($2\,\%$ at $90\,\%$ CI) but systematic differences between current waveform models can be twice as large.

(see also Purrer & Halster 19, Huang et al. 20, Gamba et al. 21)

- Crucial to make BBH model more accurate. Tidal corrections also need to be improved.

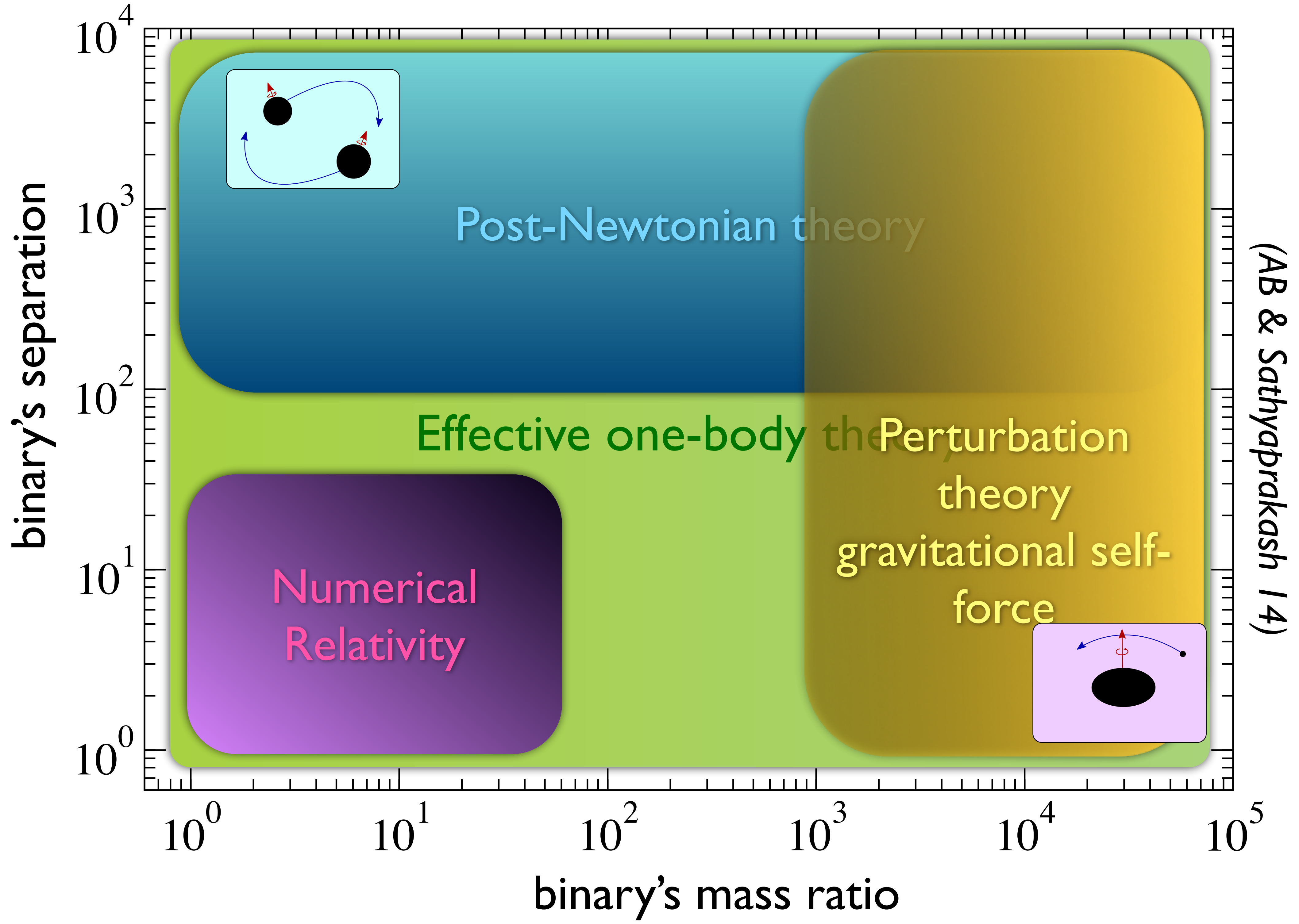
Solving Two-Body Problem in General Relativity

- **GR** is **non-linear theory**. $$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- Einstein's field equations can be solved:

-**approximately,** but **analytically** (**fast** way)

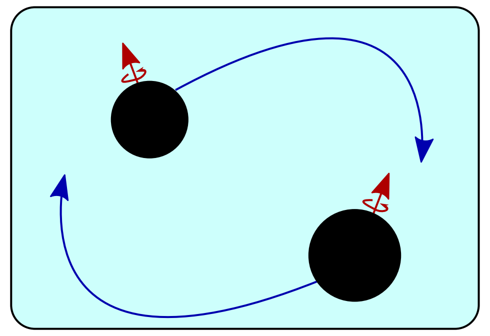
-**accurately**, but **numerically** on supercomputers (**slow** way)



- **Synergy** between **analytical** and **numerical relativity** is **crucial** to **provide GW detectors with templates** to use for **searches** and **inference analyses**.

- **Post-Newtonian** (large separation, and slow motion)

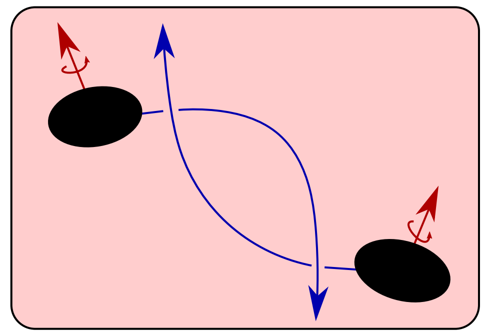
expansion in $v^2/c^2 \sim GM/rc^2$



(Droste, Lorentz, Einstein, Infeld, Hoffmann, … Blanchet, Damour, Iyer, Jaranowski, Schäfer, Will, … Goldberger, Porto, Rothstein, …)

- **Post-Minkowskian** (large separation, and fast motion)

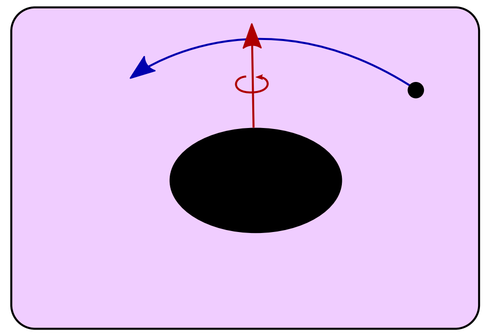
expansion in G



(Westpfahl, … Bern, Cheung, Hermmann, Parra-Martinez, Rothstein, Solon, Shen, Zeng … Khälin, Porto, … Mogull, Jakobsen, Plefka, Steinhoff …)

- **Gravitational self-force**

expansion in m_2/m_1



(Barack, Deitweiler, Mino, Poisson, Pound, Quinn, Sasaki, Tanaka, van de Meent, Wald, Warburton, Wardell, Whiting, …)

Solving Two-Body Problem in General Relativity

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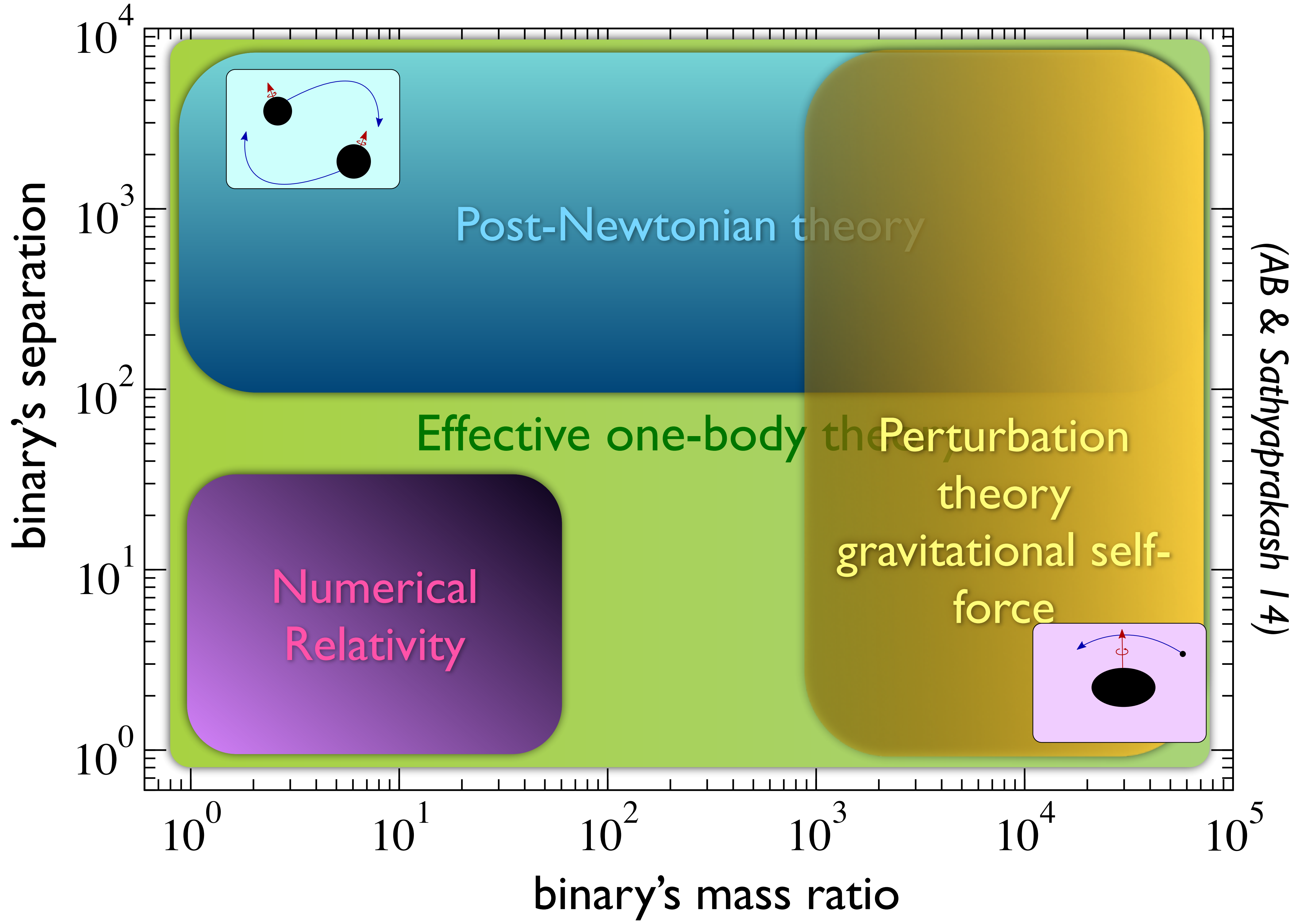
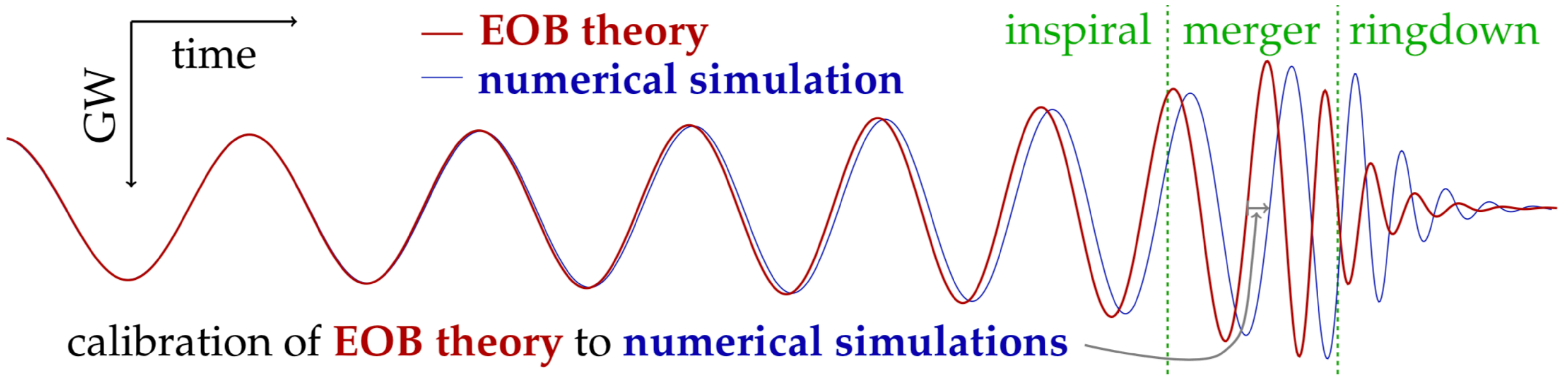


- **Synergy** between **analytical** and **numerical relativity** is **crucial** to **provide GW detectors with templates** to use for **searches** and **inference analyses**.

- **Effective-one-body** (EOB) approach **combines results** from all analytic methods, and **can be made** highly **accurate via numerical relativity.**

(AB, Damour, … Barausse, Bohé, Cotesta, Khalil, Ossokine, Pan, Pompili, Buades-Ramos, Shao, Taracchini, … Nagar, Bernuzzi, Agathos, Gamba, Messina, Rettegno, Riemenschneider,…. Iyer, Jaranowski, Schäfer)

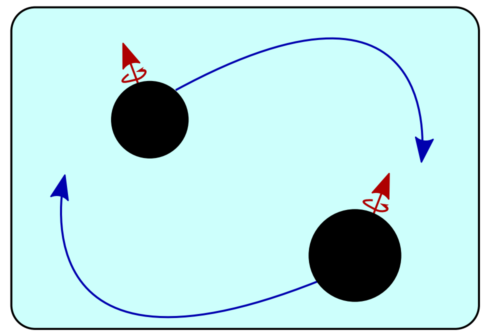


Toward High-Precision Gravitational Waves

- **Post-Newtonian, PN** (large separation, and slow motion)

  expansion in

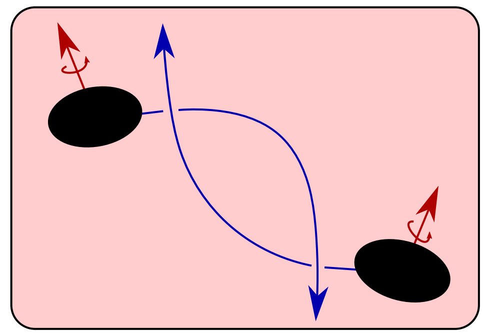
  $$v^2/c^2 \sim GM/rc^2$$



- **Post-Minkowskian, PM** (large separation, and fast motion)

  expansion in

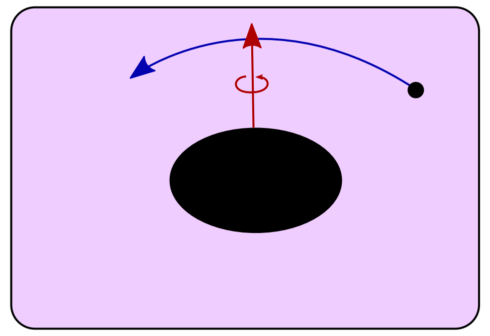
  $$G$$



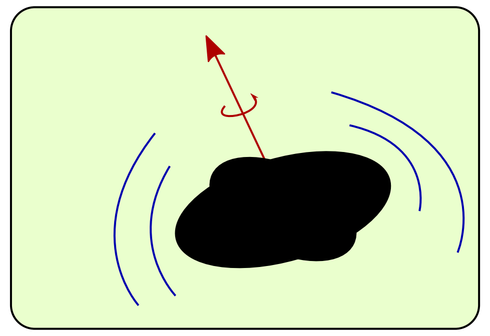
- **Small mass-ratio/ gravitational-self force, GSF**

  expansion in

  $$m_2/m_1$$



- **Perturbation theory** (e.g., **ringdown** of final object)



- **Numerical relativity**

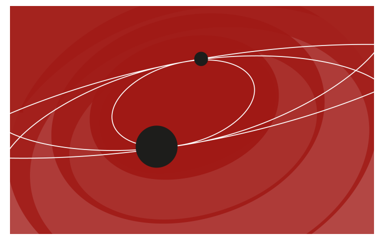
- **Waveform accuracy** would need to be **improved by two or more orders of magnitude** depending on the parameter space.

  (e.g., Pürrer & Halster 19)

PN order	0	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	need up to
no spin	N	1PN		2PN		3PN		4PN		5PN		6PN		1PM / tree
spin-orbit			LO SO		NLO SO		N2LO SO		N3LO SO		N4O SO			2PM / 1-loop
spin^2				LO S2		NLO S2		N2LO S2		N3LO S2				3PM / 2-loop
spin^3							LO S3		NLO S3					4PM / 3-loop
spin^4								LO S4		NLO S4				5PM / 4-loop
spin^5											LO S5		NLO S5	6PM / 5-loop
spin^6												LO S6		7PM / 6-loop

(credit:Vines)

- **Combine** PN, PM, GSF, perturbation theory in EOB **more effectively and in novel ways** to largely **improve analytical solution** of 2-body problem.
- **Enable numerical-relativity codes** to produce longer and more accurate waveforms, especially for **extreme parameters** (large mass ratios, spins, eccentricity)
- **Re-think at strategies to solve 2-body problem** in GR and beyond. **Unify** description of **bound and unbound** orbits.



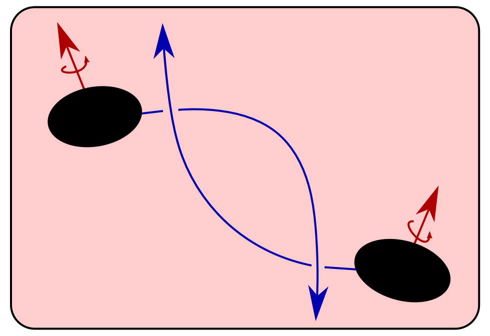
Toward Improving Waveform Accuracy: PM

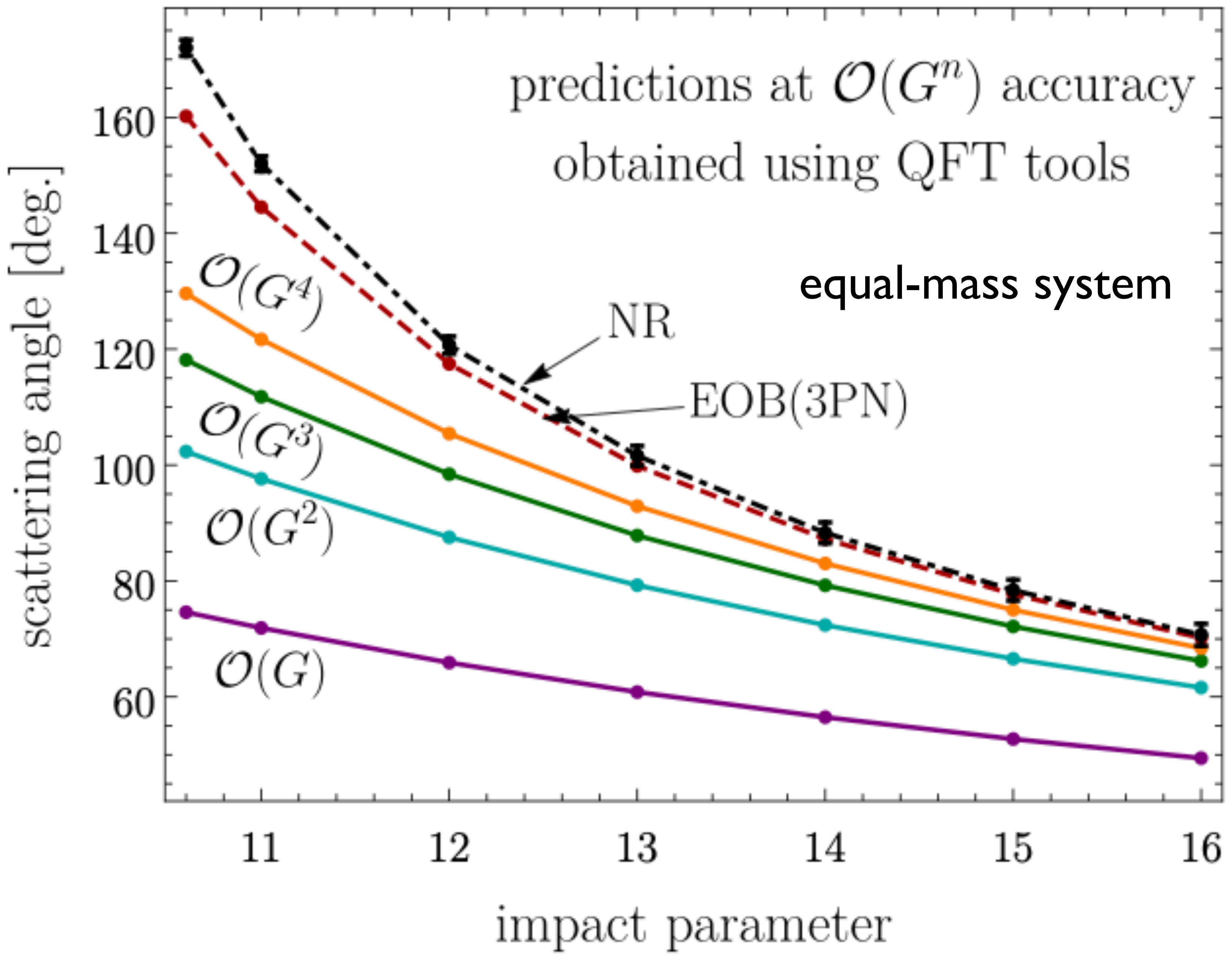
- **Conservative dynamics derived through 3PM**, it is local and valid for **generic** orbits.

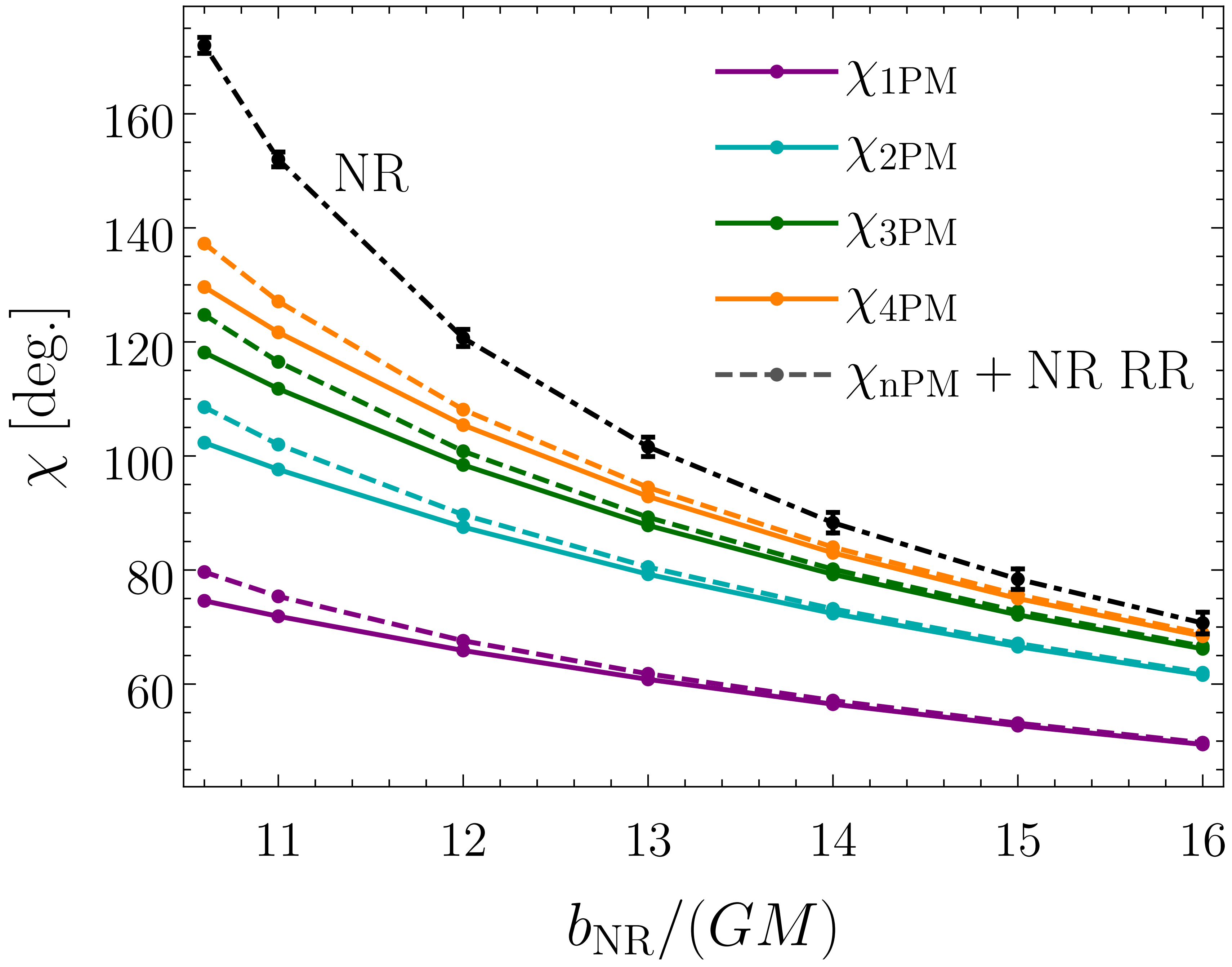
  (Cheung, Rothstein & Solon 19; Bern et al. 19; Blümlein et al. 20; Kälin, Liu & Porto 20; Cheung & Solon 20)

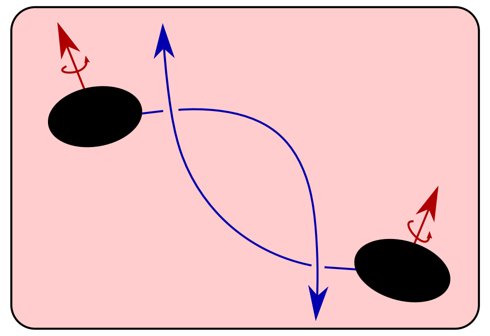
- **Conservative dynamics derived at 4PM** with non-local part for *hyperbolic* orbits.

  (Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, & Zeng 21; Dlapa, Kälin & Liu 21)









(Khalil, AB, Steinhoff & Vines 22; AB, Khalil, O'Connell, Roiban, Solon & Zeng 22)

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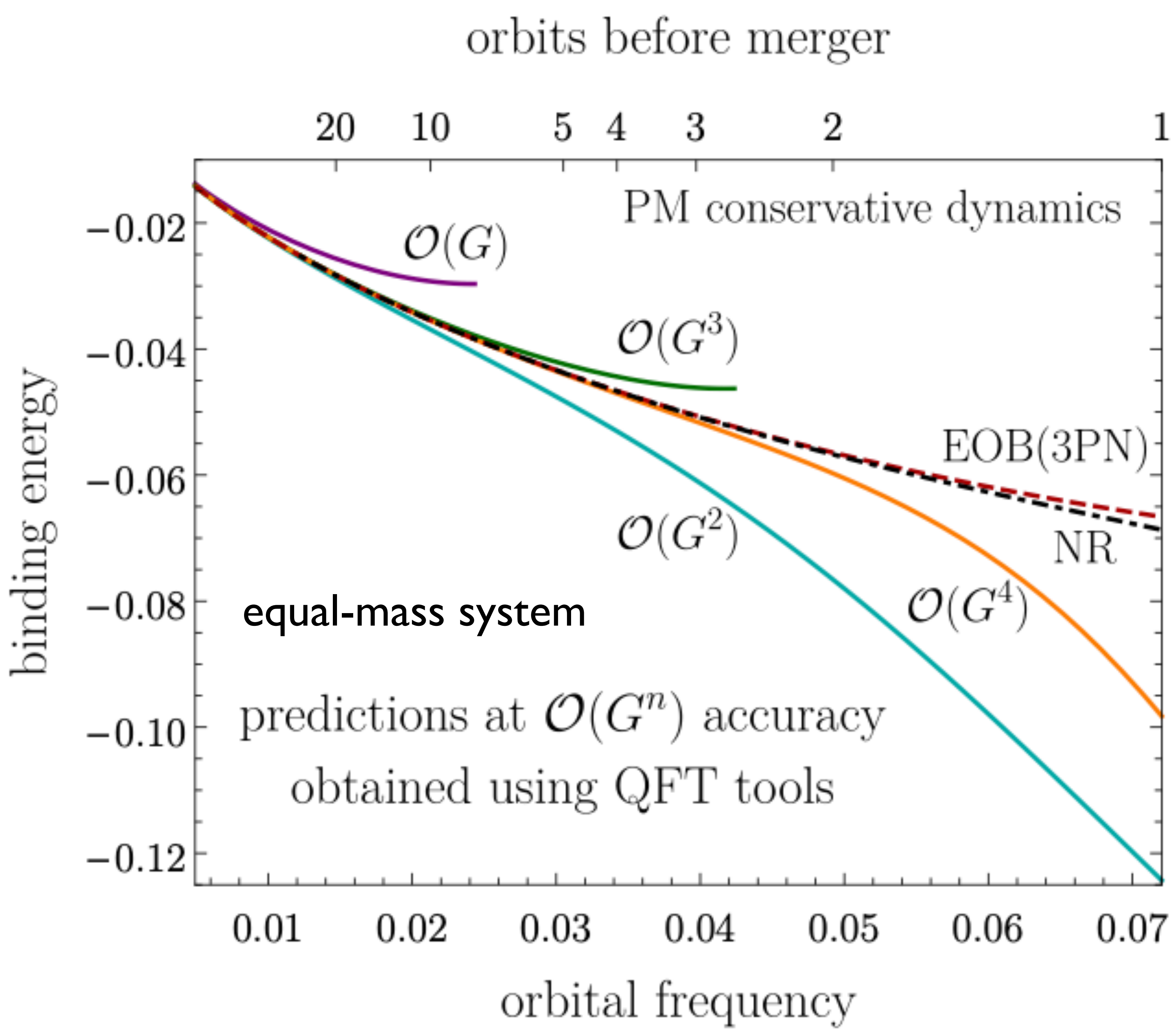
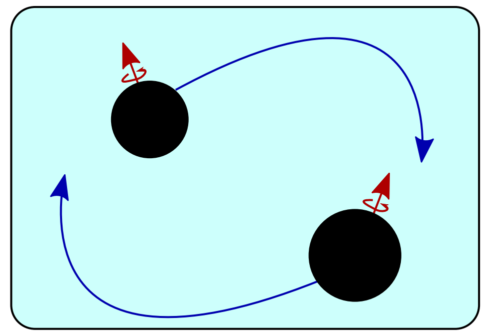


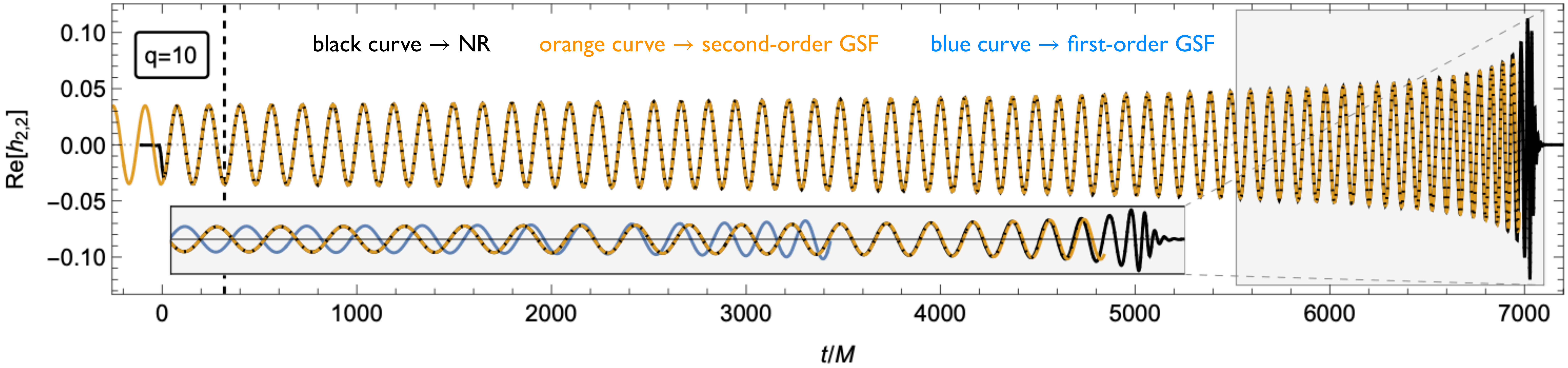


(Khalil, AB, Steinhoff & Vines 22; AB, Khalil, O'Connell, Roiban, Solon & Zeng 22)

Toward Improving Waveform Accuracy: GSF

- For **nonspinning binaries** in **quasi-circular orbits**, GSF effects **at second order in mass ratio** (all order in velocities, strong field) have been computed. *(Pound, Wardell, Warburton, Miller 20)*

- Although **GSF approximation** is **designed for** cases in which **mass ratio** is **extreme**, it also **performs remarkably well for more comparable mass ratios** including $1:10$.



(Wardell, Pound, Warburton, Miller, Durkan & Le Tiec 21)

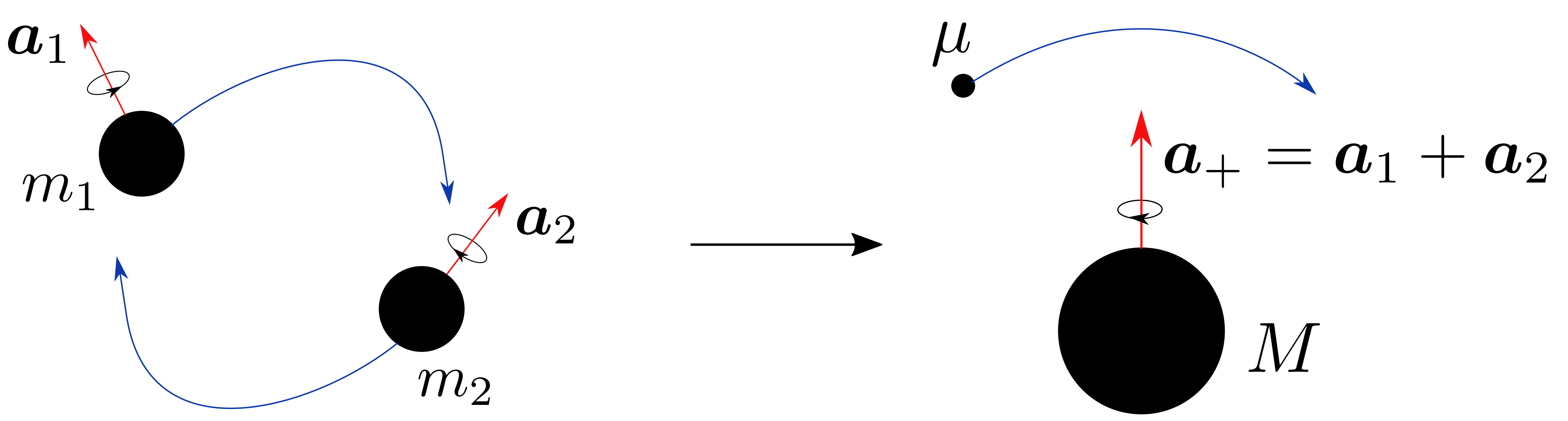
How to take advantage of new results in PN, GSF, PM, …

EOB Hamiltonian for Spinning Bodies

$\mu = m_1 m_2/M$

$M = m_1 + m_2$

$\nu = \mu/M \qquad 0 \le \nu \le 1/4$



(credit: Khalil)

$$\mathbf{a}_i = \frac{m_i}{M}\chi_i \quad i = 1{,}2$$

$$0 \le \chi_i \le 1$$

$$H_{\rm real}^{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}}{\mu} - 1\right)}$$

(AB & Damour 99; Damour 00; AB, Chen & Damour 05; Damour, Jaranowski & Schafer 08; Barausse, Racine & AB 10; Barausse & AB 11; Damour & Nagar 14; Balmelli & Damour 15; Khalil, Steinhoff, Vines & AB 20)

$$H^{\rm eff} = H^{\rm eff}_{\rm odd} + H^{\rm eff}_{\rm even}$$

odd (even) powers in BH's spin

restricted to aligned-spin, equatorial orbits

(Khalil, Steinhoff, Vines & AB 20) **@4PN order**

$a_+ = a_1 + a_2 \qquad a_- = a_1 - a_2 \qquad \delta = \sqrt{1 - 4\nu}$

$$H^{\rm eff}_{\rm even} = \sqrt{A(r, a_6)\left[\mu + p_r^2\,(1 + B_{np}(r)) + \frac{p_\phi^2}{r^2}(1 + a_+^2\,B_{npa}(r)) + Q(r, p_r)\right]}$$

$$H^{\rm eff}_{\rm odd} = \frac{\mu\, p_\phi}{a_+^2(r+2) + r^3}\left\{\nu\left[G_a^{a_+}(r, p_\phi; d_{\rm SO})\, a_+ + G_a^{a_-}(r, p_\phi)\, a_-\right] - \frac{a_+^2}{4r^2}\left(a_+ - a_-\delta\right)\right\}$$

resummation of Hamiltonian

gyro-gravitomagnetic functions

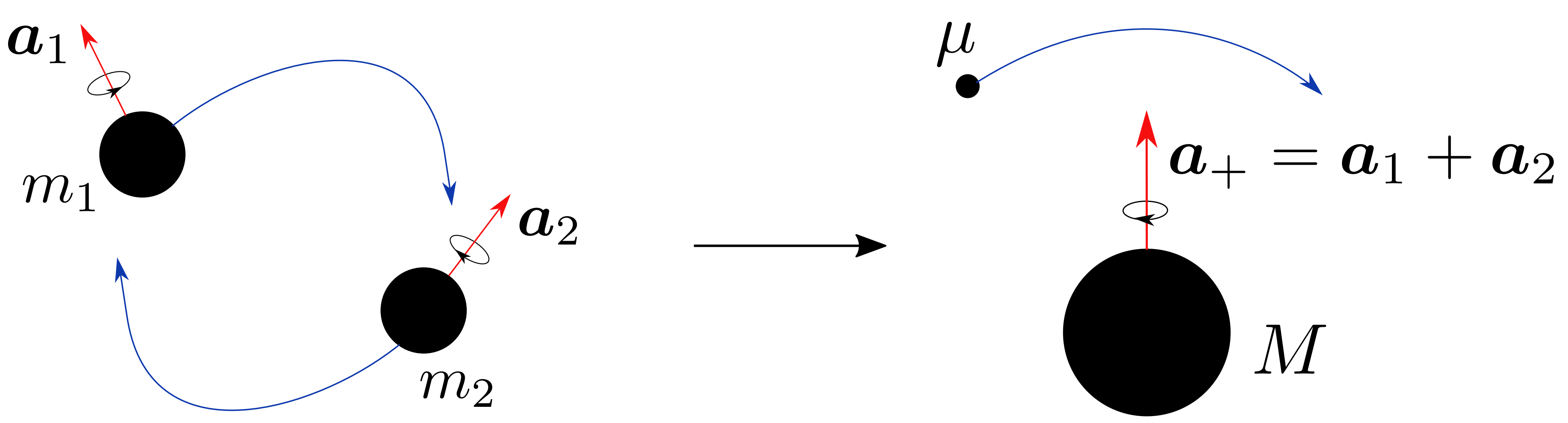
- **Non-spinning 5PN terms** are known except two coefficients, which can be fixed by second-order GSF. *(Bini, Damour & Geralico 20; Blümlein et al. 21)*
- **5.5PN SO terms** are known except for one coefficient, which can be fixed by second-order GSF. *(Khalil 22)*
- **5PN SS terms** are known for quasi-circular orbits. *(Kim, Levi & Yin 22)*

EOB EOM and RR Force for Spinning Bodies

$$\mu = m_1 m_2/M$$

$$M = m_1 + m_2$$

$$\nu = \mu/M \qquad 0 \le \nu \le 1/4$$



(credit: Khalil)

$$\mathbf{a}_i = \frac{m_i}{M}\chi_i \quad i = 1,2$$

$$0 \le \chi_i \le 1$$

$$H_{\rm real}^{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}}{\mu} - 1\right)}$$

(AB & Damour 99; Damour 00; AB, Chen & Damour 05; Damour, Jaranowski & Schafer 08; Barausse, Racine & AB 10; Barausse & AB 11; Damour & Nagar 14; Balmelli & Damour 15; Khalil, Steinhoff, Vines & AB 20)

- **EOB equations of motion:**

(AB & Damour 00; AB, Chen & Damour 05; Damour et al. 09)

$$\dot{\mathbf{r}} = \frac{\partial H_{\rm real}^{\rm EOB}(\mathbf{r}, \mathbf{p}, \mathbf{a}_i)}{\partial \mathbf{p}} \qquad \dot{\mathbf{a}}_{\mathbf{i}} = \left\{\mathbf{a}_i, H_{\rm real}^{\rm EOB}\right\}$$

$$\dot{\mathbf{p}} = -\frac{\partial H_{\rm real}^{\rm EOB}(\mathbf{r}, \mathbf{p}, \mathbf{a}_i)}{\partial \mathbf{r}} + \mathbf{F}(\mathbf{r}, \mathbf{p}, \mathbf{a}_i)$$

- **Radiation-reaction force** and gravitational **modes**:

(AB & Damour 00; Damour et al. 09; Pan, AB et al. 11)

$$F_\phi \propto \frac{dE}{dt} \propto \sum_{\ell m} (m\,\Omega)^2 \,|\, h_{\ell m}^{\rm insp}(r, \Omega)\,|^2 \longleftarrow \text{quasicircular orbits}$$

$$h_{\ell m}^{\rm insp-plunge} = h_{\ell m}^{\rm Newt}\, e^{-im\phi}\, S_{\rm eff}\, T_{\ell m}\, e^{i\delta_{\ell m}} (\rho_{\ell m})^\ell\, h_{\ell m}^{\rm NQC}$$

resummation of PN results

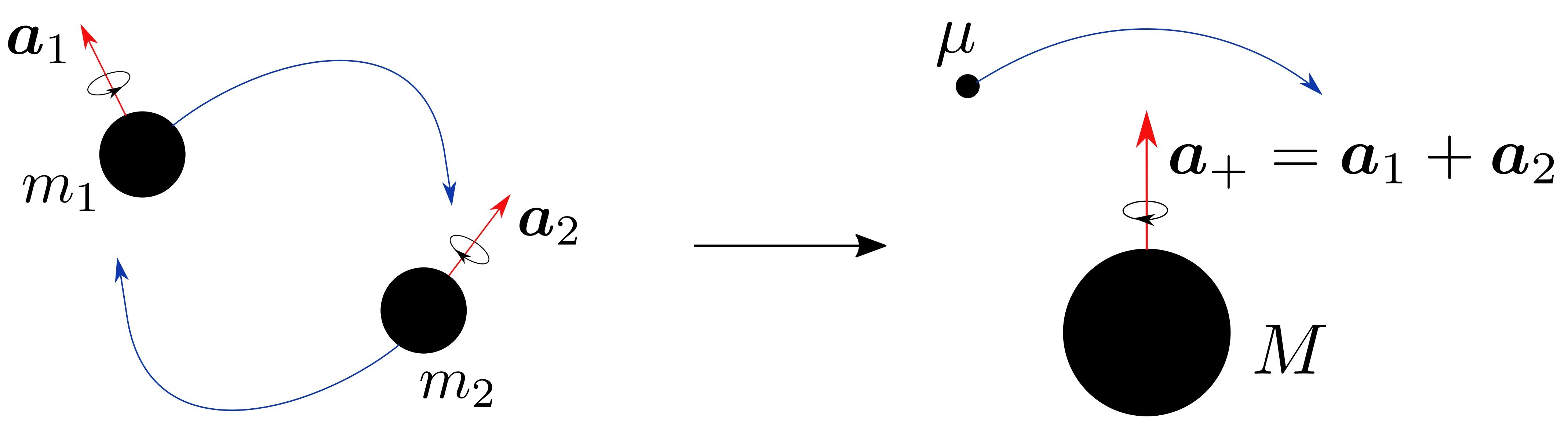
non-quasicircular (NQC) corrections

EOB Hamiltonian for Non-Spinning Bodies

$\mu = m_1 m_2 / M$

$M = m_1 + m_2$

$\nu = \mu / M \qquad 0 \leq \nu \leq 1/4$



(credit: Khalil)

$$\mathbf{a}_i = \frac{m_i}{M} \chi_i \qquad i = 1,2$$

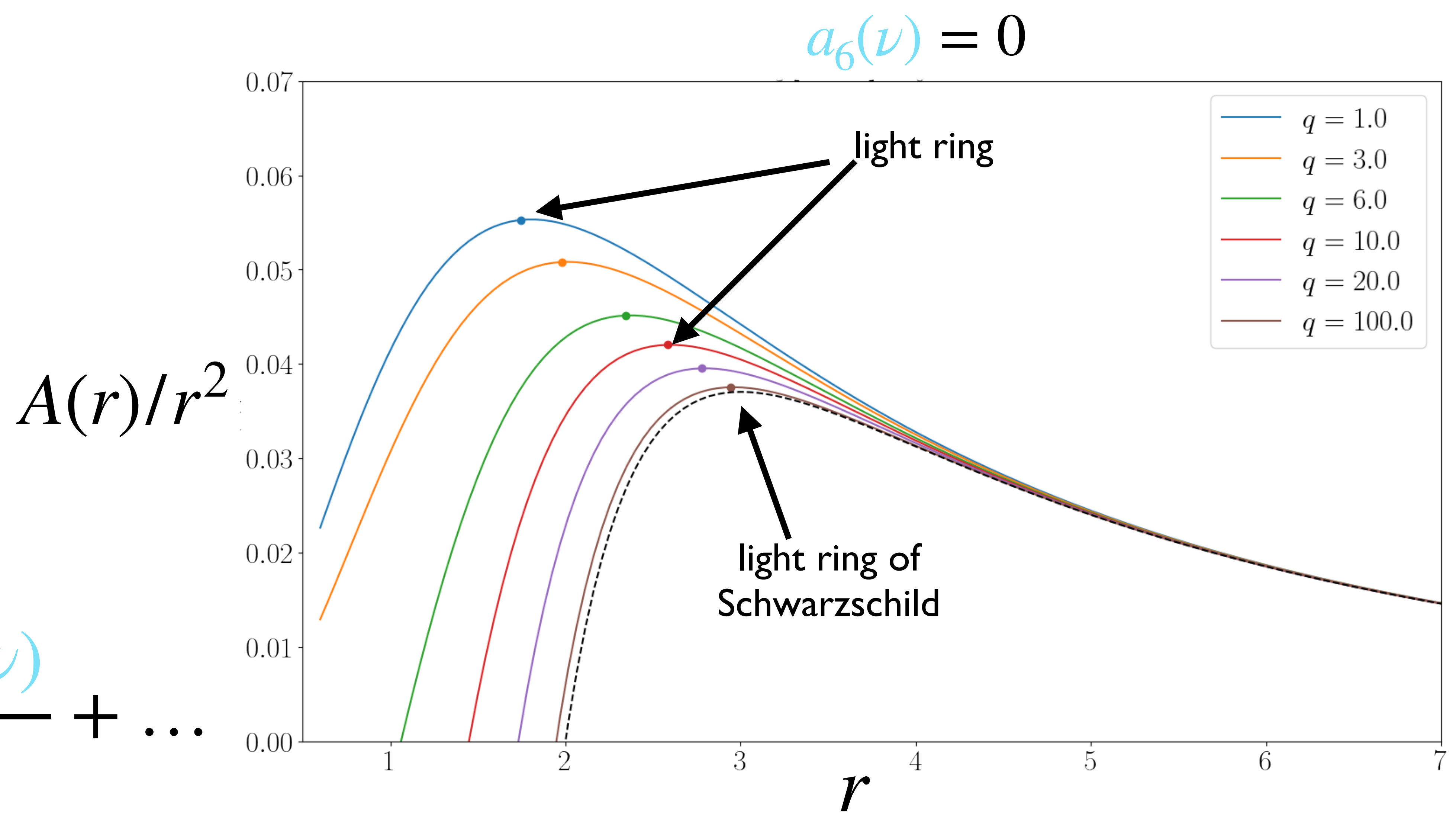
$$0 \leq \chi_i \leq 1$$

$$H_{\text{real}}^{\text{EOB}} = M \sqrt{1 + 2\nu \left(\frac{H_{\text{eff}}}{\mu} - 1 \right)}$$

(AB & Damour 99; Damour 00; AB, Chen & Damour 05; Damour, Jaranowski & Schafer 08; Barausse, Racine & AB 10; Barausse & AB 11; Damour & Nagar 14; Balmelli & Damour 15; Khalil, Steinhoff, Vines & AB 20)

$$H^{\text{eff}} = H^{\text{eff}}_{\text{even}} \qquad \mathbf{a}_i = 0 \quad i = 1,2$$

$$H^{\text{eff}}_{\text{even}} = \sqrt{A(r; a_6) \left[\mu^2 + p_r^2 B_{np}(r) + \frac{p_\phi^2}{r^2} + Q(r, p_r) \right]}$$



$$A(r, a_6) = 1 - \frac{2}{r} + \frac{2\nu}{r^3} + \left(\frac{94}{3} - \frac{41}{32} \pi^2 \right) \frac{\nu}{r^4} + [a_5(\nu) + a_5^{\log}(\nu) \log(r)] \frac{1}{r^5} + \frac{a_6(\nu)}{r^6} + \ldots$$

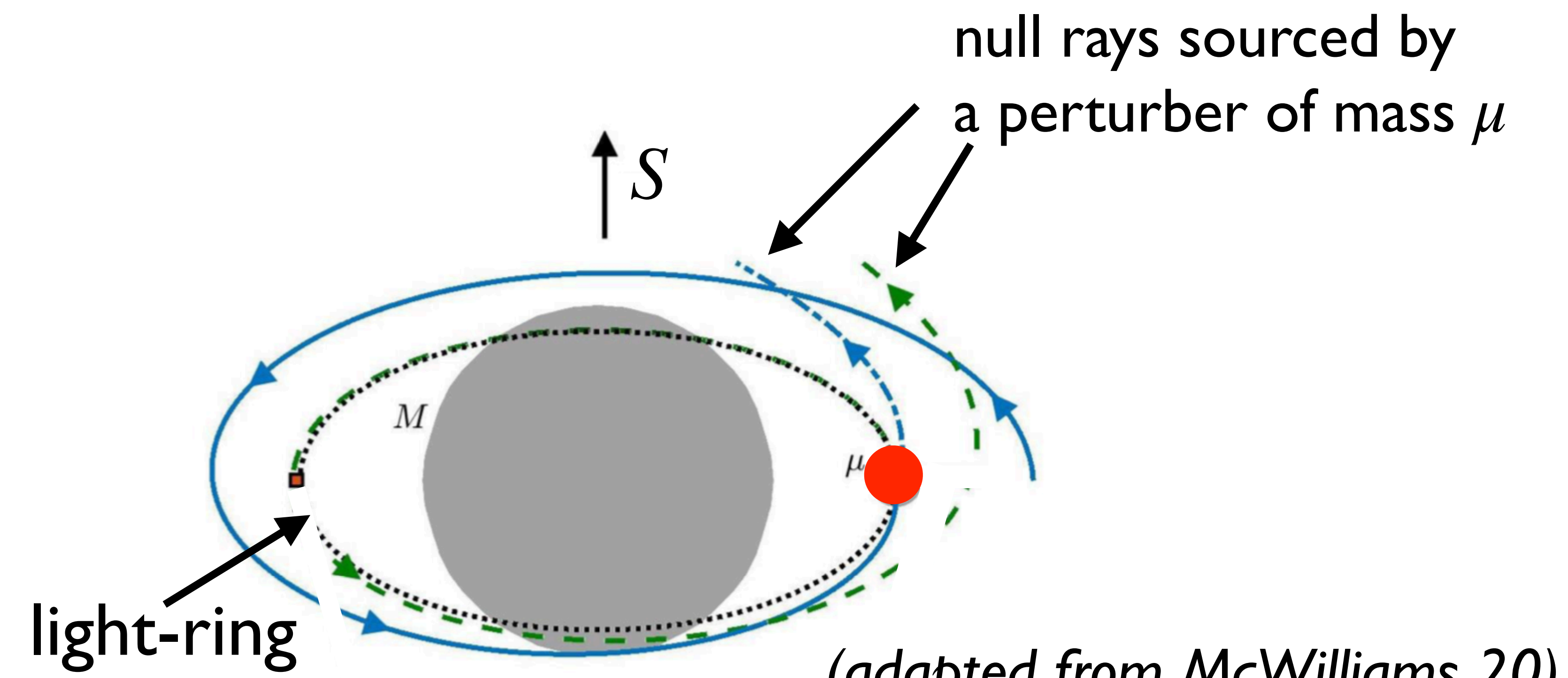
resummation of $A(r)$ potential

Inspiral-Plunge EOB Waveform & Frequency

- **EOB equations of motion**

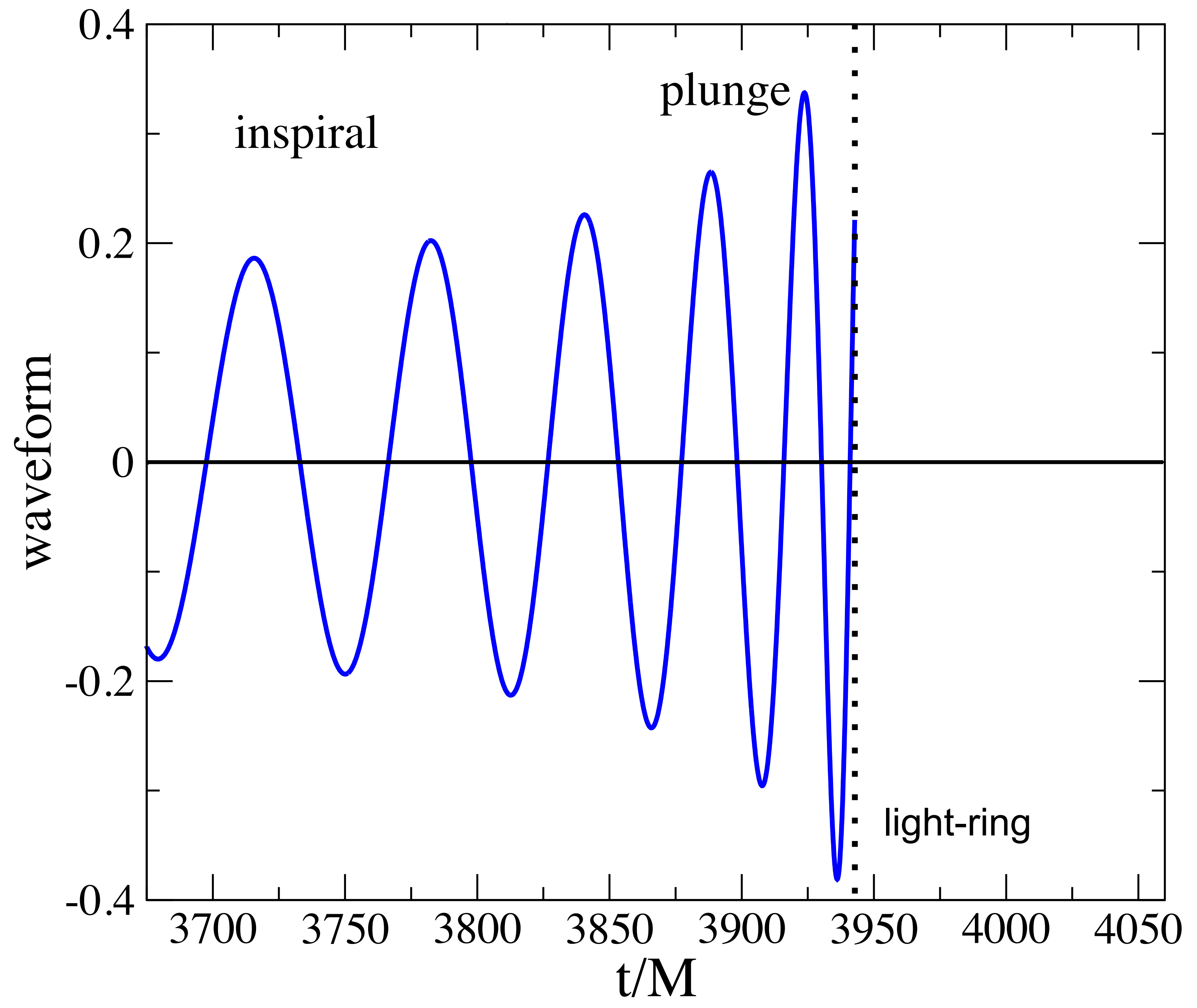
$$\dot{\mathbf{r}} = \frac{\partial H^{\mathrm{EOB}}_{\mathrm{real}}(\mathbf{r}, \mathbf{p}, \mathbf{a}_i)}{\partial \mathbf{p}}$$

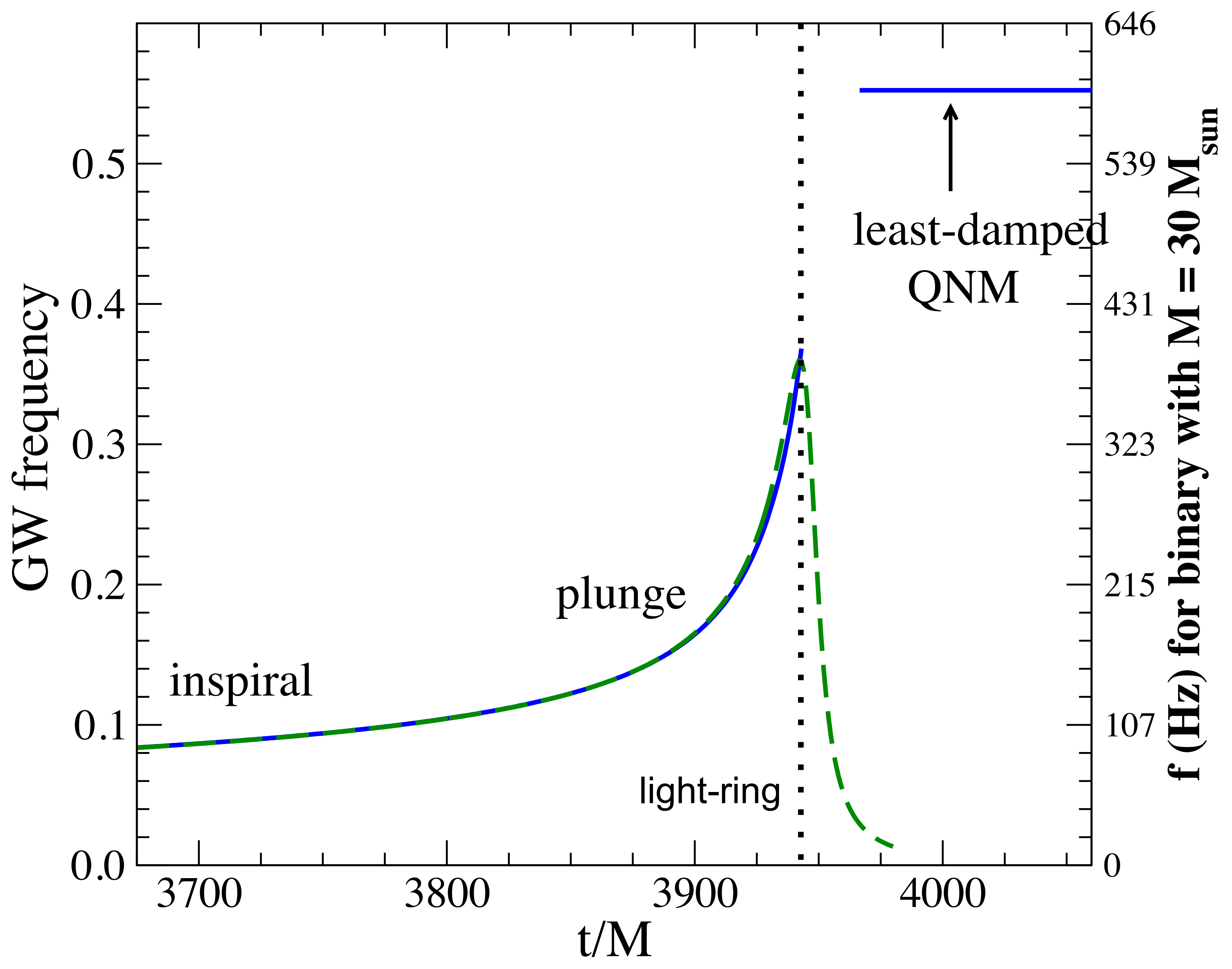
$$\dot{\mathbf{p}} = -\frac{\partial H^{\mathrm{EOB}}_{\mathrm{real}}(\mathbf{r}, \mathbf{p}, \mathbf{a}_i)}{\partial \mathbf{r}} + \mathbf{F}(\mathbf{r}, \mathbf{p}, \mathbf{a}_i)$$



(adapted from McWilliams 20)

- Evolve **two-body dynamics up to light ring** (or photon orbit) and then …





- **Quasi-normal modes** excited at **light-ring crossing.** *(Goebel 1972; Davis, Ruffini & Tiomno 1972; Ferrari et al. 1984; Price and Pullin 1994)*

Inspiral-Merger-Ringdown EOB Waveform & Frequency

- **EOB equations of motion**

$$\dot{\mathbf{r}} = \frac{\partial H^{\rm EOB}_{\rm real}(\mathbf{r},\mathbf{p},\mathbf{a}_i)}{\partial \mathbf{p}}$$

$$\dot{\mathbf{p}} = -\frac{\partial H^{\rm EOB}_{\rm real}(\mathbf{r},\mathbf{p},\mathbf{a}_i)}{\partial \mathbf{r}} + \mathbf{F}(\mathbf{r},\mathbf{p},\mathbf{a}_i)$$

- … attach a function representing **quasi-normal mode ringing** of remnant BH.

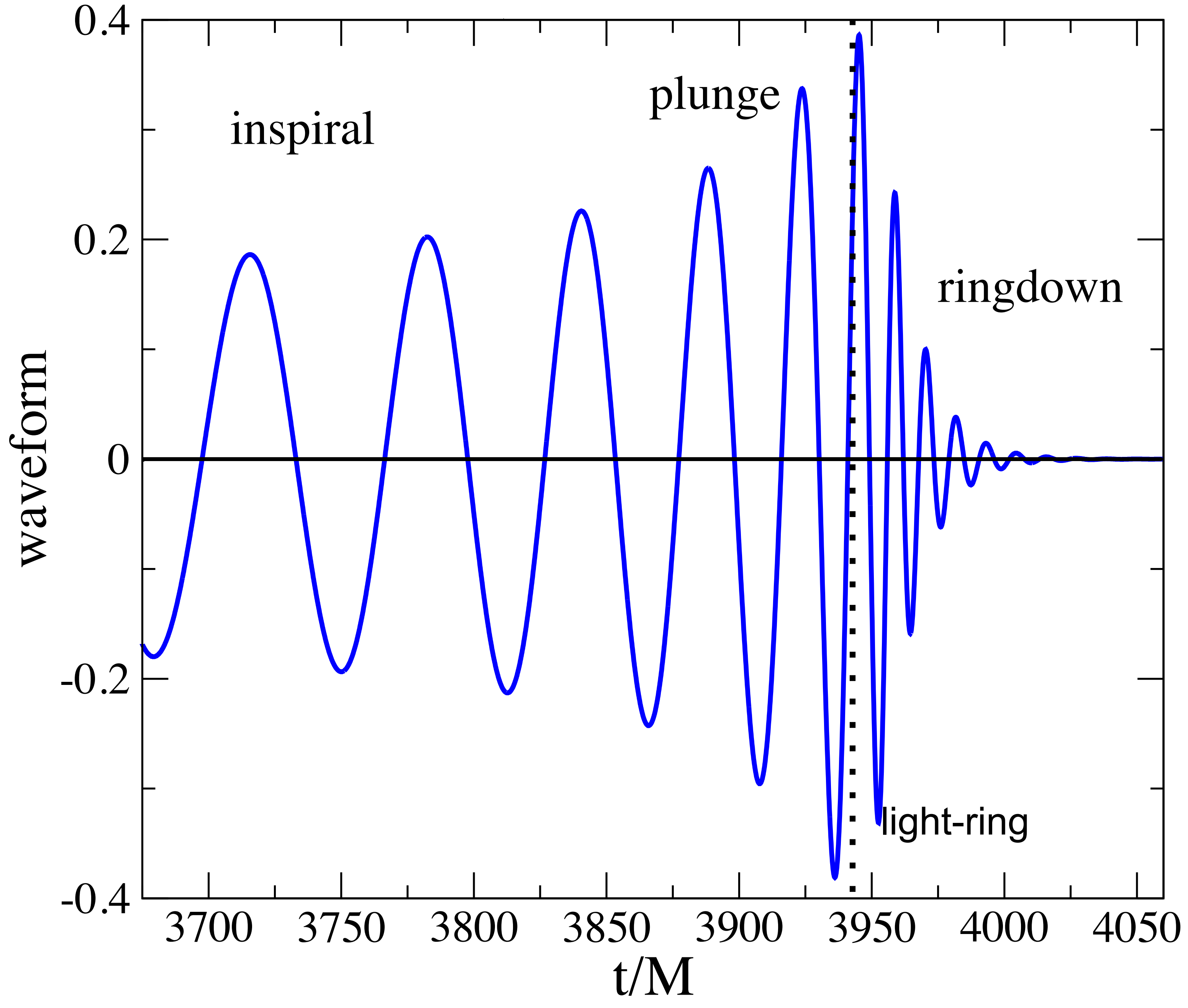
BH quasi-normal modes

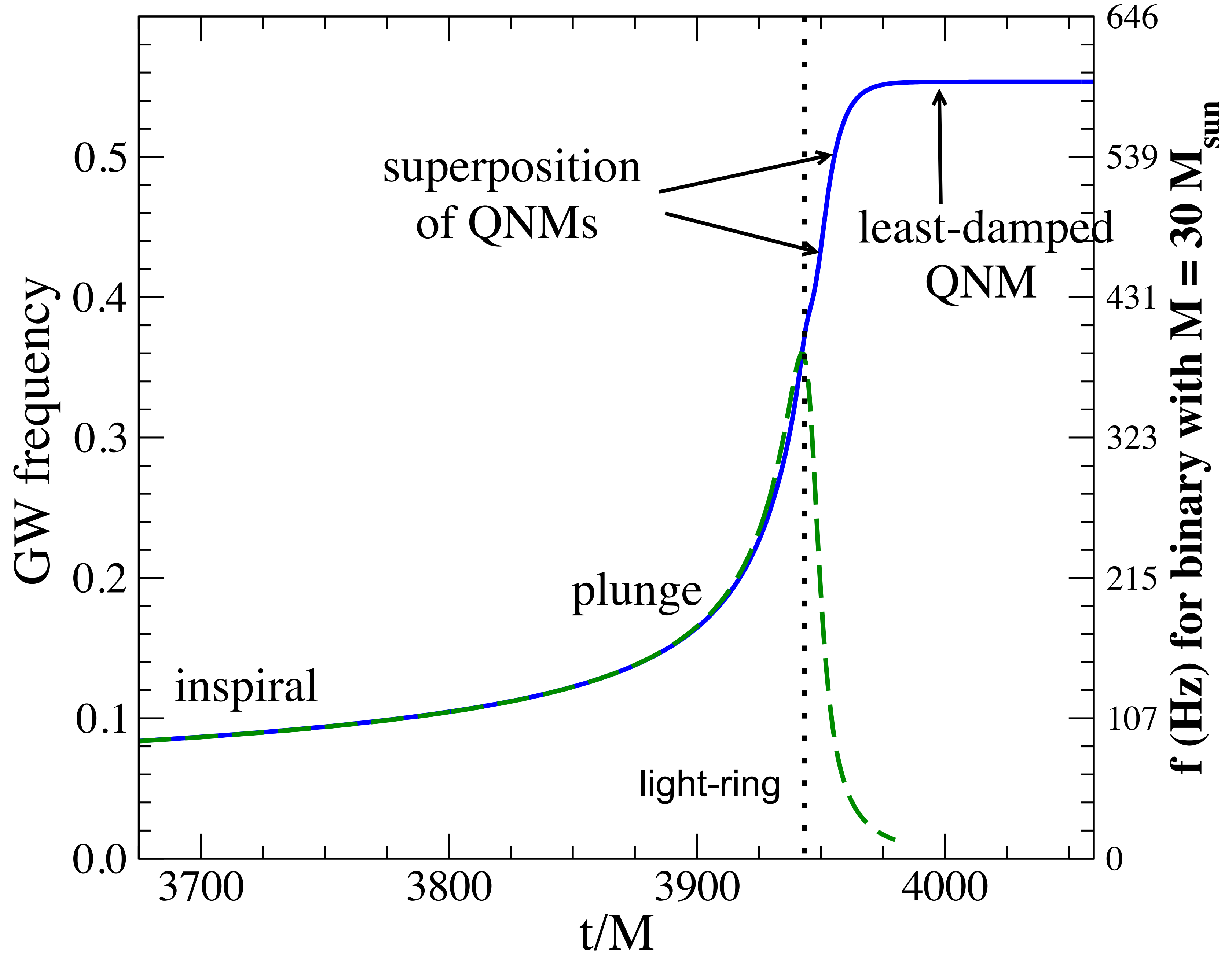
$$h^{\rm merger-RD}_{\ell m}(t) = \nu\, \tilde{A}_{\ell m}(t)\, e^{i\,\tilde{\phi}_{\ell m}(t)}\, e^{-i\sigma_{\ell m0}(t-t^{\ell m}_{\rm match})}$$

(Baker et al. 08; Damour et al.14; London et al. 14; Bohé, … AB et al. 17; Cotesta, AB et al. 19)

$$t^{\ell m}_{\rm match} = t^{\Omega}_{\rm peak} + \Delta t^{\ell m}$$

peak of orbital frequency (light ring)





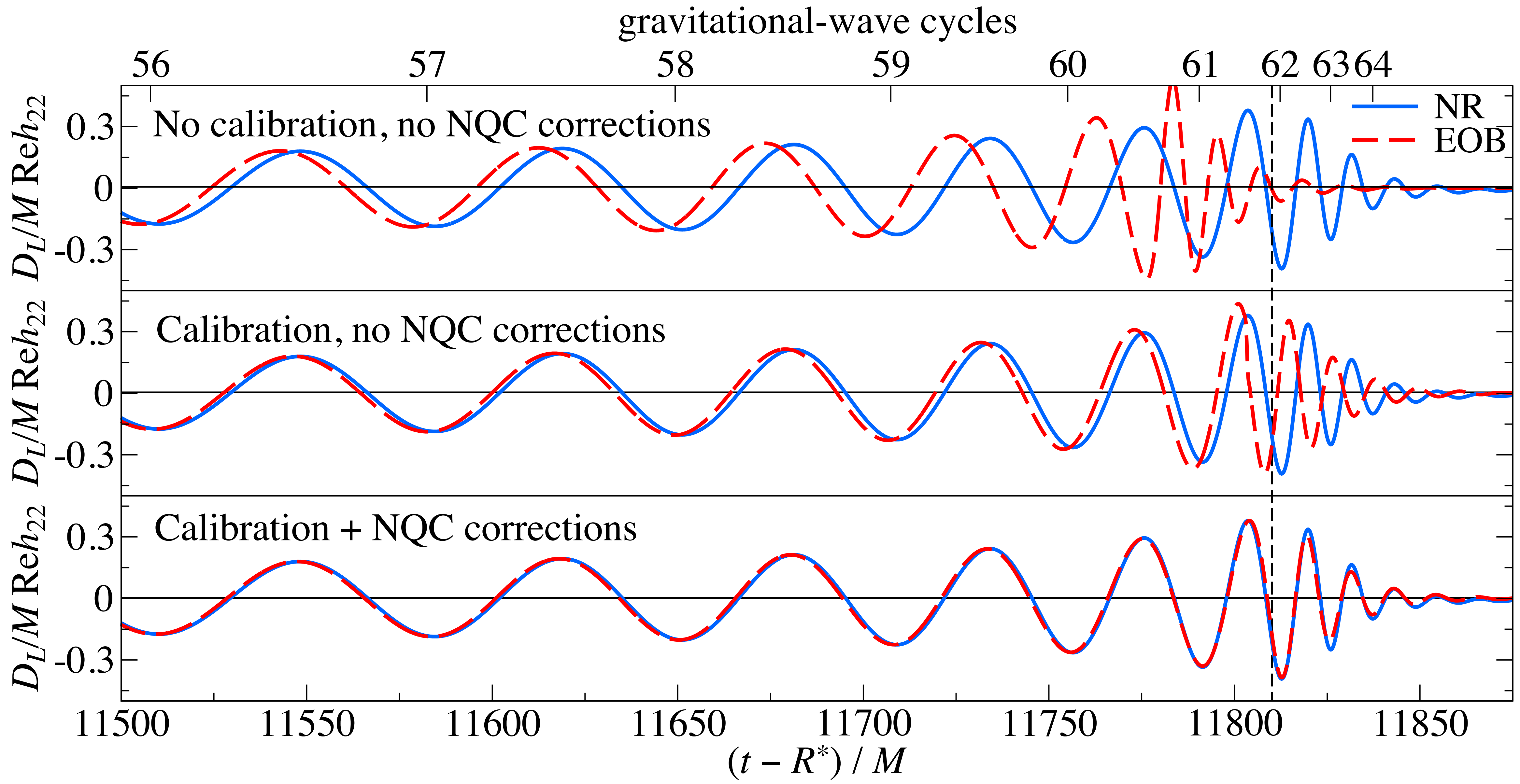
$$h_{22}(t) = h^{\rm insp-plunge}_{22}(t)\,\theta(t^{22}_{\rm match}-t) + h^{\rm merger-RD}_{22}(t)\,\theta(t-t^{22}_{\rm match})$$

(AB & Damour 00; AB, Chen & Damour 05; AB, Cook & Pretorius 07)

Calibrating EOB Waveforms to NR

- **Steps of calibration** to NR.

mass ratio = 1

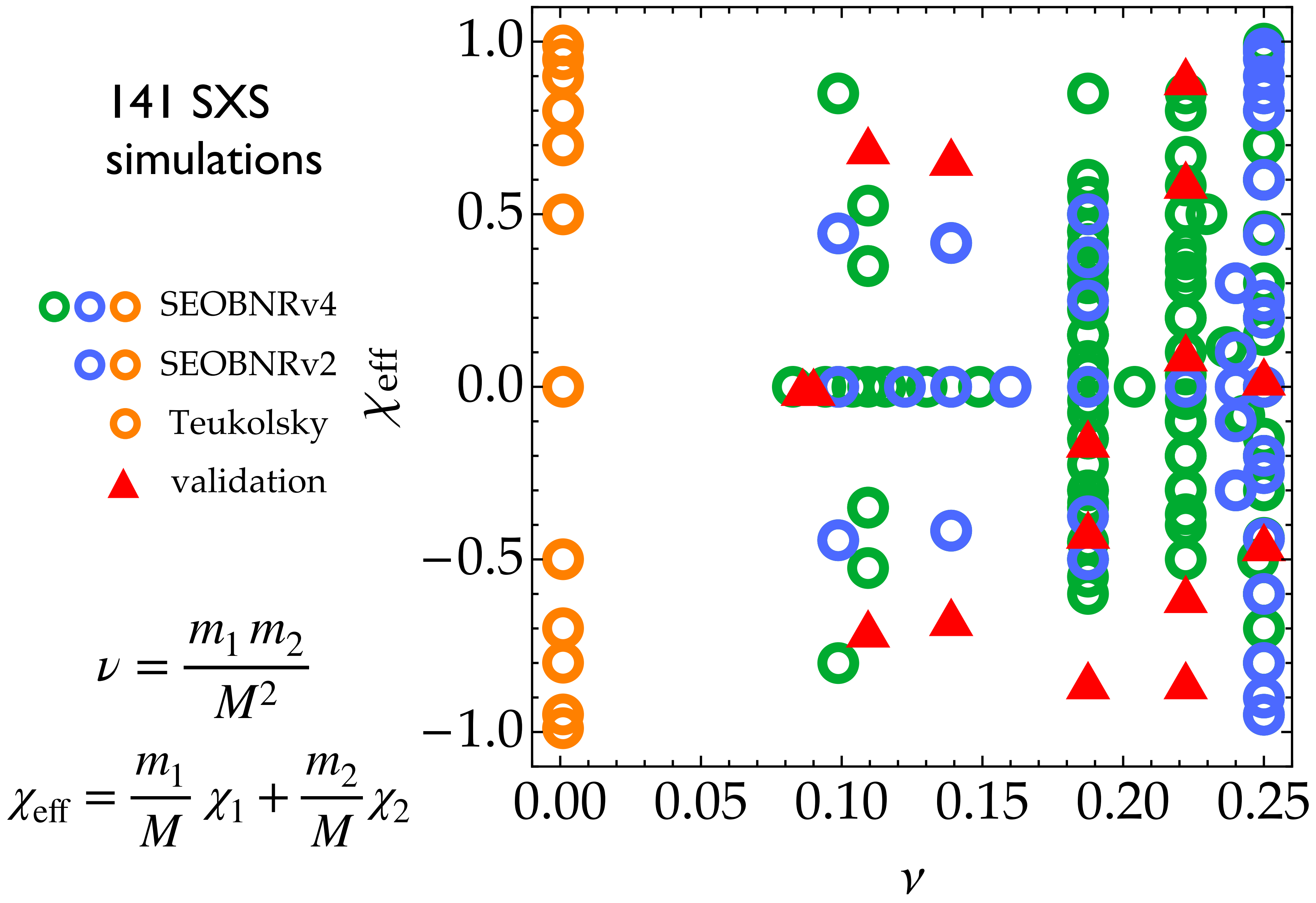


(credit: Taracchini)

(Pan, AB et al. 13; Taracchini, AB et al. 14; Pürrer 15; Bohé, Shao, Taracchini, AB et al 17; Babak et al. 16; Cotesta et al. 18, 20; Ossokine et al. 20)

(SEOBNRv4)

141 SXS simulations



$$\nu = \frac{m_1 m_2}{M^2}$$

$$\chi_{\mathrm{eff}} = \frac{m_1}{M}\chi_1 + \frac{m_2}{M}\chi_2$$

(Bohé et al. 17)

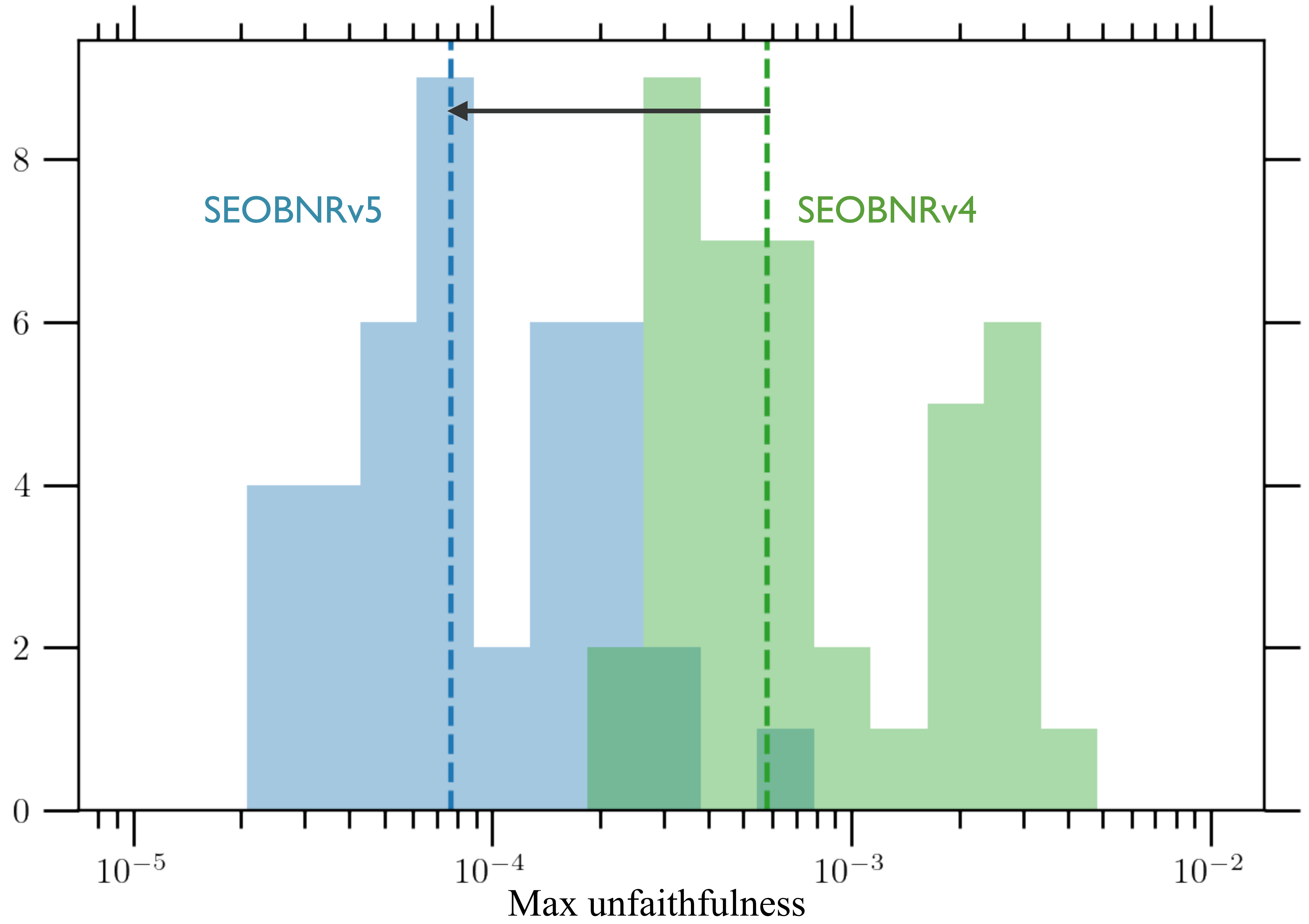
- Once calibrated, SEOBNR waveform models are employed for **LIGO-Virgo template banks**, and **inference studies** to **measure source properties** and **for tests of GR.**

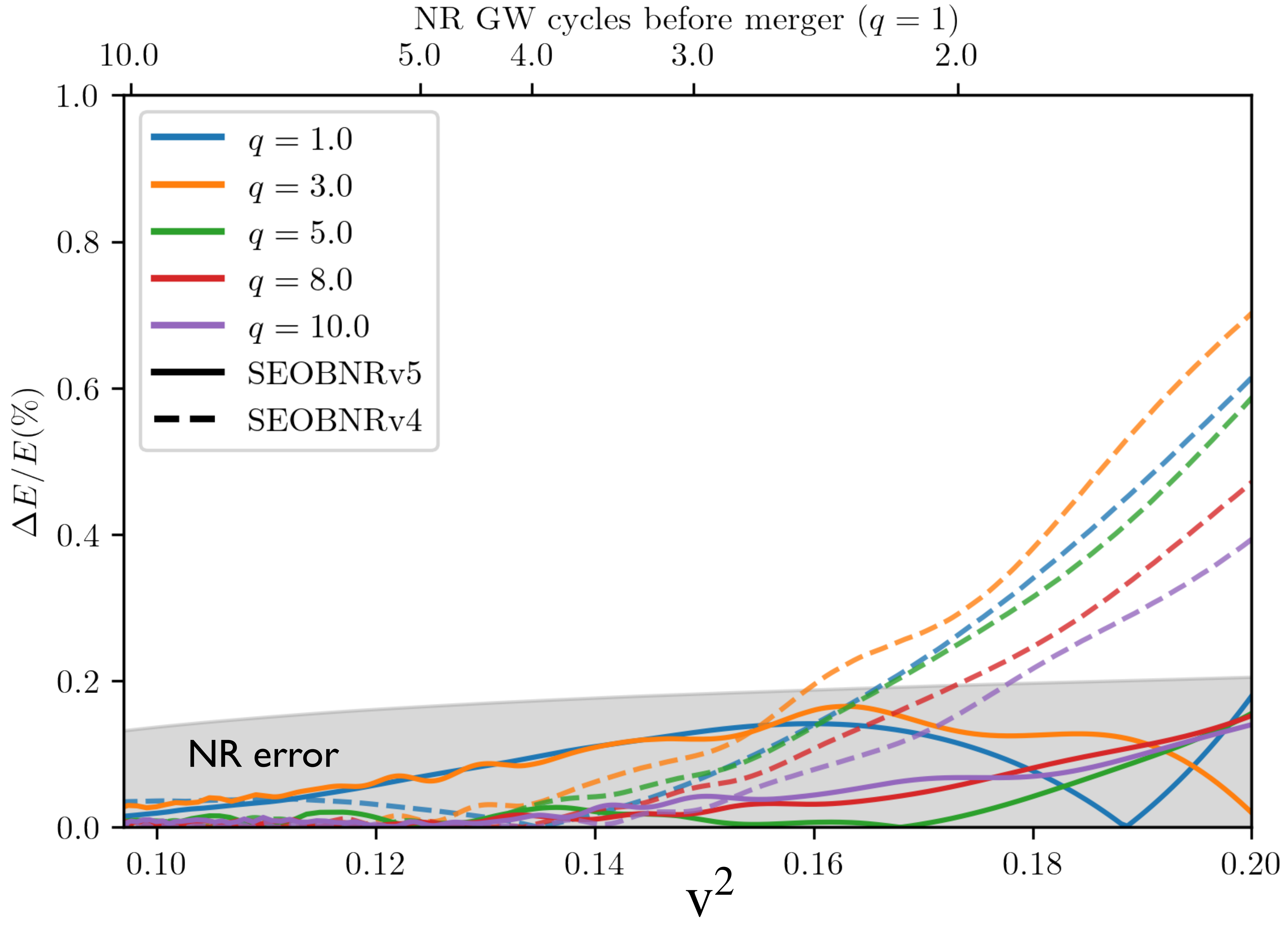
(see also other waveform models: IMRPhenom, NRSur, TEOBResumS)

More Accurate EOBNR Model for O4 run

Non-spinning sector.

$$\mathcal{U} = 1 - \max_{t_0,\phi_0} \frac{(h_1, h_2)}{\sqrt{(h_1,h_1)(h_2,h_2)}} \qquad (h_1, h_2) = 4\mathrm{Re}\left[\int_{f_{\min}}^{f_{\max}} \frac{h_1(f)\, h_2^*(f)\, df}{S_n(f)}\right]$$





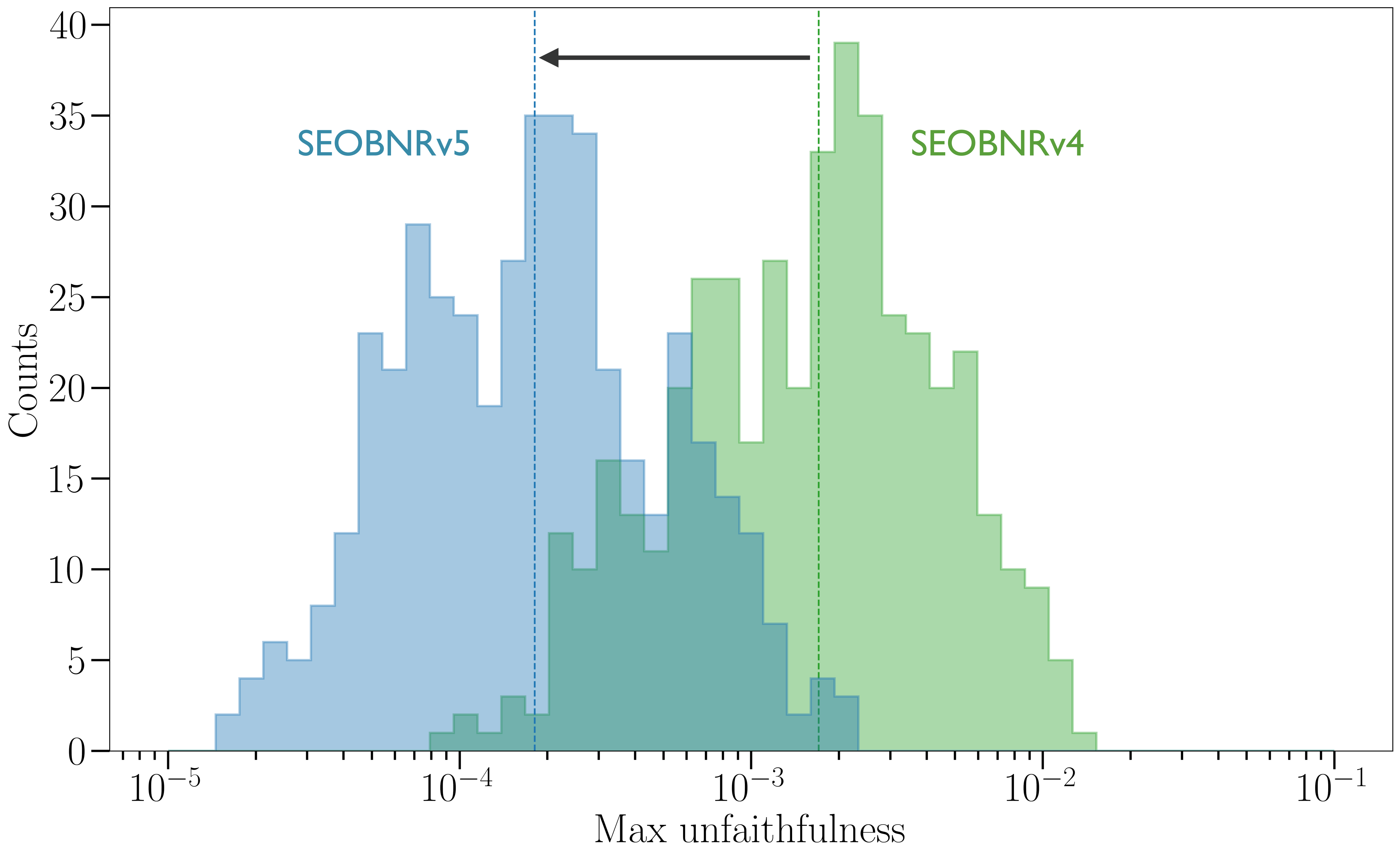
- With **improved SEOBNR** model, we will be **dominated by systematic errors at SNR higher by a factor** $\sqrt{10}$.
- SEOBNRv5 **binding energy within NR** error.

(AB, Khalil, van den Meent, Mihaylov, Pompili, Pürrer, Ramos-Buades & Ossokine in prep.)

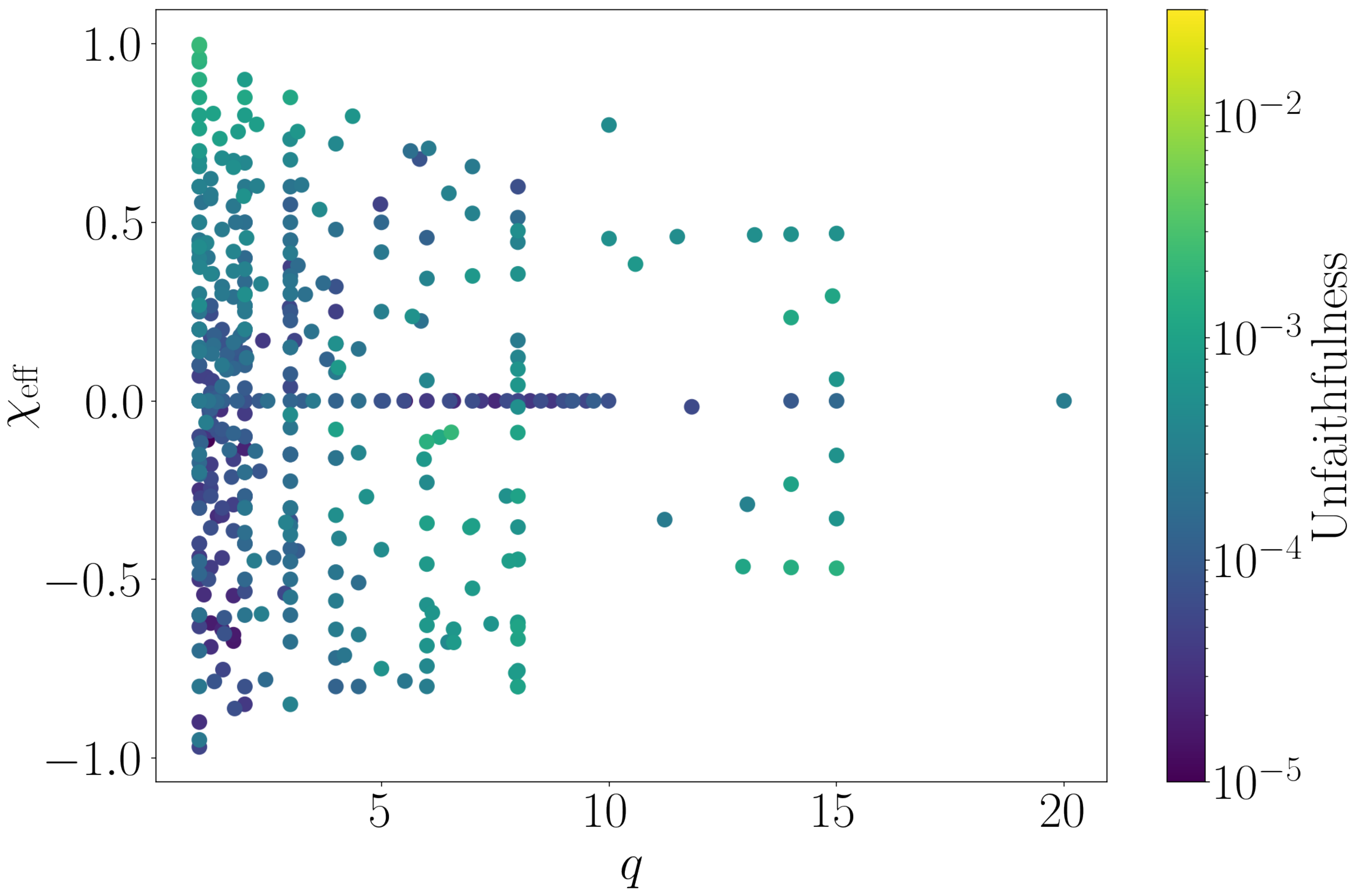
More Accurate EOBNR Model for O4 run (contd.)

Spinning sector.

- **Unfaithfulness** $= 0$ implies EOBNR & NR **match perfectly**



- Parameter-space **coverage of 442 SXS NR** waveforms



$$\chi_{\rm eff} = \frac{m_1}{M}\chi_1 + \frac{m_2}{M}\chi_2 \qquad q = m_1/m_2$$

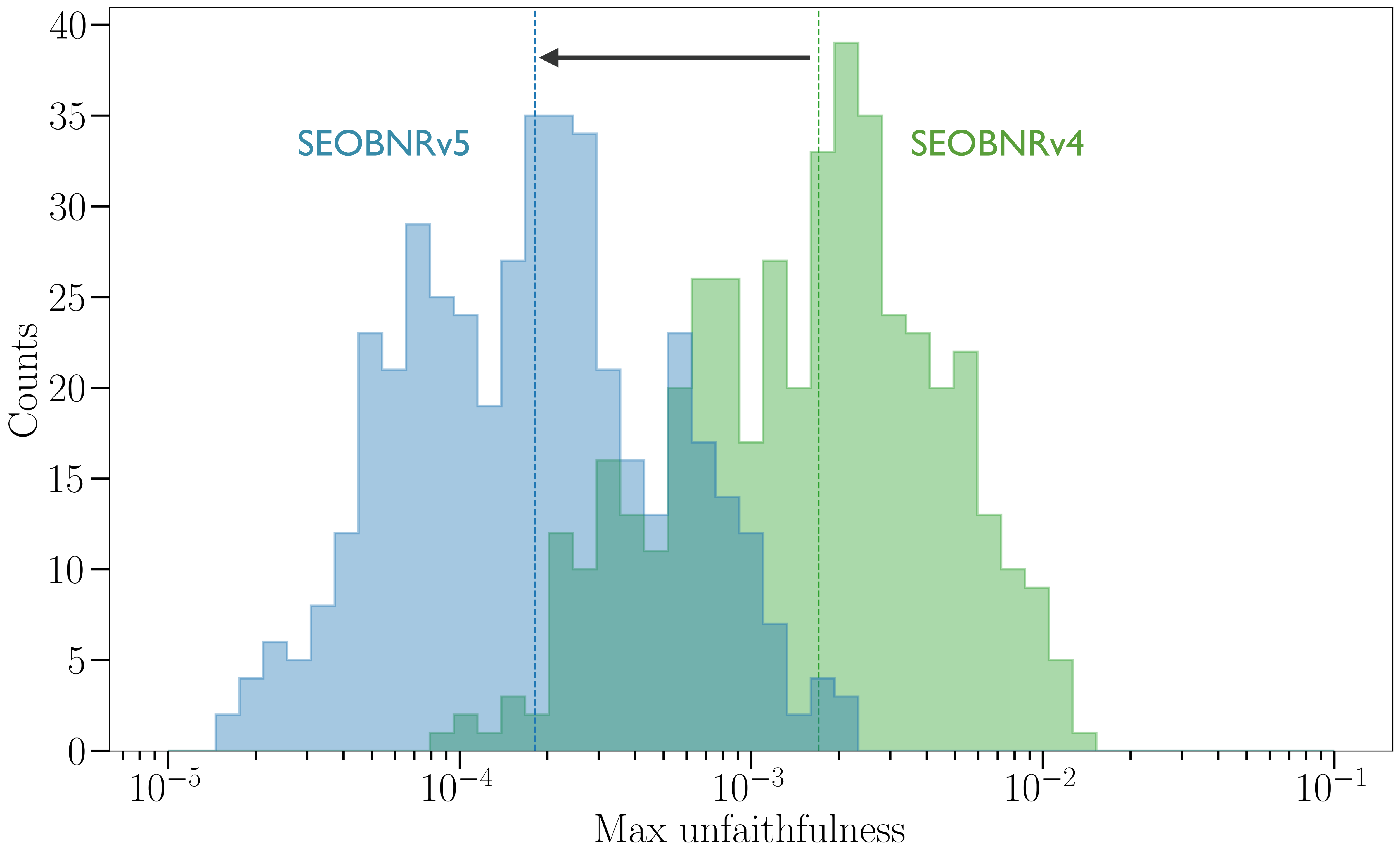
- With **improved SEOBNR** model, we will be **dominated by systematic errors at SNR higher by a factor** $\sqrt{10}$.

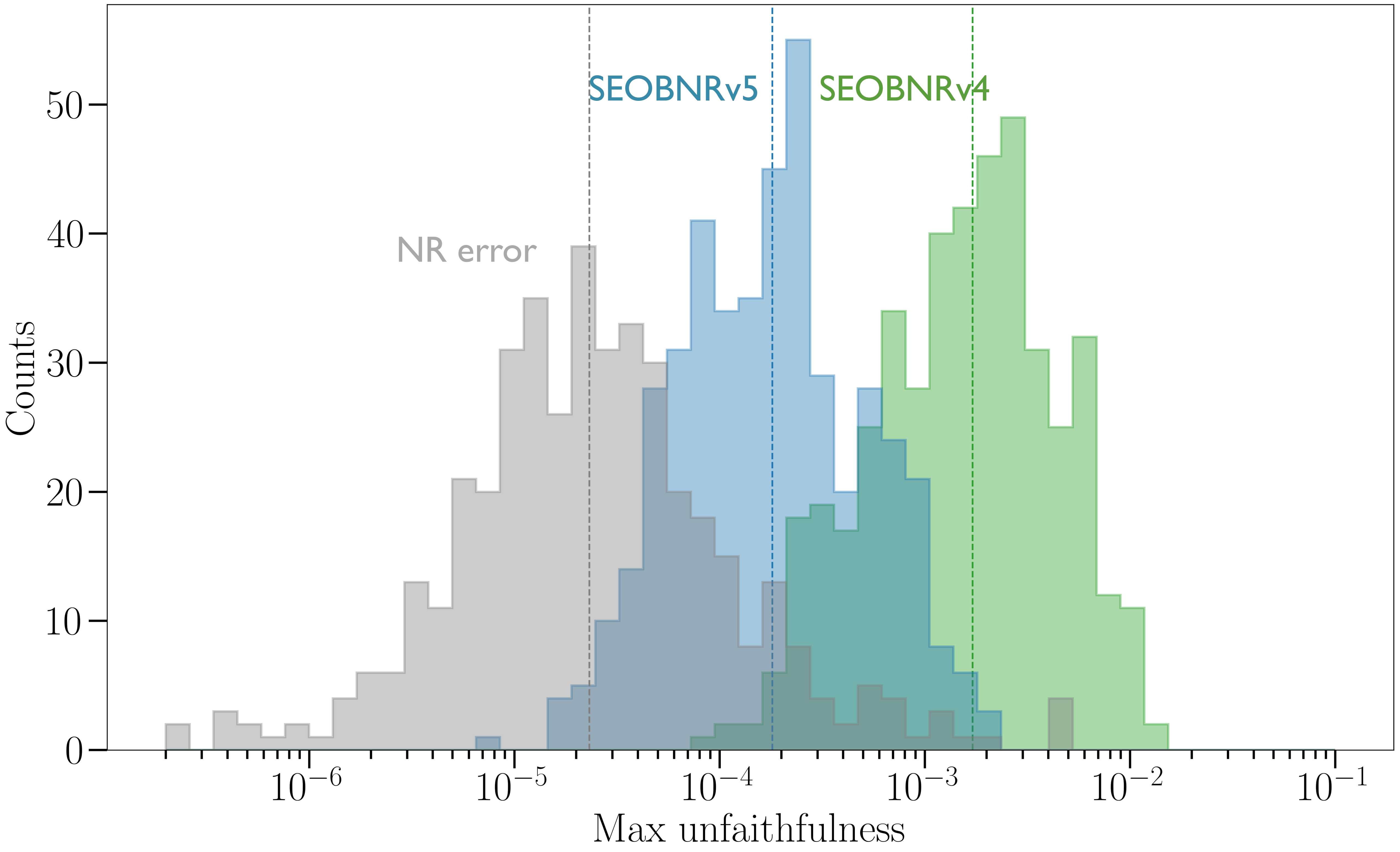
(AB, Khalil, van den Meent, Mihaylov, Pompili, Pürrer, Ramos-Buades & Ossokine in prep.)

More Accurate EOBNR Model for O4 run (contd.)

Spinning sector.

- **Unfaithfulness** $= 0$ implies EOBNR & NR **match perfectly**



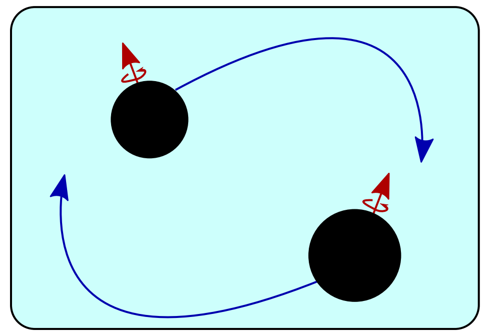


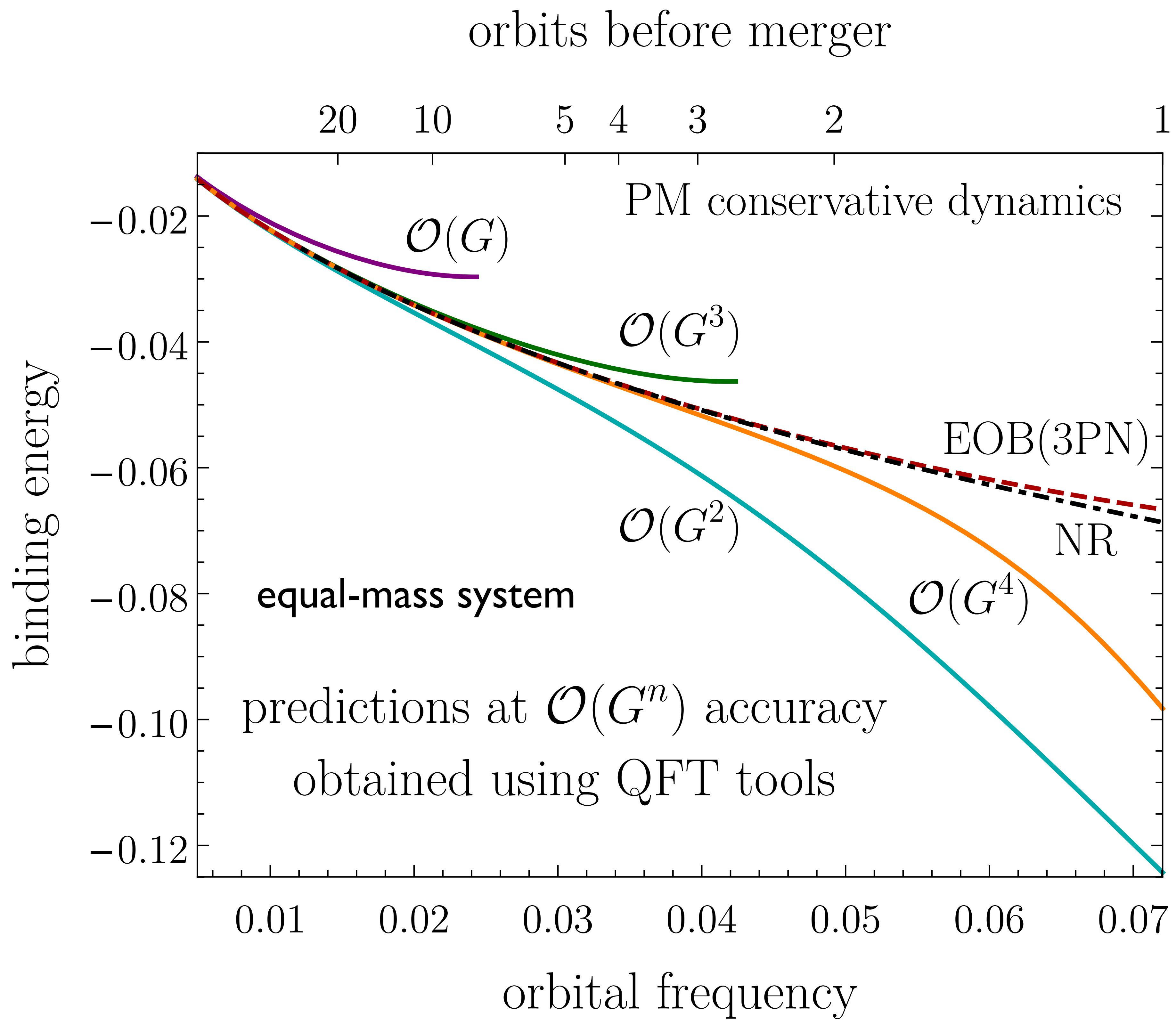
- With **improved SEOBNR** model, we will be **dominated by systematic errors at SNR higher by a factor** $\sqrt{10}$.
- **NR waveforms** would need to **be more accurate**.

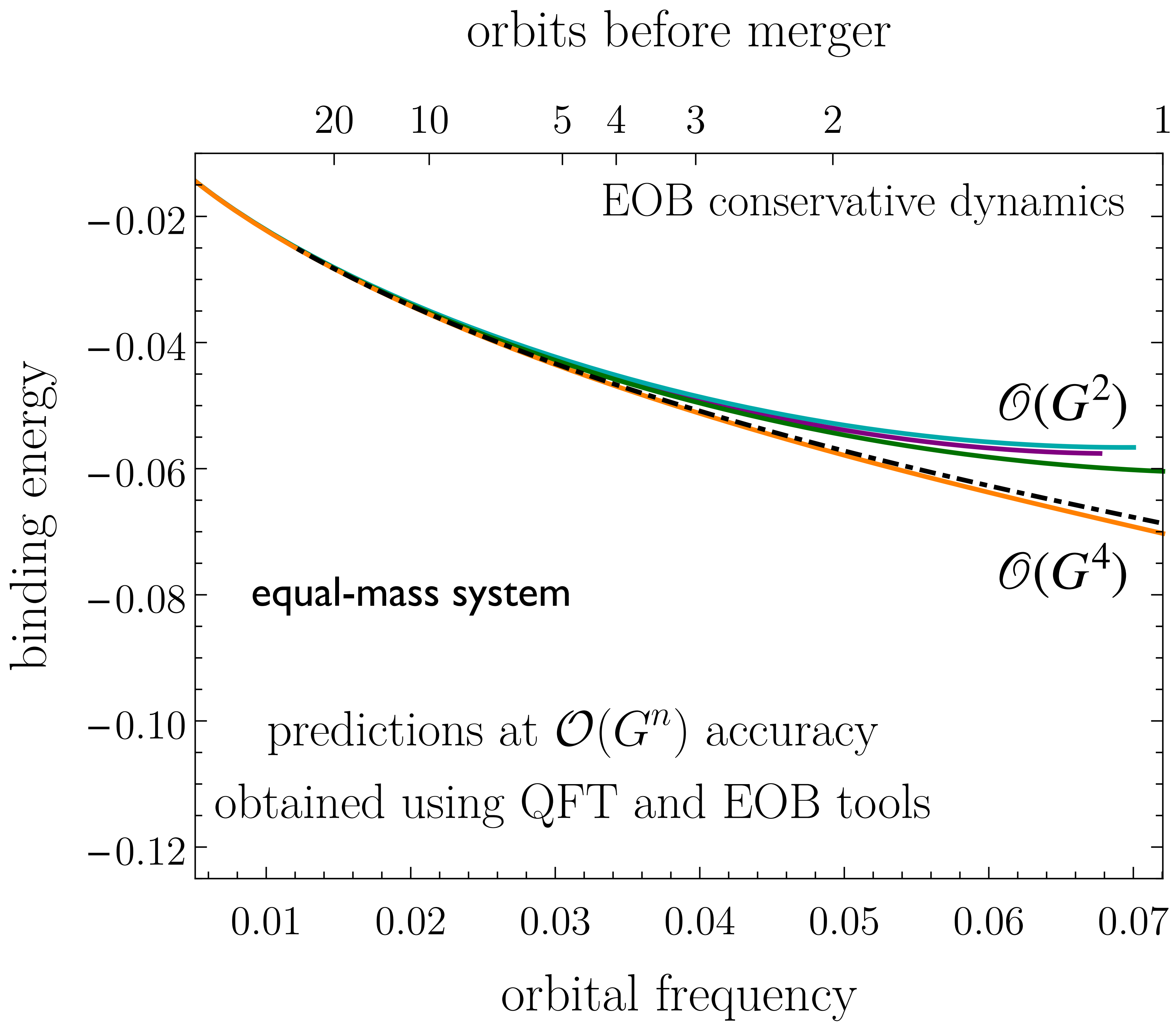
(AB, Khalil, van den Meent, Mihaylov, Pompili, Pürrer, Ramos-Buades & Ossokine in prep.)

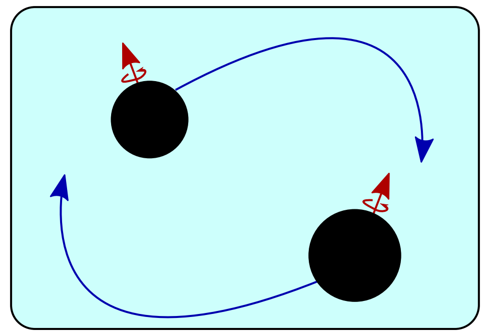
Toward Improving Waveform Accuracy: PM/EOB

- **Conservative dynamics derived through 3PM**, it is local and valid for generic orbits.
  (Cheung, Rothstein & Solon 19; Bern et al. 19; Blümlein et al. 20; Kälin, Liu & Porto 20; Cheung & Solon 20)
- **Conservative dynamics derived at 4PM** with non-local part for *hyperbolic* orbits.
  (Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, & Zeng 21; Dlapa, Kälin & Liu 21)









(Khalil, AB, Steinhoff & Vines 22)

Toward Improving Waveform Accuracy: PM/EOB (contd.)

- **Conservative dynamics derived through 3PM**, it is local and valid for generic orbits.

  (Cheung, Rothstein & Solon 19; Bern et al. 19; Blümlein et al. 20; Kälin, Liu & Porto 20; Cheung & Solon 20)

- **Conservative dynamics derived at 4PM** with non-local part for *hyperbolic* orbits.

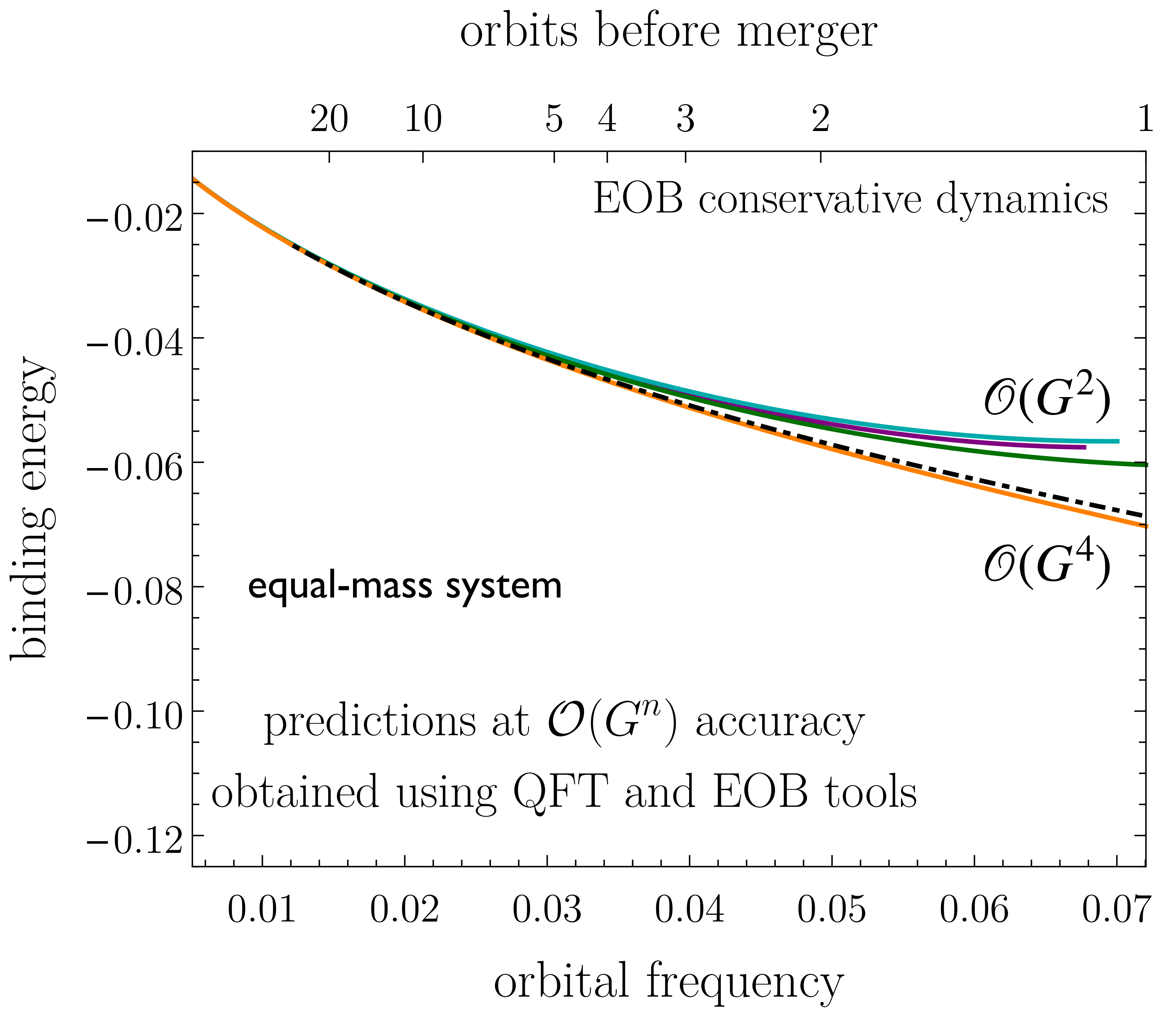
  (Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, & Zeng 21; Dlapa, Kälin & Liu 21)

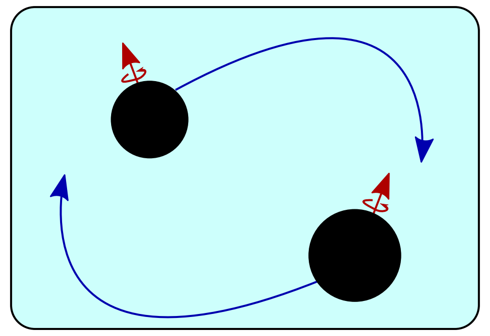
$$H_{\rm real}^{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}}{\mu} - 1\right)}$$

$$H_{\rm eff} = \sqrt{\left(1 - \frac{2}{r} + \frac{a_{\rm 2PM}}{r^2} + \frac{a_{\rm 3PM}}{r^3} + \frac{a_{\rm 4PM}}{r^3}\right)\left[\mu^2 + \left(1 - \frac{2}{r}\right)p_r^2 + \frac{p_\phi}{r^2}\right]}$$

- The coefficients $a_{\rm nPM}$ are obtained matching the scattering angle.

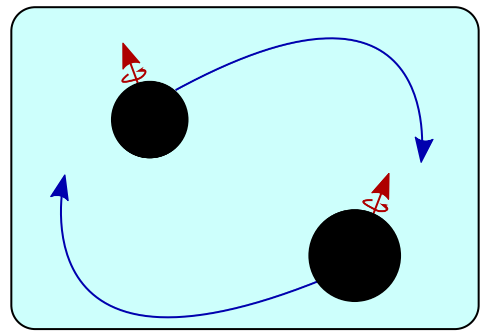
(Khalil, AB, Steinhoff & Vines 22)

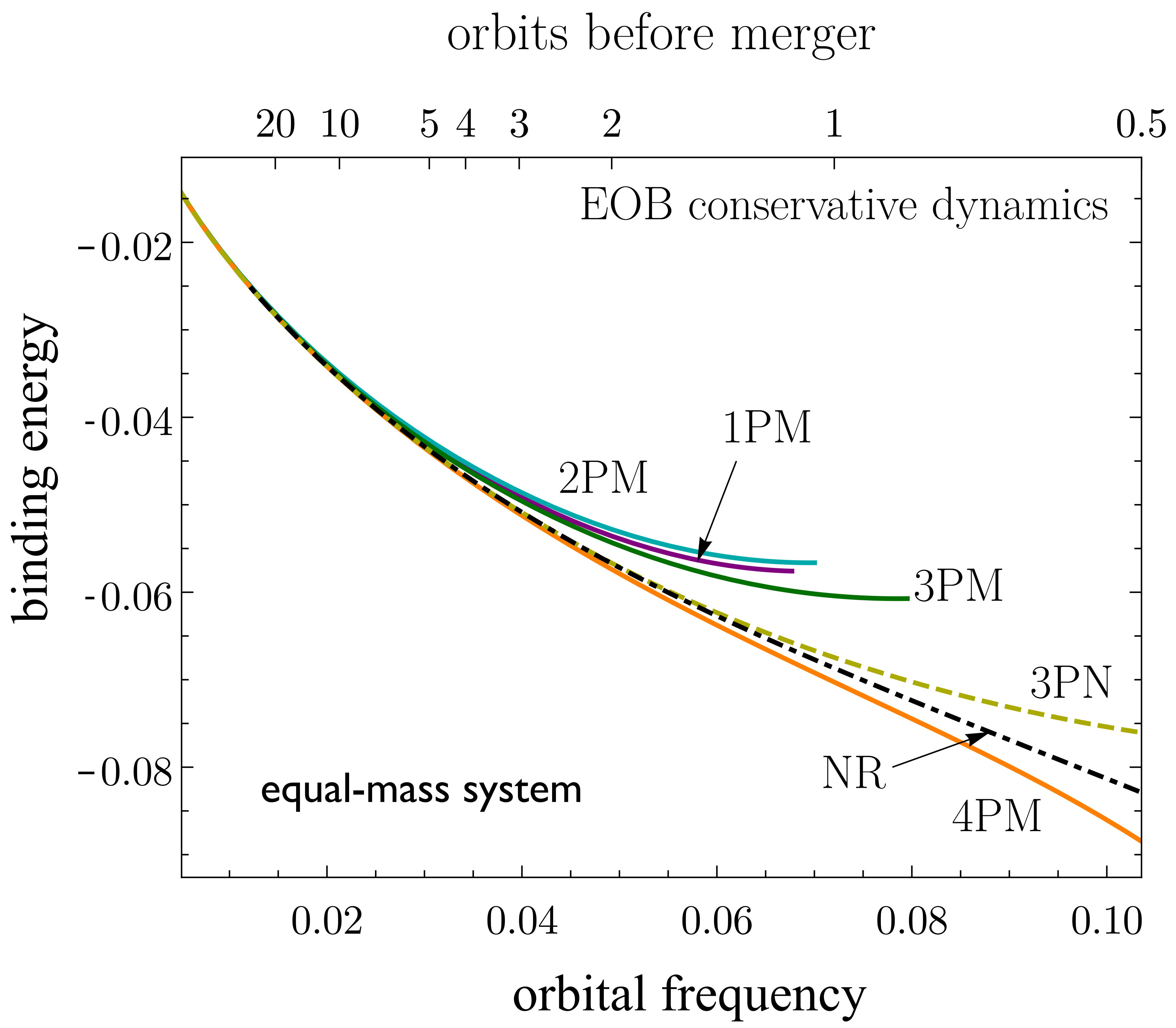


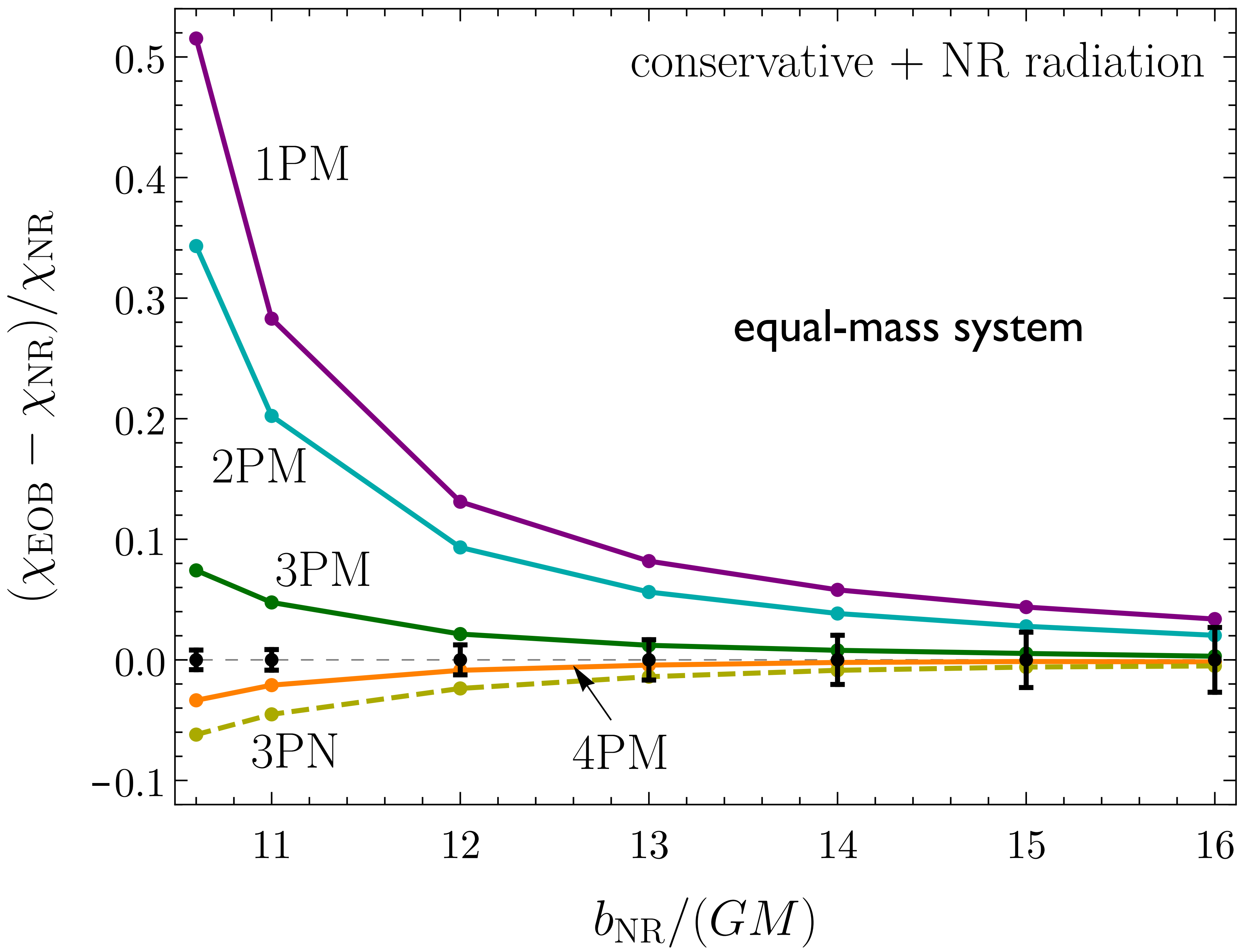


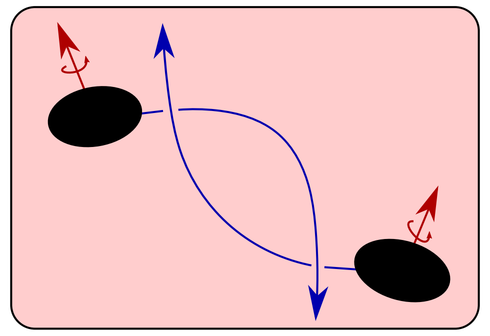
Toward Improving Waveform Accuracy: PM/EOB (contd.)

(Khalil, AB, Steinhoff & Vines 22)









- **3PN** is slightly **better for circular orbits,** but **4PM** is **better for scattering angle.**

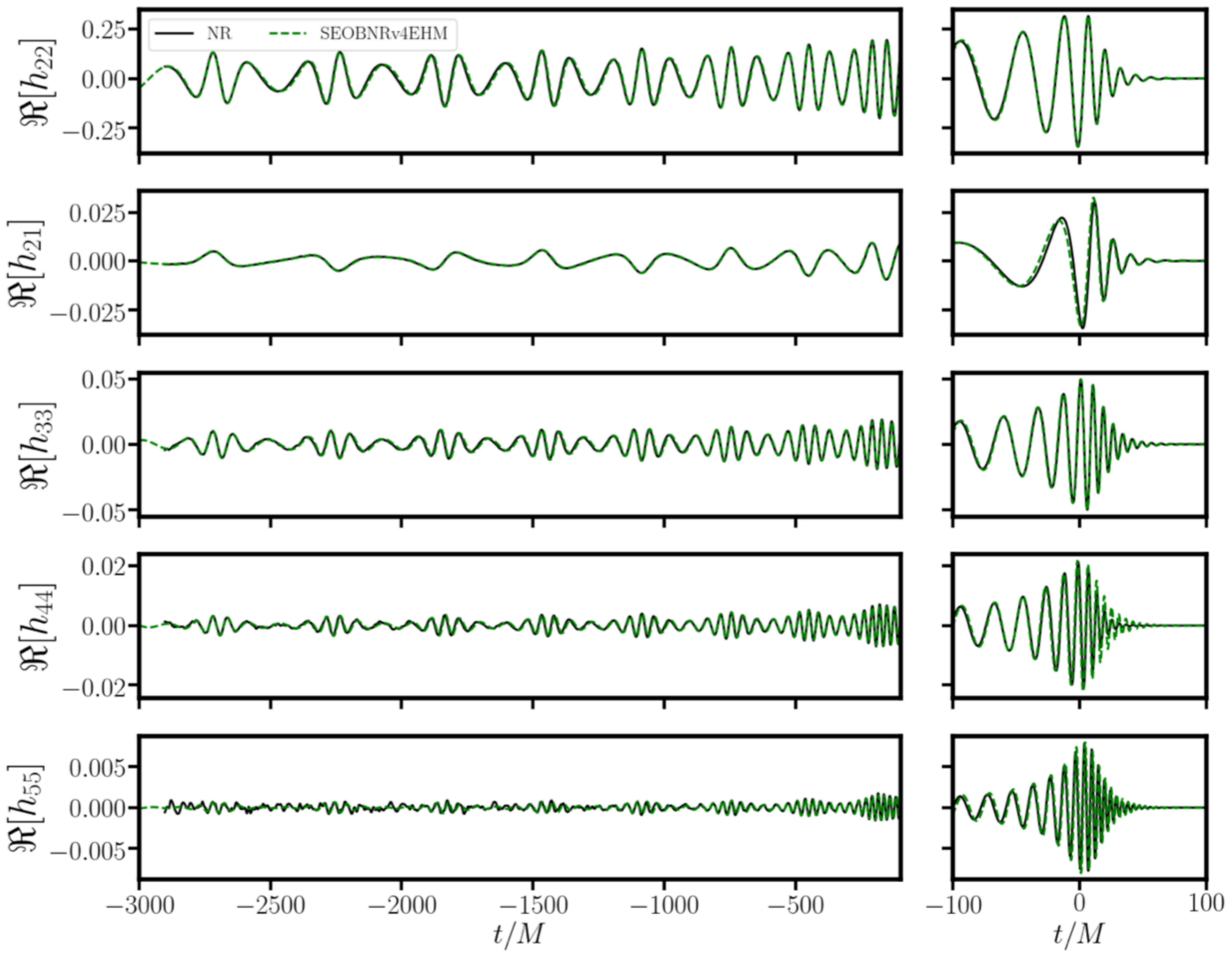
- **To assess accuracy of nPM Hamiltonians** for gravitational waveform models, **dissipative effects** need to be included, **resummation** of EOB potentials and **calibration against NR** would need to be pursued, etc.

Toward Addressing the Eccentric Problem

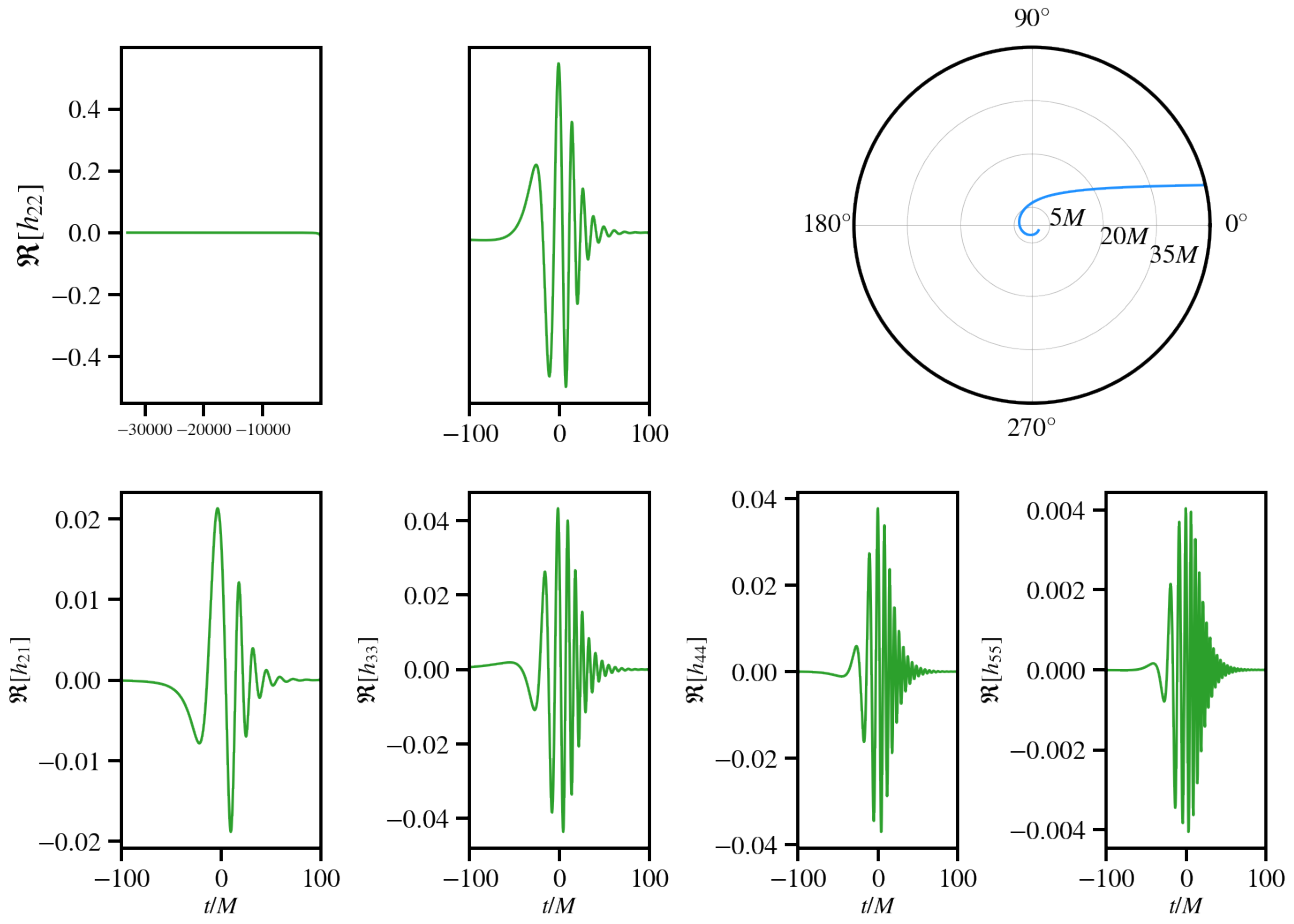
- **Measuring eccentricity** can unveil **origin of compact-binary** observed by LIGO-Virgo, and **reduce systematics.**
- **Eccentric, spinning non-precessing** SEOBNR waveforms. *(Khalil, AB, Steinhoff & Vines 21, Ramos-Buades, AB et al. 21)*

binary black-hole coalescence

mass ratio = 2, non-spinning, e = 0.06



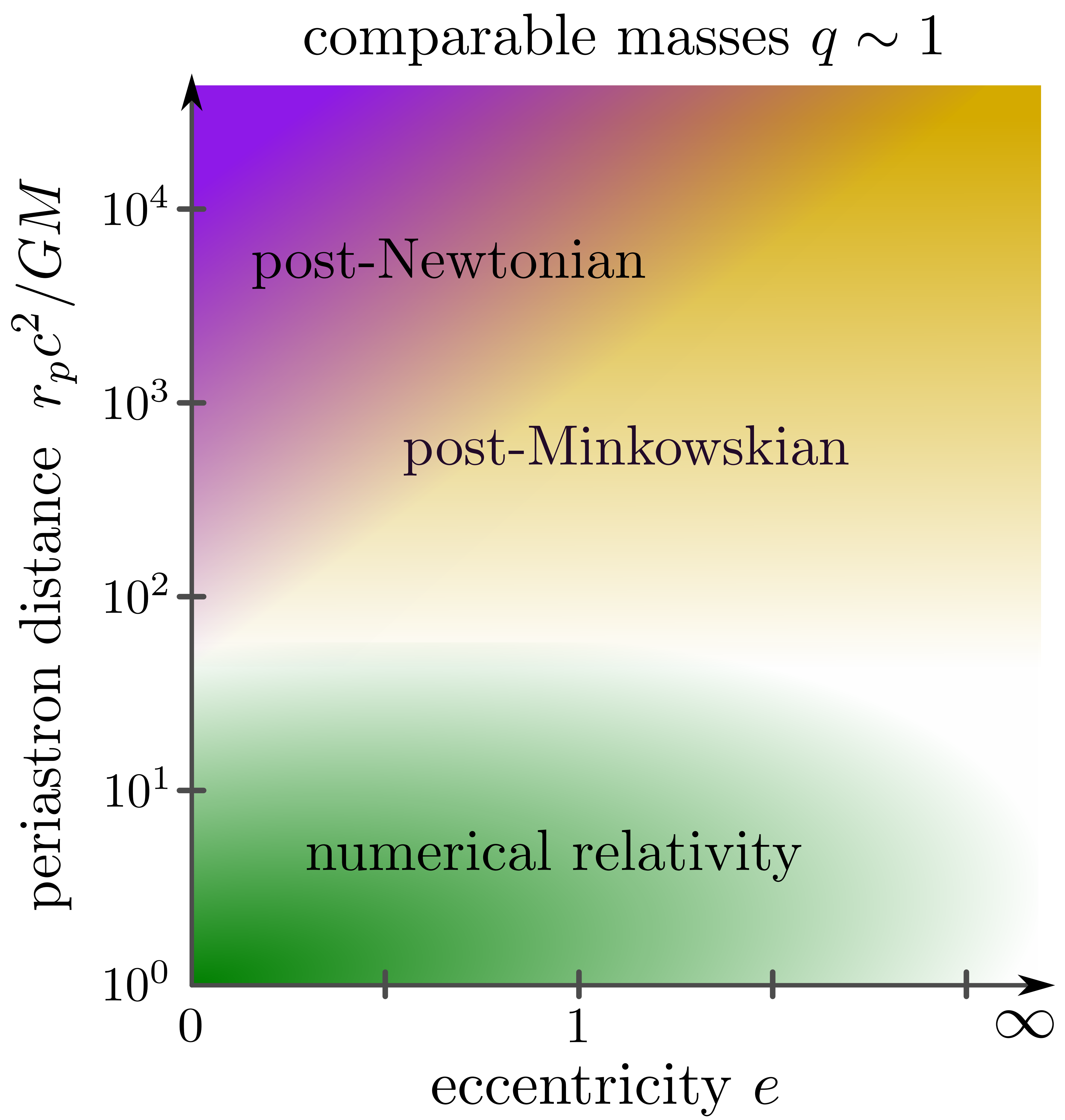
dynamical capture



(see also Huerta et al. 14-19, Hinder et al. 17, Cao & Han 17; Loutrel & Yunes 16, 17, Ireland et al. 19, Moore & Yunes 19, Tiwari et al. 19, Chiaramello & Nagar 20, Ramos-Buades et al. 20, Liu et al. 21, Nagar et al. 20, 21, Islam et al. 21, Nagar & Rettegno 21, Gamba et al. 21, Placidi et al. 21)

Toward Addressing the Eccentric Problem (contd.)

(Khalil, AB, Steinhoff & Vines 22)



- The **PM approximation is more accurate than PN approximation** for scattering encounters **at large velocities,** or equivalently **large eccentricities at fixed periastron distance**.

How Scattering Amplitudes May Improve Waveform Modeling

- **Scattering-amplitude** methods have **brought new and fresh perspectives** (and tools) to solve 2-body problem.

- Besides progress in the non-spinning case, **perturbative results in PM** have also been **extended to the spin sector** (spin-orbit and spin-spin-…). *(Jakobsen et al. 20,-22; Bern et al. 21-22; Liu et al 21; Chen et al. 21; Aoude et al. 21; Alessio & Di Vecchia 22)*

- So far, EOB Hamiltonians have been mostly based on PN results (with some contribution from GSF). Given the recent important developments in PM and GSF, natural to **explore EOB Hamiltonians based on PM, GSF and PN**. *(Damour 10; Le Tiec, Barausse & AB 12; Barausse, AB et al. 12; Antonelli, van den Meent, AB, Steinhoff & Vines 19; Antonelli, AB, et al. 19; Khalil et al. 22; Nagar et al. 22)*

- Until the full calibration of EOB waveforms against NR simulations is performed, it is **difficult to assess the actual gain of a new higher-order result in PN**. Perhaps this **may change with PM and GSF**.

- **Scattering amplitudes** may be **more effective in pushing perturbative calculations** (PM, PN,GSF) at higher order, and may suggest **new ways of resuming the building blocks** of 2-body dynamics/radiation.

- In the future **we might not need Hamiltonians and RR forces** (i.e, gauge dependent quantities) to construct **high-precision gravitational waves**, but currently this is the **only way we know how to do it.**

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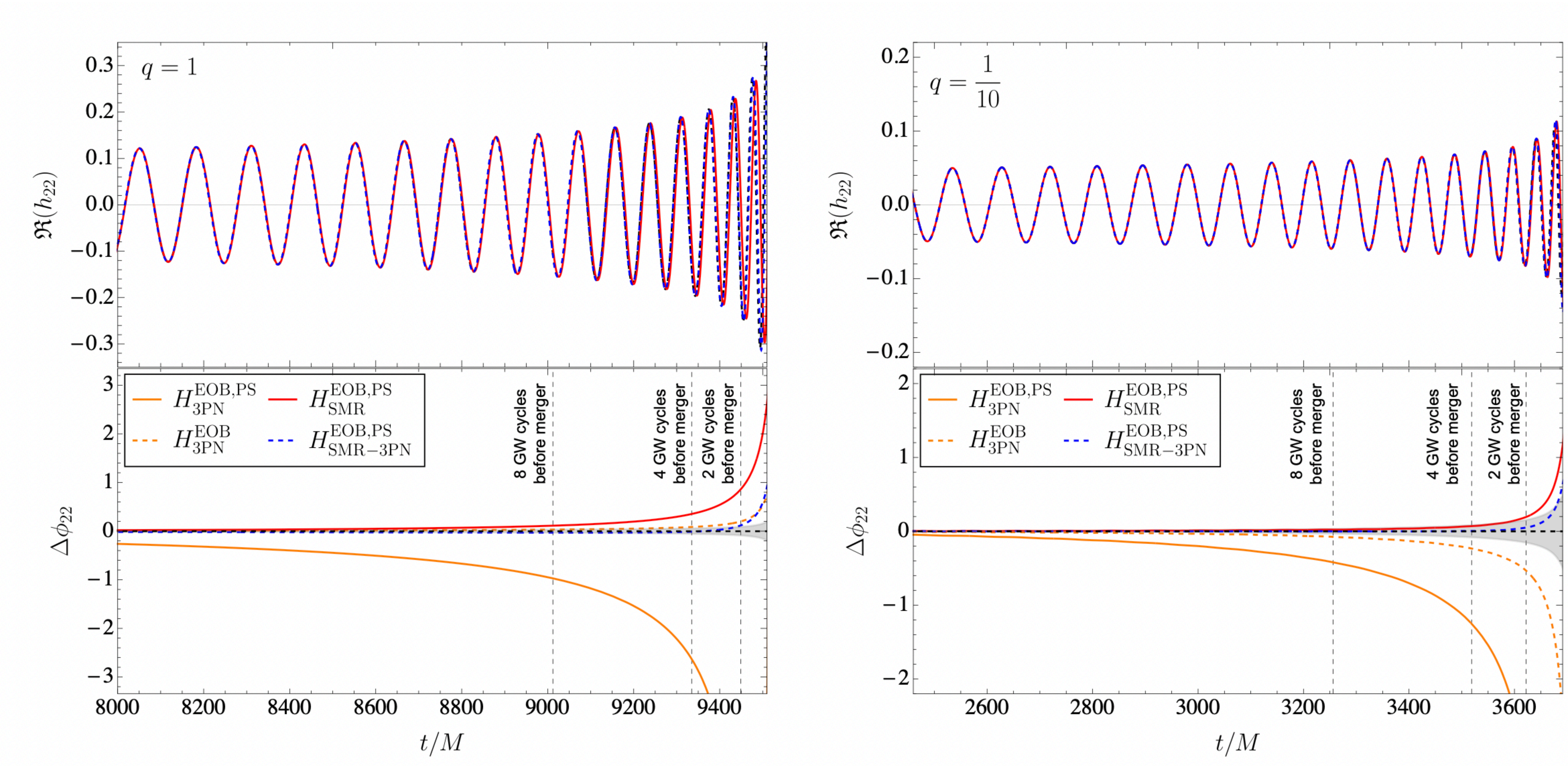
LIGO
VIRGO
KAGRA

Thank You!

Toward Improving Waveform Accuracy: GSF/EOB

(Antonelli, van den Meent, AB, Steinhoff & Vines 19)



- **Improvement of accuracy** against NR, for mass ratios larger than one, **when including GSF & PN** information in EOB Hamiltonian.