Applications of Scattering Amplitudes to Gravitational Waves

Amplitudes 2022

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Zvi Bern

ZB, J. Parra-Martinez, R. Roiban, M. Ruf. C.-H. Shen, M. Solon, M. Zeng, arXiv:2101.07254; arXiv:2112.10750

ZB, D. Kosmopoulos, A. Luna, R. Roiban, F. Teng, arXiv: 2203.06202

ZB, D. Kosmopoulos, A. Luna, R. Roiban, T. Scheopner, F. Teng, in preparation

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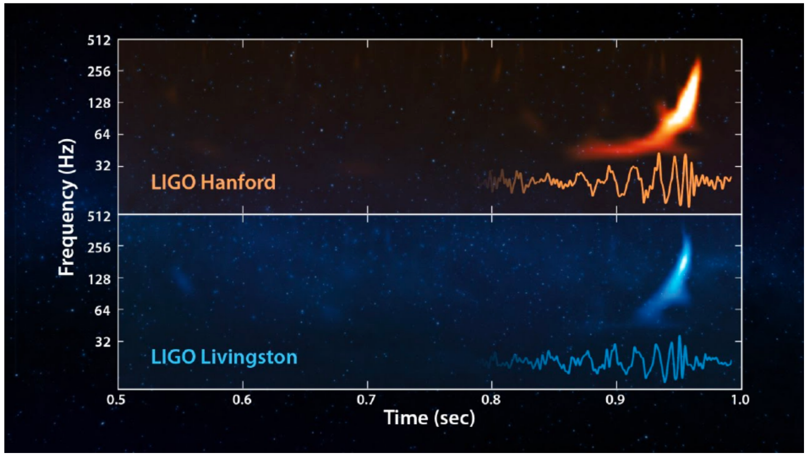
Outline

Era of gravitational-wave astronomy has begun.

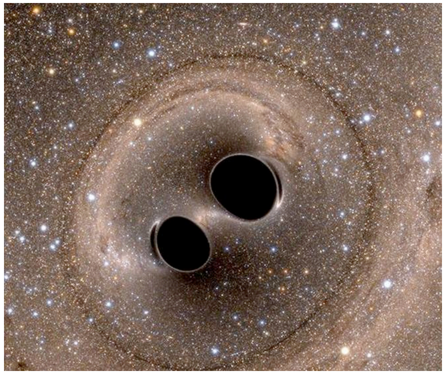


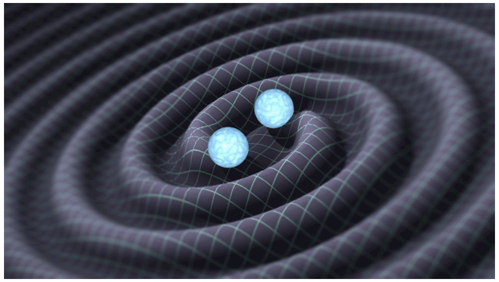
See Buonanno's talk

1. Completion of G^4 conservative part of spinless 2 body interaction.
2. New results and conjectures for spin.
4. Outlook.

Also talks from Jones, Kosower, Levi, Mogull, Shen, Travaglini

Basic Problem: Two Body Interactions





In center of mass frame:

$$m = m_A + m_B, \quad \nu = \mu/M,$$

$$\mu = m_A m_B/m, \quad P_R = P \cdot \hat{R}$$

$$\frac{H}{\mu} = \frac{P^2}{2} - \frac{Gm}{R}$$

← **Newton**

$$+ \frac{1}{c^2}\left\{ -\frac{P^4}{8} + \frac{3\,\nu\,P^4}{8} + \frac{Gm}{R}\left(-\frac{{P_R}^2\,\nu}{2} - \frac{3\,P^2}{2} - \frac{\nu\,P^2}{2}\right) + \frac{G^2 m^2}{2R^2}\right\}$$

$$+ \dots$$

1PN: Einstein, Infeld, Hoffmann; Droste, Lorentz

Hamiltonian known to 4PN order.

2PN: Ohta, Okamura, Kimura and Hiida.

3PN: Damour, Jaranowski and Schaefer; L. Blanchet and G. Faye.

4PN: Damour, Jaranowski and Schaefer; Foffa (2017), Porto, Rothstein, Sturani (2019).

PN versus PM expansion for conservative two-body dynamics

From Buonanno Amplitudes 2018

$$\mathcal{L} = -Mc^2 + \underbrace{\frac{\mu v^2}{2} + \frac{GM\mu}{r}}_{\downarrow} + \frac{1}{c^2}\Big[\dots\Big] + \frac{1}{c^4}\Big[\dots\Big] + \dots$$

$$E(v) = -\frac{\mu}{2} v^2 + \cdots$$

non-spinning compact objects

		0PN	1PN	2PN	3PN	4PN	5PN	...
0PM:	1	v^2	v^4	v^6	v^8	v^{10}	v^{12}	...
1PM:		$1/r$	v^2/r	v^4/r	v^6/r	v^8/r	v^{10}/r	...
2PM:			$1/r^2$	v^2/r^2	v^4/r^2	v^6/r^2	v^8/r^2	...
3PM:				$1/r^3$	v^2/r^3	v^4/r^3	v^6/r^3	...
4PM:					$1/r^4$	v^2/r^4	v^4/r^4	...
...						...	...	

(credit: Justin Vines)

$$1 \to Mc^2, \qquad v^2 \to \frac{v^2}{c^2}, \qquad \frac{1}{r} \to \frac{GM}{rc^2}.$$

current known PN results

current known PM results

overlap between PN & PM results

unknown

- **PM results** (Westfahl 79, Westfahl & Goller 80, Portilla 79-80, Bel et al. 81, Ledvinka et al. 10, Damour 16-17, Guevara 17, Vines 17, Bini & Damour 17-18, Vines in prep)

Importing QFT Methods

In QFT we are very good at perturbation theory.
Vast experience with gauge and gravity theories.

Highlight of imported field theory methods:

1. **Unitarity method for building integrands.** ZB, Dunbar, Dixon, Kosower
2. **Double copy: Gravity $\sim$ (gauge theory)^2** Kawai, Lewellen, Tye; ZB, Carrasco, Johansson
3. **Method of regions.** Beneke and Smirnov
4. **NRQCD and EFT methods.** Caswell and Lepage; Luke, Manohar, Rothstein
5. **IBP reductions for Feynman integrals.** Chetyrkin, Tkachov; Laporta
6. **Method of differential equations for Feynman integrals.** Kotikov, ZB, Dixon and Kosower; Gehrmann, Remiddi; Henn, Smirnov

We leverage advances in scattering amplitudes to help with gravitational-wave physics.

Various Amplitudes Methods

There are now multiple methods for using amplitudes:

- **EFT matching to extract Hamiltonian** — Cheung, Rothstein, Solon; ZB, C. Cheung, R. Roiban, C. H. Shen, M. Solon, M. Zeng
- **Calculate physical observables** — Kosower, Maybee, O'Connell (KMOC)
- **Eikonal Phase** — Amati, Ciafaloni, Veneziano; Di Vecchia, Heissenberg, Russo, Veneziano
- **Amplitude Action Relation** — ZB, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng
- **Exponential representation** — Damgaard, Plante, Vanhove
- **Heavy mass field theory** — Brandhuber, Chen, Travaglini, Wen; Damgaard, Haddad, Helset
- **Spin methods** — Arikani-Hamed, Huang, O'Connell; Guevara, Ochirov, Vines, Chen, Huang, Kim, Lee; Bautista, Geuvara, Kavanagh; ZB, Luna, Roiban, Shen, Zeng + Kosmopoulos, Teng; Febres Cordero, Kraus, Lin, Ruf, Zeng
- **World line formalisms** — Goldberger, Rothstein; Levi, Steinhoff; Dlapa, Kälin, Liu, Porto; Jakobson, Mogul, Plefka, Steinhoff; Edison, Levi

All are fine. Key issue at high orders is efficient loop integration.

Amplitude in Conservative Classical Potential Limit

ZB, Cheung, Roiban, Shen, Solon, Zeng (BCRSSZ)

The computation of O(G^3) or 3PM conservative part convinced our GR friends that amplitude methods are useful:

$$\mathcal{M}_3 = \frac{\pi G^3 \nu^2 m^4 \log \boldsymbol{q}^2}{6\gamma^2 \xi}\left[3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu\sigma^3 - \frac{48\nu\left(3 + 12\sigma^2 - 4\sigma^4\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}} - \frac{18\nu\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)}{(1+\gamma)(1+\sigma)}\right] + \frac{8\pi^3 G^3 \nu^4 m^6}{\gamma^4 \xi}\left[3\gamma\left(1-2\sigma^2\right)\left(1-5\sigma^2\right)F_1 - 32 m^2\nu^2\left(1-2\sigma^2\right)^3 F_2\right]$$

$$m = m_1 + m_2 \qquad \mu = m_A m_B/m, \qquad \nu = \mu/m, \qquad \gamma = E/m,$$

$$\xi = E_1 E_2/E^2, \qquad E = E_1 + E_2, \qquad \sigma = p_1 \cdot p_2 / m_1 m_2,$$

- **Amplitude remarkably compact.**
- **Derived conservative scattering angle has simple mass dependence.**

Observed by Antonelli, Buonanno, Steinhoff, van de Meent, Vines (1901.07102)
Comprehensive understanding: Damour

- **Extracted Hamitonian requested by our GR friends.**

Key Question: How do we know it is right?

Original check:

Compared to 4PN Hamiltonians after canonical transformation

Damour, Jaranowski, Schäfer; Bernard, Blanchet, Bohé, Faye, Marsat

Thibault Damour seriously questioned correctness.
Specific corrections proposed. Damour, arXiv:1912.02139v1

Subsequent calculations confirm our 3PM Hamiltonian:

1. **Papers confirming our result in 6PN overlap.**
   Blümlein, Maier, Marquard, Schäfer;
   Bini, Damour, Geralico
2. **Subsequent calculations reproducing our 3PM result.**
   Cheung and Solon; Kälin, Liu, Porto
3. **Scattering angle checks.** ZB, Ita, Parra-Martinez, Ruf
4. **Adding real radiation solves mass singularity puzzle.**
   Di Vecchia, Heissenberg, Russo, Veneziano; Damour

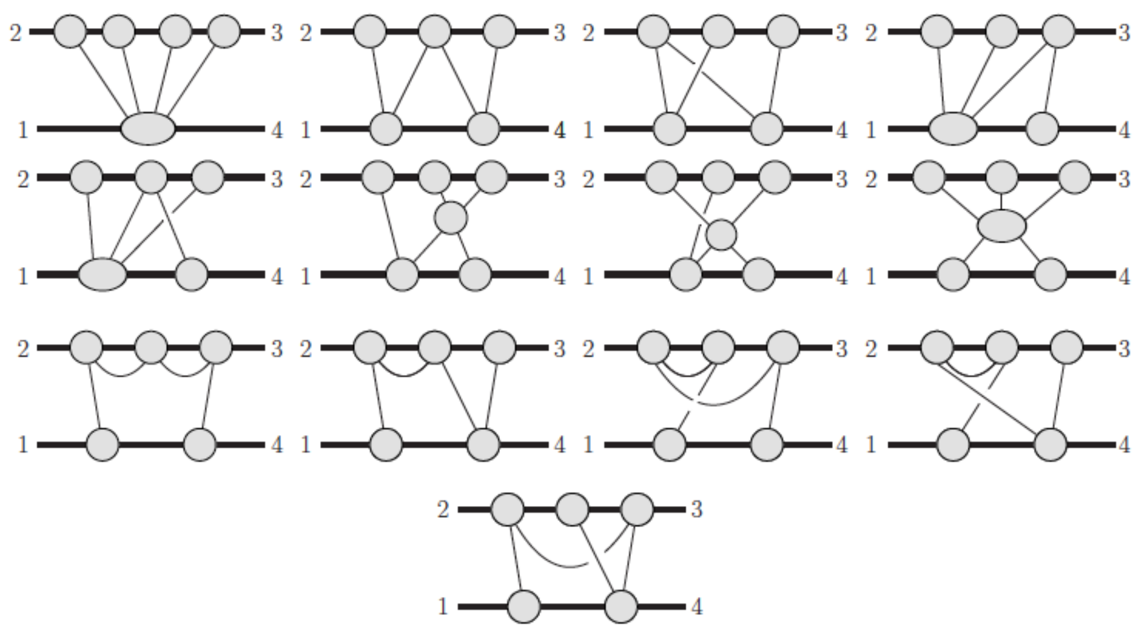
3PM results have passed highly nontrivial checks and careful scrutiny.

Higher Order Scalability: $O(G^4)$

Our GR friends wanted the next order.
Methods scale well.

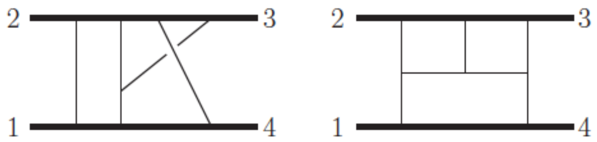
ZB, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng

At 4PM or $O(G^4)$ similar, except cuts more complicated and integrals significantly harder.



High Loop Integration

ZB, Parra-Martinez, Roiban, Ruf. Shen, Solon, Zeng



Integration more challenging beyond 2 loops.

Developed a hybrid approach that combines ideas from various methods:

1. Method of regions to separate potential and radiation.

Beneke and Smirnov

2. Nonrelativistic integration. Velocity expand and then mechanically integrate. Get first few orders in velocity. Boundary conditions.

Cheung, Rothstein, Solon

3. Integration by parts and differential equations. Imported from QCD.

Chetyrkin, Tkachov; Laporta; Kotikov, Bern, Dixon and Kosower; Gehrmann, Remiddi.

Single scale integrals!

Parra-Martinez, Ruf, Zeng

IBP: $$0 = \int \prod_i^L \frac{d^D \ell_i}{(2\pi)^D} \frac{\partial}{\partial \ell_i^\mu} \frac{N^\mu(\ell_k, p_M)}{Z_1 \dots Z_n}$$

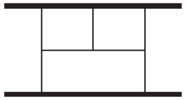
Solve linear relations between integrals in terms of master integrals.

DEs: $$\frac{\partial}{\partial s_i} I_j^{\text{master}} = \text{simplified via IBP}$$

Solve DEs either as series or basis of functions.

- **Many tools available: FIRE6 is more than sufficient at 3 loops.** Smirnov, Chuharev
- **Elliptic integrals make an appearance. Just a minor annoyance.**

The $O(G^4)$ Conservative Contributions



ZB, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng

$O(G^4)$ amplitude

$$\mathcal{M}_4^{\rm cons} = G^4 M^7 \nu^2 |\boldsymbol{q}| \pi^2 \left[\mathcal{M}_4^{\rm p} + \nu\left(4\mathcal{M}_4^{\rm t}\log\left(\tfrac{p_\infty}{2}\right) + \mathcal{M}_4^{\pi^2} + \mathcal{M}_4^{\rm rem}\right)\right] + \overbrace{\int_\ell \frac{\tilde{I}_{r,1}^4}{Z_1 Z_2 Z_3} + \int_\ell \frac{\tilde{I}_{r,1}^2 \tilde{I}_{r,2}}{Z_1 Z_2} + \int_\ell \frac{\tilde{I}_{r,1}\tilde{I}_{r,3}}{Z_1} + \int_\ell \frac{\tilde{I}_{r,2}^2}{Z_1}}^{\textbf{Iteration. No need to compute}}$$

$D = 4 - 2\epsilon$

tail effect

$$Z_j = -4E|\boldsymbol{p}|\big((\boldsymbol{\ell}_1 + \boldsymbol{\ell}_2 + \cdots + \boldsymbol{\ell}_j)\cdot\hat{\boldsymbol{z}} + i0\big)$$

Amplitude action relation: Classical amplitude is exponential of radial action.

$$i\mathcal{M}(\boldsymbol{q}) = \int_J \left(e^{iI_r(J)} - 1\right)$$

$$\tilde{I}_r(\boldsymbol{q}) = \int_J I_r(J) \equiv 4E|\boldsymbol{p}| \int \mu^{-2\epsilon} d^{D-2}\boldsymbol{b}\, e^{i\boldsymbol{q}\cdot\boldsymbol{b}}\, I_r(J)$$

$$\int_J \frac{(iI_r(J))^n}{n!} = i\int_\ell \frac{\tilde{I}_r(\boldsymbol{\ell}_1)\ldots\tilde{I}_r(\boldsymbol{\ell}_n)}{Z_1\ldots Z_{n-1}}$$

$$\mathcal{M}_4^{\rm t} = r_1 + r_2\log\left(\tfrac{\sigma+1}{2}\right) + r_3\frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}},$$

$$\mathcal{M}_4^{\pi^2} = r_4\pi^2 + r_5\,\mathrm{K}\left(\tfrac{\sigma-1}{\sigma+1}\right)\mathrm{E}\left(\tfrac{\sigma-1}{\sigma+1}\right) + r_6\,\mathrm{K}^2\left(\tfrac{\sigma-1}{\sigma+1}\right) + r_7\,\mathrm{E}^2\left(\tfrac{\sigma-1}{\sigma+1}\right),$$

$$\mathcal{M}_4^{\rm p} = -\frac{35\left(1 - 18\sigma^2 + 33\sigma^4\right)}{8\left(\sigma^2-1\right)}$$

$$\mathcal{M}_4^{\rm rem} = r_8 + r_9\log\left(\tfrac{\sigma+1}{2}\right) + r_{10}\tfrac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}} + r_{11}\log(\sigma) + r_{12}\log^2\left(\tfrac{\sigma+1}{2}\right) + r_{13}\tfrac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}}\log\left(\tfrac{\sigma+1}{2}\right) + r_{14}\tfrac{\operatorname{arccosh}^2(\sigma)}{\sigma^2-1}$$
$$+ r_{15}\,\mathrm{Li}_2\left(\tfrac{1-\sigma}{2}\right) + r_{16}\,\mathrm{Li}_2\left(\tfrac{1-\sigma}{1+\sigma}\right) + r_{17}\tfrac{1}{\sqrt{\sigma^2-1}}\left[\mathrm{Li}_2\left(-\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right) - \mathrm{Li}_2\left(\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right)\right].$$

$\nu = m_1 m_2/(m_1+m_2)^2$ $\qquad \sigma = p_1\cdot p_2/m_1 m_2,$ $\qquad r_i$ rational coefficients

- **Derived using Feynman *i* epsilon prescription.**
- **Real part identical to result using Damour's PV prescription for conservative.**

This is the *complete* conservative contribution.

How Do We Know it is Right?

ZB, Parra-Martinez, Roiban, Ruf,
Shen, Solon, Zeng

Prior to the completion of our result, Bini, Damour and Geralico presented first three terms in the velocity expansion of the scattering amplitude:

$$\mathcal{M}_4^{\text{radgrav,f}} = \frac{12044}{75}p_\infty^2 + \frac{212077}{3675}p_\infty^4 + \frac{115917979}{793800}p_\infty^6 - \frac{9823091209}{76839840}p_\infty^8 + \frac{115240251793703}{1038874636800}p_\infty^{10} + \cdots$$

First 3 terms match PN results of Bini, Damour, Geralico!

Seems there is always some controversy:

- **Some disagreement in one term with 5PN of Blumlein, Maier, Marquard, Schafer.**
- **Claim by Diapa, Kalin, Liu, Porto in their calculation that we dropped “memory” pieces. *We disagree.***

Status

1) **Disagreements with Blumlein et al and Porto et al still need to be fully resolved, though consensus is starting to form.**

Damour's "Good Polynomiality": No $1/(m_1 + m_2)$. Manifest in the scattering amplitude.

Forthcoming paper from Bjerrum-Bohr, Vanhove, Plante, also confirms our result.

2) **We give a Hamiltonian valid for (unbound) hyperbolic motion. Analytic continuation (bound) elliptic motion at $O(G^4)$ is nontrivial. "Tail Effect". This issue is similar to the one encountered in PN expansion.**

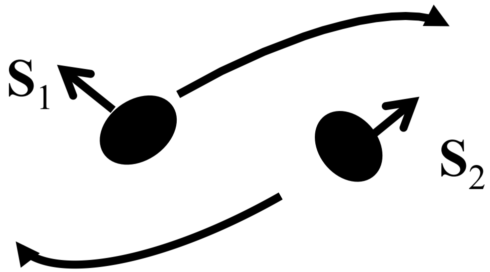
3) **Great progress on dissipative part. Odd in velocity terms computed.**

Manohar, Ridgeway, Shen

See Shen's talk.

Case of Spin

ZB, A. Luna, R. Roiban, C. H. Shen, M. Zeng
ZB, Kosmopoulos, Luna, Roiban, Teng
ZB, Kosmopoulos, Luna, Roiban, Sheoepner, Teng, to appear



Kerr black holes.

Two issues I will summarize here:

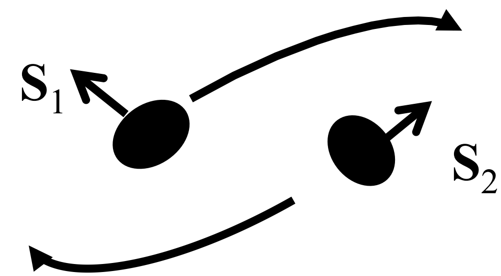
1. **How to define define EFT for Kerr black hole 2 body interactions?**

2. **How many physical parameters (Wilson coefficients) are needed to describe gravitational coupling to generic spinning objects (e.g. neutron stars)?**

   Results suggest more parameters than previously appreciated.

Basic Setup

ZB, Luna Roiban, Shen, Zeng



We start with an EFT of particle with arbitrary spin

$$\mathcal{L}_{\min} = -R - \frac{1}{2}\phi_s(\nabla^2 + m^2)\phi_s + \ldots$$

$$\mathcal{L}_C = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{\mathrm{ES}^{2n}}}{m^{2n}} \nabla_{f_{2n}} \ldots \nabla_{f_3} R_{a f_1 b f_2} \nabla^a \phi_s \mathbb{S}^{(f_1} \mathbb{S}^{f_2} \ldots \mathbb{S}^{f_{2n})} \nabla^b \phi_s$$

arbitrary spin field

$$- \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{\mathrm{BS}^{2n+1}}}{m^{2n+1}} \nabla_{f_{2n+1}} \ldots \nabla_{f_3} *R_{a f_1 b f_2} \nabla^a \phi_s \mathbb{S}^{(f_1} \mathbb{S}^{f_2} \ldots \mathbb{S}^{f_{2n+1})} \nabla^b \phi_s$$

$$\mathbb{S}^a \equiv \frac{-i}{2m} \epsilon^{abcd} M_{cd} \nabla_b$$

1-1 correspondence to worldline Lagrangian. Porto, Rothstein; Levi, Steinhoff

In classical limit Lorentz generators correspond to spin tensors :

$$\varepsilon(p_1)\{M^{a_1 b_1} M^{a_2 b_2} \ldots M^{a_j b_j}\}\varepsilon(p_2) = S(p_1)^{a_1 a_1} S(p_1)^{a_2 b_2} \cdots S(p_1)^{a_j b_j}\, \varepsilon(p_1)\cdot\varepsilon(p_2) + \cdots$$

$$S^{\alpha\beta}(p) = -\frac{1}{m}\epsilon^{\alpha\beta\mu\nu} p_\mu S_\nu(p) \qquad S^\alpha(p) = -\frac{1}{2m}\epsilon^{\alpha\beta\mu\nu} p_\beta S_{\mu\nu}(p)$$

By construction we satisfy: $p_\mu S^\mu(p) = 0$ **trivial in the rest frame**

Extra Operators

ZB, Kosmopoulos, Luna, Roiban, Teng

Infinite classes of extra operators, e.g. "*H*" class:

$$\mathcal{L}_H = -\sum_{n=1}^{\infty} \frac{(-1)^n(2n-1)}{(2n)!(2n+1)} \frac{H_{2n}}{m^{2n-2}} \nabla_{f_{2n}} \ldots \nabla_{f_3} R_{(a|f_1|b)f_2} \phi_s M^{a(f_1} M^{|b|f_2} \mathbb{S}^{f_3} \ldots \mathbb{S}^{f_{2n})} \phi_s$$

$$+\sum_{n=1}^{\infty} \frac{(-1)^n n}{(2n+1)!(n+1)} \frac{H_{2n+1}}{m^{2n-1}} \nabla_{f_{2n+1}} \ldots \nabla_{f_3} *R_{(a|f_1|b)f_2} \phi_s M^{a(f_1} M^{|b|f_2} \mathbb{S}^{f_3} \ldots \mathbb{S}^{f_{2n+1})} \phi_s$$

$$\mathcal{L}_{H_2} = \frac{H_2}{8} R_{abcd} \phi_s M^{ab} M^{cd} \phi_s \qquad \mathcal{L}_{D_2} = \frac{D_2}{m^2} R_{af_1bf_2} \nabla_i \phi_s M^{if_1} M^{bf_2} \nabla^a \phi_s$$

- **$H_2 = 1$ improves high energy behavior of Compton.**
- **Black hole solution picks $H_2 = 1$.** Cucchieri, Porrati, Deser; Chiodaroli, Johansson, Pichini
- **For 3 point matrix element in classical limit this Lagrangian looks to be redundant.** $p_\mu S^{\mu\nu} = 0$ **makes extra operators redundant. SSC.**

Besides these there are infinite classes of other operators with higher powers of Riemann,

$$\mathbb{S}^a \equiv \frac{-i}{2m} \epsilon^{abcd} M_{cd} \nabla_b$$

***R*^{2} operators at *S*^{4} and beyond:**

$$\frac{C_{\mathrm{E}^2\mathrm{S}^4}}{m^6} R_{af_1bf_2} R_{cf_3df_4} \nabla^{(a}\nabla^{c)} \phi_s \mathbb{S}^{(f_1}\mathbb{S}^{f_2}\mathbb{S}^{f_3}\mathbb{S}^{f_4)} \nabla^{(b}\nabla^{d)} \phi_s \,,$$

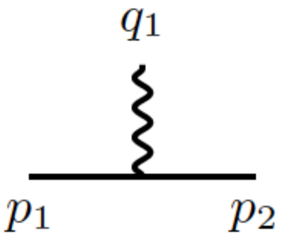
$$\frac{C_{\mathrm{B}^2\mathrm{S}^4}}{m^6} *R_{(a|f_1|b)f_2} *R_{(c|f_3|d)f_4} \nabla^{(a}\nabla^{c)} \phi_s \mathbb{S}^{(f_1}\mathbb{S}^{f_2}\mathbb{S}^{f_3}\mathbb{S}^{f_4)} \nabla^{(b}\nabla^{d)} \phi_s \,, \ldots$$

ZB, Kosmopoulos, Luna, Roiban, Teng

Our three point interactions match known results:

$$T^{\mu\nu} = \frac{p_1^\mu p_1^\nu}{m} \sum_{n=1}^{\infty} \frac{C_{2n}}{(2n)!} \left(\frac{q \cdot S}{m}\right)^n - \frac{i q_\rho p_1^{(\mu} S^{\nu)\rho}}{m} \sum_{n=1}^{\infty} \frac{C_{2n+1}}{(2n+1)!} \left(\frac{q \cdot S}{m}\right)^{2n}$$

Vines

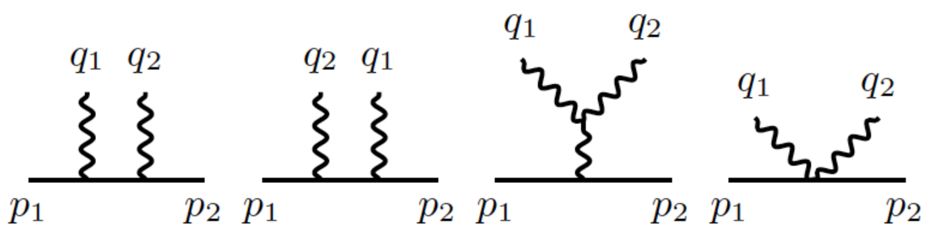


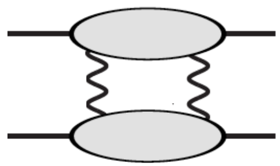
Kerr black hole is $C_n = 1$

At this level the operators in $\mathcal{L}_C$ and $\mathcal{L}_H$ are redundant.

$$C_{2n} = C_{\mathrm{ES}^{2n}} + H_{2n} \qquad C_{2n+1} = C_{\mathrm{BS}^{2n+1}} + H_{2n+1}$$

Compton





Strong check: with appropriate choices of Wilson coefficients, our Compton through S^3 matches the one recently obtained from world line.

Saketh and Vines (a few days ago)

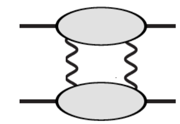
Kerr Black Holes At $O(G^2)$

Guevara, Ochirov, Vines; Chen, Huang, Kim, Lee; ZB, Luna, Roiban, Shen, Zeng, Bautista, Guevara, Kavanagh; Aoude, Haddad, Helset

spin structures

$$\mathcal{M}^{G^2}_{\text{classical}} = \frac{2\pi^2 G^2 \varepsilon_1 \cdot \varepsilon_4 \varepsilon_2 \cdot \varepsilon_3}{\sqrt{-q^2}} \sum_n \sum_i \alpha^{(n,i)} \mathcal{O}^{(n,i)}$$

coefficients



Through G^2 and S^4 only blue structures contribute to black hole interactions:

$S_1^n\, S_2^0$	i		i		i	
$\mathcal{O}^{(2,i)}$	1	$\mathcal{E}_1^2$	2	$q^2(u_2\cdot a_1)^2$	3	$(q\cdot a_1)^2$
$\mathcal{O}^{(4,i)}$	1	$\mathcal{E}_1^4$	2	$q^2(u_2\cdot a_1)^2\mathcal{E}_1^2$	3	$q^4(u_2\cdot a_1)^4$
	4	$(q\cdot a_1)^2\mathcal{E}_1^2$	5	$q^2(q\cdot a_1)^2(u_2\cdot a_1)^2$	6	$(q\cdot a_1)^4$

$$\mathcal{O}^{(3,i)} = \mathcal{E}_1 \mathcal{O}^{(2,i)}$$

$$\mathcal{E}_i = -i\epsilon^{\mu\nu\rho\lambda} u_{1\mu} u_{2\nu} q_\rho a_{i\lambda}$$

$$a_i^\mu = S_i^\mu / m_i$$

$$q = p_2 + p_3$$

$$u_i^\mu = p_i^\mu / m_i$$

QFT Kerr black hole *conjecture*: ZB, Kosmopoulos, Luna, Roiban, Teng

- **Structures must satisfy *shift symmetry*:** $a_i^\mu \to a_i^\mu + \xi_i \dfrac{q^\mu}{q^2}$
- **At high-energy amplitudes should not grow faster than tree level.**

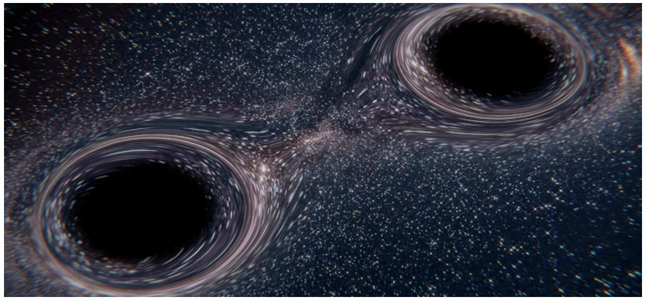
Classical limit: $p_i \cdot q = \pm q^2/2 \to 0$

Similar simultaneous conjecture from Aoude, Haddad, Helset.

We use this conjecture to define Kerr black hole 2 body interactions.

Black Holes through S^5 and Beyond

$$\mathcal{M}^{G^2}_{\text{classical}} = \frac{2\pi^2 G^2 \varepsilon_1 \cdot \varepsilon_4 \varepsilon_2 \cdot \varepsilon_3}{\sqrt{-q^2}} \sum_n \sum_i \alpha^{(n,i)} \mathcal{O}^{(n,i)}$$



Determined by shift symmetry and demanding good high-energy behavior.

$$\alpha^{(5,i)} = \frac{m_1^2 m_2^2 \sigma}{768(-1+\sigma^2)^3}\left(\gamma^{(5,i)} m_1 + \frac{1}{75}\delta^{(5,i)} m_2\right)$$

$$\gamma^{(5,1)}_{\text{Kerr}} = 16(7-13\sigma^2)\,, \quad \frac{1}{75}\delta^{(5,1)}_{\text{Kerr}} = 24(1-4\sigma^2)\,,$$

$$\gamma^{(5,2)}_{\text{Kerr}} = 32(11\sigma^2-5)\,, \quad \frac{1}{75}\delta^{(5,2)}_{\text{Kerr}} = 48(2+\sigma^2)\,,$$

$$\gamma^{(5,3)}_{\text{Kerr}} = 16(3\sigma^2-1)\,, \quad \frac{1}{75}\delta^{(5,3)}_{\text{Kerr}} = 8(12-16\sigma^2+7\sigma^4)$$

$$\sigma = \frac{p_1 \cdot p_2}{m_1 m_2}$$

Determines G^2 S^5 conservative interactions between two Kerr black holes

Matches results from Aoude, Haddad, Helset, who used similar ideas to also go to even higher powers of spin.

Extra Wilson Coefficients in Field Theory

Worked out dependence on *H* and *C* series of operators through S^5

$Z_{2,1} = C_2 + 1$	$Z_{2,2} = C_2 - 1$
$Z_{3,1} = 3C_2 + C_3$	$Z_{3,2} = C_2 - C_3$
$Z_{4,1} = 3C_2^2 + 4C_3 + C_4$ $Z_{4,2} = 3C_2^2 + C_4$	$Z_{4,3} = C_2^2 - C_4$ $Z_{4,4} = 3C_2^2 - 4C_3 + C_4$
$Z_{5,1} = 10C_2C_3 + 5C_4 + C_5$	$Z_{5,2} = 2C_2C_3 - C_4 - C_5$ $Z_{5,3} = 2C_2C_3 - 3C_4 + C_5$

i	$\gamma^{(2,i)}$	i	$\gamma^{(2,i)}$
1	$7 + 23C_2 - Z_{2,1}\sigma^2(102 - 95\sigma^2)$	3	$12Z_{2,2}(\sigma^2 - 1)^2(5\sigma^2 - 1)$
2	$5 - 11C_2 + 5Z_{2,1}\sigma^2(6 - 7\sigma^2)$		
i	$\gamma^{(3,i)}$	i	$\gamma^{(3,i)}$
1	$Z_{3,1}(5 - 9\sigma^2)$	3	$4Z_{3,2}(\sigma^2 - 1)(5\sigma^2 - 3)$
2	$Z_{3,1}(7\sigma^2 - 3)$		
i	$\gamma^{(4,i)}$	i	$\gamma^{(4,i)}$
1	$44C_3 + 59Z_{4,}$ [illegible]		[illegible]
2	$72C_3 - 78Z_{4,}$ [illegible]		[illegible]
3	$28C_3 - 9Z_4$ [illegible]		[illegible]
i	[illegible]		[illegible]
1	$Z_{5,1}$ [illegible]		[illegible]
2	$2Z_{5,1}(11\sigma^2 - 5)$	5	$12Z_{5,2}(\sigma^2 - 1)(3 - 7\sigma^2)$
3	$Z_{5,1}(3\sigma^2 - 1)$	6	$8Z_{5,3}(\sigma^2 - 1)^2(3 - 5\sigma^2)$

i	$\delta_0^{(2,i)}$	$\delta_1^{(2,i)}$	$\delta_2^{(2,i)}$	$\delta_3^{(2,i)}$
1	$8Z_{2,1}$	$-(68 + 52C_2)$	$60Z_{2,1}$	0
2	$4(3 - C_2)$	$-12Z_{2,1}$	0	0
3	$-4Z_{2,2}$	$68Z_{2,2}$	$-124Z_{2,2}$	$60Z_{2,2}$

i	$\delta_0^{(3,i)}$	$\delta_1^{(3,i)}$
1	$3(H_2 - 2)H_2 - (C_2 - 8)C_2$	$C_2(2C_2 - 13) - 2C_3 - 3(H_2 - 2)H_2$
2	$C_2(4C_2 - 5) + 2C_3 - 5(H_2 - 2)H_2$	$5(C_2 - H_2)(2 - C_2 - H_2)$
3	$(5 - 2C_2)C_2 - 2C_3 + (H_2 - 2)H_2$	$2[C_2(3C_2 - 8) + 4C_3 - (H_2 - 2)H_2]$
	$\delta_2^{(3,1)} = \delta_2^{(3,2)} = 0\,,\qquad \delta_2^{(3,3)} = (11 - 4C_2)C_2 - 6C_3 + (H_2 - 2)H_2$	

	i		i	
$\delta_0^{(4,i)}$	1	$8[45C_2^2 + 5C_3 - 5C_2(C_3 + 9H_2) + 5(H_2 - 1)H_3 - H_4]$	4	$10[4C_4 - 5C_3 + C_2(36C_2 - 11C_3 - 24H_2) - 5H_3(H_2 - 1)] - 4H_4$
	2	$-10[36C_2^2 - 7C_3 - 25C_2C_3 - 4C_4 - 48C_2H_2 + 9(H_2 - 1)H_3] - 4H_4$	5	$10[C_2(9C_2 + 10C_3 - 15H_2) + 4H_2H_3 - 2(C_3 + C_4 + 2H_3)] + 8H_4$
	3	$4H_4 - 10C_4 + 15C_2(9C_2 - 11H_2)$	6	$135C_2^2 - 5C_2(8C_3 + 9H_2) - 10[4C_3 + C_4 + 4(H_2 - 1)H_3] + 4H_4$
$\delta_1^{(4,i)}$	1	$-10[120C_2^2 + 51C_3 + 4C_4 - C_2(19C_3 + 108H_2) + 19(H_2 - 1)H_3 - 2H_4]$	4	$2[65C_3 - 825C_2^2 + 50C_4 + 5C_2(43C_3 + 99H_2) + 35(H_2 - 1)H_3 + 4H_4]$
	2	$2[615C_2^2 + 85C_3 - 50C_4 - 5C_2(49C_3 + 153H_2) + 45(H_2 - 1)H_3 + 8H_4]$	5	$10[15C_2^2 - 5C_3 - 11C_2C_3 + 16C_4 - 15C_2H_2 - 23(H_2 - 1)H_3] - 28H_4$
	3	$50[C_3 + (H_2 - 1)H_3] - 165C_2^2 - 5C_2(10C_3 - 33H_2) - 4H_4$	6	$345C_2H_2 - 825C_2^2 + 80[6C_3 + (H_2 - 1)H_3] - 24H_4$
$\delta_2^{(4,i)}$	1	$5[243C_2^2 + 78C_3 - 14C_2C_3 + 14C_4 - 201C_2H_2 + 30(H_2 - 1)H_3] - 12H_4$	4	$10[273C_2^2 - 33C_3 - 31C_2C_3 - 44C_4 - 165C_2H_2 + (H_2 - 1)H_3] - 4H_4$
	5	$-2[525C_2^2 - 55C_3 + C_2(55C_3 - 525H_2) - 175(H_2 - 1)H_3 + 6H_4]$	5	$-10[69C_2^2 - 51C_3 + 43C_2C_3 + 14C_4 - 75C_2H_2 - 69(H_2 - 1)H_3] + 32H_4$
	3	$50[(C_2 - 1)C_3 + H_3 - H_2H_3]$	6	$2[-640C_3 + 30C_4 + 5C_2(195C_2 + 32C_3 - 105H_2) + 24H_4]$
$\delta_3^{(4,i)}$	1	$5[16C_3 - C_2(57C_2 + 16C_3 - 57H_2)]$	4	$-10[183C_2^2 - 47C_3 + 23C_2C_3 - 30C_4 - 129C_2H_2 + 3(H_2 - 1)H_3]$
	2	$350[(C_2 - 1)C_3 + H_3 - H_2H_3]$	5	$10[45C_2^2 - 79C_3 + 79C_2C_3 - 45C_2H_2 - 85(H_2 - 1)H_3] - 12H_4$
	3	0	6	$-10[C_2(225C_2 + 56C_3 - 153H_2) + 8H_2H_3 + 4(-34C_3 + 2C_4 - 2H_3 + H_4)]$
$\delta_4^{(4,i)}$	1	0	4	$10[-22C_3 + C_2(39C_2 + 22C_3 - 39H_2)]$
	2	0	5	$-350[(C_2 - 1)C_3 - H_3(H_2 - 1)]$
	3	0	6	$5[3(-40C_3 + 2C_4 + C_2(85C_2 + 24C_3 - 71H_2)) + 8(H_2 - 1)H_3] + 12H_4$
$\delta_5^{(4,i)}$	1	0	4	0
	2	0	5	0
		[illegible]		[illegible] $_2 + 16C_3 - 57H_2)]$

	i	
		[illegible] $- 7E_7 - 2E_2 + E_4 + 8E_3 + 2E_1)$
		[illegible] $_5 - 14E_7 - E_2 - 31E_4 - 13E_3 + E_1$
		[illegible] $3E_2 - 33E_4 - 29E_3 - 3E_1$
		[illegible] $E_6 - 34E_7 - 11E_2 + 19E_4 + 47E_3 + 3E_1$
		[illegible] $_6 - 20E_7 - E_2 - 61E_4 - 43E_3 - E_1$
	6	$20[30H_4 - 285C_4 - 48C_5 + 5(18C_3H_2 - 8C_3^2 + 24C_4H_2 + 9C_2(5C_3 - 2H_3) + 8H_3^2 - 6H_2H_4)] + 596H_5 + 2[2E_5 + 3E_6 - 10E_7 - 2E_2 - 7(2E_4 + E_3) + E_1]$
$\delta_1^{(5,i)}$	1	$-75[228C_2C_3 + 39C_4 + 6C_5 - 76C_3H_2 - 4C_4H_2 - 72C_2H_3 + 8H_3^2 + 14(H_2 - 1)H_4] + 69H_5 - 42E_5 - 24E_6 + 42E_7 + 11E_2 + 5E_4 - 11(3E_3 + E_1)$
	2	$50[33C_4 + 6C_5 + 48C_3H_2 + 72C_4H_2 - 36C_2(C_3 - 3H_3) - 44H_3^2 - 36(H_2 - 1)H_4] - 212H_5 - 8(7E_5 + 2E_6 - 7E_7 - E_2 - 10E_4 - 3E_3 + E_1)$
	3	$-75[9C_4 - 84C_2C_3 + 2C_5 + 44C_3H_2 - 4C_4H_2 + 40C_2H_3 + 8H_3^2 + 10(H_2 - 1)H_4] - 281H_5 - 14E_5 + 8E_6 + 14E_7 - 3E_2 + 75E_4 + 57E_3 + 3E_1$
	4	$60(250C_4 + 52C_5 - 35H_4) + 50[56C_3^2 - 3H_2(6C_3 + 29C_4 - 14H_4) - 48H_3^2 + 6C_2(35H_3 - 93C_3)] - 1054H_5 - 92E_5 - 66E_6 + 124E_7 + 39E_2 - 69E_4 - 203E_3 - 17E_1$
	5	$5[3060C_3H_2 - 880C_3^2 + 60C_2(9C_3 - 34H_3) + 3(328C_5 - 5C_4(169 - 200H_2) + 80H_3^2 - 730(H_2 - 1)H_4)] - 1579H_5 - 50E_5 + 82E_7 + 9(E_2 + 19E_4 - 2E_6) + 67E_3 - 5E_1$
	6	$20[80C_3^2 + 1095C_4 + 84C_5 - 120C_3H_2 - 510C_4H_2 - 160H_3^2 + 60C_2(9H_3 - 19C_3) + 90(H_2 - 1)H_4] - 1788H_5 - 8(5E_5 + 5E_6 - 11E_7 - 2E_2 - 17E_4 - 5E_3 + 2E_1)$
$\delta_2^{(5,i)}$	1	$50[22C_3^2 + 21C_4(H_2 - 1) - 42C_3H_2 + 42C_2(2C_3 - H_3)] + 28E_5 + 16E_6 - 28E_7 - 7E_2 - 7E_4 + 17E_3 + 7E_1$
	2	$25[153C_4 + 4(147C_2C_3 - 32C_3^2 - 69C_3H_2 - 57C_4H_2 - 78C_2H_3 + 26H_3^2) + 138H_2H_4 - 6(C_5 + 23H_4) + 7H_5] + 42E_5 + 16E_6 - 42E_7 - 7E_2 - 49E_4 - 11E_3 + 7E_1$
	3	$-175[4C_3^2 + 6C_4(H_2 - 1) - 4H_3^2 + 6H_4 - 6H_2H_4 - H_5] + 14(E_5 - E_7 - 3E_4 - 2E_3)$
	4	$15(70H_4 - 815C_4 - 224C_5) + 50[56C_3^2 - 3H_2(30C_3 - 40C_4 + 7H_4) + 6C_2(87C_3 - 38H_3) + 32H_3^2] + 527H_5 + 5(26E_5 + 18E_6 + 53E_3 + 5E_1) - 146E_7 - 9(5E_2 - 9E_4)$
	5	$10[15C_4(91 - 89H_2) - 440C_3^2 - 156C_5 - 2070C_3H_2 + 90C_2(19C_3 - 3H_3) + 200(H_3^2 + 6(H_2 - 1)H_4)] + 1052H_5 + 88E_5 + 42E_6 - 104E_7 - 15E_2 - 159E_4 - 5E_3 + 13E_1$
	6	$4[75C_4(46H_2 - 91) - 120C_5 + 150C_2(65C_3 - 32H_3) + 50(20H_3^2 + 9H_4 - 9H_2(2C_3 + H_4)) + 447H_5] + 12(8E_5 + 7E_6 - 12E_7 - 2E_2 - 20E_4 - 3E_3 + 3E_1)$
$\delta_3^{(5,i)}$	4	$-50[56C_3^2 + 51C_4(H_2 - 1) - 66C_3H_2 + 6C_2(19C_3 - 11H_3)] - 56E_5 - 38E_6 + 56E_7 + 17E_2 - 31E_4 - 109E_3 - 11E_1$
	5	$25(176C_3^2 - 588C_2C_3 - 171C_4 + 24C_5 + 348C_3H_2 + 156C_4H_2 + 240C_2H_3 - 80H_3^2 - 174(H_2 - 1)H_4 - 7H_5) - 42E_5 - 22E_6 + 42E_7 + 7E_2 + 49E_4 - 19E_3 - 7E_1$
	6	$20[585C_4 - 12C_5 - 30H_4 - 10(138C_2C_3 + 8C_3^2 - 36C_3H_2 + 33C_4H_2 - 66C_2H_3 + 8H_3^2 - 3H_2H_4)] - 596H_5 - 8(11E_5 + 9E_6 - 13E_7 - 2E_2 - 23E_4 - E_3 + 4E_1)$
$\delta_4^{(5,i)}$	6	$100[8C_3^2 + 6C_4(H_2 - 1) - 30C_3H_2 + 3C_2(23C_3 - 10H_3)] + 2(14E_5 + 11E_6 - 14E_7 - 2E_2 - 26E_4 + E_3 + 5E_1)$

Starting at S^3 we have *extra* independent Wilson coefficients compared to worldline approach.

Extra Wilson Coefficients

How many Wilson coefficients are needed to describe a generic spinning object in GR? We have extra physically distinct ones.

After imposing SSC single Wilson coefficient at O(S^2) **SSC:** $p_\mu S^{\mu\nu} = 0$

Worldline 1 independent operator

$$-\int d\sigma \, \frac{C_2}{2m} \frac{E_{\mu\nu}}{\sqrt{u^2}} S^\mu S^\nu$$

$$E_{\mu\nu} \equiv R_{\mu\alpha\nu\beta} u^\alpha u^\beta$$

$$S^\alpha(p) = -\frac{1}{2m} \epsilon^{\alpha\beta\gamma\delta} p_\beta S_{\gamma\delta}(p)$$

Field theory 3 independent operators:

$$-\frac{C_{ES^2}}{2m^2} R_{afbg} \nabla^a \phi_s \mathbb{S}^{(f} \mathbb{S}^{g)} \nabla^b \phi_s$$

$$\frac{H_2}{8} R_{abcd} \phi_s M^{ab} M^{cd} \phi_s \qquad \frac{D_2}{m^2} R_{afbg} \nabla_i \phi_s M^{if} M^{bg} \nabla^a \phi_s, \qquad \mathbb{S}^a \equiv \frac{-i}{2m} \epsilon^{abcd} M_{cd} \nabla_b$$

- **Physically distinct in impulse or scattering angle starting from $O(G^2)$ S^3**
- **Kerr Black holes have definite values of Wilson coefficients.**

Tracking Source of Extra Wilson Coefficients

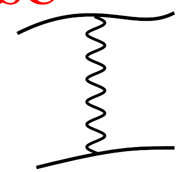
ZB, Kosmopoulos, Luna, Roiban, Trevor Scheopner, Teng, to appear

To track source of the extra Wilson coefficients:

- **Switch to E&M where mismatch with field theory start at lowest order in S^1, not S^3. Much easier!**
- **Redo the worldline but impose SSC only on initial condition.**
- **Solve equations with and without Lagrange multiplier term.**

Lagrange multiplier imposing SSC

$$S[\alpha,\beta,z,p,\Lambda,S]=\int_{-\infty}^{\infty}\left(p_\mu \dot{z}^\mu + qA_\mu \dot{z}^\mu + \frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu} + \frac{\alpha}{2}(p^2-\mathcal{M}^2) + \beta_\mu S^{\mu\nu}p_\nu\right)d\lambda$$



Formalism from Steinhoff; Vines, Kunst, Steinhoff, Hinderer

At lowest order in spin:

Extra

Thanks to Justin Vines

$$\mathcal{M}=m+\frac{qW_1}{m}S^{\mu\nu}\mathcal{F}_{\mu\nu}+\frac{qW_2}{m}\hat{p}_\mu S^{\mu\nu}\mathcal{F}_{\nu\rho}\hat{p}^\rho+\mathcal{O}(S^2)$$

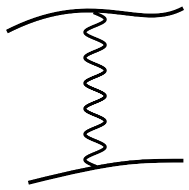
- **With Lagrange multiplier active, W_2 term effectively drops out.**
- **Dropping Lagrange multiplier term leaves one extra Wilson coefficient, matching S^1 result of field theory approach to E&M.**
- **With specific choice of W_2 can match result of enforcing SSC throughout.**

Fewer Wilson coefficients come from imposing SSC throughout.

Tracking Source of Extra Wilson Coefficients

ZB, Kosmopoulos, Luna, Roiban, Trevor Scheopner, Teng, to appear

1. **Identified source of the discrepancy in physical parameters between field theory and worldline approach as due imposing SSC throughout.**
2. **Principles of EFTs suggest our QFT count is the proper one.**
3. **Reason this was not noticed earlier: Prior to O(G^2) and $O(S^3)$, in physical quantities extra operators redundant. Beyond this distinct physical effects.**
4. **Issue of basic importance: How many physical parameters describe spinning neutron stars (or any other generic object)?**



Still to be done:

1. **Higher order in spin in E&M.**
2. **Work this out in gravity, not E&M.**

Looking forward to fully resolving this together with GR friends.

Summary

- **Scattering amplitudes give us new ways to think about problems of current interest in general relativity.**
- **Pushed state of the art: G^4 conservative part complete.**
- **Conjecture for defining Kerr black hole G^2 2 body Hamiltonian.**
  - **— Shift symmetry**
  - **— Good high-energy behavior**
- **We found extra physical parameters for describing interaction of generic spinning objects with gravitational field.**
  - **— Identified source of extra parameters on the word line.**
  - **— Complete resolution and consensus still needed.**

Amplitudes approaches are pushing the state of the art in post-Minkowskian framework and providing new insights into physical questions.

Nordita Program

Amplifying Gravity at All Scales

June 26 – July 21, 2023

Organizers:

Daniel Baumann, Zvi Bern, Alessandra Buonanno,
John Joseph Carrasco, Paolo Di Vecchia, Henrik Johansson,
Andrea Phum, Oliver Schlotterer

NORDITA

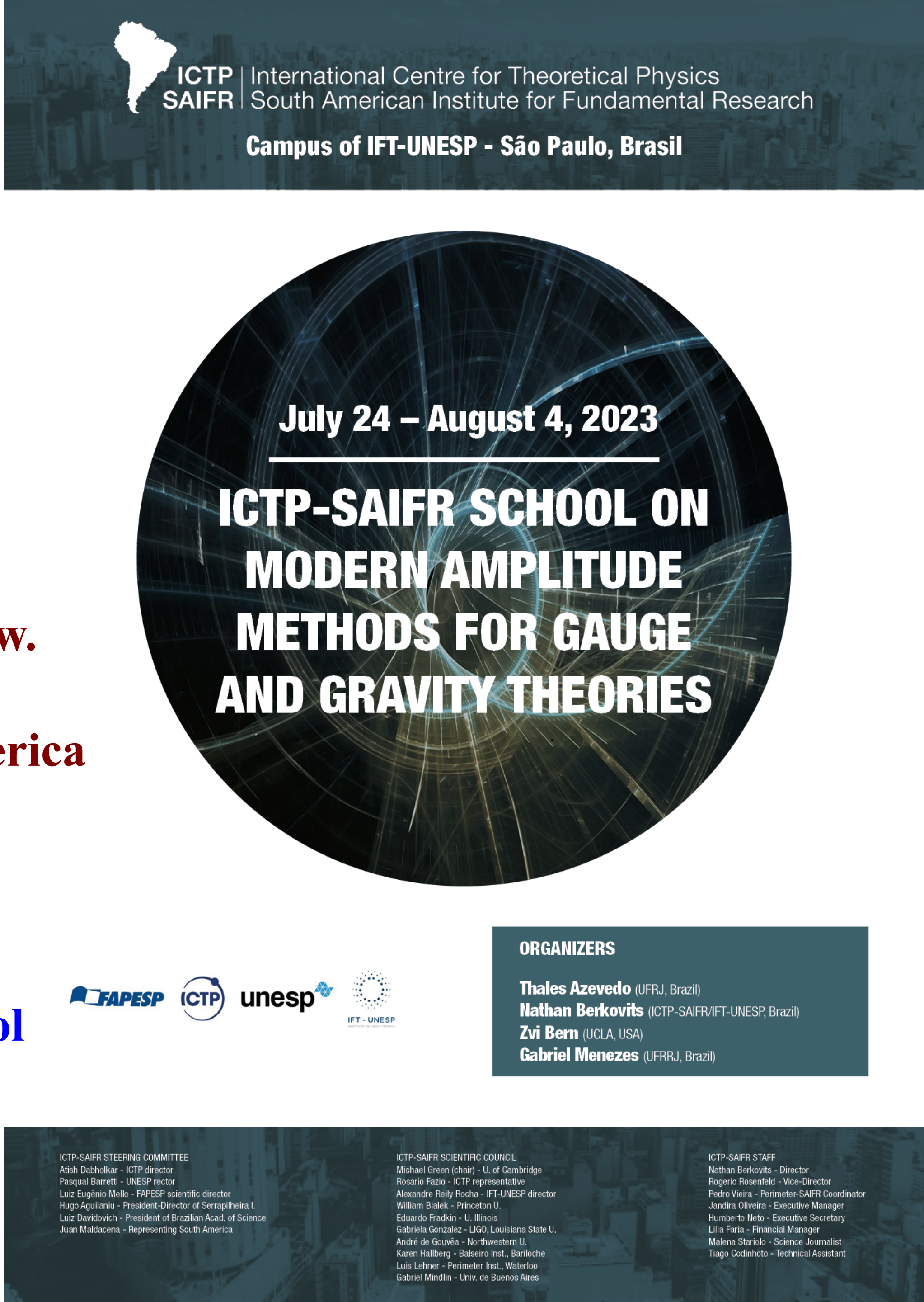
Focus event conference:

From Scattering Amplitudes to Gravitational Waves

July 24 – 28, 2023

All kinds of gravitational amplitudes and applications:
- quantum gravity amplitudes, strings, supergravity
- multiloop integration
- gravitational waves and classical GR
- EFT methods
- celestial amplitudes
- cosmology, inflation, curved space amplitudes

2023 South American School on Modern Amplitudes Methods. Sao Paulo, Brazil.



Lining up great lecturers.

"This school is going to be a blast!"

Nima

If you know students in South America please make sure they know.

If you have colleagues in South America please make sure they know to send students.

Note: Will overlap Amplitudes School at CERN next year but no problem.

EXTRA

QCD meets Gravity

University of Zurich, December 12th-16th 2022

Organized by
Charalampos Anastasiou
Johannes Broedel
Thomas Gehrmann
Harald Ita
Lucio Mayer
Leonardo Senatore

Double copy · Gravitational waves · Celestial amplitudes · Differential equations · Intersection numbers · EFT islands

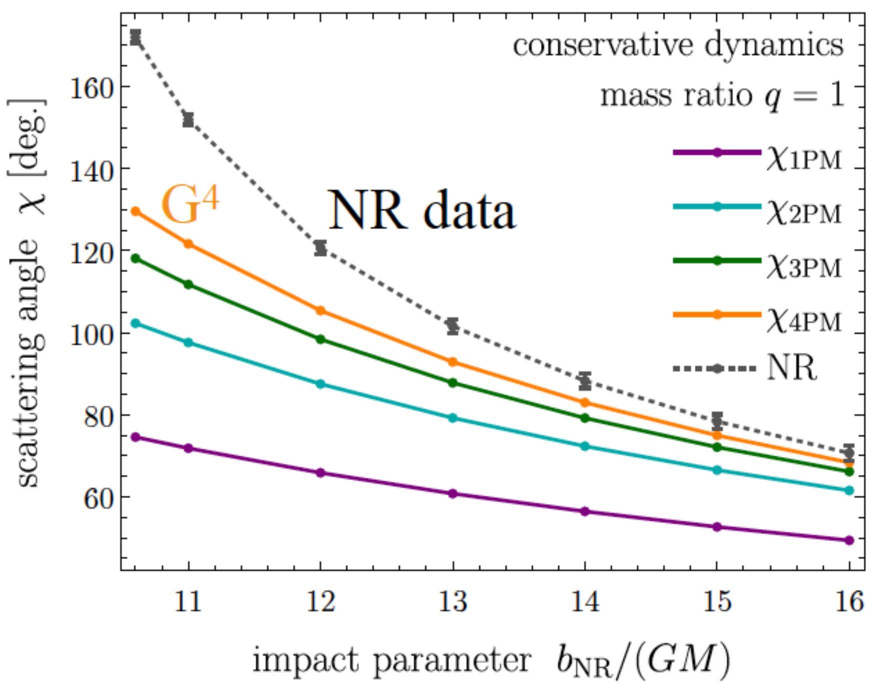
PAULI CENTER for Theoretical Studies · University of Zurich · ETH zürich · PAUL SCHERRER INSTITUT PSI

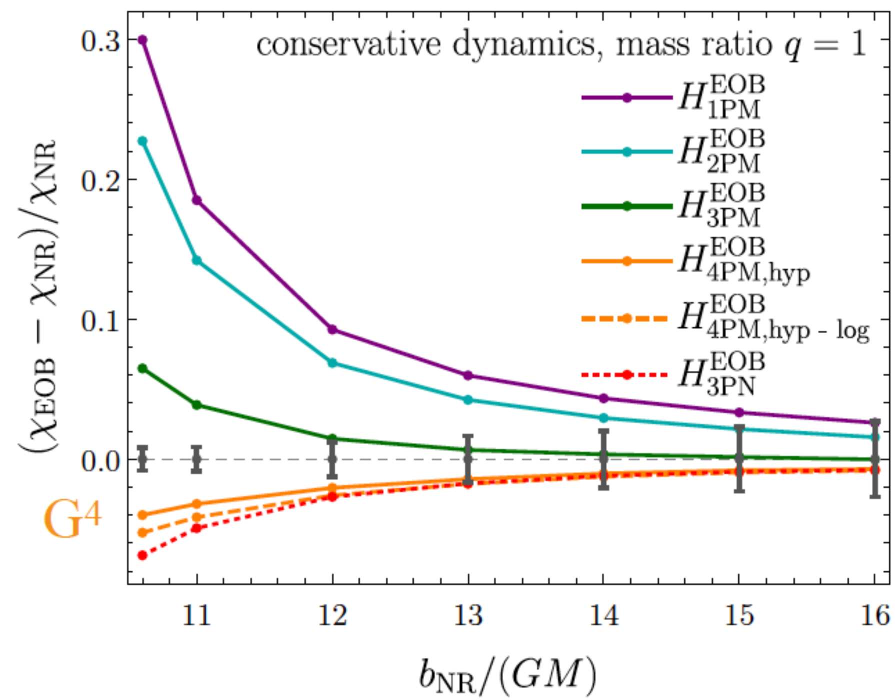
qmg22.phys.ethz.ch

Comparison with Numerical Relativity

Khalil, Buonanno, Steinhoff, Vines

As an interesting check compare to numerical relativity:





Numerical data from Damour, Guercilena, Hinder, Hopper, Nagar, Rezzolla

Surprisingly good agreement with numerical relativity.

Double Copy as a Tool for Gravity

Kawai, Lewellen, Tye
ZB, Carrasco, Johansson

gauge theory:

color factor

$$\mathcal{A}_m^{\text{tree}} = g^{m-2} \sum_j \frac{c_j n_j}{D_j}$$

kinematic numerator

Feynman propagators

$$c_i = c_j - c_k \;\Rightarrow\; n_i = n_j - n_k$$

color/kinematics duality

Color Jacobi   Kinematic Jacobi

gauge theory → gravity theory

simply take

color factor → kinematic numerator

gravity:

$$\mathcal{M}_m^{\text{tree}} = i\left(\frac{\kappa}{2}\right)^{m-2} \sum_j \frac{n_j n_j}{D_i}$$

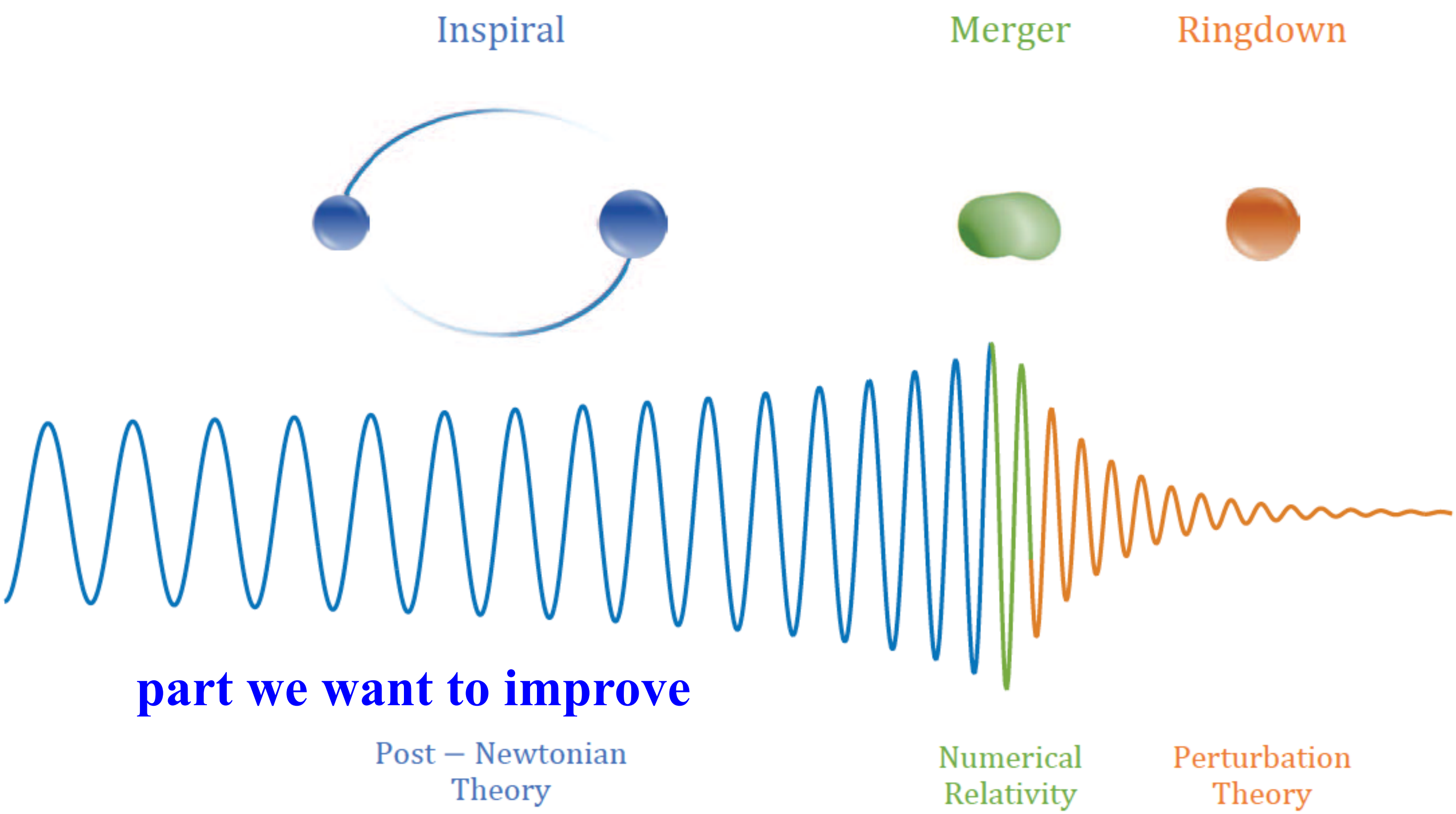
$$c_j \to n_j$$

Gravity ~ (gauge theory) x (gauge theory)

Applications: supergravity, web of theories, gravitational waves

Two Body Problem

From Antelis and Moreno, arXiv:1610.03567



part we want to improve

- **Small errors accumulate. Need for high precision.**
- **Input to EOB or other modeling to reliably approach merger.** Buonanno and Damour
- **Two primary inputs: binding energy and frequency shift.**

High-energy gravitational scattering and the general relativistic two-body problem

Thibault Damour*
Institut des Hautes Etudes Scientifiques, 35 route de Chartres, 91440 Bures-sur-Yvette, France
(Dated: October 31, 2017)

A technique for translating the classical scattering function of two gravitationally interacting bod-

"… and we urge amplitude experts to use their novel techniques to compute the 2-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian."

tum gra… amplitude of two particles, and we urge amplitude experts to use their novel techniques to compute the 2-loop scattering amplitude of scalar masses, from which one could deduce the third post-Minkowskian effective one-body Hamiltonian

Hard to resist an invitation with this kind of clarity!

nals from inspiralling and coalescing binary black holes has been significantly helped, from the theoretical side, by the availability of a large bank of waveform templates, defined [5, 6] within the analytical effective one-body (EOB) formalism [7–11]. The EOB formalism combines, in a suitably resummed format, perturbative, analytical results on the motion and radiation of compact binaries, with some non-perturbative information extracted from numerical simulations of coalescing black-hole binaries (see [12] for a review of perturbative results on binary systems, and [13] for a review of the numerical relativity of binary black holes). Until recently, the perturbative results used to define the EOB conservative dynamics were mostly based on the post-Newtonian (PN) approach to the general relativistic two-body interaction. The conservative two-body dynamics was derived, successively, at the second post-Newtonian (2PN) [14, 15], third post-

…ntly intro- … to derive … from the (gauge-invariant) *scattering function* Φ linking (half) the center of mass (c.m.) classical gravitational scattering angle χ to the total energy, $E_{\rm real} \equiv \sqrt{s}$, and the total angular momentum, J, of the system[1]

$$\frac{1}{2}\chi = \Phi(E_{\rm real}, J; m_1, m_2, G)\,. \tag{1.1}$$

The (dimensionless) scattering function can be expressed as a function of dimensionless ratios, say

$$\frac{1}{2}\chi = \Phi(h, j; \nu)\,, \tag{1.2}$$

where we denoted

$$h \equiv \frac{E_{\rm real}}{M}\,;\; j \equiv \frac{J}{G m_1 m_2} = \frac{J}{G\mu M}\,, \tag{1.3}$$

with

$$M \equiv m_1 + m_2;\ \mu \equiv \frac{m_1 m_2}{\cdots};\ \nu \equiv \frac{\mu}{\cdots} = \frac{m_1 m_2}{\cdots}.$$

EFT Approach

No need to re-invent the wheel.
Build EFT from which we can read off potential.

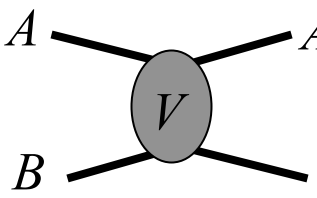
Goldberger and Rothstein
Neill, Rothstein
Cheung, Rothstein, Solon (2018)

$$L_{\rm kin} = \int_{\boldsymbol{k}} A^\dagger(-\boldsymbol{k})\left(i\partial_t + \sqrt{\boldsymbol{k}^2 + m_A^2}\right)A(\boldsymbol{k}) + \int_{\boldsymbol{k}} B^\dagger(-\boldsymbol{k})\left(i\partial_t + \sqrt{\boldsymbol{k}^2 + m_B^2}\right)B(\boldsymbol{k})$$

$$L_{\rm int} = -\int_{\boldsymbol{k},\boldsymbol{k}'} V(\boldsymbol{k},\boldsymbol{k}')\,A^\dagger(\boldsymbol{k}')A(\boldsymbol{k})B^\dagger(-\boldsymbol{k}')B(-\boldsymbol{k})$$

two body potential

***A*, *B* scalars represents spinless black holes**



Match amplitudes of this theory to the full theory in classical limit to extract a potential which can then be directly used for bound state.

The EFT is used to define the potential and 2 body Hamiltonian. This gives us binding energy.

Effective Field Theory Approach

ZB, Cheung, Roiban, Shen, Solon, Zeng

Cheung, Rothstein, Solon (2018)

Amplitudes community

Gravitational Scattering Amplitudes

Effective Field Theory Methods

EFT community

Kawai, Lewellen, Tye
ZB, Dixon, Dunbar and Kosower
ZB, Dixon, Dunbar, Perelstein, Rozowsky
ZB, Carrasco, Johansson; Etc

Beneke, Smirnov (Method of regions)
Goldberger, Rothstein;
Porto; Neill, Rothstein;
Vaydia, Foffa , Porto, Rothstein, Sturani;
Kol, Smolkin, Levi, Steinhoff, etc.

Post Minkowskian Potentials

In a form useful for bound-state problem

The EFT directly gives us a two-body Hamiltonian of a form appropriate to enter LIGO analysis pipeline (after importing into EOB or pheno models).

We prefer this method when pushing into new territory.

EFT Matching

full general relativity
(complicated)

Amplitude methods
double copy

$\downarrow$

tree amplitude

$\hbar \to 0$

generalized
unitarity

$\downarrow$

loop integrand

Loop integration
Method of regions

$\downarrow$

GR loop amplitude

identical
physics

$=$

EFT loop amplitude

effective theory
(simpler)

$\downarrow$

build
ansatz

potential

$\downarrow$

Feynman
diagrams

loop integrand

$\downarrow$

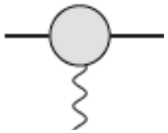
loop
integration

EFT loop amplitude

Roundabout but efficiently determines potential.

Nontrivial Double Copy and Spin

ZB, Luna Roiban, Shen, Zeng



Consider generic O(G) energy momentum tensor for spin:

$$T^{\mu\nu} = \frac{p_1^\mu p_1^\nu}{m} \sum_{n=0}^{\infty} \frac{C_{ES^{2n}}}{(2n)!} \left(\frac{q \cdot S(p_1)}{m} \right)^{2n} - \frac{i}{m} q_\rho p_1^{(\mu} S(p_1)^{\nu)\rho} \sum_{n=1}^{\infty} \frac{C_{BS^{2n+1}}}{(2n+1)!} \left(\frac{q \cdot S(p_1)}{m} \right)^{2n}$$

Closely related to worldline Lagrangian. Porto, Rothstein; Levi, Steinhoff

For $C_{ES^{2n}} = 1,\ C_{BS^{2n}} = 1$ **matches Vines' Kerr black hole tensor**

Does this have a double-copy construction? Yes!

On-shell, can factorize the energy momentum tensor:

$$T^{\mu\nu} = -i\varepsilon(s, p_2) V_{3,\mathrm{GR}}^{\mu\nu} \varepsilon(s, p_1)$$ **Gravity**

$$= [-i\varepsilon(s_L, p_2) V_3^\mu \varepsilon(s_L, p_1)][-i\varepsilon(s_R, p_2) V_3^\nu \varepsilon(s_R, p_1)]$$ **gauge theory**

$$\varepsilon(s, p) = \varepsilon(s_L, p) \otimes \varepsilon(s_R, p)$$

At O(G) energy momentum tensor factorizes
to all orders in spin! Only works on shell.