SUSY in the Sky with Gravitons

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RTG 2575:
Rethinking Quantum Field Theory

Worldline QFT (non-spinning)

[GM, Plefka, Steinhoff '20]

Our starting point is the classical worldline action:

$$S = \int d^4x \Big(\underbrace{-\tfrac{2}{\kappa^2}\sqrt{-g}R}_{S_{EH}} + \underbrace{(\partial_\nu h^{\mu\nu} - \tfrac{1}{2}\partial^\mu h^\nu{}_\nu)^2}_{S_{gf}} \Big) - \underbrace{\sum_{i=1}^{2} \int d\tau_i \frac{m_i}{2} g_{\mu\nu} \dot{x}_i^\mu \dot{x}_i^\nu}_{S_{pm}^{(i)}}$$

} Same starting point as WEFT

equivalent to $m_i \int d\tau_i$

[Kälin, Porto, Dlapa, Liu]; [Mougiakakos, Riva, Vernizzi]

where $x_i^\mu(\tau_i) = b_i^\mu + \tau_i v_i^\mu + z_i^\mu(\tau_i)$, $g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x)$

We promote both z_i^μ, $h_{\mu\nu}$ to propagating d.o.f.'s:

$$h_{\mu\nu} \xrightarrow[k]{} h_{\rho\sigma} = i \frac{P_{\mu\nu;\rho\sigma}}{(k^0+i\epsilon)^2 - \vec{k}^2} \qquad h_{\mu\nu}(k) = -\frac{i m\kappa}{2} e^{ik\cdot b}\delta(k\cdot v) v^\mu v^\nu$$

$$z^\mu \xrightarrow[\omega]{} z^\nu = -i \frac{\eta^{\mu\nu}}{m(\omega+i\epsilon)^2} \qquad h_{\mu\nu}(k),\ z^\rho(\omega) = \frac{m\kappa}{2} e^{ik\cdot b}\delta(k\cdot v)\left(2\omega v^{(\mu}\delta_\rho^{\nu)} + v^\mu v^\nu k_\rho\right)$$

In WQFT the $\hbar$ expansion IS the loop expansion... (saddle point approx). So

$$\boxed{\text{tree-level} = \text{classical physics}}$$

The $\mathcal{N}=2$ theory

[Jakobsen, GM, Plefka, Steinhoff '21]

Represent spin with Grassmann-odd ψ^a vectors $\quad D_\tau \psi^a := \dot{x}^\mu \nabla_\mu \psi^a$

$$\boxed{S = -m \int d\tau \left[\frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu + i \bar{\psi}_a \frac{D\psi^a}{D\tau} + \frac{1}{2} R_{abcd} \bar{\psi}^a \psi^b \bar{\psi}^c \psi^d \right]} \Bigg\} \text{ valid up to } O(S^2)$$

The spin tensor $S^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma} p_\rho a_\sigma$ is identified as

$$\boxed{S^{\mu\nu} = -2i \bar{\psi}^{[\mu} \psi^{\nu]}} \Bigg\} \quad \{\bar{\psi}^\mu, \psi^\nu\}_{P.B.} = -i \eta^{\mu\nu} \quad \text{(1st-order formalism)}$$

$$\Rightarrow \{S^{\mu\nu}, S^{\rho\sigma}\}_{P.B.} = \eta^{\mu\rho} S^{\nu\sigma} + \eta^{\nu\sigma} S^{\mu\rho} - \eta^{\nu\rho} S^{\mu\sigma} - \eta^{\mu\sigma} S^{\nu\rho}$$

$\mathcal{N}=2$ SUSY (flat space): $\quad \delta x^\mu = i\bar{\varepsilon}\psi^\mu + i\varepsilon\bar{\psi}^\mu, \quad \delta\psi^\mu = -\varepsilon v^\mu$

$$\Rightarrow \delta S^{\mu\nu} = -2\,\delta x^{[\mu} v^{\nu]} \Bigg\} \quad \varepsilon \text{ is Grassmann-odd}$$

Total angular momentum $\quad \boxed{J^{\mu\nu} = \underbrace{2x^{[\mu} v^{\nu]}}_{\text{orbital}} + \underbrace{S^{\mu\nu}}_{\text{spin}}} \Bigg\}$ is invariant under $x^\mu \to x^\mu + \delta x^\mu$

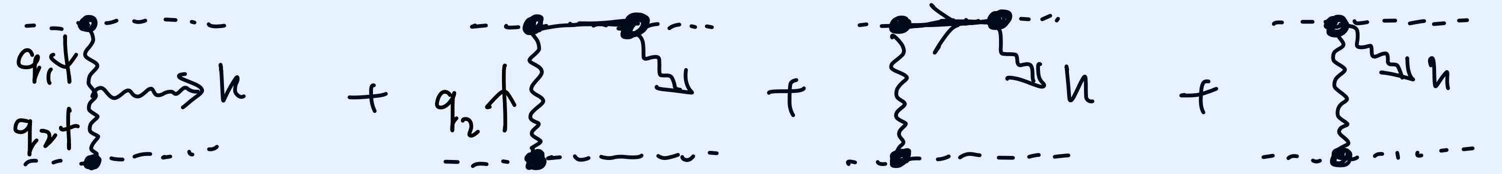
Fix SUSY using a spin-supplementary condⁿ (SSC):

$$\boxed{\begin{aligned} &p \cdot \psi = 0 \\ \Rightarrow\ &p_\mu S^{\mu\nu} = 0 \end{aligned}}$$

Spinning Waveform

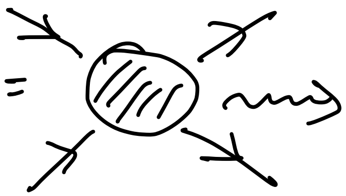
[Jakobsen, GM, Plefka, Steinhoff '21]

Sum on diagrams with an outgoing graviton. Integrate on internal lines:

$\langle h_{\mu\nu}(k)\rangle =$ 

$+\ (1 \leftrightarrow 2)$

We obtain the time-domain waveform for large $|\underline{x}| = r$. This requires integrating on the outgoing energy:

$\mathcal{M}_S =$ 

$$\frac{f_{+,\times}(u,\hat{\underline{x}})}{r} = \frac{4G}{r}\int_\Omega e^{-ik\cdot x}\,\varepsilon^{\mu\nu}_{+,\times}\langle h_{\mu\nu}(k=\Omega n)\rangle \quad \Big\}\ n^\mu=(1,\hat{\underline{x}})$$

where $k^\mu = \Omega n^\mu$, $n^\mu = (1,\hat{\underline{x}})$ points to the observer.

$k\cdot x = \Omega n\cdot x = \Omega(t-r) = \Omega u$

$u = t-r =$ retarded time

Two components: $+/\times$ polarizations.

$$f_{+,\times}(m_1, m_2, u, \underbrace{\Theta, \phi}_{\hat{\underline{x}}}, \underbrace{v}_{\gamma}, |b|)$$

3PM Scattering Observables

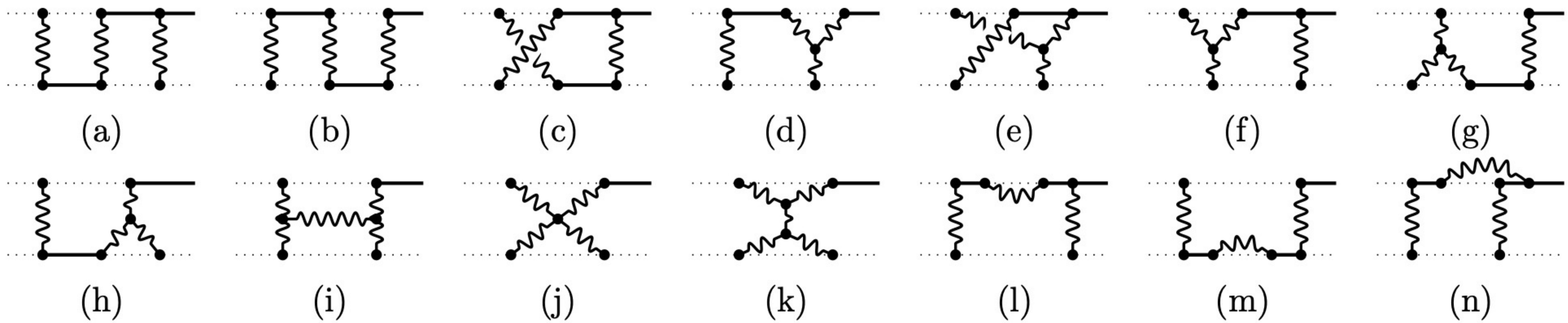
[GM, Jakobsen '22]

We calculate observables directly,

$$\Delta p_i^\mu = m_i \int_{-\infty}^{\infty} d\tau \left\langle \frac{d^2 x_i^\mu(\tau)}{d\tau^2} \right\rangle = -m_i \omega^2 \langle z_i^\mu(\omega) \rangle \Big|_{\omega=0}$$

$$\Delta \psi_i^\mu = \int_{-\infty}^{\infty} d\tau \left\langle \frac{d\psi_i^\mu(\tau)}{d\tau} \right\rangle = i\omega \langle \psi_i'^\mu(\omega) \rangle \Big|_{\omega=0}$$

Draw diagrams with outgoing z^μ / ψ'^μ lines, outgoing energy $\omega = 0$



From $\Delta \psi_i^\mu$ we can obtain the spin kick Δa_i^μ:

$$\Delta S_i^{\mu\nu} = -2i \left(\bar{\Psi}_i^{[\mu} \Delta \psi_i^{\nu]} + \Psi_i^{[\mu} \Delta \bar{\psi}_i^{\nu]} + \Delta \bar{\psi}_i^{[\mu} \Delta \psi_i^{\nu]} \right) \qquad S_i^{\mu\nu} = -2i \bar{\psi}_i^{[\mu} \psi_i^{\nu]}$$

$$\Delta a_i^\mu = \frac{1}{2} \varepsilon^\mu{}_{\nu\rho\sigma} \left(S_i^{\nu\rho} \Delta p_i^\sigma + \Delta S_i^{\nu\rho} p_i^\sigma + \Delta S_i^{\nu\rho} \Delta p_i^\sigma \right) \qquad a_i^\mu = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} S_{i,\nu\rho} p_{i,\sigma}$$

Schwinger-Keldysh In-In Formalism

Expectation values contain two copies of the evolution operator:

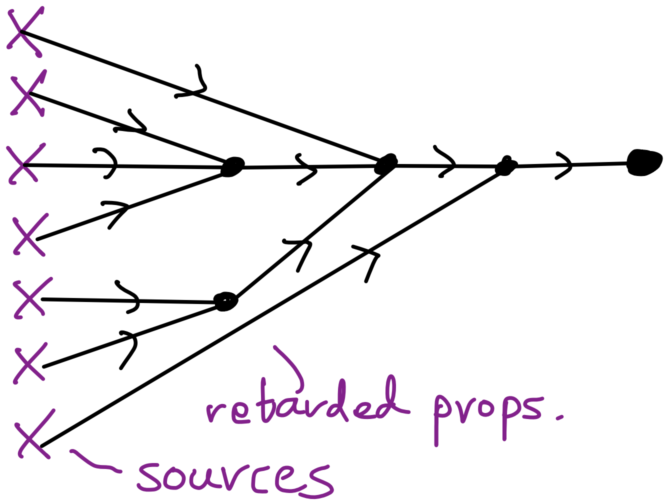
$$\langle \hat{\phi}(t,\vec{x}) \rangle_{in\text{-}in} := \langle 0 | \hat{U}(-\infty,t)\, \hat{\phi}(t,\vec{x})\, \hat{U}(t,-\infty) | 0 \rangle \quad \Big\} \; |\psi(t)\rangle = U(t,-\infty)\, |\psi(-\infty)\rangle$$

In the path integral representation, we double our theory:

$$e^{\frac{i}{\hbar} W[J_1,J_2]} = \int \mathcal{D}[\phi_1,\phi_2] \exp\left\{ \frac{i}{\hbar}\left(S[\phi_1] - S[\phi_2] + \int d^4x\, J_1(x)\phi_1(x) - J_2(x)\phi_2(x) \right) \right\}$$

Fields ϕ_1, ϕ_2 entangled via b.c.'s:

$$\phi_1(t=+\infty,\vec{x}) = \phi_2(t=+\infty,\vec{x})$$
$$\phi_1(t=-\infty,\vec{x}) = \phi_2(t=-\infty,\vec{x}) = 0$$



For tree-level one-point functions there is an enormous simplification from working in the Keldysh basis:

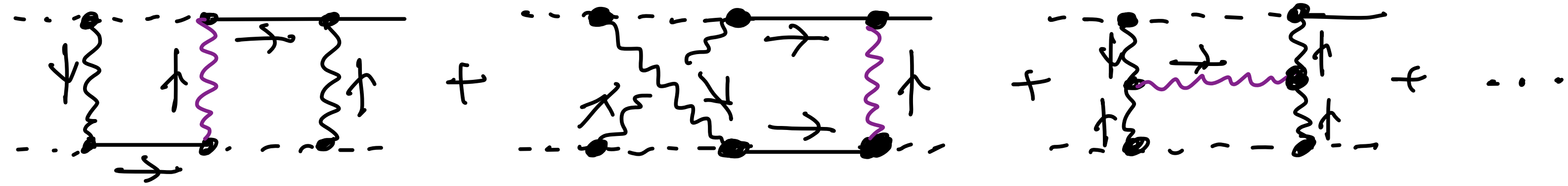
$$\phi_+ = \phi_1 + \phi_2 \quad , \quad \phi_- = \frac{1}{2}(\phi_1 - \phi_2)$$

Full Observables

[Jakobsen, GM, Plefka, Sauer '22]

Full observables including radiation-reaction involve retarded propagators:

$$\Delta p_1^{\mu} = $$



We have a simple prescription to write down observables including radiation-reaction! But... what about the new integrals?

$$\int d^D\ell_1 d^D\ell_2 \frac{\delta(\ell_1\cdot v_2)\,\delta(\ell_2\cdot v_1)}{(\ell_i\cdot v_1 \pm i\epsilon)^{n_1}(\ell_i\cdot v_2 \pm i\epsilon)^{n_2}\underbrace{\left((\ell_1+\ell_2-q)^2 \pm i\epsilon\,\mathrm{sgn}(\ell_1^0+\ell_2^0-q^0)\right)^{n_3}}_{\text{active propagator}}(\ell_1^2)^{n_4}(\ell_2^2)^{n_5}((\ell_1-q)^2)^{n_6}((\ell_2-q)^2)^{n_7}}$$

$$:= I_{n_1,n_2,\dots,n_7}$$

Most graviton propagators can't go on-shell, because $\delta(\ell_i\cdot v_j)$ prevents $\ell_i^2=0$.
And... the integrals are (pseudo) real in the physical region:

$$\left.I^*_{n_1,n_2,\dots,n_7} = (-1)^{n_1+n_2} I_{n_1,n_2,\dots,n_7}\right\} \text{ when } 1<\gamma<\infty$$

Contrast with Feynman integrals, real for $\underline{\underline{-1<\gamma<1}}$.

Performing Retarded Integrals

Our methodology is conventional: IBPs + DEs. Two subtleties:

- symmetries modified by presence of retarded $i\varepsilon$
- boundary fixing in static $v \to 0$ limit, $\gamma = \frac{1}{\sqrt{1-v^2}}$

$$\frac{\delta^{(n)}(\omega)}{(-1)^n n!} = \frac{i}{(\omega+i\varepsilon)^{n+1}} - \frac{i}{(\omega-i\varepsilon)^{n-1}}$$

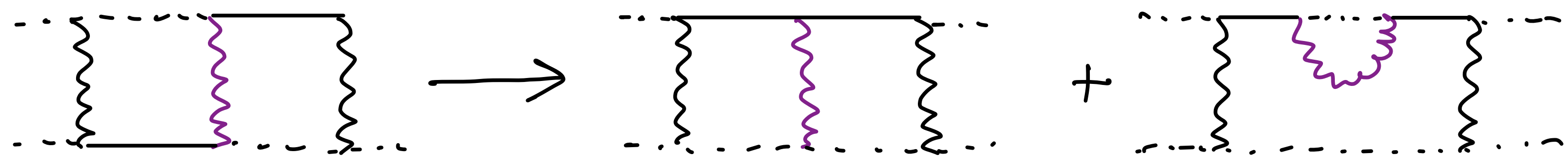
} treat $\delta(\omega)$ as a propagator from perspective of IBPs

Symmetries aren't so bad when we can ignore most $i\varepsilon$ instances. As for the $v \to 0$ limit, the behavior is characterized by one graviton:

$$k^{pot} = (k^0, \vec{k}) \sim (v, 1)$$
$$k^{rad} = (h^0, \vec{h}) \sim (v, v)$$

$$I_{n_1 n_2 \cdots n_7} = I^{pot}_{n_1 n_2 \cdots n_7} + I^{rad}_{n_1 n_2 \cdots n_7}$$

In these regions, re-express in terms of simpler integrals:



Intuitively... because

$$v_1^\mu = (1, \vec{0})$$
$$v_2^\mu = \gamma(1, \vec{v}) = v_1^\mu + \ldots$$

} identify in the static limit

Scattering Observables

Final results for Δp_i^μ, $\Delta \psi_i^\mu$ satisfy consistency checks:

1) Cancellation of all $1/\varepsilon$ poles from dim-reg.
2) Conservation of SUSY charges $p_i^2, \psi_i \cdot \bar{\psi}_i, p_i \cdot \psi_i, p_i \cdot \bar{\psi}_i$, e.g.

$$p_i \cdot \psi_i = (p_i + \Delta p_i) \cdot (\psi_i + \Delta \psi_i)$$

$$P^\mu_{rad} = -\Delta p_1^\mu - \Delta p_2^\mu$$ } agrees with results recently obtained [Riva, Vernizzi, Wong '22].

We also obtain radiated energy in the COM frame:

$$E_{rad} = \frac{(p_1+p_2)\cdot P_{rad}}{|p_1+p_2|} = \frac{G^3 M^4 \nu^2}{|b|^3} \mathcal{E}(v_i^\mu, a_i^\mu, m_i)$$

$$\frac{\mathcal{E}}{\pi} = f_1 + f_2 \log\left(\frac{1+\gamma}{2}\right) + f_3 \operatorname{arccosh}\gamma$$ } f_i rational functions of a_i^μ, v_i^μ

B2B Mapping: (aligned spins)

$$E_{rad}^{bound}(E, J) = E_{rad}^{unbound}(E, J) + E_{rad}^{unbound}(E, -J)$$

Scattering angle

Focus on aligned spins, $a_i \cdot b = a_i \cdot v_j = 0$. The scattering angle is

$$\boxed{\sin\left(\frac{\Theta}{2}\right) = \frac{|\Delta p_\perp|}{2p_\infty}} \Bigg\} \; p_\infty = \frac{\mu\gamma v}{\Gamma}, \; \Gamma = \frac{E}{M} = \sqrt{1+2\nu(\gamma-1)}, \; M = m_1+m_2, \; \mu = M\nu = \frac{m_1 m_2}{M}$$

$$\theta^{(3,0)}_{\rm cons} = 2\frac{64\gamma^6-120\gamma^4+60\gamma^2-5}{3(\gamma^2-1)^3}\Gamma^2 - \frac{8\nu\gamma(14\gamma^2+25)}{3(\gamma^2-1)} - 8\nu\frac{4\gamma^4-12\gamma^2-3}{(\gamma^2-1)^{3/2}}\operatorname{arccosh}\gamma\,, \tag{17a}$$

$$\theta^{(3,1)}_{\rm cons} = 2\gamma\frac{16\gamma^4-20\gamma^2+5}{(\gamma^2-1)^{5/2}}(5\Gamma^2 s_+ - \delta s_-) - 4\nu s_+\left(\frac{44\gamma^4+100\gamma^2+41}{(\gamma^2-1)^{3/2}} + 12\gamma\frac{(\gamma^2-6)(2\gamma^2+1)}{(\gamma^2-1)^2}\operatorname{arccosh}\gamma\right), \tag{17b}$$

$$\begin{aligned}\theta^{(3,2)}_{\rm cons} = {}& \frac{4\Gamma^2}{(\gamma^2-1)^3}\left((96\gamma^6-160\gamma^4+70\gamma^2-5)s_+^2 - \frac{1772\gamma^6-2946\gamma^4+1346\gamma^2-137}{35}s_{{\rm E},+}^2\right) - 8\delta\left(\frac{16\gamma^4-12\gamma^2+1}{(\gamma^2-1)^2}s_-s_+\right.\\ &\left.- \frac{214\gamma^4-223\gamma^2+44}{35(\gamma^2-1)^2}s_{{\rm E},-}^2\right) + 8\nu\gamma\left[\frac{2\gamma^4+86\gamma^2+87}{5(\gamma^2-1)^2}s_-^2 - \frac{298\gamma^4+834\gamma^2+853}{5(\gamma^2-1)^2}s_+^2 + \frac{3244\gamma^4+7972\gamma^2+4639}{105(\gamma^2-1)^2}s_{{\rm E},+}^2\right.\\ &\left.-\left(3s_-^2(4\gamma^4+7\gamma^2+1)+3s_+^2(8\gamma^6-68\gamma^4-63\gamma^2-9)-2s_{{\rm E},+}^2(8\gamma^6-56\gamma^4-24\gamma^2-3)\right)\frac{\operatorname{arccosh}\gamma}{\gamma(\gamma^2-1)^{5/2}}\right], \end{aligned} \tag{17c}$$

Agrees with test-body limit $\nu \to 0$, 4PN results [Levi, Steinhoff].
Radiative corrections agree with linear response [Bini, Damour]:

$$\boxed{\Theta_{\rm rad} = -\frac{1}{2}\Big(\underbrace{\frac{\partial\Theta_{\rm cons}}{\partial E}E_{\rm rad}}_{\text{starts at } G^4} + \frac{\partial\Theta_{\rm cons}}{\partial J}J_{\rm rad}\Big)} \Bigg\} \text{ we have } J_{\rm rad} \text{ from the waveform!}$$

$\hookrightarrow$ finite high-energy $\gamma \to \infty$ limit

Linear Response with Spin

We have a mechanism to generalize Bini / Damour's linear response relation:

$$\Delta X_{\text{rad}}\Big|_{\text{even-in-}v} = -\frac{1}{2}\left(\cancelto{O(G^4)}{\frac{\partial \Delta X}{\partial p^\mu} P^\mu_{\text{rad}}} + \frac{\partial \Delta X}{\partial L^\mu} J^\mu_{\text{rad}} \right) \quad \Bigg\} \; \Delta X = \Delta p_i, \Delta x_i$$

Derivatives w.r.t.

$$P^\mu = p_1^\mu + p_2^\mu$$

$$L^\mu = \frac{1}{2}\varepsilon^\mu{}_{\nu\rho\sigma} L^{\nu\rho} P^\sigma = \varepsilon^\mu{}_{\nu\rho\sigma} b^\nu p_1^\rho p_2^\sigma$$

the orbital angular momentum vector in the center-of-mass frame.

We distinguish between terms even/odd under the operation

$$v_i^\mu \longleftrightarrow -v_i^\mu \quad \Big\}$$

associates terms with families of integrals

This reduces to the "usual" Bini-Damour formula for aligned spins:

$$\Delta p_1^\mu = \underbrace{p_\infty \sin\Theta \frac{b^\mu}{|b|}}_{\text{we get this!}} + (\cos\Theta - 1)\frac{m_1 m_2}{M^2}\left[(\gamma m_2 + m_1) v_1^\mu - (\gamma m_2 + m_1) v_2^\mu\right] \underbrace{- v_2 \cdot P_{\text{rad}} \frac{\gamma v_1^\mu - v_2^\mu}{\gamma^2 - 1}}_{\text{not this!}}$$

Gravitational Potential

Make an ansatz for the conservative 2-body Hamiltonian:

$$H = \sqrt{\vec{p}^2 + m_1^2} + \sqrt{\vec{p}^2 + m_2^2} + V(\vec{r}, \vec{p}, \vec{S}_i) \qquad \vec{L} = \vec{r} \times \vec{p}$$

$$V = \sum_A \mathcal{O}^A V^A \qquad \mathcal{O}^A = \left\{ 1, \frac{\vec{L}\cdot\vec{S}_i}{r^2}, \frac{\vec{r}\cdot\vec{S}_i\,\vec{r}\cdot\vec{S}_j}{r^4}, \frac{\vec{S}_i\cdot\vec{S}_j}{r^2}, \frac{\vec{p}\cdot\vec{S}_i\,\vec{p}\cdot\vec{S}_j}{r^2} \right\}$$

$$V^A = \sum_n \left(\frac{G}{r}\right)^n c_n^A(|\vec{p}|)$$

Ansatz avoids $\vec{r}\cdot\vec{p}$, c.f. 2PM [Kosmopoulos, Luna '21]. Compare with scattering observables Δp_1^μ, Δs_1^μ by solving Hamilton's eq$^{\text{n}}$s:

$$\dot{\vec{r}} = \frac{\partial H}{\partial \vec{p}}, \quad \dot{\vec{p}} = -\frac{\partial H}{\partial \vec{r}}, \quad \dot{\vec{S}}_i = -\vec{S}_i \times \frac{\partial H}{\partial \vec{S}_i} \qquad \Big\}\ \text{get } \vec{r}(t), \vec{p}(t), \vec{S}_i(t)$$

Agrees with [Cordero, Kraus, Lin, Ruf, Zeng '22], and fills in additional terms. Interestingly, Δp_1^μ contains Δs_1^μ!

Also agrees with [Levi, Steinhoff] 4PN

Conclusions

- New results obtained using WQFT so far, all up to $O(S^2)$:
  - 2PM gravitational waveform for unbound scattering
    $\Rightarrow$ radiated $P^{\mu}_{\rm rad}$, $J^{\rm rad}_{ij}$
  - 3PM observables Δp_i^{μ}, Δa_i^{μ}, θ
    $\Rightarrow$ full knowledge of scattering angle via linear response.
    $\Rightarrow$ recently upgraded to include radiation-reaction.

- Also recently 3PM + tidal effects (with B. Sauer)

- Other recent work on the WQFT:
  - double copy [Shi, Plefka '21]
  - light bending [Bastianelli, Comberiati, de la Cruz '22]
  - WQFT for YM, eikonal [Wang '22]

Next Steps

Push WQFT to higher spins, c.f. recent amplitudes progress [Bern, Kosmopoulos, Luna, Roiban, Teng ; Aoude, Haddad, Helset], higher PM orders.

Thanks for listening!