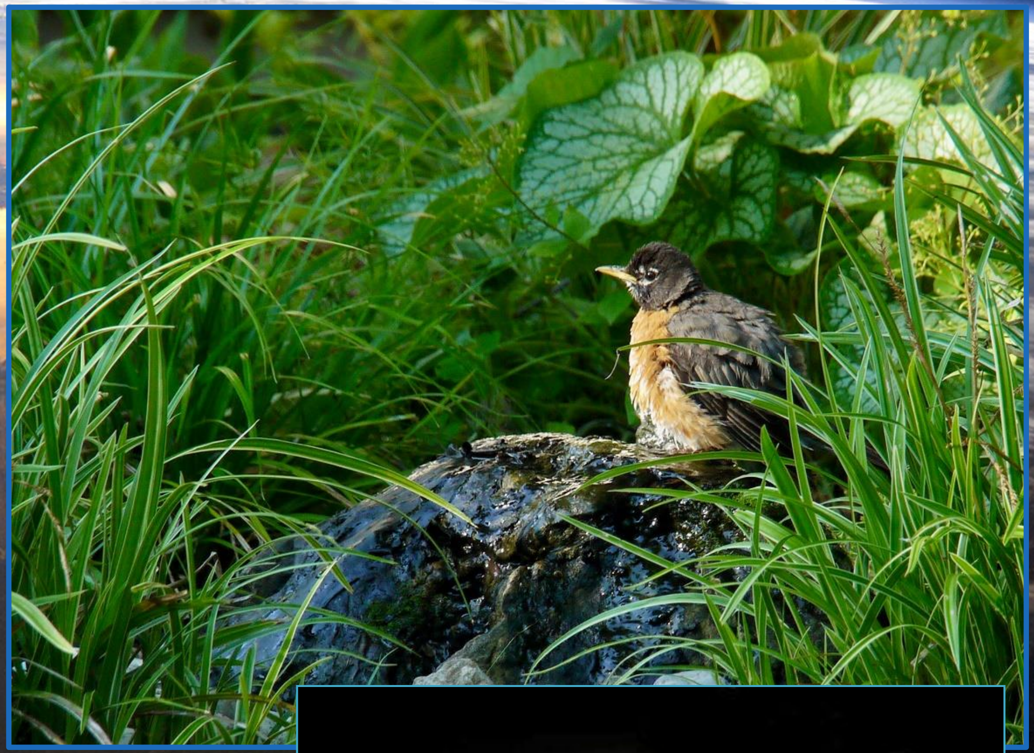
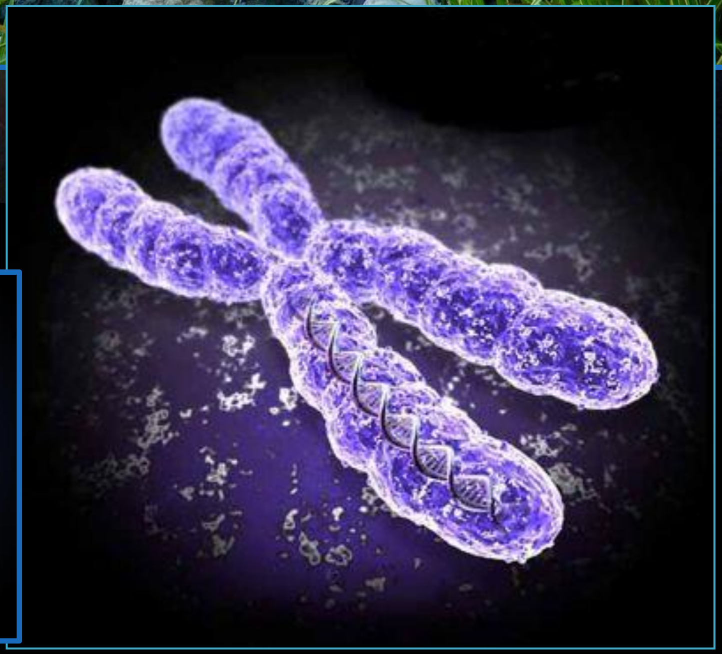
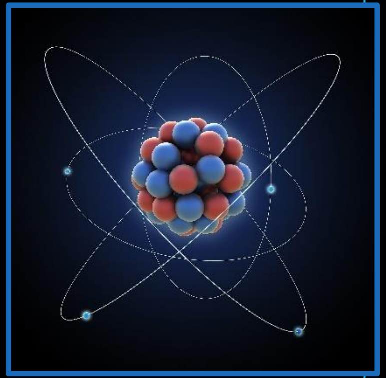
INTRODUCTION TO EFFECTIVE FIELD THEORY

THE VIEW FROM BELOW







EFFECTIVE PATHWAYS TO NEW PHYSICS, BHUBANESWAR
FEBRUARY 2022

CP BURGESS

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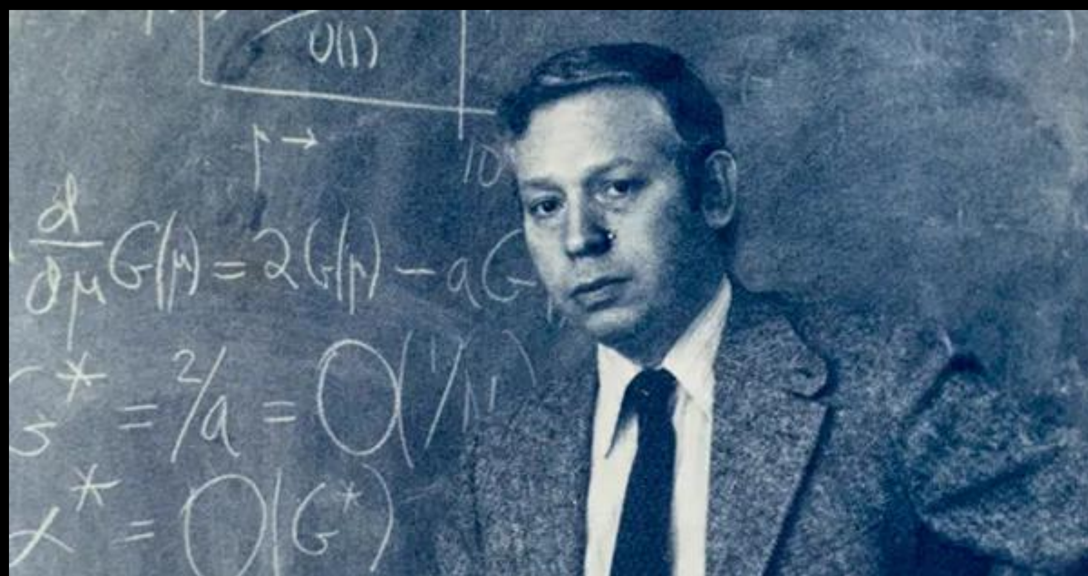
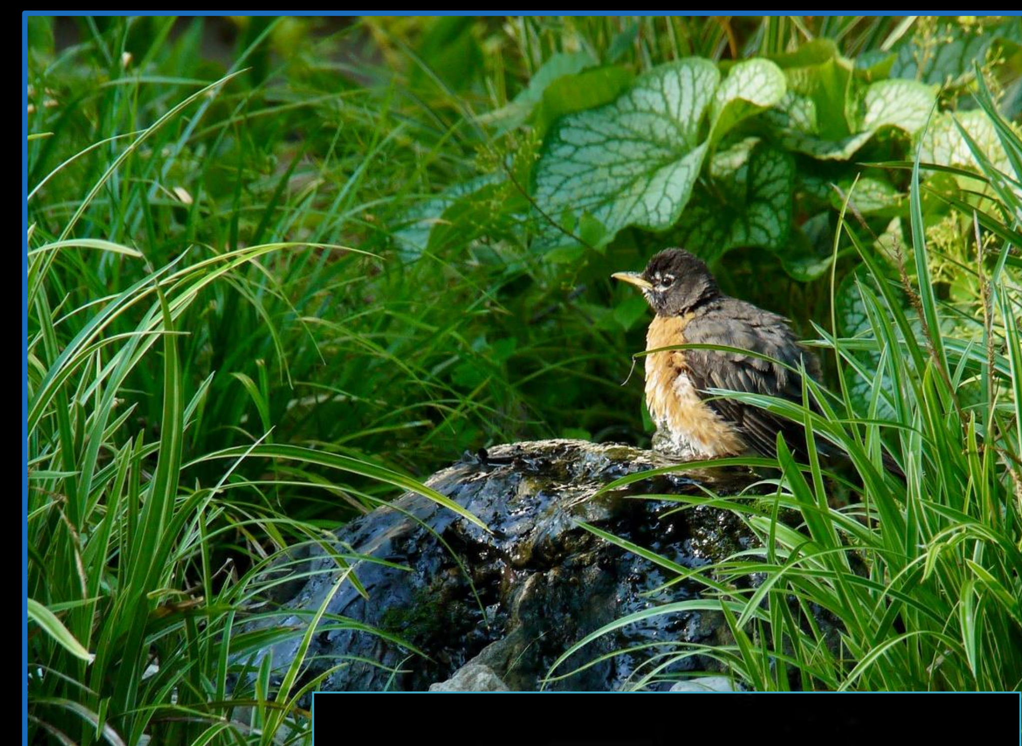
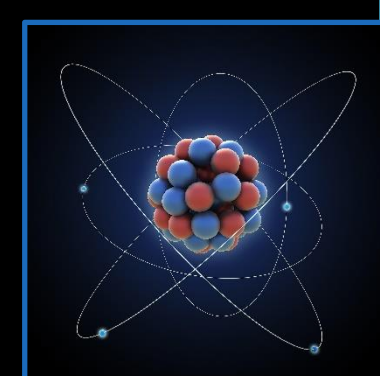
ON THE SHOULDERS OF GIANTS…



Steven Weinberg 1933-2021

Nobel Prize 1979





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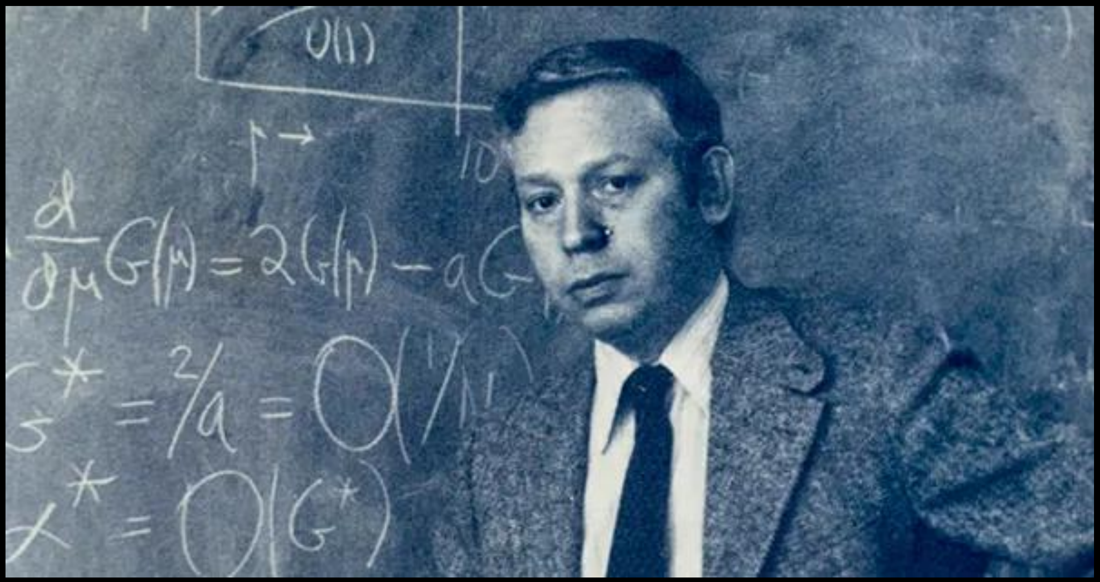
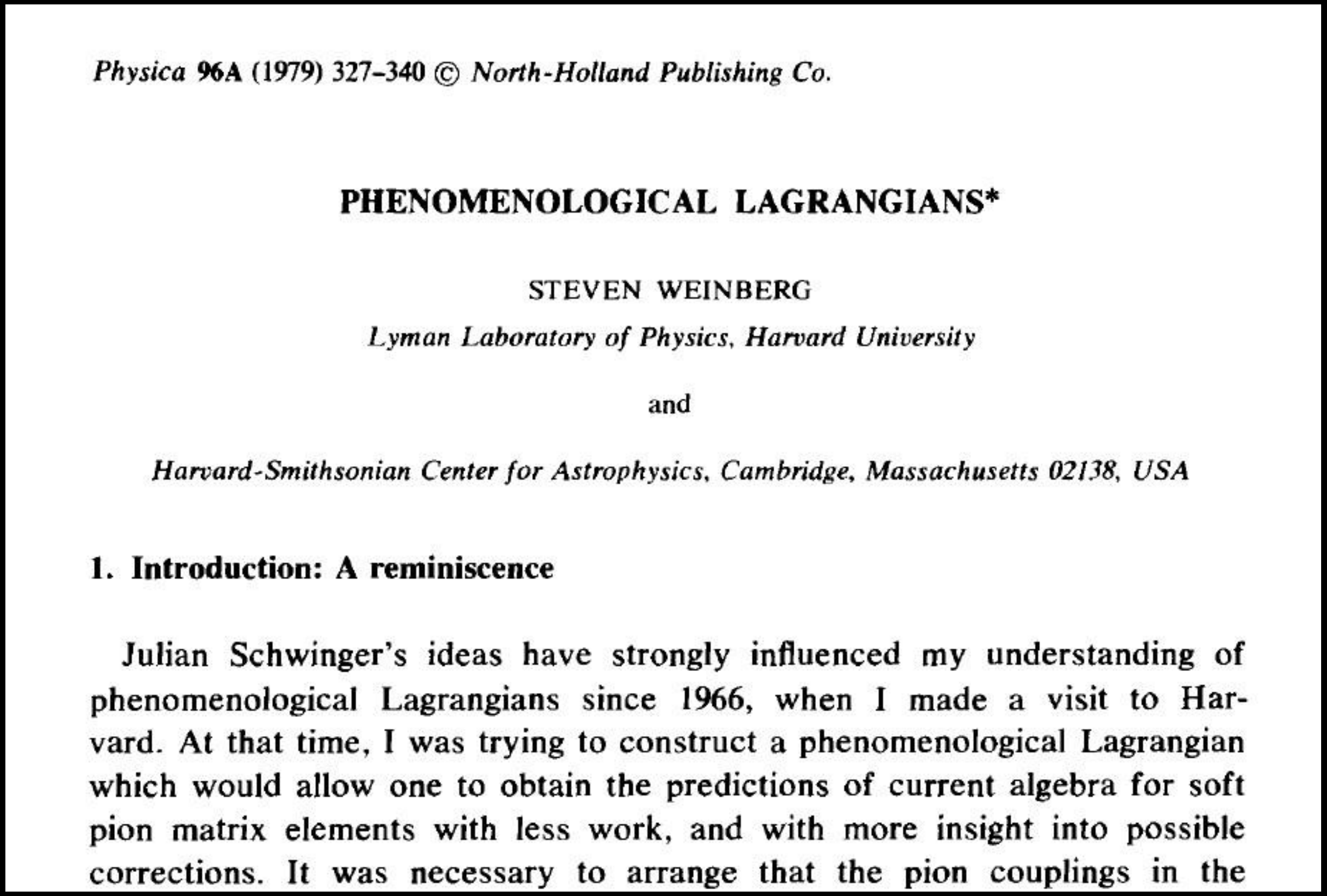




Physica **96A** (1979) 327–340 © *North-Holland Publishing Co.*

PHENOMENOLOGICAL LAGRANGIANS*

STEVEN WEINBERG

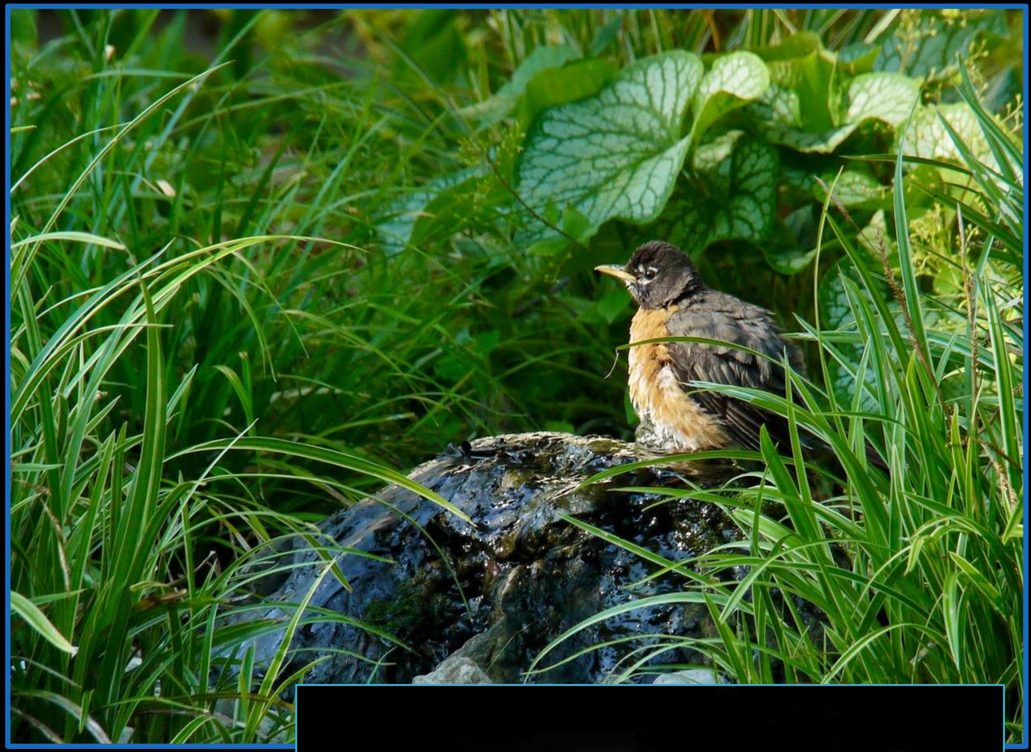
Lyman Laboratory of Physics, Harvard University

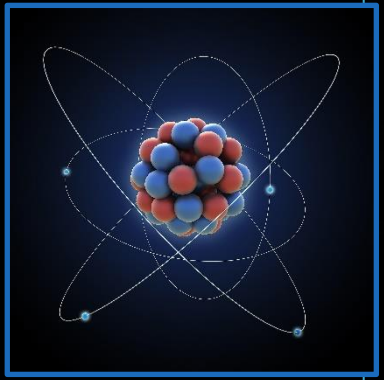
and

Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138, USA

1. Introduction: A reminiscence

Julian Schwinger's ideas have strongly influenced my understanding of phenomenological Lagrangians since 1966, when I made a visit to Harvard. At that time, I was trying to construct a phenomenological Lagrangian which would allow one to obtain the predictions of current algebra for soft pion matrix elements with less work, and with more insight into possible corrections. It was necessary to arrange that the pion couplings in the





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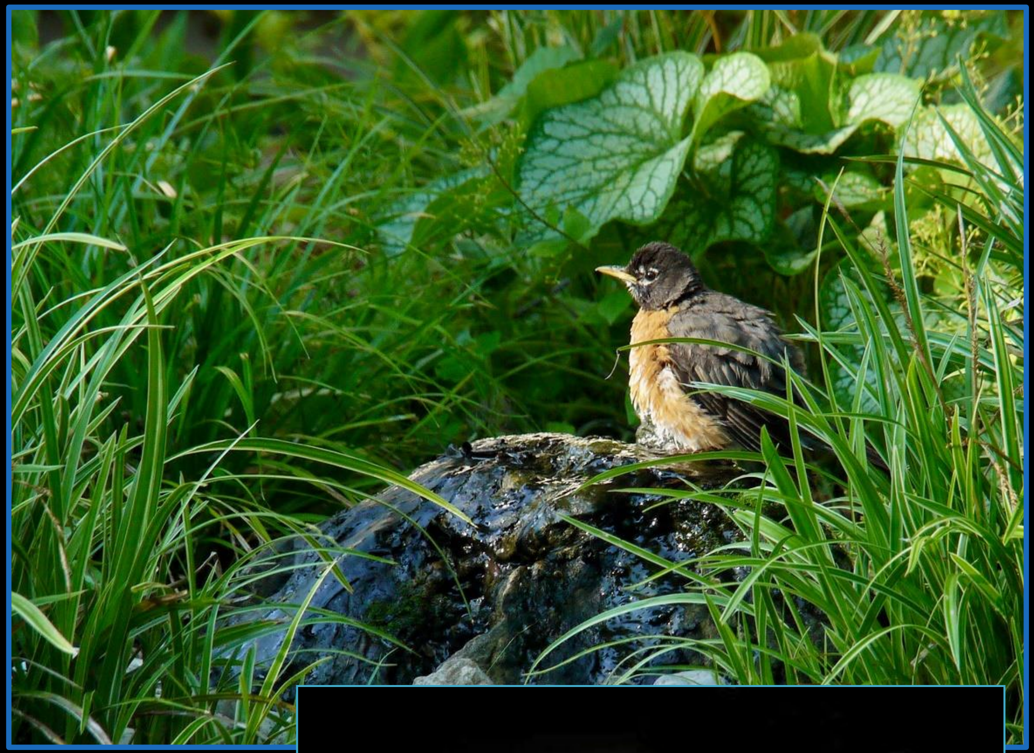
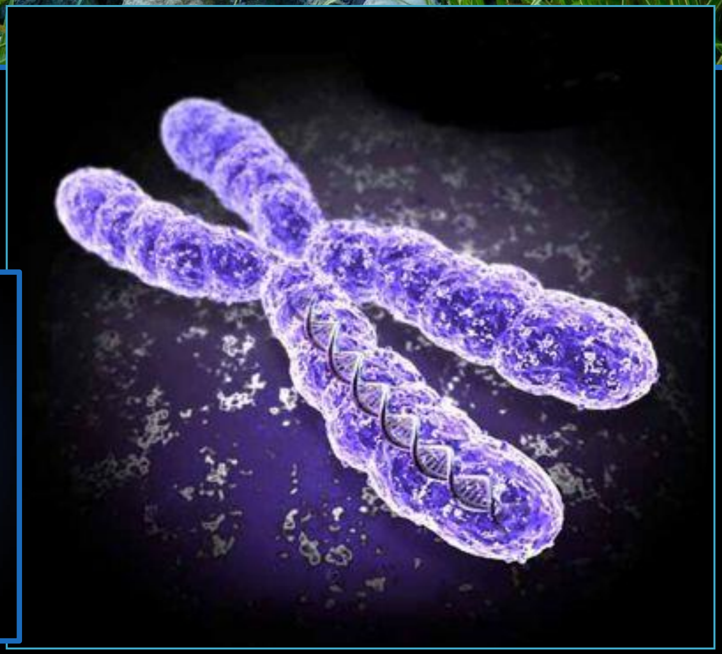
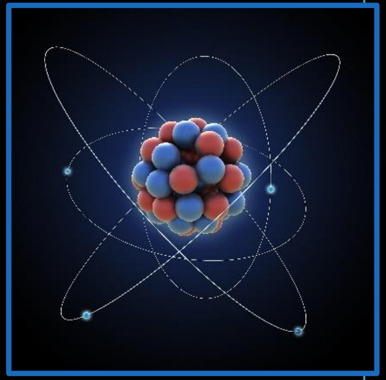
- GENERAL FRAMEWORK

- EFT APPLICATIONS







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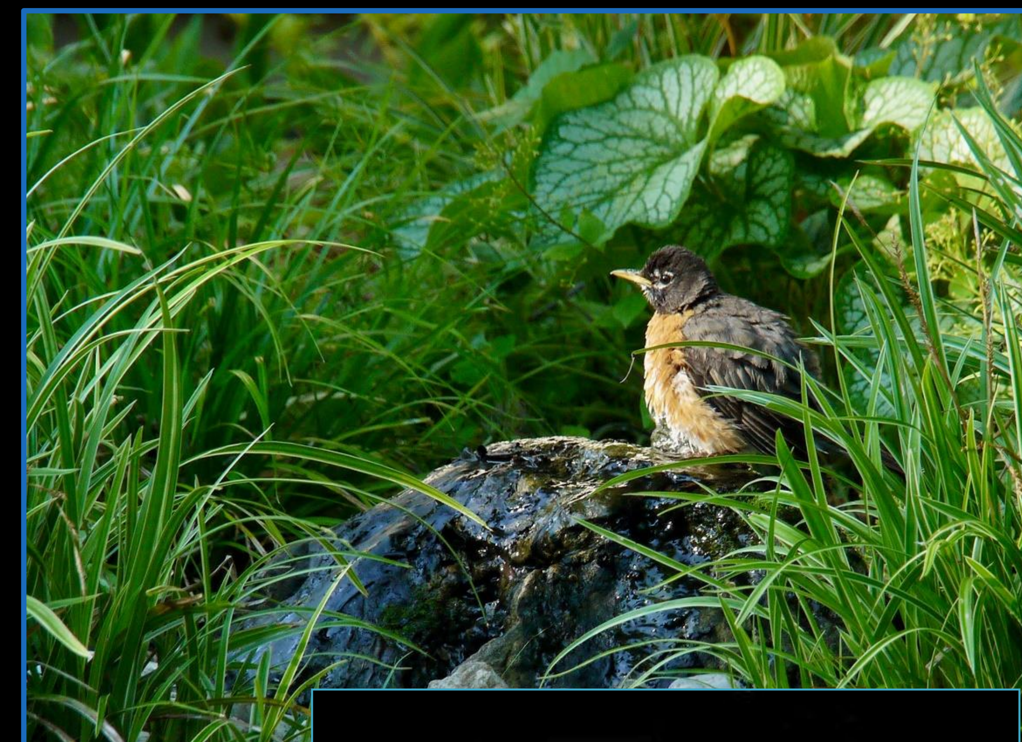
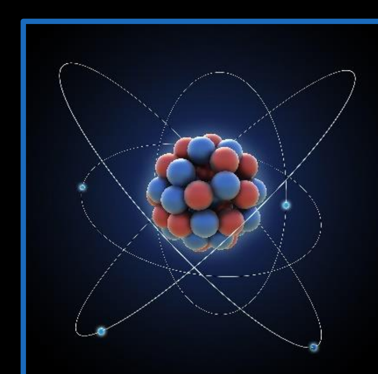
GENERAL FRAMEWORK

- DECOUPLING
- EXPLOITING HIERARCHIES
- WHY RENORMALIZATION IS A GOOD THING
- TIME DEPENDENT FIELDS





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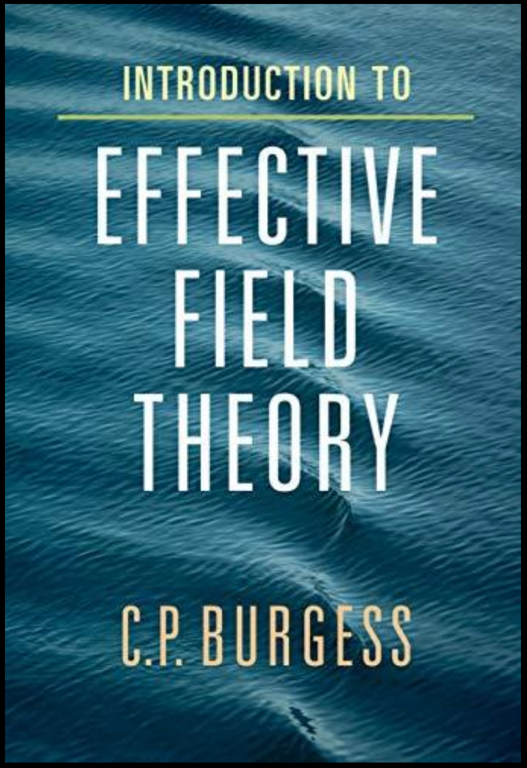
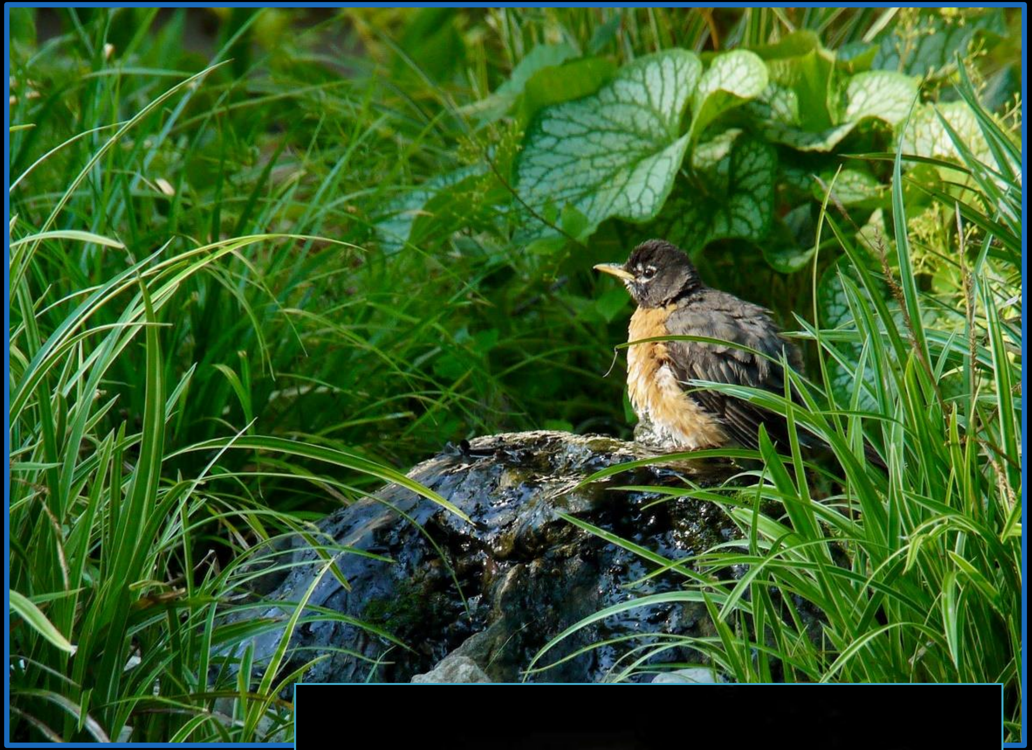
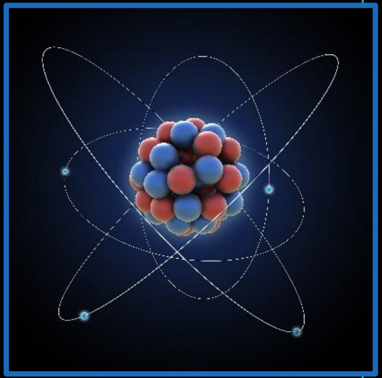
EFT APPLICATIONS

- ELECTROWEAK PHYSICS
- SUBSTRUCTURE
- NREFT
- GRAVITY



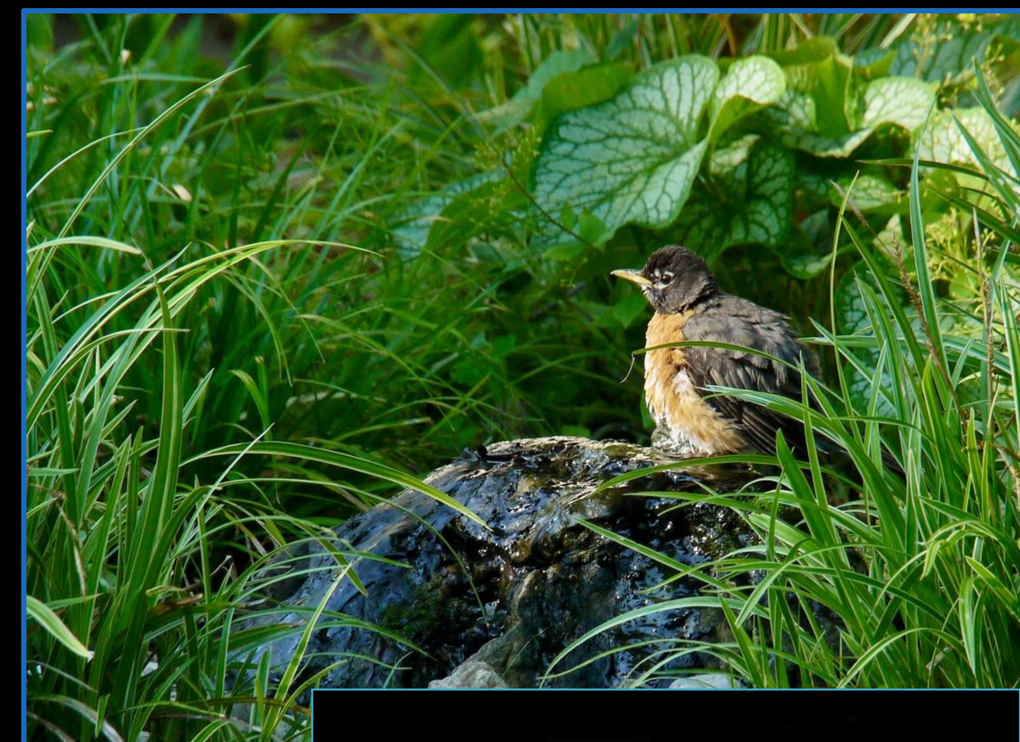


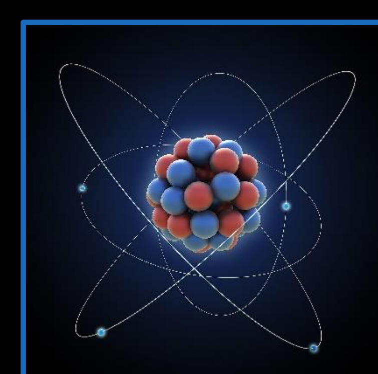


https://physics.mcmaster.ca/~cburgess/cburgess/?page_id=630 IMAGES: CB, GENEGEEK.CA, QUORA

GENERAL FRAMEWORK

DECOUPLING





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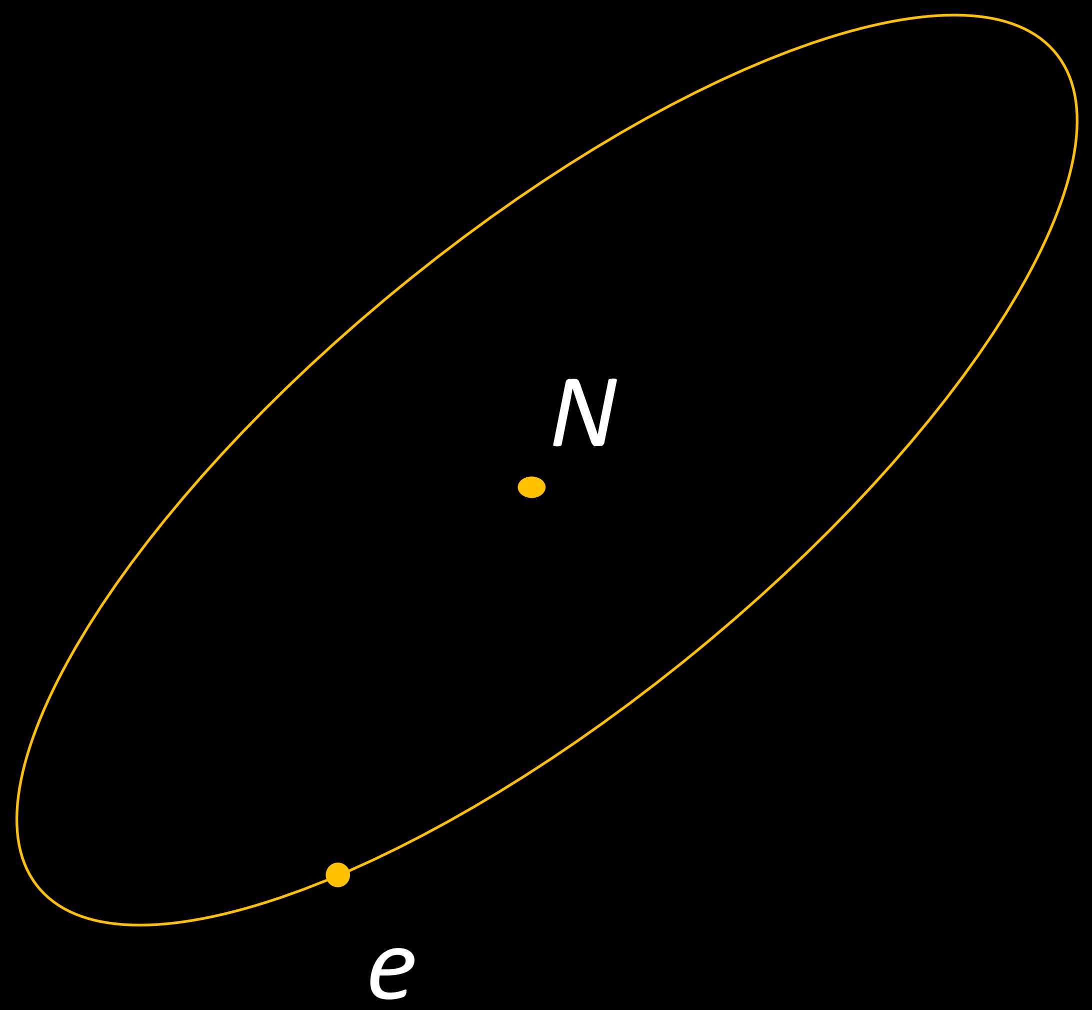
FACT: *Nature comes with many hierarchies of scale, and details of small distances are not needed to understand long distances*



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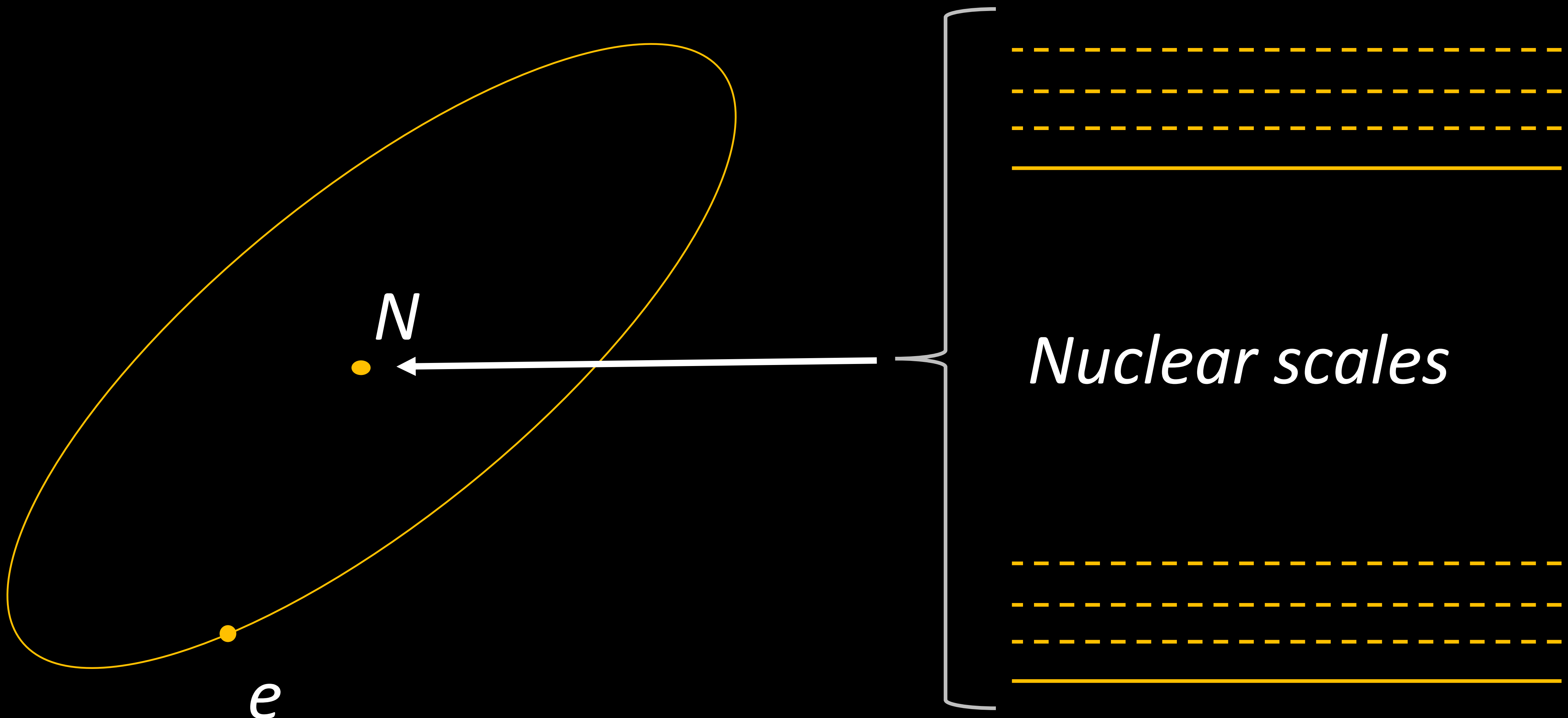
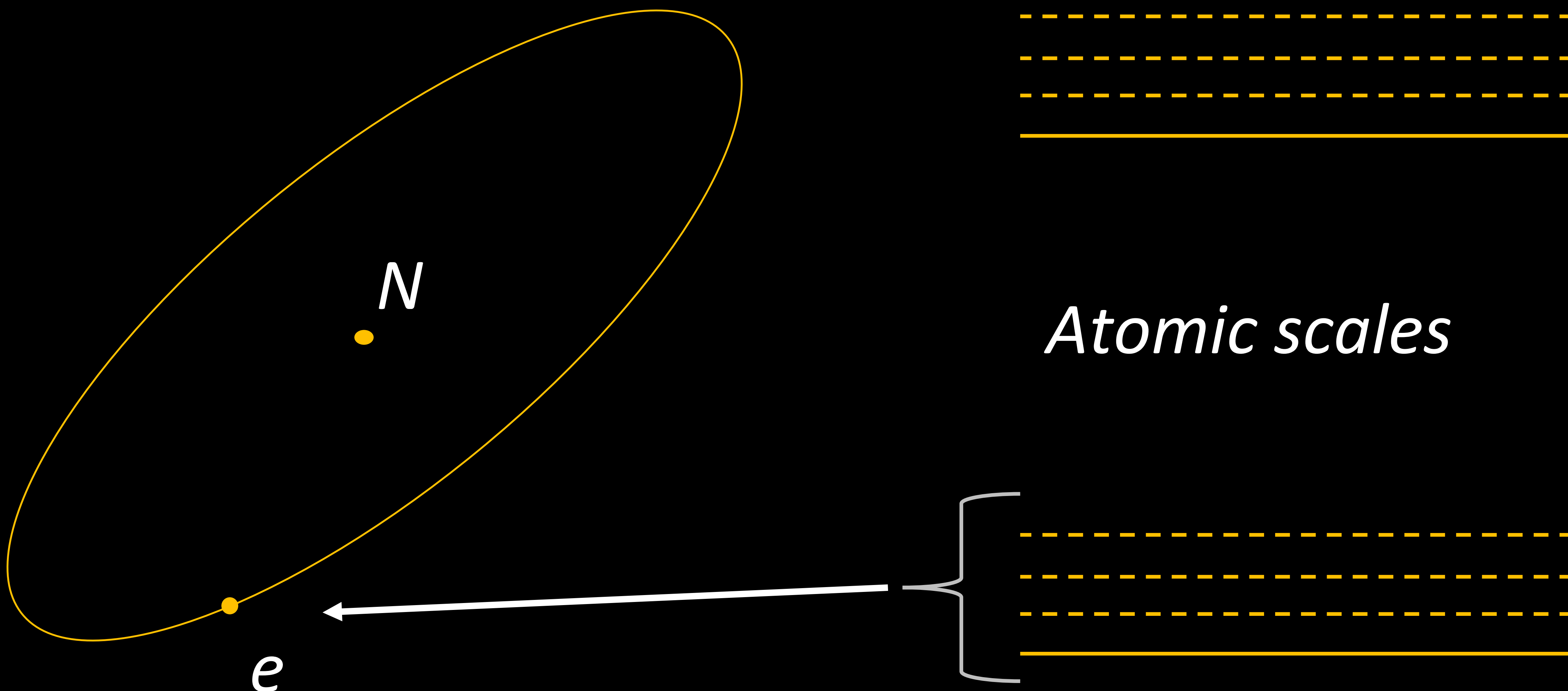
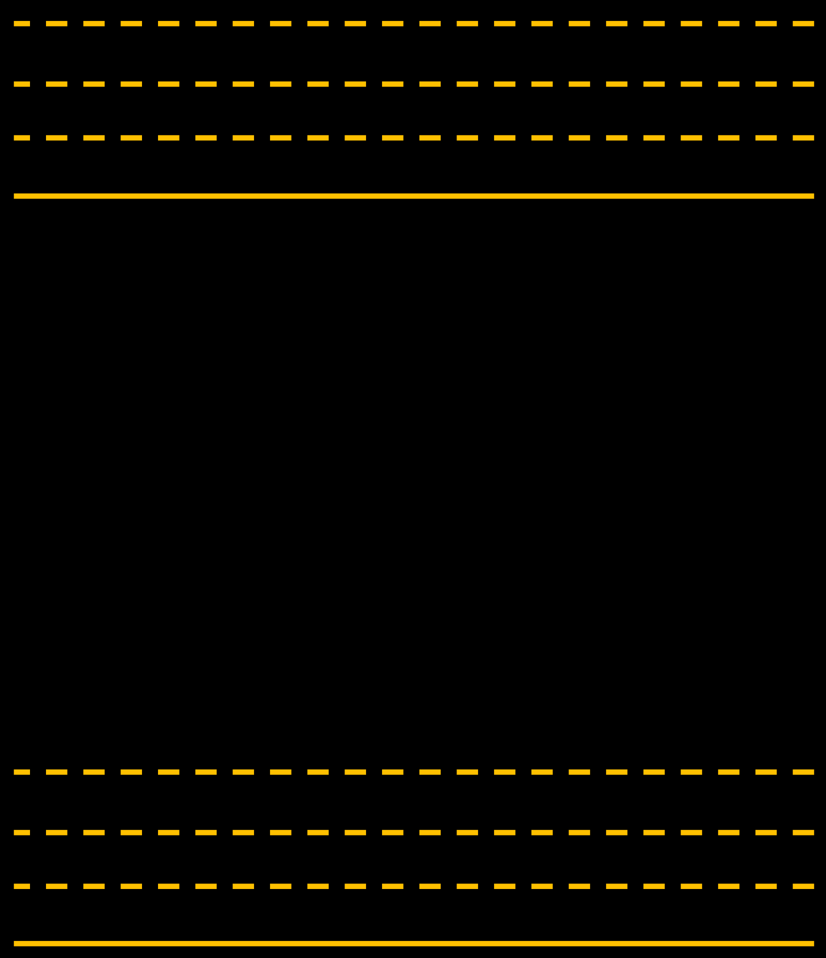


DECOUPLING

FACT: *Nature comes with many hierarchies of scale, and details of small distances are not needed to understand long distances*



FACT: *Quantum field theory shares this property that small distance physics drops out of long distance physics*



$$A(m, M, \theta) = M^p\, f\left(\tfrac{m}{M}, \theta\right)$$

$$\simeq M^p\, f(0, \theta)\Big[1 + O(m/M)\Big]$$

(modulo logarithms)

Behooves us to exploit this simplicity as early as possible in a calculation: EFTs

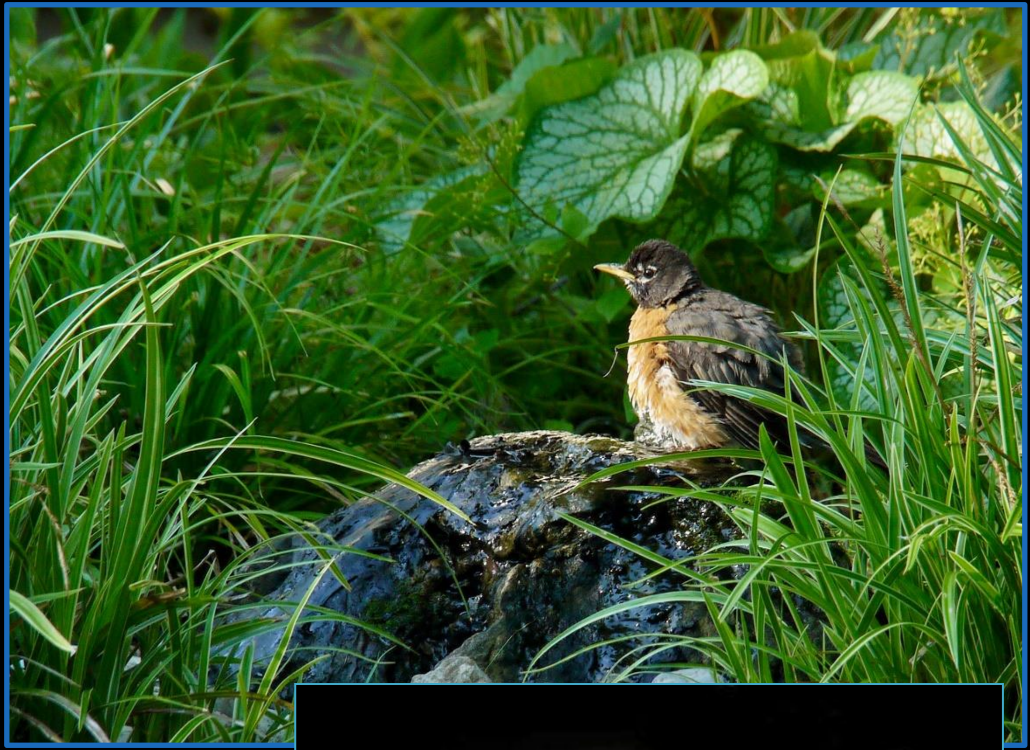
$$A(m, M, \theta) \simeq M^p\, f(0,\theta)\Big[1 + O(m/M)\Big]$$

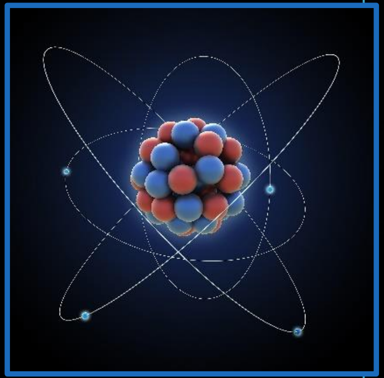
Applicable everywhere because in quantum physics we cannot help probing very short distances

$$E_k \simeq \langle k|H_I|k\rangle + \sum_n \frac{|\langle n|H_I|k\rangle|^2}{E_k - E_n} + \cdots$$

GENERAL FRAMEWORK

EXPLOITING HIERARCHIES





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Simple example: two spinless fields

$$S := -\int \mathrm{d}^4x \left[\partial_\mu \phi^* \partial^\mu \phi + V(\phi^* \phi)\right]$$

$$V(\phi^*\phi) = \tfrac{\lambda}{4}\left(\phi^*\phi - v^2\right)^2$$

Perturbative treatment: $\phi = v + \frac{1}{\sqrt{2}}\Big(R + iI\Big)$

$$S_0 := -\tfrac{1}{2}\int \mathrm{d}^4x \left[\partial_\mu R\, \partial^\mu R + \partial_\mu I\, \partial^\mu I + \lambda\, v^2 R^2\right]$$

$$S_{\rm int} := -\int \mathrm{d}^4x \left[\tfrac{\lambda v}{2\sqrt{2}}\, R(R^2 + I^2) + \tfrac{\lambda}{16}(R^2 + I^2)^2\right]$$

Simple example: two spinless fields

Particle masses

$$m_R^2 = m^2 = \lambda v^2 \qquad m_I^2 = 0$$

Hierarchy $E \ll m$

$\phi)\big]$

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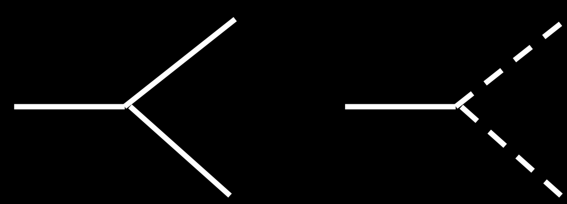
Simple example: two spinless fields

Interactions



λv

$$\left[\partial_\mu \phi^* \partial^\mu \phi + V(\phi^*\phi)\right]$$

$$= \frac{\lambda}{4}\left(\phi^*\phi - v^2\right)^2$$

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EXPLOITING HIERARCHIES

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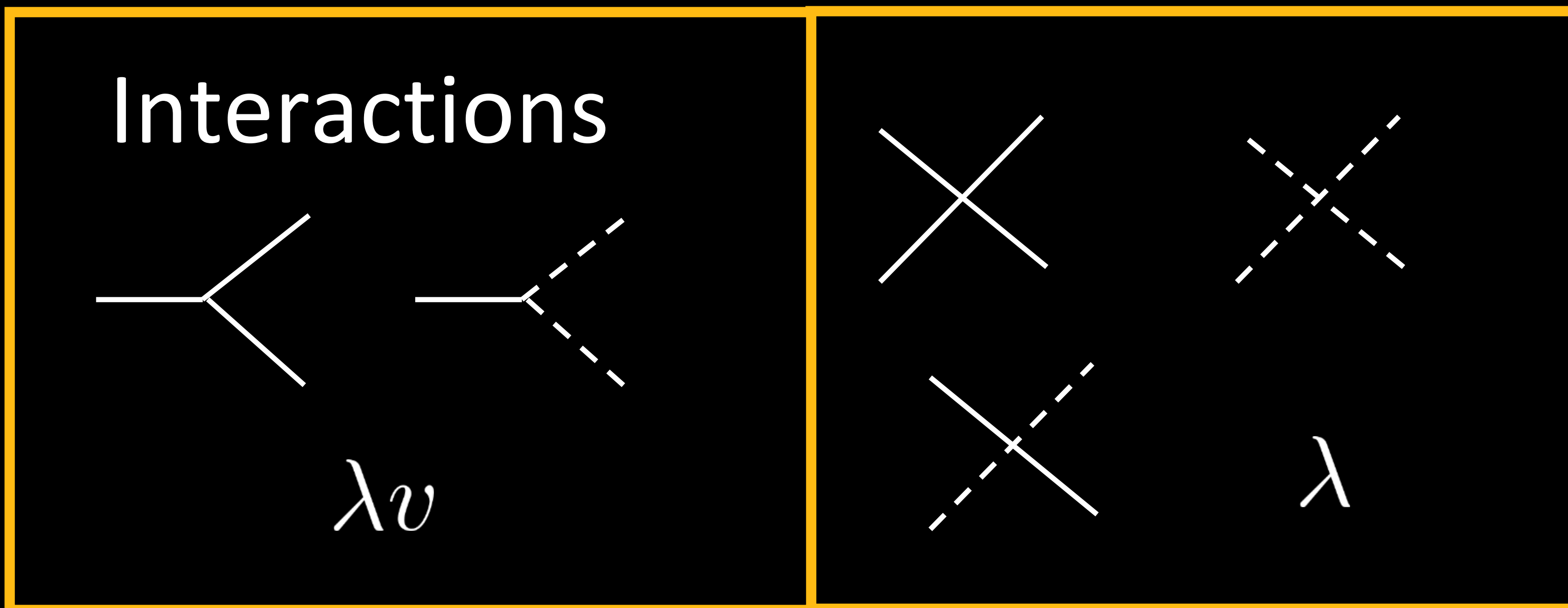

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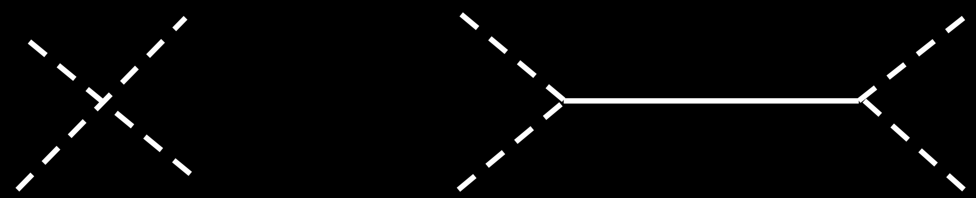
Low-energy scattering: $I(q) + I(p) \to I(q') + I(p')$



plus 'crossed' graphs

has (on shell) amplitude

$$\mathcal{A} = -\frac{3i\lambda}{2} + \frac{i(\lambda v)^2}{2}\left[\frac{1}{m^2+2p\cdot q} + \frac{1}{m^2-2q\cdot q'} + \frac{1}{m^2-2p\cdot q'}\right]$$

which at low energies becomes

$$\mathcal{A} \simeq 2i\lambda\left[\frac{(p\cdot q)^2+(p\cdot q')^2+(q\cdot q')^2}{m^4}\right] + O\left(m^{-6}\right)$$

Low-energy scattering: $I(q) + I(p) \to I(q') + I(p')$

Weaker than expected at energies $E \ll m_R$

Turns out to persist to higher orders in λ

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EXPLOITING HIERARCHIES - SYMMETRIES

Low-energy scattering: $I(q) + I(p) \to I(q') + I(p')$

Weaker than expected at energies $E \ll m_R$

Turns out to persist to higher orders in λ

has (on shell) amplitude

$\mathcal{A} =$ ... $]$

Also true for low-energy R+I scattering:

$$\mathcal{A}(R + I \to R + I) \simeq 2i\lambda \left(\frac{q \cdot q'}{m^2} \right) + O\left(m^{-4}\right)$$

whicl

$$\mathcal{A} \simeq 2i\lambda \left[\frac{(p\cdot q)^2 + (p \cdot q')^2 + (q \cdot q')^2}{m^4} \right] + O\left(m^{-6}\right)$$

Underlying symmetry $\phi \to e^{i\omega}\,\phi$

$$S := -\int \mathrm{d}^4x \,\left[\partial_\mu\phi^*\partial^\mu\phi + V(\phi^*\phi)\right]$$

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Better low-E variables: $\phi = \left(v + \frac{\chi}{\sqrt{2}}\right)\, e^{i\xi/\sqrt{2}\,v}$

$$\chi \to \chi \qquad \xi \to \xi + \sqrt{2}\,\omega$$

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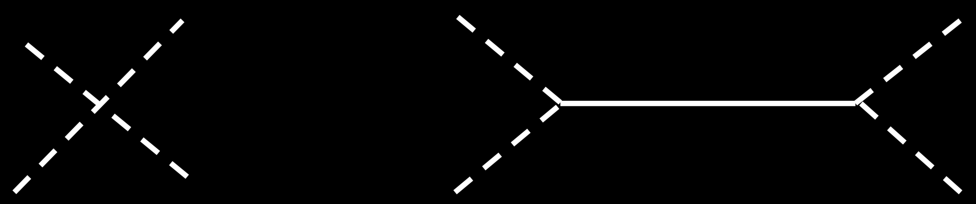

plus ‘crossed’ graphs

EXPLOITING HIERARCHIES - SYMMETRIES

Underlying symmetry $\phi \to e^{i\omega}\,\phi$

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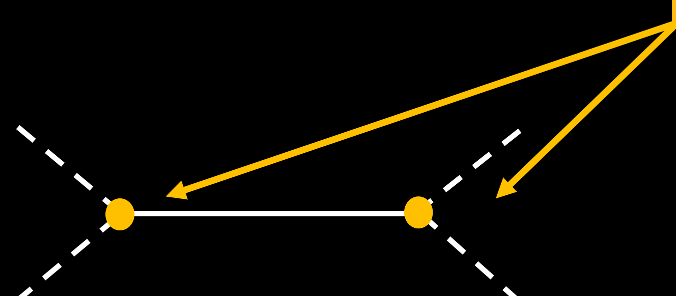
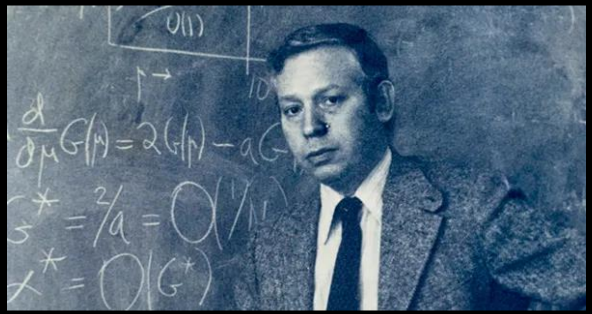


plus ‘crossed’ graphs

Moral: make symmetries manifest $\phi \to e^{i\omega}\,\phi$



You can use any variables you like, but if you use the wrong ones you will be sorry.
(one of Weinberg's 3 Laws of Theoretical Physics)

Symmetries cannot always be realized linearly when restricted to low energy variables

$$\begin{pmatrix} R \\ I \end{pmatrix} \to \begin{pmatrix} \cos\omega & -\sin\omega \\ \sin\omega & \cos\omega \end{pmatrix} \begin{pmatrix} R \\ I \end{pmatrix}$$

vs

$$\chi \to \chi \qquad \xi \to \xi + \sqrt{2}\,\omega$$

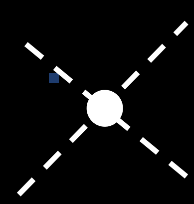
EXPLOITING HIERARCHIES - EFTS

Another Moral: the leading low-energy I-I scattering amplitude

$$\mathcal{A} \simeq 2i\lambda\left[\frac{(p\cdot q)^2+(p\cdot q')^2+(q\cdot q')^2}{m^4}\right]+O\left(m^{-6}\right)$$

is precisely as would have arisen from 'effective' interaction

$$S_{\rm eff} = \frac{\lambda}{4m^2}\int {\rm d}^4x\,(\partial_\mu I\,\partial^\mu I)^2$$

through the Feynman graph  plus 'crossed' graphs

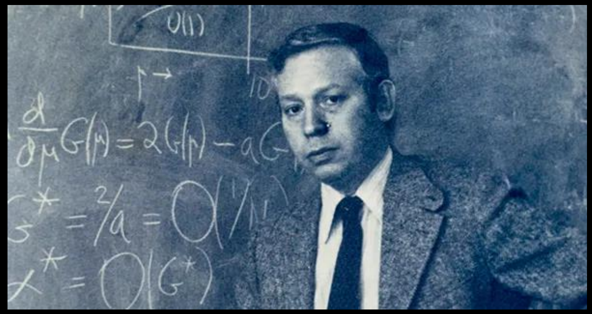
More remarkably: the order $(E/m)^4$ contributions to any low energy observable are captured by this same interaction, possibly with λ-corrected coefficient

Why does this work?

A low-energy lagrangian involving only the light field must exist because energy conservation ensures projecting onto low-energy is consistent with time-evolution

$$P_\Lambda e^{-iHt} P_\Lambda = e^{-iH_{\rm eff}t}$$

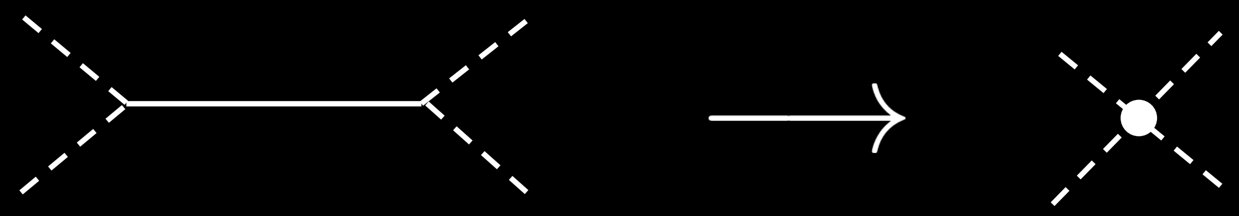
It can always be written in terms of the low energy field because this is a complete set of QFT operators at low energy



another of Weinberg's insights: QFT in itself contains no content beyond encoding things like special relativity, unitarity, cluster decomposition, etc in QM

But why is $H_{\rm eff}$ so simple? (eg why local? why no $1/m^2$ terms?)

Locality is a consequence of the uncertainty principle



Because energy conservation forbids actually producing them, heavy states can only influence light states as virtual particles.

Uncertainty relations allow temporary production of a state with energy *m* only for time intervals $\Delta t < 1/m$

$$G(x,y) = -i \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \; \frac{e^{ip(x-y)}}{p^2+m^2}$$

$$\simeq -\frac{i}{m^2} \sum_{k=0}^{\infty} \left(\frac{\partial^2}{m^2}\right)^k \; \delta^4(x-y)$$

More complicated interactions dimensionally cost more powers of 1/m, so should be less important at low energies

$$\mathcal{L}_{\rm eff} \simeq -\tfrac{1}{2}(\partial_\mu\xi\,\partial^\mu\xi) + \frac{\lambda}{m^4}(\partial_\mu\xi\,\partial^\mu\xi)^2 + \frac{a_8}{m^8}(\partial_\mu\xi\,\partial^\mu\xi)^3 + \cdots$$

so working to fixed order in E/m only involves a fixed number of interactions

Explains special role played by renormalizable interactions (unsuppressed by 1/m) in describing Nature

$$\mathcal{L}_{\rm ren} = g_3 m\,\xi^3 + g_4\xi^4$$

(renormalizable intns forbidden for toy model by symmetry)

Why not start at *1/m²*?

$$\mathcal{L} = \frac{a_1}{m^2}(\partial_\mu\xi\partial^2\partial^\mu\xi) + \frac{a_2}{m^2}(\partial_\mu\partial_\nu\xi\,\partial^\mu\partial^\nu\xi)$$

$1/m^2$ terms (and other $1/m^4$ terms) are all *redundant*

$$\mathcal{L} = \frac{(a_1-a_2)}{m^2}(\partial_\mu\xi\partial^2\partial^\mu\xi) + \boxed{\frac{a_2}{m^2}\partial_\nu(\partial_\mu\xi\,\partial^\mu\partial^\nu\xi)}$$

ie a total derivative

Why not start at *1/m²*?

$$\mathcal{L} = \frac{a_1}{m^2}(\partial_\mu \xi \partial^2 \partial^\mu \xi) + \frac{a_2}{m^2}(\partial_\mu \partial_\nu \xi \, \partial^\mu \partial^\nu \xi)$$

$1/m^2$ terms (and other $1/m^4$ terms) are all *redundant*

$$\mathcal{L} = \boxed{\frac{(a_1 - a_2)}{m^2}(\partial_\mu \xi \partial^2 \partial^\mu \xi)} + \frac{a_2}{m^2}\partial_\nu(\partial_\mu \xi \, \partial^\mu \partial^\nu \xi)$$

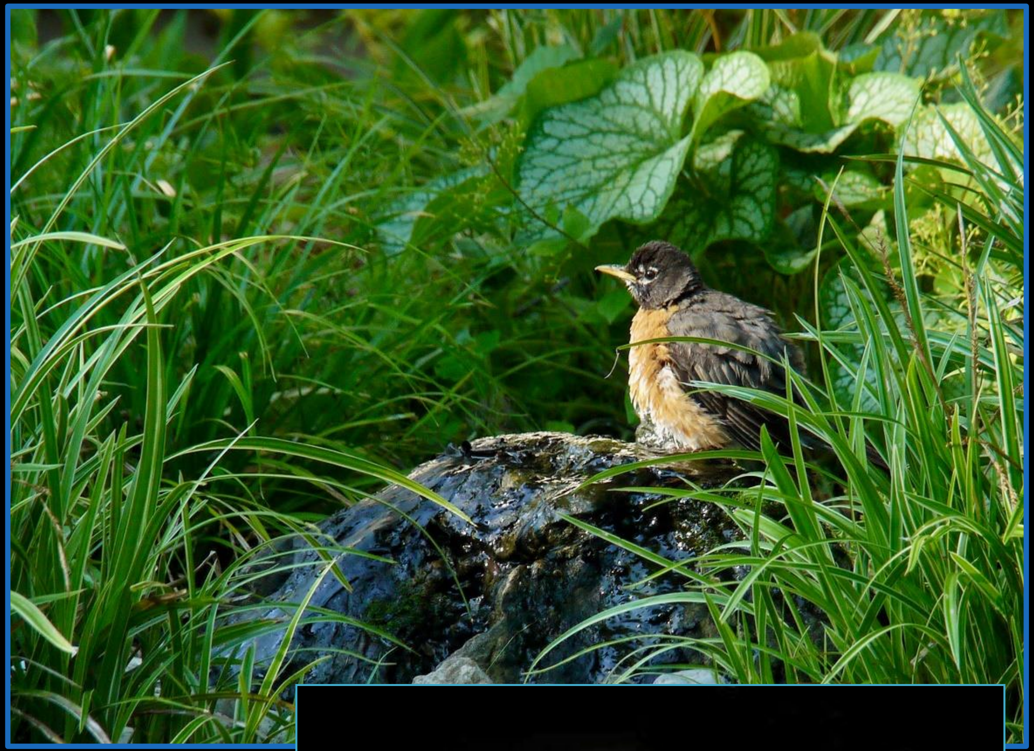
ie a total derivative or can be removed with a field redefinition

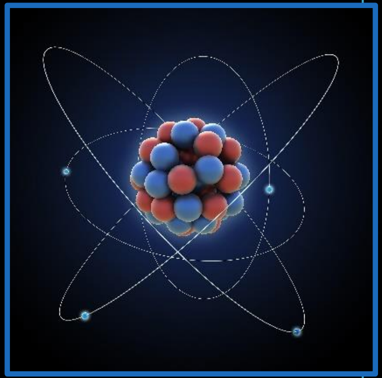
$\xi \to \xi + \frac{a_2 - a_1}{m^2} \, \partial^2 \, \xi$ for which

$$-\frac{1}{2}\,\partial_\mu \xi \partial^\mu \xi \to -\frac{1}{2}\,\partial_\mu \xi \partial^\mu \xi + \frac{a_2 - a_1}{m^2}\,(\partial_\mu \xi \partial^2 \partial^\mu \xi)$$

GENERAL FRAMEWORK

WHY RENORMALIZATION IS A GOOD THING





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WILSON ACTION

Suppose heavy and light degrees of freedom exist but only the light ones are ever measured:

$$\langle O(\ell) \rangle = \int \mathcal{D}\ell \, \mathcal{D}h \; e^{iS[\ell,h]} \, O(\ell)$$

Most simple if m/M expansion is done as early as possible

Define low-energy by $E < \Lambda$ and the Wilson action by

$$e^{iS_\Lambda[\ell]} = \int_\Lambda \mathcal{D}h \; e^{iS[\ell,h]}$$

so

$$\langle O(\ell) \rangle = \int^\Lambda \mathcal{D}\ell \; e^{iS_\Lambda[\ell]} \, O(\ell)$$

Important Properties follow from the definition:

1. Same low-energy expansion for S_Λ applies to all low-energy observables

$$\langle O(\ell)\rangle = \int^\Lambda \mathcal{D}\ell \; e^{iS_\Lambda[\ell]} \; O(\ell)$$

2a. The precise form for S_Λ depends in detail on precisely how the high- and low-energy sectors get split up

$$e^{iS_\Lambda[\ell]} = \int_\Lambda \mathcal{D}h \; e^{iS[\ell,h]}$$

2b. The details in S_Λ precisely cancel their counterparts in the measure once physical quantities are computed

$$\langle O(\ell)\rangle = \int \mathcal{D}\ell \, \mathcal{D}h \; e^{iS[\ell,h]} \; O(\ell)$$

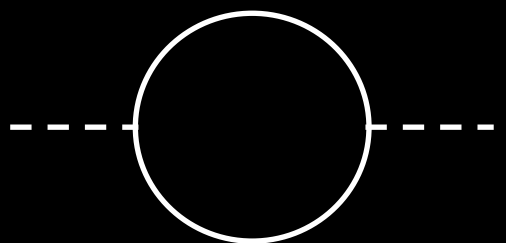
WILSON ACTION - RENORMALIZATION

Why renormalization is a good thing:

S_Λ appears in the path integral in the same way as does the traditional classical action

$$\langle O(\ell)\rangle = \int^{\Lambda} \mathcal{D}\ell \; e^{iS_\Lambda[\ell]} \; O(\ell)$$

The cancellation of Λ-dependence of integral with the dependence in S_Λ sounds exactly like traditional way of describing renormalization



$$\frac{1}{\alpha_{\mathrm{phys}}} = \frac{1}{\alpha_0(\Lambda)} + b \ln\left(\frac{\Lambda}{m}\right)$$

Surely the classical action is really just a Wilson action for a still higher UV completion?

EFTs & dimensional regularization: Can use freedom of definition to use dim-reg (rather than cutoffs like Λ) in the effective theory (with couplings fixed by ‘matching’)

Cutoff split

Dim-reg split

UV: ℓ, h

UV: ℓ, h

M

Λ

EFT: ℓ

EFT: ℓ

m

EFTs & dimensional regularization: Can use freedom of definition to use dim-reg (rather than cutoffs like Λ) in the effective theory (with couplings fixed by 'matching')

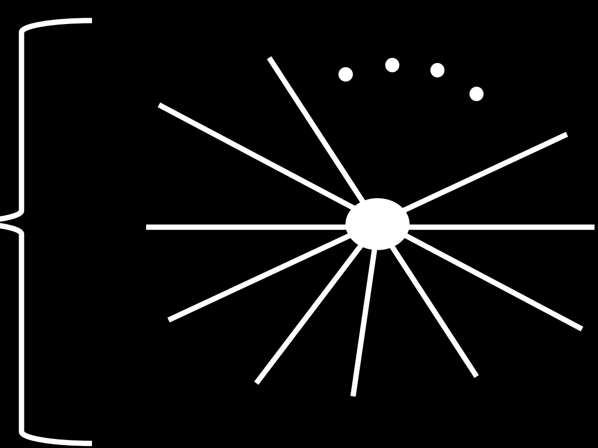
Cutoff Definition:	*Dim-Reg Definition:*
High-energy sector: all modes of *h* fields & E > Λ modes of ℓ	High-energy sector: all modes of *h*
Low-energy sector: E < Λ modes of ℓ	Low-energy sector: all modes of ℓ

These differ only in how they treat high-energy modes (E > Λ modes of ℓ) and so diff. can be absorbed into eff. couplings.

Dimensional regularization allows more precise identification of which interactions contribute at each order in energy/m.

$$\mathcal{L}_{\rm int} = \mu^4 \sum_n M^{-d_n} v^{-f_n} O(\partial^{d_n}, \phi^{f_n})$$

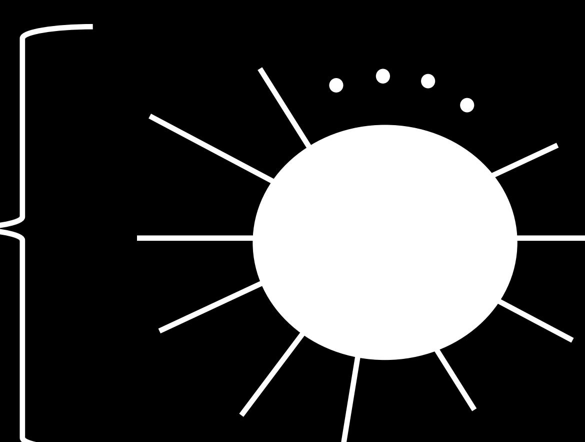
f_n fields



d_n derivatives

Use these to build a Feynman graph with E external lines, L loops and V_n vertices with f_n fields and d_n derivatives

E external lines



L loops, V_n vertices

WILSON ACTION – POWER COUNTING

Use relations amongst *E, I, L, V* coming from fact they connect together to make a graph

$$E + 2I = \sum_n f_n V_n \quad \text{(conservation of ends)}$$

$$L = 1 + I - \sum_n V_n \quad \text{(definition of \# of loops)}$$

Track factors of μ, *M* and *v* from vertices and use dimensional analysis to determine power of external energy scale *Q*

$$\mathcal{L}_{\rm int} = \mu^4 \sum_n M^{-d_n} v^{-f_n} O(\partial^{d_n}, \phi^{f_n})$$

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Track factors of μ, *M* and *v* from vertices and use dimensional analysis to determine power of external energy scale *Q*

$$\mathcal{A}_E \sim \mu^4 \left(\frac{1}{v}\right)^E \left(\frac{MQ}{4\pi\mu^2}\right)^{2L} \prod_n \left(\frac{Q}{M}\right)^{2+(d_n-2)V_n}$$

Only positive powers of external energy scale *Q* implies systematic low-energy expansion beyond leading order.

WILSON ACTION – POWER COUNTING

For example in the toy model we had $v = M = \mu = m$.

Amplitude with E external ξ particles depends on external energy scale Q by an amount

$$\mathcal{A}_E \sim m^4 \left(\frac{1}{m}\right)^E \left(\frac{Q}{4\pi m}\right)^{2L} \prod_n \left(\frac{Q}{m}\right)^{2+(d_n-2)V_n}$$

where all interactions satisfy $d_n \geq f_n \geq 4$

When $E = 4$ largest contribution has: $L = 0$ and $V_n = 1$

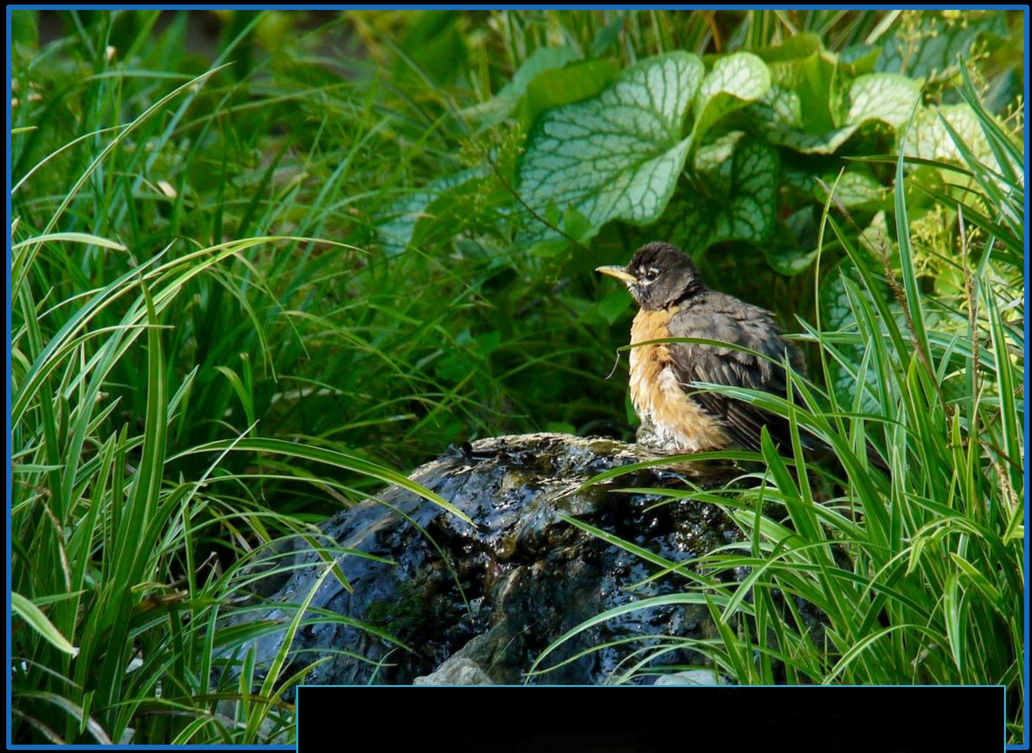
only if $d_n = f_n = 4$

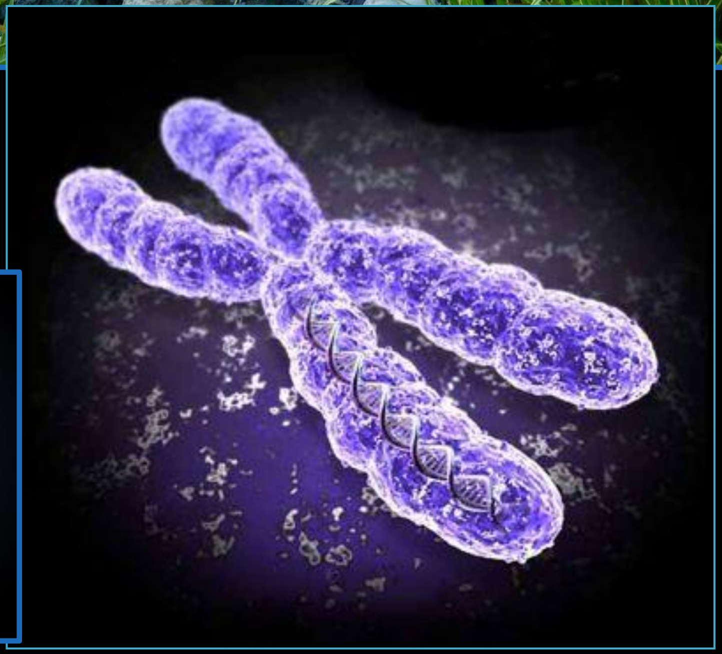
for which

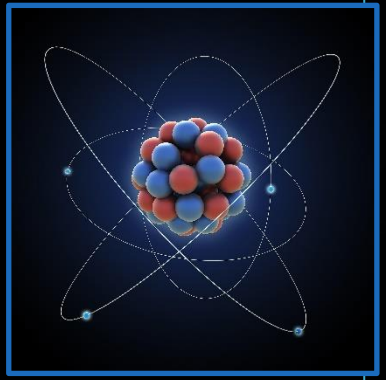
$$\mathcal{A}_E \sim \left(\frac{Q}{m}\right)^4$$

GENERAL FRAMEWORK

TIME-DEPENDENT FIELDS







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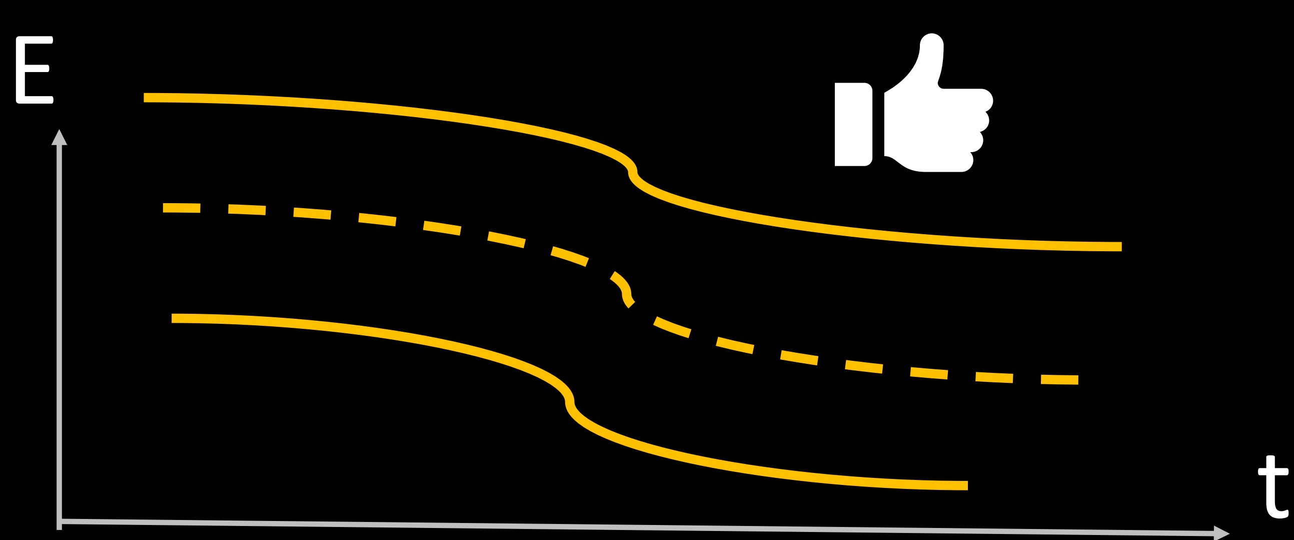
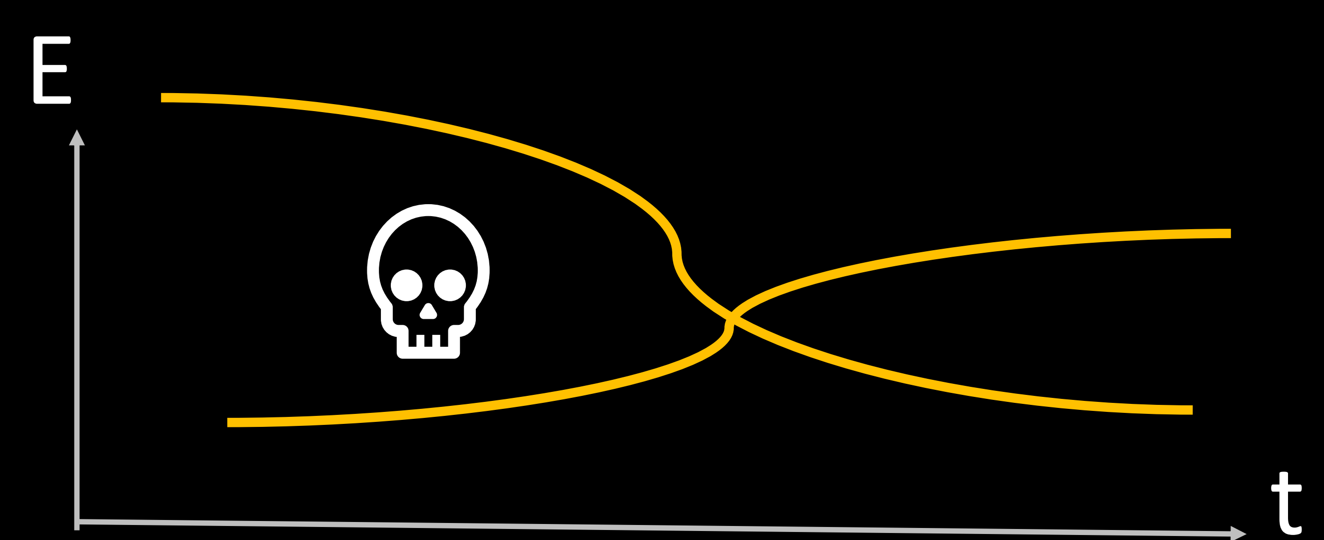
EVOLVING BACKGROUND FIELDS

Time-dependent background fields: Can EFTs be used with time-dependent backgrounds?

Naively not, because energy conservation is central while energy for fluctuations need not be conserved for time-dependent backgrounds.

They can if evolution is adiabatic, so $A^{-1}\, dA/dt \ll$ UV scales.
Then energy levels vary parametrically with time, $E_n = E_n(t)$.
Must also demand high and low energy levels do not cross.





Toy model of two spinless fields

$$S := -\int d^4x \left[\partial_\mu \phi^* \partial^\mu \phi + V(\phi^*\phi)\right]$$

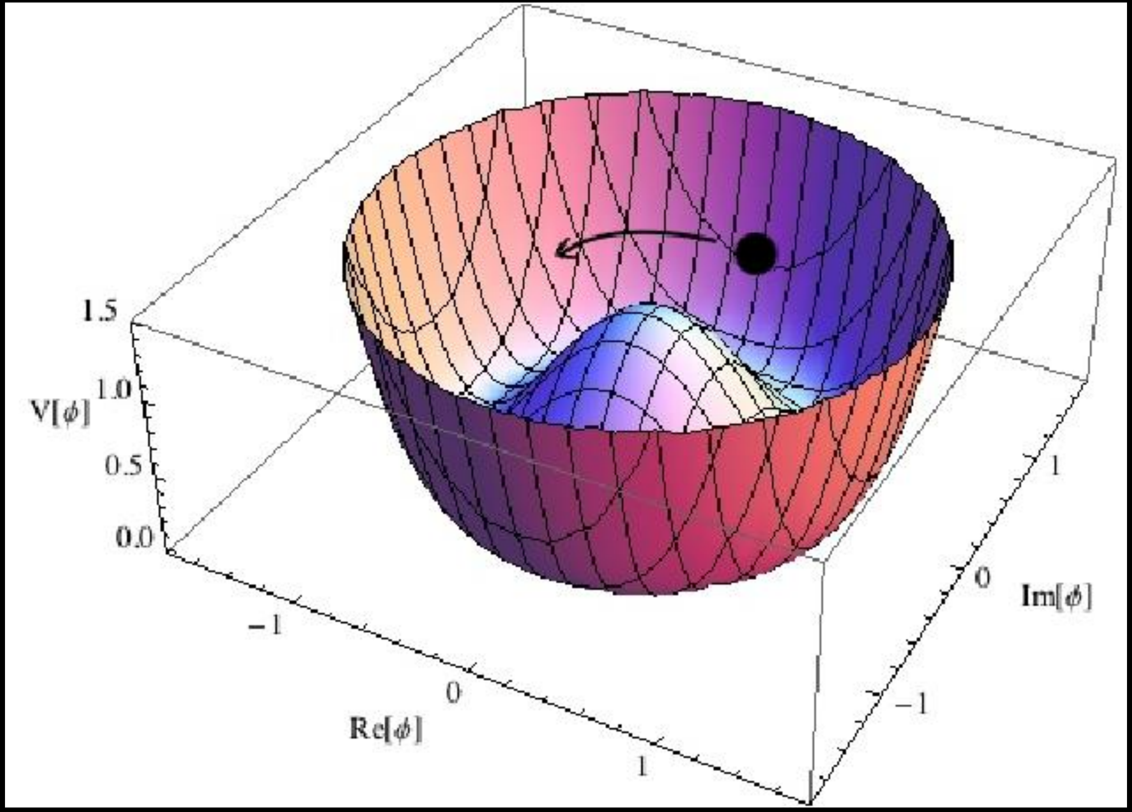
$$V(\phi^*\phi) = \tfrac{\lambda}{4}\left(\phi^*\phi - v^2\right)^2$$

Low energy EFT: $$S_{\text{eff}} = \frac{\lambda}{4m^2}\int d^4x \, (\partial_\mu \xi \, \partial^\mu \xi)^2$$

Time dependent classical solution:

$$\phi_{\text{cl}} = \rho_0 \, e^{i\omega t}$$

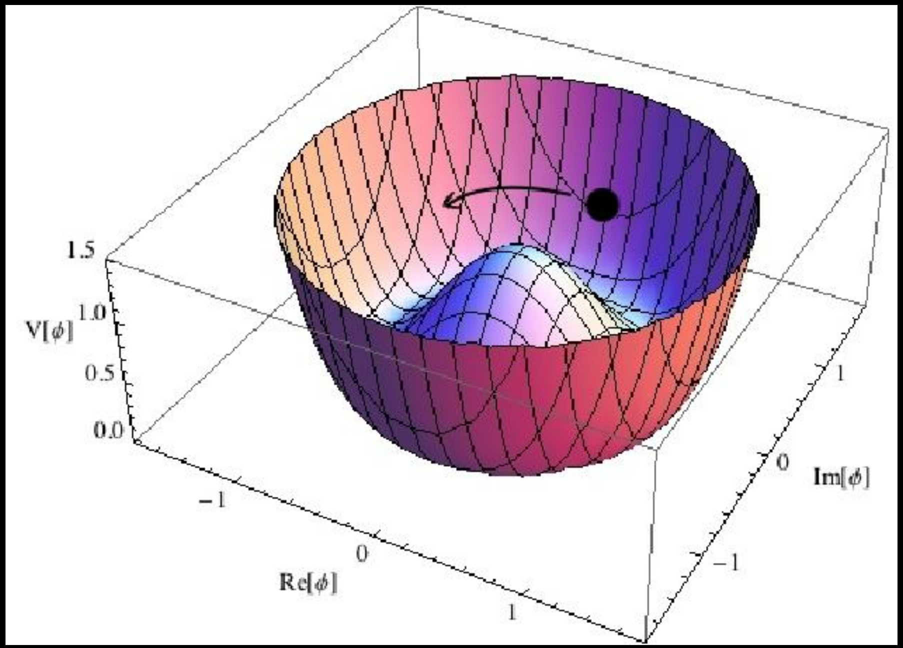
EOM: $$\rho_0 = \sqrt{v^2 + \frac{2\omega^2}{\lambda}}$$



Classical energy partly kinetic and partly due to field climbing the potential

$$\varepsilon = \dot{\phi}^* \dot{\phi} + V = \omega^2 \left(v^2 + \frac{3\omega^2}{\lambda} \right)$$

The EFT has no V, so how does it account for the potential energy?



$$\varepsilon = \frac{1}{2}\dot{\xi}^2 + \frac{3\lambda}{4m^4}\dot{\xi}^4$$
$$= \omega^2 \left(v^2 + \frac{3\omega^2}{\lambda} \right)$$

Effective interaction (with same coupling as needed for scattering) provides precisely the required classical energy

Classical energy partly kinetic and partly due to field climbing the potential

$$\varepsilon = \dot{\phi}^* \dot{\phi} + V = \omega^2 \left(v^2 + \frac{3\omega^2}{\lambda} \right)$$

The EFT has no V, so how does it account for the potential energy?

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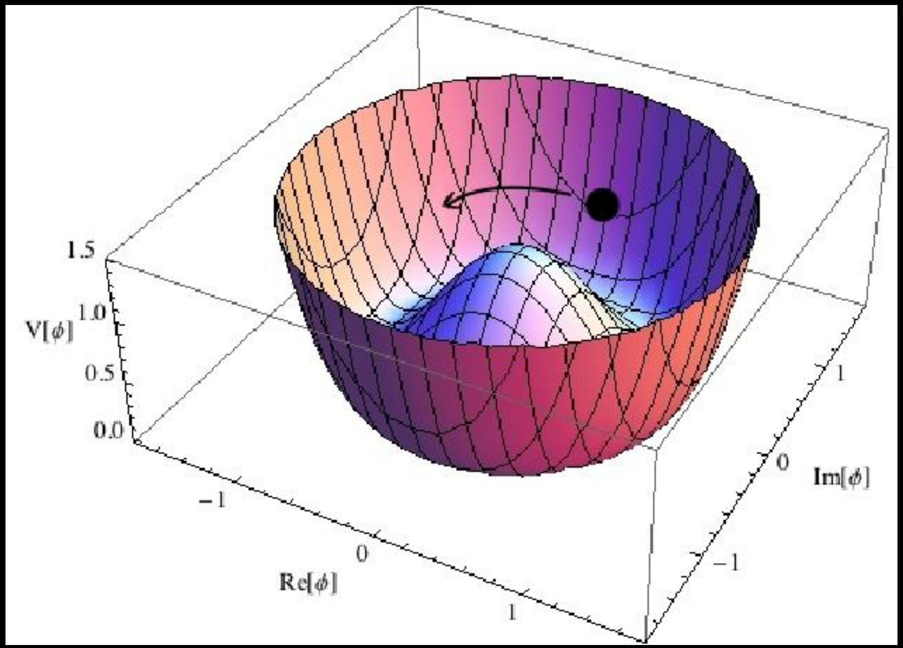
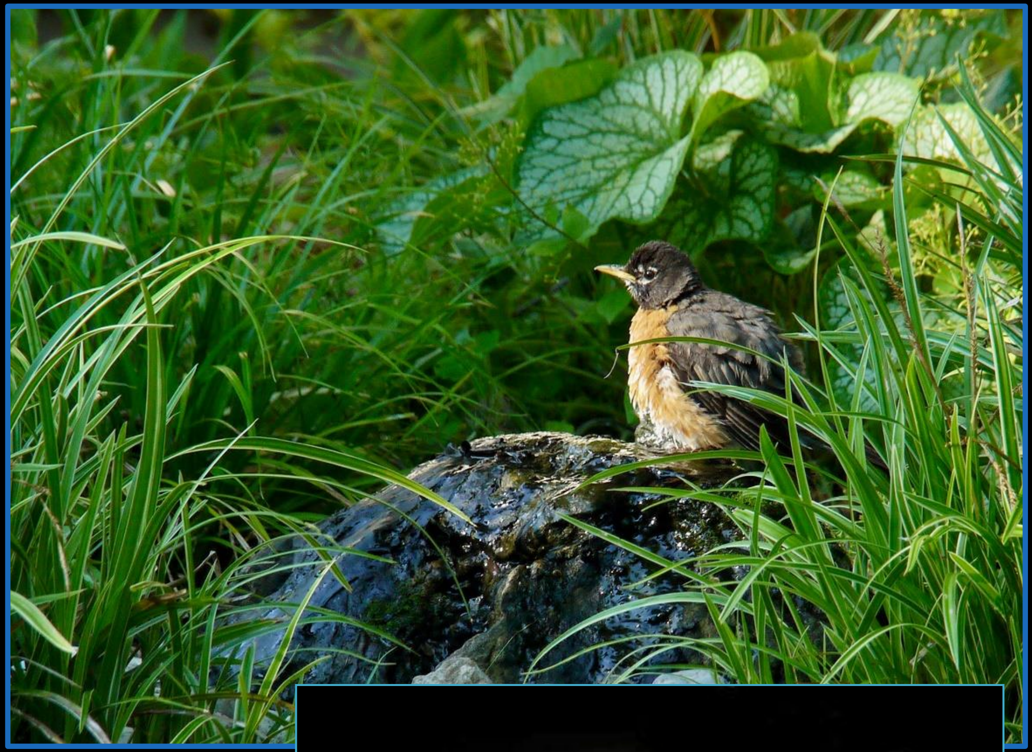
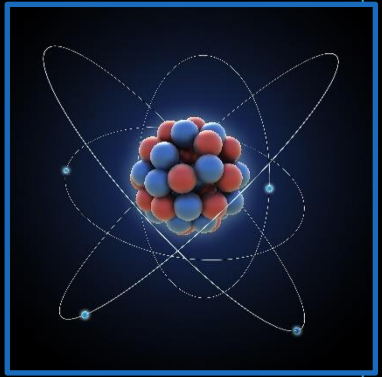


This is why EFT methods can be applied eg in cosmological applications (which proves important for consistency in GR)

EFT EXAMPLES

ELECTROWEAK PHYSICS





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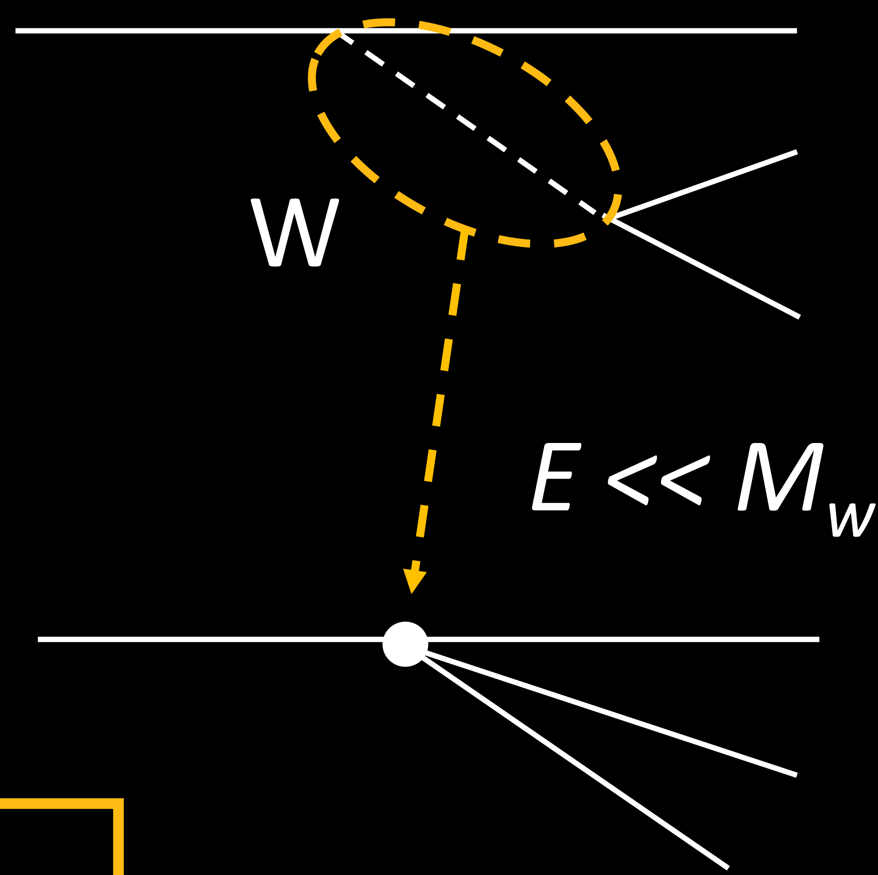
ELECTROWEAK EFTS – FERMI THEORY

The Standard Model provides many examples of hierarchies of scale (some of which helped EFT methods to be discovered).

Weak decays proceed through W exchange

$$\mathcal{L}_w = \frac{g}{2\sqrt{2}} W_\mu J^\mu + \text{c.c.}$$



Fermi theory describes weak interactions at low energies

$$\mathcal{L}_F = \frac{G_F}{\sqrt{2}} J^*_\mu J^\mu$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_w^2}$$

Massless spin-1 particle

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

A_μ not a 4-vector!

$$A_\mu \to A_\mu + \partial_\mu \omega$$

Free action

$$\mathcal{L}_0 = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Nonderivative couplings to other matter

$$S_{\rm mat} = e \int \mathrm{d}^4 x \, A_\mu J^\mu(\psi) \quad \Rightarrow \quad \partial_\mu J^\mu(\psi) = 0$$

Gauge invariance and conserved current required by Lorentz invariance!

At low energies renormalizable interactions should dominate. What are possibilities for electron – photon?

$$\mathcal{L}_{\rm ren} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \overline{\psi}(\gamma^\mu\partial_\mu + m)\psi$$
$$-ieA_\mu\overline{\psi}\gamma^\mu\psi$$

All other interactions involve couplings with dimensions of inverse length (so are not renormalizable).

QED emerges at low energies as the most general form allowed for static massless spin-one coupling to matter

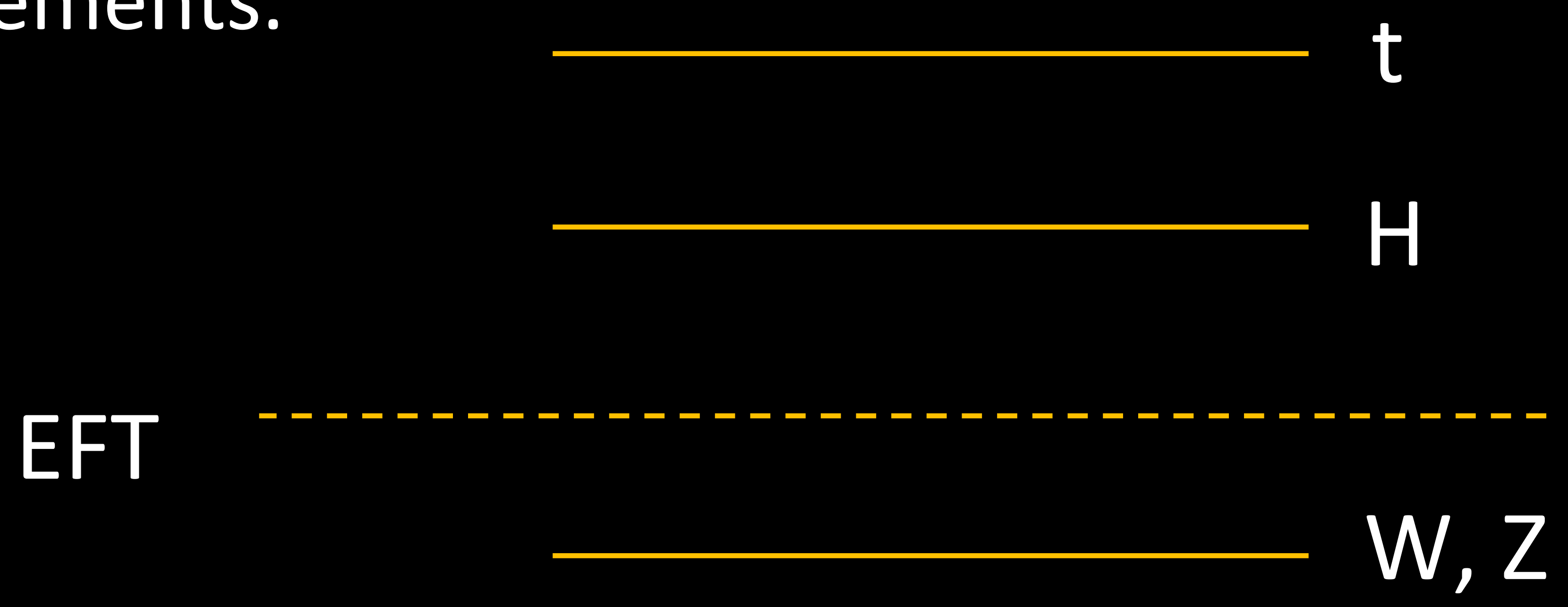
At low energies renormalizable interactions should dominate. What are possibilities once neutrino included?

$$\mathcal{L}_{\rm ren} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \overline{\psi}(\gamma^\mu\partial_\mu + m)\psi$$
$$-\overline{\nu}(\gamma^\mu\partial_\mu + m')\nu - ieA_\mu\overline{\psi}\gamma^\mu\psi$$

All other interactions involve couplings with dimensions of inverse length (so are not renormalizable).

Given their quantum numbers it should be no surprise that neutrinos only interact very weakly at low energies

EFTs identify dominant dependence of top and Higgs masses in precision measurements.



Lowest-dimension interactions are most sensitive to large masses. Lowest dimension interactions in EFT below Higgs mass are gauge boson masses

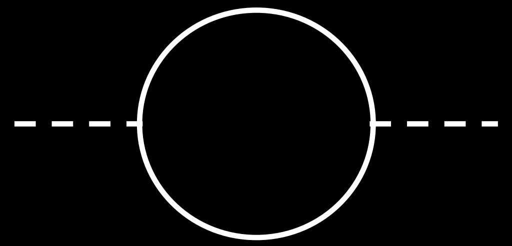
$$\mathcal{L}_2 = M_w^2 W_\mu^* W^\mu + \frac{1}{2} M_z^2 Z_\mu Z^\mu$$

Are there top/Higgs loops that contribute large effects to these masses and that can be measured? *YES!*

Both W and Z masses come from one source in SM itself

$$\mathcal{L}_2 \in D_\mu H^\dagger D^\mu H \Rightarrow M_w = M_z \cos\theta_w$$

Large top-bottom corrections are measurable if they change this relationship between W and Z masses

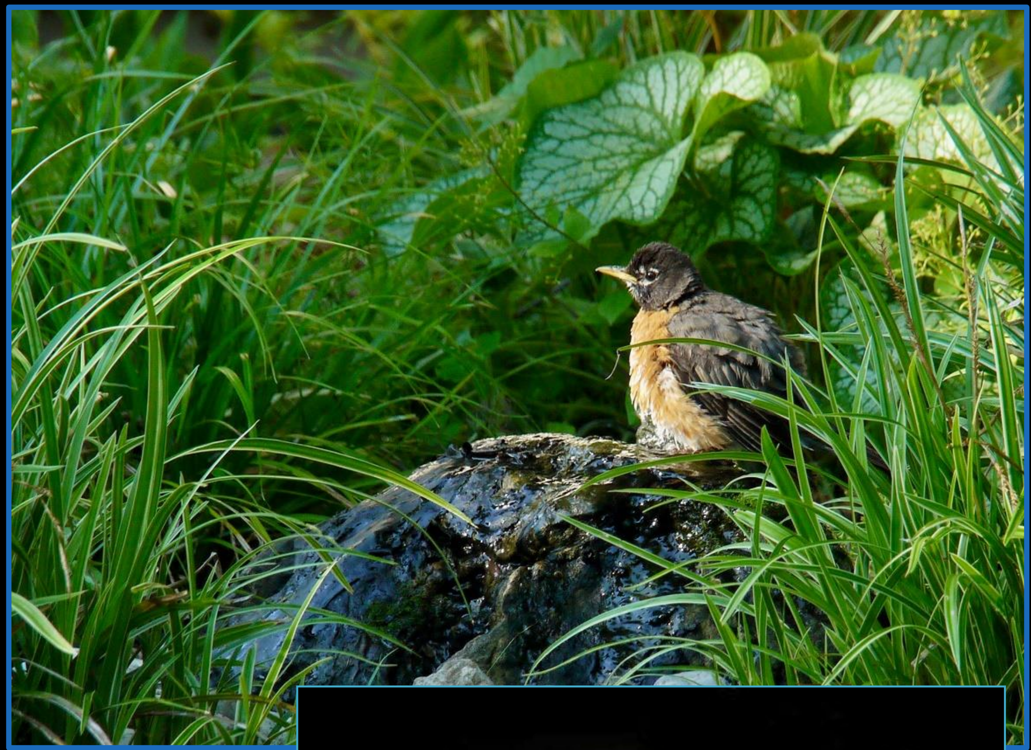


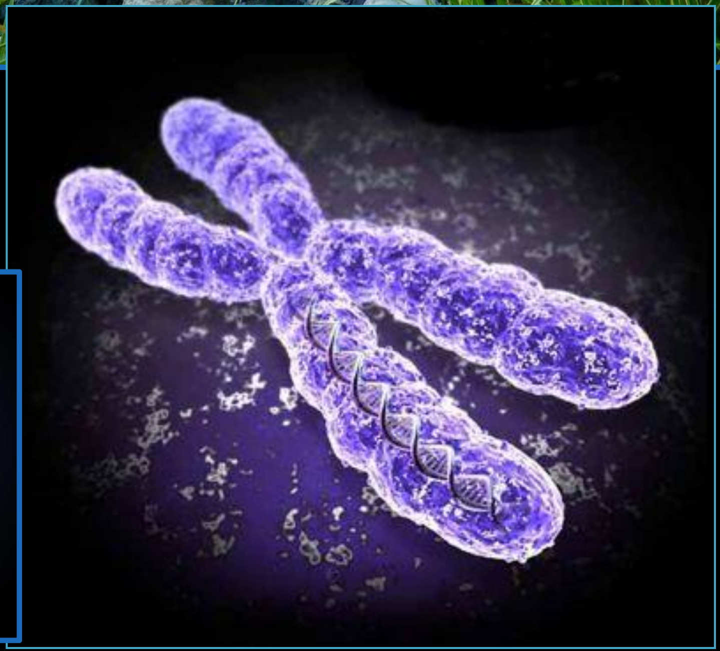
$$\frac{\delta M_w^2}{M_w^2} - \frac{\delta M_z^2}{M_z^2} \simeq \frac{3\alpha m_t^2}{16\pi \sin^2\theta_w M_w^2}$$

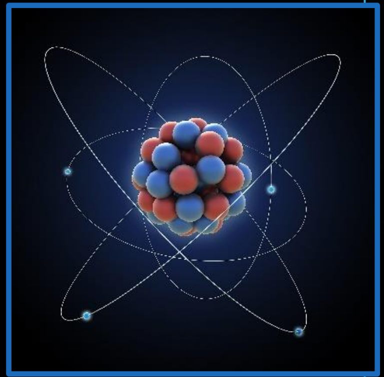
Measurements of this combination provided limit on top mass before it was found. Higgs only contributes logarithmically due to an accidental symmetry.

EFT EXAMPLES

SUBSTRUCTURE







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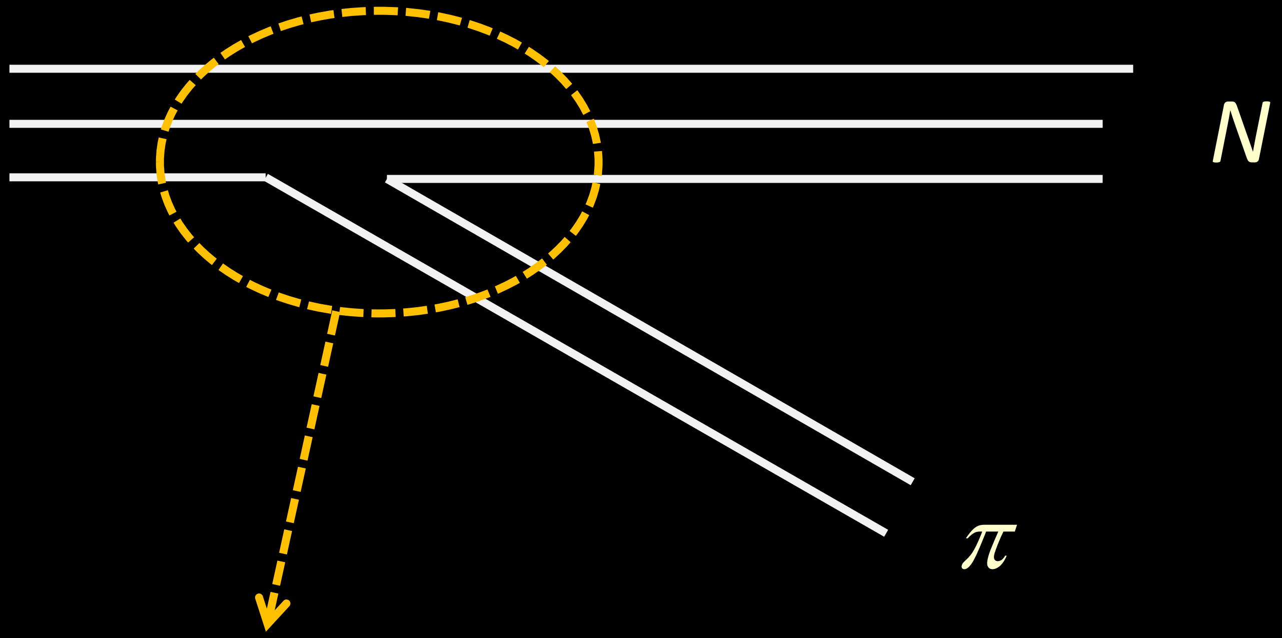
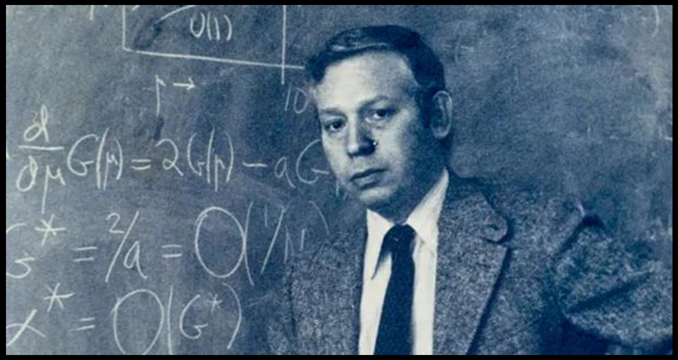
Fields in an EFT need not represent just fundamental particles. Nor must the underlying physics be weakly coupled.

For instance quarks and gluons are replaced by nucleons and mesons in an EFT at energies below 1 GeV



$$\mathcal{L}_{\text{eff}} \simeq \frac{1}{F_\pi}\,\partial_\mu \pi (\overline{N}\gamma^\mu N)$$

Because pions are Goldstone bosons for an approximate SU(2) x SU(2) symmetry of QCD their low-energy interactions are model-independent



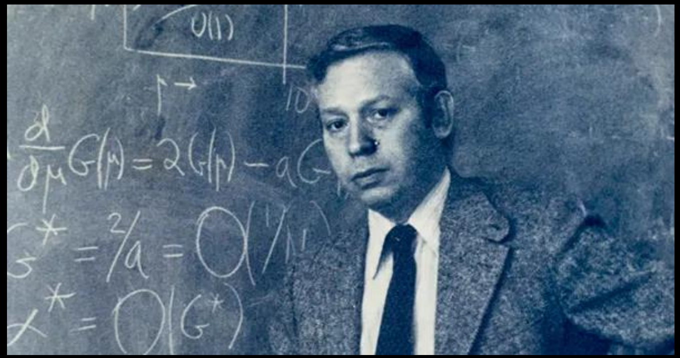
$$\mathcal{L}_{\text{eff}} \simeq -\frac{F^2}{2}\, g_{ab}(\theta)\, \partial_\mu \theta^a\, \partial^\mu \theta^b$$

with

$$g_{ab}(\theta) = \delta_{ab}\left(\frac{\sin^2\theta}{\theta^2}\right) + \theta_a\theta_b\left(\frac{\theta^2 - \sin^2\theta}{\theta^4}\right)$$

Because the symmetry is broken by quark masses can develop low energy predictions as a series in $E/(4\pi F)$ and $m_q/(4\pi F)$ ('chiral perturbation theory')

SUBSTRUCTURE – CHIRAL PERTURBATION THEORY



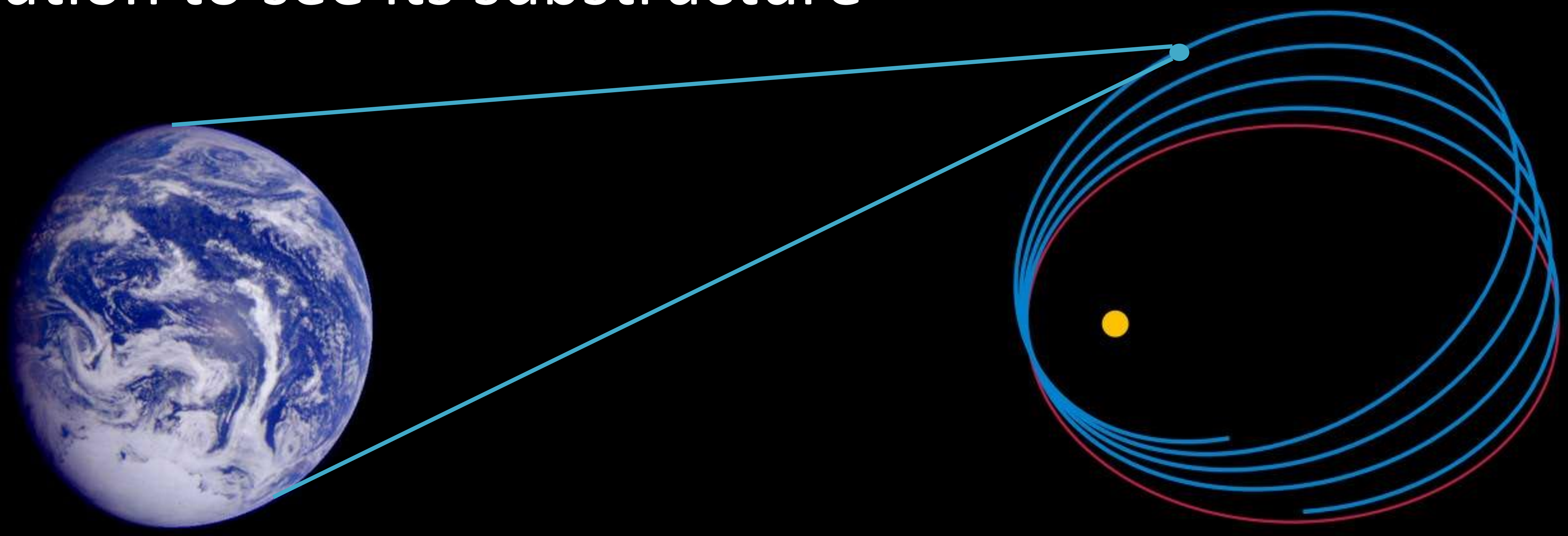
Pion self-interactions at leading order in the expansion are controlled by a single parameter *F*

$$\mathcal{L}_{\rm eff} \simeq -\frac{1}{2}\,\partial_\mu \vec\pi\,\partial^\mu \vec\pi - \frac{1}{2F^2}(\vec\pi\cdot\partial_\mu\vec\pi)(\vec\pi\cdot\partial^\mu\vec\pi) + O(\pi^6)$$

Implies relations amongst the low-energy pion scattering amplitudes (‘soft-pion theorems’)

Better yet, the parameter *F* can also be measured using the measured lifetime for charged-pion decays $\pi^+ \to \mu^+ \nu$

Any composite body can also be treated as elementary in this way when probed at energies and momenta with insufficient resolution to see its substructure

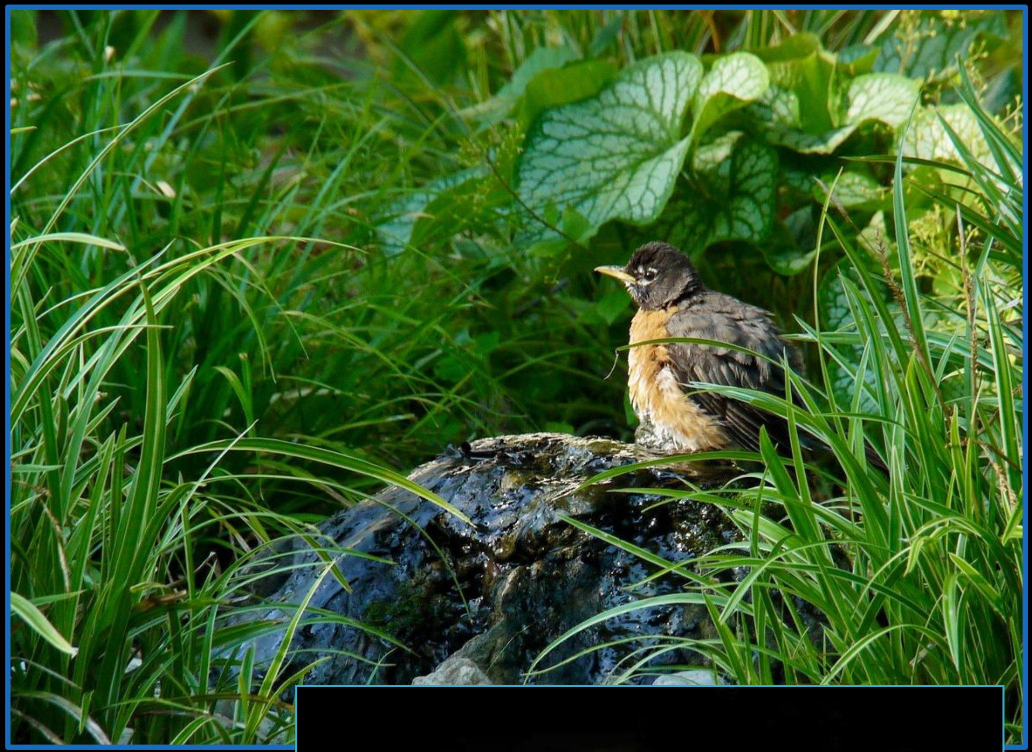


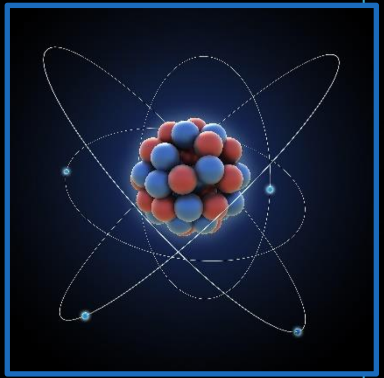
$$S_{\text{eff}} = \int_C \mathrm{d}\tau \sqrt{-g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}\left(m + c\mathbf{E}^2 + \cdots\right)$$

Centre of mass position becomes low-energy variable, leading to ordinary single particle QM as the EFT

EFT EXAMPLES

NREFT





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NREFT – NONRELATIVISTIC EFTS

For energies below a particle mass, any appearance of this particle in the low-energy theory must be nonrelativistic.

Q: why should we be allowed to keep a particle in the EFT for scales below the particle's mass?

This can be allowed if the rest mass of the heavy particle is inaccessible, such as if it is stable and in the absence of antiparticles

Practical examples include EFTs describing atoms for which energies of interest are much smaller than electron/nuclear rest mass

Can be defined as low-energy limit of a sector of fixed conserved charge (electric, or baryonic etc)

Effective nonrelativistic theory obtained in two steps:

Integrating out the antiparticle

$$\phi(x) = \sum_p \left(a_p\, e^{ipx} + \bar{a}_p^\star\, e^{-ipx} \right) \quad \text{(relativistic)}$$

$$\left.\begin{aligned} \Phi(x) &= \sum_p a_p\, e^{ipx} \\ \overline{\Phi}(x) &= \sum_p \bar{a}_p\, e^{ipx} \end{aligned}\right\} \quad \text{(nonrelativistic)}$$

Reset the zero of energy to exclude the rest mass

$$\phi(x) \to (2m)^{-1/2} e^{-imt} \left[\Phi(x) + \overline{\Phi}^\star(x) \right]$$

The leading NR EFT is Schrodinger field theory:

$$-\left[\partial_\mu \phi^* \partial^\mu \phi + m^2 \phi^* \phi\right] = \frac{i}{2}(\Phi^* \partial_t \Phi - \Phi \partial_t \Phi^*) - \frac{1}{2m} \nabla \Phi^* \cdot \nabla \Phi + \frac{1}{2m} |\partial_t \Phi|^2 + \cdots$$

Starting from the Dirac action leads to a similar result for the 2-component spin-half field $\Psi(x)$ such as used in eg HQET

Isgur, Wise

Nonrelativistic systems have multiple low-energy scales, whose small dimensionless ratio is the particle velocity *v*

$$m \gg p = |\mathbf{p}| = O(mv) \gg E = \frac{p^2}{2m} = O(mv^2)$$

More generally can perform usual matching procedure by choosing effective couplings in NR theory such that it agrees with low-energy limit of relativistic high-energy theory.

Example NRQED:

Caswell, Lepage

$$\mathcal{L}_0 = i\,\Psi^\dagger \partial_t \Psi + \boxed{e_q A_0 (\Psi^\dagger \Psi)}$$

(electrostatic)

More generally can perform usual matching procedure by choosing effective couplings in NR theory such that it agrees with low-energy limit of relativistic high-energy theory.

Example NRQED:

Caswell, Lepage

$$\mathcal{L}_0 = i\,\Psi^\dagger \partial_t \Psi + e_q A_0 (\Psi^\dagger \Psi)$$

(dipole)

$$\mathcal{L}_1 = \frac{1}{2m} \Psi^\dagger \nabla^2 \Psi + \boxed{\frac{ie_q}{2m}\, \mathbf{A} \cdot \left[(\nabla \Psi^\dagger) \Psi - \Psi^\dagger \nabla \Psi \right]}$$

$$- \frac{e_q^2}{2m} \mathbf{A}^2 (\Psi^\dagger \Psi) + \frac{e_q}{2m}\, c_F\, \mathbf{B} \cdot (\Psi^\dagger \sigma\, \Psi)$$

More generally can perform usual matching procedure by choosing effective couplings in NR theory such that it agrees with low-energy limit of relativistic high-energy theory.

Example NRQED:

Caswell, Lepage

$$\mathcal{L}_0 = i\,\Psi^\dagger \partial_t \Psi + e_q A_0 (\Psi^\dagger \Psi)$$

$$\mathcal{L}_1 = \frac{1}{2m}\Psi^\dagger \nabla^2 \Psi + \frac{i e_q}{2m}\,\mathbf{A}\cdot\left[(\nabla\Psi^\dagger)\Psi - \Psi^\dagger \nabla\Psi\right] - \frac{e_q^2}{2m}\mathbf{A}^2(\Psi^\dagger\Psi) + \boxed{\frac{e_q}{2m}\,c_F\,\mathbf{B}\cdot(\Psi^\dagger \sigma\,\Psi)}$$

(Pauli)

More generally can perform usual matching procedure by choosing effective couplings in NR theory such that it agrees with low-energy limit of relativistic high-energy theory.

Example NRQED:

Caswell, Lepage

$$\mathcal{L}_0 = i\,\Psi^\dagger \partial_t \Psi + e_q A_0 (\Psi^\dagger \Psi)$$

$$\mathcal{L}_1 = \frac{1}{2m}\Psi^\dagger \nabla^2 \Psi + \frac{ie_q}{2m}\,\mathbf{A}\cdot\left[(\nabla\Psi^\dagger)\Psi - \Psi^\dagger\nabla\Psi\right] - \frac{e_q^2}{2m}\mathbf{A}^2(\Psi^\dagger\Psi) + \frac{e_q}{2m}\,c_F\,\mathbf{B}\cdot(\Psi^\dagger\sigma\,\Psi)$$

$$\mathcal{L}_2 = \boxed{\frac{e_q}{8m^2}\,c_D(\Psi^\dagger\Psi)(\nabla\cdot\mathbf{E})} \qquad \text{(Darwin)}$$

$$- \frac{ie_q}{8m^2}\,c_S\,\Psi^\dagger\sigma\cdot\Big(\mathbf{D}\times\mathbf{E} - \mathbf{E}\times\mathbf{D}\Big)\Psi + \frac{d_1}{m^2}(\Psi^\dagger\sigma\,\Psi)\cdot(\Psi^\dagger\sigma\,\Psi) + \frac{d_2}{m^2}(\Psi^\dagger\Psi)(\Psi^\dagger\Psi)$$

More generally can perform usual matching procedure by choosing effective couplings in NR theory such that it agrees with low-energy limit of relativistic high-energy theory.

Example NRQED:

Caswell, Lepage

$$\mathcal{L}_0 = i\,\Psi^\dagger \partial_t \Psi + e_q A_0 (\Psi^\dagger \Psi)$$

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(spin-orbit)

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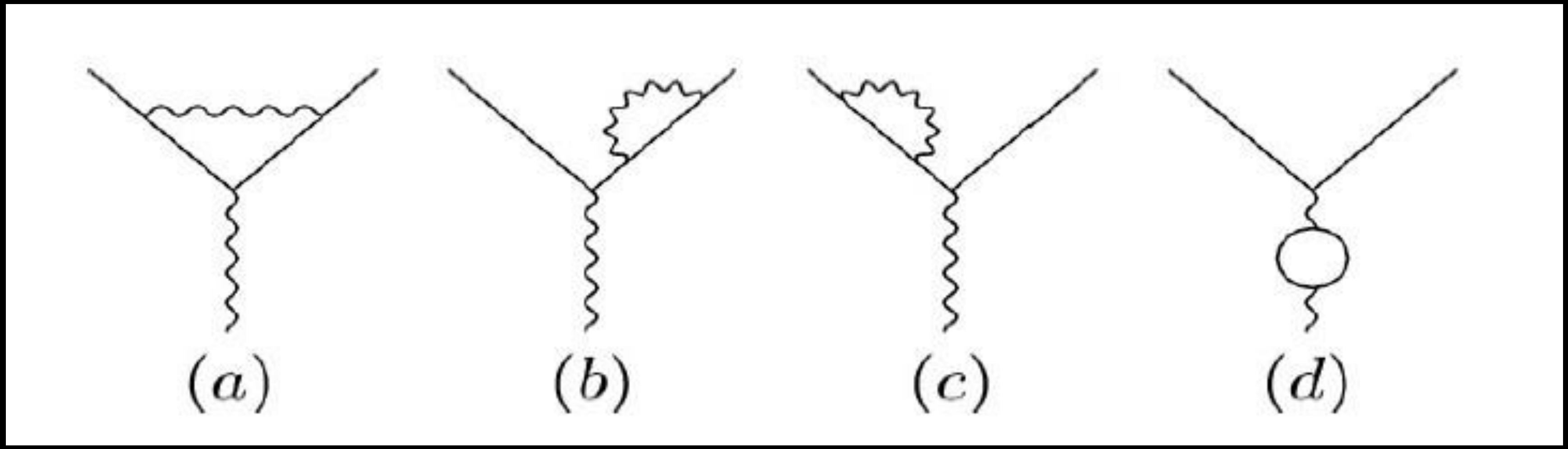
$$\mathcal{L}_2 = \frac{e_q}{8m^2}\, c_D (\Psi^\dagger \Psi)(\nabla \cdot \mathbf{E}) - \frac{i e_q}{8m^2}\, c_S\, \Psi^\dagger \sigma \cdot \Big(\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D} \Big) \Psi$$

(contact)

$$+ \frac{d_1}{m^2} (\Psi^\dagger \sigma\, \Psi) \cdot (\Psi^\dagger \sigma\, \Psi) + \frac{d_2}{m^2} (\Psi^\dagger \Psi)(\Psi^\dagger \Psi)$$

Matching to QED at high energies obtained by comparing (for small momentum transfer) with



obtained by comparing (for small momentum transfer) with

$$c_F = 1 + \frac{\alpha}{2\pi} + O(\alpha^2)$$

$$c_D = 1 + \frac{4\alpha}{3\pi} \ln \frac{m^2}{\mu^2} + O(\alpha^2)$$

$$c_S = 1 + \frac{\alpha}{\pi} + O(\alpha^2)$$

Similarly for positronium matching gives the contact interactions

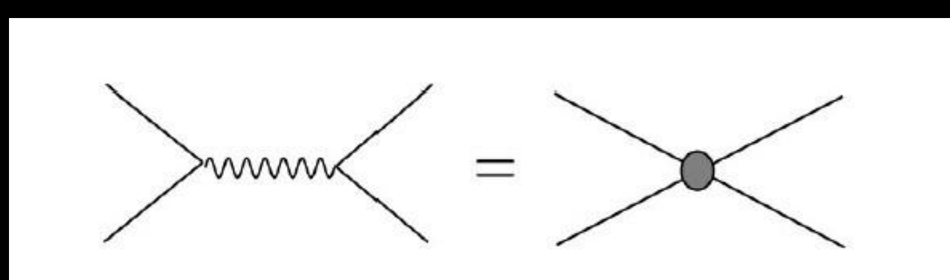
$$\mathcal{L}_c = \frac{d_v}{m^2}(\Psi^\dagger \sigma\, \Psi) \cdot (\Phi^\dagger \sigma\, \Phi) + \frac{d_s}{m^2}(\Psi^\dagger \Psi)(\Phi^\dagger \Phi)$$

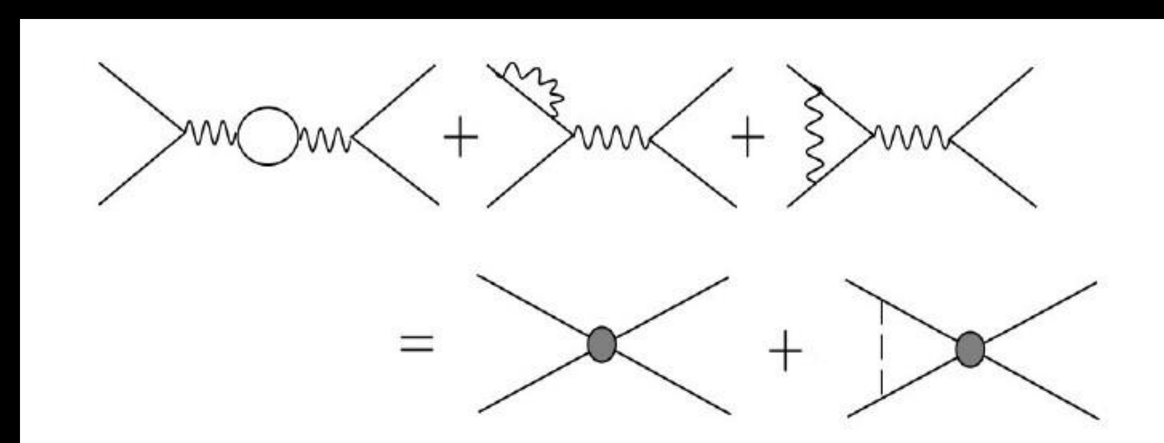
with

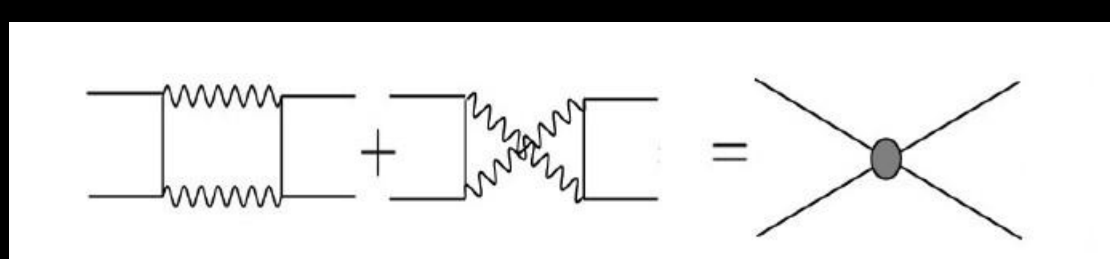
$$d_s = \frac{3\pi\alpha}{2} - \alpha^2 \left[\ln \frac{m^2}{\mu^2} + \frac{23}{3} - \ln 2 + \frac{i\pi}{2}\right] + O(\alpha^3)$$

$$d_v = -\frac{\pi\alpha}{2} + \alpha^2 \left[\frac{22}{9} + \ln 2 - \frac{i\pi}{2}\right] + O(\alpha^3)$$

using







and so on

Similarly for positronium matching gives the contact interactions

$$\mathcal{L}_c = \frac{d_v}{m^2}(\Psi^\dagger \sigma \Psi) \cdot (\Phi^\dagger \sigma \Phi) + \frac{d_s}{m^2}(\Psi^\dagger \Psi)(\Phi^\dagger \Phi)$$

with

$$d_s = \frac{3\pi\alpha}{2} - \alpha^2 \left[\ln \frac{m^2}{\mu^2} + \frac{23}{3} - \ln 2 + \frac{i\pi}{2} \right] + O(\alpha^3)$$

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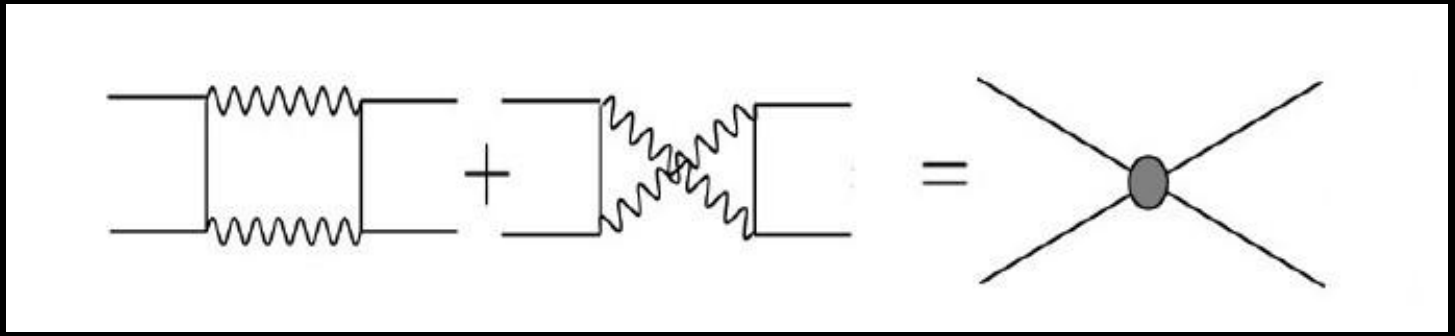


Imaginary contributions coming from evaluating annihilation graphs at threshold

Can use Schrodinger atomic wavefunctions to evaluate observables like positronium energy shifts or decay rates

$$\Delta E_{\rm hfs}(\ell = 0) = \delta E_n(S=1) - \delta E_n(S=0)$$
$$= \frac{m\,\alpha^3}{n^3}\left(\frac{\alpha c_F^2}{3} - \frac{1}{2\pi}\,\mathrm{Re}\,d_v\right)$$
$$= \frac{m\alpha^4}{2n^3}\left[\frac{7}{6} - \frac{\alpha}{\pi}\left(\ln 2 + \frac{16}{9}\right)\right]$$

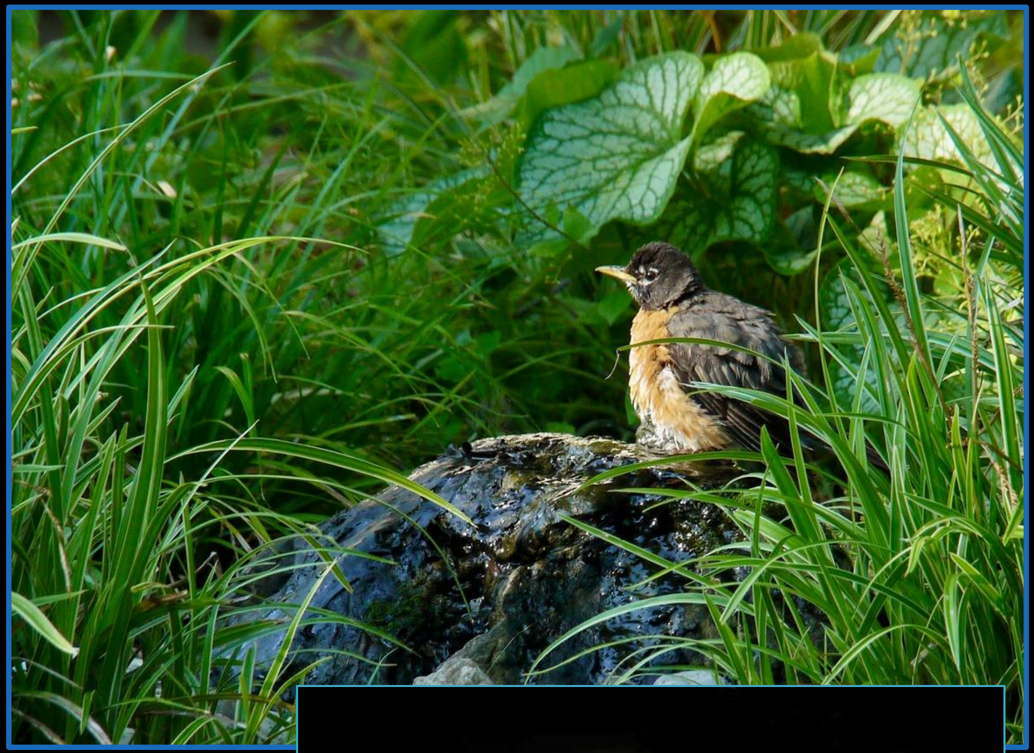
$$\Gamma_n(\ell = S = 0) = -\frac{m\,\alpha^3}{4\pi n^3}\,\mathrm{Im}\left(d_s + 3d_v\right) = \frac{m\,\alpha^5}{2n^3}$$

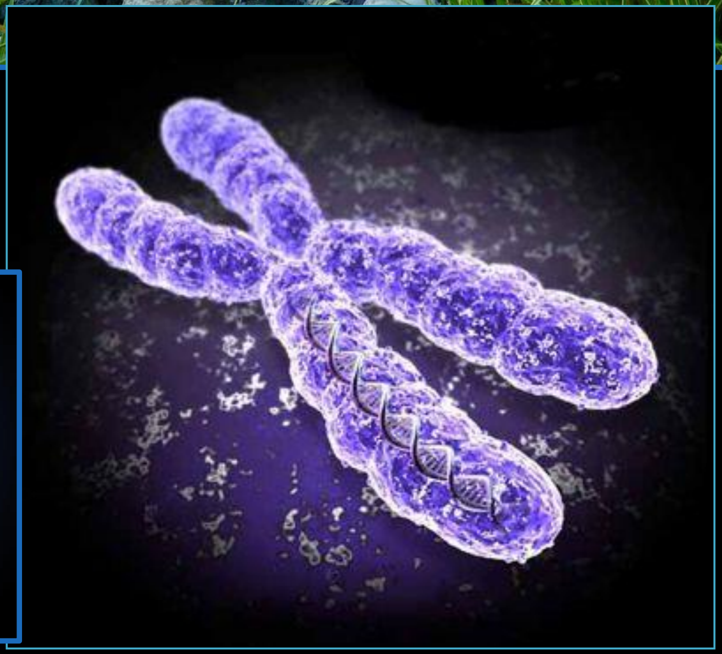
$$\Gamma_n(\ell = 0, S = 1) \propto \mathrm{Im}(d_v - d_s) = 0$$

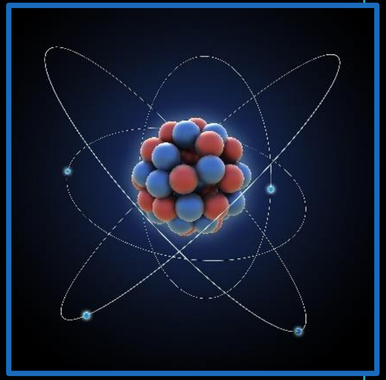
Efficiently combines relativistic quantum field theory and Schrodinger treatment of nonrelativistic bound states

EFT EXAMPLES

GRAVITY







IMAGES: CB, GENEGEEK.CA, QUORA

Massless spin-2 particle

$$h_{\mu\nu} = h_{\nu\mu} \qquad h_{\mu\nu} \to h_{\nu\mu} + \partial_\mu V_\nu + \partial_\nu V_\mu$$

Free action

$$\mathcal{L}_0 = -\tfrac{1}{2}\left[\partial^\alpha h^{\mu\nu}\partial_\alpha h_{\mu\nu} - \partial^\mu h^\alpha_\alpha \partial_\mu h^\beta_\beta\right] + \partial^\alpha h_{\alpha\mu}\partial_\beta h^{\beta\mu} - \partial^\alpha h_{\alpha\mu}\partial^\mu h^\beta_\beta$$

Nonderivative couplings to other matter

$$S_{\text{mat}} = \kappa \int \mathrm{d}^4x\, h_{\mu\nu} T^{\mu\nu}(\psi) \quad \Rightarrow \quad \partial_\mu T^{\mu\nu}(\psi) = 0$$

Conserved stress energy requires nonrenormalizable coupling $\kappa \propto 1/M_p$

Deser
Weinberg

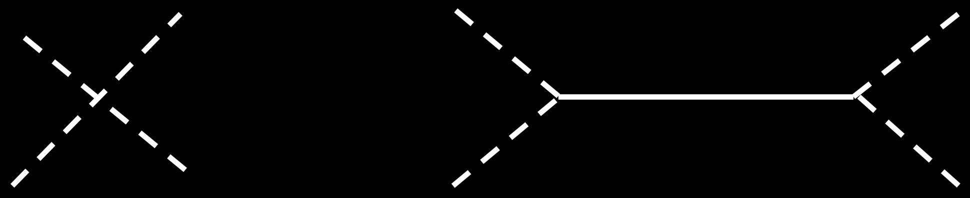
Symmetries dictate nonlinear couplings at two-derivative level to be those of General Relativity

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2h_{\mu\nu}}{M_p}$$

with lagrangian density

$$\mathcal{L} = -\tfrac{1}{2}\, M_p^2 \sqrt{-g}\, \mathcal{R}$$

$$= (\partial h)^2 + \frac{1}{M_p}\, h(\partial h)^2 + \frac{1}{M_p^2} h^2 (\partial h)^2 + \cdots$$

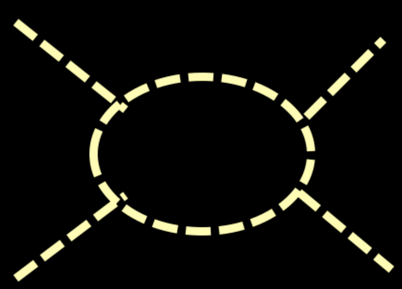


plus 'crossed' graphs

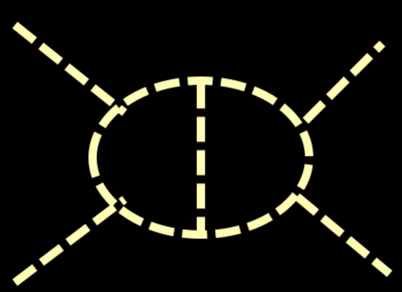
$$\mathcal{A} = \frac{8\pi i G s^3}{tu} \sim \frac{Q^2}{M_p^2}$$

DeWitt

Loop integrals diverge, and higher loops diverge more and more (because the coupling has dimensions of negative powers of mass)



$$A_{1-\text{loop}} \sim \frac{Q^2}{M_p^4} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \, \frac{p^6}{(p^2+Q^2)^4} + \cdots$$

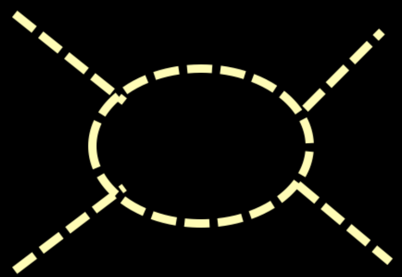


$$A_{2-\text{loop}} \sim \frac{Q^2}{M_p^4} \int \left(\frac{\mathrm{d}^4 p}{(2\pi)^4} \right)^2 \frac{p^{10}}{(p^2+Q^2)^7} + \cdots$$

Corrections modify Newton's constant

$$\delta G \simeq G^2 \Lambda^2 + G^3 \Lambda^4 + \cdots$$

Loop integrals also introduce divergences with a new dependence on external momenta



$$A_{1-\text{loop}} \sim \frac{Q^2}{M_p^4}\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\,\frac{p^6}{(p^2+Q^2)^4} + \frac{Q^4}{M_p^4}\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\,\frac{p^4}{(p^2+Q^2)^4} + \cdots$$

Some corrections *cannot* be absorbed into Newton's constant (this is what it means to say that GR is not renormalizable)

WHY DOES CLASSICAL GR WORK?

- If quantum contributions to gravity cannot be quantified, why is it meaningful to compare GR with experiment?

Quantum field theory is a precision science:

e.g. QED:

$$a_\mu = 1159652188.4(4.3)\ 10^{-12}\ \text{(exp)}$$

$$a_\mu = 1159652140(27.1)\ 10^{-12}\quad \text{(th)}$$

QED's renormalizability is an important part of its calculability, and so also underpins the theory error

WHY DOES CLASSICAL GR WORK?

- If quantum contributions to gravity cannot be quantified, why is it meaningful to compare GR with experiment?

General Relativity is also a precision science:

e.g. solar system tests, binary pulsar, ...

$$dP/dt = -2.408(10)\,10^{-12} \quad \text{(exp)}$$

$$dP/dt = -2.40243(5)\,10^{-12} \quad \text{(th)}$$

Quantum corrections are controlled once it is recognized GR is only the leading part of the low-energy GR EFT

$$\frac{L}{\sqrt{-g}} = \Lambda + \frac{M_p^2}{2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \frac{c_3}{m^2} R^3 + \cdots$$

Each curvature involves two derivatives of the metric

$$\Gamma \sim g^{-1} \partial g \qquad\qquad R \sim \partial \Gamma + \Gamma\Gamma$$

Scale m need not be as big as M_p

Using

$$\frac{L}{\sqrt{-g}} = \Lambda + \frac{M_p^2}{2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \frac{c_3}{m^2} R^3 + \cdots$$

to power-count amplitude for scattering *E* gravitons at *L* loops using V_n vertices involving f_n fields and d_n derivatives gives

$$A_E(Q) \propto \left(\frac{Q^2}{M_p^{E-2}}\right)\left(\frac{Q}{4\pi M_p}\right)^{2L} \prod_{i;k>2} \left(\frac{Q}{M_p}\right)^{2V_{ik}} \left(\frac{Q}{m}\right)^{(k-4)V_{ik}}$$

Leading contribution: L = 0 and $V_n = 0$ unless $d_n=2$ (ie classical GR)

Next-to-leading: L=1 and $V_n = 0$ unless $d_n = 2$ (ie one-loop GR)

or L=0 and V=1 for $d_n = 4$ (ie tree with one R^2 term)

Using

$$\frac{L}{\sqrt{-g}} = \Lambda + \frac{M_p^2}{2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \frac{c_3}{m^2} R^3 + \cdots$$

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$$\cdots\Big)^{2L} \prod_{i;k>2} \left(\frac{Q}{M_p}\right)^{2V_{ik}} \left(\frac{Q}{m}\right)^{(k-4)V_{ik}}$$

Divergences in these are absorbed into these

Leading contribution: L = 0 and V_n = 0 unless d_n=2 (ie classical GR)

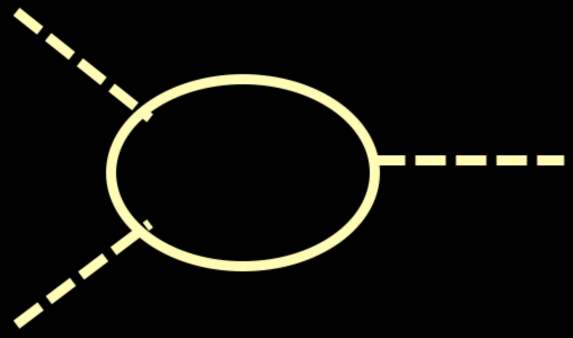
Next-to-leading: L=1 and V_n = 0 unless d_n = 2 (ie one-loop GR)

or L=0 and V=1 for d_n = 4 (ie tree with one R^2 term)

Non-GR terms likely arise as high-energy states are integrated out (such as in string theory).

$$\frac{L}{\sqrt{-g}} = \Lambda + \frac{M_p^2}{2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \frac{c_3}{m^2} R^3 + \cdots$$

As would be obtained at low energies from loops of particles with m >> Q



Notice smallest m dominates in the sum over virtual particles

EFFECTIVE FIELD THEORY

View of physics as effective quantum theory has many practical and conceptual benefits

Very efficient way to compute given hierarchy of scale

Only known way to compute in quantum way using nonrenormalizable interactions.

Einstein and Maxwell's equations are most general low-energy description of massless spin-two and spin-one particles.

Standard Model has most general low-energy interactions allowed given particle content.

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