Institute of Theoretical Physics
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EFT Operators and UV Correspondences

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International Meeting on Effective Pathways to New Physics

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Outline

- Why higher dimensional operators?
- Amplitude Basis construction for EFT operators

ITP-CAS group

Hao-Lin Li, Zhe Ren, Jing Shu, Ming-Lei Xiao, **JHY**, Yu-Hui Zheng, 2005.00008, PRD
Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **JHY**, Yu-Hui Zheng, 2007.07899, PRD
Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **JHY**, Yu-Hui Zheng, 2012.09188, JHEP
Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **JHY**, Yu-Hui Zheng, 2105.09329, JHEP
Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **JHY**, Yu-Hui Zheng, 2201.04639

- Complete UV resonances from bottom-up approach

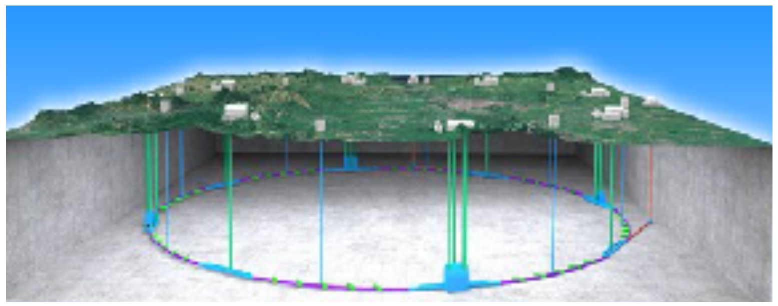
ITP-CAS group

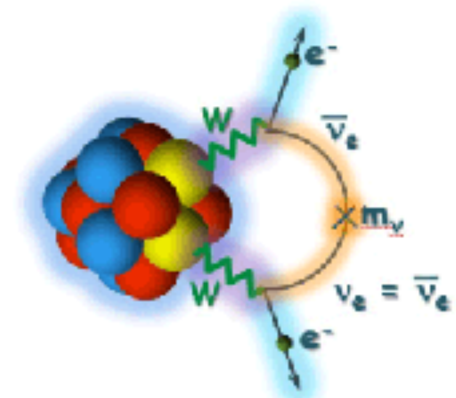
Hao-Lin Li, Yu-Han Ni, Ming-Lei Xiao, **JHY**, 2202.xxxxx
Hao-Lin Li, Yu-Han Ni, Ming-Lei Xiao, **JHY**, 2203.xxxxx
Hao-Lin Li, Yu-Han Ni, Ming-Lei Xiao, **JHY**, 2203.xxxxx

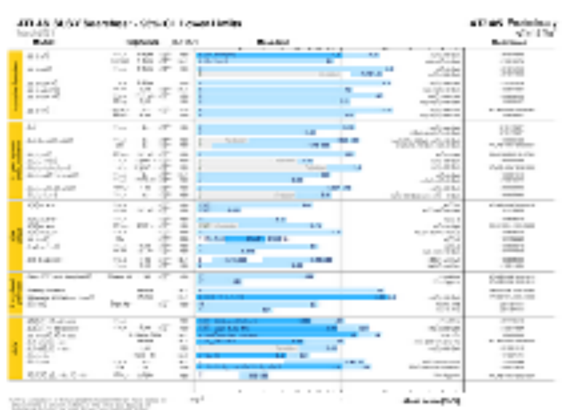
- Summary and outlook

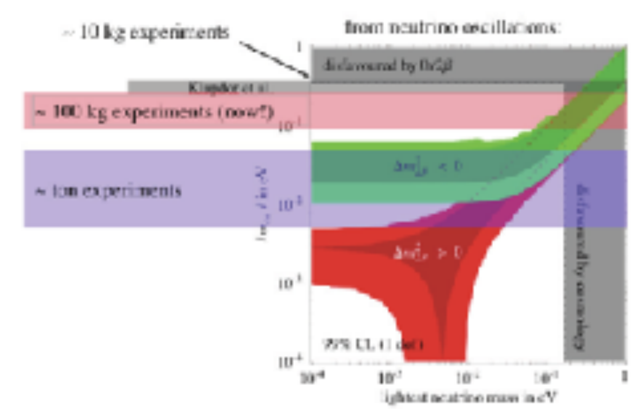
Search For New Physics

We are searching for new physics at both high energy and low energy scales









Three ways describing new physics

New Physics Scenarios → **Effective Operators**

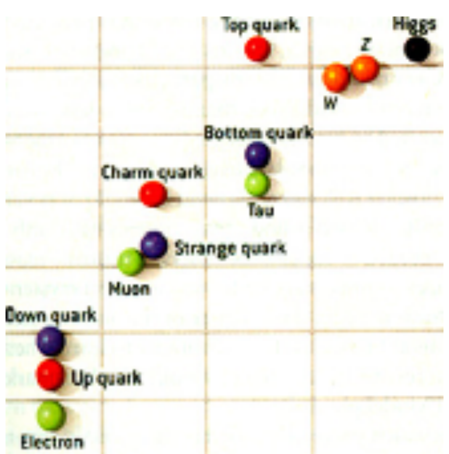
Simplified Models

Top-Down Approach

Given new physics models, integrate out heavy particles and match to SMEFT

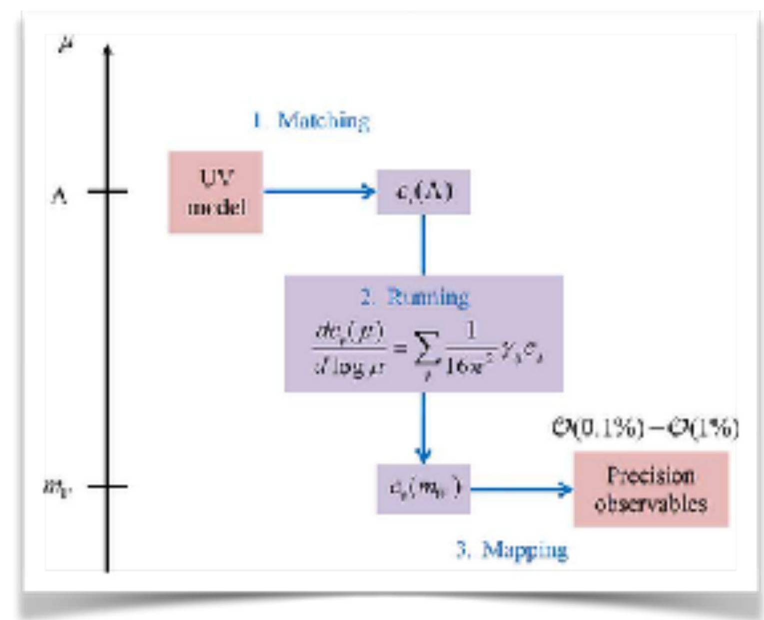
New Physics Models

Integrate out and matching/running



EFT Operators

$$\mathscr{L}_{\mathrm{EFT}} = \mathscr{L}_{\mathscr{D}\leq 4} + \frac{\mathscr{L}_5}{\Lambda} + \frac{\mathscr{L}_6}{\Lambda^2} + \dots$$

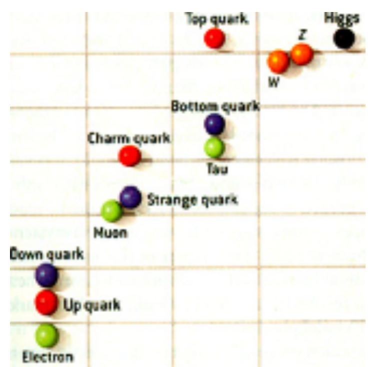


Decoupling theorem

Bottom-up Approach

Investigate new physics effects assuming no new particles

General but harder way: new physics without new particle



EFT Operators

[Weinberg 1933 - 2021]

a folk theorem: "if one writes down the most general possible Lagrangian, including *all* terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S-matrix consistent with perturbative unitarity, analyticity, cluster decomposition, and the assumed symmetry properties."

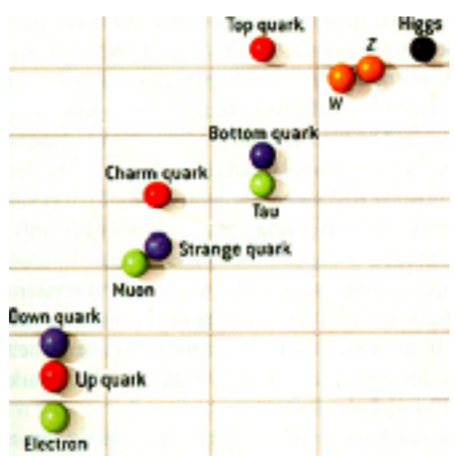
Weinberg's Folk theorem, 1979

SMEFT

Standard model effective field theory (SMEFT)

UV model unspecified

LHC probing scale



SMEFT

Most general Lagrangian with Lorentz and SM gauge symmetry

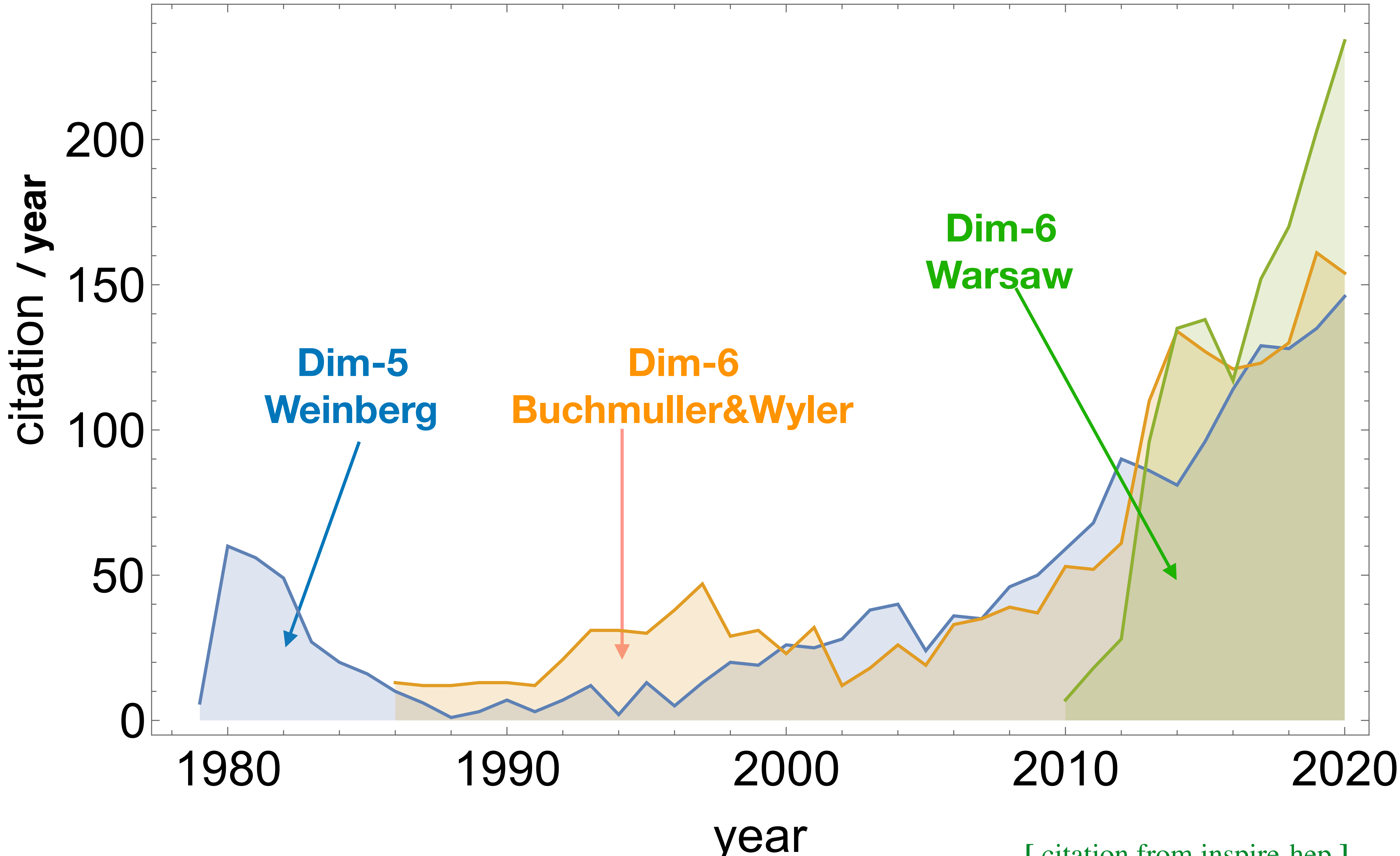
$$\mathscr{L}_{\mathrm{EFT}} = \mathscr{L}_{\mathscr{D}\leq 4} + \frac{\mathscr{L}_5}{\Lambda} + \frac{\mathscr{L}_6}{\Lambda^2} + \dots$$

power counting: canonical dim.

SMEFT provides systematic parametrization of

… all possible Lorentz inv. new physics!

SMEFT Operators

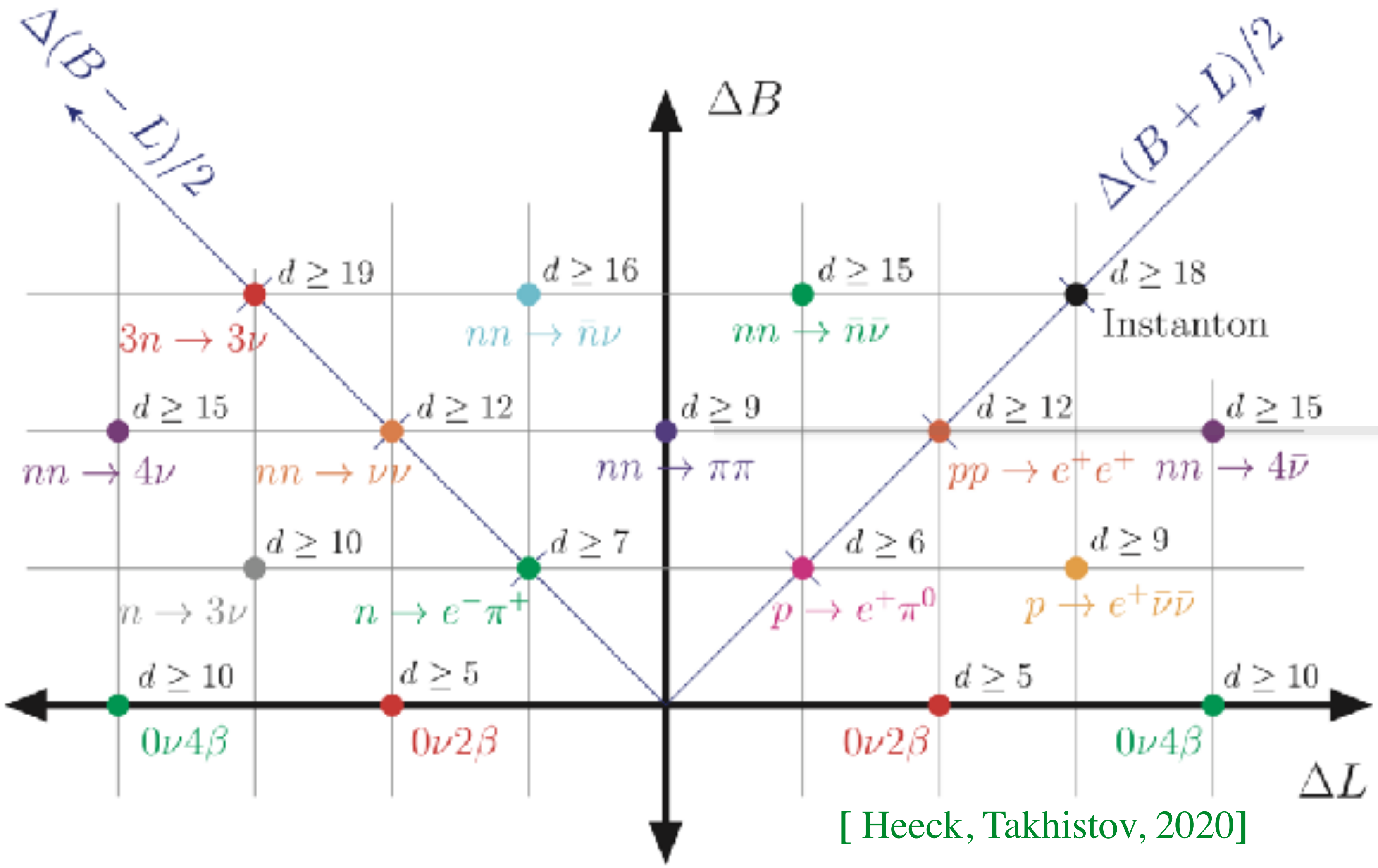


[citation from inspire-hep]

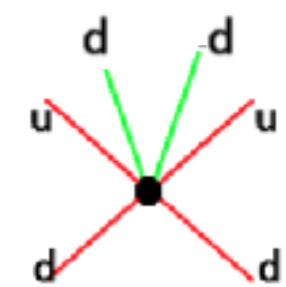
Higher Dim Operators

new physics without new particle: neutrino masses and baryon asymmetry

B and L violation



n-nbar oscillation



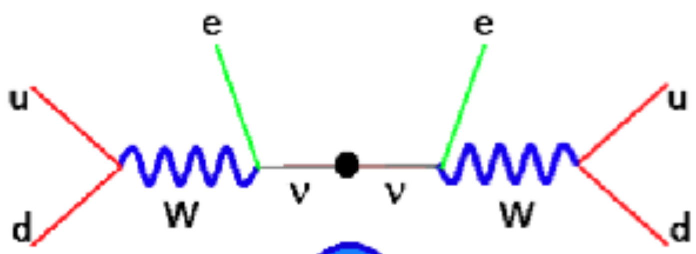
Dim-9

dim-8: neutral triple gauge couplings

SMEFT Operators

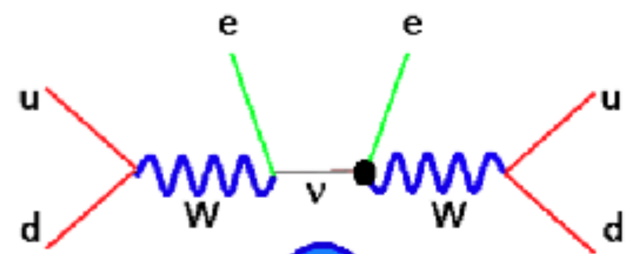
Very different types of operators contribute to the same process

$e_R = e_C^\dagger, u_R = u_C^\dagger, d_R = d_C^\dagger$



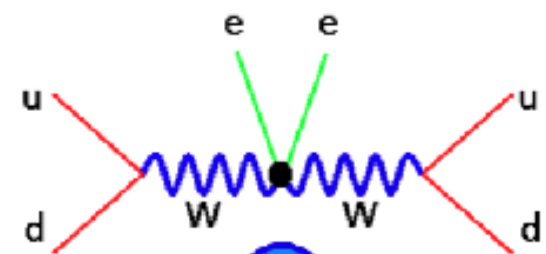
A **Dim-5**

$H^2 L^2$ $H^3 H^\dagger L^2$



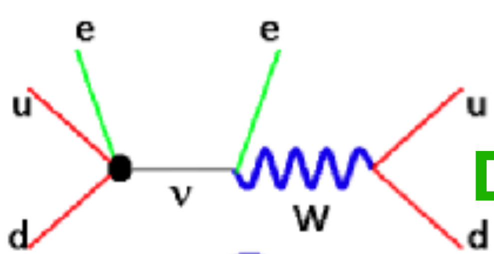
B **Dim-7**

$D\, ec\, H^{\dagger 3} L^\dagger$ $H^{\dagger 2} L^{\dagger 2}\, WR$



C **Dim-9**

$D^2 H^{\dagger 2} L^{\dagger 2}$ $D^2 H^{\dagger 2} L^{\dagger 2}\, WL$

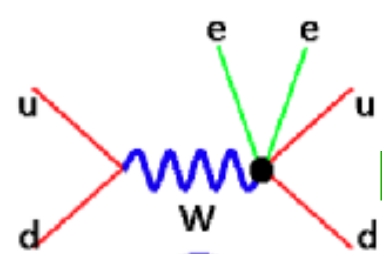


Dim-7

D

$dc^\dagger H^\dagger L^{\dagger 2} Q^\dagger$

$D\, dc^\dagger L^{\dagger 2}\, uc$

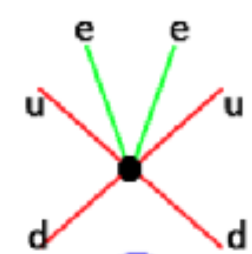


Dim-9

E

$D\, dc^\dagger L^{\dagger 2}\, uc$

$dc^\dagger\, ec\, H^\dagger L^\dagger\, uc\, WL$



Dim-9

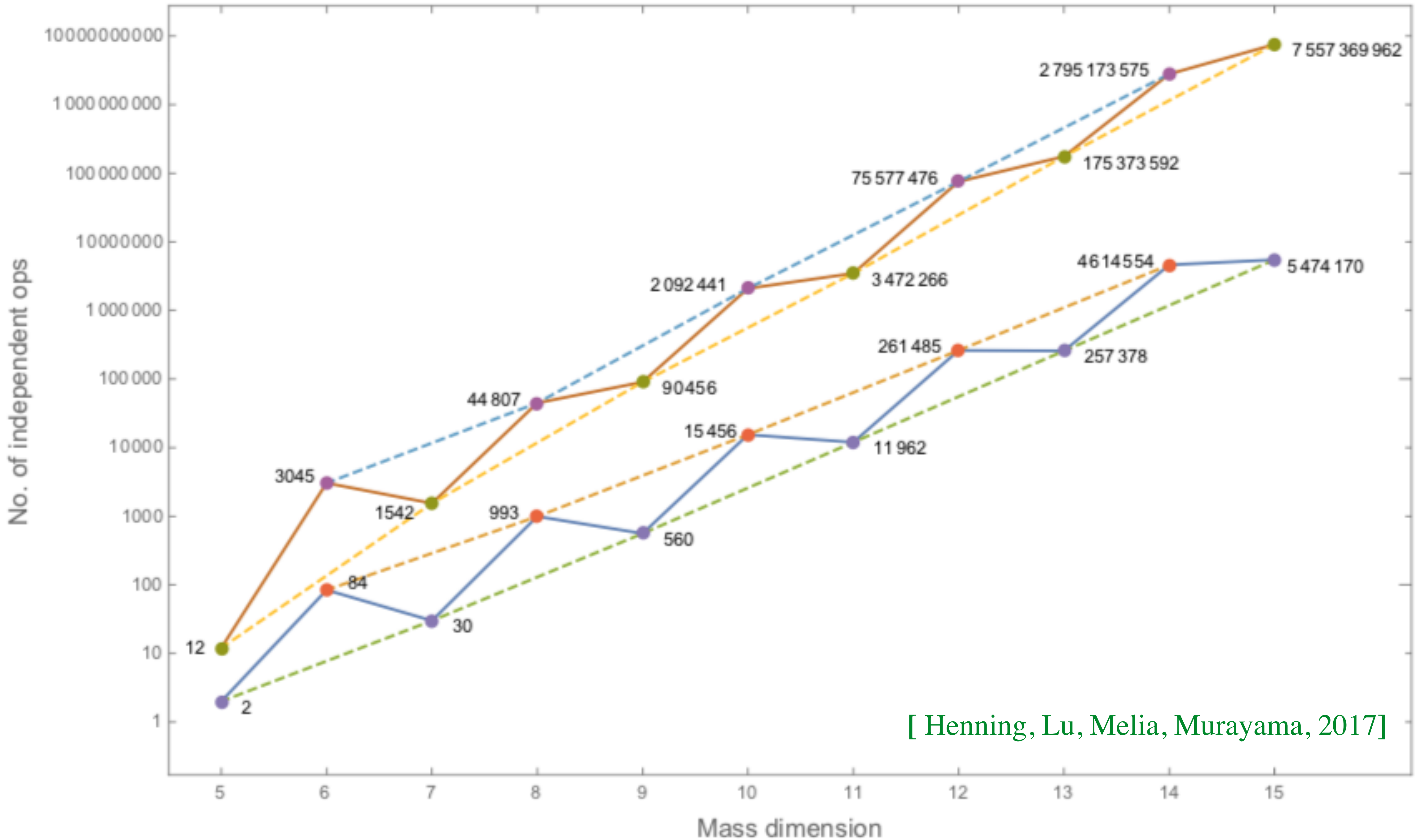
F

$dc^2 L^2 Q^2$, $dc^2\, dc^\dagger L^2\, uc^\dagger$, $dc\, L^2\, uc\, uc^{\dagger 2}$, $dc^2\, ec^\dagger L\, Q\, uc^\dagger$,

$dc^{\dagger 2}\, ec^2\, uc^2$, $dc\, L^2\, Q\, Q^-\, uc^\dagger$, $dc^\dagger\, ec\, L^\dagger Q\, uc^2$, $L^{-2} Q^2\, uc^2$

Need write down complete set of operators up to dim-9

SMEFT Operators



[Henning, Lu, Melia, Murayama, 2017]

Also [Lehman, Martin 2015]

Main Difficulties

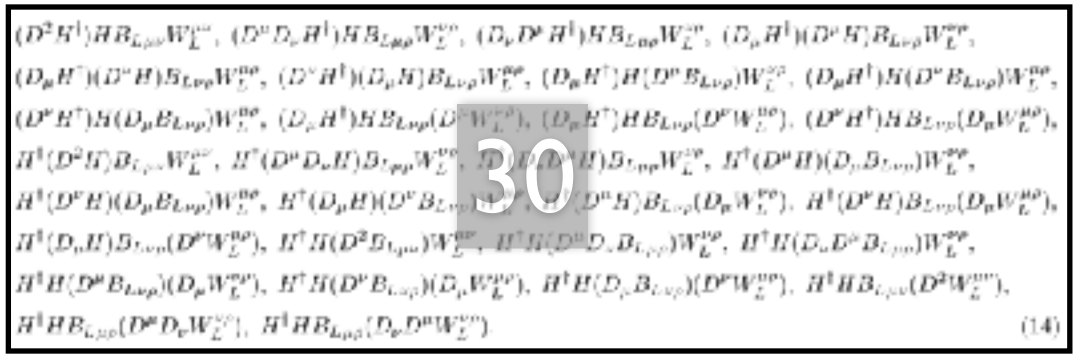
Given numbers of independent operators

Still difficult to write down explicit form of operators!

Derivatives

$BWHH^\dagger D^2$

2

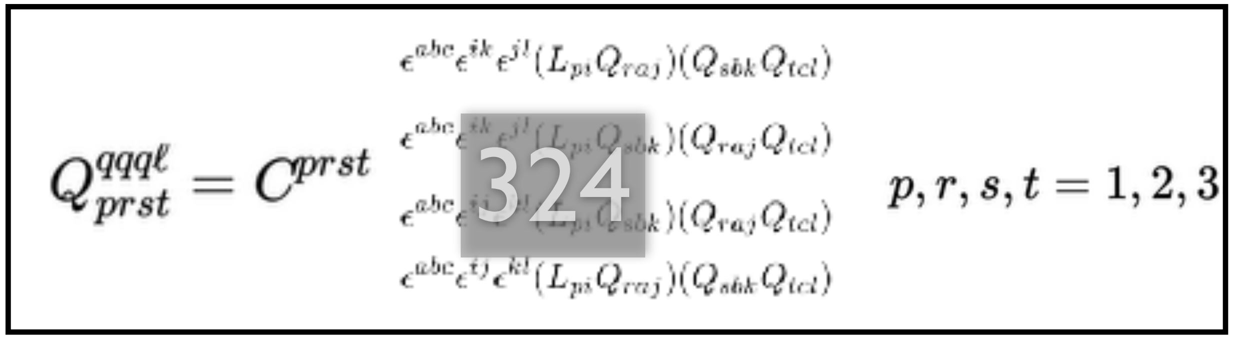


Which 2 should be picked up?

Repeated fields

$QQQL$

57

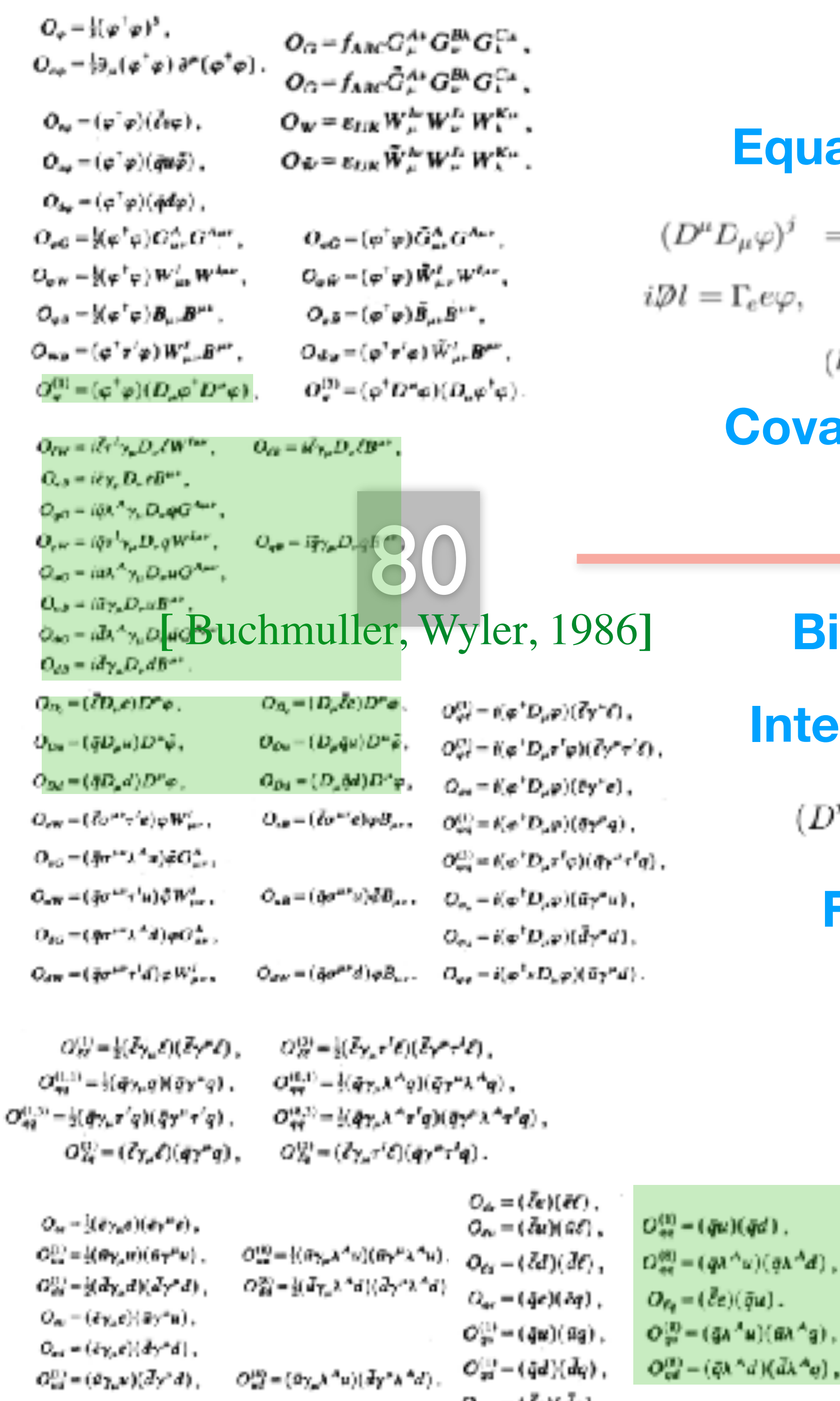


What flavor relations should be imposed?

SMEFT Dim-6 Operators

First write group-inv. operators (over-complete) and then remove redundancies

tedious and prone-to-error



[Buchmuller, Wyler, 1986]

Equation of motion (Field redefinition)

$$(D^\mu D_\mu \varphi)^j = m^2 \varphi^j - \lambda (\varphi^\dagger \varphi) \varphi^j - \bar{e}\,\Gamma_e^\dagger l^j + \varepsilon_{jk} \bar{q}^k \Gamma_u u - \bar{d}\,\Gamma_d^\dagger q^j$$

$$i\not{D} l = \Gamma_e e \varphi, \quad i\not{D} e = \Gamma_e^\dagger \varphi^\dagger l, \quad i\not{D} q = \Gamma_u u \tilde{\varphi} + \Gamma_d d \varphi, \quad i\not{D} u = \Gamma_u^\dagger \tilde{\varphi}^\dagger q,$$

$$(D^\rho W_{\rho\mu})^I = \frac{g}{2}\left(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi + \bar{l}\gamma_\mu \tau^I l + \bar{q}\gamma_\mu \tau^I q\right),$$

Covariant derivative commutator

$$[D_\rho, D_\alpha] \sim X_{\rho\alpha}$$

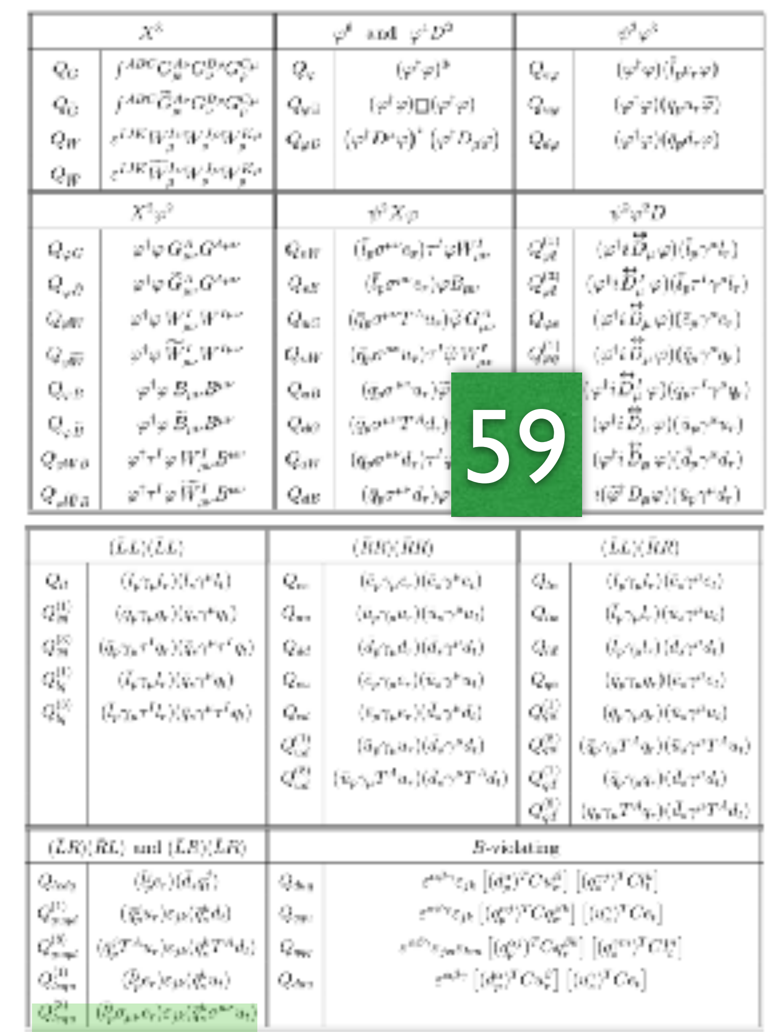
Bianchi identity $D_{[\rho} X_{\mu\nu]} = 0$

Integration by part (total derivatives)

$$(D^n \varphi)^\dagger (D^m \varphi) = -(D^{n+1}\varphi)^\dagger (D^{m-1}\varphi) + \partial\left[(D^n \varphi)^\dagger (D^{m-1}\varphi)\right]$$

Fierz identity $T^A_{\alpha\beta} T^A_{\kappa\lambda} = \frac{1}{2}\delta_{\alpha\lambda}\delta_{\kappa\beta} - \frac{1}{6}\delta_{\alpha\beta}\delta_{\kappa\lambda}$

$$\tau^I_{jk}\tau^I_{mn} = 2\delta_{jn}\delta_{mk} - \delta_{jk}\delta_{mn}$$



[Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010]

80-1-16-5+1 = 59

Operator as Spinor Tensor

Each field belongs to Lorentz irrep:

SO(3,1)	SL(2,C) $SU(2)_l \times SU(2)_r$	Spinor-helicity
ϕ	$\phi \in (0,0)$	
ψ	$\psi_\alpha \in (1/2, 0)$ $\psi^\dagger_{\dot\alpha} \in (0, 1/2)$,	λ_α
$F_{\mu\nu}$	$F_{\mathrm{L}\alpha\beta} = \frac{i}{2} F_{\mu\nu}\sigma^{\mu\nu}_{\alpha\beta} \in (1,0)$ $F_{\mathrm{R}\dot\alpha\dot\beta} = -\frac{i}{2} F_{\mu\nu}\bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta} \in (0,1)$.	$\lambda_\alpha\lambda_\beta$
$R_{\mu\nu\rho\sigma}$	$C_{\alpha\beta\gamma\delta} = C_{\mu\nu\rho\sigma}\sigma^{\mu\nu}_{\alpha\beta}\sigma^{\rho\sigma}_{\gamma\delta} \in (2,0)$	$\lambda_\alpha\lambda_\beta\lambda_\gamma\lambda_\delta$
D_μ	$D_{\alpha\dot\alpha} = D_\mu \sigma^\mu_{\alpha\dot\alpha} \in (1/2, 1/2)$,	$\lambda_\alpha\tilde\lambda_{\dot\alpha}$

Operator with explicit spinor indices

$$W_{\mu\lambda}\left(e_{cp}\sigma^{\nu\lambda}D^\mu L_r\right)D_\nu H^\dagger \quad \rightarrow \quad F_1^{\alpha\beta}\psi_2^{\gamma}\left(D\psi_3\right)_{\alpha\beta\dot\alpha}\left(D\phi_4\right)_\gamma{}^{\dot\alpha}$$

$$\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_2\alpha_4}\epsilon^{\dot\alpha_3\dot\alpha_4}F_1^{\alpha_1^2}\psi_2^{\alpha_2}\left(D\psi_3\right)^{\alpha_3^2}_{\dot\alpha_3}\left(D\phi_4\right)^{\alpha_4}_{\dot\alpha_4}$$

Easier to find more symmetries of the operator with spinor indices

Equation of Motion

For fields with derivatives, symmetric and antisymmetric indices:

$$(D\psi)_{\alpha\beta\dot{\alpha}} = -\frac{1}{2}\epsilon_{\alpha\beta}(\not{D}\psi)_{\dot{\alpha}} + \frac{1}{2}(D\psi)_{(\alpha\beta)\dot{\alpha}}$$

$$\left(\frac{1}{2},\frac{1}{2}\right)\otimes\left(\frac{1}{2},0\right) = \left(0,\frac{1}{2}\right)\ominus\left(1,\frac{1}{2}\right)$$

$\left(\frac{1}{2},\frac{1}{2}\right)\times\left(\frac{1}{2},0\right)$ (0,1/2) (1,1/2)

$$(D^2\phi)_{\alpha\beta\dot{\alpha}\dot{\beta}} = \frac{1}{2}\epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}}D^\mu D_\mu\phi - \frac{i}{4}\epsilon_{\dot{\alpha}\dot{\beta}}\sigma^{\mu\nu}_{\alpha\beta}[D_\mu, D_\nu]\phi - \frac{i}{4}\epsilon_{\alpha\beta}\bar{\sigma}^{\mu\nu}_{\dot{\alpha}\dot{\beta}}[D_\mu, D_\nu]\phi + \frac{1}{4}(D^2\phi)_{(\alpha\beta)(\dot{\alpha}\dot{\beta})}$$

$\left(\frac{1}{2},\frac{1}{2}\right)\times\left(\frac{1}{2},\frac{1}{2}\right)$ (0,0) (1,0) (0,1) (1,1)

Only take the **symmetric indices** part for field with derivatives

EOM, covariant derivative commutator, Bianchi identity all removed

$$D^w\Psi \in \left(j_l + \frac{w}{2}, j_r + \frac{w}{2}\right) \oplus \text{~~lower weights~~}$$

with totally symmetric spinor indices

[Similar treatment: EOM removed by taking highest weight rep.]

Also [Lehman, Martin 2016]

Spinor Tensor Transformation

Any operator can be written with totally symmetric spinor indices:

$$\mathcal{O} = (\epsilon^{\alpha_i\alpha_j})^n (\tilde{\epsilon}_{\dot\alpha_i\dot\alpha_j})^{\tilde n} \prod_{i=1}^{N} (D^{r_i-|h_i|}\Phi_i)^{\dot\alpha_i^{r_i+h_i}}_{\alpha_i^{r_i-h_i}}$$

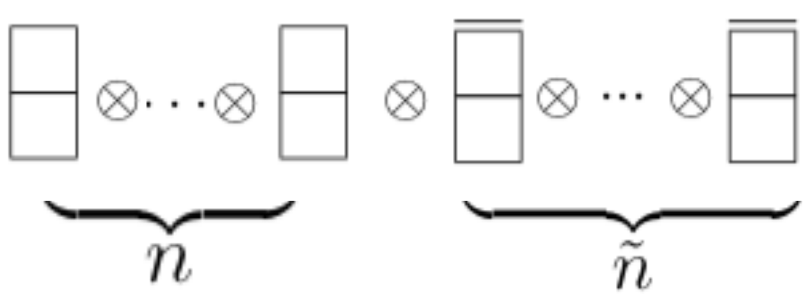
$$\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_2\alpha_4}\tilde{\epsilon}^{\dot\alpha_3\dot\alpha_4} F_1^{\alpha_1^2}\psi_2^{\alpha_2}(D\psi_3)^{\alpha_3^2}_{\dot\alpha_3}(D\phi_4)^{\alpha_4}_{\dot\alpha_4} \qquad F_1^{\alpha\beta}\psi_2^{\gamma}(D\psi_3)_{\alpha\beta\dot\alpha}(D\phi_4)_{\gamma}{}^{\dot\alpha}$$

Epsilon tensor transformations under SL(2,C) x SU(N)

$$\epsilon^{\alpha_i\alpha_j} \to \sum_{k,l} \mathcal{U}^i_k \mathcal{U}^j_l \epsilon^{\alpha_k\alpha_l}, \quad \tilde{\epsilon}_{\dot\alpha_i\dot\alpha_j} \to \sum_{k,l} \mathcal{U}^{\dagger k}_{\ i} \mathcal{U}^{\dagger l}_{\ j} \tilde{\epsilon}_{\dot\alpha_k\dot\alpha_l}. \qquad i,j,k,l = 1 \text{ to } N$$

 $= [1^2]$  $= [1^{N-2}]$

$\epsilon^{\alpha_i\alpha_j}$ $\qquad \mathcal{E}^{ijk_1,\ldots,k_{N-2}}\tilde{\epsilon}_{\dot\alpha_i\dot\alpha_j}$



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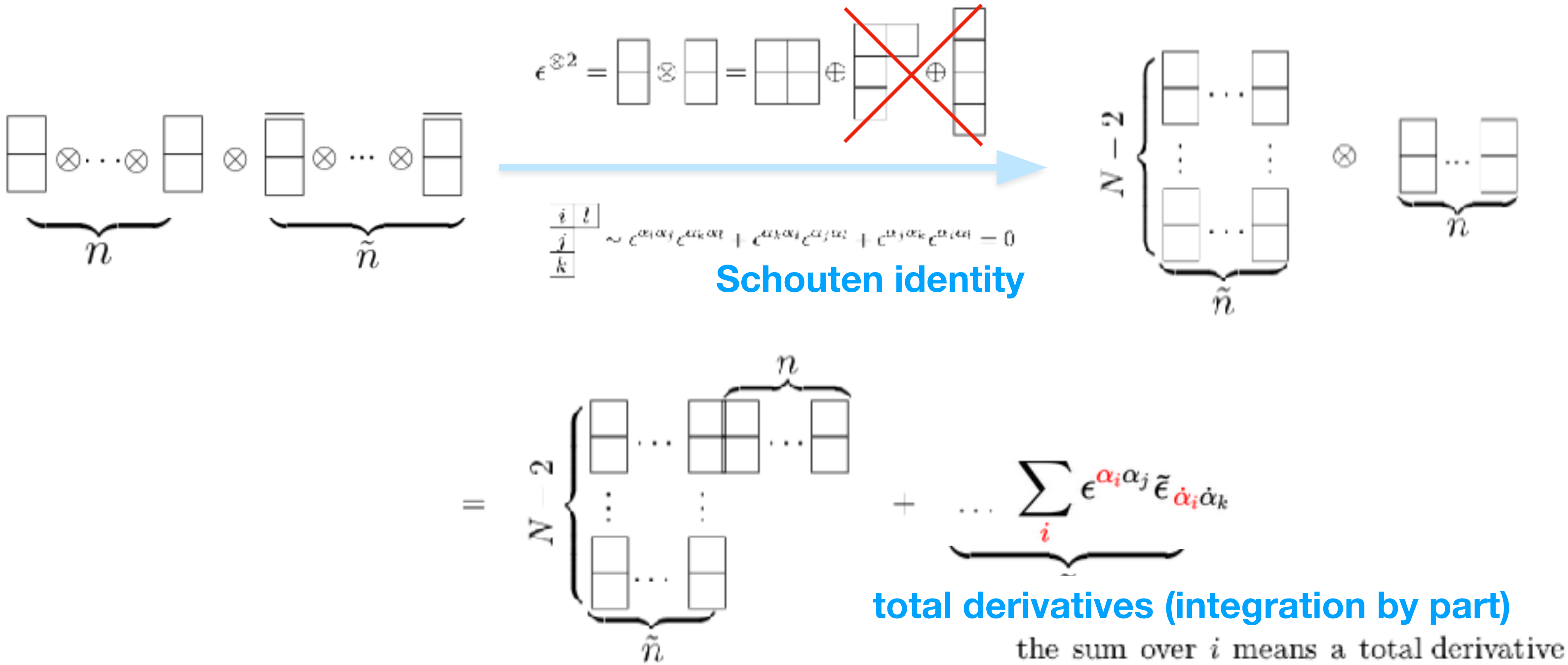
Integration-by-part



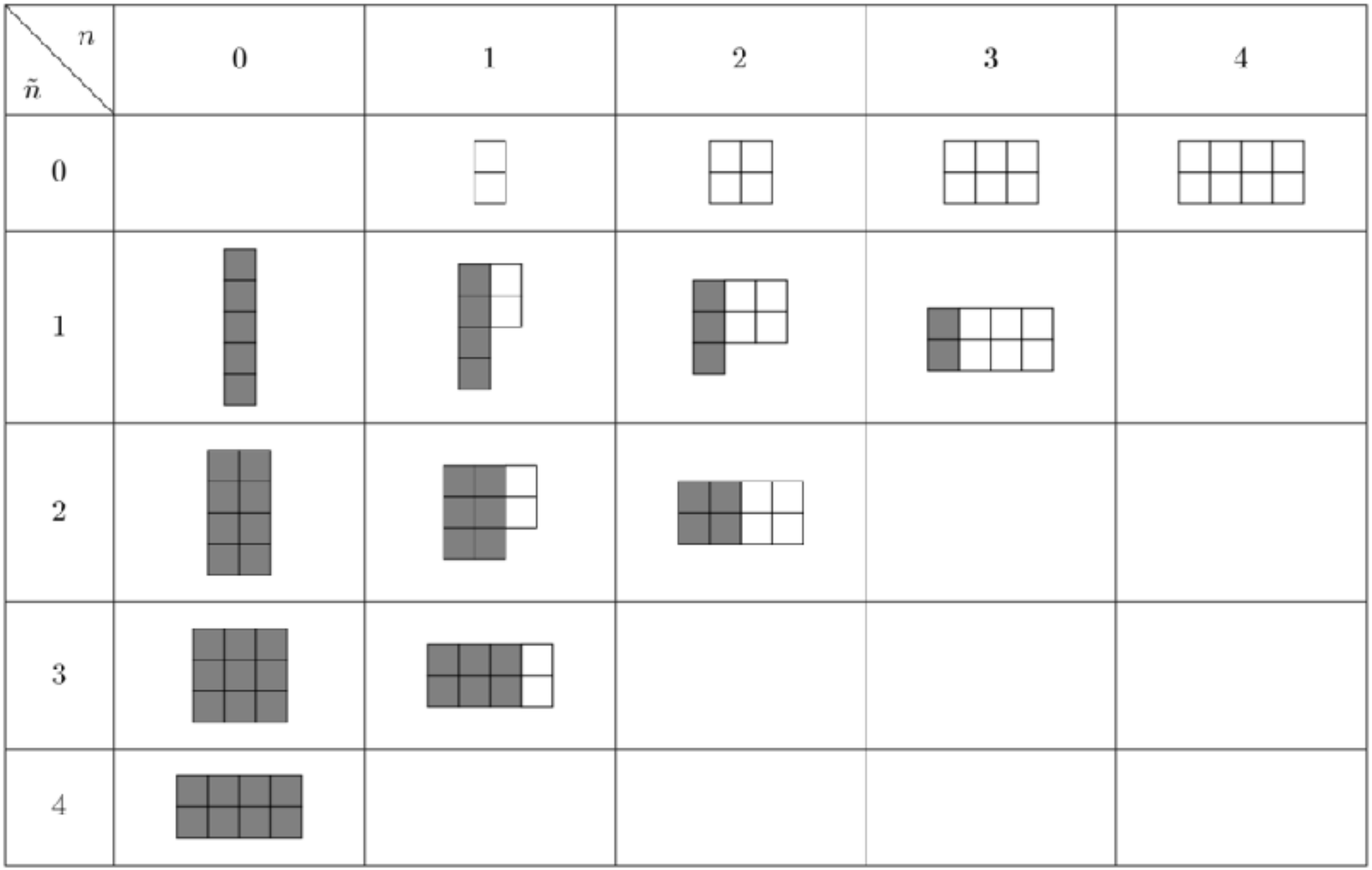
We obtain Young diagram using epsilon tensor transformation

[Such Young diagram also obtained from conformal K harmonics]

Also [Henning, Melia, 2019]

Young Diagrams

Dim-8 operators: 993 (44807) operators for 1 (3) generations



Young Diagrams

Dim-8 operators: 993 (44807) operators for 1 (3) generations

N = 7 N = 6 N = 5 N = 4

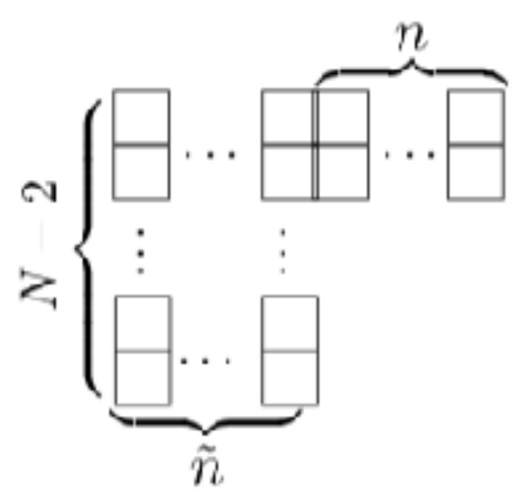
$\bar{n}$ \ n	0	1	2	3	4
0	ϕ^8	$\psi^2\phi^5$	$\psi^4\phi^2$, $F_{\rm L}\psi^2\phi^3$, $F_{\rm L}^2\phi^4$	$F_{\rm L}\psi^4$, $F_{\rm L}^2\psi^2\phi$, $F_{\rm L}^3\phi^2$	$F_{\rm L}^4$
1	$\psi^{\dagger 2}\phi^5$	$\psi^{\dagger 2}\psi^2\phi^2$, $\psi^\dagger\psi\phi^4 D$, $\phi^6 D^2$	$F_{\rm L}\psi^{\dagger 2}\psi^2$, $F_{\rm L}^2\psi^{\dagger 2}\phi$, $\psi^\dagger\psi^3\phi D$, $F_{\rm L}\psi^\dagger\psi\phi^2 D$, $\psi^2\phi^3 D^2$, $F_{\rm L}\phi^4 D^2$	$F_{\rm L}^2\psi^\dagger\psi D$, $\psi^4 D^2$, $F_{\rm L}\psi^2\phi D^2$, $F_{\rm L}^2\phi^2 D^2$	
2	$\psi^{\dagger 4}\phi^2$, $F_{\rm R}\psi^{\dagger 2}\phi^3$, $F_{\rm R}^2\phi^4$	$F_{\rm R}\psi^{\dagger 2}\psi^2$, $F_{\rm R}^2\psi^2\phi$, $\psi^{\dagger 3}\psi\phi D$, $F_{\rm R}\psi^\dagger\psi\phi^2 D$, $\psi^{\dagger 2}\phi^3 D^2$, $F_{\rm R}\phi^4 D^2$	$F_{\rm R}^2 F_{\rm L}^2$, $F_{\rm R}F_{\rm L}\psi^\dagger\psi D$, $\psi^{\dagger 2}\psi^2 D^2$, $F_{\rm R}\psi^2\phi D^2$, $F_{\rm L}\psi^{\dagger 2}\phi D^2$, $F_{\rm R}F_{\rm L}\phi^2 D^2$, $\phi^4 D^4$, $\psi^\dagger\psi\phi^2 D^3$		
3	$F_{\rm R}\psi^{\dagger 4}$, $F_{\rm R}^2\psi^{\dagger 2}\phi$, $F_{\rm R}^3\phi^2$	$F_{\rm R}^2\psi^\dagger\psi D$, $\psi^{\dagger 4}D^2$, $F_{\rm R}\psi^{\dagger 2}\phi D^2$, $F_{\rm R}^2\phi^2 D^2$			
4	$F_{\rm R}^4$				

Lorentz Structure

We invent a Young diagram filling procedure to obtain independent Lorentz

Semi-standard Young tableau (SSYT)

$Y_{N,n,\tilde{n}} =$



$$\{\overbrace{1,\ldots,1}^{\#1},\overbrace{2,\ldots,2}^{\#2},\ldots,\overbrace{N,\ldots,N}^{\#N}\}$$

$$\#i = \tilde{n} - 2h_i$$

Semi-standard Young tableau forms a **independent and complete basis** for a type

$(\tilde{n} = 1, n = 3)$

$F^2\bar{\psi}\psi D,\ \psi^4 D^2,$
$F\psi^2\phi D^2,\ F^2\phi^2 D^2$

$\#1 = 3, \#2 = \#3 = 2, \#4 = 1.$

1	1	1	2
2	3	3	4

1	1	1	3
2	2	3	4

$$F_{\rm L1}^{\alpha\beta}\psi_2^{\gamma}\,(D\psi_3)_{\alpha\beta\dot{\alpha}}\,(D\phi_4)_{\gamma}{}^{\dot{\alpha}}$$

$$F_{\rm L1}^{\alpha\beta}\psi_{2\alpha}\,(D\psi_3)_{\beta}{}^{\gamma}{}_{\dot{\alpha}}\,(D\phi_4)_{\gamma}{}^{\dot{\alpha}}$$

$\#1 = 3,\ \#2 = 3,\ \#3 = 1$ and $\#4 = 1$

1	1	1	3
2	2	2	4

1	1	1	2
2	2	3	4

$$F_{\rm L1}{}^{\alpha\beta}F_{\rm L2\alpha\beta}(D\phi_3)^{\gamma}{}_{\dot{\alpha}}(D\phi_4)_{\gamma}{}^{\dot{\alpha}}$$

$$F_{\rm L1}{}^{\alpha\beta}F_{\rm L2\alpha}{}^{\gamma}(D\phi_3)_{\beta\dot{\alpha}}(D\phi_4)_{\gamma}{}^{\dot{\alpha}}$$

On-shell Amplitude Basis

EFT operator = Contact amplitude = Group invariant + little group scaling

ψ_α	λ_α
$\psi^\dagger_{\dot\alpha}$	$\tilde\lambda_{\dot\alpha}$
$F^-_{\alpha\beta} = F_{\mu\nu}\sigma^{\mu\nu}_{\alpha\beta}$	$\lambda_\alpha\lambda_\beta$
$F^+_{\dot\alpha\dot\beta} = F_{\mu\nu}\bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta}$	$\tilde\lambda_\alpha\tilde\lambda_\beta$

$$\langle ij\rangle = \tilde\lambda_i\epsilon\tilde\lambda_j$$
$$[ij] = \lambda_i\epsilon\lambda_j$$

$$(\bar l_p\sigma^{\mu\nu}e_r)\tau^I\varphi W^I_{\mu\nu} \longrightarrow \lambda_{1\alpha}\lambda_{2\beta}\lambda_4^\alpha\lambda_4^\beta\tau^I \longrightarrow [14][24]\tau^I$$

[Shadmi, Weiss, 2018]
[Ma, Shu, Xiao, 2019]
[Durieux, Kitahara, Machado, Shadmi, Weiss, 2019]

Cannot deal with IBP for more than 4 particles!

We provide a systematical algorithm to deal with EOM/IBP redundancies

Young tableau provides on-shell amplitude basis for effective operators

1	1	1	2
2	3	3	4

1	1	1	3
2	2	3	4

$\begin{array}{|c|}\hline i \\ \hline j \\ \hline\end{array} \sim \langle ij\rangle$

$$\langle 13\rangle\langle 13\rangle\langle 24\rangle[34]$$
$$\langle 12\rangle\langle 13\rangle\langle 34\rangle[34]$$

ψ_α	λ_α
$\psi^\dagger_{\dot\alpha}$	$\tilde\lambda_{\dot\alpha}$
$F^-_{\alpha\beta} = F_{\mu\nu}\sigma^{\mu\nu}_{\alpha\beta}$	$\lambda_\alpha\lambda_\beta$
$F^+_{\dot\alpha\dot\beta} = F_{\mu\nu}\bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta}$	$\tilde\lambda_\alpha\tilde\lambda_\beta$

$$(\tau^I)^i_j W^I_{\mu\nu}(\epsilon_{cp}D^\mu L_{ri})D^\nu H^{\dagger j}$$
$$F_{\rm L1}^{\alpha\beta}\psi_2^\gamma(D\psi_3)_{\alpha\beta\dot\alpha}(D\phi_4)_\gamma{}^{\dot\alpha}$$
$$F_{\rm L1}^{\alpha\beta}\psi_{2\alpha}(D\psi_3)_\beta{}^\gamma{}_{\dot\alpha}(D\phi_4)_\gamma{}^{\dot\alpha}$$
$$(\tau^I)^i_j W^I_{\mu\lambda}(\epsilon_{cp}\sigma^{\nu\lambda}L_{ri})D^\mu D_\nu H^{\dagger j}$$

Amplitude-Operator correspondance

Different Operator Bases

Effective operators can be written in different bases, even redundant basis

Buchmuller&Wyler:

$$\mathcal{O}_{\partial H} = \frac{1}{2} D_\mu (H^\dagger H) D^\mu (H^\dagger H), \quad \mathcal{O}_H^{(1)} = (H^\dagger H)(D_\mu H^\dagger D^\mu H), \quad \mathcal{O}_H^{(2)} = (H^\dagger D^\mu H)(D_\mu H^\dagger H).$$

Warsaw: $\mathcal{O}_{H\Box} = (H^\dagger H)\Box(H^\dagger H)\,, \quad \mathcal{O}_{HD} = (H^\dagger D_\mu H)^\dagger (H^\dagger D^\mu H)\,.$

SILH: $\mathcal{O}^{(1)}_{SILH} = D^\mu (H^\dagger H) D_\mu (H^\dagger H), \quad \mathcal{O}^{(2)}_{SILH} = (H^\dagger \overleftrightarrow{D}^\mu H)(H^\dagger \overleftrightarrow{D}_\mu H),$

Amplitude-basis (ITP-basis)

$\mathcal{O}^{(p)}_{H^2 H^{\dagger 2} D^2,1}$	$H_i H_j (D_\mu D_\nu H^{\dagger i})(D^\mu D^\nu H^{\dagger j})$
$\mathcal{O}^{(p)}_{H^2 H^{\dagger 2} D^2,2}$	$H_i H^{\dagger i} (D_\mu D_\nu H_j)(D^\mu D^\nu H^{\dagger j})$
$\mathcal{O}^{(p)}_{H^2 H^{\dagger 2} D^2,3}$	$H_i (D_\mu H_j)(D_\nu H^{\dagger i})(D^\mu D^\nu H^{\dagger j})$

traditional-basis (Murphy)

$\mathcal{O}^{(1)}_{H^4}$	$(D_\mu H^{\dagger i} D_\nu H_i)(D^\nu H^{\dagger j} D^\mu H_j)$
$\mathcal{O}^{(2)}_{H^4}$	$(D_\mu H^{\dagger i} D_\nu H_i)(D^\mu H^{\dagger j} D^\nu H_j)$
$\mathcal{O}^{(3)}_{H^4}$	$(D^\mu H^{\dagger i} D_\mu H_i)(D^\nu H^{\dagger j} D_\nu H_j)$

How to perform the basis conversion? (useful for CDE, etc)

Basis conversion can be easily done in our amplitude basis

On-shell Amplitude Basis

Any operator (non-SSYT) can be converted to the SSYT basis systematically

Our on-shell amplitude basis = SSYT basis

$$\begin{pmatrix} \mathcal{O}_{\partial H} \\ \mathcal{O}_H^{(1)} \\ \mathcal{O}_H^{(2)} \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \mathcal{O}_{H\Box} \\ \mathcal{O}_{HD} \end{pmatrix}$$

Warsaw

$$\begin{pmatrix} \mathcal{O}_{SILH}^{(1)} \\ \mathcal{O}_{SILH}^{(2)} \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} \mathcal{O}_{H\Box} \\ \mathcal{O}_{HD} \end{pmatrix}$$

SILH **Warsaw**

Reduce any operators to our ITP-basis

$D^2LL^\dagger QQ^\dagger$	
$\mathcal{O}^{(1)}$	$(D_\mu L_{pi} D^\mu Q_{raj})(L_s^{\dagger i} Q_t^{\dagger aj})$
$\mathcal{O}^{(2)}$	$(D_\mu L_{pi} D^\mu Q_{raj})(L_s^{\dagger j} Q_t^{\dagger ai})$
$\mathcal{O}^{(3)}$	$(D_\mu L_{pi} Q_{raj})(D^\mu L_s^{\dagger i} Q_t^{\dagger aj})$
$\mathcal{O}^{(4)}$	$(D_\mu L_{pi} Q_{raj})(D^\mu L_s^{\dagger j} Q_t^{\dagger ai})$
$\mathcal{O}^{(5)}$	$(D_\mu L_{pi} D_\nu Q_{raj})(L_s^{\dagger i} \bar\sigma^{\mu\nu} Q_t^{\dagger aj})$
$\mathcal{O}^{(6)}$	$(D_\mu L_{pi} D_\nu Q_{raj})(L_s^{\dagger j} \bar\sigma^{\mu\nu} Q_t^{\dagger ai})$
$\mathcal{O}^{(7)}$	$(D^\nu L_{pi} \sigma_\mu L_s^{\dagger i})(D^\mu Q_{raj} \sigma_\nu Q_t^{\dagger aj})$
$\mathcal{O}^{(8)}$	$(D^\nu L_{pi} \sigma_\mu Q_t^{\dagger aj})(D^\mu Q_{raj} \sigma_\nu L_s^{\dagger i})$
$\mathcal{O}^{(9)}$	$(D^\mu L_{pi} \sigma_\mu L_s^{\dagger i})(D^\nu Q_{raj} \sigma_\nu Q_t^{\dagger aj})$.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

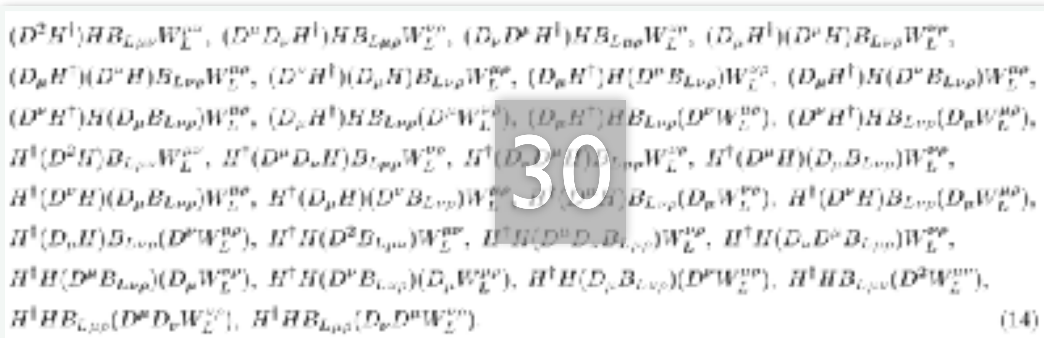
M-basis	
$\mathcal{O}_1^{(m)}$	$(L_{pi} Q_{raj})(D_\mu L_s^{\dagger i} D^\mu Q_t^{\dagger aj})$
$\mathcal{O}_2^{(m)}$	$(L_{pi} \sigma_{\mu\nu} Q_{raj})(D^\mu L_s^{\dagger i} D^\nu Q_t^{\dagger aj})$
$\mathcal{O}_3^{(m)}$	$(L_{pi} Q_{raj})(D_\mu L_s^{\dagger j} D^\mu Q_t^{\dagger ai})$
$\mathcal{O}_4^{(m)}$	$(L_{pi} \sigma_{\mu\nu} Q_{raj})(D^\mu L_s^{\dagger j} D^\nu Q_t^{\dagger ai})$

Young Tensor Method

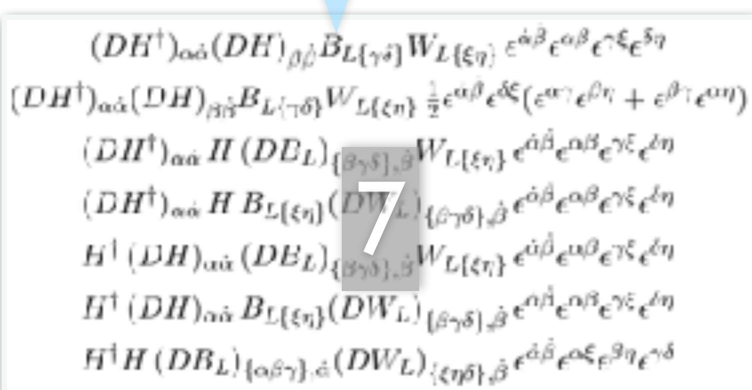
Traditional method

$\boxed{BWHH^\dagger D^2}$

[Hays, Martin, Sanz, Setford, 2018]



EOM



IBP

$$B_L{}^{\alpha\beta} W_{L\alpha\beta} \left(DH^\dagger\right)^{\gamma}{}_{\dot\alpha} \left(DH\right)_{\gamma}{}^{\dot\alpha}$$
$$B_L{}^{\alpha\beta} W_{L\alpha}{}^{\gamma} \left(DH^\dagger\right)_{\beta\dot\alpha} \left(DH\right)_{\gamma}{}^{\dot\alpha}$$

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Young tensor method (No need EOM&IBP)

$\boxed{BWHH^\dagger D^2}$ $\quad \#1 = 3, \#2 = 3, \#3 = 1, \#4 = 1$

1	1	1	3
2	2	2	4

$$\tilde\epsilon_{\dot\alpha_3\dot\alpha_4}\epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_3\alpha_4}$$

$$B_L{}^{\alpha\beta} W_{L\alpha\beta} \left(DH^\dagger\right)^{\gamma}{}_{\dot\alpha} \left(DH\right)_{\gamma}{}^{\dot\alpha},$$

$$\langle 13\rangle \langle 13\rangle \langle 24\rangle [34]$$

2

1	1	1	2
2	2	3	4

$$\tilde\epsilon_{\dot\alpha_3\dot\alpha_4}\epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_1\alpha_3}\epsilon^{\alpha_2\alpha_4}$$

$$B_L{}^{\alpha\beta} W_{L\alpha}{}^{\gamma} \left(DH^\dagger\right)_{\beta\dot\alpha} \left(DH\right)_{\gamma}{}^{\dot\alpha}$$

$$\langle 12\rangle \langle 13\rangle \langle 34\rangle [34]$$

On-shell amplitude basis easily and systematically

How about gauge structure?

Gauge Structure

Gauge structure (internal sym) is easier than Lorentz structure (spacetime sym)

Dim-6 four fermion B-conserving operators: 25

[Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010]

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu} u_t)$		

Buchmuller&Wyler wrote 29: 5 redundant operators (Fierz) + 1 missing

$$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{l}_s\tau^I\gamma^\mu l_t) = 2Q_{ll}^{ptsr} - Q_{ll}^{prst}$$

Fierz identity for SU(N):

$$\sum_a (T_a)_{ij}(T_a)_{kl} = \delta_{il}\delta_{kj} - \frac{1}{N}\delta_{ij}\delta_{kl}$$

U(N) trace relation

Gauge Young Tensor

How to obtain independent and complete gauge structure systematically?

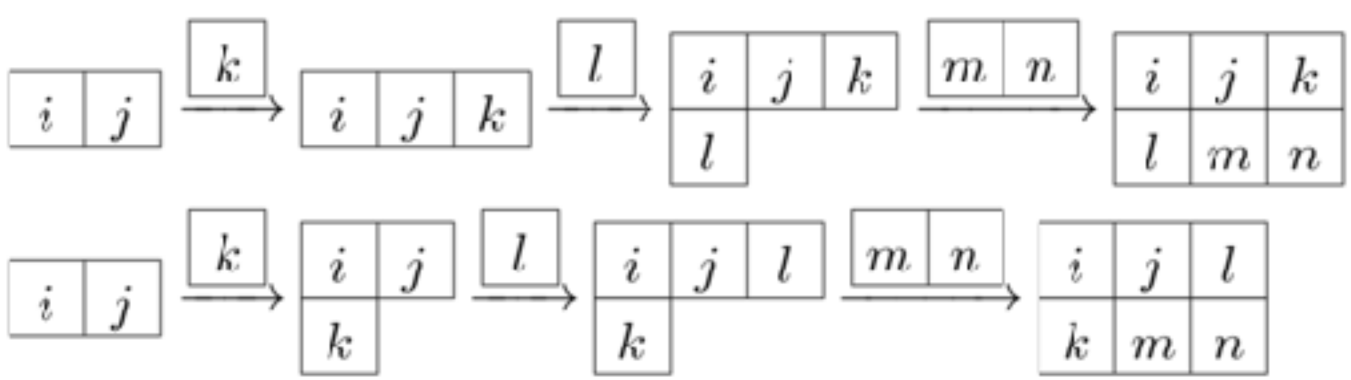
g-2 dim 8 operator

$(\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu}$ $\xrightarrow{\text{Add } H^\dagger H}$ $W e_{\mathbb{C}} L H H^{\dagger 2}$ $\quad (\tau^I)^k_j W^I_{\mu\nu} (e_{\mathbb{C}p}\sigma^{\mu\nu} L_{rk}) H^{\dagger j} (H^\dagger H)$

?

We invent Littlewood-Richardson method at Young tableau level

$\tau^I{}_{ij} W^I$: $\boxed{i\,|\,j}$, L_k: $\boxed{k}$, H_l: $\boxed{l}$, $H^\dagger_m H^\dagger_n$: $\boxed{m\,|\,n}$



$\epsilon^{il}\epsilon^{jm}\epsilon^{kn}$ $\quad W^I L_k H^{\dagger k} (H^\dagger \tau H)$

$\epsilon^{ik}\epsilon^{jm}\epsilon^{ln}$ $\quad (\tau^I)^k_j W^I L_k H^{\dagger j} (H^\dagger H)$

Find another g-2 dim 8 operator:

$W^I_{\mu\nu} (e_{\mathbb{C}p}\sigma^{\mu\nu} L_{ri}) H^{\dagger i} (H^\dagger \tau^I H)$

Flavor Structure

Concerning 3 flavors, operator forms a flavor tensor in flavor space

$$O_i^{(d)} \to O_{i,pr}^{(d)} \qquad O_{pr} = \boxed{\left(\tau^I\right)_j^i W_{\mu\nu}^I \left(e_{\mathbb{C}p} D^\mu L_{ri}\right) D^\nu H^{\dagger j}}$$

If repeated fields, flavor tensor obeys certain symmetry structure

$$O_{pr} = \epsilon_{ik}\epsilon_{jl}\epsilon_{\alpha\beta} L_p^{\alpha i} L_r^{\beta j} H^k H^l$$

Symmetric under flavor permutation

$$\begin{aligned} \epsilon_{ik}\epsilon_{jl}\epsilon_{\alpha\beta} L_p^{\alpha i} L_r^{\beta j} H^k H^l &= \epsilon_{jl}\epsilon_{ik}\epsilon_{\alpha\beta} L_p^{\alpha j} L_r^{\beta i} H^l H^k \\ &= \epsilon_{ik}\epsilon_{jl}\epsilon_{\alpha\beta} L_p^{\alpha j} L_r^{\beta i} H^k H^l \\ &= -\epsilon_{ik}\epsilon_{jl}\epsilon_{\alpha\beta} L_p^{\beta j} L_r^{\alpha i} H^k H^l \\ &= \epsilon_{ik}\epsilon_{jl}\epsilon_{\alpha\beta} L_r^{\alpha i} L_p^{\beta j} H^k H^l , \end{aligned}$$

Impose flavor relations

$$O_{pr}^{LH} = O_{rp}^{LH}$$

Flavor Relations QQQL

This is the second difficulty to write down the independent EFT operators

	B-violating
Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}(\tau^I\varepsilon)_{jk}(\tau^I\varepsilon)_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

[Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010]

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$$Q_{prst}^{qqq\ell\,(1)} = -(Q_{prst}^{qqq\ell} + Q_{rpst}^{qqq\ell})$$
$$Q_{prst}^{qqq\ell\,(3)} = -(Q_{prst}^{qqq\ell} - Q_{rpst}^{qqq\ell})$$

[Grzadkowski, et.al. v3 2017]

	B-violating
Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

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[Alonso, Chang, Jenkins, Manohar, Shotwell 2014]

$$Q_{prst}^{qqq\ell} + Q_{rpst}^{qqq\ell} = Q_{sprt}^{qqq\ell} + Q_{srpt}^{qqq\ell}$$

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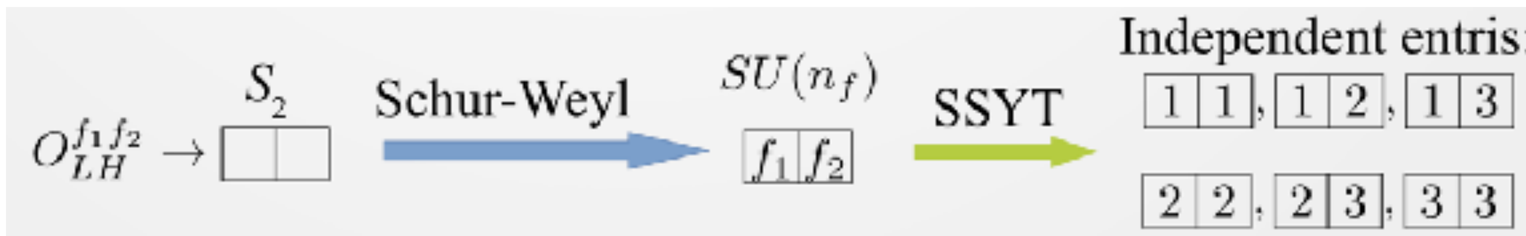
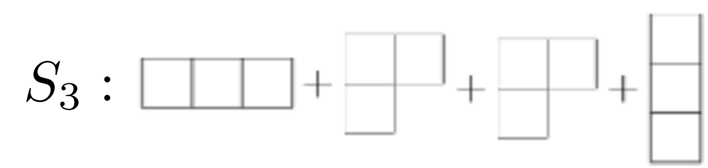
Flavor relations not easy task!

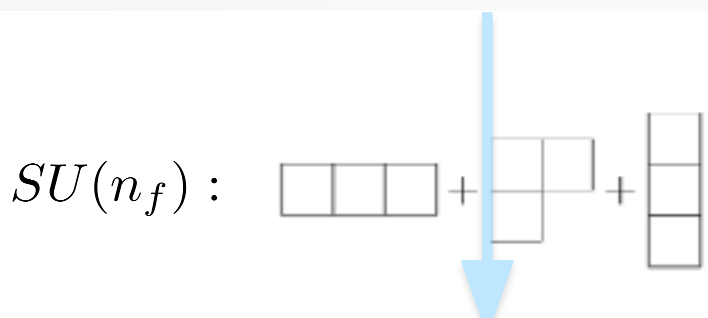
Flavor Symmetry

According to Schur-Weyl theorem, flavor tensor decomposed via S(nf) symmetry

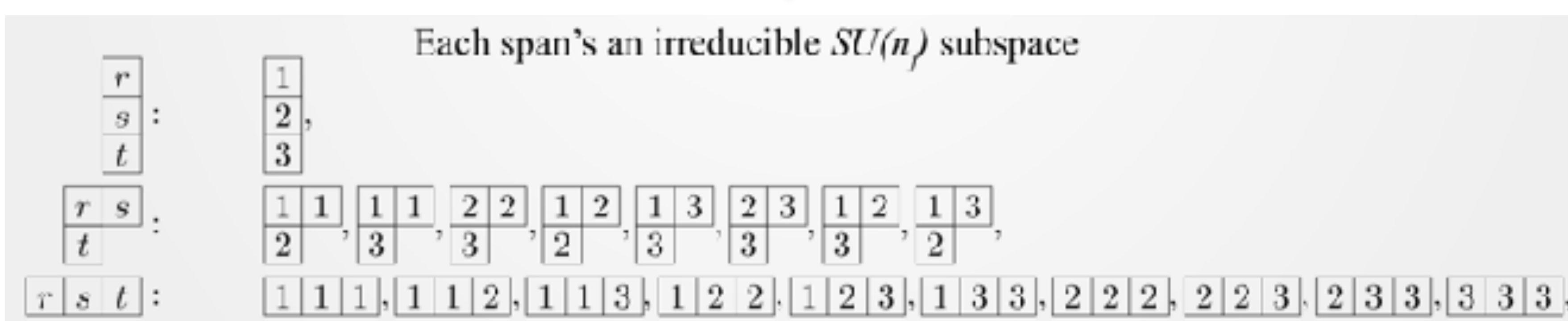


$$O_{qqql}^{p,rst}\,\epsilon^{abc}\epsilon_{ji}\epsilon_{km}[(q_r^{aj})^T C q_s^{bk}][(q_t^{cm})^T C l_p^i]$$





19 x 3 = 57



How to recognize SU(nf) symmetry according to Sn?

Spin-Statistics

N-identical particle operator obeys spin-statistics:

$$\pi\circ\mathcal{O}(1,2,\dots,m,\dots,N)=\mathcal{O}(\pi(1),\pi(2),\dots,\pi(m),\dots,N)=\pm\mathcal{O}(1,2,\dots,m,\dots,N)$$

Amplitude view:

$$Amp(1,\dots,N)\sim C_{f_1,\dots,f_N}\,T_C^{a_1,\dots a_N}\,B^{(d)}(h_1,\dots,h_N)$$

$$\lambda_{Amp}=[m]\ \text{or}\ [1^m]\in\lambda_C\odot\lambda_G\odot\lambda_B$$

$$\lambda_C^{(T)}=\lambda_G\odot\lambda_B$$

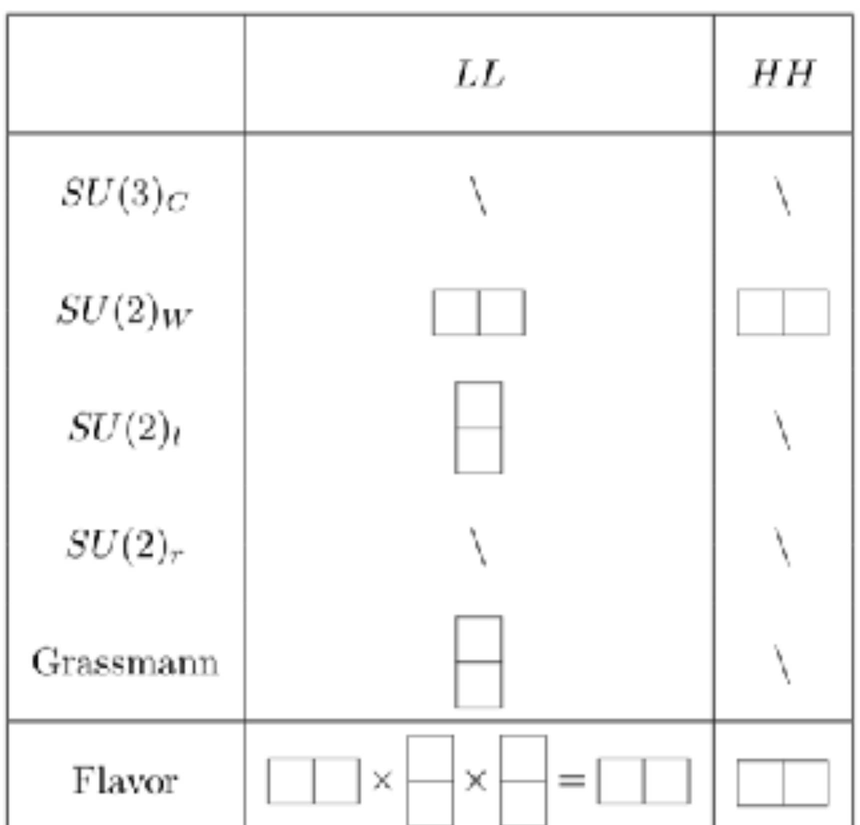
Operator view:

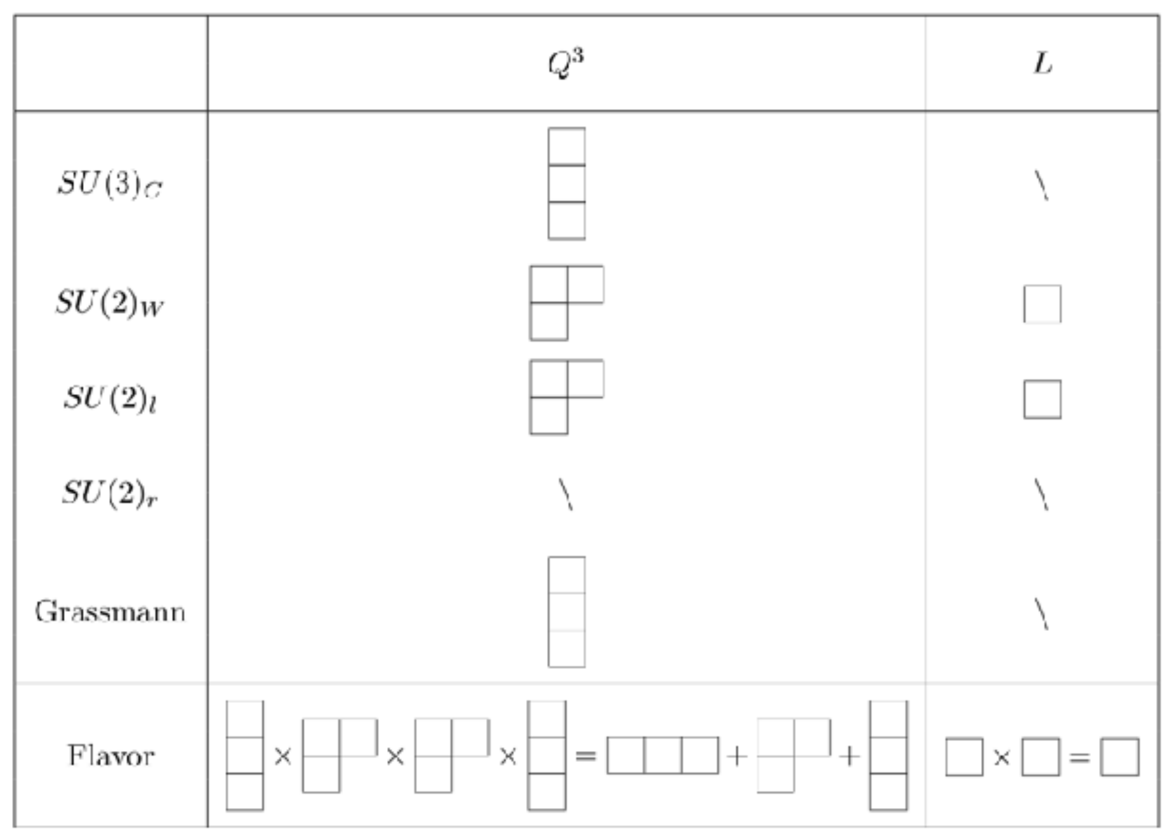
$$\underbrace{\pi\circ\mathcal{O}^{\{f_k,\dots\}}}_{\text{permute flavor}}=T_{SU3}^{\{g_k,\dots\}}T_{SU2}^{\{h_k,\dots\}}\mathcal{M}^{\{f_{\pi(k)},\dots\}}_{\{g_k,\dots\},\{h_k,\dots\}}$$

$$=T_{SU3}^{\{g_{\pi(k)},\dots\}}T_{SU2}^{\{h_{\pi(k)},\dots\}}\mathcal{M}^{\{f_{\pi(k)},\dots\}}_{\{g_{\pi(k)},\dots\},\{h_{\pi(k)},\dots\}}$$

$$=\underbrace{\left(\pi\circ T_{SU3}^{\{g_k,\dots\}}\right)\left(\pi\circ T_{SU2}^{\{h_k,\dots\}}\right)}_{\text{permute gauge}}\underbrace{\left(\pi\circ\mathcal{M}^{\{f_k,\dots\}}_{\{g_k,\dots\},\{h_k,\dots\}}\right)}_{\text{permute Lorentz}}$$

[Li, Ren, Xiao, **Yu**, Zheng, **2020**] **Also** [Fonseca, 2020]





$\times 3 = 57$

Operator Y Basis

For QQQL, the Young tableau for Lorentz and gauge structure give the Y-basis

$$\left(\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array} + \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline \end{array} \right) \times \begin{array}{|c|}\hline a \\ \hline b \\ \hline c \\ \hline \end{array} \times \left(\begin{array}{|c|c|}\hline i & j \\ \hline k & l \\ \hline \end{array} + \begin{array}{|c|c|}\hline i & k \\ \hline j & l \\ \hline \end{array} \right) =$$

$$\mathcal{M} = (L_{pi}Q_{raj})(Q_{sbk}Q_{tcl}),$$
$$(L_{pi}Q_{sbk})(Q_{raj}Q_{tcl})$$

$$T_G = \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}, \ \epsilon^{abc}\epsilon^{ij}\epsilon^{kl}$$

$$O_1 = \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}(L_{pi}Q_{raj})(Q_{sbk}Q_{tcl})$$
$$O_2 = \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}(L_{pi}Q_{sbk})(Q_{raj}Q_{tcl})$$
$$O_3 = \epsilon^{abc}\epsilon^{ij}\epsilon^{kl}(L_{pi}Q_{sbk})(Q_{raj}Q_{tcl})$$
$$O_4 = \epsilon^{abc}\epsilon^{ij}\epsilon^{kl}(L_{pi}Q_{raj})(Q_{sbk}Q_{tcl})$$

Y-Basis = Young tensor basis

Reorganize the operator using the Sn Young symmetrizer:

$$\mathcal{Y}_1^{[3]} = \mathcal{Y}\left[\begin{array}{|c|c|c|}\hline 2 & 3 & 4 \\ \hline \end{array}\right] = \begin{pmatrix} -\frac{1}{6} & \frac{1}{3} & \frac{1}{3} & -\frac{1}{6} \\ -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{6} & \frac{1}{3} & \frac{1}{3} & -\frac{1}{6} \end{pmatrix}, \quad \mathcal{Y}_1^{[2,1]} = \mathcal{Y}\left[\begin{array}{|c|c|}\hline 2 & 3 \\ \hline 4 \\ \cline{1-1} \end{array}\right] = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 \\ \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix},$$

$$\mathcal{Y}_2^{[2,1]} = (3\ 4)\mathcal{Y}\left[\begin{array}{|c|c|}\hline 2 & 3 \\ \hline 4 \\ \cline{1-1} \end{array}\right] = \begin{pmatrix} -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ 0 & 0 & 0 & 0 \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \end{pmatrix}, \quad \mathcal{Y}_1^{[1,1,1]} = \mathcal{Y}\left[\begin{array}{|c|}\hline 2 \\ \hline 3 \\ \hline 4 \\ \hline \end{array}\right] = \begin{pmatrix} \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}.$$

$$\mathcal{Y}\left[\begin{array}{|c|c|c|}\hline r & s & t \\ \hline \end{array}\right] \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}\,(L_{pi}Q_{raj})\,(Q_{sbk}Q_{tcl})$$
$$\mathcal{Y}\left[\begin{array}{|c|c|}\hline r & s \\ \hline t \\ \cline{1-1} \end{array}\right] \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}\,(L_{pi}Q_{raj})\,(Q_{sbk}Q_{tcl})$$
$$(s\ t)\mathcal{Y}\left[\begin{array}{|c|c|}\hline r & s \\ \hline t \\ \cline{1-1} \end{array}\right] \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}\,(L_{pi}Q_{raj})\,(Q_{sbk}Q_{tcl})$$
$$\mathcal{Y}\left[\begin{array}{|c|}\hline r \\ \hline s \\ \hline t \\ \hline \end{array}\right] \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}\,(L_{pi}Q_{raj})\,(Q_{sbk}Q_{tcl})$$

Caveat: Sn symmetry for repeated field, but not SU(nf) symmetry for nf flavor yet

From Y Basis to P Basis

The P-basis operator is viewed as flavor tensor in the SU(nf) group

$$
\begin{aligned}
O_1 &= \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}(L_{pi}Q_{raj})(Q_{sbk}Q_{tcl})\\
O_2 &= \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}(L_{pi}Q_{sbk})(Q_{raj}Q_{tcl})\\
O_3 &= \epsilon^{abc}\epsilon^{ij}\epsilon^{kl}(L_{pi}Q_{sbk})(Q_{raj}Q_{tcl})\\
O_4 &= \epsilon^{abc}\epsilon^{ij}\epsilon^{kl}(L_{pi}Q_{raj})(Q_{sbk}Q_{tcl})
\end{aligned}
$$

Y-Basis

S(N)

Young Operator

$$
\begin{pmatrix} O^{[3],1} \\ O^{[2,1],1} \\ O^{[2,1],2} \\ O^{[1^3],1} \end{pmatrix}
= \underbrace{\begin{pmatrix} -1 & 2 & 2 & -1 \\ 2 & -1 & -1 & 2 \\ -1 & -1 & 2 & -1 \\ -1 & 0 & 0 & 1 \end{pmatrix}}_{\mathcal{K}^{py}_{ji}}
\begin{pmatrix} O_1 \\ O_2 \\ O_3 \\ O_4 \end{pmatrix}
$$

p-basis $\qquad \mathcal{K}^{py}_{ji} \qquad$ y-basis

$O^{[2,1],1}$ $O^{[2,1],2}$ span the same $SU(n_f)$ space.

Flavor specified permutation basis (P-Basis) operator

$$
\begin{array}{l|l}
\mathcal{O}^{(p')}_{LQ^3,1} & \mathcal{Y}\left[\begin{array}{|c|c|c|}\hline r & s & t \\ \hline\end{array}\right] \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}\,(L_{pi}Q_{raj})\,(Q_{sbk}Q_{tcl}) \\
\mathcal{O}^{(p')}_{LQ^3,2} & \mathcal{Y}\left[\begin{array}{|c|c|}\hline r & s \\ \hline t \\ \cline{1-1}\end{array}\right] \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}\,(L_{pi}Q_{raj})\,(Q_{sbk}Q_{tcl}) \\
\mathcal{O}^{(p')}_{LQ^3,3} & \mathcal{Y}\left[\begin{array}{|c|}\hline r \\ \hline s \\ \hline t \\ \hline\end{array}\right] \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}\,(L_{pi}Q_{raj})\,(Q_{sbk}Q_{tcl})
\end{array}
$$

$$
\begin{aligned}
[1^3] &: \quad \Theta^{\{p,rst\}}_{[1^3]} = \mathcal{Y}[1^3]\,O^{p,rst}_{qqql} = O^{p,rst}_{qqql} + (-1)^{\mathrm{sgn}(rst)}(\text{perm } r,s,t)\\
[2,1] &: \quad \Theta^{\{p,rst\}}_{[2,1]} = \mathcal{Y}[2,1]\,O^{p,rst}_{qqql} = O^{p,rst}_{qqql} + O^{p,srt}_{qqql} - O^{p,trs}_{qqql} - O^{p,tsr}_{qqql}\\
[3] &: \quad \Theta^{\{p,rst\}}_{[3]} = \mathcal{Y}[3]\,O^{p,rst}_{qqql} = O^{p,rst}_{qqql} + (\text{perm } r,s,t)
\end{aligned}
$$

Final expression: P-Basis

Mathematica Code: ABC4EFT

Amplitude Basis Construction for Effective Field Theory

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Welcome to the HEPForge Project: ABC4EFT

This is the website for the Mathematica package: Amplitude Basis Construction for Effective Field Theories (ABC4EFT).

Package

This package has the following features:

- It provides a general procedure to construct the independent and complete operator bases for generic Lorentz invariant effective field theory, given any kind of gauge symmetry and field content, up to any mass dimension.
- Various operator bases have been systematically constructed to emphasize different aspects: operator independence (y-basis), flavor relation (p-basis) and conserved quantum number (j-basis).
- It provides a systematic way to convert any operator into our on-shell amplitude basis and the basis conversion can be easily done.

Authors

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- Jiang-Hao Yu (professor at ITP-CAS)
- Yu-Hui Zheng (5th-year graduate student at ITP-CAS)

https://abc4eft.hepforge.org/

```
<< ABC4EFT
                 --------------------------
                       ABC4EFT 1.0.0
                 --------------------------

                 A Mathematica Package for
     Amplitude Basis Construction for Effective Field Theories

        Authors: Hao-Lin Li, lihaolin1991@gmail.com
                 Zhe Ren, renzhe@itp.ac.cn
                 Ming-Lei Xiao, minglei.xiao@northwestern.edu
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                 Yu-Hui Zheng, zhengyuhui@itp.ac.cn

             The package is available at hepforge
           For the latest version, see the GitHub
            If you use this package in your research,
     Please cite: arXiv: 2201.04639, 2005.00008, 2007.07899
```
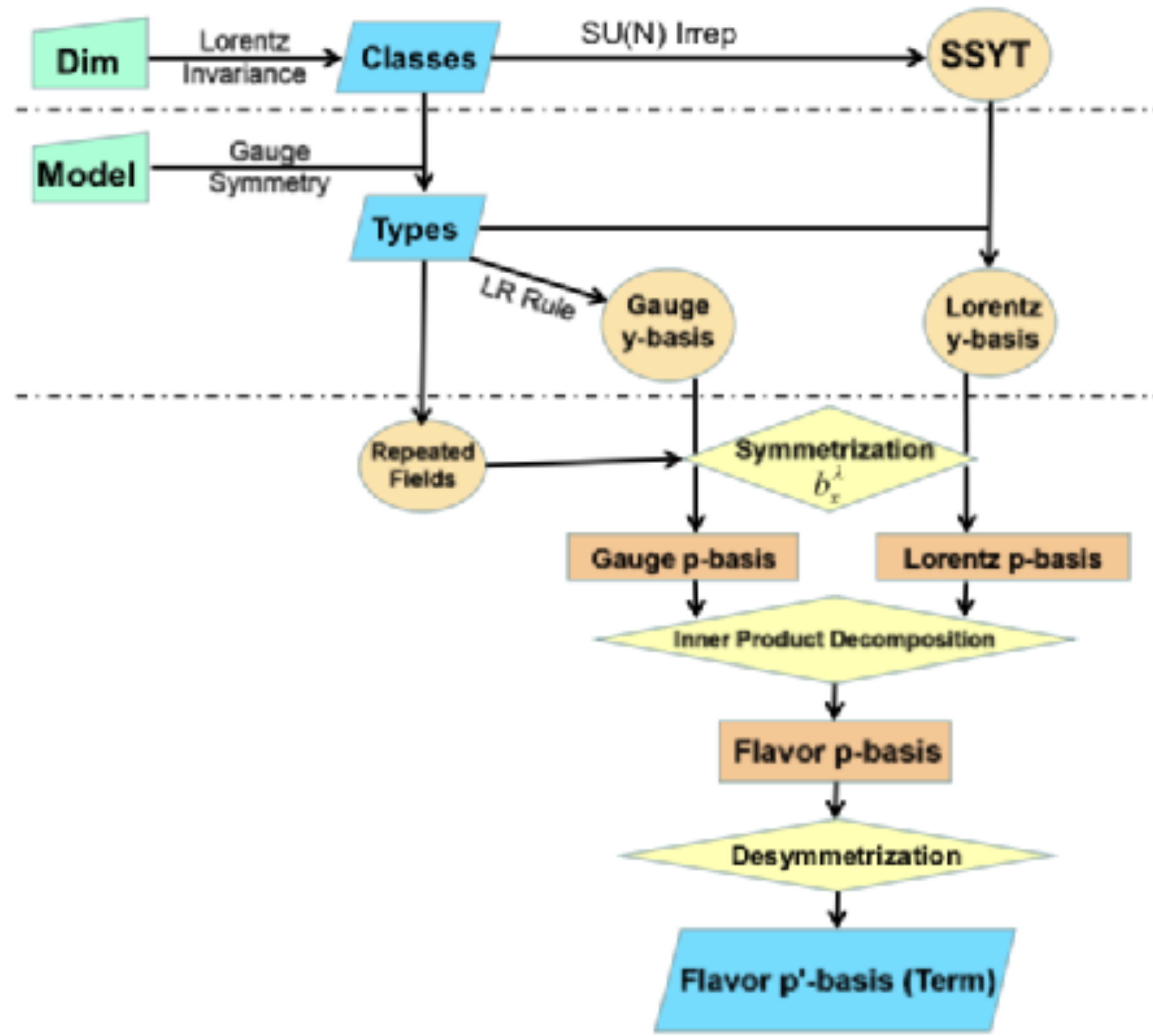



Mathematica Code: ABC4EFT

https://abc4eft.hepforge.org/

Li, Ren, Xiao, **JHY**, Zheng, 2201.04639

```
DefSMEFT[model_,nf_:3]:= Module[{},
ModelIni[model];
AddGroup[model,"U1b"];
AddGroup[model,"U1l"];
AddGroup[model,"SU3c",GaugeBoson->"G"];
AddGroup[model,"SU2w",GaugeBoson->"W"];
AddGroup[model,"U1y",GaugeBoson->"B"];
AddField[model,"Q",-1/2,{"SU3c"->{1,0},"SU2w"->{1},"U1y"->1/6,"U1b"->1/3},Flavor->nf];
AddField[model,"uc",-1/2,{"SU3c"->{0,1},"U1y"->-2/3,"U1b"->-1/3},Flavor->nf];
AddField[model,"dc",-1/2,{"SU3c"->{0,1},"U1y"->1/3,"U1b"->-1/3},Flavor->nf];
AddField[model,"L",-1/2,{"SU2w"->{1},"U1y"->-1/2,"U1l"->1},Flavor->nf];
AddField[model,"ec",-1/2,{"U1y"->1,"U1l"->-1},Flavor->nf];
AddField[model,"H",0,{"SU2w"->{1},"U1y"->1/2}]
]
```

```
In[6]:= AllTypesC[SMEFT,6]

Out[6]= <|FL^3→{DL^2,DLWL^2,WL^3,DLGL^2,GL^3},ψ^4→{dcecuc^2,ecLQuc,dcQ^2uc,LQ^3},
FLϕψ^2→{BLecH†L,BLdcH†Q,BLHQuc,ecH†LWL,dcH†QWL,HQucWL,dcGLH†Q,GLHQuc},
FL^2ϕ^2→{BL^2HH†,BLH†HWL,HH†WL^2,GL^2HH†},ψ^2ψ†^2→{ec^2ec†^2,ecec†LL†,dcec†LQ†,dcdc†LL†,
LL†ucuc†,ecec†QQ†,dcec†L†Q,L^2L†^2,dcdc†ecec†,ecec†ucuc†,dc^2dc†^2,dcdc†ucuc†,uc^2uc†^2,
ecQ†^2uc,dcL†Q†uc,dc†LQuc†,ec†Q^2uc†,LL†QQ†,dcdc†QQ†,QQ†ucuc†,Q^2Q†^2},
Dϕ^2ψψ†→{[illegible]},
D^2ϕ^4→{D^2H^2H†^2},ϕ^3ψ^2→{ecHH†^2L,dcHH†^2Q,H^2H†Quc},ϕ^6→{H^3H†^3}|>
```

```
In[8]:= GetBasisForType[SMEFT,"D"^4"H"^2"H†"^2]

Out[8]= <|"basis"→{H_i H_j (D_μ D_ν H†^i)(D^μ D^ν H†^j), H_i H†^i (D_μ D_ν H_j)(D^μ D^ν H†^j),
H_i (D_μ H_j)(D_ν H†^i)(D^μ D^ν H†^j), H_i H_j (D_μ D_ν H†^j)(D^μ D^ν H†^i), H_i H†^j (D_μ D_ν H_j)(D^μ D^ν H†^i),
H_i (D_μ H_j)(D_ν H†^j)(D^μ D^ν H†^i)},"p-basis"→<|{H→{2},H†→{2}}→{{1/2,0,0,1/2,0,0},
{0,1/2,0,1/2,1/2,1},{0,0,1/2,-1/2,0,-1/2}}|>|>
```

```
In[5]:= StatResult[SMEFT,8];

Done! time used: 0.2870018
number of real types→541
number of real terms→1266
number of real operators→44807
```

```
In[10]:= FindYCoord[SMEFT,del2["i","k"] del2["j","l"] DC[DC["H†"["k"],"mu"],"nu"]
 DC[DC["H†"["l"],"mu"],"nu"] "H"["i"] "H"["j"]]

Out[10]= <|SU2→{1, 0}, SU3→{1}, Lor→{1/4, 0, 0}, TProduct→{1/4, 0, 0, 0, 0, 0}, type→H^2H†^2D^4 |>
```

```
In[7]:= GenerateOperatorList[SMEFT,5]

Generating types of operators ...
Time spent: 0.1098517

Out[7]= <|ϕ^2ψ^2→<|H^2L^2→<|{L→{2},H→{2}}→{ϵ^ik ϵ^jl H_k H_l (L_pi L_rj)}|>|>|>
```

SMEFT Operators

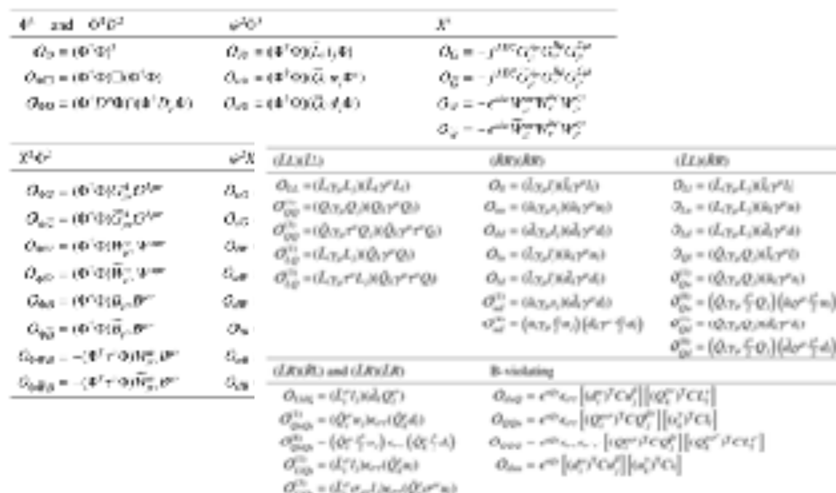
Dimension-5

$$\epsilon_{ij}\epsilon_{mn}(L^iCL^m)H^jH^n$$

[Weinberg, 1979]

2

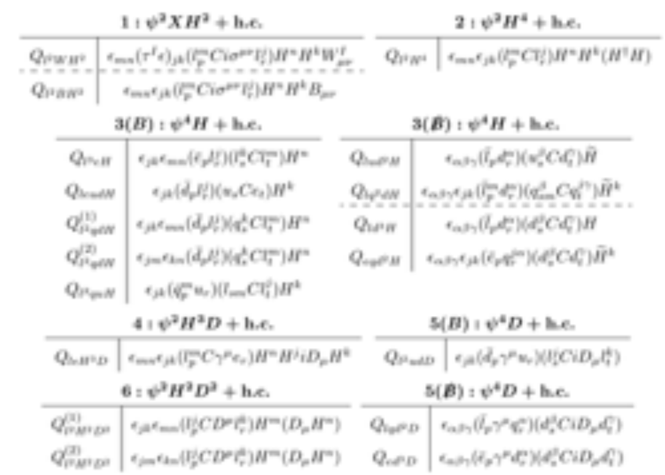
Dimension-6



[Buchmuller, Wyler, 1986] [Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010]

84

Dimension-7



[Lehman, 2014]

[Henning, Lu, Melia, Murayama, 2015]

[Liao, Ma, 2016]

Dimension-8

[Li, Ren, Shu, Xiao, **Yu**, Zheng, **2020**]

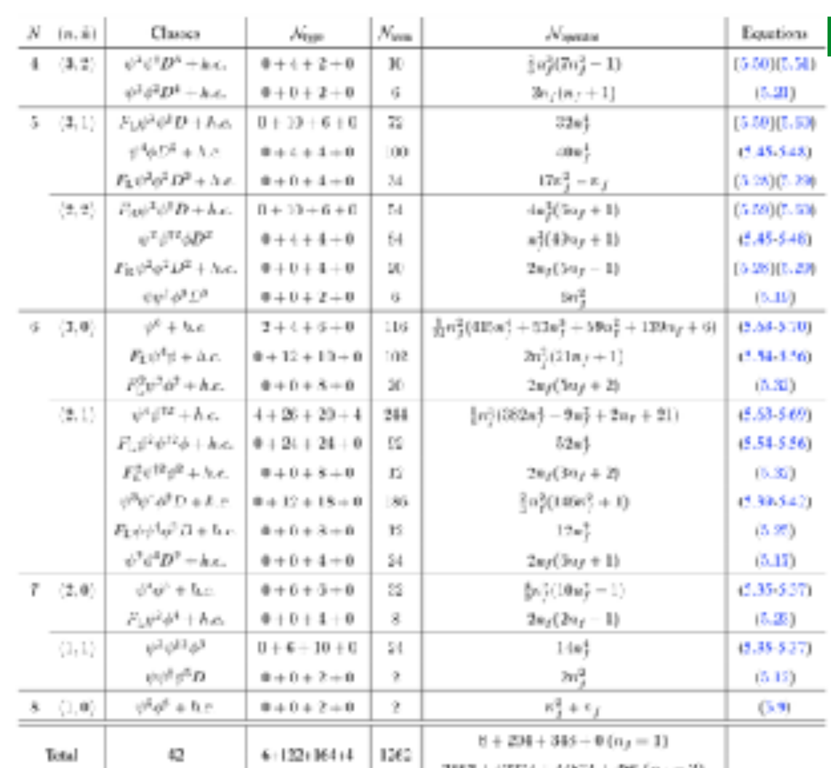
[Murphy, 2020]

993

Dimension-9

[Li, Ren, Xiao, **Yu**, Zheng, **2020**]

[Liao, Ma, 2020]



LEFT Operators

Dimension-5

Dim-5 operators

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$
3	(2,0)	$F_{\rm L}\psi_{\rm L}^2 + h.c.$	$10+0+2+0$

10

Dimension-6

Dim-6 operators

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$
3	(3,0)	$F_{\rm L}^3 + h.c.$	$2+0+0+0$	2
4	(2,0)	$\psi_{\rm L}^4 + h.c.$	$14+12+8+2$	78
	(1,1)	$\psi_{\rm L}^2\psi_{\rm R}^2$	$40+20+12+0$	84
Total		5	$56+32+20+2$	164

120

[Jenkins, Manohar, Stoffer, 2017]

Dimension-7

Dim-7 operators

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$
4	(3,0)	$F_{\rm L}^2\psi_{\rm L}^2 + h.c.$	$16+0+4+0$	32
	(2,1)	$F_{\rm L}^2\psi_{\rm R}^2 + h.c.$	$16+0+4+0$	24
		$\psi_{\rm L}^3\psi_{\rm R} D + h.c.$	$50+32+22+$	
Total		6	$82+32+30+$	

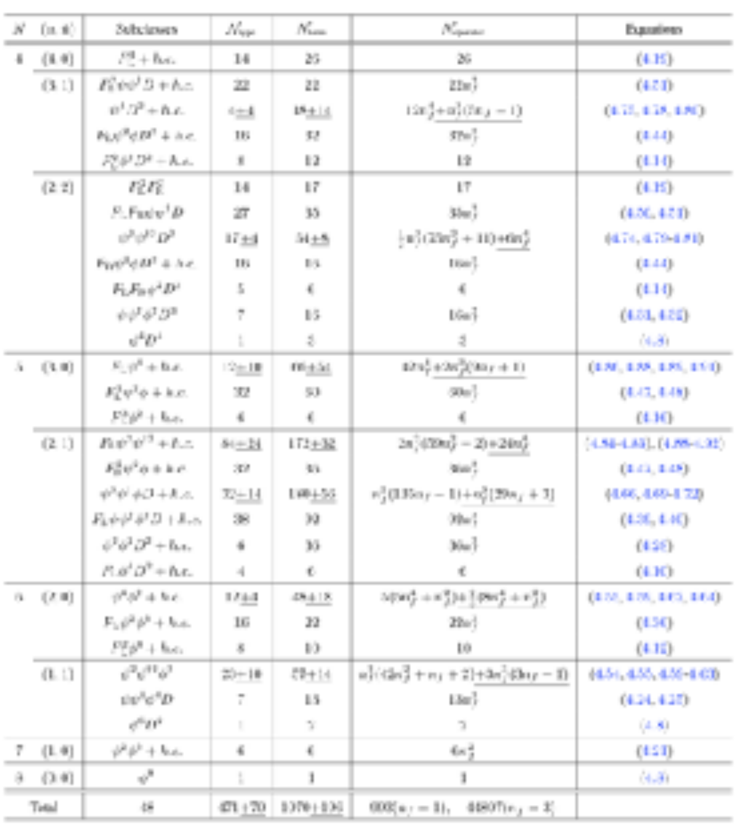
166

[Liao, Ma, Wang, 2020]

Dimension-8

[Li, Ren, Xiao, **Yu**, Zheng, 2020]

[Murphy, 2020]



783

Dimension-9

[Li, Ren, Xiao, **Yu**, Zheng, 2020]

3774

vSMEFT and vLEFT

Dimension-5

Dim-5 operato

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$
3	(2, 0)	$F_L \nu^2 + h.c.$	$0+0+2+0$	2
4	(1, 0)	$\nu^2\phi^2 + h.c.$	$0+0+2+0$	2
Total		4	$0+0+4+0$	4

2

Dimension-6

Dim-6 operato

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$
4	(2, 0)	$\psi^4 + h.c.$	$4+2+0+2$	14
		$F_L\psi^2\phi + h.c.$	$4+0+0+0$	4
	(1, 1)	$\psi^2\psi^{\dagger 2}$	$10+2+0+0$	12
		$\psi\psi^\dagger\phi^2 D$	$3+0+0+0$	3
5	(1, 0)	$\psi^2\phi^3 + h.c.$	$2+0+0+0$	2
Total		8	$23+4+0+2$	35

29

[Aguila, Bar-Shalom, Soni, Wudka, 2009]

Dimension-7

Dim-7 operato

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$
4	(3, 0)	$F_L^2\nu^2 + h.c.$	$0+0+6+0$	6
	(2, 1)	$F_R^2\psi^{\dagger 2} + h.c.$	$0+0+6+0$	6
		$\psi^3\psi^\dagger D + h.c.$	$0+4+20+0$	24
		$F_L\psi\psi^\dagger\phi D + h.c.$	$0+0+8+0$	8
		$\psi^2\phi^2 D^2 + h.c.$	$0+0+4+0$	6
5	(2, 0)	$\psi^4\phi + h.c.$	$0+2+10+0$	24
		$F_L\psi^2\phi^2 + h.c.$	$0+0+6+0$	6
	(1, 1)	$\psi^2\psi^{\dagger 2}\phi$	$0-4-22+0$	30
		$\psi\psi^\dagger\phi^2 D$	$0+0+3+0$	4
6	(1, 0)	$\psi^2\phi^4 + h.c.$	$0+0+2+0$	2
Total		18	$0+10+56+0$	116

[Bhattacharya, Wudka, 2016]

[Liao, Ma, 2017]

Dimension-8

[Li, Ren, Xiao, **Yu**, Zheng, 2021]

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$
4	(3, 1)	$\psi^4 D^2 + h.c.$	$4+0+2+2$	22
		$F_L\psi^2\phi D^2 + h.c.$	$4+0+0+0$	8
	(2, 2)	$F_L F_R\psi\psi^\dagger D$	$3+0+0+0$	3
		$\psi^2\psi^{\dagger 2} D^2$	$10+2+0+0$	24
		$F_R\psi^2\phi D^2 + h.c.$	$4+0+0+0$	4
		$\psi\psi^\dagger\phi^2 D^3$	$3+0+0+0$	4
5	(3, 0)	$F_L\psi^4 + h.c.$	$10+4+0+2$	50
		$F_L^2\psi^2\phi + h.c.$	$8+0+0+0$	12
	(2, 1)	$F_L\psi^2\psi^{\dagger 2} + h.c.$	$42+12+0+0$	58
		$F_L^2\psi^{\dagger 2}\phi + h.c.$	$8+0+0+0$	8
		$\psi^3\psi^\dagger\phi D + h.c.$	$24+6+0+2$	108
		$F_L\psi\psi^\dagger\phi^2 D + h.c.$	$12+0+0+0$	16
		$\psi^2\phi^3 D^2 + h.c.$	$2+0+0+0$	12
6	(2, 0)	$\psi^4\phi^2 + h.c.$	$8+2+0+2$	30
		$F_L\psi^2\phi^3 + h.c.$	$4+0+0+0$	6
	(1, 1)	$\psi^2\psi^{\dagger 2}\phi^2$	$16+4+0+2$	28
		$\psi\psi^\dagger\phi^4 D$	$3+0+0+0$	3
7	(1, 0)	$\psi^2\phi^5 + h.c.$	$2+0+0+0$	2
Total		37	$167+30-2+10$	398

323

Dimension-9

[Li, Ren, Xiao, **Yu**, Zheng, 2021]

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$
4	(4, 1)	$F_L^2\psi^2 D^2 + h.c.$	$0+6+0+0$	12
	(3, 2)	$F_L F_R\psi^2 D^2 + h.c.$	$0+6+0+0$	6
		$F_L^2\psi^{\dagger 2} D^2 + h.c.$	$0+6+0+0$	6
		$\psi^3\psi^\dagger D^3 + h.c.$	$4+20+0+0$	40
		$F_L\psi\psi^\dagger\phi D^2 + h.c.$	$0+8+0+0$	16
		$\psi^2\phi^2 D^4 + h.c.$	$0+1+0+0$	8
5	(4, 0)	$F_L^3\psi^2 + h.c.$	$0+10+0+0$	15
	(3, 1)	$F_L^3\psi^{\dagger 2} + h.c.$	$0+4+0+0$	4
		$F_L\psi^3\psi^\dagger D + h.c.$	$10+42+8+0$	222
		$F_L^2\psi\psi^\dagger\phi D + h.c.$	$0+10+0+0$	32
		$\psi^4\phi D^2 + h.c.$	$9+10+0+0$	130
		$F_L\psi^2\phi^2 D^2 + h.c.$	$0+8+0+0$	42
	(2, 2)	$F_L F_R^2\psi^2 + h.c.$	$0+12+0+0$	12
		$F_R\psi^3\psi^\dagger D + h.c.$	$10+42+8+0$	166
		$F_L F_R\psi\psi^\dagger\phi D$	$0+10+0+0$	24
		$\psi^2\psi^{\dagger 2}\phi D^2$	$4+22+0+0$	210
		$F_R\psi^2\phi^2 D^2 + h.c.$	$0+8+0+0$	24
		$\psi\psi^\dagger\phi^3 D^3$	$0+2+0+0$	20
6	(3, 0)	$\psi^6 + h.c.$	$6+10+6+2$	130
		$F_L\psi^4\phi + h.c.$	$6+26+0+0$	110
		$F_L^2\psi^2\phi^2 + h.c.$	$0+12+0+0$	18
	(2, 1)	$\psi^4\psi^{\dagger 2} + h.c.$	$40+106+14+0$	474
		$F_L\psi^2\psi^{\dagger 2}\phi + h.c.$	$24+106+0+0$	176
		$F_L^2\psi^{\dagger 2}\phi^2 + h.c.$	$0+10+0+0$	10
		$\psi^3\psi^\dagger\phi^2 D + h.c.$	$10+44+8+0$	268
		$F_L\psi\psi^\dagger\phi^3 D + h.c.$	$0+8+0+0$	32
		$\psi^2\phi^4 D^2 + h.c.$	$0+4+0+0$	20
7	(2, 0)	$\psi^4\phi^3 + h.c.$	$2+12+0+0$	28
		$F_L\psi^2\phi^4 + h.c.$	$0+6+0+0$	6
	(1, 1)	$\psi^2\psi^{\dagger 2}\phi^3$	$4+22+0+0$	34
		$\psi\psi^\dagger\phi^5 D$	$0+2+0+0$	[illegible]

1358

GRSMEFT Operators

GR Lorentz tensor rewritten as flat space spinor tensor with Vielbein formalism

Curved space | **Tangent space**

$$g^{\mu\nu}(x) \;=\; e^{\mu}{}_{a}(x)\, e^{\nu}{}_{b}(x)\, \eta^{ab}$$

$$\sigma^{\mu} = e^{\nu}{}_{a}(x)\, \sigma^{a}$$

Curved space:

$$\sigma^{\mu}_{\alpha\dot{\alpha}}\tilde{\sigma}^{\nu\dot{\alpha}\beta} + \sigma^{\nu}{}_{\alpha\dot{\alpha}}\tilde{\sigma}^{\mu\dot{\alpha}\beta} = -2g^{\mu\nu}(x)\mathbf{1}_{\alpha}{}^{\beta}$$

$$\mathcal{R}_{\alpha\beta\gamma\delta} = R_{\mu\nu\rho\sigma}\sigma^{\mu\nu}_{\alpha\beta}\sigma^{\rho\sigma}_{\gamma\delta}$$

Tangent space:

$$\sigma^{a}_{\alpha\dot{\alpha}}\tilde{\sigma}^{b\dot{\alpha}\beta} + \sigma^{b}{}_{\alpha\dot{\alpha}}\tilde{\sigma}^{a\dot{\alpha}\beta} = -2\eta^{ab}\mathbf{1}_{\alpha}{}^{\beta}$$

$$\mathcal{R}_{\alpha\beta\gamma\delta} = R_{abcd}\sigma^{ab}_{\alpha\beta}\sigma^{cd}_{\gamma\delta}$$

Rienmann tensor decomposed into Irreps

$$R_{\mu\nu\rho\sigma} = \underbrace{g_{\mu[\rho}R_{\sigma]\nu} - g_{\nu[\rho}R_{\sigma]\mu} - \frac{1}{2}g_{\mu[\rho}g_{\sigma]\nu}R}_{(1,1)} + \underbrace{C_{\mathrm{L}\mu\nu\rho\sigma}}_{(2,0)} + \underbrace{C_{\mathrm{R}\mu\nu\rho\sigma}}_{(0,2)} + \underbrace{\frac{1}{6}g_{\mu[\rho}g_{\sigma]\nu}R}_{(0,0)}.$$

Similar procedure to remove EOM and IBP, obtain GREFT operators

[Ruhdorfer, Serra, Weiler, 2020]

$$S = \int d^4x\sqrt{-g}\Big[-\frac{M_{\rm pl}^2}{2}R + \frac{c_1}{\Lambda^2}\mathcal{I} + \frac{c_2}{\Lambda^2}\tilde{\mathcal{I}} + \frac{d_1}{\Lambda^4}\mathcal{C}^2 + \frac{d_2}{\Lambda^4}\mathcal{C}\tilde{\mathcal{C}} + \frac{d_3}{\Lambda^4}\tilde{\mathcal{C}}^2$$

$$+ \frac{e_1}{\Lambda^6}\mathcal{I}\mathcal{C} + \frac{e_2}{\Lambda^6}\tilde{\mathcal{I}}\mathcal{C} + \frac{e_3}{\Lambda^6}\mathcal{I}\tilde{\mathcal{C}} + \frac{e_4}{\Lambda^6}\tilde{\mathcal{I}}\tilde{\mathcal{C}} + \frac{e_5}{\Lambda^6}\mathcal{F}\mathcal{C} + \frac{e_6}{\Lambda^6}\mathcal{F}\tilde{\mathcal{C}} + \frac{e_7}{\Lambda^6}\tilde{\mathcal{F}}\tilde{\mathcal{C}} + \dots\Big]$$

$$\mathcal{I} = C_{\mu\nu}{}^{\rho\sigma}C^{\mu\nu\alpha\beta}C_{\alpha\beta\rho\sigma},$$

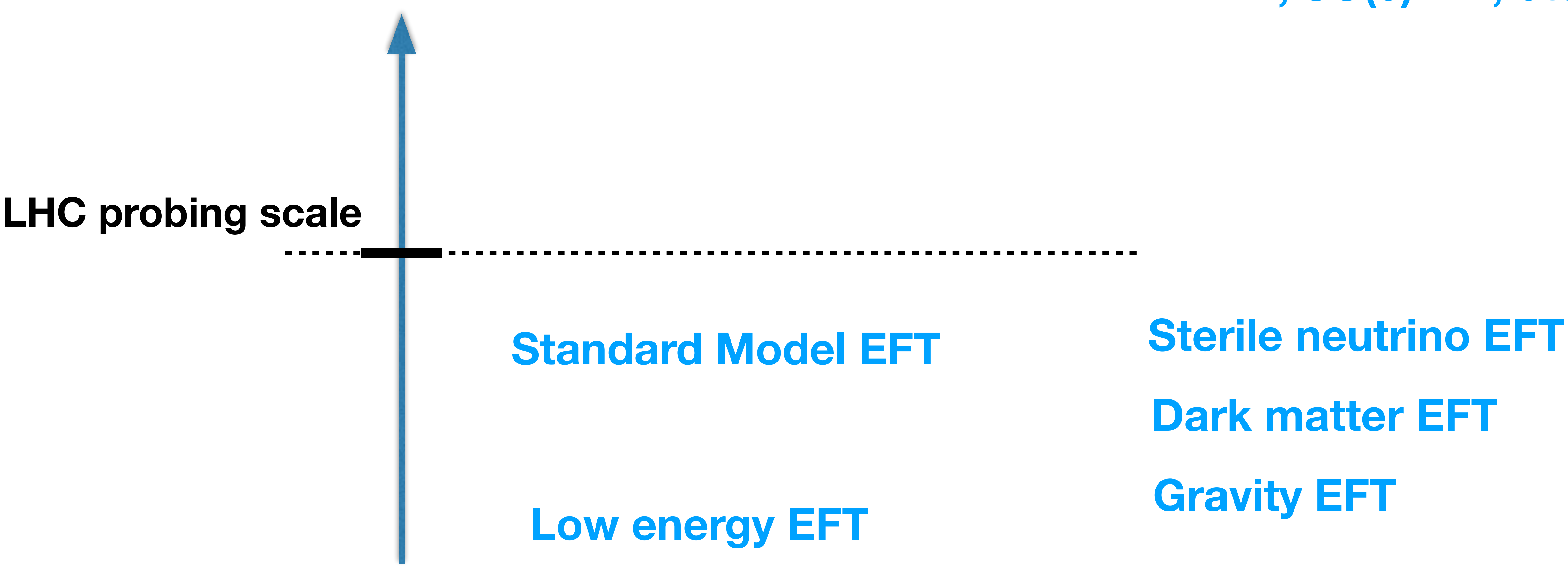
$$\mathcal{C} = C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma},$$

$$\mathcal{F} = (\nabla_\alpha C_{\mu\nu\rho\sigma})(\nabla^\alpha C^{\mu\nu\rho\sigma}),$$

Generic EFT at any Dimension

Any Lorentz inv. EFT with any gauge symmetries up to any mass dimension

2HDMEFT, SU(5)EFT, etc



EFT Ladders

Energy

UV models

many models …..

$$\mathcal{L} = \mathcal{L}^{\rm eff}_{\rm Gravity} + \underbrace{\mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm QED} + \mathcal{L}_{\rm EW}}_{\mathcal{L}_{SM}} + \mathcal{L}^{NP}_{\rm heavy}$$

TeV scale

Standard model EFT

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \frac{C_i^{(5)}}{\Lambda_{NP}} Q_i^{(5)} + \frac{C_i^{(5)}}{\Lambda_{NP}^2} Q_i^{(6)} + \cdots$$

Higgs precision, TGC, di-Higgs

EW scale

Low energy EFT

$$\mathcal{L}_{\rm LEFT} = \mathcal{L}_{\rm QED+QCD} + \frac{C_i^{(5)}}{M_W} O_i^{(5)} + \frac{C_i^{(6)}}{M_W^2} O_i^{(6)} + \cdots$$

Flavor physics, leptonic decays

$\Lambda_{\rm QCD}$

Chiral Lagrangian + Heavy B EFT

$$\mathcal{L}_\chi = \mathcal{L}_\pi(s, p, l_\mu, r_\mu) + \mathcal{L}_{\pi N}(s, p, l_\mu, r_\mu) + \cdots$$

Kaon, pion, and nucleon decays

MeV scale

Nuclear EFT (nuclear matrix elements)

$0\nu\beta\beta$ CEνNS LFV

NSI DM direct detection

eV scale

Atomic physics (NRQED)

Example: EFT for Neutrino NSI

[Du, Li, Tang, Vihonen, **Yu, 2020**]

[Du, Li, Tang, Vihonen, **Yu, 2021**]

UV models

many models …..

TeV scale

Standard model EFT

$$(\bar{l}^j_\alpha e_\beta)\epsilon_{jk}(\bar{q}^k_s u_t),\quad (\bar{l}^j_\alpha \sigma_{\mu\nu} e_\beta)\epsilon_{jk}(\bar{q}^k_s \sigma^{\mu\nu} u_t),\quad (\bar{l}^j_\alpha e_\beta)(\bar{d}_s q_{tj}),\quad (\bar{l}_\alpha \gamma^\mu \tau^I l_\beta)(\bar{q}_s \gamma_\mu \tau^I q_t)$$

$$(H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{l}_\alpha \tau^I \gamma^\mu l_\beta),\quad (H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{q}_p \tau^I \gamma^\mu q_r),\quad (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu d_r)$$

$$(H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{l}_2 \tau^I \gamma^\mu l_2),\quad (H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{l}_1 \tau^I \gamma^\mu l_1),\quad (\bar{l}_2 \gamma^\mu l_1)(\bar{l}_1 \gamma_\mu l_2),\quad (\bar{l}_1 \gamma^\mu l_2)(\bar{l}_2 \gamma_\mu l_1)$$

EW scale

Low energy EFT

$$\{[\epsilon_{L(R)}(\mu)]^{ij}_{\alpha\beta}\,(\bar{u}_i \gamma^\mu P_{L(R)} d_j)\,(\bar{\ell}_\alpha \gamma_\mu P_L \nu_\beta)\,,$$

$$[\epsilon_{S(P)}(\mu)]^{ij}_{\alpha\beta}(\bar{u}_i \mathbf{1}(\gamma_5) d_j)\,(\bar{\ell}_\alpha P_L \nu_\beta)\,,$$

$$[\epsilon_T(\mu)]^{ij}_{\alpha\beta}\,(\bar{u}_i \sigma^{\mu\nu} P_L d_j)\,(\bar{\ell}_\alpha \sigma_{\mu\nu} P_L \nu_\beta)\}$$

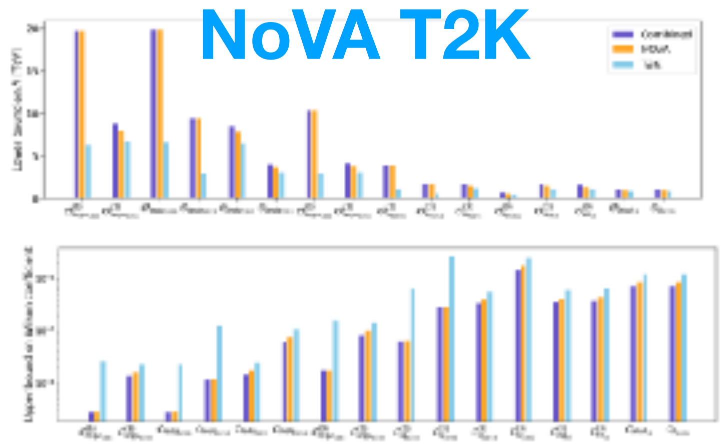
$\Lambda_{\rm QCD}$

Chiral Lagrangian

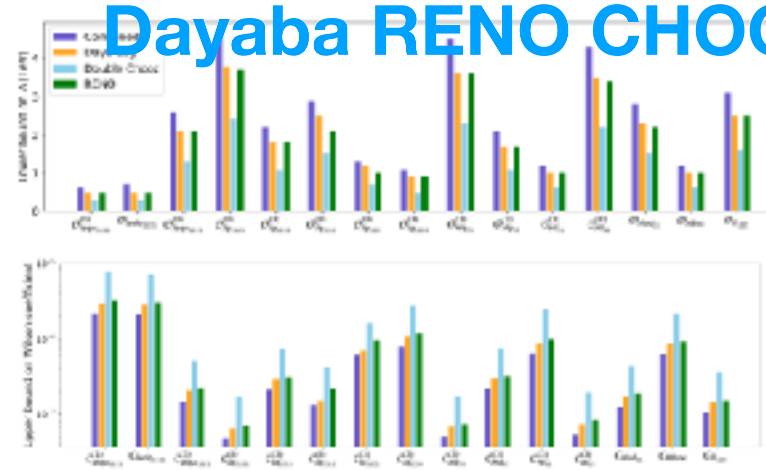
$$[\epsilon_{L(R)}]^{ij}_{\alpha\beta},\ [\epsilon_{S(P)}]^{ij}_{\alpha\beta},\ [\epsilon_T]^{ij}_{\alpha\beta}$$

MeV scale

NoVA T2K

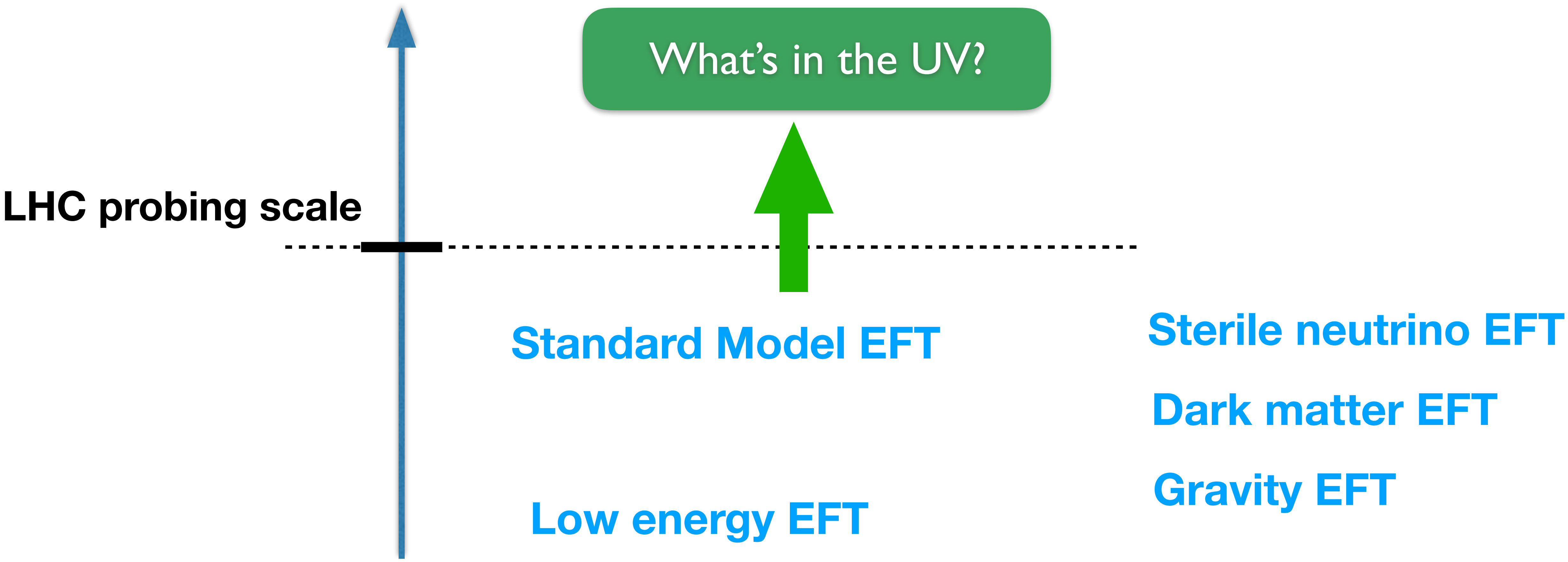


Dayaba RENO CHOOZ



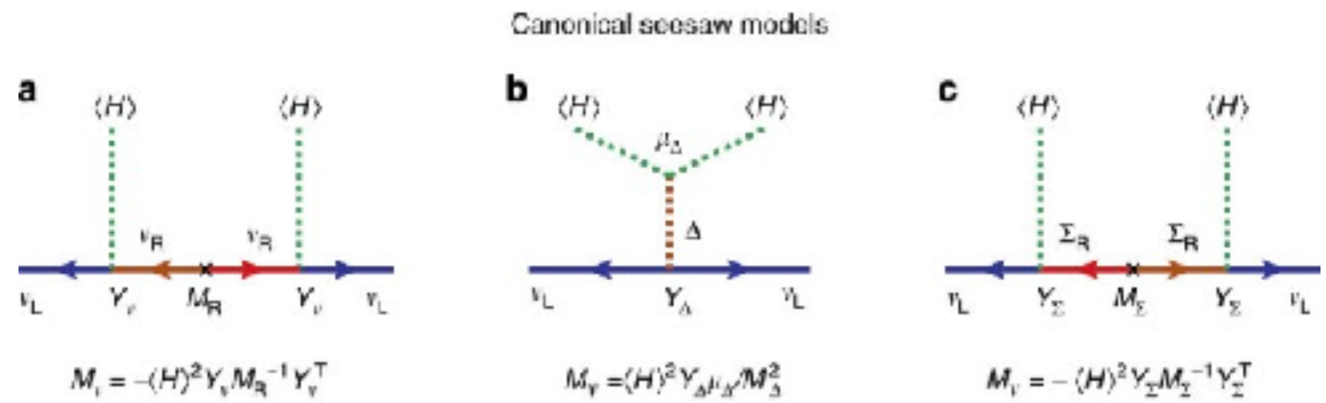
What's in the UV?

Any Lorentz inv. EFT with any gauge symmetries up to any mass dimension



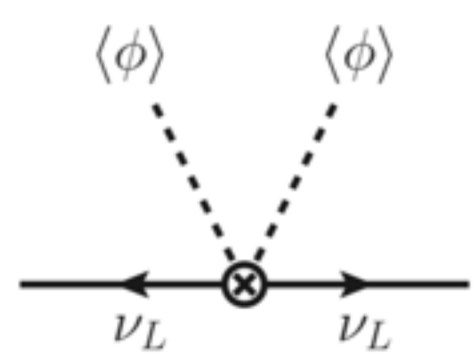
Dim-5 Weinberg Operator

Top-down approach



Choose/build a UV model

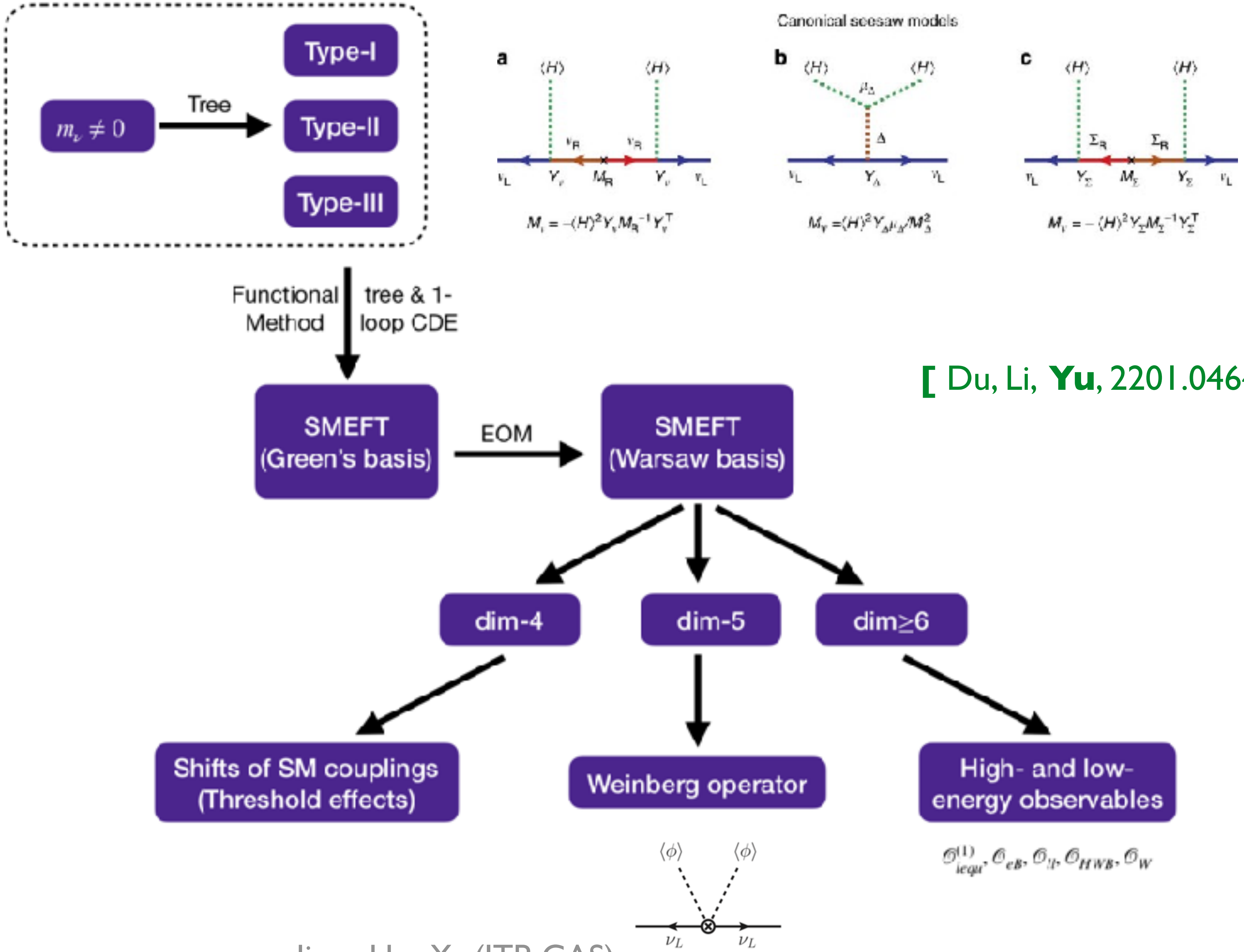
Integrate out heavy particles



Weinberg Operator LLHH

Matching and Running

Systematically done at one-loop CDE in the top-down approach

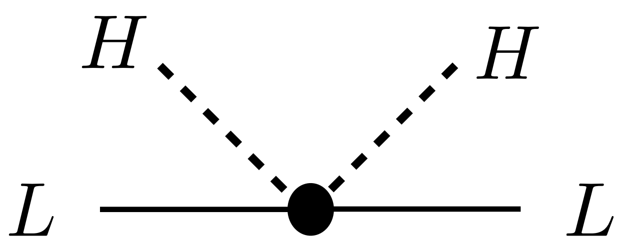


[Du, Li, **Yu**, 2201.04646]

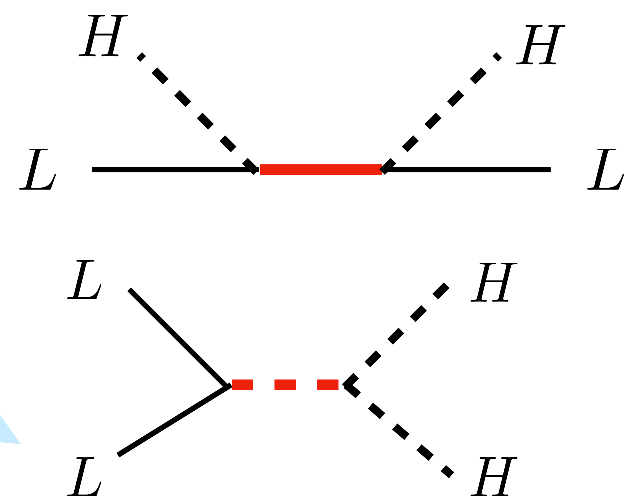
More UV for Seesaw?

Can we know whether more UV realization for canonical seesaw mechanism?

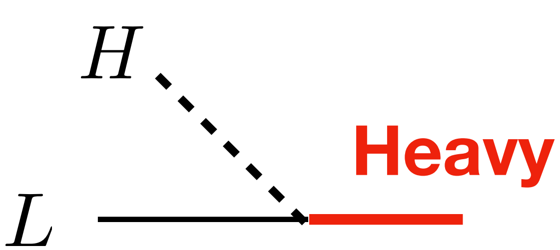
For an operator



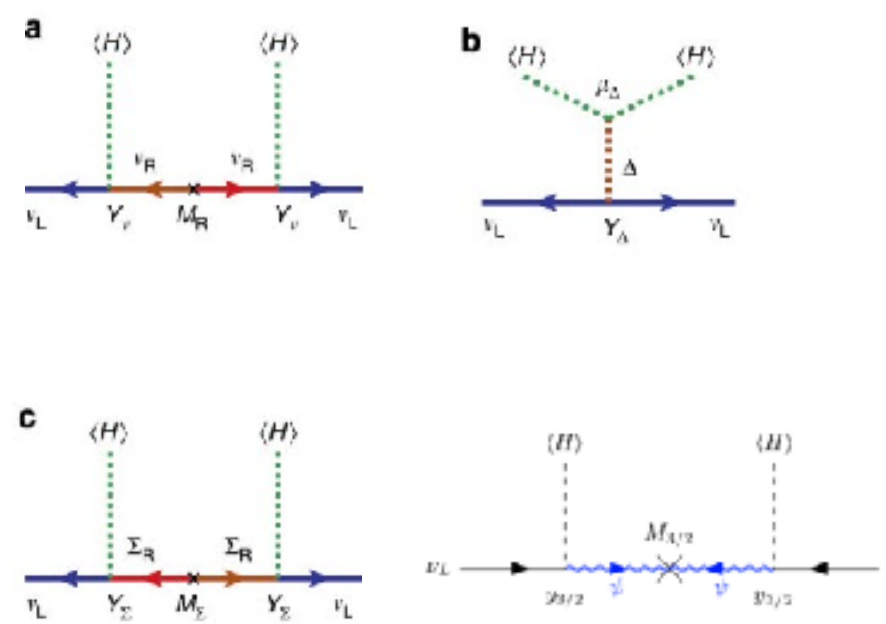
Find all topologies (exhaust all spin)



List all possible BSM Lag

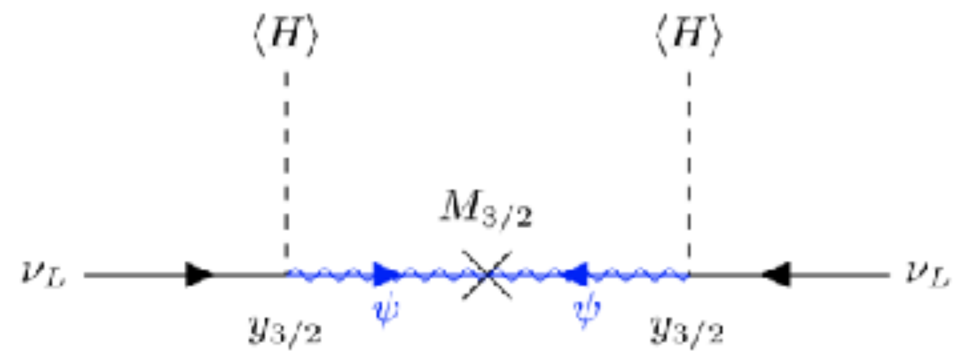


Allowed by SM Gauge symmetry



More UV for Seesaw?

Can we know whether more UV realization for canonical seesaw mechanism?



spin-1/2, 3/2, 5/2, …?

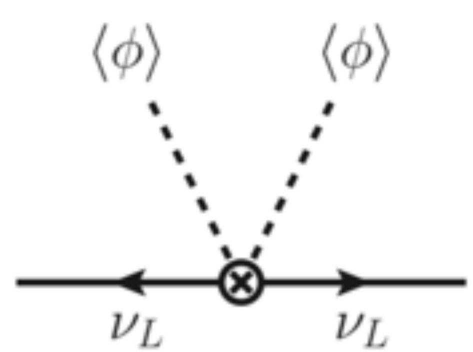
Type-3/2 Seesaw Mechanism

[Demir, Karahan, Sargm, 2021]

Durmuş Demir,[1] Canan Karahan,[2,*] and Ozan Sargın[3]

?

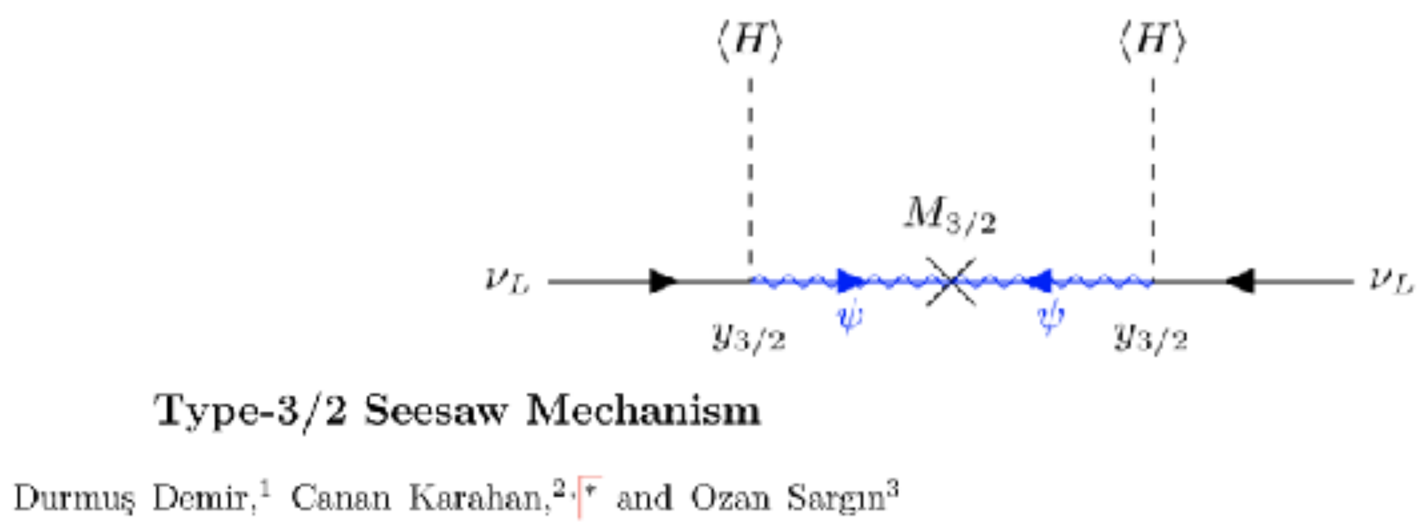
LLHH



Need matching to cross check!

Bottom-Up Approach?!

Can we know **all possible** UV realization for canonical seesaw mechanism?

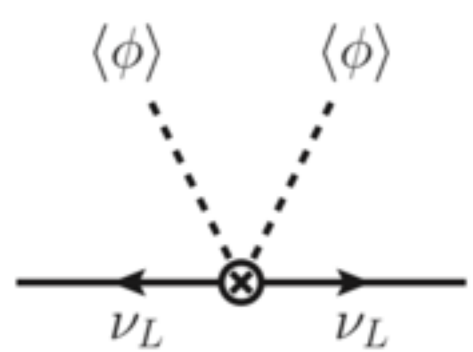


?

Start with symmetry for operators (not symmetry for BSM Lag)

Determine all possible UV

LLHH

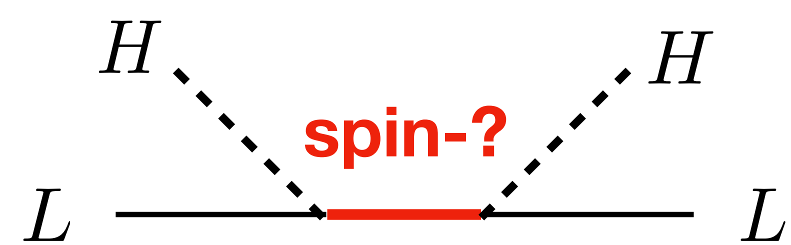


No need to know BSM Lagrangian!

Amplitude J-Basis

EFT operator = Contact amplitude = Group invariant + little group scaling

$$\langle p_L, h_L; p_H, h_H | p'_L, h'_L; p'_H, h'_H \rangle$$



$$\langle p_L, h_L; p_H, h_H | P, J, J_z \rangle \qquad \langle P, J, J_z | p'_L, h'_L; p'_H, h'_H \rangle$$

$$P_\mu \sigma^\mu_{\alpha\dot\alpha} = |\chi\rangle^I_\alpha [\chi|_{\dot\alpha I}$$

spin J	$\mathcal{C}^J(\psi_1, \phi_3)$	$\mathcal{C}^J(\psi_2, \phi_4)$
$J = 1/2$	$\langle 1\chi \rangle$	$\langle \chi 2 \rangle$
$J = 3/2$	$-\langle 1\chi \rangle [\chi \| p_3 \| \chi\rangle$	$-\langle \chi 2 \rangle [\chi \| p_4 \| \chi\rangle$
$J = 5/2$	$-\langle 1\chi \rangle [\chi \| p_3 \| \chi\rangle^2$	$-\langle \chi 2 \rangle [\chi \| p_4 \| \chi\rangle^2$

$$\langle p_L, h_L; p_H, h_H | p'_L, h'_L; p'_H, h'_H \rangle = \sum_{J, J_z} \langle p_L, h_L; p_H, h_H | P, J, J_z \rangle \langle P, J, J_z | p'_L, h'_L; p'_H, h'_H \rangle = \sum_J \mathcal{O}^{\mathrm{J}}$$

J-basis operator corresponds to the UV resonances!!!

$J = 1/2$	$\langle 12 \rangle$	dim-5
$J = 3/2$	$3 s_{34} \langle 12 \rangle + 2[34]\langle 13 \rangle \langle 24 \rangle$	dim-7
$J = 5/2$	$10 s_{34}^2 \langle 12 \rangle + 3[34] s_{24} \langle 13 \rangle \langle 24 \rangle + 12[34] s_{34} \langle 13 \rangle \langle 24 \rangle$	dim-9

Pauli-Lubanski Casimir

The above decomposition needs to assume j and then obtain UV resonances

Propose a bottom-up way in which j is obtained rather than assumed

SO(3) tensor rep

$$\mathbf{J}^2|J,M\rangle = J(J+1)|J,M\rangle$$

$$\mathbf{L}^2\langle\theta,\phi|l,m\rangle = -\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}+\frac{1}{\sin^2\theta}\frac{\partial}{\partial\phi}\right]Y_l^m(\theta,\phi)$$
$$= l(l+1)Y_l^m(\theta,\phi)$$

Poincare spinor rep

$$\mathbf{W}^2|P,J,M\rangle = -P^2J(J+1)|P,J,M\rangle$$

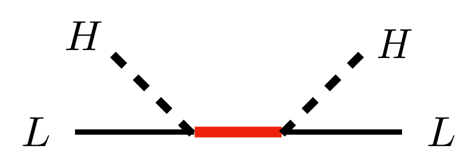
$$\mathbf{W}^2 = \frac{s}{8}\sum_{i,j=1}^{N}\Big(\langle i,\partial_j\rangle\langle j,\partial_i\rangle + [i,\partial_j][j,\partial_i]\Big) - \frac{1}{4}\sum_{i,j,k,l}[i,j]\langle j,\partial_k\rangle\langle k,l\rangle[l,\partial_i]$$

$$\mathbf{W}^2\langle p_L,h_L;p_H,h_H|P,J,J_z\rangle = -P^2J(J+1)\langle p_L,h_L;p_H,h_H|P,J,J_z\rangle$$

$$\mathbf{W}^2\ \langle p_L,h_L;p_H,h_H|p'_L,h'_L;p'_H,h'_H\rangle = -s\sum_J J(J+1)\mathcal{O}^{\mathrm{J}}$$

$L_1L_2H_3H_4$ $\quad \begin{array}{|c|}\hline 1\\ \hline 2\\ \hline\end{array}$ $\quad \mathcal{B}^y = \langle 12\rangle$ $\quad W^2_{\{1,3\}}\mathcal{B}^y = -\frac{3}{4}s_{13}\langle 12\rangle$

Young Tableau



Can only spin-1/2!

Canonical Seesaw Mechanism

$\begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline \end{array}$ $\mathcal{B}^y = \langle 12 \rangle$

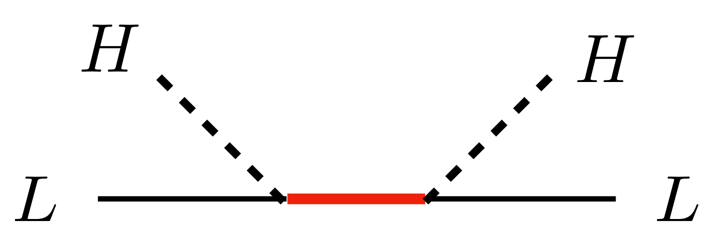
$\begin{array}{|c|c|}\hline i & j \\ \hline k & l \\ \hline \end{array}$ $\begin{array}{|c|c|}\hline i & k \\ \hline j & l \\ \hline \end{array}$ $\mathcal{B}_1^R = \epsilon^{ik}\epsilon^{jl}$, $\mathcal{B}_2^R = \epsilon^{ij}\epsilon^{kl}$

$$\mathbf{W}^2\mathcal{B}^J = -sJ(J+1)\mathcal{B}^J \quad \mathbf{C}^2\mathcal{B}^R = r(r+1)\mathcal{B}^R$$

$$\Rightarrow \mathcal{B}^{\mathbf{R}} = \begin{cases} \epsilon^{ik}\epsilon^{jl} & \mathbf{R}=1 \\ \epsilon^{ik}\epsilon^{jl} - 2\epsilon^{ij}\epsilon^{kl} & \mathbf{R}=3 \end{cases}$$

$LH \to LH$ channel

$$W^2_{\{1,3\}}\mathcal{B}^y = -\frac{3}{4}s_{13}\langle 12 \rangle$$

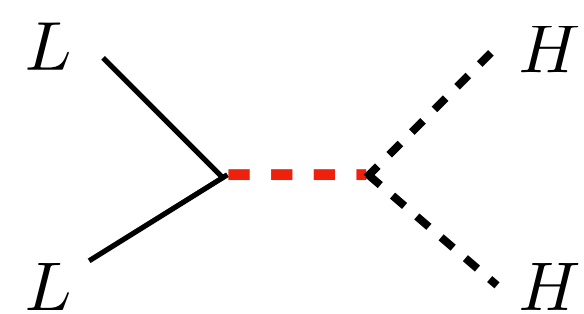


$J = \frac{1}{2}$

Type-I and III: SU(2) **single** and **triplet**

$LL \to HH$ channel

$$W^2_{\{1,2\}}\mathcal{B}^y = 0$$



$J = 0$

Type-II: SU(2) **triplet,** or singlet (excluded by repeated field)

j-basis	Model
$\mathcal{O}^{(1/2,1)}_{HL\to HL} = \mathcal{O}^S + \mathcal{O}^A$	type I
$\mathcal{O}^{(1/2,3)}_{HL\to HL} = \mathcal{O}^S - 3\mathcal{O}^A$	type III

j-basis	Model
$\mathcal{O}^{(0,1)}_{HH\to LL} = \mathcal{O}^A$	N/A
$\mathcal{O}^{(0,3)}_{HH\to LL} = \mathcal{O}^S$	type II

$$\mathcal{O}^S = (HL)(HL), \quad \mathcal{O}^A = (HH)(LL)$$

Spin-3/2 UV Resonance?

How to obtain the spin-3/2 UV resonance?

$LLHHD^2$

$$\mathcal{B}^{y}_{\psi^2\phi^2 D^2} = \begin{pmatrix} s_{34}\langle 12\rangle \\ [34]\langle 13\rangle\langle 24\rangle \end{pmatrix}$$

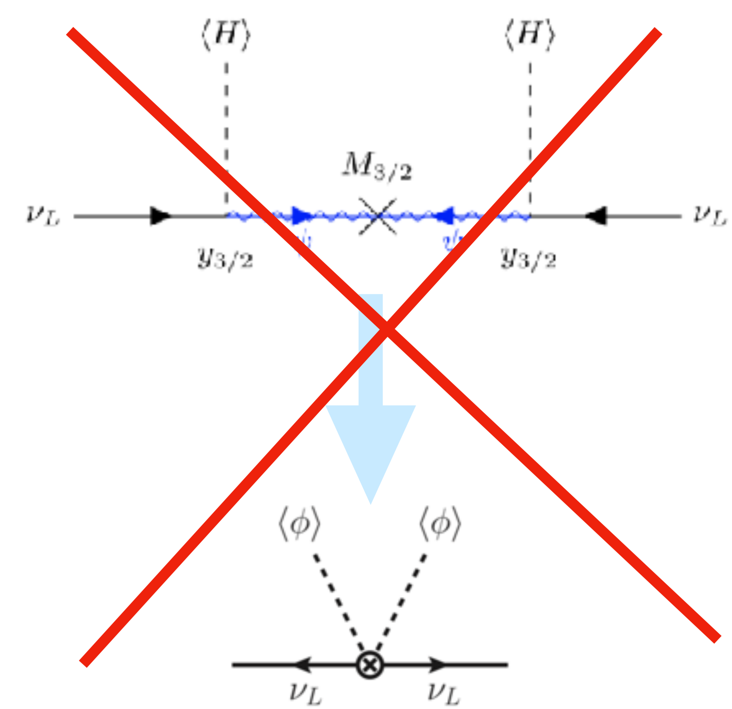
$$\mathcal{B}^{m}_{LLHH} = \begin{pmatrix} \epsilon^{ik}\epsilon^{jl} \\ \epsilon^{ij}\epsilon^{kl} \end{pmatrix}$$

$$\mathbf{W}^2\mathcal{B}^{\mathrm{y}} = -s\mathcal{W}.\mathcal{B}^{\mathrm{y}} \xrightarrow[\mathcal{K}.\mathcal{W}.\mathcal{K}^{-1} = \mathrm{diag}\{J(J+1)\}]{\mathcal{B}^{\mathrm{j}} = \mathcal{K}.\mathcal{B}^{\mathrm{y}}} \mathbf{W}^2_{\mathrm{initial/final}}\mathcal{B}^J = -sJ(J+1)\mathcal{B}^J$$

$$W^2_{\{1,3\}}\mathcal{B}^y = s_{24}\begin{pmatrix} -\frac{15}{4} & 2 \\ 0 & -\frac{3}{4} \end{pmatrix}\mathcal{B}^y$$

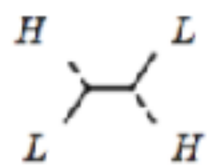
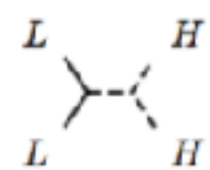
$$\Rightarrow \mathcal{B}^j = \begin{cases} 3s_{34}\langle 12\rangle + 2[34]\langle 13\rangle\langle 24\rangle & J = \frac{3}{2} \\ \langle 13\rangle\langle 24\rangle & J = \frac{1}{2} \end{cases}$$

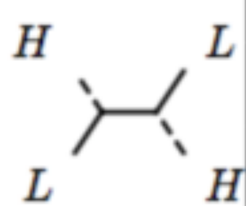
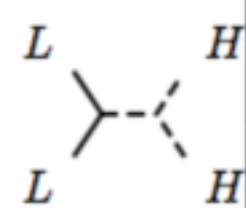
$$C_{21,3}\mathcal{B}^m = \begin{pmatrix} 0 & 0 \\ -1 & 2 \end{pmatrix}\mathcal{B}^m \quad \Rightarrow \mathcal{B}^{\mathbf{R}} = \begin{cases} \epsilon^{ik}\epsilon^{jl} & \mathbf{R} = 1 \\ \epsilon^{ik}\epsilon^{jl} - 2\epsilon^{ij}\epsilon^{kl} & \mathbf{R} = 3 \end{cases}$$



J-basis operator can be obtained from Y-basis!

From LLHH to LLHHDD

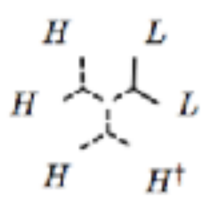
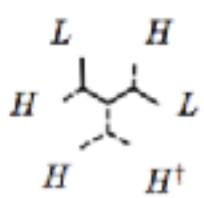
Topology	j-basis	Quantum numbers $\{J, \mathbf{R}, Y\}$	Model
	$\mathcal{B}^{J=1/2,\mathbf{R}=1}_{\{13\}} = \mathcal{B}^p_S + \mathcal{B}^p_A.$	$\{\frac{1}{2}, 1, 0\}$	Type I
	$\mathcal{B}^{J=1/2,\mathbf{R}=3}_{\{13\}} = -\mathcal{B}^p_S + 3\mathcal{B}^p_A,$	$\{\frac{1}{2}, 3, 0\}$	Type III
	$\mathcal{B}^{J=0,\mathbf{R}=3}_{\{12\}} = -2\mathcal{B}^p_S,$	$\{0, 3, -1\}$	Type II
	$\mathcal{B}^{J=0,\mathbf{R}=1}_{\{12\}} = 2\mathcal{B}^p_A.$	$\{0, 1, -1\}$	N/A

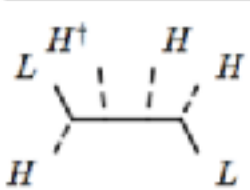
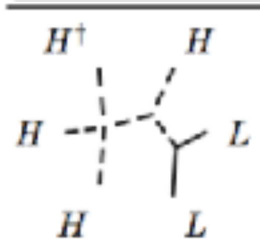
Topology	j-basis	Quantum numbers $\{J, \mathbf{R}, Y\}$
	$\mathcal{B}_{\{13\},1} = 3\mathcal{B}^p_1 + 6\mathcal{B}^p_2 - 9\mathcal{B}^p_3 - 2\mathcal{B}^p_4,$	$\{\frac{3}{2}, 3, 0\}$
	$\mathcal{B}_{\{13\},2} = 3\mathcal{B}^p_2 - \mathcal{B}^p_4,$	$\{\frac{1}{2}, 3, 0\}$
	$\mathcal{B}_{\{13\},3} = -3\mathcal{B}^p_1 + 2\mathcal{B}^p_2 - 3\mathcal{B}^p_3 + 2\mathcal{B}^p_4,$	$\{\frac{3}{2}, 1, 0\}$
	$\mathcal{B}_{\{13\},4} = \mathcal{B}^p_2 + \mathcal{B}^p_4.$	$\{\frac{1}{2}, 1, 0\}$
	$\mathcal{B}_{\{12\},1} = 2\mathcal{B}^p_1 - 4\mathcal{B}^p_4,$	$\{1, 3, -1\}$
	$\mathcal{B}_{\{12\}} = -2\mathcal{B}^p_1,$	$\{0, 3, -1\}$
	$\mathcal{B}_{\{12\}} = 4\mathcal{B}^p_2 - 2\mathcal{B}^p_3,$	$\{1, 1, -1\}$
	$\mathcal{B}_{\{12\}} = 2\mathcal{B}^p_3.$	$\{0, 1, -1\}$ N/A

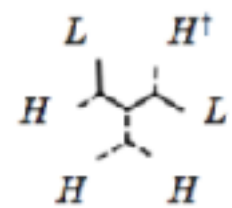
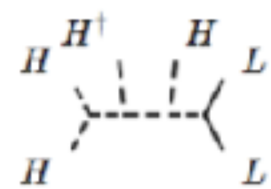
Genuine dim-7 Seesaw

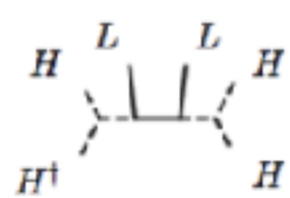
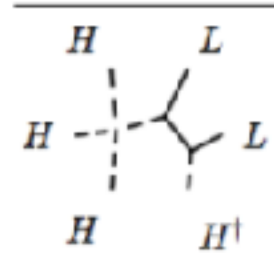
Tree-level seesaw at dim-7: among 19 topologies, one genuine dim-7 seesaw

LLHHHH

Topology	j-basis
	$\mathcal{B}_{\{12\|34\|56\},1} = 2\mathcal{B}^p_1 - 4\mathcal{B}^p_2,$
	$\mathcal{B}_{\{12\|34\|56\},2} = 2\mathcal{B}^p_2 + 4\mathcal{B}^p_3,$
	$\mathcal{B}_{\{12\|34\|56\},3} = 12\mathcal{B}^p_4,$
	$\mathcal{B}_{\{12\|34\|56\},4} = 4\mathcal{B}^p_1 + 4\mathcal{B}^p_2,$
	$\mathcal{B}_{\{12\|34\|56\},5} = 2\mathcal{B}^p_4 + 4\mathcal{B}^p_5.$
	$\mathcal{B}_{\{13\|24\|56\},1} = -\mathcal{B}^p_2 - 2\mathcal{B}^p_3 + 3\mathcal{B}^p_4,$
	$\mathcal{B}_{\{13\|24\|56\},2} = -\mathcal{B}^p_1 + 3\mathcal{B}^p_2 + 2\mathcal{B}^p_3 + 3\mathcal{B}^p_4,$
	$\mathcal{B}_{\{13\|24\|56\},3} = -\mathcal{B}^p_1 + \mathcal{B}^p_2 - 2\mathcal{B}^p_3 - 3\mathcal{B}^p_4,$
	$\mathcal{B}_{\{13\|24\|56\},4} = -\mathcal{B}^p_1 - \mathcal{B}^p_2 + 3\mathcal{B}^p_4 + 6\mathcal{B}^p_5,$
	$\mathcal{B}_{\{13\|24\|56\},5} = \mathcal{B}^p_1 + \mathcal{B}^p_2 + \mathcal{B}^p_4 + 2\mathcal{B}^p_5.$

	$\mathcal{B}_{\{13\|135\|24\},1} = \mathcal{B}^p_1 - 2\mathcal{B}^p_3 - 6\mathcal{B}^p_5,$
	$\mathcal{B}_{\{13\|135\|24\},2} = -\mathcal{B}^p_1 - 3\mathcal{B}^p_2 - 4\mathcal{B}^p_3 + 9\mathcal{B}^p_4 + 6\mathcal{B}^p_5,$
	$\mathcal{B}_{\{13\|135\|24\},3} = \mathcal{B}^p_1 - \mathcal{B}^p_2 + 2\mathcal{B}^p_3 + 3\mathcal{B}^p_4,$
	$\mathcal{B}_{\{13\|135\|24\},4} = \mathcal{B}^p_1 - 3\mathcal{B}^p_2 - 2\mathcal{B}^p_3 - 3\mathcal{B}^p_4,$
	$\mathcal{B}_{\{13\|135\|24\},5} = \mathcal{B}^p_1 + \mathcal{B}^p_2 + \mathcal{B}^p_4 + 2\mathcal{B}^p_5.$
	$\mathcal{B}_{\{12\|123\},1} = -\mathcal{B}^p_1 + 4\mathcal{B}^p_2 + 4\mathcal{B}^p_3,$
	$\mathcal{B}_{\{12\|123\},2,3} = \begin{pmatrix} -2\mathcal{B}^p_1 - 2\mathcal{B}^p_3 \\ 4\mathcal{B}^p_1 + 2\mathcal{B}^p_2 + 2\mathcal{B}^p_3 \end{pmatrix},$
	$\mathcal{B}_{\{12\|123\},4,5} = \begin{pmatrix} -2\mathcal{B}^p_4 + 2\mathcal{B}^p_5 \\ -4\mathcal{B}^p_4 - 2\mathcal{B}^p_5 \end{pmatrix}.$

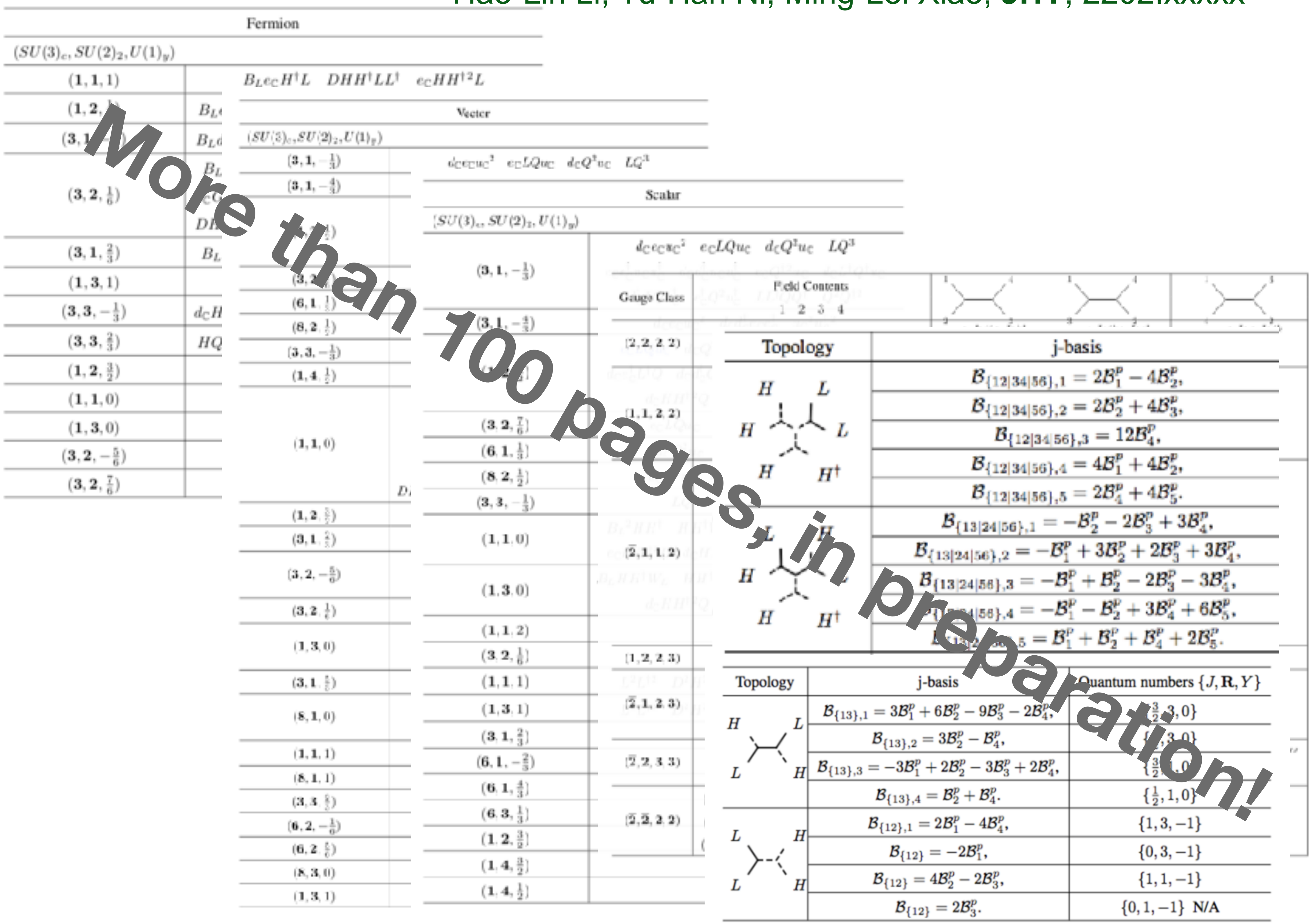
	$\mathcal{B}_{\{16\|23\|45\},1} = 2\mathcal{B}^p_1 - 2\mathcal{B}^p_2 - 2\mathcal{B}^p_3 + 6\mathcal{B}^p_4 + 6\mathcal{B}^p_5,$
	$\mathcal{B}_{\{16\|23\|45\},2} = -2\mathcal{B}^p_1 - \mathcal{B}^p_2 - \mathcal{B}^p_3 + 3\mathcal{B}^p_4 + 3\mathcal{B}^p_5,$
	$\mathcal{B}_{\{16\|23\|45\},3} = 3\mathcal{B}^p_2 + 3\mathcal{B}^p_3 + 3\mathcal{B}^p_4 + 3\mathcal{B}^p_5,$
	$\mathcal{B}_{\{16\|23\|45\},4} = \mathcal{B}^p_2 - \mathcal{B}^p_3 - 3\mathcal{B}^p_4 + 3\mathcal{B}^p_5,$
	$\mathcal{B}_{\{16\|23\|45\},5} = \mathcal{B}^p_2 - \mathcal{B}^p_3 + \mathcal{B}^p_4 - \mathcal{B}^p_5.$
	$\mathcal{B}_{\{12\|125\|34\},1} = \mathcal{B}^p_1 + 4\mathcal{B}^p_2,$
	$\mathcal{B}_{\{12\|125\|34\},2} = -8\mathcal{B}^p_1 + 4\mathcal{B}^p_2,$
	$\mathcal{B}_{\{12\|125\|34\},3} = -12\mathcal{B}^p_4,$
	$\mathcal{B}_{\{12\|125\|34\},4} = -2\mathcal{B}^p_2 - 4\mathcal{B}^p_3,$
	$\mathcal{B}_{\{12\|125\|34\},5} = -2\mathcal{B}^p_4 - 4\mathcal{B}^p_5.$

	$\mathcal{B}_{\{34\|134\|56\},1} = -\mathcal{B}^p_1 + 2\mathcal{B}^p_2 + 6\mathcal{B}^p_4,$
	$\mathcal{B}_{\{34\|134\|56\},2} = 4\mathcal{B}^p_1 - 8\mathcal{B}^p_2 + 12\mathcal{B}^p_4,$
	$\mathcal{B}_{\{34\|134\|56\},3} = 2\mathcal{B}^p_2 + 4\mathcal{B}^p_3,$
	$\mathcal{B}_{\{34\|134\|56\},4} = -4\mathcal{B}^p_1 - 4\mathcal{B}^p_2,$
	$\mathcal{B}_{\{34\|134\|56\},5} = 2\mathcal{B}^p_4 + 4\mathcal{B}^p_5.$
	$\mathcal{B}_{\{16\|126\},1} = 6\mathcal{B}^p_1,$
	$\mathcal{B}_{\{16\|126\},2,3} = \begin{pmatrix} \mathcal{B}^p_2 - \mathcal{B}^p_3 - 3\mathcal{B}^p_4 + 3\mathcal{B}^p_5 \\ -2\mathcal{B}^p_2 - \mathcal{B}^p_3 + 6\mathcal{B}^p_4 + 3\mathcal{B}^p_5 \end{pmatrix},$
	$\mathcal{B}_{\{16\|126\},4,5} = \begin{pmatrix} -\mathcal{B}^p_2 - 2\mathcal{B}^p_3 - \mathcal{B}^p_4 - 2\mathcal{B}^p_5 \\ -2\mathcal{B}^p_2 - \mathcal{B}^p_3 - 2\mathcal{B}^p_4 - \mathcal{B}^p_5 \end{pmatrix}.$

[Babu, Nandi, Tavartkiladze, 2009]

Complete Dim-6/7/8 UVs

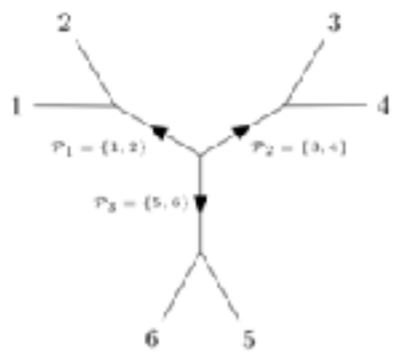
Hao-Lin Li, Yu-Han Ni, Ming-Lei Xiao, **JHY**, 2202.xxxxx

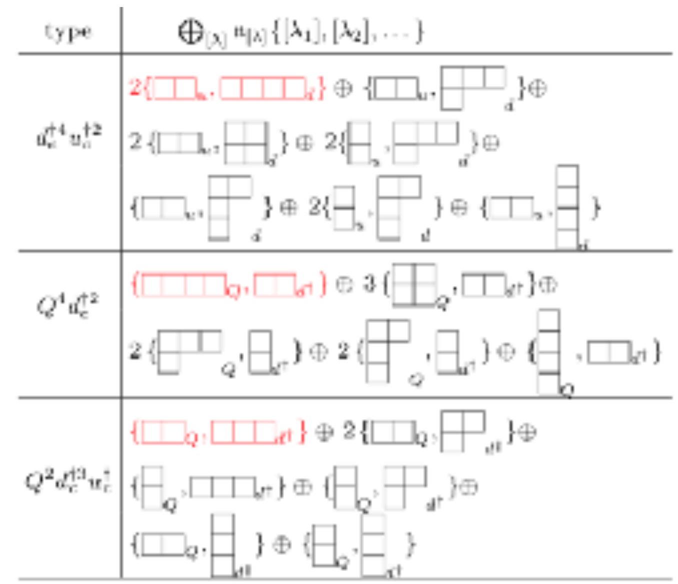


Topology	j-basis	Quantum numbers $\{J, \mathbf{R}, Y\}$
	$\mathcal{B}_{\{13\},1} = 3\mathcal{B}^p_1 + 6\mathcal{B}^p_2 - 9\mathcal{B}^p_3 - 2\mathcal{B}^p_4,$	$\{\frac{3}{2},3,0\}$
	$\mathcal{B}_{\{13\},2} = 3\mathcal{B}^p_2 - \mathcal{B}^p_4,$	$\{\frac{1}{2},3,0\}$
	$\mathcal{B}_{\{13\},3} = -3\mathcal{B}^p_1 + 2\mathcal{B}^p_2 - 3\mathcal{B}^p_3 + 2\mathcal{B}^p_4,$	$\{\frac{3}{2},1,0\}$
	$\mathcal{B}_{\{13\},4} = \mathcal{B}^p_2 + \mathcal{B}^p_4.$	$\{\frac{1}{2},1,0\}$
	$\mathcal{B}_{\{12\},1} = 2\mathcal{B}^p_1 - 4\mathcal{B}^p_4,$	$\{1,3,-1\}$
	$\mathcal{B}_{\{12\}} = -2\mathcal{B}^p_1,$	$\{0,3,-1\}$
	$\mathcal{B}_{\{12\}} = 4\mathcal{B}^p_2 - 2\mathcal{B}^p_3,$	$\{1,1,-1\}$
	$\mathcal{B}_{\{12\}} = 2\mathcal{B}^p_3.$	$\{0,1,-1\}$ N/A

Contains complete UVs and corresponding combinations of operators

Dim-9: n-nbar oscillation





$(\mathbf{r}_i, J_i)$	$(1,1,1)$	$(0,1,1)$	$(1,0,1)$	$(1,1,0)$	$(0,0,0)$
$(\mathbf{6},\mathbf{6},\mathbf{6})$	0	$3\mathcal{O}_1+8\mathcal{O}_2$	0	0	$\mathcal{O}_1-8\mathcal{O}_2$
$(\mathbf{6},\bar{\mathbf{3}},\bar{\mathbf{3}})$	0	$\mathcal{O}_2$	0	0	$\mathcal{O}_2$
$(\bar{\mathbf{3}},\mathbf{6},\bar{\mathbf{3}})$	$3\mathcal{O}_1+8\mathcal{O}_2$	0	$\mathcal{O}_1-8\mathcal{O}_2$	$3\mathcal{O}_1+8\mathcal{O}_2$	0
$(\bar{\mathbf{3}},\bar{\mathbf{3}},\mathbf{6})$	$3\mathcal{O}_1+8\mathcal{O}_2$	0	$3\mathcal{O}_1+8\mathcal{O}_2$	$\mathcal{O}_1-8\mathcal{O}_2$	0
$(\bar{\mathbf{3}},\bar{\mathbf{3}},\bar{\mathbf{3}})$	$-3\mathcal{O}_1+8\mathcal{O}_2$	0	$3\mathcal{O}_1+8\mathcal{O}_2$	$3\mathcal{O}_1+8\mathcal{O}_2$	0

[Babu, Mohapatra, Nasri, 2006]

$$\mathcal{O}_1^{\rm P} = \epsilon^{ace}\epsilon^{bdf}(d_{Ra}d_{Rb})(d_{Rc}d_{Rd})(u_{Re}u_{Rf}),$$
$$\mathcal{O}_2^{\rm P} = \epsilon^{acd}\epsilon^{bef}(d_{Ra}d_{Rb})(d_{Rc}u_{Re})(d_{Rd}u_{Rf}).$$

A Bigger Picture

Conformal algebra:

$$
\begin{aligned}
[D, P_\mu] &= -iP_\mu, \\
[D, K_\mu] &= iK_\mu, \\
[K_\mu, P_\nu] &= 2i(\eta_{\mu\nu} D + M_{\mu\nu}), \\
[M_{\mu\nu}, K_\rho] &= i(\eta_{\mu\rho} K_\nu - \eta_{\nu\rho} K_\mu),
\end{aligned}
$$

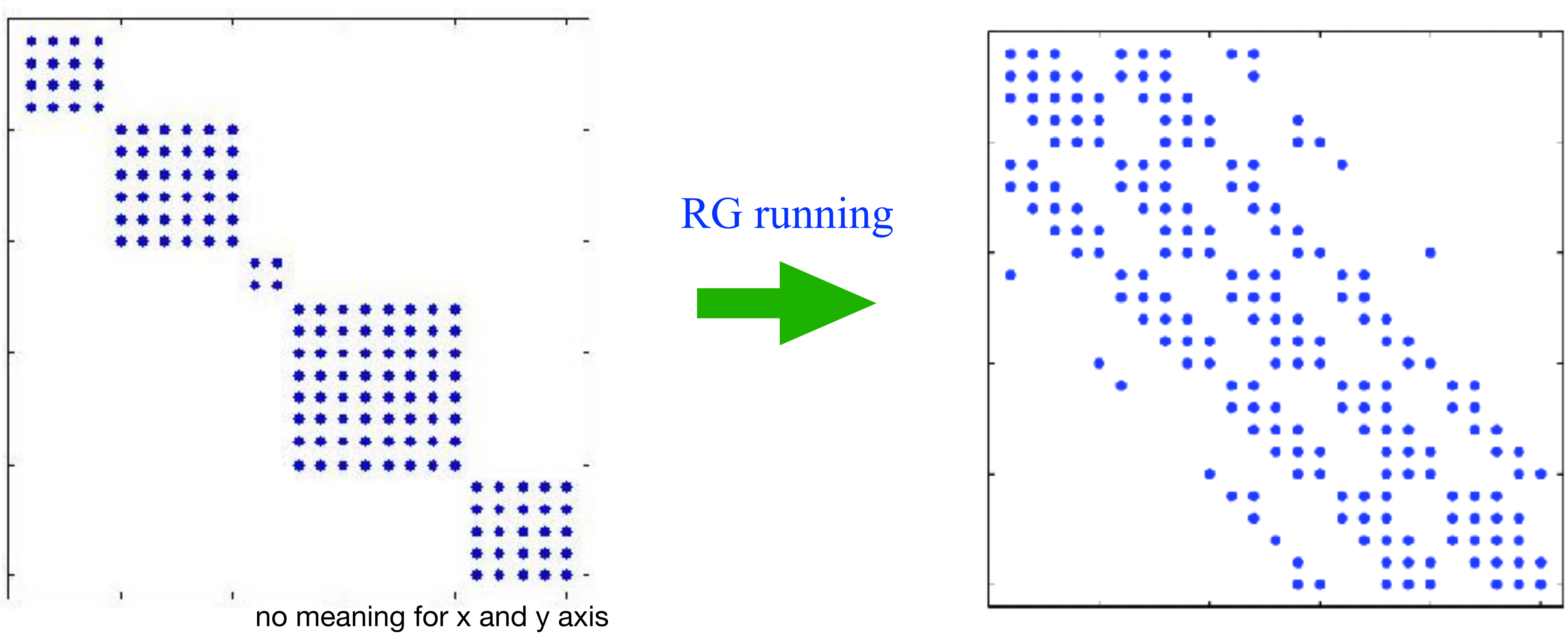
Spinor representation

$$
\begin{aligned}
P^{\alpha\dot\alpha} &= \sum_i \lambda_i^\alpha \tilde\lambda_i^{\dot\alpha} & -iD &= n + \frac{1}{2}\sum_i \left(\lambda_i^\alpha \partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha} \bar\partial_{i\dot\alpha}\right), \\
K_{\alpha\dot\alpha} &= -4\sum \partial_{i\alpha}\bar\partial_{i\dot\alpha} & -iM_{\alpha\beta} &= \sum_i \lambda_{i\alpha}\partial_{i\beta} + \lambda_{i\beta}\partial_{i\alpha}, \\
W_{\alpha\dot\alpha} &= \frac{i}{2}\left(P_{\alpha\dot\beta} \bar M^{\dot\beta}{}_{\dot\alpha} - M_\alpha{}^\beta P_{\beta\dot\alpha}\right) & -i\bar M_{\dot\alpha\dot\beta} &= \sum_i \tilde\lambda_{i\dot\alpha}\bar\partial_{i\dot\beta} + \tilde\lambda_{i\dot\beta}\bar\partial_{i\dot\alpha}.
\end{aligned}
$$

Special conformal K	Pauli-Lubanski W	Dilatation D
Amplitude-basis	**UV resonances**	**Anomalous dim**

RGE in On-shell Basis

RG running (anomalous dilatation) mix among classes of operators



Operator space

Operator space

Young tensor basis provides a preferred basis to perform RG Running!

ITP-Basis: Young Tensor

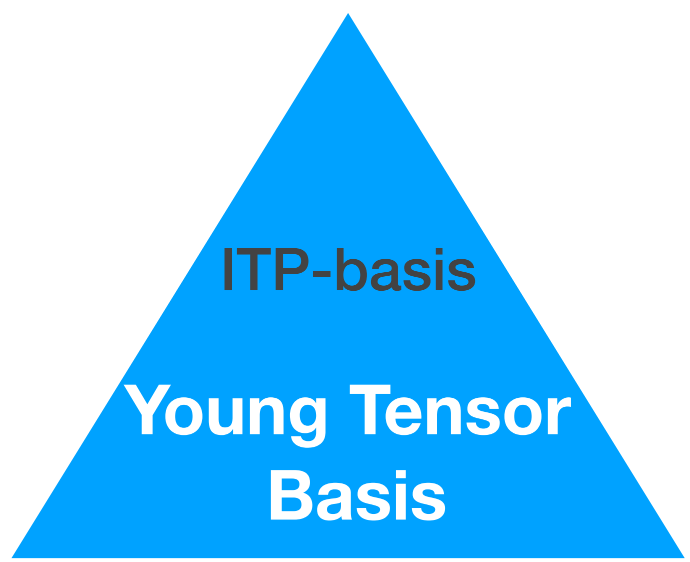
Modern view: operator as on-shell amplitude, with spin-statistics included

$$\begin{array}{|c|c|}\hline 1 & 1 \\ \hline 2 & 3 \\ \hline \end{array} \qquad \epsilon_{\alpha_1\alpha_2}\epsilon_{\alpha_1\alpha_3} \qquad \langle 12\rangle\langle 13\rangle$$

Operator amplitude correspondance

Any operator could be uniquely converted to this basis!

Eliminate Feynman rule



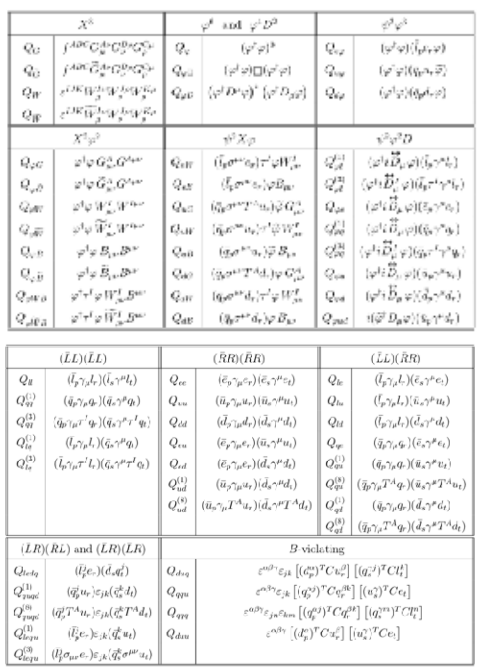
UV resonance **RGE**

Summary

Take home message 1: From Warsaw dim-6 to any-dim amplitude basis systematically

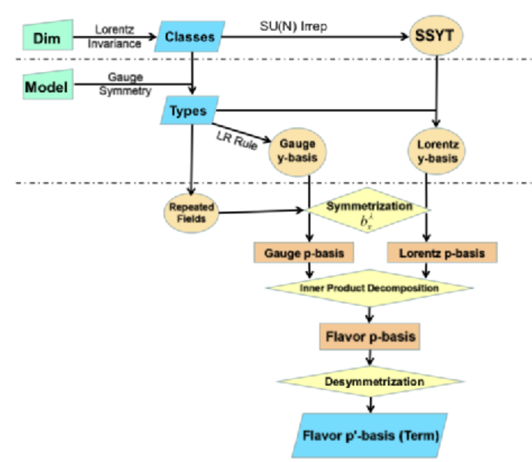
ITP-basis

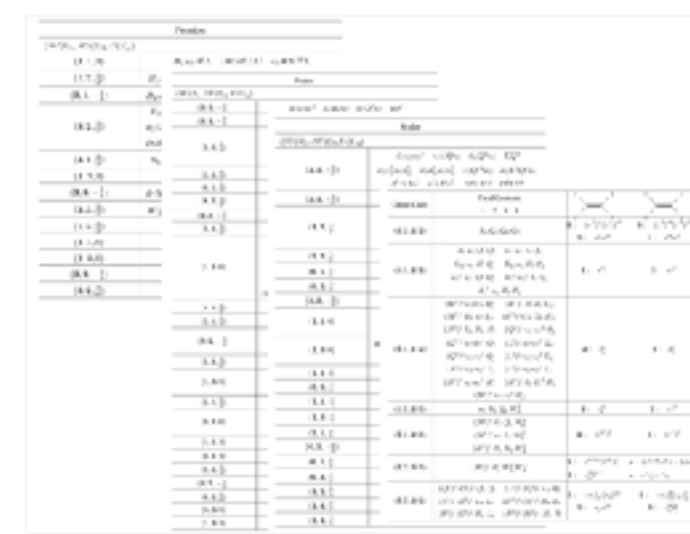
Warsaw basis → Young Tensor Basis → Complete UVs



Any operator

to any mass dimension





Take home message 2: Amplitude ITP-basis is a preferred basis to perform on-shell calculation, identify UV resonances and calculate RGE

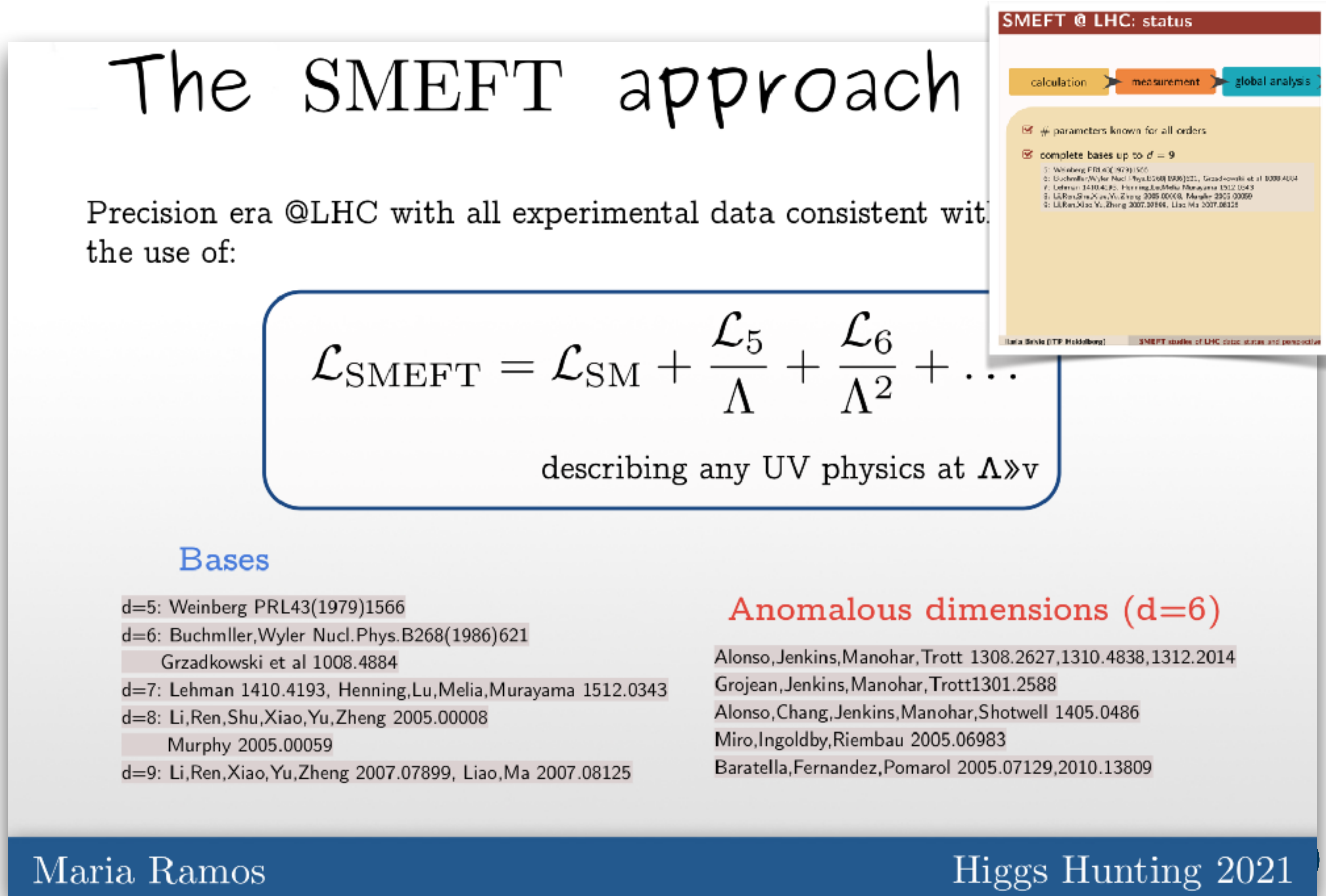
Thanks for your listening!

Backup Slides

Some slides from my talk at **All things EFT Series** online seminar

Some materials are prepared by Zhe Ren and Yu-Han Ni

Literatures on SMEFT



Notations for this Talk

$\psi_\alpha \in (1/2, 0), \quad \psi^\dagger_{\dot\alpha} \in (0, 1/2),$ $\qquad H_i \in (0,0)\,, \quad H^{\dagger i} \in (0,0)\,,$

$F_{\mathrm{L}\alpha\beta} = \frac{i}{2} F_{\mu\nu}\sigma^{\mu\nu}_{\alpha\beta} \in (1,0), \quad F_{\mathrm{R}\dot\alpha\dot\beta} = -\frac{i}{2} F_{\mu\nu}\bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta} \in (0,1).$ $\qquad X^{\mu\nu}_{L,R} = \frac{1}{2}(X^{\mu\nu} \mp i\widetilde{X}^{\mu\nu})$

$h = j_r - j_l$

Fields	$SU(2)_l \times SU(2)_r$	h	$SU(3)_C$	$SU(2)_W$	$U(1)_Y$	Flavor
$G^A_{\mathrm{L}\alpha\beta}$	$(1,0)$	-1	$\mathbf{8}$	$\mathbf{1}$	0	1
$W^I_{\mathrm{L}\alpha\beta}$	$(1,0)$	-1	$\mathbf{1}$	$\mathbf{3}$	0	1
$B_{\mathrm{L}\alpha\beta}$	$(1,0)$	-1	$\mathbf{1}$	$\mathbf{1}$	0	1
$L_{\alpha i}$	$(\frac{1}{2},0)$	$-1/2$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	n_f
$e_{\mathrm{c}\alpha}$	$(\frac{1}{2},0)$	$-1/2$	$\mathbf{1}$	$\mathbf{1}$	1	n_f
$Q_{a\alpha i}$	$(\frac{1}{2},0)$	$-1/2$	$\mathbf{3}$	$\mathbf{2}$	$1/6$	n_f
$u^a_{\mathrm{c}\alpha}$	$(\frac{1}{2},0)$	$-1/2$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$	n_f
$d^a_{\mathrm{c}\alpha}$	$(\frac{1}{2},0)$	$-1/2$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$1/3$	n_f
H_i	$(0,0)$	0	$\mathbf{1}$	$\mathbf{2}$	$1/2$	1

Hermitian conjugate $\qquad (F_{\mathrm{L}\alpha\beta})^\dagger = F_{\mathrm{R}\dot\alpha\dot\beta}$

$H^\dagger$ (and $L^\dagger, Q^\dagger$) as a 2 of $SU(2)$ $\qquad \epsilon^{ij} H^\dagger_i H_j. \quad H^\dagger_{\mathbf{2}} = \epsilon H^\dagger_{\bar{\mathbf{2}}}$ $\qquad e_R = e^\dagger_C, u_R = u^\dagger_C, d_R = d^\dagger_C$

$\tilde{H} = \epsilon H^\dagger$

Fierz and Schouten Identities

$$\Psi = \begin{pmatrix} \xi_\alpha \\ \chi^{\dagger\dot\alpha} \end{pmatrix}, \quad \bar\Psi = \Psi^\dagger\gamma^0 = \left(\chi^\alpha,\ \xi^\dagger_{\dot\alpha}\right)$$

$$u_c\sigma^\mu u_c^\dagger = \bar u\gamma^\mu u, \quad e_c L = \bar e\, l, \quad u_c^\dagger d_c^\dagger = u^T C d\,.$$

Fierz identity

$$\begin{pmatrix} \delta_{ij}\delta_{kl} \\ (\gamma^\mu)_{ij}(\gamma_\mu)_{kl} \\ \frac{1}{2}(\sigma^{\mu\nu})_{ij}(\sigma_{\mu\nu})_{kl} \\ (\gamma^\mu\gamma_5)_{ij}(\gamma_\mu\gamma_5)_{kl} \\ (\gamma_5)_{ij}(\gamma_5)_{kl} \end{pmatrix} = \begin{pmatrix} 1/4 & 1/4 & 1/4 & -1/4 & 1/4 \\ 1 & -1/2 & 0 & -1/2 & -1 \\ 3/2 & 0 & -1/2 & 0 & 3/2 \\ -1 & -1/2 & 0 & -1/2 & 1 \\ 1/4 & -1/4 & 1/4 & 1/4 & 1/4 \end{pmatrix} \begin{pmatrix} \delta_{il}\delta_{kj} \\ (\gamma^\mu)_{il}(\gamma_\mu)_{kj} \\ \frac{1}{2}(\sigma^{\mu\nu})_{il}(\sigma_{\mu\nu})_{kj} \\ (\gamma^\mu\gamma_5)_{il}(\gamma_\mu\gamma_5)_{kj} \\ (\gamma_5)_{il}(\gamma_5)_{kj} \end{pmatrix}$$

SO(3,1) trace part

Schouten identity

$$g_{\mu\nu}\sigma^\mu_{\alpha\dot\alpha}\sigma^\nu_{\beta\dot\beta} = 2\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta},$$
$$\epsilon^{\alpha\beta}\delta^\gamma_\kappa + \epsilon^{\beta\gamma}\delta^\alpha_\kappa + \epsilon^{\gamma\alpha}\delta^\beta_\kappa = 0.$$
$$\tilde\epsilon_{\dot\alpha\dot\beta}\delta^{\dot\kappa}_{\dot\gamma} + \tilde\epsilon_{\dot\beta\dot\gamma}\delta^{\dot\kappa}_{\dot\alpha} + \tilde\epsilon_{\dot\gamma\dot\alpha}\delta^{\dot\kappa}_{\dot\beta} = 0$$

$$|i\rangle\langle jk\rangle + |j\rangle\langle ki\rangle + |k\rangle\langle ij\rangle = 0\,.$$

$$\langle ri\rangle\langle jk\rangle + \langle rj\rangle\langle ki\rangle + \langle rk\rangle\langle ij\rangle = 0\,.$$

$$\begin{aligned}(\bar d l)(\bar l d) &= -\frac{1}{4}(\bar d d)(\bar l l) - \frac{1}{4}(\bar d\gamma^\mu d)(\bar l\gamma_\mu l) - \frac{1}{8}(\bar d\sigma^{\mu\nu}d)(\bar l\sigma_{\mu\nu}l) + \frac{1}{4}(\bar d\gamma^\mu\gamma_5 d)(\bar l\gamma_\mu\gamma_5 l) - \frac{1}{4}(\bar d\gamma_5 d)(\bar l\gamma_5 l) \\ &= -\frac{1}{2}(\bar d\gamma^\mu d)(\bar l\gamma_\mu l)\,, \end{aligned}$$

$$\begin{aligned}(\bar l C\bar q)(lCq) &= -\frac{1}{4}(\bar l l)(\bar q q) + \frac{1}{4}(\bar l\gamma^\mu l)(\bar q\gamma_\mu q) + \frac{1}{8}(\bar l\sigma^{\mu\nu}l)(\bar q\sigma_{\mu\nu}q) + \frac{1}{4}(\bar l\gamma^\mu\gamma_5 l)(\bar q\gamma_\mu\gamma_5 q) - \frac{1}{4}(\bar l\gamma_5 l)(\bar q\gamma_5 q) \\ &= \frac{1}{2}(\bar l\gamma^\mu l)(\bar q\gamma_\mu q)\,. \end{aligned}$$

Schur-Weyl Theorem

Three 1/2-particle with 3 flavors:

$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$$

SU(3) 1-d irrrep

SU(3) 8-d irrrep

SU(3) 10-d irrrep

S3 1-dim irrrep [111]

S3 2-dim irrrep [21]

S3 1-dim irrrep [3]

$$|1\rangle \quad |2\rangle \quad |3\rangle \quad |4\rangle \quad |5\rangle \quad |6\rangle \quad |7\rangle \quad |8\rangle$$

$$|1'\rangle \quad |2'\rangle \quad |3'\rangle \quad |4'\rangle \quad |5'\rangle \quad |6'\rangle \quad |7'\rangle \quad |8'\rangle$$

SU(3) 8-dim irrrep

S3 2-dim irrrep [2 1]

Schur-Weyl theorem:

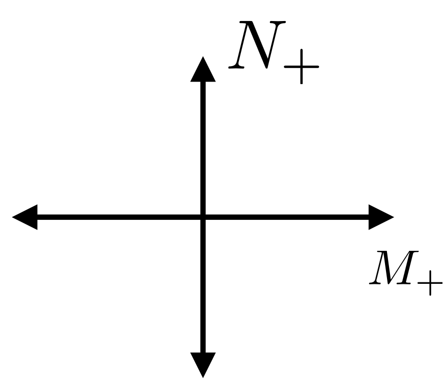
$$\underbrace{(\{1\}\otimes(1))\otimes(\{1\}\otimes(1))\otimes\cdots\otimes(\{1\}\otimes(1))}_{N\ \text{factors}} = \sum_{\lambda\vdash N}\{\lambda\}\otimes(\lambda)$$

Lorentz/Poincare/Conformal

SO(3,1)

$$\tfrac{1}{2}M^{\mu\nu}M_{\mu\nu} = \mathbf{J}^2 - \mathbf{K}^2 \,,$$
$$\tfrac{1}{2}\epsilon^{\mu\nu\sigma\tau}M_{\mu\nu}M_{\sigma\tau} = -\mathbf{J}\cdot\mathbf{K} \,,$$

$$M_i = \tfrac{1}{2}(J_i + iK_i) \,,$$
$$N_i = \tfrac{1}{2}(J_i - iK_i) \,,$$



Fields

$(0,0), (1/2,0), (0,1/2),$
$(1/2,1/2), (1,0), (0,1)$

$$K_i = -i\frac{1}{2}\sigma_i, \qquad J_i = iK_i = \frac{1}{2}\sigma_i.$$

Poincare

Semidirect product of a semisimple (Lorentz) and an Abelian group (translations)

$$P^2 = P_\mu P^\mu$$
$$W^2 = W_\mu W^\mu$$

$$W_\mu = \epsilon_{\mu\nu\sigma\tau}M^{\nu\sigma}P^\tau$$

$$W_0 = \mathbf{P}\cdot\mathbf{J} \,,$$
$$\mathbf{W} = P_0\,\mathbf{J} - \mathbf{P}\times\mathbf{K} \,.$$

P invariant subgroup:

$$[w_0, w_\pm] = \pm m w_\pm$$
$$[w_+, w_-] = 2m w_0$$

States

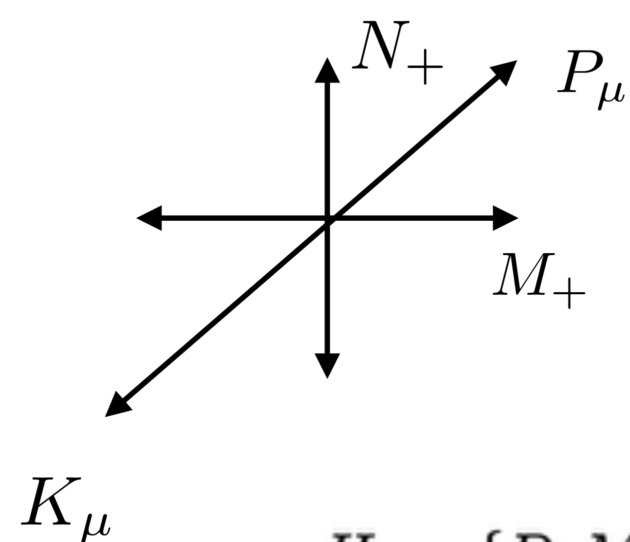
$|m, \mathbf{p}, j, \sigma\rangle$

SO(4,2)

3 casimirs

$$\tfrac{1}{2}M^{\mu\nu}M_{\mu\nu} + D(D-d) \; -P^\mu K_\mu$$

$$E_+ = \{P_\mu, M_+, N_+\} \qquad E_- = \{K_\mu, M_-, N_-\}$$



$$H_0 = \{D, M_3, N_3\}$$

(Δ, ℓ_1, ℓ_2)

$$P_\mu : (\Delta, \ell) \to \left(\Delta + 1, \ell \otimes \left(\frac{1}{2}, \frac{1}{2}\right)\right)$$
$$\ell = (\ell_1, \ell_2)$$

J-Basis Operators

J-Basis is flavor blind basis, not independent if repeated fields

$$\mathcal{A}^{(6)}(H_{1,i}, H_3^{\dagger,k} \to H_{2,j}, H_4^{\dagger,l}) = \sum_{J,\mathbf{r}} C^{J,\mathbf{r}} \mathcal{B}^J T^{\mathbf{r}}$$

$\phi^4 D^2$

$$\mathcal{B}^{\mathrm{y}} = \begin{cases} s_{12} \\ s_{13} \end{cases}$$

$$\mathbf{W}_{(1,3)}{}^2 \mathcal{B}^{\mathrm{y}} = -s_{13} \begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix} . \mathcal{B}^{\mathrm{y}}$$

$$\Rightarrow \mathcal{B}^{\mathrm{j}} = \begin{cases} s_{13} & J = 0 \\ s_{12} - s_{14} & J = 1 \end{cases}$$

Lorentz j-basis (partial waves)	Gauge j-basis
$\mathcal{B}^{J=0} = s_{13}/\Lambda^2$	$T^{\mathbf{r}=\mathbf{1}} = \delta_i^k \delta_j^l$
$\mathcal{B}^{J=1} = (s_{12} - s_{14})/\Lambda^2$	$T^{\mathbf{r}=\mathbf{3}} = (\tau^I)_i^k (\tau^I)_j^l$

$$\mathcal{B}^{\mathrm{j}} = \mathcal{K}.\mathcal{B}^{\mathrm{y}}$$

$$C^{J,\mathbf{r}} \to C^{\mathrm{j}} = (\mathcal{K}^{\mathrm{pj}})^{\intercal}.C^{\mathrm{p}}$$

$$\mathcal{K}^{\mathrm{pj}} = \mathcal{K}^{\mathrm{py}}.(\mathcal{K}^{\mathrm{jy}})^{-1}$$

$$\underbrace{\begin{pmatrix} C^{0,\mathbf{1}} \\ C^{0,\mathbf{3}} \\ C^{1,\mathbf{1}} \\ C^{1,\mathbf{3}} \end{pmatrix}}_{C^{\mathrm{j}}} = \underbrace{\begin{pmatrix} 3 & 0 \\ -1 & 1 \\ -1 & -1 \\ -1 & 0 \end{pmatrix}}_{(\mathcal{K}^{\mathrm{pj}})^{\intercal}} \cdot \underbrace{\begin{pmatrix} C_{H\Box} \\ C_{HD} \end{pmatrix}}_{C^{\mathrm{p}} \text{ in the Warsaw basis}}$$

$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$

$(\varphi^\dagger D^\mu\varphi)^* (\varphi^\dagger D_\mu\varphi)$