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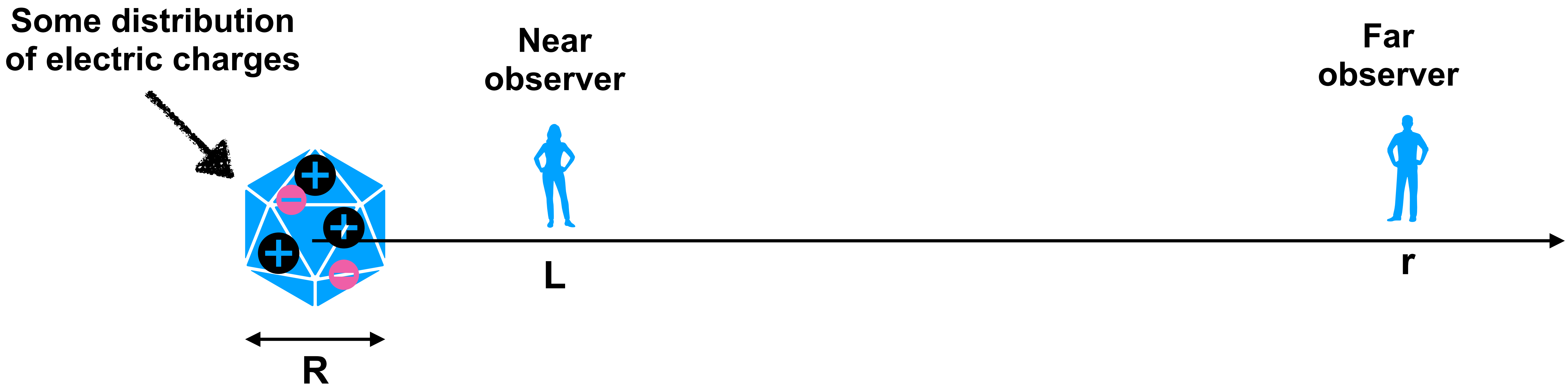
Lectures on SMEFT

**Lectures given during
the IMEPNP workshop in Bhubaneswar
on 11-12 February 2022**

Part 1

Brief Philosophy of EFT

Role of scale in physical problems



<u>Near observer</u>, L~R, needs to know the position of every charge to describe electric field in her proximity

<u>Far observer</u>, r >> R, can instead use multipole expansion: $V(\vec{r}) = \frac{q}{r} + \frac{\overrightarrow{d} \cdot \vec{r}}{r^3} + \frac{Q_{ij} r_i r_j}{r^5} + \dots$

~1/r ~R/r^2 ~R^2/r^3

Higher order terms in the multipole expansion are suppressed by powers of the small parameter (R/r).
One can truncate the expansion at some order depending on the value of (R/r) and experimental precision

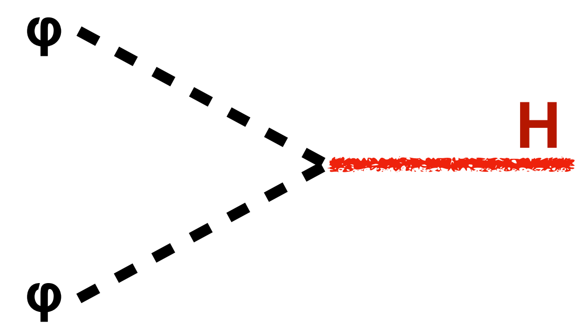
Far observer is able to describe electric field in his vicinity using just a few parameters:
the total electric charge, eventually the dipole moment ….

On the other hand, far observer can only guess the "fundamental" distributions of the charges,
as many distinct distributions lead to the same first few moments

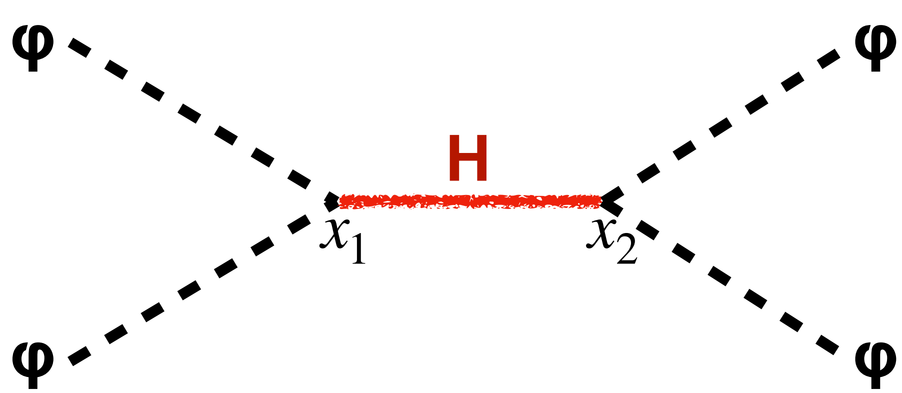
Far observer, like Molière's Mr. Jourdain,
discovers that he has been using EFT all his life

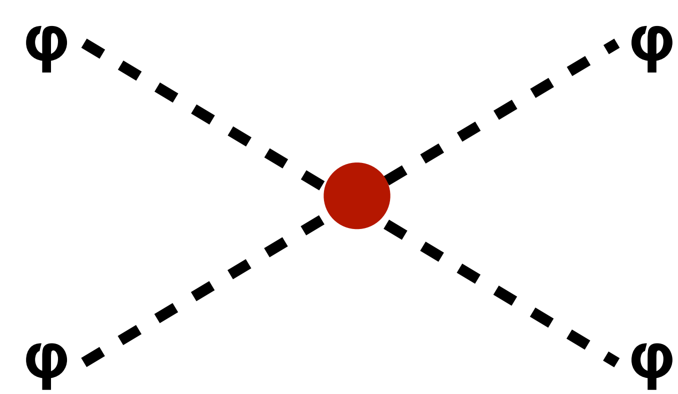
Scale in quantum field theory

Consider a theory of a light particle φ interacting with a heavy particle H



Heavy particle H propagator in coordinate space: $P(x_1, x_2) \sim \exp(-m_H |x_1 - x_2|)$





At small distance scales, $|x_1-x_2| \ll 1/m_H$, the heavy particle H propagates. Force acting between light particles φ

At large distance scales, $|x_1-x_2| \gg 1/m_H$, propagation of the heavy particle H suppressed. Interaction looks like a delta function potential

$$m_H \sim \Delta E \ll \frac{1}{|x_1 - x_2|} \sim \frac{1}{\Delta t} \quad \Rightarrow \quad \Delta E \Delta t \ll 1$$

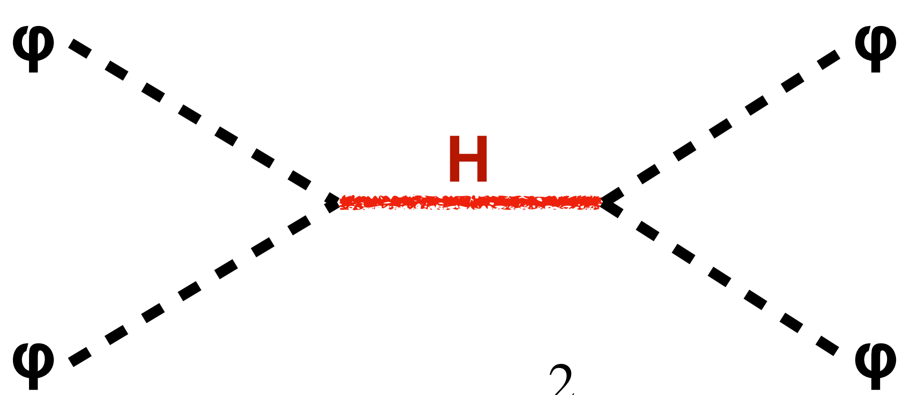
$$m_H \sim \Delta E \gg \frac{1}{|x_1 - x_2|} \sim \frac{1}{\Delta t} \quad \Rightarrow \quad \Delta E \Delta t \gg 1$$

Scale in quantum field theory

Consider a theory of a light particle φ interacting with a heavy particle H

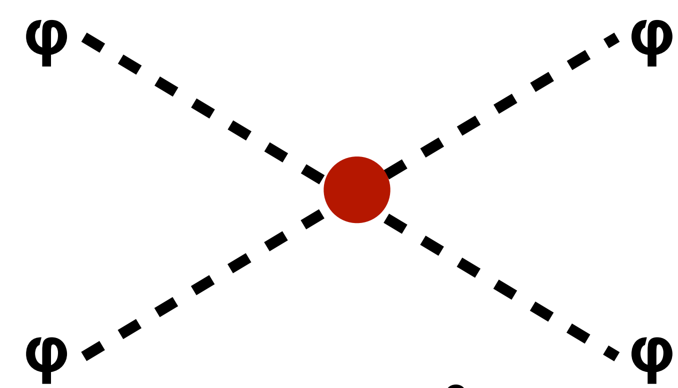
Heavy particle H propagator in momentum space:

$$P(p^2) \sim \frac{1}{p^2 - m_H^2} = \begin{cases} \dfrac{1}{p^2} & p^2 \gg m_H^2 \\ -\dfrac{1}{m_H^2} & p^2 \ll m_H^2 \end{cases}$$



$$\mathscr{M} \sim \frac{g^2}{p^2}$$

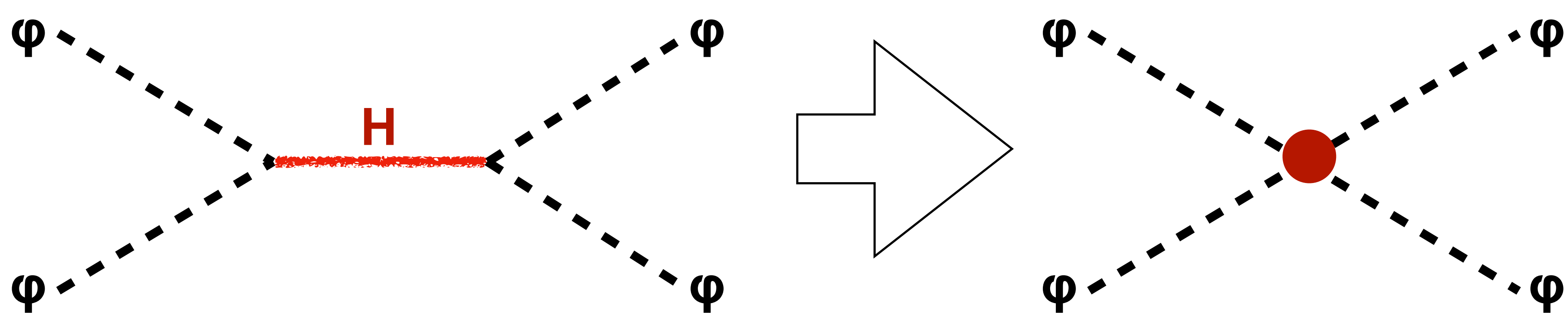
At large momentum scales, $p^2 >> m_H^2$, we see propagation of the heavy particle H. Long range force acting between light particles φ



$$\mathscr{M} \sim \frac{g^2}{m_H^2}$$

At small momentum scales, $p^2 << m_H^2$, propagation of the heavy particle H effectively leads to a contact interaction between light particles φ

Scale in particle theory

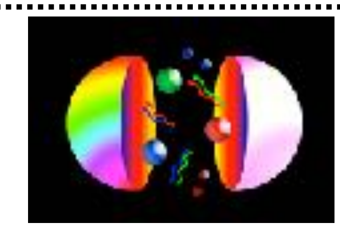
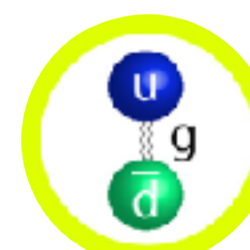


- Processes probing distance scales >> $1/m_H$, equivalently energies scales << m_H, cannot resolve the propagation of H

- Then, intuitively, exchange of heavy particle H between light particles φ should be indistinguishable from a contact interaction of φ

- In other words, the effective theory describing φ interactions should be well approximated by a local Lagrangian, that is, by a polynomial in φ and its derivatives

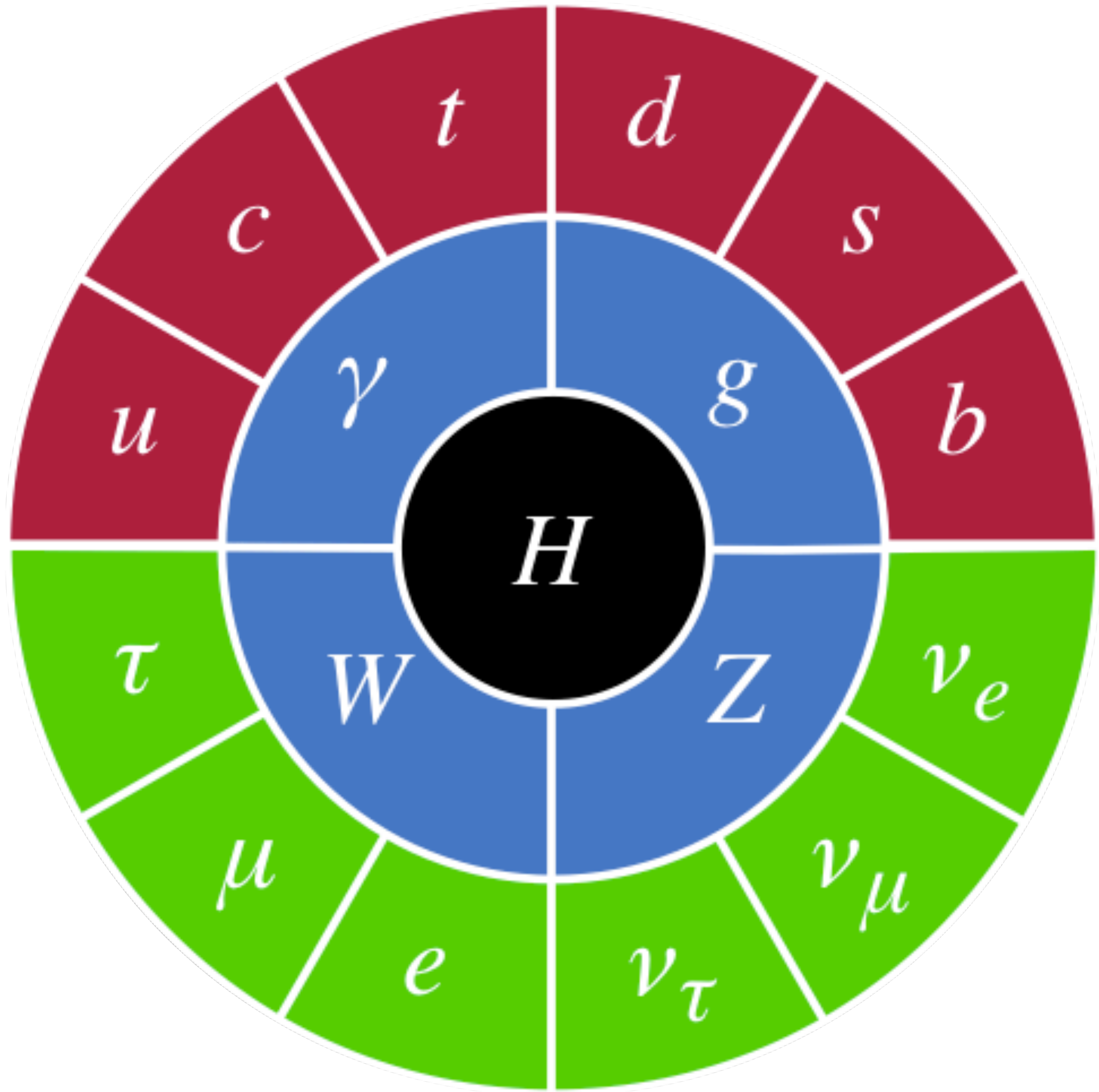
This is the generic way how the effective theory description arise in particle physics, which will be repeated in many examples that follow

UV	100 TeV	**???**	
SMEFT	100 GeV	$\gamma, g, W, Z, \nu_i, e, \mu, \tau$ + u, d, s, c, b, t + h	
WEFT5	5 GeV	$\gamma, g, \nu_i, e, \mu, \tau$ + u, d, s, c, b	
WEFT4	2 GeV	$\gamma, g, \nu_i, e, \mu, \tau$ + u, d, s, c	
ChRT	1 GeV	γ, ν_i, e, μ + hadrons	
ChPT	100 MeV	γ, ν_i, e, μ + pions and kaons	
QED	1 MeV	γ, ν_i, e	
EH		γ, ν_i	

Part 2

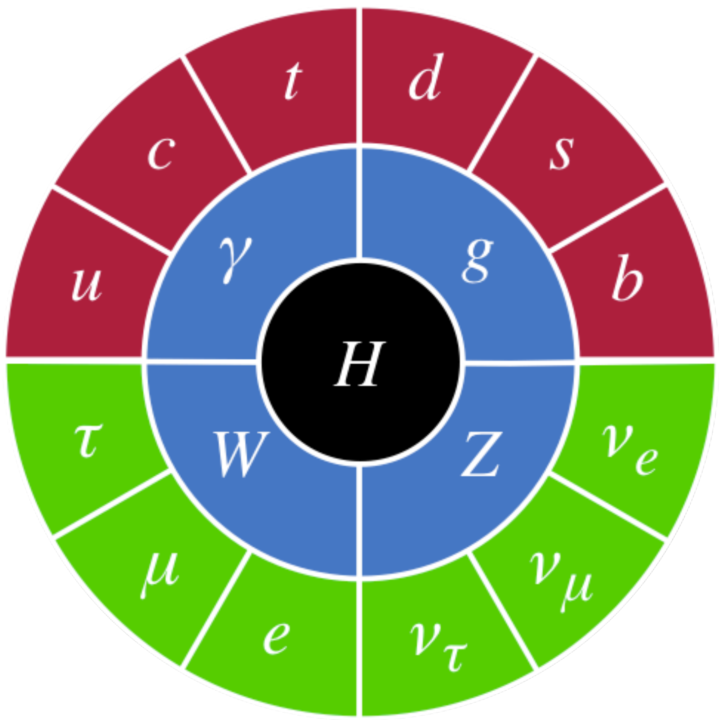
Introducing the SMEFT

Elementary particles we know today



All these particles are propagating degrees of freedom right above the electroweak scale, that is at $E \sim$ 100 GeV - 1 TeV

SMEFT



SMEFT is an effective theory for these degrees of freedom incorporating certain physical assumptions:

1. **Locality, unitarity, Poincaré symmetry**
2. **Mass gap: absence of non-SM degrees of freedom at or below the electroweak scale**
3. **Gauge symmetry: local SU(3)xSU(2)xU(1) symmetry strictly respected by all interactions and spontaneously broken to SU(3)xU(1) by a VEV of the Higgs field**

	G^a_μ	W^k_μ	B_μ	**1**
	$SU(3)_C$	$SU(2)_W$	$U(1)_Y$	Spin
$Q = (u_L, d_L)$	3	2	1/6	1/2
u_R	3	1	2/3	1/2
d_R	3	1	-1/3	1/2
$L = (\nu_L, e_L)$	1	2	-1/2	1/2
e_R	1	1	-1	1/2
H	1	2	1/2	0

$$\langle H \rangle = \begin{pmatrix} 0 \\ \mathrm{v}/\sqrt{2} \end{pmatrix}$$

SMEFT

1. Locality, unitarity, Poincaré symmetry
2. Mass gap: absence of non-SM degrees of freedom at or below the electroweak scale
3. Gauge symmetry: local SU(3)xSU(2)xU(1) symmetry strictly respected by all interactions

If these assumptions are true we can organize the EFT as an expansion in 1/Λ, where Λ is identified with the mass scale of the UV completion of the SMEFT, and each term is a linear combination of SU(3)xSU(2)xU(1) invariant operators of a given canonical dimension D

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{D=2} + \mathscr{L}_{D=3} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \dots$$

SM Lagrangian

Higher-dimensional $SU(3)_C$ x $SU(2)_L$ x $U(1)_Y$ invariant interactions added to the SM

In the spirit of EFT, each $\mathscr{L}_D$ should include a <u>complete</u> and <u>non-redundant</u> set of interactions

SMEFT

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=3} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \dots$$

$$\mathscr{L}_{D=2} = \mu_H^2 H^\dagger H$$

Experiment: $\mu_H \sim 100$ GeV

Unsolved mystery why $\mu_H^2 \ll \Lambda^2$,
which is called the hierarchy problem

$$\mathscr{L}_{D=3} = 0$$

Simply, no gauge invariant operators made of SM fields exist at canonical dimension D=3

$$\mathscr{L}_{D=4} = -\frac{1}{4} \sum_{V \in B, W^i, G^a} V_{\mu\nu} V^{\mu\nu} + \sum_{f \in q,u,d,L,e} i\bar{f}\gamma^\mu D_\mu f$$
$$- \left(\bar{u} Y_u Q H + \bar{d} Y_d H^\dagger Q + \bar{e} Y_e H^\dagger L + \mathrm{h.c.} \right)$$
$$+ D_\mu H^\dagger D^\mu H - \lambda (H^\dagger H)^2 + \tilde{\theta} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}$$

$$V^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu + g f^{abc} V^b_\mu V^c_\nu$$

$$D_\mu f = \partial_\mu f - i g_s G^a_\mu T^a f - i g_L W^i_\mu \frac{\sigma^i}{2} f - i g_Y B_\mu Y f$$

$$\tilde{G}^a_{\mu\nu} \equiv \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} G^{\alpha\beta\ a}$$

Experiment: all interactions at D=2 and D=4 above have been observed, except for $\tilde{\theta}$

Strictly speaking, λ has not been observed directly. Its value is known within SM hypothesis, but not within SMEFT, without additional assumptions. Observation of double Higgs production (receiving contribution from cubic Higgs coupling) will be a direct proof that λ is there in the Lagrangian.

Note that $\theta_B B_{\mu\nu} \tilde{B}_{\mu\nu}$ is not physical, while $\theta_W W^k_{\mu\nu} \tilde{W}^k_{\mu\nu}$ can be eliminated by chiral rotation

SMEFT at dimension-5

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$

Weinberg (1979)
Phys. Rev. Lett. 43, 1566

$$\frac{c_{ij}}{\Lambda}(L_i H)(L_j H) + \mathrm{h.c.} \to c_{ij}\frac{\mathrm{v}^2}{\Lambda}\nu_i \nu_j + \mathrm{h.c.}$$

$$H \to \begin{pmatrix} 0 \\ \mathrm{v}/\sqrt{2} \end{pmatrix} \qquad L_i \to \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}$$

- At dimension 5, the only gauge-invariant operators one can construct are the so-called Weinberg operators, which break the lepton number

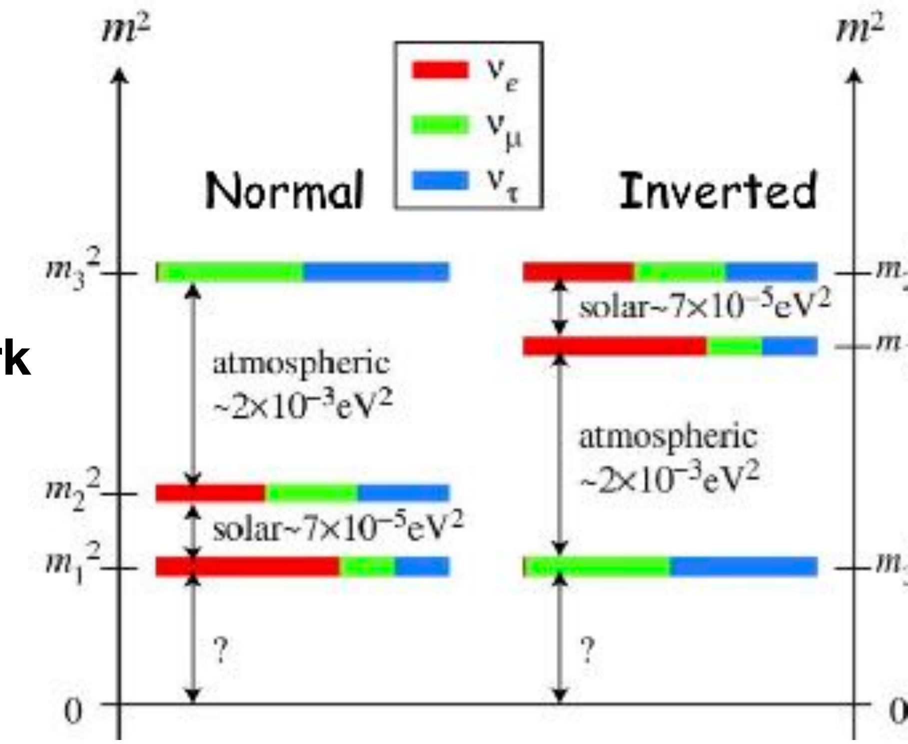
- After electroweak symmetry breaking they give rise to Majorana mass terms for the SM (left-handed) neutrinos

- Neutrino oscillation experiments strongly suggest that these operators are present (unless neutrino masses are of the Dirac type)

**This is a huge success of the SMEFT paradigm:
corrections to the SM Lagrangian predicted at the next order in the EFT expansion, are indeed observed in experiment!**

SMEFT at dimension-5

$$\mathscr{L}_{\text{SMEFT}} \supset c_{ij}\frac{\text{v}^2}{\Lambda}\nu_i\nu_j + \text{h.c.}$$



Neutrino masses or most likely in the 0.01 eV - 0.1 eV ballpark (while the lightest neutrino may even be massless)

It follows that $\dfrac{\Lambda}{c_{ij}} \sim 10^{15}$ **GeV**

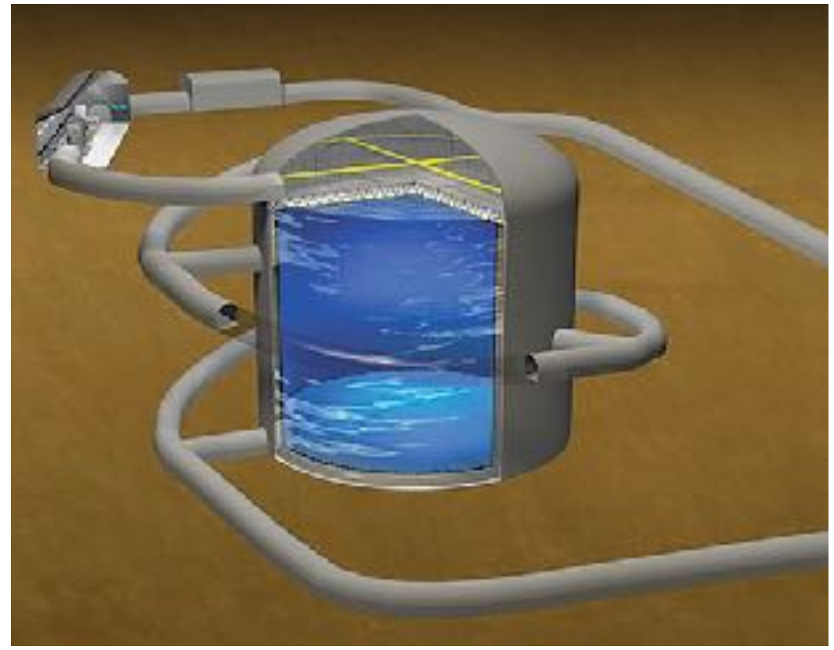
One problem now:

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$

Naively: $\mathscr{L}_{D=5} \sim \dfrac{1}{\Lambda}$ **and then** $\mathscr{L}_{D=6} \sim \dfrac{1}{\Lambda^2}$**,** $\mathscr{L}_{D=7} \sim \dfrac{1}{\Lambda^3}$**, and so on**

If this is really the correct estimate, then we will never see any other effects of higher-dimensional operators, except possibly of baryon-number violating ones :/

Career opportunities



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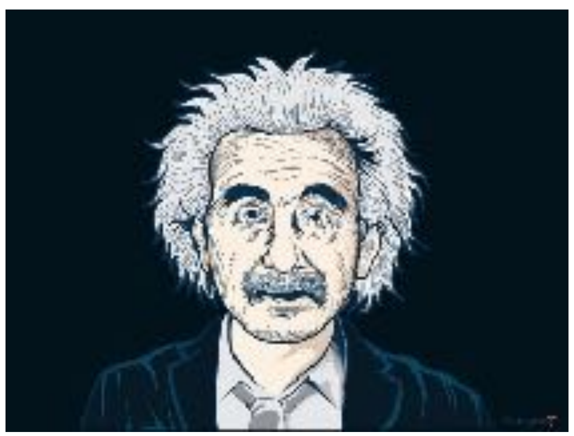
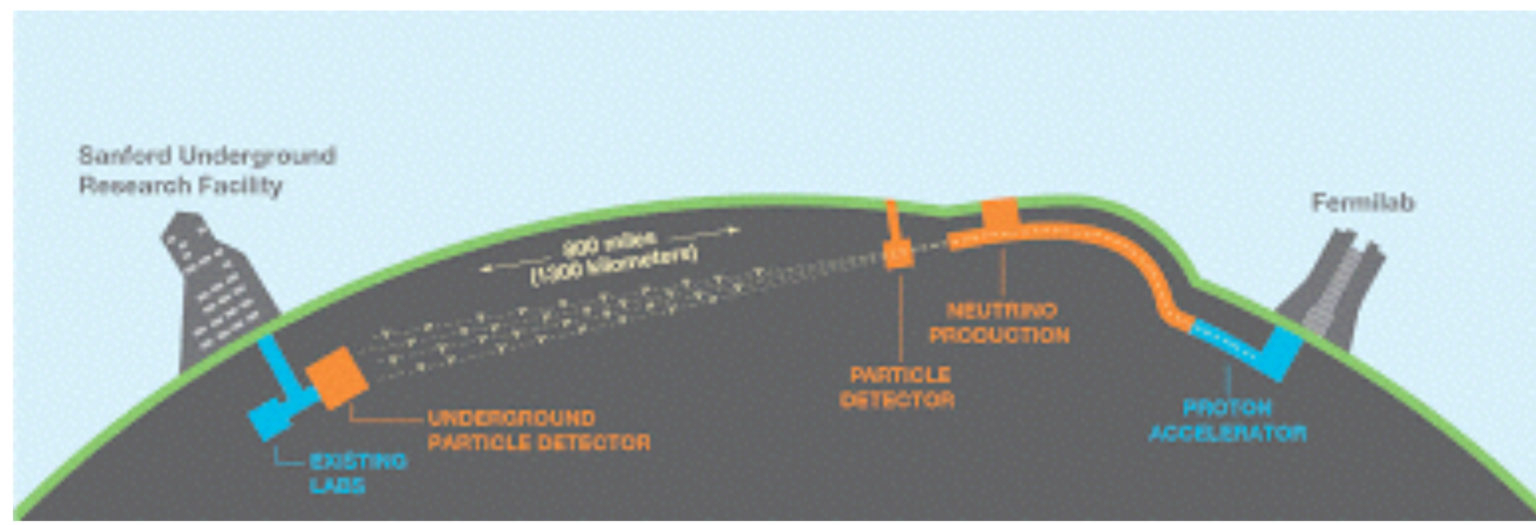
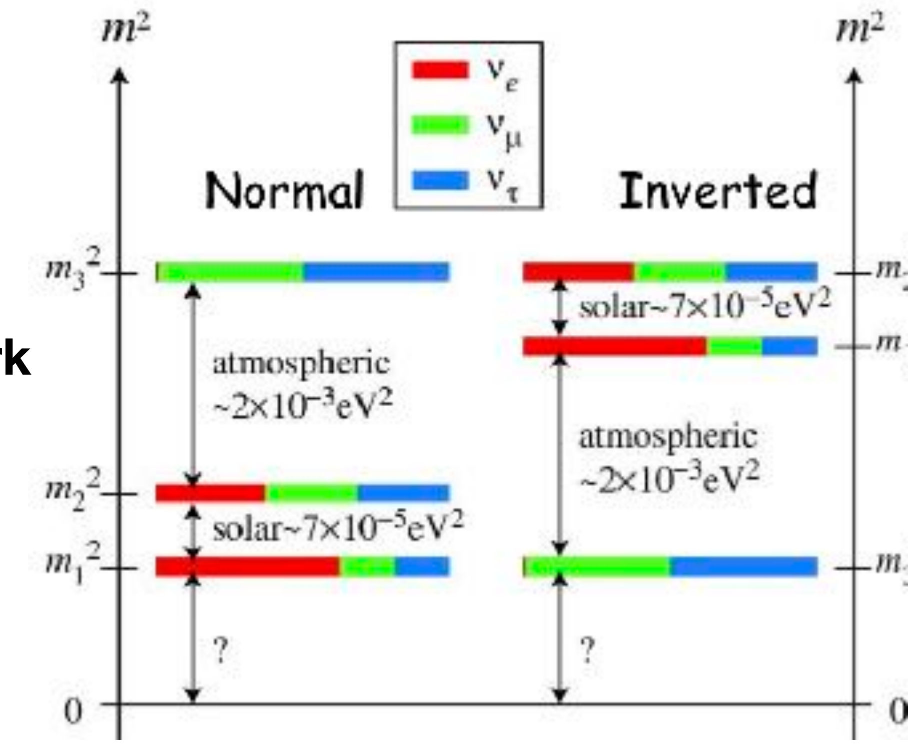




SMEFT at dimension-5

$$\mathscr{L}_{\text{SMEFT}} \supset c_{ij}\frac{\mathrm{v}^2}{\Lambda}\nu_i\nu_j + \mathrm{h.c.}$$



Neutrino masses or most likely in the 0.01 eV - 0.1 eV ballpark (while the lightest neutrino may even be massless)

It follows that $\frac{\Lambda}{c_{ij}} \sim 10^{15}$ **GeV**

One problem now:

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$

Naively: $\mathscr{L}_{D=5} \sim \frac{1}{\Lambda}$ **and then** $\mathscr{L}_{D=6} \sim \frac{1}{\Lambda^2}$, $\mathscr{L}_{D=7} \sim \frac{1}{\Lambda^3}$, **and so on**

It is however possible that Λ **is not far from TeV, but instead** $c_{ij} \ll 1$

Alternatively, it is possible (and likely) that there is more than one mass scale of new physics

Dimension-5 interactions are special because they violate <u>lepton number</u> L.
If we assume that the mass scale of new particles with L-violating interactions is Λ_L,
and there is also L-conserving new physics at the scale $\Lambda \ll \Lambda_L$, **then the estimate is**

$$\mathscr{L}_{D=5} \sim \frac{1}{\Lambda_L},\ \mathscr{L}_{D=6} \sim \frac{1}{\Lambda^2},\ \mathscr{L}_{D=7} \sim \frac{1}{\Lambda_L^3},\ \mathscr{L}_{D=8} \sim \frac{1}{\Lambda^4},$$ **and so on**

SMEFT at dimension-6

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$

At dimension-6 all hell breaks loose

Grzadkowski et al
arXiv:1008.4884

Bosonic CP-even	
O_H	$(H^\dagger H)^3$
$O_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
O_{HD}	$\left\lvert H^\dagger D_\mu H\right\rvert^2$
O_{HG}	$H^\dagger H\, G^a_{\mu\nu} G^a_{\mu\nu}$
O_{HW}	$H^\dagger H\, W^i_{\mu\nu} W^i_{\mu\nu}$
O_{HB}	$H^\dagger H\, B_{\mu\nu} B_{\mu\nu}$
O_{HWB}	$H^\dagger \sigma^i H\, W^i_{\mu\nu} B_{\mu\nu}$
O_W	$\epsilon^{ijk} W^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}$
O_G	$f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$

Bosonic CP-odd	
$O_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}^a_{\mu\nu} G^a_{\mu\nu}$
$O_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}^i_{\mu\nu} W^i_{\mu\nu}$
$O_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu} B_{\mu\nu}$
$O_{H\widetilde{W}B}$	$H^\dagger \sigma^i H\, \widetilde{W}^i_{\mu\nu} B_{\mu\nu}$
$O_{\widetilde{W}}$	$\epsilon^{ijk} \widetilde{W}^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}$
$O_{\widetilde{G}}$	$f^{abc} \widetilde{G}^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$

Yukawa	
$[O^\dagger_{eH}]_{IJ}$	$H^\dagger H e^c_I H^\dagger \ell_J$
$[O^\dagger_{uH}]_{IJ}$	$H^\dagger H u^c_I \widetilde{H}^\dagger q_J$
$[O^\dagger_{dH}]_{IJ}$	$H^\dagger H d^c_I H^\dagger q_J$

Vertex		Dipole	
$[O^{(1)}_{H\ell}]_{IJ}$	$i\bar{\ell}_I \bar{\sigma}_\mu \ell_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{eW}]_{IJ}$	$e^c_I \sigma_{\mu\nu} H^\dagger \sigma^i \ell_J W^i_{\mu\nu}$
$[O^{(3)}_{H\ell}]_{IJ}$	$i\bar{\ell}_I \sigma^i \bar{\sigma}_\mu \ell_J H^\dagger \sigma^i \overleftrightarrow{D}_\mu H$	$[O^\dagger_{eB}]_{IJ}$	$e^c_I \sigma_{\mu\nu} H^\dagger \ell_J B_{\mu\nu}$
$[O_{He}]_{IJ}$	$i e^c_I \sigma_\mu \bar{e}^c_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{uG}]_{IJ}$	$u^c_I \sigma_{\mu\nu} T^a \widetilde{H}^\dagger q_J\, G^a_{\mu\nu}$
$[O^{(1)}_{Hq}]_{IJ}$	$i\bar{q}_I \bar{\sigma}_\mu q_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{uW}]_{IJ}$	$u^c_I \sigma_{\mu\nu} \widetilde{H}^\dagger \sigma^i q_J\, W^i_{\mu\nu}$
$[O^{(3)}_{Hq}]_{IJ}$	$i\bar{q}_I \sigma^i \bar{\sigma}_\mu q_J H^\dagger \sigma^i \overleftrightarrow{D}_\mu H$	$[O^\dagger_{uB}]_{IJ}$	$u^c_I \sigma_{\mu\nu} \widetilde{H}^\dagger q_J\, B_{\mu\nu}$
$[O_{Hu}]_{IJ}$	$i u^c_I \sigma_\mu \bar{u}^c_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{dG}]_{IJ}$	$d^c_I \sigma_{\mu\nu} T^a H^\dagger q_J\, G^a_{\mu\nu}$
$[O_{Hd}]_{IJ}$	$i d^c_I \sigma_\mu \bar{d}^c_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{dW}]_{IJ}$	$d^c_I \sigma_{\mu\nu} \bar{H}^\dagger \sigma^i q_J\, W^i_{\mu\nu}$
$[O_{Hud}]_{IJ}$	$i u^c_I \sigma_\mu \bar{d}^c_J \tilde{H}^\dagger D_\mu H$	$[O^\dagger_{dB}]_{IJ}$	$d^c_I \sigma_{\mu\nu} H^\dagger q_J\, B_{\mu\nu}$

Table 2.3: Two-fermion D=6 operators in the Warsaw basis. The flavor indices denoted by I, J. For complex operators (O_{Hud} and all Yukawa and dipole operat the corresponding complex conjugate operator is implicitly included.

$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
O_{ee}	$\eta(e^c\sigma_\mu\bar{e}^c)(e^c\sigma_\mu\bar{e}^c)$	$O_{\ell e}$	$(\bar{\ell}\bar{\sigma}_\mu\ell)(e^c\sigma_\mu\bar{e}^c)$
O_{uu}	$\eta(u^c\sigma_\mu\bar{u}^c)(u^c\sigma_\mu\bar{u}^c)$	$O_{\ell u}$	$(\bar{\ell}\bar{\sigma}_\mu\ell)(u^c\sigma_\mu\bar{u}^c)$
O_{dd}	$\eta(d^c\sigma_\mu\bar{d}^c)(d^c\sigma_\mu\bar{d}^c)$	$O_{\ell d}$	$(\bar{\ell}\bar{\sigma}_\mu\ell)(d^c\sigma_\mu\bar{d}^c)$
O_{eu}	$(e^c\sigma_\mu\bar{e}^c)(u^c\sigma_\mu\bar{u}^c)$	O_{eq}	$(e^c\sigma_\mu\bar{e}^c)(\bar{q}\bar{\sigma}_\mu q)$
O_{ed}	$(e^c\sigma_\mu\bar{e}^c)(d^c\sigma_\mu\bar{d}^c)$	O_{qu}	$(\bar{q}\bar{\sigma}_\mu q)(u^c\sigma_\mu\bar{u}^c)$
O_{ud}	$(u^c\sigma_\mu\bar{u}^c)(d^c\sigma_\mu\bar{d}^c)$	O'_{qu}	$(\bar{q}\bar{\sigma}_\mu T^a q)(u^c\sigma_\mu T^a\bar{u}^c)$
O'_{ud}	$(u^c\sigma_\mu T^a\bar{u}^c)(d^c\sigma_\mu T^a\bar{d}^c)$	O_{qd}	$(\bar{q}\bar{\sigma}_\mu q)(d^c\sigma_\mu\bar{d}^c)$
		O'_{qd}	$(\bar{q}\bar{\sigma}_\mu T^a q)(d^c\sigma_\mu T^a\bar{d}^c)$

$(\bar{L}L)(\bar{L}L)$		$(\bar{L}R)(\bar{L}R)$	
$O_{\ell\ell}$	$\eta(\bar{\ell}\bar{\sigma}_\mu\ell)(\bar{\ell}\bar{\sigma}_\mu\ell)$	O_{quqd}	$(u^c q^j)\epsilon_{jk}(d^c q^k)$
O_{qq}	$\eta(\bar{q}\bar{\sigma}_\mu q)(\bar{q}\bar{\sigma}_\mu q)$	O'_{quqd}	$(u^c T^a q^j)\epsilon_{jk}(d^c T^a q^k)$
O'_{qq}	$\eta(\bar{q}\bar{\sigma}_\mu\sigma^i q)(\bar{q}\bar{\sigma}_\mu\sigma^i q)$	$O_{\ell equ}$	$(e^c\ell^j)\epsilon_{jk}(u^c q^k)$
$O_{\ell q}$	$(\bar{\ell}\bar{\sigma}_\mu\ell)(\bar{q}\bar{\sigma}_\mu q)$	$O'_{\ell equ}$	$(e^c\bar{\sigma}_{\mu\nu}\ell^j)\epsilon_{jk}(u^c\bar{\sigma}^{\mu\nu}q^k)$
$O'_{\ell q}$	$(\bar{\ell}\bar{\sigma}_\mu\sigma^i\ell)(\bar{q}\bar{\sigma}_\mu\sigma^i q)$	$O_{\ell edq}$	$(\bar{\ell}\bar{e}^c)(d^c q)$

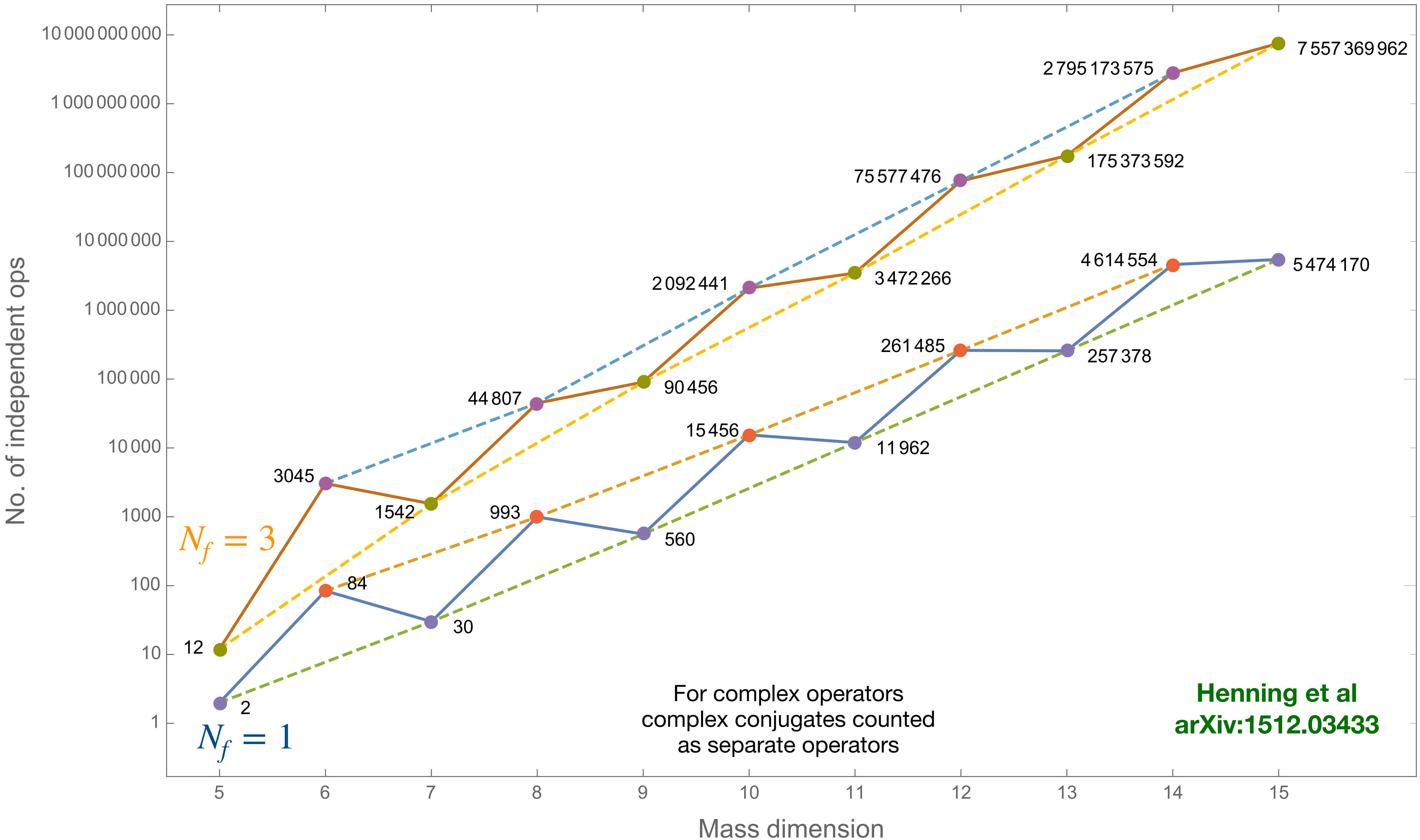
Table 2.4: Four-fermion D=6 operators in the Warsaw basis. Flavor indices a suppressed here to reduce the clutter. The factor η is equal to 1/2 when all flav indices are equal (e.g. in $[O_{ee}]_{1111}$), and $\eta = 1$ otherwise. For each complex operat the complex conjugate should be included.

$$O_{duq} = (d^c u^c)(\bar{Q}\bar{\ell})$$
$$O_{qqu} = (qq)(\bar{u}^c\bar{e}^c)$$
$$O_{qqq} = (qq)(q\ell)$$
$$O_{duu} = (d^c u^c)(u^c e^c)$$

$|H|^2 B_{\mu\nu}\widetilde{B}_{\mu\nu}$
$|H|^2 G^a_{\mu\nu}\widetilde{G}^a_{\mu\nu}$
$|H|^2 W^a_{\mu\nu}\widetilde{W}^a_{\mu\nu}$
$|H|^2 W^a_{\mu\nu}W^a_{\mu\nu}$
$|H|^2 B_{\mu\nu}B_{\mu\nu}$
$G^a_{\mu\nu}G^a_{\nu\rho}\widetilde{G}^a_{\rho\mu}$
$|H|^6$
$|H|^2 G^a_{\mu\nu}G^a_{\mu\nu}$

SMEFT at higher dimensions

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{D=2} + \mathcal{L}_{D=4} + \mathcal{L}_{D=5} + \mathcal{L}_{D=6} + \mathcal{L}_{D=7} + \mathcal{L}_{D=8} + \ldots$$



Exponential growth of the number of operators with the canonical dimension D

SMEFT at higher dimensions

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Number of baryon-number-conserving operators as function of D and number of generations N_f

	N_f=0	N_f=1	N_f=2	N_f=3	...
Dimension-5	0	2	6	12	...
Dimension-6	15	76	582	2499	...
Dimension-7	0	22	212	948	...
Dimension-8	89	895	8251	36971	...
...	...	...	...	...	...

SMEFT at higher dimensions

SMEFT at dimension-5: Weinberg (1979) Phys. Rev. Lett. 43, 1566

SMEFT at dimension-6: Grzadkowski et al arXiv: 1008.4884

SMEFT at dimension-7: Lehman arXiv: 1410.4193

SMEFT at dimension-8: Li et al arXiv: 2005.00008

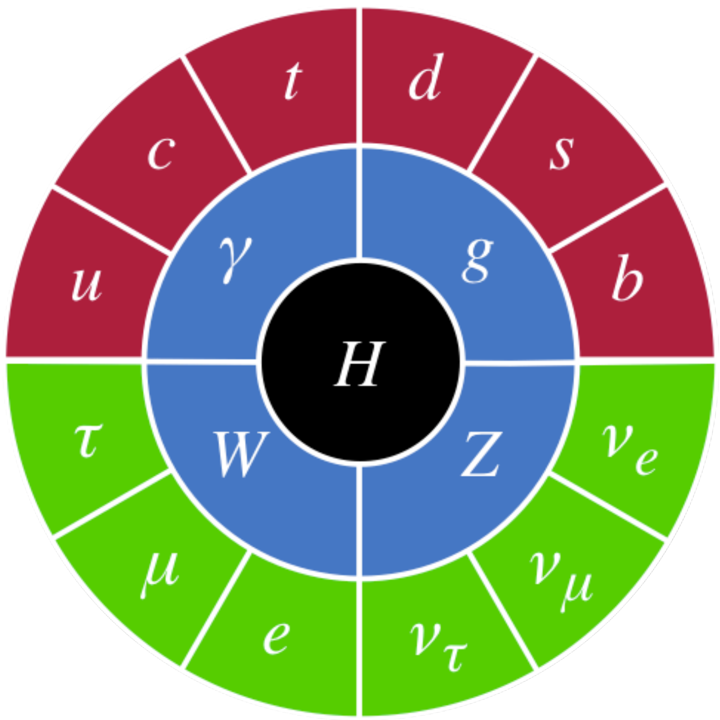
SMEFT at dimension-9: Li et al arXiv: 2012.09188

Code to generate a basis at arbitrary dimension in SMEFT: Li et al arXiv:2201.04639

Part 3

Assumptions behind the SMEFT

SMEFT



But are these assumptions true?

1. **Locality, unitarity, Poincaré symmetry**
2. **Mass gap: absence of non-SM degrees of freedom at or below the electroweak scale**
3. **Gauge symmetry: local SU(3)xSU(2)xU(1) symmetry strictly respected by all interactions and spontaneously broken to SU(3)xU(1) by a VEV of the Higgs field**

	G^a_μ	W^k_μ	B_μ	1
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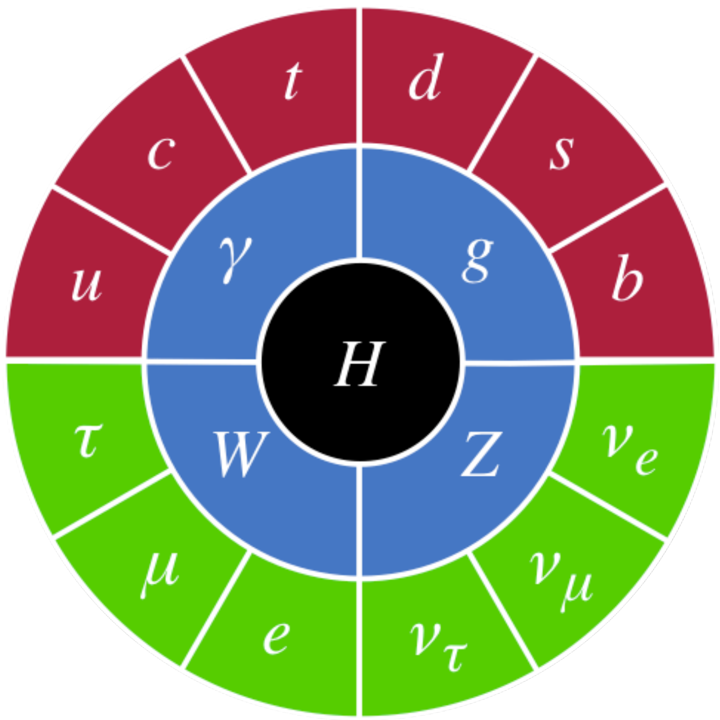
SMEFT



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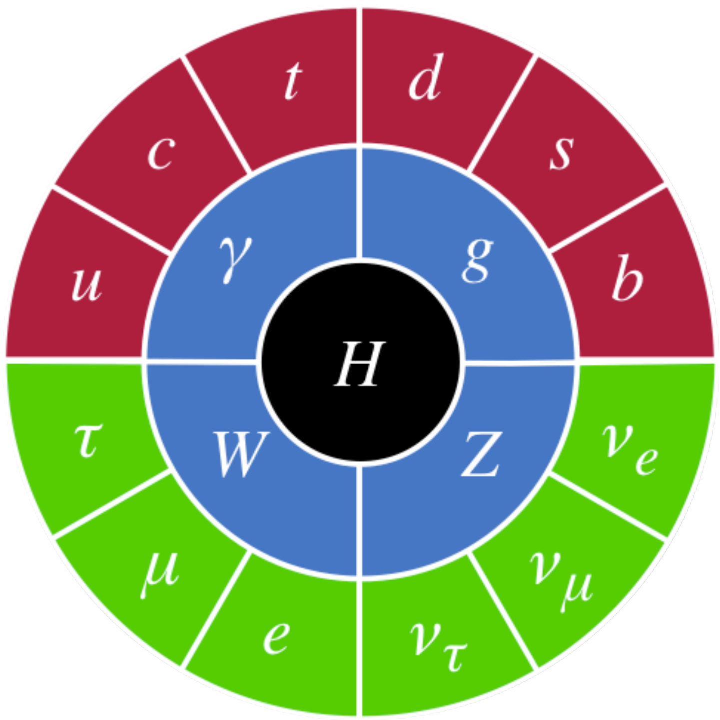

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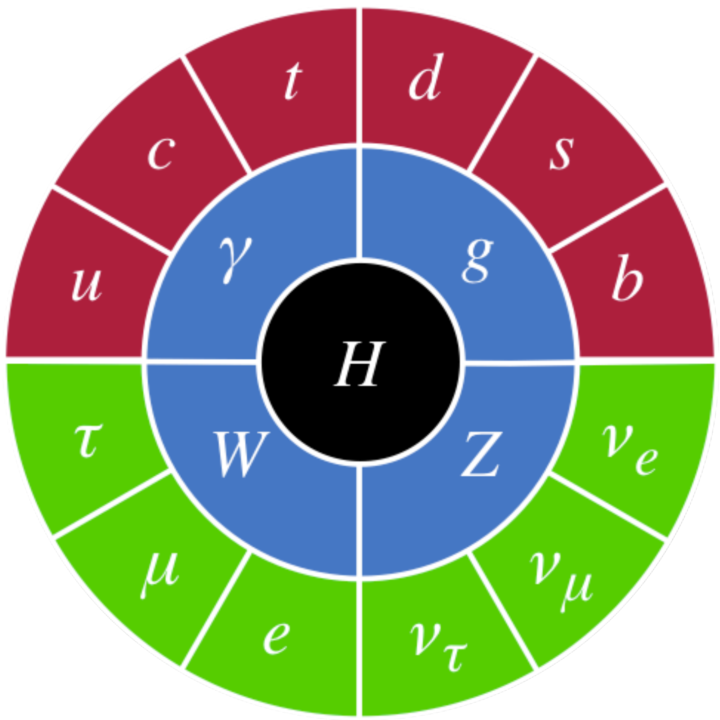

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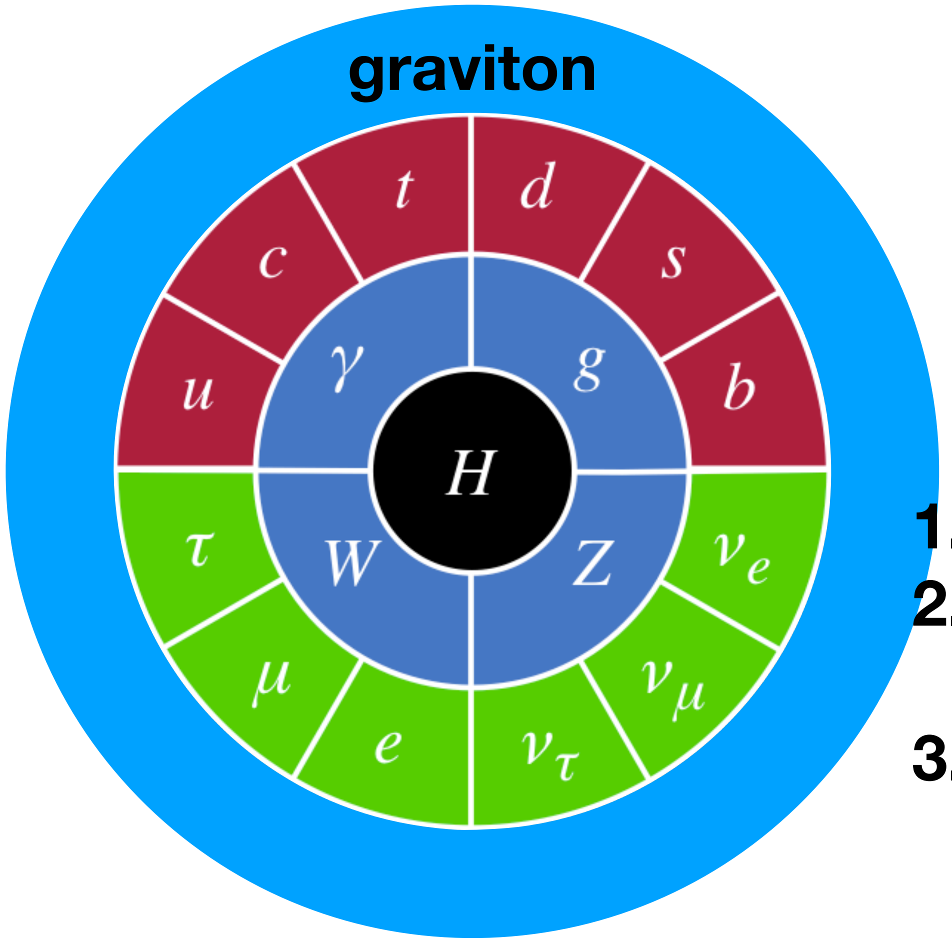

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$h_{\mu\nu}$

Certainly not,

because gravity exists !

GR-SMEFT

In principle, the relevant effective theory at the electroweak scale is GR-SMEFT rather than SMEFT

First of all, Einstein's GR can be generalised to an effective theory: GR-EFT

$$\mathscr{L}_{\mathrm{GR-EFT}} = \sqrt{-g}\left\{ \Lambda_c^4 + \frac{1}{2} M_{\mathrm{Planck}}^2 \sqrt{-g} R \right\} + \frac{1}{\Lambda^2}\left\{ c_1 C_{\mu\nu\alpha\beta} C^{\alpha\beta\rho\sigma} C_{\rho\sigma}{}^{\mu\nu} + c_2 C_{\mu\nu\alpha\beta} C^{\alpha\beta\rho\sigma} \tilde{C}_{\rho\sigma}{}^{\mu\nu} \right\} + \dots$$

GR **EFT corrections**

Ruhdorfer et al arXiv:1908.08050

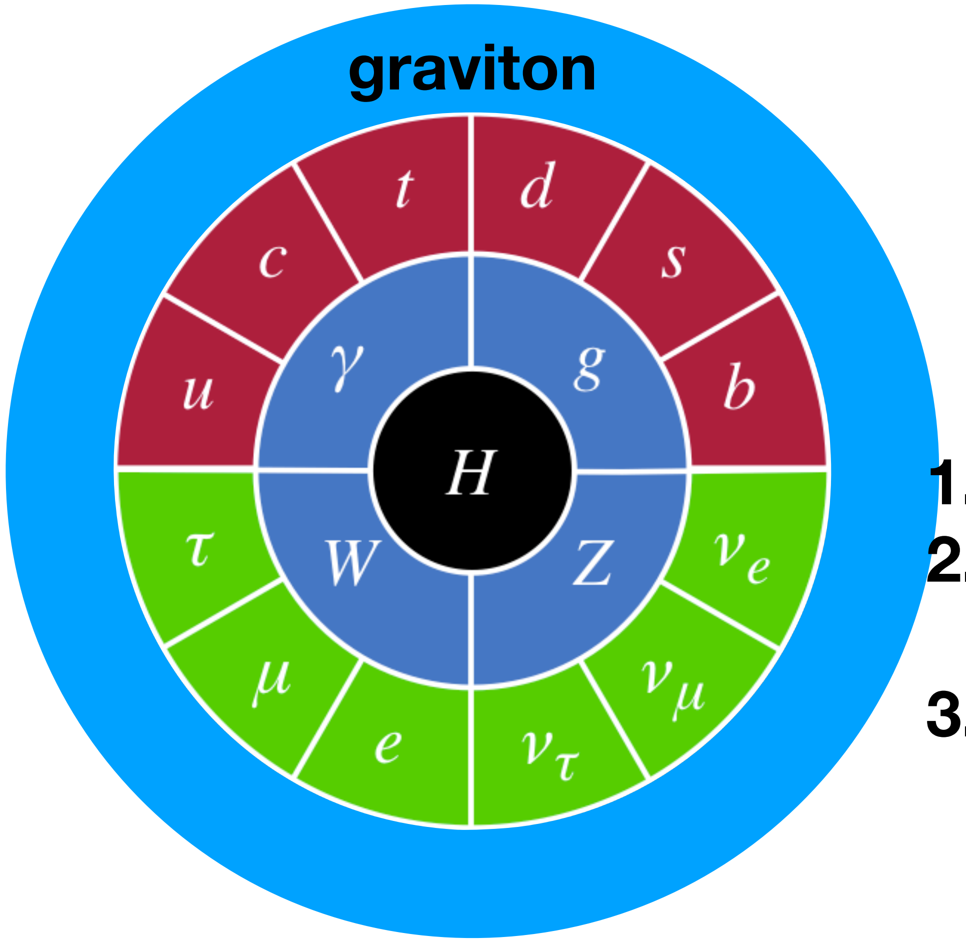
Weyl tensor $C_{\mu\nu\rho\sigma}$ is the $(2,0) \oplus (0,2)$ part in decomposition of Riemann tensor:

$$\begin{aligned} R_{\alpha\beta\gamma\delta} &= C_{\alpha\beta\gamma\delta} \\ &+ \frac{1}{2}\left(g_{\alpha\gamma} R_{\delta\beta} + g_{\beta\delta} R_{\gamma\alpha} - g_{\alpha\delta} R_{\gamma\beta} - g_{\beta\gamma} R_{\delta\alpha} \right) \\ &- \frac{1}{6}\left(g_{\alpha\gamma} g_{\delta\beta} - g_{\alpha\delta} g_{\gamma\beta} \right) R \end{aligned}$$

Furthermore, one can consider the EFT of SM degrees of freedom coupled to gravity: GR-SMEFT. At lowest order, graviton couples to the energy-momentum tensor of matter, without any free parameters. At higher order one can construct effective operators with arbitrary Wilson coefficients, for example at dimension-6 in the gauge-gravity sector one has:

$$\begin{aligned} \mathscr{L}_{D=6} \subset\ & \frac{c_1}{\Lambda^2} C_{\mu\nu}{}^{\rho\sigma} C^{\mu\nu\alpha\beta} C_{\alpha\beta\rho\sigma} + \frac{\tilde{c}_1}{\Lambda^2} C_{\mu\nu}{}^{\rho\sigma} C^{\mu\nu\alpha\beta} \tilde{C}_{\alpha\beta\rho\sigma} \\ & + \frac{c_2}{\Lambda^2} H^\dagger H C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \frac{\tilde{c}_2}{\Lambda^2} H^\dagger H C_{\mu\nu\rho\sigma} \tilde{C}^{\mu\nu\rho\sigma} \\ & + \frac{c_3}{\Lambda^2} B^{\mu\nu} B^{\rho\sigma} C_{\mu\nu\rho\sigma} + \frac{\tilde{c}_3}{\Lambda^2} B^{\mu\nu} B^{\rho\sigma} \tilde{C}_{\mu\nu\rho\sigma} + \frac{c_4}{\Lambda^2} G^{\mu\nu} G^{\rho\sigma} C_{\mu\nu\rho\sigma} + \frac{\tilde{c}_4}{\Lambda^2} G^{\mu\nu} G^{\rho\sigma} \tilde{C}_{\mu\nu\rho\sigma} \\ & + \frac{c_5}{\Lambda^2} W^{\mu\nu} W^{\rho\sigma} C_{\mu\nu\rho\sigma} + \frac{\tilde{c}_5}{\Lambda^2} W^{\mu\nu} W^{\rho\sigma} \tilde{C}_{\mu\nu\rho\sigma} \,. \end{aligned}$$

SMEFT



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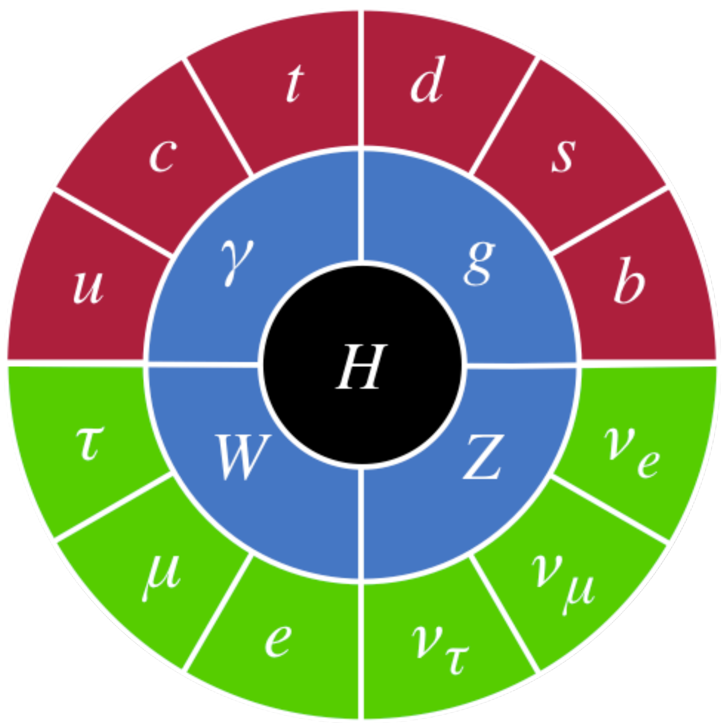
H	1	2	1/2	0

$h_{\mu\nu}$

However, unless something weird happens at the level of higher-dimensional operators, we expect graviton couplings to matter to be very suppressed, likely by powers of $M_{\rm Planck}$
In such a case, the GR- part of GR-SMEFT has tiny impact on collider or low-energy experiments.
For the sake of these applications, we can safely ignore the graviton and focus on the SMEFT.
On the other hand, for applications like black hole scattering/inspiral, weak gravity conjecture, early cosmology, etc. GR-SMEFT remains relevant

SMEFT



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e_R	1	1	-1	1/2
H	1	2	1/2	0
N	1	1	1	1/2

Maybe not, because light right-handed neutrinos might as well exist

R-SMEFT

$$\mathcal{L}_{\rm R-SMEFT} = \mathcal{L}_{D=2} + \mathcal{L}_{D=3} + \mathcal{L}_{D=4} + \mathcal{L}_{D=5} + \mathcal{L}_{D=6} + \mathcal{L}_{D=7} + \mathcal{L}_{D=8} + \ldots$$

Effective theory should include all interactions with singlet neutrinos:

$$\mathcal{L}_{D=2} = \underline{\mu_H^2 H^\dagger H}$$

$$\mathcal{L}_{D=3} = -M_N(NN) + \mathrm{h.c.}$$

$$\mathcal{L}_{D=4} = -\frac{1}{4}\sum_{V\in B,W^i,G^a} V_{\mu\nu}V^{\mu\nu} + \sum_{f\in q,u,d,L,e,N} i\bar{f}\gamma^\mu D_\mu f$$
$$-\left(\bar{u}Y_u QH + \bar{d}Y_d H^\dagger Q + \bar{e}Y_e H^\dagger L + \underline{\bar{N}Y_N HL} + \mathrm{h.c.}\right)$$
$$+D_\mu H^\dagger D^\mu H - \lambda(H^\dagger H)^2 + \tilde{\theta} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}$$

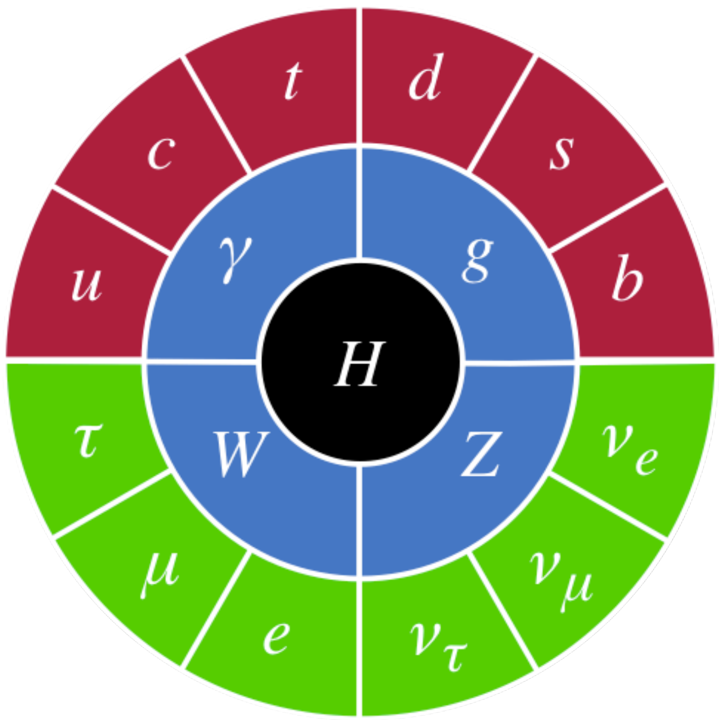
$$\mathcal{L}_{D=5} = \frac{1}{\Lambda}\left\{ c_1(LH)(LH) + \underline{c_2(NN)H^\dagger H + c_3(N\sigma_{\mu\nu}N)B^{\mu\nu}} \right\} + \mathrm{h.c.}$$

$$\mathcal{L}_{D=6} =$$

ψ^2H^3		ψ^2H^2D		$\psi^2HX\,(+\mathrm{h.c.})$	
$\mathcal{O}_{LNH}\,(+\mathrm{h.c.})$	$(\bar{L}N)\tilde{H}(H^\dagger H)$	$\mathcal{O}_{HN}$ $\mathcal{O}_{HNe}\,(+\mathrm{h.c.})$	$(\bar{N}\gamma^\mu N)(H^\dagger i\overleftrightarrow{D}_\mu H)$ $(\bar{N}\gamma^\mu e)(\tilde{H}^\dagger iD_\mu H)$	$\mathcal{O}_{NB}$ $\mathcal{O}_{NW}$	$(\bar{L}\sigma_{\mu\nu}N)\tilde{H}B^{\mu\nu}$ $(\bar{L}\sigma_{\mu\nu}N)\tau^I\tilde{H}W^{I\mu\nu}$
$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$		$(\bar{L}R)(\bar{L}R)\,(+\mathrm{h.c.})$	
$\mathcal{O}_{NN}$ $\mathcal{O}_{eN}$ $\mathcal{O}_{uN}$ $\mathcal{O}_{dN}$ $\mathcal{O}_{duNe}\,(+\mathrm{h.c.})$	$(\bar{N}\gamma^\mu N)(\bar{N}\gamma_\mu N)$ $(\bar{e}\gamma^\mu e)(\bar{N}\gamma_\mu N)$ $(\bar{u}\gamma^\mu u)(\bar{N}\gamma_\mu N)$ $(\bar{d}\gamma^\mu d)(\bar{N}\gamma_\mu N)$ $(\bar{d}\gamma^\mu u)(\bar{N}\gamma_\mu e)$	$\mathcal{O}_{LN}$ $\mathcal{O}_{QN}$	$(\bar{L}\gamma^\mu L)(\bar{N}\gamma_\mu N)$ $(\bar{Q}\gamma^\mu Q)(\bar{N}\gamma_\mu N)$	$\mathcal{O}_{LNLe}$ $\mathcal{O}_{LNQd}$ $\mathcal{O}_{LdQN}$	$(\bar{L}N)\varepsilon(\bar{L}e)$ $(\bar{L}N)\varepsilon(\bar{Q}d))$ $(\bar{L}d)\varepsilon(\bar{Q}N)$
$(\bar{L}R)(\bar{R}L)$		$(\not{L}\cap B)\,(+\mathrm{h.c.})$		$(\not{L}\cap \not{B})\,(+\mathrm{h.c.})$	
$\mathcal{O}_{QuNL}\,(+\mathrm{h.c.})$	$(\bar{Q}u)(\bar{N}L)$	$\mathcal{O}_{NNNN}$	$(NCN)(NCN)$	$\mathcal{O}_{QQdN}$ $\mathcal{O}_{uddN}$	$\varepsilon_{ij}\varepsilon_{\alpha\beta\sigma}(Q^i_\alpha CQ^j_\beta)(d_\sigma CN)$ $\varepsilon_{\alpha\beta\sigma}(u_\alpha Cd_\beta)(d_\sigma CN)$

Liao, Ma
arXiv:1612.04527

SMEFT



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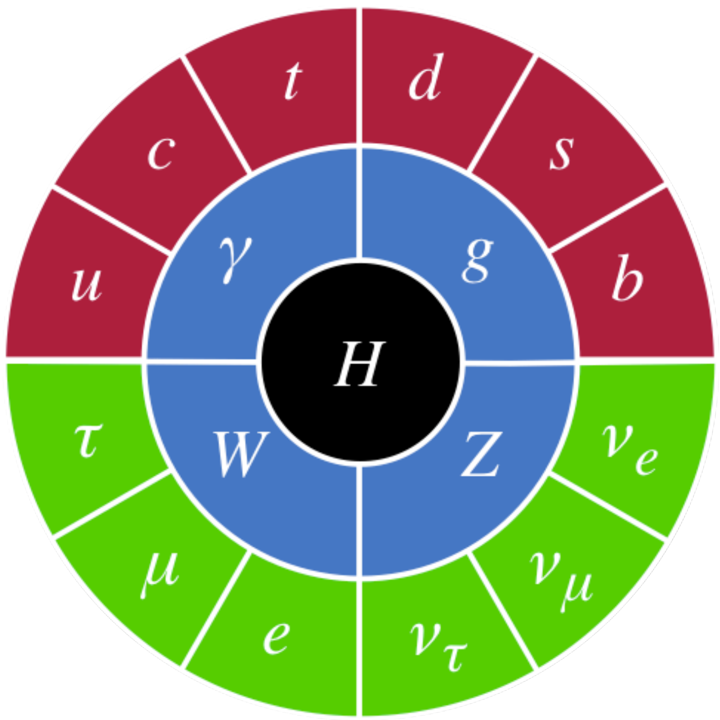
axions

might as well exist

SMEFT



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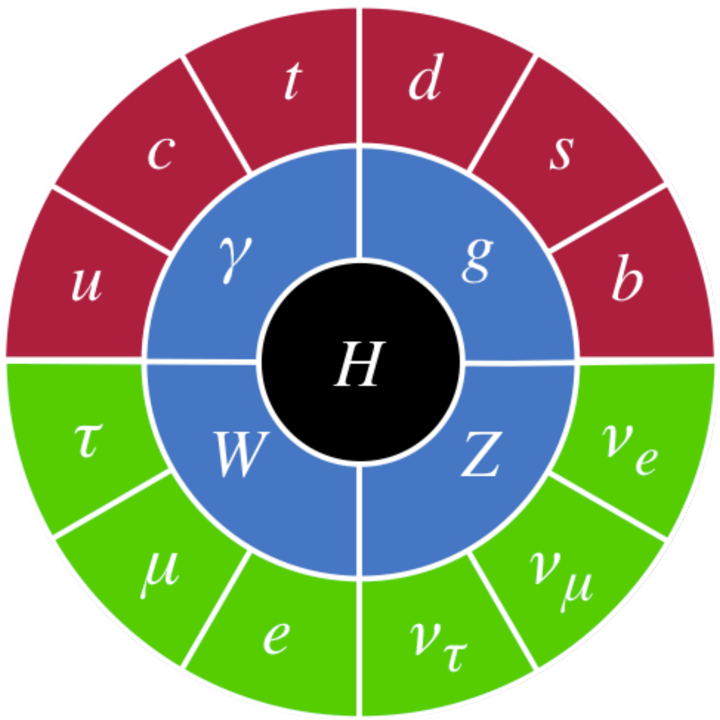
DM	1	?	?	?

Maybe not, because light dark matter might as well exist

SMEFT



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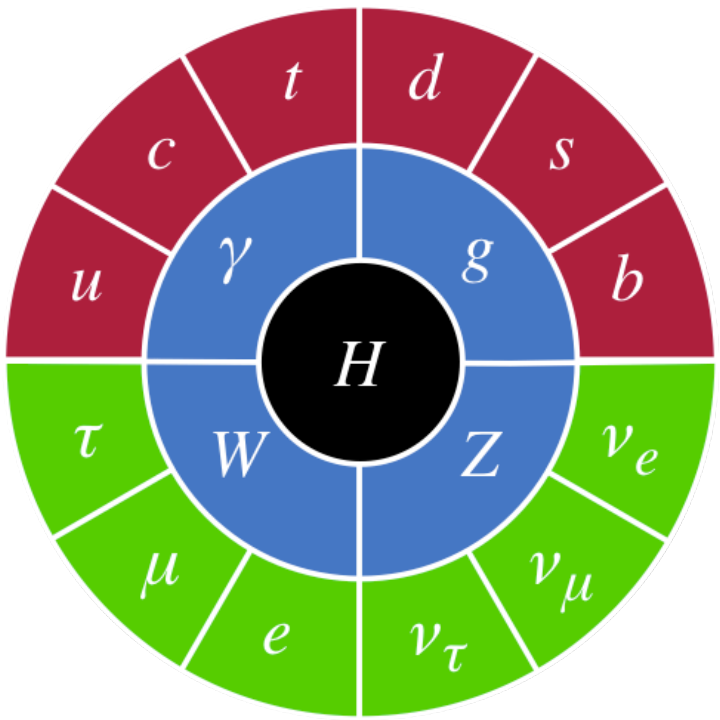
?	?	?	?	?

All in all,

assumption #2 is reasonable,

but it is a serious leap of faith

SMEFT



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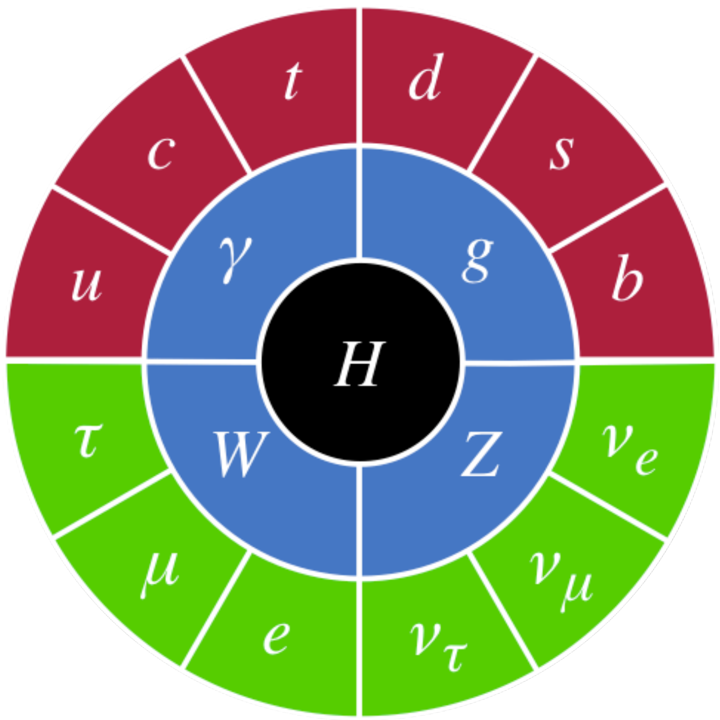

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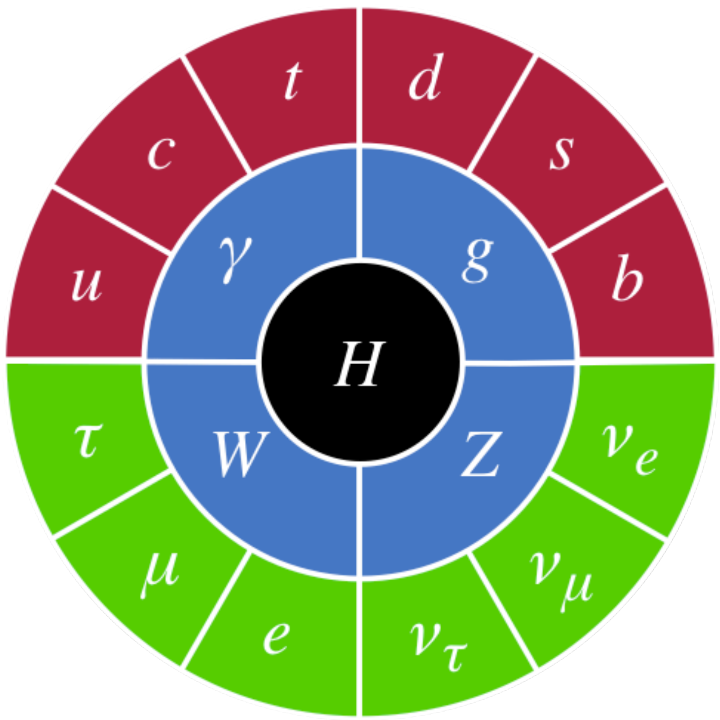
Why should physics beyond SM respect the SM gauge symmetry?

Gauge symmetry

- Gauge symmetries are not real symmetries, in the sense that they do not relate distinct physical states (unlike global symmetries)
- Instead, gauge symmetries are now understood as a redundancy of our theoretical description of fundamental interactions
- As explained e.g. in Weinberg's QFT vol 1 sec 5.9, this redundancy is inevitable if one wants to write down a Lagrangian containing massless gauge bosons in a manifestly Lorentz-invariant way
- Since we need a gauge symmetry for each massless gauge bosons, thus the EFT for SM degrees of freedom must have at least SU(3)xU(1) symmetry

HEFT



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We can work with HEFT

where only SU(3)xU(1)

is linearly realized

Linear vs non-linear

Two mathematical formulations for effective theories with SM spectrum

Linearly realized electroweak symmetry

Non-linearly realized electroweak symmetry

$SU(3)_C \times SU(2)_L \times U(1)_Y$

$SU(3)_C \times U(1)_{em}$

$L \in SU(2)_L \quad R \in U(1)_Y$

$$H \to LH$$

$$U \to LUR^\dagger \qquad h \to h$$

125 GeV Higgs boson

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} iG_1 + G_2 \\ \mathrm{v} + h + iG_3 \end{pmatrix}$$

Goldstone bosons eaten by W and Z

$$U = \exp\left(\frac{i\pi^a \sigma^a}{\mathrm{v}}\right)$$

In general, the two formulations lead to two distinct effective theories

Higgs VEV
$v \approx 246$ GeV

SMEFT $\subset$ **HEFT**

Expansion parameter
$v \approx 246$ GeV

Linear vs non-linear: Higgs self-couplings

In the SM self-coupling completely fixed...

$$\mathscr{L}_{\text{SM}} \supset m^2|H|^2 - \lambda|H|^4$$

$$\to -\frac{1}{2}m_h^2 h^2 - \frac{m_h^2}{2\text{v}}h^3 - \frac{m_h^2}{8\text{v}^2}h^4$$

...but they can be deformed by BSM effects

SMEFT

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} - \frac{c_6}{\Lambda^2}|H|^6 + \mathscr{O}(\Lambda^{-4})$$

$$\mathscr{L}_{\text{SMEFT}} \supset -\frac{m_h^2}{2\text{v}}(1+\delta\lambda_3)h^3 - \frac{m_h^2}{8\text{v}^2}(1+\delta\lambda_4)h^4 - \frac{\lambda_5}{\text{v}}h^5 - \frac{\lambda_6}{\text{v}^2}h^6$$

$$\delta\lambda_3 = \frac{2c_6\text{v}^4}{m_h^2\Lambda^2},\ \delta\lambda_4 = \frac{12c_6\text{v}^4}{m_h^2\Lambda^2},\ \lambda_5 = \frac{3c_6\text{v}^2}{4\Lambda^2},\ \lambda_6 = \frac{c_6\text{v}^2}{8\Lambda^2}$$

SMEFT: Predicts correlations between self-couplings as long as Λ >> v

HEFT

$$\mathscr{L}_{\text{HEFT}} \supset -c_3\frac{m_h^2}{2\text{v}}h^3 - c_4\frac{m_h^2}{8\text{v}^2}h^4 - \frac{c_5}{\text{v}}h^5 - \frac{c_6}{\text{v}^2}h^6 + \ldots$$

HEFT: no correlations between self-couplings

Linear vs non-linear

AA, Rattazzi
arXiv:1902.05936

- SMEFT and HEFT lead to a vastly different phenomenology at the electroweak scale

- Choosing SMEFT or HEFT implicitly entails an assumption about a class of BSM theories that we want to characterize

- SMEFT is appropriate to describe BSM theories which can be parametrically decoupled, that is to say, where the mass scale of the new particles depends on a free parameter(s) that can be taken to infinity

- Conversely, HEFT is appropriate to describe non-decoupling BSM theories, where the masses of the new particles vanish in the limit $v \to 0$

Example: cubic Higgs deformation

Consider a toy EFT model where Higgs cubic (and only that) deviates from the SM

$$\mathscr{L} = \mathscr{L}_{\mathrm{SM}} - \Delta_3 \frac{m_h^2}{2\mathrm{v}} h^3$$

$$V(h) = \frac{m_h^2}{2} h^2 + \frac{m_h^2}{2\mathrm{v}} \left(1 + \Delta_3\right) h^3 + \frac{m_h^2}{8\mathrm{v}^2} h^4$$

This EFT belongs to the HEFT but not SMEFT parameter space

HEFT = Non-analytic Higgs potential

$$V(h) = \frac{m_h^2}{2}h^2 + \frac{m_h^2}{2\mathrm{v}}\left(1+\Delta_3\right)h^3 + \frac{m_h^2}{8\mathrm{v}^2}h^4 \qquad \textbf{(1)}$$

Given a Lagrangian for Higgs boson h, one can always uplift it to a manifestly SU(2)xU(1) invariant form by replacing

$$h \to \sqrt{2H^\dagger H} - \mathrm{v}$$

After this replacement, Higgs potential contains terms non-analytic at H=0

$$V(H) = \frac{m_h^2}{8\mathrm{v}^2}\left(2H^\dagger H - \mathrm{v}^2\right)^2 + \Delta_3 \frac{m_h^2}{2\mathrm{v}}\left(\sqrt{2H^\dagger H} - \mathrm{v}\right)^3 \qquad \textbf{(2)}$$

(1) and (2) are equal in the unitary gauge

$$H \to \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \mathrm{v}+h \end{pmatrix}$$

<u>Thus, (1) and (2) describe the same physics</u>

Non-analytic Higgs potential

$$V(H) = \frac{m_h^2}{8\mathrm{v}^2}\left(2H^\dagger H - \mathrm{v}^2\right)^2 + \Delta_3 \frac{m_h^2}{2\mathrm{v}}\left(\sqrt{2H^\dagger H} - \mathrm{v}\right)^3$$

In the unitary gauge, the Higgs potential looks totally healthy and renormalizable…

Going away from the unitary gauge:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} iG_1 + G_2 \\ \mathrm{v} + h + iG_3 \end{pmatrix} \quad \Rightarrow \quad V \supset \Delta_3 \frac{m_h^2}{2\mathrm{v}}\left(\sqrt{(h+\mathrm{v})^2 + G^2} - \mathrm{v}\right)^3$$

$$G^2 \equiv \sum_i G_i^2$$

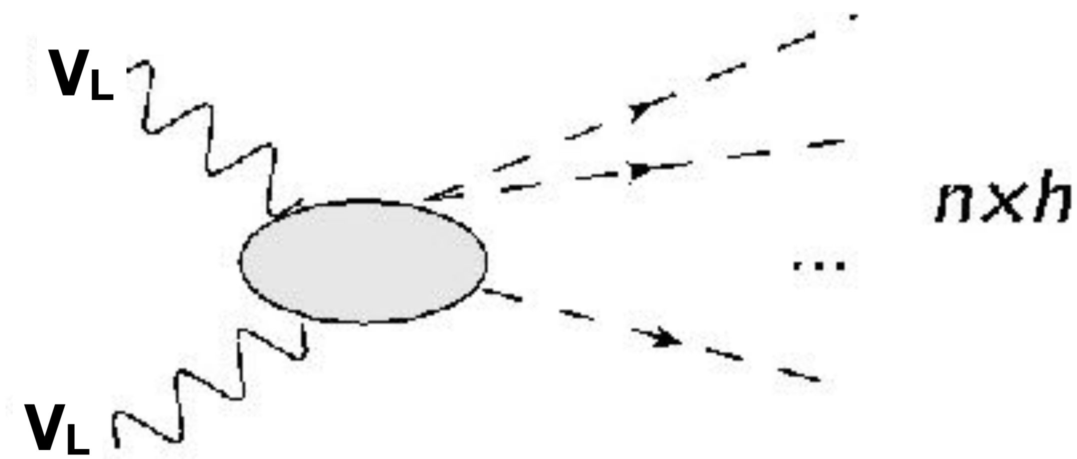
Away from the unitary gauge, it becomes clear that the Higgs potential contains non-renormalizable interactions suppressed only by the EW scale v

$$V \supset \Delta_3 \frac{3m_h^2}{4\mathrm{v}} \frac{G^2 h^2}{h + \mathrm{v}} + \mathcal{O}(G^4) = \Delta_3 \frac{3m_h^2}{4} G^2 \sum_{n=2}^{\infty}\left(\frac{-h}{\mathrm{v}}\right)^n + \mathcal{O}(G^4)$$

Multi-Higgs production

Consider VBF production of n ≥ 2 Higgs bosons: $V_L V_L \to n \times h$

By the equivalence theorem, at high energies the same as $GG \to n \times h$



Expanded potential contains interactions

$$V \supset = \Delta_3 \frac{3m_h^2}{4} G^2 \sum_{n=2}^{\infty} \left(\frac{-h}{\mathrm{v}}\right)^n$$

leading to interaction vertices with arbitrary number of Higgs bosons

$$\mathcal{M}(GG \to \underbrace{h \ldots h}_{n}) \sim \Delta_3 \frac{n! m_h^2}{\mathrm{v}^n}$$

Amplitudes for multi-Higgs production in W/Z boson fusion are only suppressed by the scale v and do not decay with growing energy, leading to unitarity loss at some scale right above v

Unitarity primer

S matrix unitarity $S^\dagger S = 1$

symmetry factor for n-body final state

implies relation between forward scattering amplitude, and elastic and inelastic production cross sections

$$2\mathrm{Im}\mathscr{M}(p_1p_2 \to p_1p_2) = S_2 \int d\Pi_2 \, |\mathscr{M}^{\mathrm{elastic}}(p_1p_2 \to k_1k_2)|^2 + \sum S_n \int d\Pi_n \, |\mathscr{M}^{\mathrm{inelastic}}(p_1p_2 \to k_1 \ldots k_n)|^2$$

Equation is "diagonalized" after initial and final 2-body state are projected into partial waves

$$a_l(s) = \frac{S_2}{16\pi}\sqrt{1-\frac{4m^2}{s}} \int_{-1}^{1} d\cos\theta P_l(\cos\theta)\mathcal{M}(s,\cos\theta),$$

$$2\mathrm{Im}a_l = a_l^2 + \sum S_n \int d\Pi_n \, |\mathscr{M}_l^{\mathrm{inelastic}}|^2$$

This can be rewritten as the Argand circle equation

$$(\mathrm{Re}a_l)^2 + (\mathrm{Im}a_l - 1)^2 = R_l^2, \qquad R_l^2 = 1 - \sum S_n \int d\Pi_n \, |\mathscr{M}_l^{\mathrm{inelastic}}|^2$$

Unitarity primer

Argand circle equation

$$(\mathrm{Re}\,a_l)^2 + (\mathrm{Im}\,a_l - 1)^2 = R_l^2, \qquad R_l^2 = 1 - \sum S_n \int d\Pi_n |\mathscr{M}_l^{\text{inelastic}}|^2$$

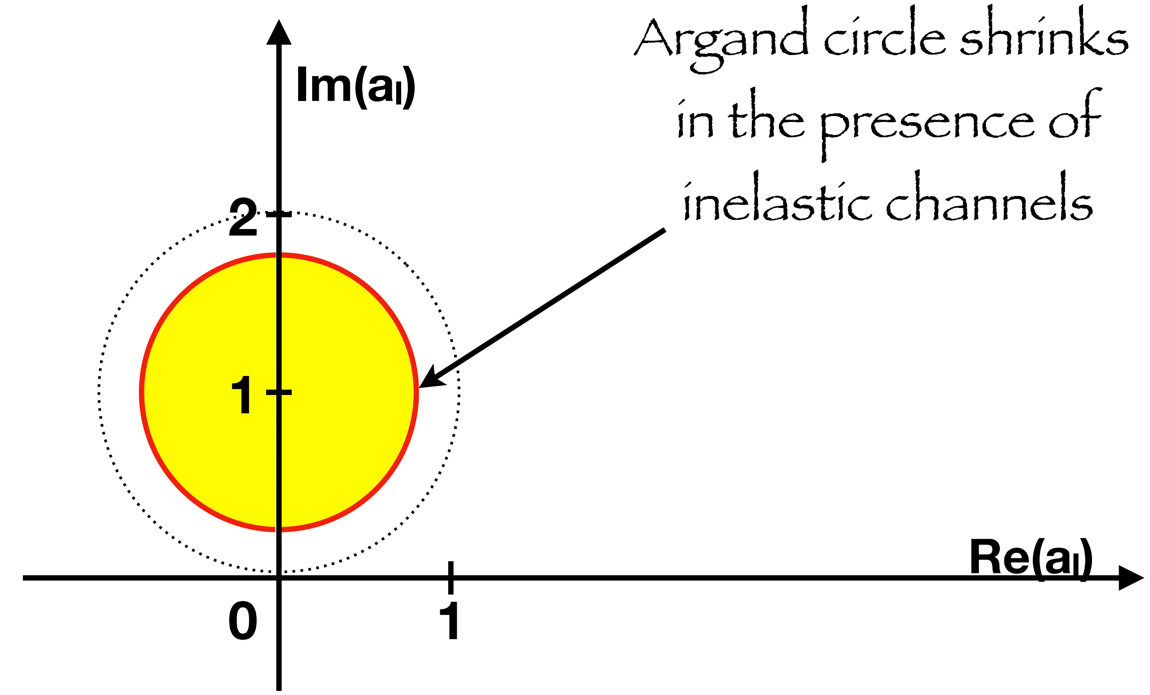
implies constraints on <u>both</u> elastic and inelastic amplitudes

Often used

$$|\mathrm{Re}\,a_l| \leq 1$$

$$\sum S_n \int d\Pi_n |\mathscr{M}_l^{\text{inelastic}}|^2 \leq 1$$

Often forgotten



Unitarity constraints on inelastic channels

Unitarity (strong coupling) constraint on inelastic multi-Higgs production

$$\sum_{n=2}^{\infty} \frac{1}{n!} \int d\Pi_n \, |\mathcal{M}(GG \to h^n)|^2 = \sum_{n=2}^{\infty} \frac{1}{n!} V_n(\sqrt{s}) \, |\mathcal{M}(GG \to h^n)|^2 \lesssim \mathcal{O}(1)$$

Volume of phase space in the massless limit:

$$V_n(\sqrt{s}) = \int d\Pi_n = \frac{s^{n-2}}{2(n-1)!(n-2)!(4\pi)^{2n-3}} \sim \frac{s^{n-2}}{(n!)^2(4\pi)^{2n}}$$

In a fundamental theory,

2 → n amplitude must decay as $1/s^{n/2-1}$

in order to maintain unitarity up to arbitrary high scales

Process	*Unitarity limit*
2 → 2	**1**
2 → 3	**$1/s^{1/2}$**
2 → 4	**$1/s$**
...	...

Unitarity constraints on HEFT

Unitarity equation

$$\sum_{n=2}^{\infty}\frac{1}{n!}V_n(\sqrt{s})\,|\mathscr{M}(GG\to h^n)|^2 \lesssim \mathcal{O}(1)$$

Our amplitude

$$\mathscr{M}(GG\to \underbrace{h\ldots h}_{n}) \sim \Delta_3\frac{n!m_h^2}{\mathrm{v}^n}$$

$$\mathcal{O}(1) \gtrsim \sum_{n=2}^{\infty}\frac{1}{n!}V_n(\sqrt{s})\,|\mathscr{M}(GG\to h^n)|^2 \sim \sum_{n=2}^{\infty}\frac{1}{n!}\frac{s^{n-2}}{(n!)^2(4\pi)^{2n}}\Delta_3^2\frac{(n!)^2m_h^4}{\mathrm{v}^{2n}} \sim \frac{\Delta_3^2 m_h^4}{s^2}\exp\left[\frac{s}{(4\pi\mathrm{v})^2}\right]$$

In model with deformed Higgs cubic, multi-Higgs amplitude do not decay with energy leading to unitarity loss at a finite value of energy

$$\boxed{\Lambda \lesssim (4\pi\mathrm{v})\log^{1/2}\left(\frac{4\pi\mathrm{v}}{m_h|\Delta_3|^{1/2}}\right)}$$

Unless Δ_3 is unobservably small, unitarity loss happens at the scale 4 π v ~ 3 TeV !

Perspective on HEFT

Example of UV model leading to non-analytic terms in low-energy effective theory

$$\mathscr{L}_{\mathrm{UV}} = \mathscr{L}_{\mathrm{SM}} - \frac{\kappa}{2}|\Phi|^4 + \mu^2(\Phi^\dagger H + \mathrm{h.c.})$$

Eqs of motion: $$\Phi = \left(\frac{\mu^2}{\kappa H^\dagger H}\right)^{1/3} H$$

Effective Lagrangian: $$\mathscr{L}_{\mathrm{EFT}} \approx \mathscr{L}_{\mathrm{SM}} + \frac{3\mu^{8/3}}{2\kappa^{1/3}}\left(H^\dagger H\right)^{2/3}$$

Non-analyticity appears because of integrating out particle that would be massless in the absence of EW symmetry breaking

Perspective on HEFT

More familiar example is integrating out 4th chiral generation at one loop, which produces Log|H|^2 terms in the Coleman-Weinberg potential

Below a similar example with scalar instead of fermion:

Cohen et al arXiv:2008.08597

$$\mathcal{L}_{\mathrm{UV}} = |\partial H|^2 + \mu_H^2 |H|^2 - \lambda_H |H|^4 + \frac{1}{2} S\left(-\partial^2 - m^2 - \kappa |H|^2\right) S$$

Integrating out the scalar S at one produces the CW potential:

$$\int \mathrm{d}^4 x\, \mathcal{L}_{\text{Eff, 1-loop}}(H) = \frac{i}{2} \log \det{}_S\left(\partial^2 + m^2 + \kappa |H|^2\right)$$

Effective Lagrangian at zero and two derivative levels:

$$\mathcal{L}_{\mathrm{Eff}}^{(0)} = \mu_H^2 |H|^2 - \lambda_H |H|^4 + \frac{1}{64\pi^2}\left(m^2 + \kappa |H|^2\right)^2 \left(\ln \frac{\mu^2}{m^2 + \kappa |H|^2} + \frac{3}{2}\right)$$

$$\mathcal{L}_{\mathrm{Eff}}^{(2)} = |\partial H|^2 + \frac{1}{384\pi^2} \frac{\kappa^2}{m^2 + \kappa |H|^2}\left(\partial |H|^2\right)^2 .$$

For $m^2 \gg \kappa \mathrm{v}^2$ we can expand in powers if $1/m^2$, which leads to analytic SMEFT Lagrangian

For $m^2 \ll \kappa \mathrm{v}^2$ we cannot expand, and effective Lagrangian is non-analytic, which corresponds to HEFT

Linear vs non-linear summary

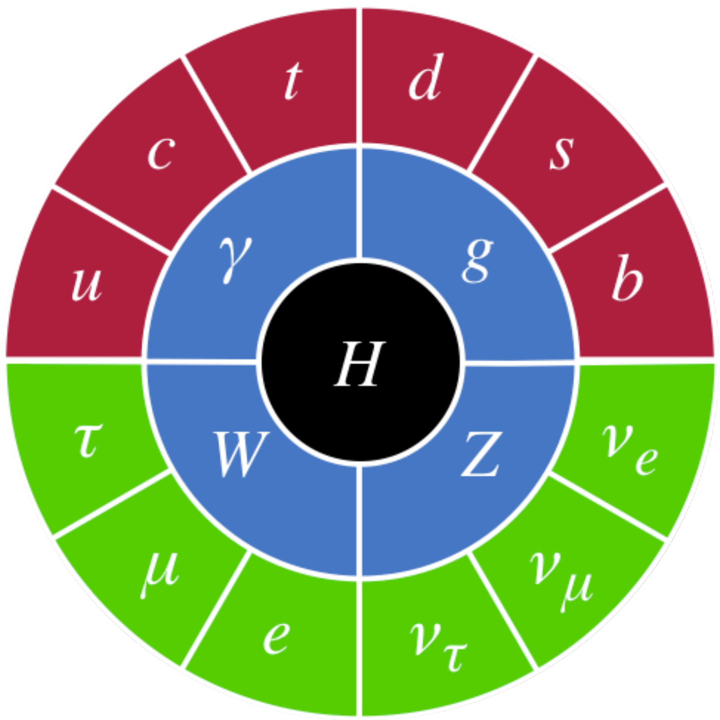
- EFT with non-linearly realized electroweak symmetry (aka HEFT) is equivalent to EFT with linearly realized electroweak symmetry but whose Lagrangian is a non-polynomial function of the Higgs field that is non-analytic at $H=0$

- This non-analyticity leads to explosion of multi-Higgs amplitudes at the scale $4\pi v$. For this reason, the validity regime of HEFT is limited below the scale of order $4\pi v \sim 3$ TeV

- HEFT is useful to approximate BSM theories where new particles' masses vanish in the limit $v \to 0$, e.g. SM + a 4th generation of chiral fermions or when most of the new particle mass comes from EW symmetry breaking

- On the other hand, an EFT with linearly realized electroweak symmetry and the Lagrangian polynomial in the Higgs field (aka SMEFT) is useful to approximate BSM theories where new particles' masses do not vanish in the limit $v \to 0$, and are parametrically larger than the electroweak scale, e.g. SM + vector-like fermions

SMEFT



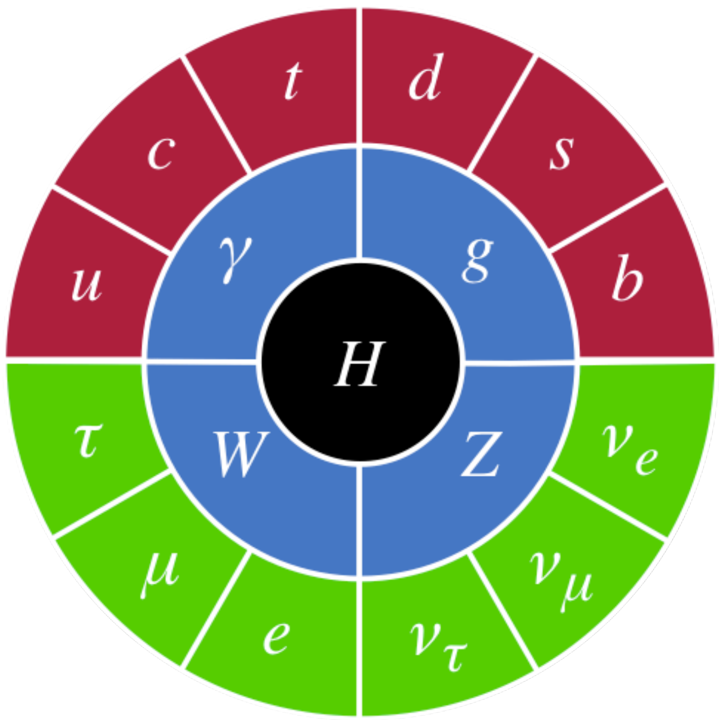
But are these assumptions true?

1. **Unitarity, locality, Poincaré symmetry**
2. **Mass gap: absence of non-SM degrees of freedom at or below the electroweak scale**
3. **Gauge symmetry: local SU(3)xSU(2)xU(1) symmetry strictly respected by all interactions and spontaneously broken to SU(3)xU(1) by a VEV of the Higgs field**

	G_μ^a	W_μ^k	B_μ	1
	$SU(3)_C$	$SU(2)_W$	$U(1)_Y$	Spin
$Q = (u_L,d_L)$	3	2	1/6	1/2
u_R	3	1	2/3	1/2
d_R	3	1	-1/3	1/2
$L = (\nu_L,e_L)$	1	2	-1/2	1/2
e_R	1	1	-1	1/2
H	1	2	1/2	0

This is a very reasonable assumption, given we haven't seen non-SM particles around the EW scale

SMEFT



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	G_μ^a	W_μ^k	B_μ	**1**
	SU(3)$_C$	**SU(2)$_W$**	**U(1)$_Y$**	**Spin**
Q = (u$_L$,d$_L$)	3	2	1/6	1/2
u$_R$	3	1	2/3	1/2
d$_R$	3	1	-1/3	1/2
L = (ν$_L$,e$_L$)	1	2	-1/2	1/2
e$_R$	1	1	-1	1/2
H	1	2	1/2	0

In assumptions #2 and #3 are related

#3 means that the mass gap is such that

$$\Lambda \gtrsim 4\pi v \sim 3\ \mathrm{TeV}$$

Part 4

Bases of SMEFT

Bases of EFT

- Quantum field theories formulated in terms of fields and Lagrangians have an important redundancy, in addition to the gauge symmetry redundancy

- The point is that quantum fields are not physical observables, but merely tools in our computations, akin to integration variables under the integral

- Continuing with this analogy, changing variables, that is field redefinitions, do not change the physical content of the theory. However they do change the Lagrangian!

- Therefore Lagrangian parameters are "measurable" only after (redundant) operators in the same equivalence classes are eliminated. This can be done in practice by eliminating certain terms using equations of motion for the EFT fields, as this is equivalent to using field redefinitions.

- Since the elimination of redundant operators can be performed in many different manners, a single EFT corresponds to an infinite number of Lagrangians that lead to equivalent results. For a given canonical dimension, these different Lagrangians are called <u>bases</u>.

- Thus, the SMEFT has an infinite number of equivalent bases, at each canonical dimension. They are multi-dimensional, e.g. at dimension 6 each basis has 3045 different interaction terms

- To illustrate the concept of the basis, let us first consider a simpler toy example, where the dimension-6 basis has one element

Toy model EFT Lagrangian

Consider an EFT of a single real scalar field ϕ with Z2 symmetry $\phi \to -\phi$
By general arguments, the EFT Lagrangian must have the following form

$$\mathscr{L}_{\mathrm{EFT}} = \frac{1}{2}\left[(\partial_\mu \phi)^2 - m^2\phi^2\right] - C_4\frac{\phi^4}{4!} - \frac{C_6}{\Lambda^2}\frac{\phi^6}{6!} + \mathcal{O}(\Lambda^{-4})$$

In this discussion we truncate the Lagrangian at order Λ^{-4},
ignoring all operators with dimension higher than six

Operators with odd dimensions do not appear in this EFT because of the Z2 symmetry

What about other dimension-6 operators, e.g.

$$\hat{O}_6 \equiv (\Box\phi)^2, \quad \tilde{O}_6 \equiv \phi\,\Box\,\phi^3, \quad \tilde{O}_6' \equiv \phi^2\,\Box\,\phi^2, \quad \tilde{O}_6'' \equiv \phi^2\partial_\mu\phi\partial_\mu\phi, \quad \ldots$$

These are all <u>redundant</u>, that is to say,
they can be expressed by the operators already present in $\mathscr{L}_{\mathrm{EFT}}$ by using equations of motion, fields redefinitions, and integration by parts

Redundant operators

$$\mathscr{L}_{\rm EFT} = \frac{1}{2}\left[(\partial_\mu\phi)^2 - m^2\phi^2\right] - C_4\frac{\phi^4}{4!} - \frac{C_6}{\Lambda^2}\frac{\phi^6}{6!} + \mathcal{O}(\Lambda^{-4})$$

$$O_6 \equiv \phi^6, \quad \hat{O}_6 \equiv (\Box\phi)^2, \quad \tilde{O}_6 \equiv \phi^3\Box\phi, \quad \tilde{O}_6' \equiv \phi^2\Box\phi^2, \quad \tilde{O}_6'' \equiv \phi^2\partial_\mu\phi\partial_\mu\phi, \quad \ldots$$

Use Leibniz rule + integration by parts:

$$\phi^2\partial_\mu\phi\partial_\mu\phi = -2\phi\partial_\mu\phi\partial_\mu\phi\phi - \phi^3\Box\phi \qquad \Rightarrow \qquad \tilde{O}_6'' = -\frac{1}{3}\phi^3\Box\phi = -\frac{1}{3}\tilde{O}_6$$

$$\phi^2\Box\phi^2 = 2\phi^2\partial_\mu(\phi\partial_\mu\phi) = 2\phi^3\Box\phi + 2\phi^2(\partial_\mu\phi)^2 \qquad \Rightarrow \qquad \tilde{O}_6' = 2\tilde{O}_6 + 2\tilde{O}_6'' = \frac{4}{3}\tilde{O}_6$$

Use equations of motion: $\Box\phi = -m^2\phi - \dfrac{C_4}{6}\phi^3 + \mathcal{O}(\Lambda^{-2})$

This is relevant only if we want to keep track of dimension-8 operators

$$\tilde{O}_6 \equiv \phi^3\Box\phi = -m^2\phi^4 - \frac{C_4}{6}\phi^6 = -m^2 O_4 - \frac{C_4}{6}O_6$$

$$\hat{O}_6 \equiv (\Box\phi)^2 = m^4\phi^2 + \frac{m^2C_4}{3}\phi^4 + \frac{C_4^2}{36}\phi^6 = m^4 O_2 + \frac{m^2C_4}{3}O_4 + \frac{C_4^2}{36}O_6$$

$O_2 \equiv \phi^2$

$O_4 \equiv \phi^4$

Bases of operators

"Unbox basis"

$$\mathscr{L}_{\mathrm{EFT}} = \frac{1}{2}\left[(\partial_\mu\phi)^2 - m^2\phi^2\right] - C_4\frac{\phi^4}{4!} - \frac{C_6}{\Lambda^2}\frac{\phi^6}{6!} + \mathcal{O}(\Lambda^{-4})$$

We can equivalently use an EFT Lagrangian where O_6 is absent, and replaced by another equivalent operator

$$O_2 \equiv \phi^2$$

$$O_4 \equiv \phi^4$$

$$O_6 \equiv \phi^6$$

$$\tilde{O}_6 \equiv \phi^3\Box\phi = -m^2 O_4 - \frac{C_4}{6}O_6 \Rightarrow O_6 = -\frac{6}{C_4}\phi^3\Box\phi - \frac{6m^2}{C_4}O_4$$

"Box basis"

$$\mathscr{L}_{\mathrm{EFT}} = \frac{1}{2}\left[(\partial_\mu\phi)^2 - \tilde{m}^2\phi^2\right] - \tilde{C}_4\frac{\phi^4}{4!} - \frac{\tilde{C}_6}{\Lambda^2}\frac{\phi^3\Box\phi}{4!} + \mathcal{O}(\Lambda^{-4})$$

Map between the Wilson coefficients in the two bases:

$$\tilde{C}_6 = -\frac{C_6}{5C_4}$$

$$\tilde{C}_4 = C_4 - \frac{m^2}{\Lambda^2}\frac{C_6}{5C_4}$$

$$\tilde{m} = m$$

Bases of operators

"Unbox basis"

$$\mathscr{L}_{\text{EFT}} = \frac{1}{2}\left[(\partial_\mu\phi)^2 - m^2\phi^2\right] - C_4\frac{\phi^4}{4!} - \frac{C_6}{\Lambda^2}\frac{\phi^6}{6!} + \mathcal{O}(\Lambda^{-4})$$

We can equivalently use an EFT Lagrangian where O_6 is absent, and replaced by another equivalent operators

$$O_2 \equiv \phi^2$$
$$O_4 \equiv \phi^4$$
$$O_6 \equiv \phi^6$$

$$\hat{O}_6 \equiv (\Box\phi)^2 = m^4\phi^2 + \frac{m^2C_4}{3}\phi^4 + \frac{C_4^2}{36}\phi^6 \quad\Rightarrow\quad O_6 = \frac{36}{C_4^2}\hat{O}_6 - 12\frac{m^2}{C_4}O_4 + \frac{36m^4}{C_4^2}O_2$$

"Double-Box basis"

$$\mathscr{L}_{\text{EFT}} = \frac{1}{2}\left[(\partial_\mu\phi)^2 - \hat{m}^2\phi^2\right] - \hat{C}_4\frac{\phi^4}{4!} - \frac{\hat{C}_6}{\Lambda^2}\frac{(\Box\phi)^2}{2} + \mathcal{O}(\Lambda^{-4})$$

Map between the Wilson coefficients in the two bases

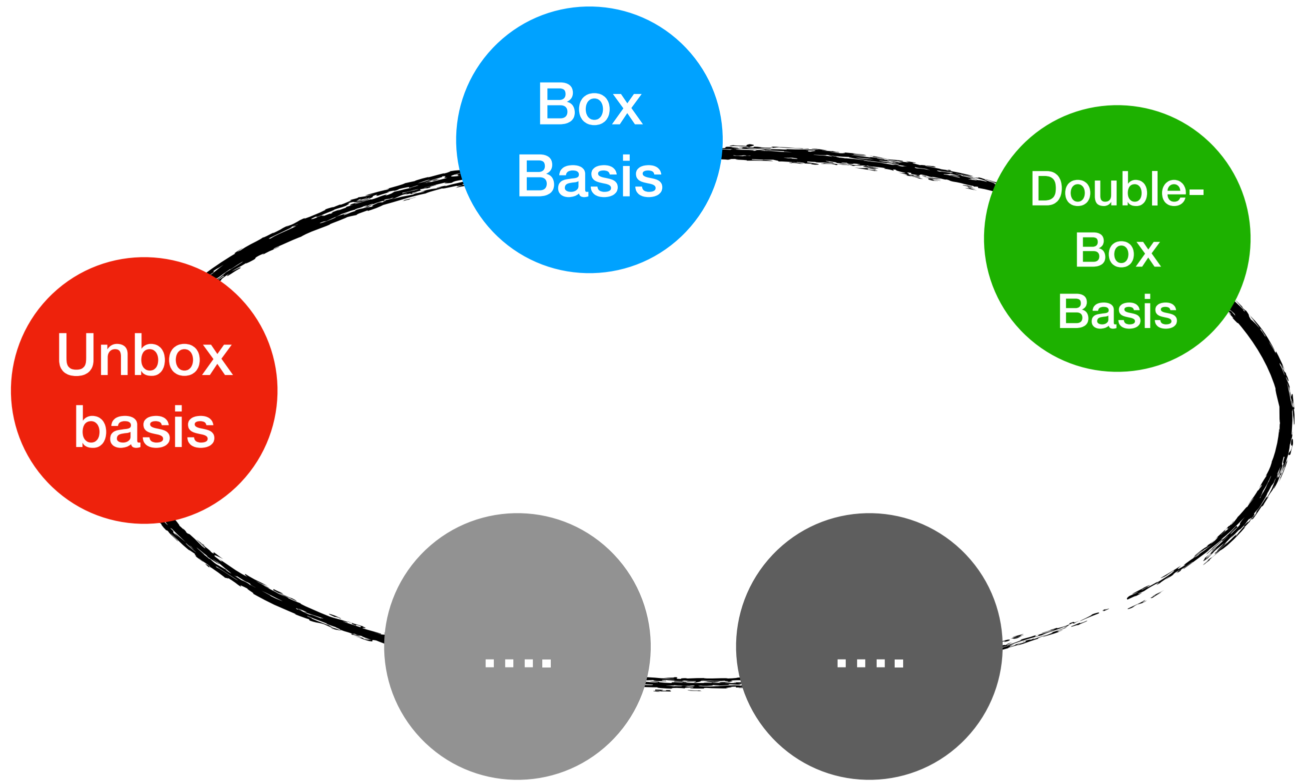
$$\hat{C}_6 = -\frac{C_6}{10C_4^2}$$

$$\hat{C}_4 = C_4 - \frac{m^2}{\Lambda^2}\frac{2C_6}{5C_4}$$

$$\hat{m}^2 = m^2 - \frac{m^4}{\Lambda^2}\frac{C_6}{30C_4^2}$$

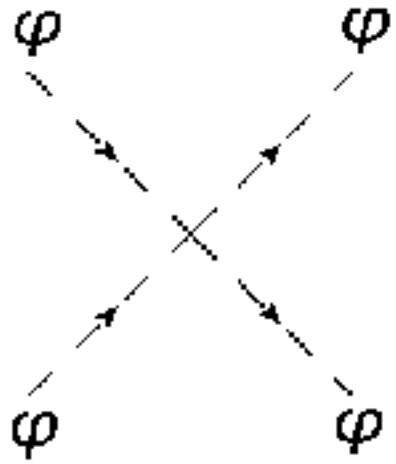
Bases of operators

Every EFT has an infinite number of equivalent bases



Physics is independent of which basis we use,
but the Lagrangian and intermediate calculations look different in different bases!

In our toy example, a basis of dimension-6 operators is one dimensional
(to be compared e.g. with the -dimensional basis of dimension-6 operators in the SMEFT)



Bases of operators

Consider 2-to-2 scattering in the box and unbox bases

“Unbox basis”

$$\mathscr{L}_{\text{EFT}} = \frac{1}{2}\left[(\partial_\mu \phi)^2 - m^2\phi^2\right] - C_4 \frac{\phi^4}{4!} - \frac{C_6}{\Lambda^2}\frac{\phi^6}{6!} + \mathcal{O}(\Lambda^{-4})$$

$$\mathscr{M}^{\text{unbox}}_{EFT} = -C_4 + \mathcal{O}(\Lambda^{-4})$$

“Box basis”

$$\mathscr{L}_{\text{EFT}} = \frac{1}{2}\left[(\partial_\mu \phi)^2 - m^2\phi^2\right] - \tilde{C}_4 \frac{\phi^4}{4!} - \frac{\tilde{C}_6}{\Lambda^2}\frac{\phi^3 \Box \phi}{4!} + \mathcal{O}(\Lambda^{-4})$$

$$\mathscr{M}^{\text{box}}_{EFT} = -\tilde{C}_4 + \frac{\tilde{C}_6}{4\Lambda^2}\left(p_1^2 + p_2^2 + p_3^2 + p_4^2\right) + \mathcal{O}(\Lambda^{-4})$$

$$= -\tilde{C}_4 + \tilde{C}_6 \frac{m^2}{\Lambda^2} + \mathcal{O}(\Lambda^{-4})$$

Map

$$\boxed{\begin{aligned} \tilde{C}_6 &= -\frac{C_6}{5C_4} \\ \tilde{C}_4 &= C_4 - \frac{m^2}{M^2}\frac{C_6}{5C_4} \end{aligned}}$$

on-shell

$$\mathscr{M}^{\text{unbox}}_{EFT} = \mathscr{M}^{\text{box}}_{EFT} + \mathcal{O}(\Lambda^{-4})$$

SMEFT at dimension-6

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \dots$$

Grzadkowski et al
arXiv:1008.4884

<u>Warsaw basis</u> of B-conserving dimension-6 operators

Bosonic CP-even		Bosonic CP-odd	
O_H	$(H^\dagger H)^3$		
$O_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$		
O_{HD}	$\left\lvert H^\dagger D_\mu H \right\rvert^2$		
O_{HG}	$H^\dagger H\, G^a_{\mu\nu} G^a_{\mu\nu}$	$O_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}^a_{\mu\nu} G^a_{\mu\nu}$
O_{HW}	$H^\dagger H\, W^i_{\mu\nu} W^i_{\mu\nu}$	$O_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}^i_{\mu\nu} W^i_{\mu\nu}$
O_{HB}	$H^\dagger H\, B_{\mu\nu} B_{\mu\nu}$	$O_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu} B_{\mu\nu}$
O_{HWB}	$H^\dagger \sigma^i H\, W^i_{\mu\nu} B_{\mu\nu}$	$O_{H\widetilde{W}B}$	$H^\dagger \sigma^i H\, \widetilde{W}^i_{\mu\nu} B_{\mu\nu}$
O_W	$\epsilon^{ijk} W^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}$	$O_{\widetilde{W}}$	$\epsilon^{ijk} \widetilde{W}^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}$
O_G	$f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$	$O_{\widetilde{G}}$	$f^{abc} \widetilde{G}^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$

Yukawa	
$[O^\dagger_{eH}]_{IJ}$	$H^\dagger H e^c_I H^\dagger \ell_J$
$[O^\dagger_{uH}]_{IJ}$	$H^\dagger H u^c_I \widetilde{H}^\dagger q_J$
$[O^\dagger_{dH}]_{IJ}$	$H^\dagger H d^c_I H^\dagger q_J$

Vertex		Dipole	
$[O^{(1)}_{H\ell}]_{IJ}$	$i\bar{\ell}_I \bar{\sigma}_\mu \ell_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{eW}]_{IJ}$	$e^c_I \sigma_{\mu\nu} H^\dagger \sigma^i \ell_J W^i_{\mu\nu}$
$[O^{(3)}_{H\ell}]_{IJ}$	$i\bar{\ell}_I \sigma^i \bar{\sigma}_\mu \ell_J H^\dagger \sigma^i \overleftrightarrow{D}_\mu H$	$[O^\dagger_{eB}]_{IJ}$	$e^c_I \sigma_{\mu\nu} H^\dagger \ell_J B_{\mu\nu}$
$[O_{He}]_{IJ}$	$ie^c_I \sigma_\mu \bar{e}^c_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{uG}]_{IJ}$	$u^c_I \sigma_{\mu\nu} T^a \widetilde{H}^\dagger q_J\, G^a_{\mu\nu}$
$[O^{(1)}_{Hq}]_{IJ}$	$i\bar{q}_I \bar{\sigma}_\mu q_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{uW}]_{IJ}$	$u^c_I \sigma_{\mu\nu} \widetilde{H}^\dagger \sigma^i q_J\, W^i_{\mu\nu}$
$[O^{(3)}_{Hq}]_{IJ}$	$i\bar{q}_I \sigma^i \bar{\sigma}_\mu q_J H^\dagger \sigma^i \overleftrightarrow{D}_\mu H$	$[O^\dagger_{uB}]_{IJ}$	$u^c_I \sigma_{\mu\nu} \widetilde{H}^\dagger q_J\, B_{\mu\nu}$
$[O_{Hu}]_{IJ}$	$iu^c_I \sigma_\mu \bar{u}^c_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{dG}]_{IJ}$	$d^c_I \sigma_{\mu\nu} T^a H^\dagger q_J\, G^a_{\mu\nu}$
$[O_{Hd}]_{IJ}$	$id^c_I \sigma_\mu \bar{d}^c_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{dW}]_{IJ}$	$d^c_I \sigma_{\mu\nu} \bar{H}^\dagger \sigma^i q_J\, W^i_{\mu\nu}$
$[O_{Hud}]_{IJ}$	$iu^c_I \sigma_\mu \bar{d}^c_J \tilde{H}^\dagger D_\mu H$	$[O^\dagger_{dB}]_{IJ}$	$d^c_I \sigma_{\mu\nu} H^\dagger q_J\, B_{\mu\nu}$

Table 2.3: Two-fermion D=6 operators in the Warsaw basis. The flavor indices denoted by I, J. For complex operators (O_{Hud} and all Yukawa and dipole operat the corresponding complex conjugate operator is implicitly included.

$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
O_{ee}	$\eta(e^c\sigma_\mu \bar{e}^c)(e^c\sigma_\mu \bar{e}^c)$	$O_{\ell e}$	$(\bar{\ell}\bar{\sigma}_\mu \ell)(e^c\sigma_\mu \bar{e}^c)$
O_{uu}	$\eta(u^c\sigma_\mu \bar{u}^c)(u^c\sigma_\mu \bar{u}^c)$	$O_{\ell u}$	$(\bar{\ell}\bar{\sigma}_\mu \ell)(u^c\sigma_\mu \bar{u}^c)$
O_{dd}	$\eta(d^c\sigma_\mu \bar{d}^c)(d^c\sigma_\mu \bar{d}^c)$	$O_{\ell d}$	$(\bar{\ell}\bar{\sigma}_\mu \ell)(d^c\sigma_\mu \bar{d}^c)$
O_{eu}	$(e^c\sigma_\mu \bar{e}^c)(u^c\sigma_\mu \bar{u}^c)$	O_{eq}	$(e^c\sigma_\mu \bar{e}^c)(\bar{q}\bar{\sigma}_\mu q)$
O_{ed}	$(e^c\sigma_\mu \bar{e}^c)(d^c\sigma_\mu \bar{d}^c)$	O_{qu}	$(\bar{q}\bar{\sigma}_\mu q)(u^c\sigma_\mu \bar{u}^c)$
O_{ud}	$(u^c\sigma_\mu \bar{u}^c)(d^c\sigma_\mu \bar{d}^c)$	O'_{qu}	$(\bar{q}\bar{\sigma}_\mu T^a q)(u^c\sigma_\mu T^a \bar{u}^c)$
O'_{ud}	$(u^c\sigma_\mu T^a \bar{u}^c)(d^c\sigma_\mu T^a \bar{d}^c)$	O_{qd}	$(\bar{q}\bar{\sigma}_\mu q)(d^c\sigma_\mu \bar{d}^c)$
		O'_{qd}	$(\bar{q}\bar{\sigma}_\mu T^a q)(d^c\sigma_\mu T^a \bar{d}^c)$

$(\bar{L}L)(\bar{L}L)$		$(\bar{L}R)(\bar{L}R)$	
$O_{\ell\ell}$	$\eta(\bar{\ell}\bar{\sigma}_\mu \ell)(\bar{\ell}\bar{\sigma}_\mu \ell)$	O_{quqd}	$(u^c q^j)\epsilon_{jk}(d^c q^k)$
O_{qq}	$\eta(\bar{q}\bar{\sigma}_\mu q)(\bar{q}\bar{\sigma}_\mu q)$	O'_{quqd}	$(u^c T^a q^j)\epsilon_{jk}(d^c T^a q^k)$
O'_{qq}	$\eta(\bar{q}\bar{\sigma}_\mu \sigma^i q)(\bar{q}\bar{\sigma}_\mu \sigma^i q)$	$O_{\ell equ}$	$(e^c \ell^j)\epsilon_{jk}(u^c q^k)$
$O_{\ell q}$	$(\bar{\ell}\bar{\sigma}_\mu \ell)(\bar{q}\bar{\sigma}_\mu q)$	$O'_{\ell equ}$	$(e^c \bar{\sigma}_{\mu\nu} \ell^j)\epsilon_{jk}(u^c \bar{\sigma}^{\mu\nu} q^k)$
$O'_{\ell q}$	$(\bar{\ell}\bar{\sigma}_\mu \sigma^i \ell)(\bar{q}\bar{\sigma}_\mu \sigma^i q)$	$O_{\ell edq}$	$(\bar{\ell}\bar{e}^c)(d^c q)$

Table 2.4: Four-fermion D=6 operators in the Warsaw basis. Flavor indices a suppressed here to reduce the clutter. The factor η is equal to 1/2 when all flav indices are equal (e.g. in $[O_{ee}]_{1111}$), and $\eta = 1$ otherwise. For each complex operat the complex conjugate should be included.

SMEFT at dimension-6

$(H^\dagger H)(D_\mu H^\dagger D_\mu H)$ why it's not in Warsaw basis?

Integration by parts

$$(H^\dagger H)(D_\mu H^\dagger D_\mu H) = \frac{1}{2}(H^\dagger H)\left[\Box(H^\dagger H) - H^\dagger \Box H - \Box H^\dagger H\right]$$

SM equations of motion

$$\begin{aligned}
\partial_\nu B_{\nu\mu} &= -\frac{ig_Y}{2} H^\dagger \overleftrightarrow{D}_\mu H - g_Y j_\mu^Y, \\
(\partial_\nu W^i_{\nu\mu} + \epsilon^{ijk} g_L W^j_\nu W^k_{\nu\mu}) = D_\nu W^i_{\nu\mu} &= -\frac{i}{2} g_L H^\dagger \sigma^i \overleftrightarrow{D}_\mu H - g_L j^i_\mu, \\
D_\nu G^a_{\nu\mu} &= -g_s j^a_\mu,
\end{aligned}$$

$$D_\mu D_\mu H = \mu_H^2 H - 2\lambda (H^\dagger H) H - f^c y_f F$$

$$(H^\dagger H)(D_\mu H^\dagger D_\mu H) = \underbrace{\frac{1}{2}(H^\dagger H)\Box(H^\dagger H)}_{O_{H\Box}} \underbrace{- \mu_H^2 (H^\dagger H)^2}_{\text{SM}} + \underbrace{2\lambda (H^\dagger H)^3}_{O_H}$$

$$+ \underbrace{\frac{1}{2}(H^\dagger H)\left[f^c y_f H^\dagger F + \text{h.c.}\right]}_{O_{fH}^\dagger}$$

SMEFT at dimension-6

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$

Giudice et al hep-ph/0703164
Contino et al 1303.3876

SILH basis of B-conserving dimension-6 operators

O_H	$\frac{1}{2v^2}\left[\partial_\mu(H^\dagger H)\right]^2$		
O_T	$\frac{1}{2v^2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2$		
O_6	$-\frac{\lambda}{v^2}(H^\dagger H)^3$		
O_g	$\frac{g_s^2}{m_W^2} H^\dagger H\, G^a_{\mu\nu} G^a_{\mu\nu}$	$\widetilde{O}_g$	$\frac{g_s^2}{m_W^2} H^\dagger H\, \widetilde{G}^a_{\mu\nu} G^a_{\mu\nu}$
O_γ	$\frac{g'^2}{m_W^2} H^\dagger H\, B_{\mu\nu} B_{\mu\nu}$	$\widetilde{O}_\gamma$	$\frac{g'^2}{m_W^2} H^\dagger H\, \widetilde{B}_{\mu\nu} B_{\mu\nu}$
O_W	$\frac{ig}{2m_W^2}\left(H^\dagger \sigma^i \overleftrightarrow{D}_\mu H\right) D_\nu W^i_{\mu\nu}$		
O_B	$\frac{ig'}{2m_W^2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right) \partial_\nu B_{\mu\nu}$		
O_{HW}	$\frac{ig}{m_W^2}\left(D_\mu H^\dagger \sigma^i D_\nu H\right) W^i_{\mu\nu}$	$\widetilde{O}_{HW}$	$\frac{ig}{m_W^2}\left(D_\mu H^\dagger \sigma^i D_\nu H\right) \widetilde{W}^i_{\mu\nu}$
O_{HB}	$\frac{ig'}{m_W^2}\left(D_\mu H^\dagger D_\nu H\right) B_{\mu\nu}$	$\widetilde{O}_{HB}$	$\frac{ig}{m_W^2}\left(D_\mu H^\dagger D_\nu H\right) \widetilde{B}_{\mu\nu}$
O_{2W}	$\frac{1}{m_W^2} D_\mu W^i_{\mu\nu} D_\rho W^i_{\rho\nu}$		
O_{2B}	$\frac{1}{m_W^2} \partial_\mu B_{\mu\nu} \partial_\rho B_{\rho\nu}$		
O_{2G}	$\frac{1}{m_W^2} D_\mu G^a_{\mu\nu} D_\rho G^a_{\rho\nu}$		
O_{3W}	$\frac{g^3}{m_W^2} \epsilon^{ijk} W^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}$	$\widetilde{O}_{3W}$	$\frac{g^3}{m_W^2} \epsilon^{ijk} \widetilde{W}^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}$
O_{3G}	$\frac{g_s^3}{m_W^2} f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$	$\widetilde{O}_{3G}$	$\frac{g_s^3}{m_W^2} f^{abc} \widetilde{G}^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$

Table 98: Two-fermion dimension-6 operators in the SILH basis. They are the same as in the Warsaw basis, except that the operators $[O_{H\ell}]_{11}$, $[O'_{H\ell}]_{11}$ are absent by definition. We define $\sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2$. In this table, e, u, d are always right-handed fermions, while ℓ and q are left-handed. For complex operators the complex conjugate operator is implicit.

Vertex		Yukawa and Dipole	
$[O_{H\ell}]_{ij}$	$\frac{i}{v^2}\bar{\ell}_i\gamma_\mu\ell_j H^\dagger \overleftrightarrow{D}_\mu H$	$[O_e]_{ij}$	$\frac{\sqrt{2m_{e_i}m_{e_j}}}{v^3} H^\dagger H \bar{\ell}_i H e_j$
$[O'_{H\ell}]_{ij}$	$\frac{i}{v^2}\bar{\ell}_i\sigma^k\gamma_\mu\ell_j H^\dagger \sigma^k \overleftrightarrow{D}_\mu H$	$[O_u]_{ij}$	$\frac{\sqrt{2m_{u_i}m_{u_j}}}{v^3} H^\dagger H \bar{q}_i \widetilde{H} u_j$
$[O_{He}]_{ij}$	$\frac{i}{v^2}\bar{e}_i\gamma_\mu\bar{e}_j H^\dagger \overleftrightarrow{D}_\mu H$	$[O_d]_{ij}$	$\frac{\sqrt{2m_{d_i}m_{d_j}}}{v^3} H^\dagger H \bar{q}_i H d_j$
$[O_{Hq}]_{ij}$	$\frac{i}{v^2}\bar{q}_i\gamma_\mu q_j H^\dagger \overleftrightarrow{D}_\mu H$	$[O_{eW}]_{ij}$	$\frac{g}{m_W^2}\frac{\sqrt{2m_{e_i}m_{e_j}}}{v} \bar{\ell}_i \sigma^k H \sigma_{\mu\nu} e_j W^k_{\mu\nu}$
$[O'_{Hq}]_{ij}$	$\frac{i}{v^2}\bar{q}_i\sigma^k\gamma_\mu q_j H^\dagger \sigma^k \overleftrightarrow{D}_\mu H$	$[O_{eB}]_{ij}$	$\frac{g'}{m_W^2}\frac{\sqrt{2m_{e_i}m_{e_j}}}{v} \bar{\ell}_i H \sigma_{\mu\nu} e_j B_{\mu\nu}$
$[O_{Hu}]_{ij}$	$\frac{i}{v^2}\bar{u}_i\gamma_\mu u_j H^\dagger \overleftrightarrow{D}_\mu H$	$[O_{uG}]_{ij}$	$\frac{g_s}{m_W^2}\frac{\sqrt{2m_{u_i}m_{u_j}}}{v} \bar{q}_i \tilde{H} \sigma_{\mu\nu} T^a u_j G^a_{\mu\nu}$
$[O_{Hd}]_{ij}$	$\frac{i}{v^2}\bar{d}_i\gamma_\mu d_j H^\dagger \overleftrightarrow{D}_\mu H$	$[O_{uW}]_{ij}$	$\frac{g}{m_W^2}\frac{\sqrt{2m_{u_i}m_{u_j}}}{v} \bar{q}_i \sigma^k \tilde{H} \sigma_{\mu\nu} u_j W^k_{\mu\nu}$
$[O_{Hud}]_{ij}$	$\frac{i}{v^2}\bar{u}_i\gamma_\mu d_j \tilde{H}^\dagger D_\mu H$	$[O_{uB}]_{ij}$	$\frac{g'}{m_W^2}\frac{\sqrt{2m_{u_i}m_{u_j}}}{v} \bar{q}_i \tilde{H} \sigma_{\mu\nu} u_j B_{\mu\nu}$
		$[O_{dG}]_{ij}$	$\frac{g_s}{m_W^2}\frac{\sqrt{2m_{d_i}m_{d_j}}}{v} \bar{q}_i H \sigma_{\mu\nu} T^a d_j G^a_{\mu\nu}$
		$[O_{dW}]_{ij}$	$\frac{g}{m_W^2}\frac{\sqrt{2m_{d_i}m_{d_j}}}{v} \bar{q}_i \sigma^k H \sigma_{\mu\nu} d_j W^k_{\mu\nu}$
		$[O_{dB}]_{ij}$	$\frac{g'}{m_W^2}\frac{\sqrt{2m_{d_i}m_{d_j}}}{v} \bar{q}_i H \sigma_{\mu\nu} d_j B_{\mu\nu}$

Table 99: Four-fermion operators in the SILH basis. They are the same as in the Warsaw basis [614], except that the operators $[O_{\ell\ell}]_{1221}$, $[O_{\ell\ell}]_{1122}$, $[O_{uu}]_{3333}$ are absent by definition. In this table, e, u, d are always right-handed fermions, while ℓ and q are left-handed. A flavour index is implicit for each fermion field. For complex operators the complex conjugate operator is implicit.

$(\bar{L}L)(\bar{L}L)$ and $(\bar{L}R)(\bar{L}R)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$O_{\ell\ell}$	$\frac{1}{v^2}(\bar{\ell}\gamma_\mu\ell)(\bar{\ell}\gamma_\mu\ell)$	O_{ee}	$\frac{1}{v^2}(\bar{e}\gamma_\mu e)(\bar{e}\gamma_\mu e)$	$O_{\ell e}$	$\frac{1}{v^2}(\bar{\ell}\gamma_\mu\ell)(\bar{e}\gamma_\mu e)$
O_{qq}	$\frac{1}{v^2}(\bar{q}\gamma_\mu q)(\bar{q}\gamma_\mu q)$	O_{uu}	$\frac{1}{v^2}(\bar{u}\gamma_\mu u)(\bar{u}\gamma_\mu u)$	$O_{\ell u}$	$\frac{1}{v^2}(\bar{\ell}\gamma_\mu\ell)(\bar{u}\gamma_\mu u)$
O'_{qq}	$\frac{1}{v^2}(\bar{q}\gamma_\mu\sigma^i q)(\bar{q}\gamma_\mu\sigma^i q)$	O_{dd}	$\frac{1}{v^2}(\bar{d}\gamma_\mu d)(\bar{d}\gamma_\mu d)$	$O_{\ell d}$	$\frac{1}{v^2}(\bar{\ell}\gamma_\mu\ell)(\bar{d}\gamma_\mu d)$
$O_{\ell q}$	$\frac{1}{v^2}(\bar{\ell}\gamma_\mu\ell)(\bar{q}\gamma_\mu q)$	O_{eu}	$\frac{1}{v^2}(\bar{e}\gamma_\mu e)(\bar{u}\gamma_\mu u)$	O_{eq}	$\frac{1}{v^2}(\bar{q}\gamma_\mu q)(\bar{e}\gamma_\mu e)$
$O'_{\ell q}$	$\frac{1}{v^2}(\bar{\ell}\gamma_\mu\sigma^i\ell)(\bar{q}\gamma_\mu\sigma^i q)$	O_{ed}	$\frac{1}{v^2}(\bar{e}\gamma_\mu e)(\bar{d}\gamma_\mu d)$	O_{qu}	$\frac{1}{v^2}(\bar{q}\gamma_\mu q)(\bar{u}\gamma_\mu u)$
O_{quqd}	$\frac{1}{v^2}(\bar{q}^j u)\epsilon_{jk}(\bar{q}^k d)$	O_{ud}	$\frac{1}{v^2}(\bar{u}\gamma_\mu u)(\bar{d}\gamma_\mu d)$	O'_{qu}	$\frac{1}{v^2}(\bar{q}\gamma_\mu T^a q)(\bar{u}\gamma_\mu T^a u)$
O'_{quqd}	$\frac{1}{v^2}(\bar{q}^j T^a u)\epsilon_{jk}(\bar{q}^k T^a d)$	O'_{ud}	$\frac{1}{v^2}(\bar{u}\gamma_\mu T^a u)(\bar{d}\gamma_\mu T^a d)$	O_{qd}	$\frac{1}{v^2}(\bar{q}\gamma_\mu q)(\bar{d}\gamma_\mu d)$
$O_{\ell equ}$	$\frac{1}{v^2}(\bar{\ell}^j e)\epsilon_{jk}(\bar{q}^k u)$			O'_{qd}	$\frac{1}{v^2}(\bar{q}\gamma_\mu T^a q)(\bar{d}\gamma_\mu T^a d)$
$O'_{\ell equ}$	$\frac{1}{v^2}(\bar{\ell}^j \sigma_{\mu\nu} e)\epsilon_{jk}(\bar{q}^k \sigma^{\mu\nu} u)$				
$O_{\ell edq}$	$\frac{1}{v^2}(\bar{\ell}^j e)(\bar{d} q^j)$				

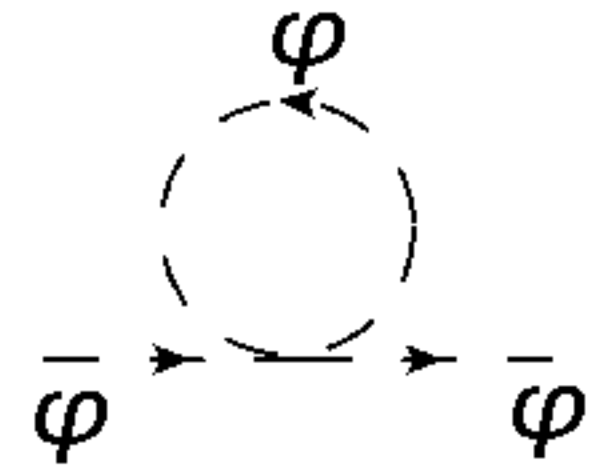
Part 5

Running in SMEFT

Back to toy model example

$$\mathcal{L}_{\rm EFT} = \frac{1}{2}\left[(\partial_\mu\phi)^2 - m^2\phi^2\right] - C_4\frac{\phi^4}{4!} - \frac{C_6}{\Lambda^2}\frac{\phi^6}{6!}$$

In this EFT, there is a single diagram contributing to ϕ mass at one loop



$$\delta M_2^{\rm EFT} = -\frac{C_4}{2}\int\frac{d^dk}{(2\pi)^d}\frac{i}{k^2-m^2}$$

$$= C_4\frac{m^2}{32\pi^2}\left[\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{m^2}\right) + 1\right]$$

$$1/\bar{\epsilon} \equiv 1/\epsilon + \gamma_E + \log(4\pi)$$

Note that we use dimensional regularization, which is very convenient in the EFT context, as it does not introduce new mass scale, so it does not mess up the EFT power counting

Furthermore, we will use the MSbar renormalization, simply dropping all $\frac{1}{\bar{\epsilon}}$ poles

The physical ϕ mass in the EFT at one loop:

$$m_{\rm phys}^2 = m^2 - C_4\frac{m^2}{32\pi^2}\left[\log\left(\frac{\mu^2}{m^2}\right) + 1\right]$$

Running of the mass parameter

The physical mass is an observable in this model, therefore it cannot depend on the arbitrary parameter μ

$$\frac{dm^2_{\mathrm{phys}}}{d\log\mu} = 0$$

This means that the Lagrangian mass parameter, up to higher-loop corrections, must satisfy

$$\frac{dm^2}{d\log\mu} = C_4 \frac{m^2}{16\pi^2}$$

The solution is

$$m^2(\mu) = m^2(\Lambda)\left(\frac{\mu}{\Lambda}\right)^{\frac{C_4}{16\pi^2}}$$

We can interpret μ as the renormalization group scale

This also shows that naive scaling of EFT parameters with Λ is modified by loop effects therefore the exponent is called *the anomalous dimension*

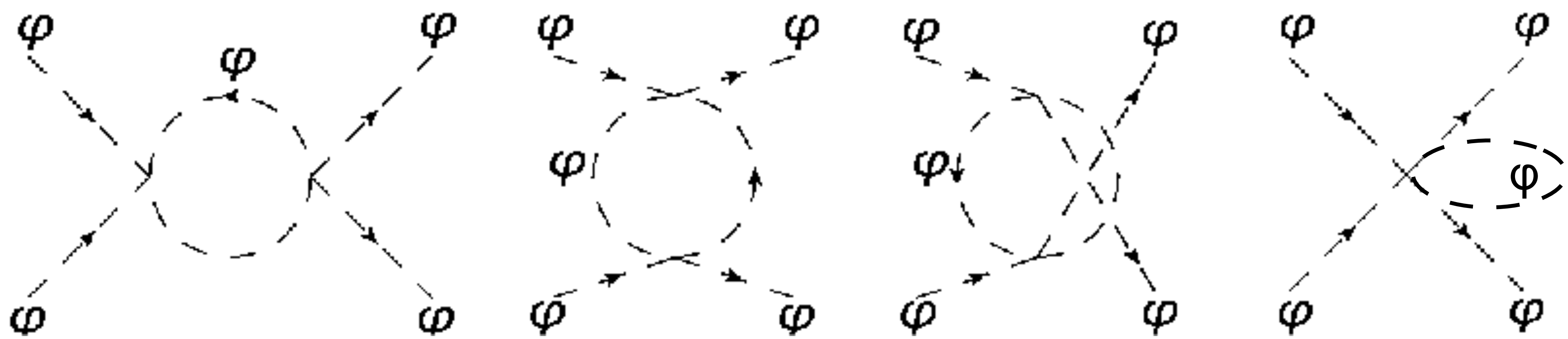
Note that, <u>within the EFT</u>, there is no hierarchy problem, that is to say, if $m^2(\Lambda) \ll \Lambda^2$ then $m^2(\mu) \ll \Lambda^2$ at all scales

Running of the quartic coupling

We move to one-loop matching of the quartic coupling

EFT calculation

$$\mathscr{L}_{\text{EFT}} = \frac{1}{2}\left[(\partial_\mu \phi)^2 - m^2\phi^2\right] - C_4\frac{\phi^4}{4!} - \frac{C_6}{\Lambda^2}\frac{\phi^6}{6!}$$



Answer

$$f(s,m) \equiv \sqrt{1-\frac{4m^2}{s}}\log\left(\frac{2m^2 - s + \sqrt{s(s-m^2)}}{2m^2}\right)$$

$$M_4^{\text{EFT}} = -C_4 + \frac{C_4^2}{32\pi^2}\left[f(s,m) + f(t,m) + f(u,m)\right]$$
$$+\frac{3C_4^2}{32\pi^2}\left(\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{m^2}\right) + 2\right) + \frac{C_6 m^2}{32\pi^2\Lambda^2}\left(\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{m^2}\right) + 1\right)$$

Running of the EFT quartic coupling

$$M_4^{\rm EFT} = -C_4 + \frac{C_4^2}{32\pi^2}\left[f(s,m)+f(t,m)+f(u,m)\right] + \frac{3C_4^2}{32\pi^2}\left(\log\left(\frac{\mu^2}{m^2}\right)+2\right) + \frac{C_6 m^2}{32\pi^2\Lambda^2}\left(\log\left(\frac{\mu^2}{m^2}\right)+1\right)$$

The observable is $S_4^{\rm EFT} \equiv \dfrac{M_4^{\rm EFT}}{(1+\delta_\phi)^2}$ **where δ_ϕ is wave function renormalization**

$S_4^{\rm EFT}$ **can be related to the cross section, so it must not depend on μ**

One can show that $\delta_\phi = 0$ at one loop in the unbox basis

Therefore $M_4^{\rm EFT}$ cannot depend on the arbitrary parameter μ: $\dfrac{dM_4^{\rm EFT}}{d\log\mu} = 0$

This means that the Lagrangian parameters, up to higher-loop corrections, must satisfy

$$\frac{dC_4}{d\log\mu} = \frac{3C_4^2}{16\pi^2} + \frac{C_6 m^2}{16\pi^2\Lambda^2}$$

Note that higher-order Wilson coefficients affect the running of lower-order Wilson coefficients, but not vice versa

Running in EFT

$$f(s,m) \equiv \sqrt{1 - \frac{4m^2}{s}} \log\left(\frac{2m^2 - s + \sqrt{s(s-m^2)}}{2m^2}\right)$$

$$M_4^{\mathrm{EFT}} = -C_4(\mu) + \frac{C_4^2}{32\pi^2}\left[f(s,m) + f(t,m) + f(u,m)\right] + \frac{3C_4^2}{32\pi^2}\left(\log\left(\frac{\mu^2}{m^2}\right) + 2\right) + \frac{C_6 m^2}{32\pi^2\Lambda^2}\left(\log\left(\frac{\mu^2}{m^2}\right) + 1\right)$$

$$\frac{dC_4}{d\log\mu} = \frac{3C_4^2}{16\pi^2} + \frac{C_6 m^2}{16\pi^2\Lambda^2} \quad \Longrightarrow \quad C_4(\mu) \approx C_4(m) + \frac{3C_4^2}{16\pi^2}\log\left(\frac{\mu}{m}\right) + \frac{C_6 m^2}{16\pi^2\Lambda^2}\log\left(\frac{\mu}{m}\right)$$

$$M_4^{\mathrm{EFT}} = -C_4(m) + \frac{C_4^2}{32\pi^2}\left[f(s,m) + f(t,m) + f(u,m) + 6\right] + \frac{C_6 m^2}{32\pi^2\Lambda^2}$$

No large logarithms in the EFT if we use
the couplings evolve down to the characteristic mass scale
The potentially problematic $\log(\mu/\Lambda)$ **terms are all hidden (resummed)**
in the running Wilson coefficient C_4(m)

Running in SMEFT

Running of dimension-6 Wilson coefficients

$$\frac{dC_k}{d\log\mu} = [\gamma]_{kl} C_l$$

C_k **is a vector of all dimension-6 Wilson coefficients in a given basis**

$[\gamma]$ **is the matrix of anomalous dimensions**

For Warsaw basis complete matrix $[\gamma]$ **written down in series of papers:**

Jenkins et al
arXiv: 1308.2627, 1310.4838,1312.2014

For results in the SILH basis see

Elias-Miro et al
1308.1879

Running in SMEFT

Example: running of SILH operators most relevant for Higgs physics:

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$$\mathcal{O}_H = \tfrac{1}{2}(\partial^\mu |H|^2)^2$$
$$\mathcal{O}_T = \tfrac{1}{2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2$$
$$\mathcal{O}_6 = \lambda |H|^6$$

$$\mathcal{O}_W = \frac{ig}{2}\left(H^\dagger \sigma^a \overleftrightarrow{D}^\mu H\right) D^\nu W^a_{\mu\nu}$$
$$\mathcal{O}_B = \frac{ig'}{2}\left(H^\dagger \overleftrightarrow{D}^\mu H\right) \partial^\nu B_{\mu\nu}$$
$$\mathcal{O}_{2W} = -\tfrac{1}{2}(D^\mu W^a_{\mu\nu})^2$$
$$\mathcal{O}_{2B} = -\tfrac{1}{2}(\partial^\mu B_{\mu\nu})^2$$
$$\mathcal{O}_{2G} = -\tfrac{1}{2}(D^\mu G^A_{\mu\nu})^2$$

$$\mathcal{O}_{BB} = g'^2 |H|^2 B_{\mu\nu} B^{\mu\nu}$$
$$\mathcal{O}_{GG} = g_s^2 |H|^2 G^A_{\mu\nu} G^{A\mu\nu}$$
$$\mathcal{O}_{HW} = ig(D^\mu H)^\dagger \sigma^a (D^\nu H) W^a_{\mu\nu}$$
$$\mathcal{O}_{HB} = ig'(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$$
$$\mathcal{O}_{3W} = \tfrac{1}{3!} g \epsilon_{abc} W^{a\,\nu}_{\mu} W^b_{\nu\rho} W^{c\,\rho\mu}$$
$$\mathcal{O}_{3G} = \tfrac{1}{3!} g_s f_{ABC} G^{A\,\nu}_{\mu} G^B_{\nu\rho} G^{C\,\rho\mu}$$

$$16\pi^2 \gamma_{c_H} = \left[4N_c y_t^2 + 24\lambda - \frac{3}{2}(3g^2 + 2g'^2)\right] c_H + 12 N_c y_t^2 c_L^{(3)} \,, \quad (54$$

$$\begin{aligned} 16\pi^2 \gamma_{\lambda c_6} = {} & 6\left[N_c y_t^2 + 18\lambda - \frac{3}{4}(3g^2 + g'^2)\right] \lambda c_6 + 2(40\lambda - 3g^2)\lambda c_H \\ & -16 N_c \lambda y_t^2 c_L^{(3)} + 8 N_c y_t^2 (\lambda - y_t^2) c_{y_t} \,, \end{aligned} \quad (55$$

$$\begin{aligned} 16\pi^2 \gamma_{c_{y_t}} = {} & \left[(4N_c + 9)y_t^2 + 24\lambda - \frac{3}{2}(3g^2 + g'^2)\right] c_{y_t} + \left(3y_t^2 + 2\lambda - \frac{3}{2}g^2\right) c_H \\ & +(2y_t^2 + 4\lambda - 3g^2 - g'^2)c_R - 2(y_t^2 + 2\lambda + 2g'^2)c_L \\ & +4(-N_c y_t^2 + 3\lambda + g'^2)c_L^{(3)} + 8(y_t^2 - \lambda)\left[c_{LR} + C_F c_{LR}^{(8)}\right] \,, \end{aligned} \quad (56$$

$$\begin{aligned} 16\pi^2 \gamma_{c_{y_b}} = {} & \left[2(N_c + 1)y_t^2 + 24\lambda - \frac{3}{2}(3g^2 + g'^2)\right] c_{y_b} + \left(2\lambda - \frac{3}{2}g^2\right) c_H + (2N_c - 1)y_t^2 c_{y_t} \\ & +2(2\lambda + g'^2)c_L + 2\left[(3 - 2N_c)y_t^2 + 6\lambda + g'^2\right] c_L^{(3)} - 4\frac{y_t^2}{g_*^2}\left(y_t^2 + 2\lambda - \frac{3}{2}g^2\right) c_R^{tb} \\ & +2\frac{y_t^2}{g_*^2}(\lambda - y_t^2)\left[(2N_c + 1)\, c_{y_t y_b} + C_F c_{y_t y_b}^{(8)}\right] \,, \end{aligned} \quad (57$$

$$\begin{aligned} 16\pi^2 \gamma_{c_{y_\tau}} = {} & \left[2N_c y_t^2 + 24\lambda - \frac{3}{2}(3g^2 + g'^2)\right] c_{y_\tau} + \left(2\lambda - \frac{3}{2}g^2\right) c_H + 2N_c y_t^2 [c_{y_t} - 2c_L^{(3)}] \\ & -2\frac{y_t^2}{g_*^2} N_c (\lambda - y_t^2)\left(2c_{y_t y_\tau} + c'_{y_t y_\tau}\right) \,, \end{aligned} \quad (58$$

Running in SMEFT

Example: running of SILH operators most relevant for Higgs physics:

Elias-Miro et al 1308.1879

$\mathcal{O}_{y_u} = y_u\|H\|^2\bar{Q}_L\widetilde{H}u_R$	$\mathcal{O}_{y_d} = y_d\|H\|^2\bar{Q}_L H d_R$	$\mathcal{O}_{y_e} = y_e\|H\|^2\bar{L}_L H e_R$
$\mathcal{O}_R^u = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{u}_R\gamma^\mu u_R)$	$\mathcal{O}_R^d = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{d}_R\gamma^\mu d_R)$	$\mathcal{O}_R^e = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{e}_R\gamma^\mu e_R)$
$\mathcal{O}_L^q = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{Q}_L\gamma^\mu Q_L)$		$\mathcal{O}_L^l = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{L}_L\gamma^\mu L_L)$
$\mathcal{O}_L^{(3)\,q} = (iH^\dagger\sigma^a \overleftrightarrow{D}_\mu H)(\bar{Q}_L\gamma^\mu\sigma^a Q_L)$		$\mathcal{O}_L^{(3)\,l} = (iH^\dagger\sigma^a \overleftrightarrow{D}_\mu H)(\bar{L}_L\gamma^\mu\sigma^a L_L)$
$\mathcal{O}_{LR}^u = (\bar{Q}_L\gamma^\mu Q_L)(\bar{u}_R\gamma_\mu u_R)$	$\mathcal{O}_{LR}^d = (\bar{Q}_L\gamma^\mu Q_L)(\bar{d}_R\gamma_\mu d_R)$	$\mathcal{O}_{LR}^e = (\bar{L}_L\gamma^\mu L_L)(\bar{e}_R\gamma_\mu e_R)$
$\mathcal{O}_{LR}^{(8)\,u} = (\bar{Q}_L\gamma^\mu T^A Q_L)(\bar{u}_R\gamma_\mu T^A u_R)$	$\mathcal{O}_{LR}^{(8)\,d} = (\bar{Q}_L\gamma^\mu T^A Q_L)(\bar{d}_R\gamma_\mu T^A d_R)$	
$\mathcal{O}_{RR}^u = (\bar{u}_R\gamma^\mu u_R)(\bar{u}_R\gamma_\mu u_R)$	$\mathcal{O}_{RR}^d = (\bar{d}_R\gamma^\mu d_R)(\bar{d}_R\gamma_\mu d_R)$	$\mathcal{O}_{RR}^e = (\bar{e}_R\gamma^\mu e_R)(\bar{e}_R\gamma_\mu e_R)$
$\mathcal{O}_{LL}^q = (\bar{Q}_L\gamma^\mu Q_L)(\bar{Q}_L\gamma_\mu Q_L)$		$\mathcal{O}_{LL}^l = (\bar{L}_L\gamma^\mu L_L)(\bar{L}_L\gamma_\mu L_L)$
$\mathcal{O}_{LL}^{(8)\,q} = (\bar{Q}_L\gamma^\mu T^A Q_L)(\bar{Q}_L\gamma_\mu T^A Q_L)$		
$\mathcal{O}_{LL}^{ql} = (\bar{Q}_L\gamma^\mu Q_L)(\bar{L}_L\gamma_\mu L_L)$		
$\mathcal{O}_{LL}^{(3)\,ql} = (\bar{Q}_L\gamma^\mu\sigma^a Q_L)(\bar{L}_L\gamma_\mu\sigma^a L_L)$		
$\mathcal{O}_{LR}^{qe} = (\bar{Q}_L\gamma^\mu Q_L)(\bar{e}_R\gamma_\mu e_R)$		
$\mathcal{O}_{LR}^{lu} = (\bar{L}_L\gamma^\mu L_L)(\bar{u}_R\gamma_\mu u_R)$	$\mathcal{O}_{LR}^{ld} = (\bar{L}_L\gamma^\mu L_L)(\bar{d}_R\gamma_\mu d_R)$	
$\mathcal{O}_{RR}^{ud} = (\bar{u}_R\gamma^\mu u_R)(\bar{d}_R\gamma_\mu d_R)$		
$\mathcal{O}_{RR}^{(8)\,ud} = (\bar{u}_R\gamma^\mu T^A u_R)(\bar{d}_R\gamma_\mu T^A d_R)$		
$\mathcal{O}_{RR}^{ue} = (\bar{u}_R\gamma^\mu u_R)(\bar{e}_R\gamma_\mu e_R)$	$\mathcal{O}_{RR}^{de} = (\bar{d}_R\gamma^\mu d_R)(\bar{e}_R\gamma_\mu e_R)$	
$\mathcal{O}_R^{ud} = y_u^\dagger y_d(i\widetilde{H}^\dagger \overleftrightarrow{D}_\mu H)(\bar{u}_R\gamma^\mu d_R)$		
$\mathcal{O}_{y_u y_d} = y_u y_d(\bar{Q}_L^r u_R)\epsilon_{rs}(\bar{Q}_L^s d_R)$		
$\mathcal{O}_{y_u y_d}^{(8)} = y_u y_d(\bar{Q}_L^r T^A u_R)\epsilon_{rs}(\bar{Q}_L^s T^A d_R)$		
$\mathcal{O}_{y_u y_e} = y_u y_e(\bar{Q}_L^r u_R)\epsilon_{rs}(\bar{L}_L^s e_R)$		
$\mathcal{O}'_{y_u y_e} = y_u y_e(\bar{Q}_L^{r\,\alpha} e_R)\epsilon_{rs}(\bar{L}_L^s u_R^\alpha)$		
$\mathcal{O}_{y_e y_d} = y_e y_d^\dagger(\bar{L}_L e_R)(\bar{d}_R Q_L)$		
$\mathcal{O}_{DB}^u = y_u\bar{Q}_L\sigma^{\mu\nu}u_R\,\widetilde{H}g'B_{\mu\nu}$	$\mathcal{O}_{DB}^d = y_d\bar{Q}_L\sigma^{\mu\nu}d_R\,Hg'B_{\mu\nu}$	$\mathcal{O}_{DB}^e = y_e\bar{L}_L\sigma^{\mu\nu}e_R\,Hg'B_{\mu\nu}$
$\mathcal{O}_{DW}^u = y_u\bar{Q}_L\sigma^{\mu\nu}u_R\,\sigma^a\widetilde{H}gW_{\mu\nu}^a$	$\mathcal{O}_{DW}^d = y_d\bar{Q}_L\sigma^{\mu\nu}d_R\,\sigma^a HgW_{\mu\nu}^a$	$\mathcal{O}_{DW}^e = y_e\bar{L}_L\sigma^{\mu\nu}e_R\,\sigma^a HgW_{\mu\nu}^a$
$\mathcal{O}_{DG}^u = y_u\bar{Q}_L\sigma^{\mu\nu}T^A u_R\,\widetilde{H}g_s G_{\mu\nu}^A$	$\mathcal{O}_{DG}^d = y_d\bar{Q}_L\sigma^{\mu\nu}T^A d_R\,Hg_s G_{\mu\nu}^A$	

Part 6

Excerpts from SMEFT phenomenology

From operators to observables

Two main kinds of effects of higher-dimensional SMEFT operators

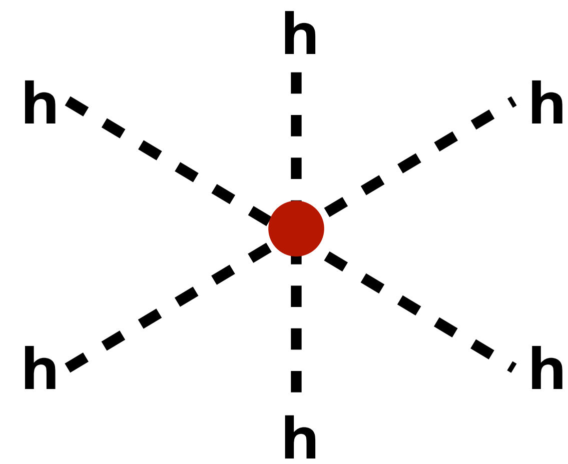
New interactions not present in SM Lagrangian

Corrections to coupling strength of SM interactions

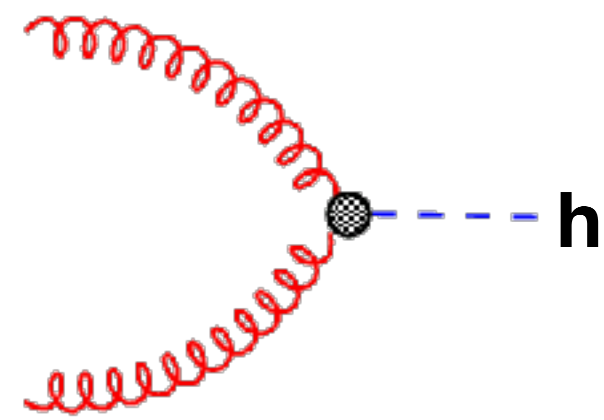
New interactions

1. New "harder" vertices

$$\text{e.g.}\quad \frac{1}{\Lambda^2}|H|^6 \to \frac{h^6}{8\Lambda^2} + \frac{3vh^5}{4\Lambda^2} + \dots$$

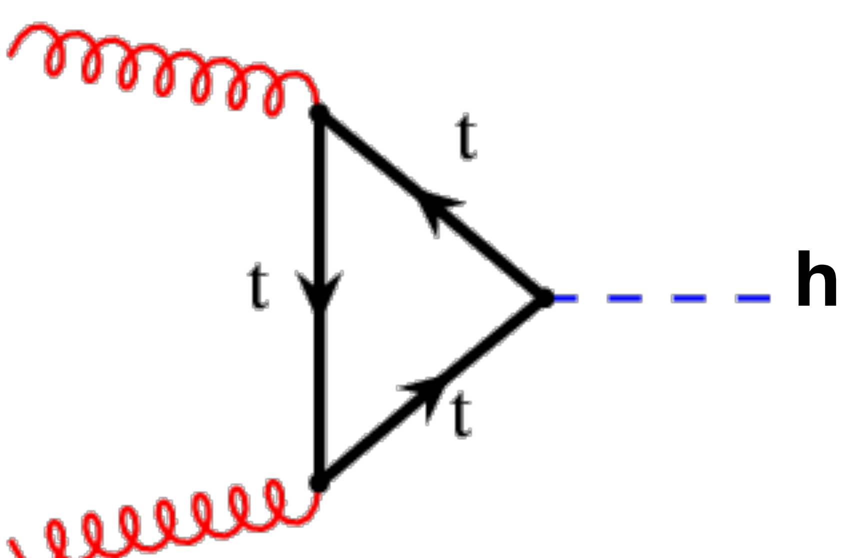


$$\text{or}\quad \frac{1}{\Lambda^2}|H|^2 G^a_{\mu\nu}G^a_{\mu\nu} \to \frac{h^2}{2\Lambda^2}G^a_{\mu\nu}G^a_{\mu\nu} + \frac{vh}{\Lambda^2}G^a_{\mu\nu}G^a_{\mu\nu} + \dots$$



These examples do not lead to new processes compared to the SM, but they introduce "harder" contributions to amplitudes, leading to different energy/pT dependence

Diagrams borrowed from R. Harlander's web page



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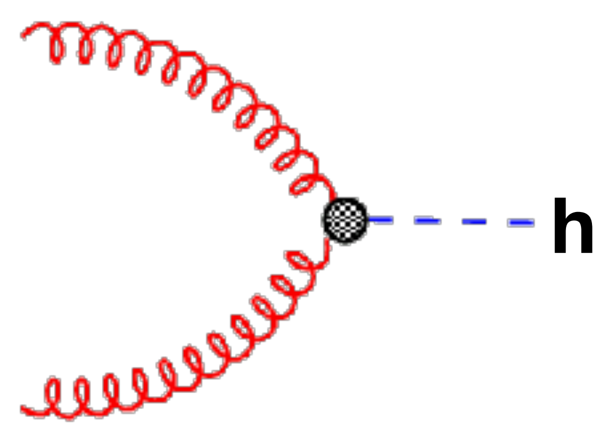


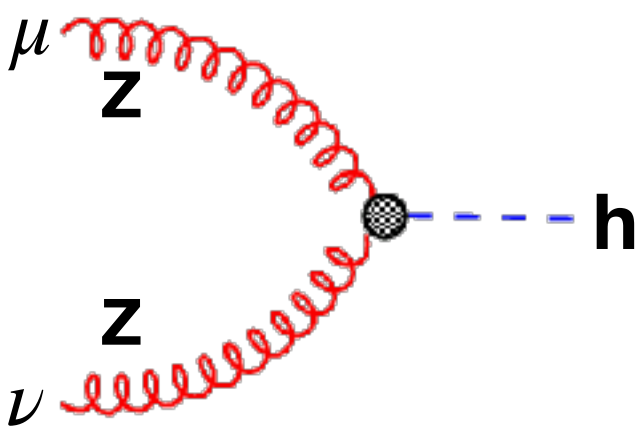
New interactions

2. New Lorentz structures

$$\text{e.g.} \quad \frac{1}{\Lambda^2}|H|^2 B_{\mu\nu}B_{\mu\nu} \to \frac{2\text{v}\sin^2\theta_W}{\Lambda^2} hZ_{\mu\nu}Z_{\mu\nu} + \ldots$$

in addition to $\frac{h}{\text{v}} m_Z^2 Z_\mu Z_\mu$ **present in the SM**

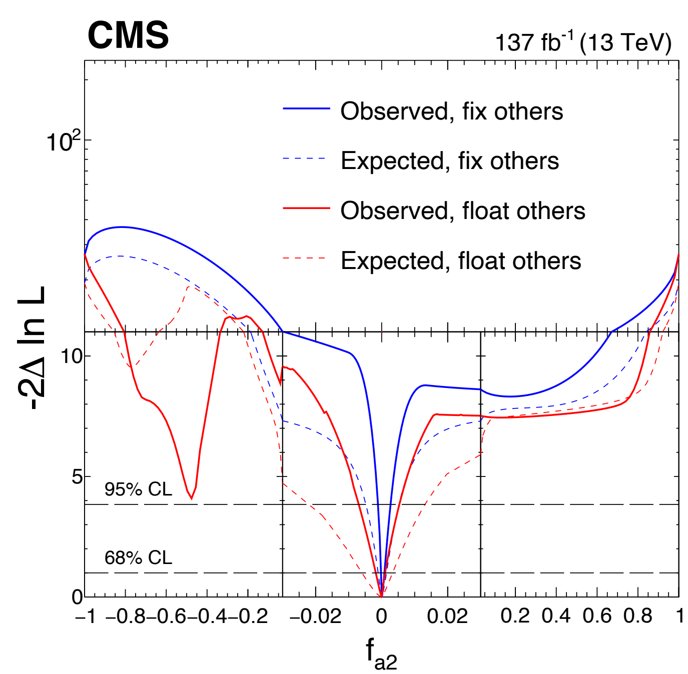
hZZ vertex



$$\frac{2i}{\text{v}}\left[m_Z^2\eta^{\mu\nu} - \frac{c_{HB}\text{v}^2}{\Lambda^2}\left(p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - 2p_1p_2\eta^{\mu\nu}\right)\right]$$

The presence of the new structure affects e.g. the CM energy depends of Zh productions

But most cleanly it can be extracted from angular distributions in $h \to ZZ^* \to \ell^+\ell^-\ell^+\ell^-$ **decays**



CMS 2104.12152

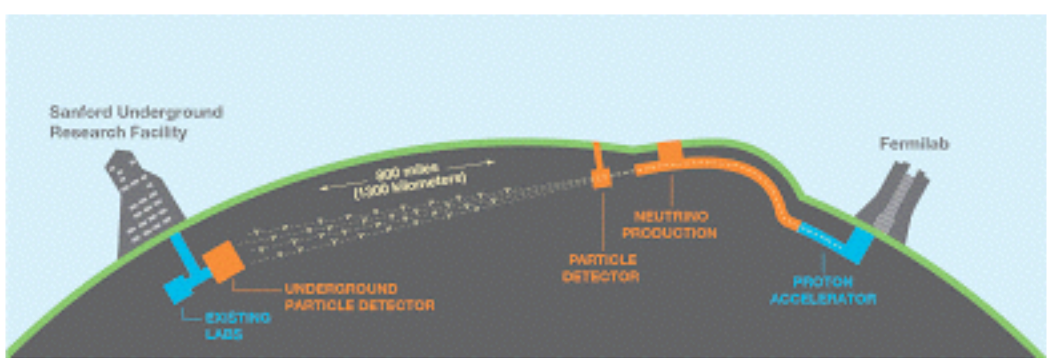
New interactions

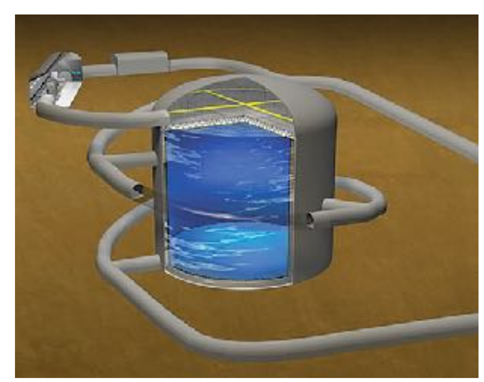
3. Violation of accidental symmetries of the SM

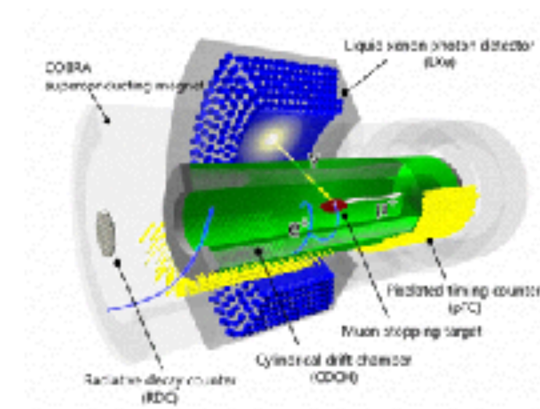
Lepton number violation, e.g. $\frac{1}{\Lambda}(HL)(HL)$ **-> neutrino oscillations probing** $\Lambda \sim 10^{15}$ **GeV**

Baryon number violation, e.g. $\frac{1}{\Lambda^2}u^c d^c u^c e^c$ **-> proton decay probing** $\Lambda \sim 10^{15}$ **GeV**

Individual lepton number violation, e.g. $\frac{1}{\Lambda^2}e_1^c \sigma^{\mu\nu} H^\dagger L_2 B_{\mu\nu}$ **->** $\mu \to e\gamma$ **decay probing** $\Lambda \sim 10^8$ **GeV**





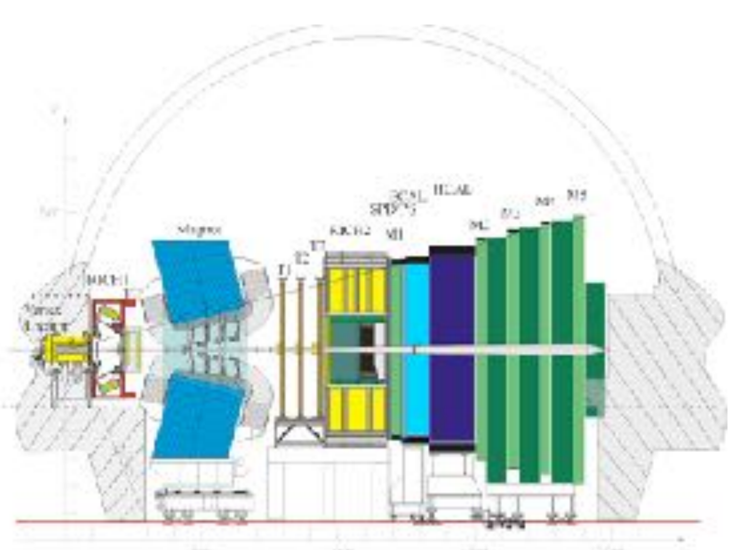


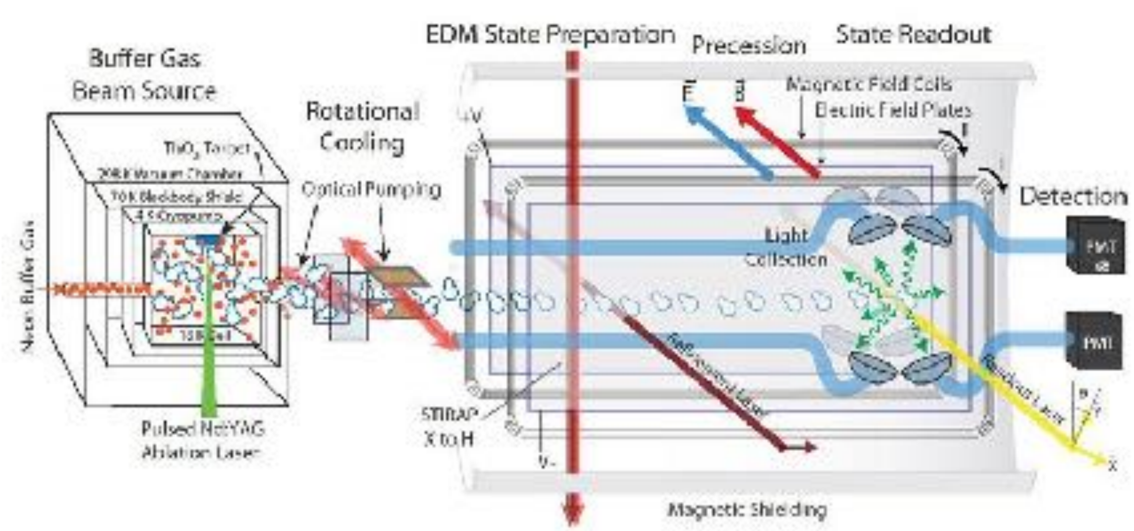
The characteristic feature of these processes is that they probe scales far above TeV, sometime even close to the GUT scale

4. Violation of approximate symmetries of the SM

Flavour changing neutral currents, e.g. $\frac{1}{\Lambda^2}(\bar{s}\gamma^\alpha b)(\bar{\mu}\gamma_\alpha \mu)$ **probes** $\Lambda \sim 30$ **TeV**

CP violation, e.g. $\frac{1}{\Lambda^2}\bar{e}_R \sigma^{\mu\nu} H L F_{\mu\nu}$ **with complex Wilson coefficient probes** $\Lambda \sim 10^6$ **TeV**





These often probe Λ **far above TeV as well**

From operators to observables

Two main kinds of effects of higher-dimensional SMEFT operators

New interactions not present in SM Lagrangian

Corrections to coupling strength of SM interactions

Modified interaction strength

There are 3 ways higher-dimensional operators may modify SM interaction strength

1. **Directly: after electroweak symmetry breaking, an operator contributes to a gauge or Yukawa interaction already present in the SM**

2. **Indirectly: after electroweak symmetry breaking, an operator contributes to the kinetic term of a SM field, thus effectively shifting the strength of all interactions of that field**

3. **Stealthily: after electroweak symmetry breaking, an operator contributes to an experimental observable from which some SM parameter is extracted**

Modified interaction strength: directly

Example:

$$\frac{i}{\Lambda^2}\bar{e}_R\gamma^\mu e_R(H^\dagger D_\mu H - D_\mu H^\dagger H)$$

After electroweak symmetry breaking $i(H^\dagger D_\mu H - D_\mu H^\dagger H) \to -\frac{\mathrm{v}^2}{2}\sqrt{g_L^2 + g_Y^2} Z_\mu + \ldots$

$$\frac{ic_{He}}{\Lambda^2}\bar{e}_R\gamma^\mu e_R(H^\dagger D_\mu H - D_\mu H^\dagger H) \to -c_{He}\frac{\mathrm{v}^2\sqrt{g_L^2 + g_Y^2}}{2\Lambda^2}\bar{e}_R\gamma^\mu e_R Z_\mu$$

This adds up to the weak interaction in the SM $\sqrt{g_L^2 + g_Y^2}\left(T_f^3 - \sin^2\theta_W Q_f + \delta g^{Zf}\right)\bar{f}\gamma^\mu f Z_\mu$

$$\boxed{\delta g_R^{Ze} = -c_{He}\frac{\mathrm{v}^2}{2\Lambda^2}}$$

Thus c_{He} can be constrained, e.g., form LEP-1 Z-pole data

Modified interaction strength: directly

$$\boxed{\delta g_R^{Ze} = -\, c_{He}\frac{\mathrm{v}^2}{2\Lambda^2}}$$

Observable	Experimental value	SM prediction	Definition
Γ_Z [GeV]	2.4955 ± 0.0023 [4, 28]	2.4941	$\sum_f \Gamma(Z \to f\bar{f})$
σ_{had} [nb]	41.4802 ± 0.0325 [4, 28]	41.4842	$\frac{12\pi}{m_Z^2}\frac{\Gamma(Z\to e^+e^-)\Gamma(Z\to q\bar{q})}{\Gamma_Z^2}$
R_e	20.804 ± 0.050 [4]	20.734	$\frac{\sum_q \Gamma(Z\to q\bar{q})}{\Gamma(Z\to e^+e^-)}$
R_μ	20.785 ± 0.033 [4]	20.734	$\frac{\sum_q \Gamma(Z\to q\bar{q})}{\Gamma(Z\to \mu^+\mu^-)}$
R_τ	20.764 ± 0.045 [4]	20.781	$\frac{\sum_q \Gamma(Z\to q\bar{q})}{\Gamma(Z\to \tau^+\tau^-)}$
$A_{\text{FB}}^{0,e}$	0.0145 ± 0.0025 [4]	0.0162	$\frac{3}{4}A_e^2$
$A_{\text{FB}}^{0,\mu}$	0.0169 ± 0.0013 [4]	0.0162	$\frac{3}{4}A_e A_\mu$
$A_{\text{FB}}^{0,\tau}$	0.0188 ± 0.0017 [4]	0.0162	$\frac{3}{4}A_e A_\tau$
R_b	0.21629 ± 0.00066 [4]	0.21581	$\frac{\Gamma(Z\to b\bar{b})}{\sum_q \Gamma(Z\to q\bar{q})}$
R_c	0.1721 ± 0.0030 [4]	0.17222	$\frac{\Gamma(Z\to c\bar{c})}{\sum_q \Gamma(Z\to q\bar{q})}$
A_b^{FB}	0.0996 ± 0.0016 [4, 29]	0.1032	$\frac{3}{4}A_e A_b$
A_c^{FB}	0.0707 ± 0.0035 [4]	0.0736	$\frac{3}{4}A_e A_c$
A_e	0.1516 ± 0.0021 [4]	0.1470	$\frac{\Gamma(Z\to e_L^+e_L^-)-\Gamma(Z\to e_R^+e_R^-)}{\Gamma(Z\to e^+e^-)}$
A_μ	0.142 ± 0.015 [4]	0.1470	$\frac{\Gamma(Z\to \mu_L^+\mu_L^-)-\Gamma(Z\to \mu_R^+\mu_R^-)}{\Gamma(Z\to \mu^+\mu^-)}$
A_τ	0.136 ± 0.015 [4]	0.1470	$\frac{\Gamma(Z\to \tau_L^+\tau_L^-)-\Gamma(Z\to \tau_R^+\tau_R^-)}{\Gamma(Z\to \tau^+\tau^-)}$
A_e	0.1498 ± 0.0049 [4]	0.1470	$\frac{\Gamma(Z\to e_L^+e_L^-)-\Gamma(Z\to e_R^+e_R^-)}{\Gamma(Z\to e^+e^-)}$
A_τ	0.1439 ± 0.0043 [4]	0.1470	$\frac{\Gamma(Z\to \tau_L^+\tau_L^-)-\Gamma(Z\to \tau_R^+\tau_R^-)}{\Gamma(Z\to \tau^+\tau^-)}$
A_b	0.923 ± 0.020 [4]	0.935	$\frac{\Gamma(Z\to b_L\bar{b}_L)-\Gamma(Z\to b_R\bar{b}_R)}{\Gamma(Z\to b\bar{b})}$
A_c	0.670 ± 0.027 [4]	0.668	$\frac{\Gamma(Z\to c_L\bar{c}_L)-\Gamma(Z\to c_R\bar{c}_R)}{\Gamma(Z\to c\bar{c})}$
A_s	0.895 ± 0.091 [30]	0.936	$\frac{\Gamma(Z\to s_L\bar{s}_L)-\Gamma(Z\to s_R\bar{s}_R)}{\Gamma(Z\to s\bar{s})}$
R_{uc}	0.166 ± 0.009 [9]	0.1722	$\frac{\Gamma(Z\to u\bar{u})+\Gamma(Z\to c\bar{c})}{2\sum_q \Gamma(Z\to q\bar{q})}$

$$\begin{pmatrix} \delta g_L^{We} \\ \delta g_L^{W\mu} \\ \delta g_L^{W\tau} \\ \delta g_L^{Ze} \\ \delta g_L^{Z\mu} \\ \delta g_L^{Z\tau} \\ \delta g_R^{Ze} \\ \delta g_R^{Z\mu} \\ \delta g_R^{Z\tau} \\ \delta g_L^{Zu} \\ \delta g_R^{Zu} \\ \delta g_L^{Zd} \\ \delta g_R^{Zd} \\ \delta g_L^{Zs} \\ \delta g_R^{Zs} \\ \delta g_L^{Zc} \\ \delta g_R^{Zc} \\ \delta g_L^{Zb} \\ \delta g_R^{Zb} \\ \delta m_w \end{pmatrix} = \begin{pmatrix} -1.2 \pm 3.2 \\ -2.7 \pm 2.6 \\ 1.5 \pm 4.0 \\ -0.20 \pm 0.28 \\ 0.1 \pm 1.2 \\ -0.09 \pm 0.59 \\ -0.43 \pm 0.27 \\ 0.0 \pm 1.4 \\ 0.62 \pm 0.62 \\ -12 \pm 23 \\ -4 \pm 31 \\ -19 \pm 36 \\ -30 \pm 130 \\ 11 \pm 28 \\ 32 \pm 48 \\ -1.5 \pm 3.6 \\ -3.3 \pm 5.3 \\ 3.1 \pm 1.7 \\ 21.9 \pm 8.8 \\ 0.29 \pm 0.16 \end{pmatrix} \times 10^{-3}.$$

Breso-Pla et al 2103.12074

It follows $$\frac{\Lambda}{|c_{He}|^{1/2}} \gtrsim 5.6 \text{ TeV}$$

Modified interaction strength: indirectly

Example: $$(H^\dagger H)\,\Box\,(H^\dagger H)$$

This contributes to the kinetic term of the Higgs boson

$$\frac{c_{H\Box}}{\Lambda^2}(H^\dagger H)\,\Box\,(H^\dagger H) \to -\frac{c_{H\Box}\mathrm{v}^2}{\Lambda^2}(\partial_\mu h)^2$$

Together with the SM kinetic term:

$$\mathscr{L}_{\mathrm{SMEFT}} \supset \frac{1}{2}(\partial_\mu h)^2\left(1 - \frac{2c_{H\Box}\mathrm{v}^2}{\Lambda^2}\right)$$

To restore canonical normalization, we need to rescale the Higgs boson field:

$$h \to h\left(1 + \frac{c_{H\Box}\mathrm{v}^2}{\Lambda^2}\right)$$

This restore canonical normalization of the Higgs boson field, up to terms of order $1/\Lambda^4$, which we ignore here

Modified interaction strength: indirectly

$$h \to h\left(1 + \frac{c_{H\Box} \mathrm{v}^2}{\Lambda^2}\right)$$

After this rescaling, the dimension-6 contribution vanishes from the Higgs boson kinetic term

However, it resurfaces in all Higgs boson couplings present in the SM !

$$\frac{h}{\mathrm{v}}\left[2m_W^2 W_\mu^+ W_\mu^- + m_Z^2 Z_\mu Z_\mu\right] \to \frac{h}{\mathrm{v}}\left(1 + \frac{c_{H\Box} \mathrm{v}^2}{\Lambda^2}\right)\left[2m_W^2 W_\mu^+ W_\mu^- + m_Z^2 Z_\mu Z_\mu\right]$$

$$\frac{h}{\mathrm{v}} m_f \bar{f} f \to \frac{h}{\mathrm{v}}\left(1 + \frac{c_{H\Box} \mathrm{v}^2}{\Lambda^2}\right) m_f \bar{f} f$$

Hence, the Higgs boson interaction strength predicted by the SM is universally shifted

LHC measurements of the Higgs signal strength provide a bound on the Wilson coefficient

$$\mu = 1.09 \pm 0.11 \quad \Rightarrow \quad \frac{c_{H\Box} \mathrm{v}^2}{\Lambda^2} = 0.09 \pm 0.11$$

or, equivalently

$$\frac{c_{H\Box}}{\Lambda^2} = \frac{1}{(820\mathrm{GeV})^2} \pm \frac{1}{(740\mathrm{GeV})^2}$$

Higgs measurements only probe new physics scale of order a TeV

Modified interaction strength: stealthily

Consider the dimension-6 operator $|H^\dagger D_\mu H|^2$

After electroweak symmetry breaking:

$$\frac{c_{HD}}{\Lambda^2}|H^\dagger D_\mu H|^2 \to \frac{c_{HD}\mathrm{v}^2}{2\Lambda^2}\frac{(g_L^2+g_Y^2)\mathrm{v}^2}{8}Z_\mu Z_\mu + \ldots$$

Thus it modifies the Z boson mass: $$m_Z^2 = \frac{(g_L^2+g_Y^2)\mathrm{v}^2}{4}\left(1+\frac{c_{HD}\mathrm{v}^2}{2\Lambda^2}\right)$$

We have this very precise O(10^{-4}) measurement of the Z boson mass

$$m_Z = (91.1876 \pm 0.0021)\ \mathrm{GeV}$$

From which we find the very stringent constraint

$$\frac{|c_{HD}|}{\Lambda^2} \leq \frac{1}{(26\ \mathrm{TeV})^2}$$

Modified interaction strength: stealthily

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From which we find the very stringent constraint

$$\frac{|c_{HD}|}{\Lambda^2} \leq \frac{1}{(26\ \mathrm{TeV})^2}$$

No!

Non!

Nein!

Nie!

Нет!

Ni!

Modified interaction strength: stealthily

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After electroweak symmetry breaking:

$$\frac{c_{HD}}{\Lambda^2}|H^\dagger D_\mu H|^2 \to \frac{c_{HD}\mathrm{v}^2}{2\Lambda^2}\frac{(g_L^2+g_Y^2)\mathrm{v}^2}{8}Z_\mu Z_\mu + \dots$$

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$$m_Z^2 = \frac{(g_L^2+g_Y^2)\mathrm{v}^2}{4}\left(1+\frac{c_{HD}\mathrm{v}^2}{2\Lambda^2}\right)$$

We cannot use the Z-boson mass measurement to constrain new physics because, it is one of the inputs to determine the electroweak parameters of the SM

In the SM:

$$G_F = \frac{1}{\sqrt{2}\mathrm{v}^2}$$

$$\alpha = \frac{g_L^2 g_Y^2}{4\pi(g_L^2+g_Y^2)}$$

$$m_Z^2 = \frac{(g_L^2+g_Y^2)\mathrm{v}^2}{4}$$

$$\Rightarrow$$

$$g_L = 0.6485$$

$$g_Y = 0.3580$$

$$\mathrm{v} = 246.22\ \mathrm{GeV}$$

with very small errors

Modified interaction strength: stealthily

$|H^\dagger D_\mu H|^2$

In the presence of our dimension-6 operators, the relation between electroweak couplings and observables is disrupted

$$G_F = \frac{1}{\sqrt{2}\mathrm{v}^2} \qquad \alpha = \frac{g_L^2 g_Y^2}{4\pi(g_L^2 + g_Y^2)} \qquad m_Z^2 = \frac{(g_L^2 + g_Y^2)\mathrm{v}^2}{4}\left(1 + \frac{c_{HD}\mathrm{v}^2}{2\Lambda^2}\right)$$

Now we cannot assign numerical values to the electroweak parameters, because they depend on c_{HD}

A useful trick is to get rid of the dimension-6 pollution in the input equations by redefining the SM electroweak parameters

$$g_L \to \tilde{g}_L\left(1 - \frac{c_{HD} g_L^2 \mathrm{v}^2}{4(g_L^2 - g_Y^2)\Lambda^2}\right) \qquad g_Y \to \tilde{g}_Y\left(1 + \frac{c_{HD} g_Y^2 \mathrm{v}^2}{4(g_L^2 - g_Y^2)\Lambda^2}\right)$$

For the twiddle electroweak parameter, we can now assign numerical values

$$G_F = \frac{1}{\sqrt{2}\mathrm{v}^2}$$

$$\alpha = \frac{\tilde{g}_L^2 \tilde{g}_Y^2}{4\pi(\tilde{g}_L^2 + \tilde{g}_Y^2)}$$

$$m_Z^2 = \frac{(\tilde{g}_L^2 + \tilde{g}_Y^2)\mathrm{v}^2}{4}$$

$$\tilde{g}_L = 0.6485$$

$$\tilde{g}_Y = 0.3580$$

$$\mathrm{v} = 246.22\ \mathrm{GeV}$$

same as in the SM

Modified interaction strength: stealthily

Z mass cannot be used to constrain new physics, because it was already used to set numerical values for the twiddle electroweak parameter

But new physics emerges now in other observables, e.g. in the W mass

$$m_W = \frac{g_L \mathrm{v}}{2} = \frac{\tilde{g}_L \mathrm{v}}{2}\left(1 - \frac{c_{HD} g_L^2 \mathrm{v}^2}{4(g_L^2 - g_Y^2)\Lambda^2}\right) = \frac{\tilde{g}_L \mathrm{v}}{2}\left(1 - \frac{c_{HD} \tilde{g}_L^2 \mathrm{v}^2}{4(\tilde{g}_L^2 - \tilde{g}_Y^2)\Lambda^2}\right)$$

We can now use the experimental measurement of the W mass

$$m_W = \left(80.379 \pm 0.012\right)\ \mathrm{GeV}$$

to constrain the Wilson coefficients

$$-\frac{1}{(7\ \mathrm{TeV})^2} \leq \frac{c_{HD}}{\Lambda^2} \leq -\frac{1}{(12\ \mathrm{TeV})^2}$$

at 1 sigma

Numerically very different constraint than what one would (incorrectly) obtain from Z mass!

Modified interaction strength: stealthily

Flavor observables are another class of experiments that depends on a priori unknown parameters

$$\mathscr{L}_{\mathrm{SMEFT}} \supset V_{u_k d_l} \bar{u}_k \gamma^\mu P_L d_l + \mathrm{h.c.}$$

CKM matrix:

$$\begin{aligned} V &= \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \\ &= \begin{pmatrix} 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4 & \lambda & A\lambda^3(1+\frac{1}{2}\lambda^2)(\bar{\rho}-i\bar{\eta}) \\ -\lambda+A^2\lambda^5(\frac{1}{2}-\bar{\rho}-i\bar{\eta}) & 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4(1+4A^2) & A\lambda^2 \\ A\lambda^3(1-\bar{\rho}-i\bar{\eta}) & -A\lambda^2+A\lambda^4(\frac{1}{2}-\bar{\rho}-i\bar{\eta}) & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix} + \mathcal{O}(\lambda^6) \end{aligned}$$

The 4 Wolfenstein parameters need to be taken from experiment

PDG says:

$$\lambda = 0.22650 \pm 0.00048\,, \qquad A = 0.790^{+0.017}_{-0.012}\,,$$
$$\bar{\rho} = 0.141^{+0.016}_{-0.017}\,, \qquad \bar{\eta} = 0.357 \pm 0.011$$

But you cannot use it in the SMEFT !

Measurements from which Wolfenstein parameters are normally extracted depend on the CKM parameters, and at the same time on many Wilson coefficients of dimension-6 operators

Modified interaction strength: stealthily

Much as for electroweak parameters,
in the SMEFT one needs and input scheme for the CKM parameters

Descotes-Genon
arXiv:1812.08163

Example, extracting the Cabibbo angle λ from pseudoscalar decays

Decay constant **Known radiative corrections**

$$\Gamma(P^- \to \ell^- \bar{\nu}_\ell) = |V_{uq}|^2 \frac{f_P^2\, m_P m_\ell^2}{16\pi \tilde{v}^4} \left(1 - \frac{m_\ell^2}{m_P^2}\right)^2 (1 + \delta_{P\ell})\,(1 + \Delta_{P\ell 2}) \,,$$

True CKM element **SMEFT corrections depending on dimension-6 Wilson coefficient**

$$\frac{\Gamma(K \to \mu\nu)}{\Gamma(\pi \to \mu\nu)} = \frac{\lambda^2}{1-\lambda^2} \frac{f_K^2}{f_\pi^2} \frac{m_K}{m_\pi} \frac{(1 - m_\mu^2/m_K^2)^2}{(1 - m_\mu^2/m_\pi^2)^2} \left(1 + \delta_{K\ell} - \delta_{\pi\ell}\right)\left(1 + \Delta_{K\ell 2} - \Delta_{\pi\ell 2}\right)$$

Introduce polluted Cabibbo angle $\tilde{\lambda}$ defined by $\frac{\lambda^2}{1-\lambda^2}\left(1 + \Delta_{K\ell 2} - \Delta_{\pi\ell 2}\right) = \frac{\tilde{\lambda}^2}{1-\tilde{\lambda}^2}$ ➡ $\lambda = \tilde{\lambda}(1 + \Delta_\lambda)$

$$\frac{\Gamma(K \to \mu\nu)}{\Gamma(\pi \to \mu\nu)} = \frac{\tilde{\lambda}^2}{1-\tilde{\lambda}^2} \frac{f_K^2}{f_\pi^2} \frac{m_K}{m_\pi} \frac{(1 - m_\mu^2/m_K^2)^2}{(1 - m_\mu^2/m_\pi^2)^2} \left(1 + \delta_{K\ell} - \delta_{\pi\ell}\right)$$

Everything in this equation is known (experimentally or theoretically) except for $\tilde{\lambda}$
thus we can use it to assign a numerical value to $\tilde{\lambda}$

Modified interaction strength: stealthily

Continuing this procedure, polluted Wolfenstein parameters can be extracted e.g. from the following 4 observables

$$\boxed{\Gamma(K \to \mu\nu_\mu)/\Gamma(\pi \to \mu\nu_\mu), \quad \Gamma(B \to \tau\nu_\tau), \quad \Delta M_d, \quad \Delta M_s.}$$

$$\tilde{\lambda} = 0.22537 \pm 0.00046$$

$$\tilde{A} = 0.828 \pm 0.021$$

$$\tilde{\rho} = 0.194 \pm 0.024$$

$$\tilde{\eta} = 0.391 \pm 0.048$$

All other flavor observables can then be use for constraining dimension-6 SMEFT operators after replacing:

$$\begin{pmatrix} \lambda \\ A \\ \bar{\rho} \\ \bar{\eta} \end{pmatrix} \to \begin{pmatrix} \tilde{\lambda}(1+\Delta_\lambda) \\ \tilde{A}(1+\Delta_A) \\ \tilde{\rho}(1+\Delta_\rho) \\ \tilde{\eta}(1+\Delta_\eta) \end{pmatrix}$$

where Δ_x are know linear combinations of SMEFT Wilson coefficients

Part 7

Exercise in
UV matching of SMEFT

Matching new physics to D=6 Lagrangian

Example: Vector Triplet Resonance

Why vector triplet?

- Predicted by technicolor and composite Higgs models
- Nice simple model leading to higher-derivative Higgs boson couplings at tree level

Vector Triplet Resonance

A new SU(2) triplet of heavy vector bosons, coupled to SM SU(2) Higgs and fermionic currents:

$$\Delta\mathcal{L} = -\frac{1}{4}V^i_{\mu\nu}V^i_{\mu\nu} + \frac{m_V^2}{2}V^i_\mu V^i_\mu + \frac{i}{2}\kappa_H V^i_\mu H^\dagger \sigma^i \overleftrightarrow{D}_\mu H + \frac{1}{2}\kappa_F V^i_\mu \sum_{f\in\ell,q} \bar{f}\sigma^i\bar{\sigma}_\mu f$$

For, simplicity, couplings to fermions assumed universal.
Thus, model has 3 free parameters: mV, κH, and κF.
This time we identify mV with EFT expansion parameter Λ.
Solving equations of motions to leading order in $1/\Lambda$:

$$V^i_\mu = -\frac{1}{\Lambda^2}\left(\frac{i}{2}\kappa_H H^\dagger\sigma^i\overleftrightarrow{D}_\mu H + \frac{1}{2}\kappa_F \sum_{f\in\ell,q}\bar{f}\sigma^i\bar{\sigma}_\mu f\right)$$

Effective Lagrangian

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} - \frac{1}{8\Lambda^2}\left(i\kappa_H H^\dagger\sigma^i\overleftrightarrow{D}_\mu H + \kappa_F \sum_{f\in\ell,q}\bar{f}\sigma^i\bar{\sigma}_\mu f\right)^2$$

Vector Triplet Resonance

Effective Lagrangian can also be obtained another way by 1st shifting: $W_\mu^i \to W_\mu^i - \frac{\kappa_F}{g_L} V_\mu^i$

$$\Delta\mathcal{L} = -\frac{1}{4} V_{\mu\nu}^i V_{\mu\nu}^i + \frac{m_V^2}{2} V_\mu^i V_\mu^i + \frac{i}{2}(\kappa_H - \kappa_F) V_\mu^i H^\dagger \sigma^i \overleftrightarrow{D}_\mu H + \frac{\kappa_F}{g_L} V_\mu^i D_\nu W_{\mu\nu}^i + \dots$$

Note that the new vector field does not couple to fermions anymore.
Solving equations of motions to leading order in $1/\Lambda$, and plugging back, we obtain the effective Lagrangian:

$$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_{\mathrm{SM}} - \frac{1}{2\Lambda^2}\left(\frac{i}{2}(\kappa_H - \kappa_F) H^\dagger \sigma^i \overleftrightarrow{D}_\mu H + \frac{\kappa_F}{g_L} D_\nu W_{\mu\nu}^i\right)^2$$

As compared to

$$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_{\mathrm{SM}} - \frac{1}{8\Lambda^2}\left(i\kappa_H H^\dagger \sigma^i \overleftrightarrow{D}_\mu H + \kappa_F \sum_{f\in \ell,q} \bar{f}\sigma^i \bar{\sigma}_\mu f\right)^2$$

Which one is right? Answer: both!
The equivalence can be proven by using the SM equations of motion:

$$D_\nu W_{\mu\nu}^i = \frac{i g_L}{2} H^\dagger \sigma^i \overleftrightarrow{D}_\mu H + \frac{g_L}{2} \sum_{f\in \ell,q} \bar{f}\sigma^i \bar{\sigma}_\mu f$$

Vector Triplet Resonance

Bosonic CP-even		Bosonic CP-odd	
O_H	$(H^\dagger H)^3$		
$O_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$		
O_{HD}	$\left\|H^\dagger D_\mu H\right\|^2$		
O_{HG}	$H^\dagger H\, G^a_{\mu\nu}G^a_{\mu\nu}$	$O_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}^a_{\mu\nu}G^a_{\mu\nu}$
O_{HW}	$H^\dagger H\, W^i_{\mu\nu}W^i_{\mu\nu}$	$O_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}^i_{\mu\nu}W^i_{\mu\nu}$
O_{HB}	$H^\dagger H\, B_{\mu\nu}B_{\mu\nu}$	$O_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu}B_{\mu\nu}$
O_{HWB}	$H^\dagger\sigma^i H\, W^i_{\mu\nu}B_{\mu\nu}$	$O_{H\widetilde{W}B}$	$H^\dagger\sigma^i H\, \widetilde{W}^i_{\mu\nu}B_{\mu\nu}$
O_W	$\epsilon^{ijk}W^i_{\mu\nu}W^j_{\nu\rho}W^k_{\rho\mu}$	$O_{\widetilde{W}}$	$\epsilon^{ijk}\widetilde{W}^i_{\mu\nu}W^j_{\nu\rho}W^k_{\rho\mu}$
O_G	$f^{abc}G^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}$	$O_{\widetilde{G}}$	$f^{abc}\widetilde{G}^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}$

Table 2.2: Bosonic D=6 operators in the Warsaw basis.

$$(H^\dagger\sigma^i \overleftrightarrow{D}_\mu H)(H^\dagger\sigma^i \overleftrightarrow{D}_\mu H)$$?

$$(\sigma^i)_{mn}(\sigma^i)_{kl} = 2\delta_{ml}\delta_{nk} - \delta_{mn}\delta_{kl}$$

$$\left(\partial_\mu(H^\dagger H)\right)^2 - 4(H^\dagger H)(D_\mu H^\dagger D_\mu H)$$?

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} - \frac{1}{8\Lambda^2}\left(i\kappa_H H^\dagger\sigma^i\overleftrightarrow{D}_\mu H + \kappa_F \sum_{f\in\ell,q}\bar{f}\sigma^i\bar{\sigma}_\mu f\right)^2$$

Vector Triplet Resonance

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O_{HW}	$H^\dagger H\, W^i_{\mu\nu}W^i_{\mu\nu}$	$O_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}^i_{\mu\nu}W^i_{\mu\nu}$
O_{HB}	$H^\dagger H\, B_{\mu\nu}B_{\mu\nu}$	$O_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu}B_{\mu\nu}$
O_{HWB}	$H^\dagger\sigma^i H\, W^i_{\mu\nu}B_{\mu\nu}$	$O_{H\widetilde{W}B}$	$H^\dagger\sigma^i H\, \widetilde{W}^i_{\mu\nu}B_{\mu\nu}$
O_W	$\epsilon^{ijk}W^i_{\mu\nu}W^j_{\nu\rho}W^k_{\rho\mu}$	$O_{\widetilde{W}}$	$\epsilon^{ijk}\widetilde{W}^i_{\mu\nu}W^j_{\nu\rho}W^k_{\rho\mu}$
O_G	$f^{abc}G^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}$	$O_{\widetilde{G}}$	$f^{abc}\widetilde{G}^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}$

Table 2.2: Bosonic D=6 operators in the Warsaw basis.

?

$$(H^\dagger\sigma^i \overleftrightarrow{D}_\mu H)(H^\dagger\sigma^i \overleftrightarrow{D}_\mu H)$$

$$\left(\partial_\mu(H^\dagger H)\right)^2 - 4(H^\dagger H)(D_\mu H^\dagger D_\mu H)$$

?

$$(H^\dagger H)(D_\mu H^\dagger D_\mu H) = \frac{1}{2}(H^\dagger H)\Box(H^\dagger H) - \mu_H^2(H^\dagger H)^2 + 2\lambda(H^\dagger H)^3 + \frac{1}{2}(H^\dagger H)\left[f^c y_f H^\dagger F + \text{h.c.}\right]$$

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} - \frac{1}{8\Lambda^2}\left(i\kappa_H H^\dagger\sigma^i\overleftrightarrow{D}_\mu H + \kappa_F \sum_{f\in\ell,q}\bar{f}\sigma^i\bar{\sigma}_\mu f\right)^2$$

Vector Triplet Resonance

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$O_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$		
O_{HD}	$\left\lvert H^\dagger D_\mu H\right\rvert^2$		
O_{HG}	$H^\dagger H\, G^a_{\mu\nu}G^a_{\mu\nu}$	$O_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}^a_{\mu\nu}G^a_{\mu\nu}$
O_{HW}	$H^\dagger H\, W^i_{\mu\nu}W^i_{\mu\nu}$	$O_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}^i_{\mu\nu}W^i_{\mu\nu}$
O_{HB}	$H^\dagger H\, B_{\mu\nu}B_{\mu\nu}$	$O_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu}B_{\mu\nu}$
O_{HWB}	$H^\dagger\sigma^i H\, W^i_{\mu\nu}B_{\mu\nu}$	$O_{H\widetilde{W}B}$	$H^\dagger\sigma^i H\, \widetilde{W}^i_{\mu\nu}B_{\mu\nu}$
O_W	$\epsilon^{ijk}W^i_{\mu\nu}W^j_{\nu\rho}W^k_{\rho\mu}$	$O_{\widetilde{W}}$	$\epsilon^{ijk}\widetilde{W}^i_{\mu\nu}W^j_{\nu\rho}W^k_{\rho\mu}$
O_G	$f^{abc}G^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}$	$O_{\widetilde{G}}$	$f^{abc}\widetilde{G}^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}$

Table 2.2: Bosonic D=6 operators in the Warsaw basis.

Yukawa

$[O^\dagger_{eH}]_{IJ}$	$H^\dagger H e^c_I H^\dagger \ell_J$
$[O^\dagger_{uH}]_{IJ}$	$H^\dagger H u^c_I \widetilde{H}^\dagger q_J$
$[O^\dagger_{dH}]_{IJ}$	$H^\dagger H d^c_I H^\dagger q_J$

Vertex		Dipole	
$[O^{(1)}_{H\ell}]_{IJ}$	$i\bar\ell_I\bar\sigma_\mu\ell_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{eW}]_{IJ}$	$e^c_I\sigma_{\mu\nu}H^\dagger\sigma^i\ell_J W^i_{\mu\nu}$
$[O^{(3)}_{H\ell}]_{IJ}$	$i\bar\ell_I\sigma^i\bar\sigma_\mu\ell_J H^\dagger\sigma^i \overleftrightarrow{D}_\mu H$	$[O^\dagger_{eB}]_{IJ}$	$e^c_I\sigma_{\mu\nu}H^\dagger\ell_J B_{\mu\nu}$
$[O_{He}]_{IJ}$	$ie^c_I\sigma_\mu\bar e^c_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{uG}]_{IJ}$	$u^c_I\sigma_{\mu\nu}T^a\widetilde{H}^\dagger q_J\, G^a_{\mu\nu}$
$[O^{(1)}_{Hq}]_{IJ}$	$i\bar q_I\bar\sigma_\mu q_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{uW}]_{IJ}$	$u^c_I\sigma_{\mu\nu}\widetilde{H}^\dagger\sigma^i q_J\, W^i_{\mu\nu}$
$[O^{(3)}_{Hq}]_{IJ}$	$i\bar q_I\sigma^i\bar\sigma_\mu q_J H^\dagger\sigma^i \overleftrightarrow{D}_\mu H$	$[O^\dagger_{uB}]_{IJ}$	$u^c_I\sigma_{\mu\nu}\widetilde{H}^\dagger q_J\, B_{\mu\nu}$
$[O_{Hu}]_{IJ}$	$iu^c_I\sigma_\mu\bar u^c_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{dG}]_{IJ}$	$d^c_I\sigma_{\mu\nu}T^a H^\dagger q_J\, G^a_{\mu\nu}$
$[O_{Hd}]_{IJ}$	$id^c_I\sigma_\mu\bar d^c_J H^\dagger \overleftrightarrow{D}_\mu H$	$[O^\dagger_{dW}]_{IJ}$	$d^c_I\sigma_{\mu\nu}\bar H^\dagger\sigma^i q_J\, W^i_{\mu\nu}$
$[O_{Hud}]_{IJ}$	$iu^c_I\sigma_\mu\bar d^c_J\tilde H^\dagger D_\mu H$	$[O^\dagger_{dB}]_{IJ}$	$d^c_I\sigma_{\mu\nu}H^\dagger q_J\, B_{\mu\nu}$

Table 2.3: Two-fermion D=6 operators in the Warsaw basis. The flavor indices are denoted by I, J. For complex operators (O_{Hud} and all Yukawa and dipole operators) the corresponding complex conjugate operator is implicitly included.

Matching:

$$c_H = \frac{3v^2\kappa_H^2}{8\Lambda^2}$$

$$c_{6H} = -\frac{\lambda v^2\kappa_H^2}{\Lambda^2}$$

$$[c'_{H\ell}]_{JJ} = [c'_{Hq}]_{JJ} = -\kappa_H\kappa_F\frac{v^2}{4\Lambda^2}$$

$$[c_f]_{IJ} = \frac{v^2\kappa_H^2}{2\sqrt{2}\Lambda^2}\delta_{IJ}$$

+ 4 fermion operators

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} - \frac{1}{8\Lambda^2}\left(i\kappa_H H^\dagger\sigma^i\overleftrightarrow{D}_\mu H + \kappa_F\sum_{f\in\ell,q}\bar f\sigma^i\bar\sigma_\mu f\right)^2$$

Vector Triplet Resonance

SILH Basis

Table 97: Bosonic D=6 operators in the SILH basis.

Bosonic CP-even		Bosonic CP-odd	
O_H	$\frac{1}{2v^2}\left[\partial_\mu(H^\dagger H)\right]^2$		
O_T	$\frac{1}{2v^2}\left(H^\dagger \overleftrightarrow{D_\mu} H\right)^2$		
O_6	$-\frac{\lambda}{v^2}(H^\dagger H)^3$		
O_g	$\frac{g_s^2}{m_W^2} H^\dagger H\, G^a_{\mu\nu} G^a_{\mu\nu}$	$\widetilde{O}_g$	$\frac{g_s^2}{m_W^2} H^\dagger H\, \widetilde{G}^a_{\mu\nu} G^a_{\mu\nu}$
O_γ	$\frac{g'^2}{m_W^2} H^\dagger H\, B_{\mu\nu} B_{\mu\nu}$	$\widetilde{O}_\gamma$	$\frac{g'^2}{m_W^2} H^\dagger H\, \widetilde{B}_{\mu\nu} B_{\mu\nu}$
O_W	$\frac{ig}{2m_W^2}\left(H^\dagger \sigma^i \overleftrightarrow{D_\mu} H\right) D_\nu W^i_{\mu\nu}$		
O_B	$\frac{ig'}{2m_W^2}\left(H^\dagger \overleftrightarrow{D_\mu} H\right) \partial_\nu B_{\mu\nu}$		
O_{HW}	$\frac{ig}{m_W^2}\left(D_\mu H^\dagger \sigma^i D_\nu H\right) W^i_{\mu\nu}$	$\widetilde{O}_{HW}$	$\frac{ig}{m_W^2}\left(D_\mu H^\dagger \sigma^i D_\nu H\right) \widetilde{W}^i_{\mu\nu}$
O_{HB}	$\frac{ig'}{m_W^2}\left(D_\mu H^\dagger D_\nu H\right) B_{\mu\nu}$	$\widetilde{O}_{HB}$	$\frac{ig}{m_W^2}\left(D_\mu H^\dagger D_\nu H\right) \widetilde{B}_{\mu\nu}$
O_{2W}	$\frac{1}{m_W^2} D_\mu W^i_{\mu\nu} D_\rho W^i_{\rho\nu}$		
O_{2B}	$\frac{1}{m_W^2} \partial_\mu B_{\mu\nu} \partial_\rho B_{\rho\nu}$		
O_{2G}	$\frac{1}{m_W^2} D_\mu G^a_{\mu\nu} D_\rho G^a_{\rho\nu}$		
O_{3W}	$\frac{g^3}{m_W^2}\epsilon^{ijk} W^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}$	$\widetilde{O}_{3W}$	$\frac{g^3}{m_W^2}\epsilon^{ijk} \widetilde{W}^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}$
O_{3G}	$\frac{g_s^3}{m_W^2} f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$	$\widetilde{O}_{3G}$	$\frac{g_s^3}{m_W^2} f^{abc} \widetilde{G}^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$

Yukawa	
$[O^\dagger_{eH}]_{IJ}$	$H^\dagger H e^c_I H^\dagger \ell_J$
$[O^\dagger_{uH}]_{IJ}$	$H^\dagger H u^c_I \widetilde{H}^\dagger q_J$
$[O^\dagger_{dH}]_{IJ}$	$H^\dagger H d^c_I H^\dagger q_J$

Vertex		Dipole	
$[O^{(1)}_{H\ell}]_{IJ}$	$i\bar{\ell}_I \bar{\sigma}_\mu \ell_J H^\dagger \overleftrightarrow{D_\mu} H$	$[O^\dagger_{eW}]_{IJ}$	$e^c_I \sigma_{\mu\nu} H^\dagger \sigma^i \ell_J W^i_{\mu\nu}$
$[O^{(3)}_{H\ell}]_{IJ}$	$i\bar{\ell}_I \sigma^i \bar{\sigma}_\mu \ell_J H^\dagger \sigma^i \overleftrightarrow{D_\mu} H$	$[O^\dagger_{eB}]_{IJ}$	$e^c_I \sigma_{\mu\nu} H^\dagger \ell_J B_{\mu\nu}$
$[O_{He}]_{IJ}$	$i e^c_I \sigma_\mu \bar{e}^c_J H^\dagger \overleftrightarrow{D_\mu} H$	$[O^\dagger_{uG}]_{IJ}$	$u^c_I \sigma_{\mu\nu} T^a \widetilde{H}^\dagger q_J\, G^a_{\mu\nu}$
$[O^{(1)}_{Hq}]_{IJ}$	$i\bar{q}_I \bar{\sigma}_\mu q_J H^\dagger \overleftrightarrow{D_\mu} H$	$[O^\dagger_{uW}]_{IJ}$	$u^c_I \sigma_{\mu\nu} \widetilde{H}^\dagger \sigma^i q_J\, W^i_{\mu\nu}$
$[O^{(3)}_{Hq}]_{IJ}$	$i\bar{q}_I \sigma^i \bar{\sigma}_\mu q_J H^\dagger \sigma^i \overleftrightarrow{D_\mu} H$	$[O^\dagger_{uB}]_{IJ}$	$u^c_I \sigma_{\mu\nu} \widetilde{H}^\dagger q_J\, B_{\mu\nu}$
$[O_{Hu}]_{IJ}$	$i u^c_I \sigma_\mu \bar{u}^c_J H^\dagger \overleftrightarrow{D_\mu} H$	$[O^\dagger_{dG}]_{IJ}$	$d^c_I \sigma_{\mu\nu} T^a H^\dagger q_J\, G^a_{\mu\nu}$
$[O_{Hd}]_{IJ}$	$i d^c_I \sigma_\mu \bar{d}^c_J H^\dagger \overleftrightarrow{D_\mu} H$	$[O^\dagger_{dW}]_{IJ}$	$d^c_I \sigma_{\mu\nu} \bar{H}^\dagger \sigma^i q_J\, W^i_{\mu\nu}$
$[O_{Hud}]_{IJ}$	$i u^c_I \sigma_\mu \bar{d}^c_J \tilde{H}^\dagger D_\mu H$	$[O^\dagger_{dB}]_{IJ}$	$d^c_I \sigma_{\mu\nu} H^\dagger q_J\, B_{\mu\nu}$

Table 2.3: Two-fermion D=6 operators in the Warsaw basis. The flavor indices are denoted by I, J. For complex operators (O_{Hud} and all Yukawa and dipole operators) the corresponding complex conjugate operator is implicitly included.

Exercise: find Wilson coefficients in the SILH basis

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} - \frac{1}{2\Lambda^2}\left(\frac{i}{2}(\kappa_H - \kappa_F) H^\dagger \sigma^i \overleftrightarrow{D_\mu} H + \frac{\kappa_F}{g_L} D_\nu W^i_{\mu\nu}\right)^2$$

Vector Triplet Resonance

Bosonic CP-even		Bosonic CP-odd	
O_H	$(H^\dagger H)^3$		
$O_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$		
O_{HD}	$\left\|H^\dagger D_\mu H\right\|^2$		
O_{HG}	$H^\dagger H\, G^a_{\mu\nu} G^a_{\mu\nu}$	$O_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}^a_{\mu\nu} G^a_{\mu\nu}$
O_{HW}	$H^\dagger H\, W^i_{\mu\nu} W^i_{\mu\nu}$	$O_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}^i_{\mu\nu} W^i_{\mu\nu}$
O_{HB}	$H^\dagger H\, B_{\mu\nu} B_{\mu\nu}$	$O_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu} B_{\mu\nu}$
O_{HWB}	$H^\dagger \sigma^i H\, W^i_{\mu\nu} B_{\mu\nu}$	$O_{H\widetilde{W}B}$	$H^\dagger \sigma^i H\, \widetilde{W}^i_{\mu\nu} B_{\mu\nu}$
O_W	$\epsilon^{ijk} W^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}$	$O_{\widetilde{W}}$	$\epsilon^{ijk} \widetilde{W}^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}$
O_G	$f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$	$O_{\widetilde{G}}$	$f^{abc} \widetilde{G}^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$

Table 2.2: Bosonic D=6 operators in the Warsaw basis.

?

$$(H^\dagger \sigma^i \overleftrightarrow{D}_\mu H)(H^\dagger \sigma^i \overleftrightarrow{D}_\mu H)$$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} - \frac{1}{8\Lambda^2}\left(i\kappa_H H^\dagger \sigma^i \overleftrightarrow{D}_\mu H + \kappa_F \sum_{f\in \ell,q} \bar{f}\sigma^i \bar{\sigma}_\mu f\right)^2$$

Vector Triplet Resonance

Yukawa

$[O_{eH}^\dagger]_{IJ}$	$H^\dagger H e_I^c H^\dagger \ell_J$
$[O_{uH}^\dagger]_{IJ}$	$H^\dagger H u_I^c \widetilde{H}^\dagger q_J$
$[O_{dH}^\dagger]_{IJ}$	$H^\dagger H d_I^c H^\dagger q_J$

Vertex

$[O_{H\ell}^{(1)}]_{IJ}$	$i\bar{\ell}_I \bar{\sigma}_\mu \ell_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O_{H\ell}^{(3)}]_{IJ}$	$i\bar{\ell}_I \sigma^i \bar{\sigma}_\mu \ell_J H^\dagger \sigma^i \overleftrightarrow{D}_\mu H$
$[O_{He}]_{IJ}$	$ie_I^c \sigma_\mu \bar{e}_J^c H^\dagger \overleftrightarrow{D}_\mu H$
$[O_{Hq}^{(1)}]_{IJ}$	$i\bar{q}_I \bar{\sigma}_\mu q_J H^\dagger \overleftrightarrow{D}_\mu H$
$[O_{Hq}^{(3)}]_{IJ}$	$i\bar{q}_I \sigma^i \bar{\sigma}_\mu q_J H^\dagger \sigma^i \overleftrightarrow{D}_\mu H$
$[O_{Hu}]_{IJ}$	$iu_I^c \sigma_\mu \bar{u}_J^c H^\dagger \overleftrightarrow{D}_\mu H$
$[O_{Hd}]_{IJ}$	$id_I^c \sigma_\mu \bar{d}_J^c H^\dagger \overleftrightarrow{D}_\mu H$
$[O_{Hud}]_{IJ}$	$iu_I^c \sigma_\mu \bar{d}_J^c \tilde{H}^\dagger D_\mu H$

Dipole

$[O_{eW}^\dagger]_{IJ}$	$e_I^c \sigma_{\mu\nu} H^\dagger \sigma^i \ell_J W_{\mu\nu}^i$
$[O_{eB}^\dagger]_{IJ}$	$e_I^c \sigma_{\mu\nu} H^\dagger \ell_J B_{\mu\nu}$
$[O_{uG}^\dagger]_{IJ}$	$u_I^c \sigma_{\mu\nu} T^a \widetilde{H}^\dagger q_J G_{\mu\nu}^a$
$[O_{uW}^\dagger]_{IJ}$	$u_I^c \sigma_{\mu\nu} \widetilde{H}^\dagger \sigma^i q_J W_{\mu\nu}^i$
$[O_{uB}^\dagger]_{IJ}$	$u_I^c \sigma_{\mu\nu} \widetilde{H}^\dagger q_J B_{\mu\nu}$
$[O_{dG}^\dagger]_{IJ}$	$d_I^c \sigma_{\mu\nu} T^a H^\dagger q_J G_{\mu\nu}^a$
$[O_{dW}^\dagger]_{IJ}$	$d_I^c \sigma_{\mu\nu} \bar{H}^\dagger \sigma^i q_J W_{\mu\nu}^i$
$[O_{dB}^\dagger]_{IJ}$	$d_I^c \sigma_{\mu\nu} H^\dagger q_J B_{\mu\nu}$

Table 2.3: Two-fermion D=6 operators in the Warsaw basis. The flavor indices are denoted by I, J. For complex operators (O_{Hud} and all Yukawa and dipole operators) the corresponding complex conjugate operator is implicitly included.

Matching:

$$[c'_{H\ell}]_{JJ} = [c'_{Hq}]_{JJ} = -\kappa_H \kappa_F \frac{v^2}{4\Lambda^2}$$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} - \frac{1}{8\Lambda^2}\left(i\kappa_H H^\dagger \sigma^i \overleftrightarrow{D}_\mu H + \kappa_F \sum_{f \in \ell, q} \bar{f} \sigma^i \bar{\sigma}_\mu f \right)^2$$

Vector Triplet Resonance

Lessons learned:

- A subset of all possible dimension-6 operators appear in the low-energy EFT for vector triplet model at tree-level
- But different models would give different subset of operators
- Therefore, to be model independent, one should simultaneously constrain *all* dimension-6 operators
- Matching to dimension-6 operators to UV theory is not always trivial. One needs to use equations of motion and other trick to reduce to operator set in given basis
- However, SM EFT approach is basis independent - results can always be transformed from one basis to another, provided all independent operators are taken into account. Predictions for physical observable do not depend on which bases you use

Thank You