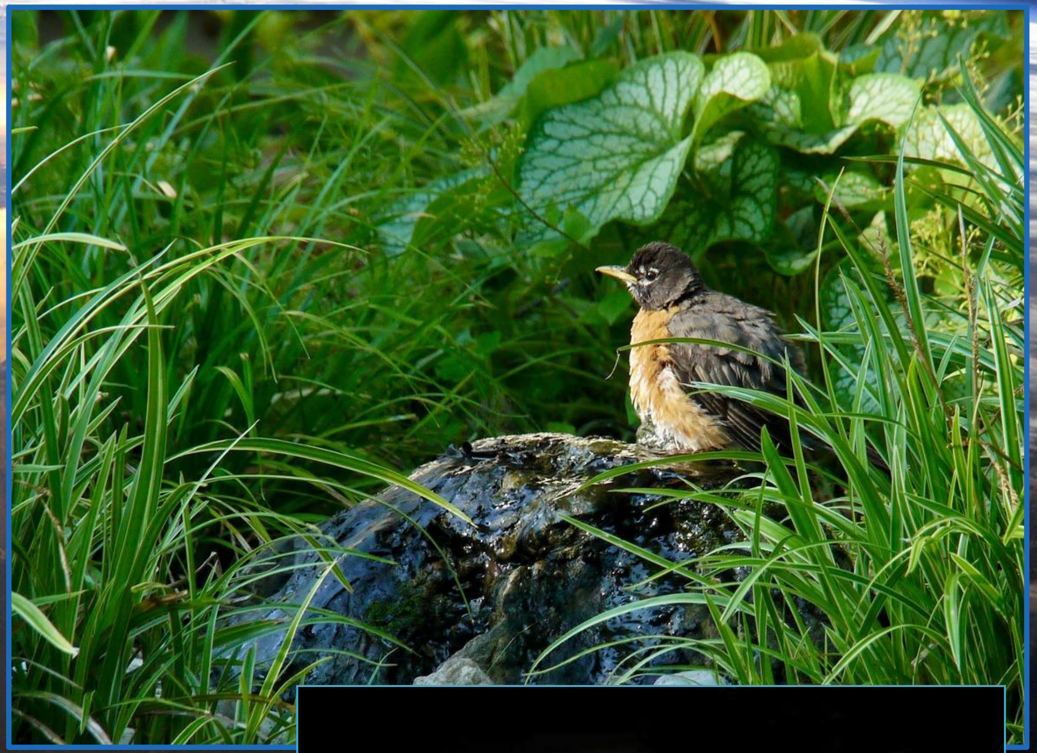
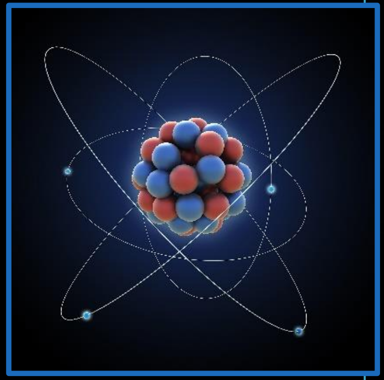
INTRODUCTION TO EFFECTIVE FIELD THEORY

THE VIEW FROM BELOW



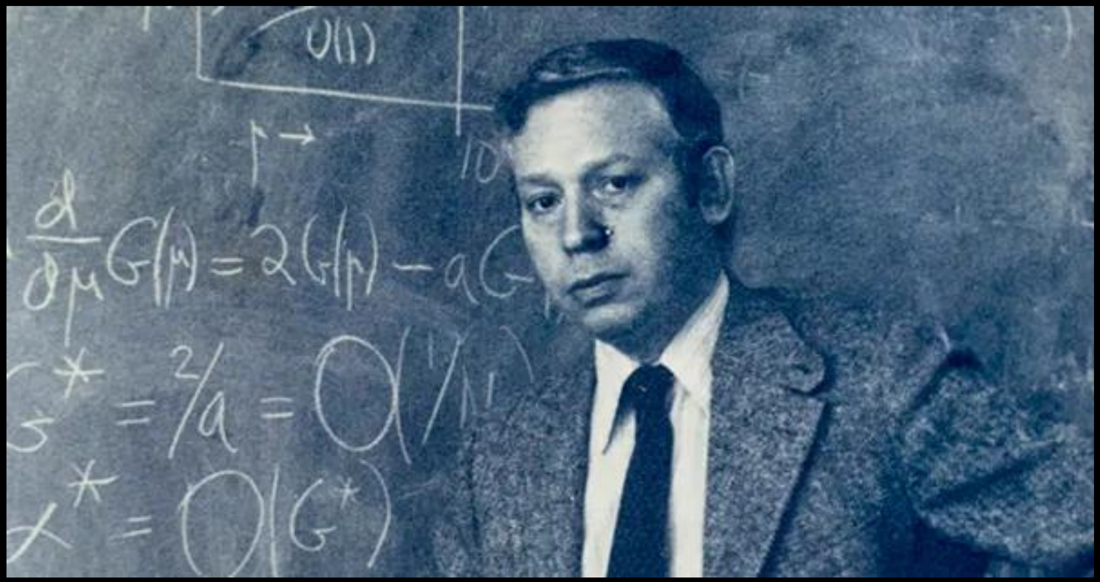
EFFECTIVE PATHWAYS TO NEW PHYSICS, BHUBANESWAR
FEBRUARY 2022



CP BURGESS

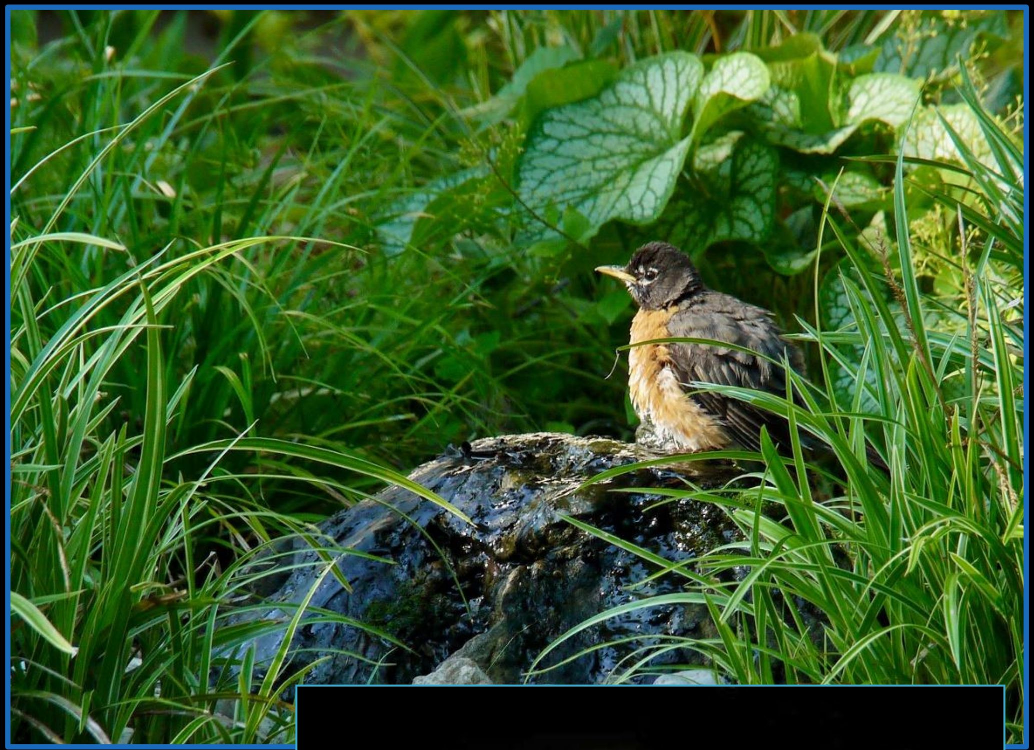
IMAGES: CB, NASA, GENEGEEK.CA, QUORA

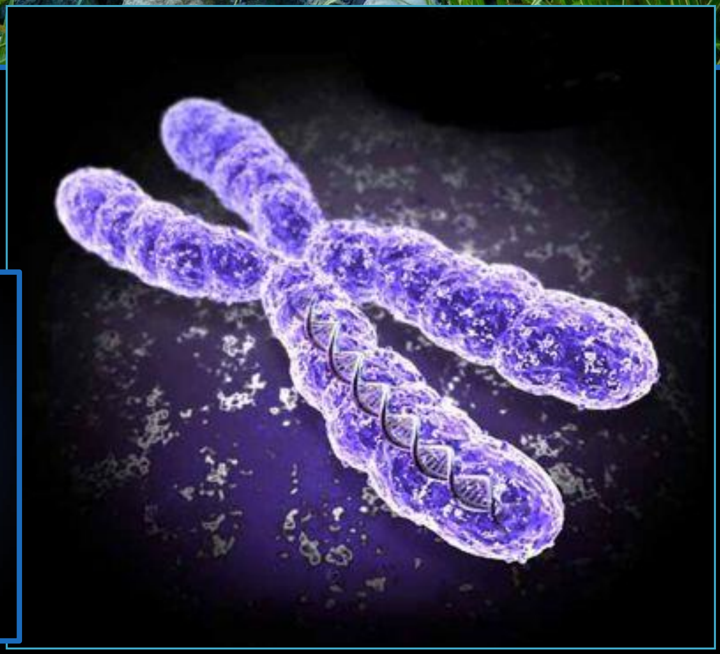
ON THE SHOULDERS OF GIANTS…

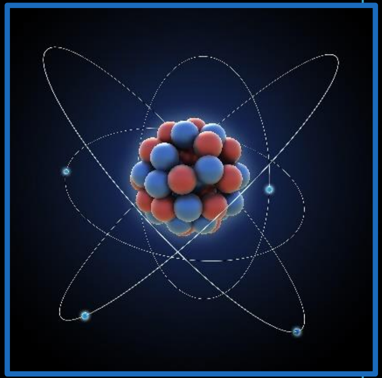


Steven Weinberg 1933-2021

Nobel Prize 1979

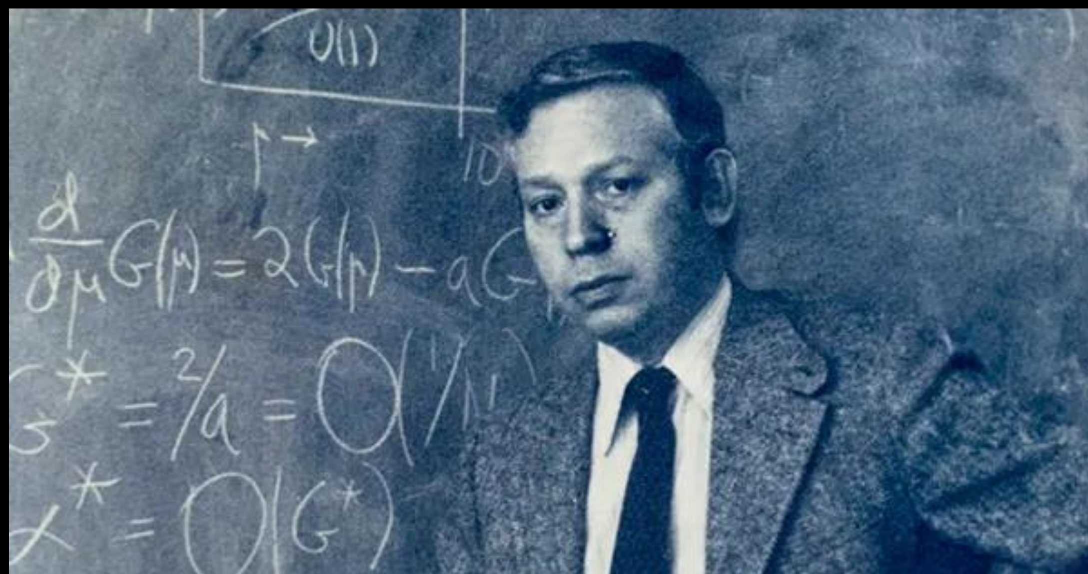


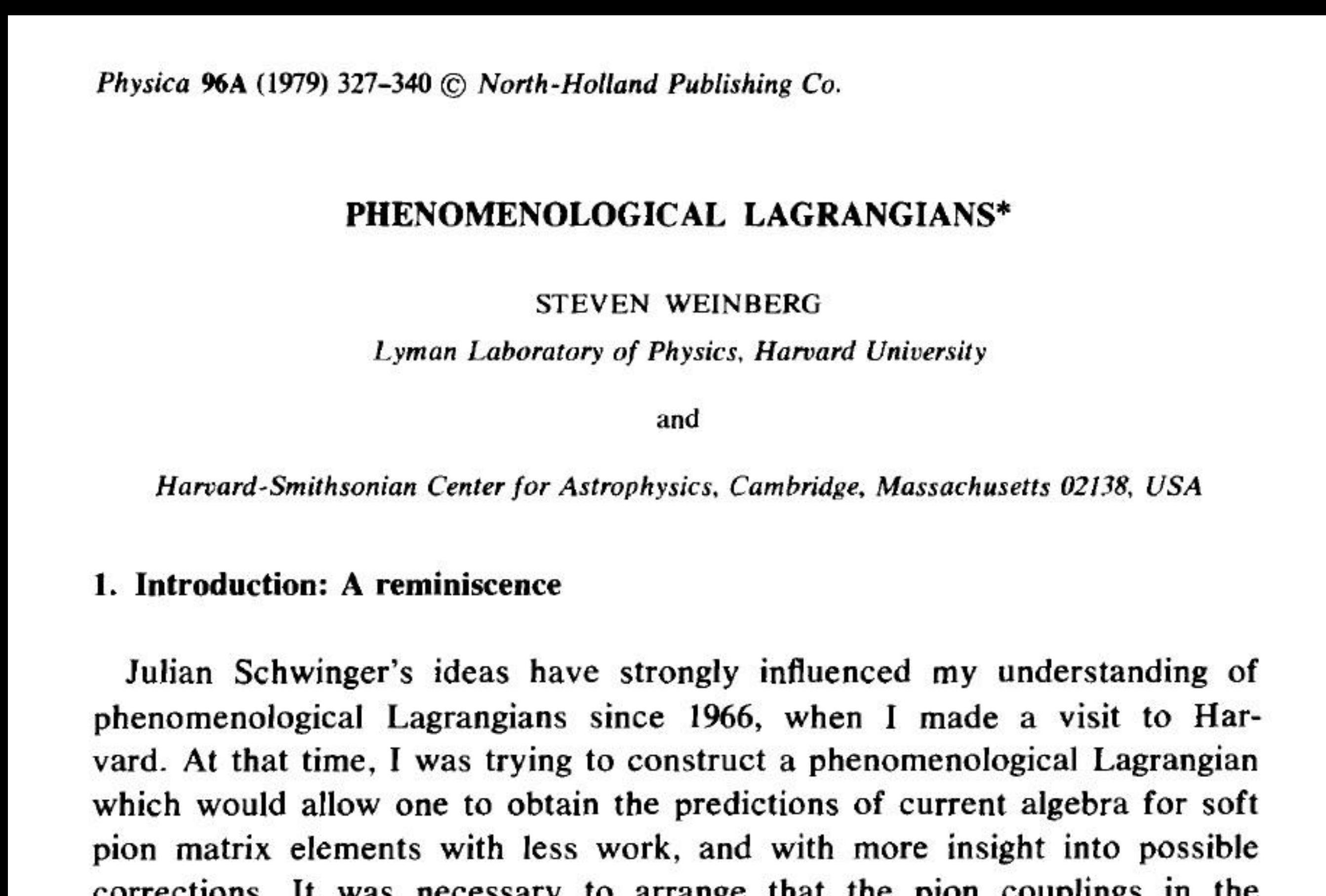




IMAGES: CB, GENEGEEK.CA, QUORA

ON THE SHOULDERS OF GIANTS…





Physica **96A** (1979) 327–340 © *North-Holland Publishing Co.*

PHENOMENOLOGICAL LAGRANGIANS*

STEVEN WEINBERG

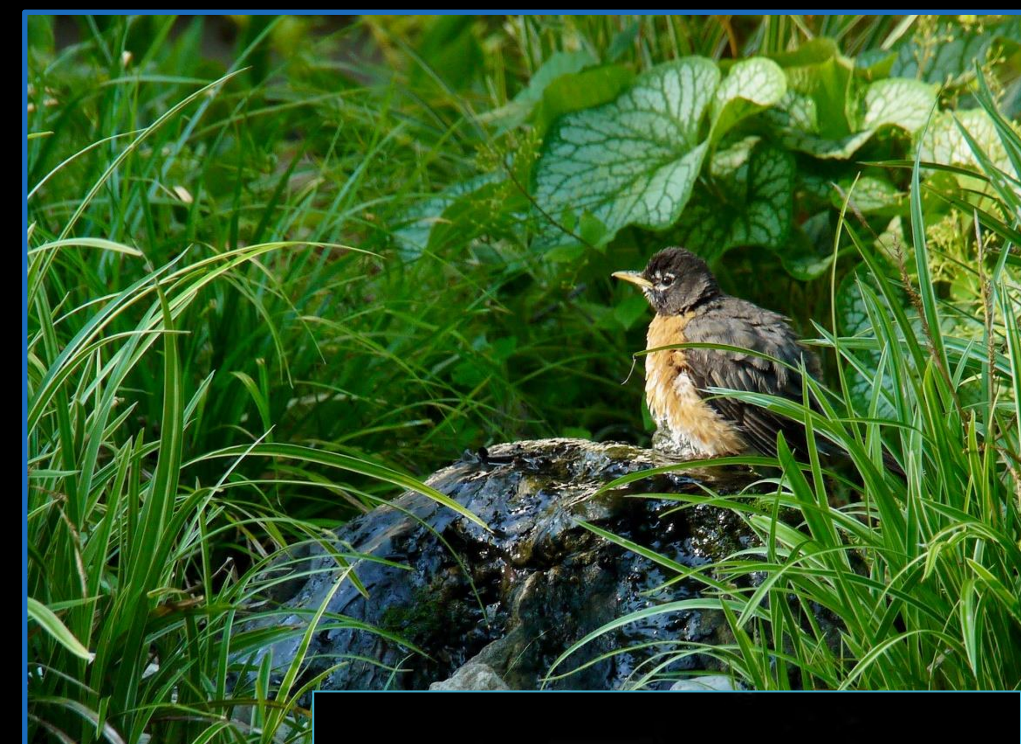
Lyman Laboratory of Physics, Harvard University

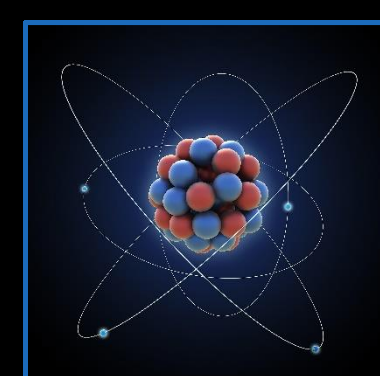
and

Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138, USA

1. Introduction: A reminiscence

Julian Schwinger's ideas have strongly influenced my understanding of phenomenological Lagrangians since 1966, when I made a visit to Harvard. At that time, I was trying to construct a phenomenological Lagrangian which would allow one to obtain the predictions of current algebra for soft pion matrix elements with less work, and with more insight into possible corrections. It was necessary to arrange that the pion couplings in the





IMAGES: CB, GENEGEEK.CA, QUORA

CONTENTS

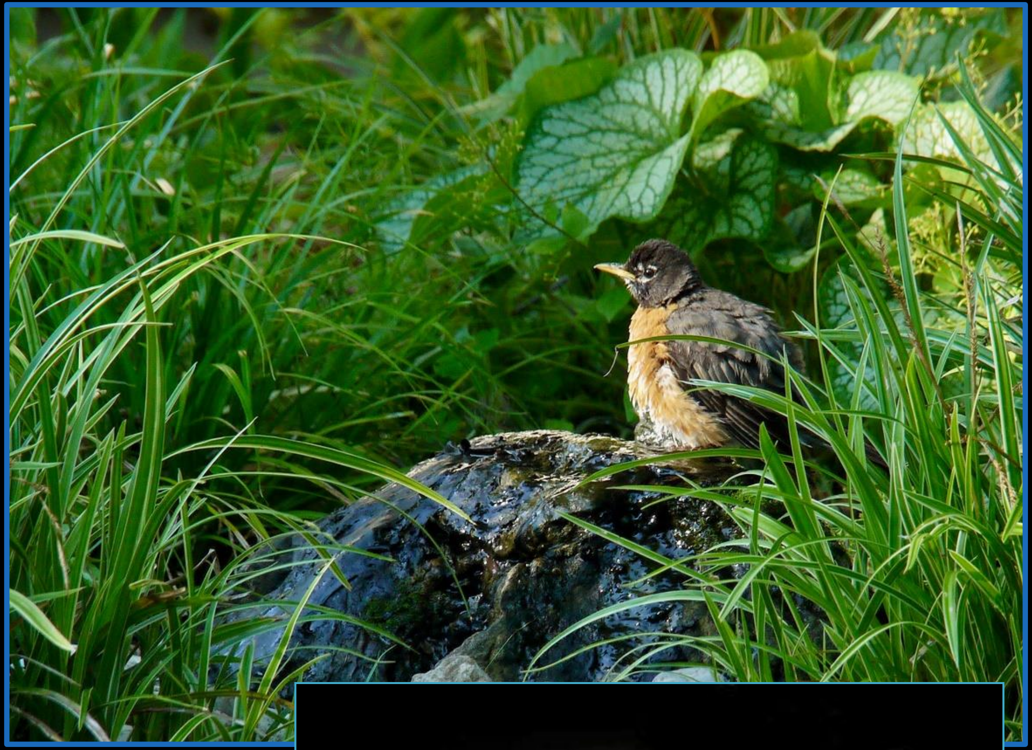
- GENERAL FRAMEWORK

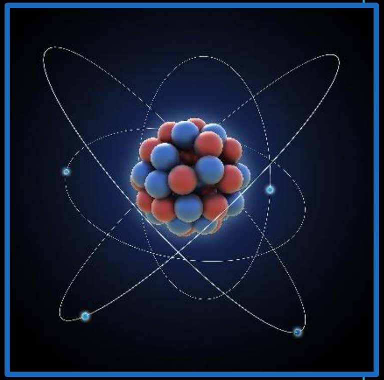
- EFT APPLICATIONS

IMAGES: CB, GENEGEEK.CA, QUORA

GENERAL FRAMEWORK

- DECOUPLING
- EXPLOITING HIERARCHIES
- WHY RENORMALIZATION IS A GOOD THING
- TIME DEPENDENT FIELDS

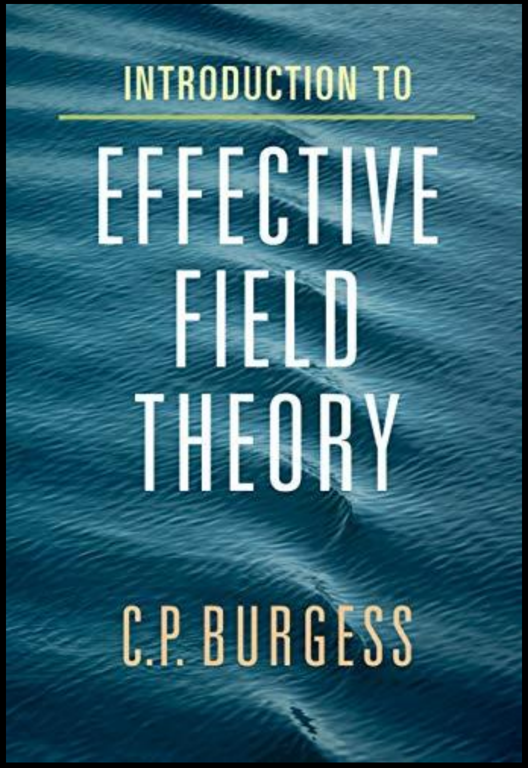


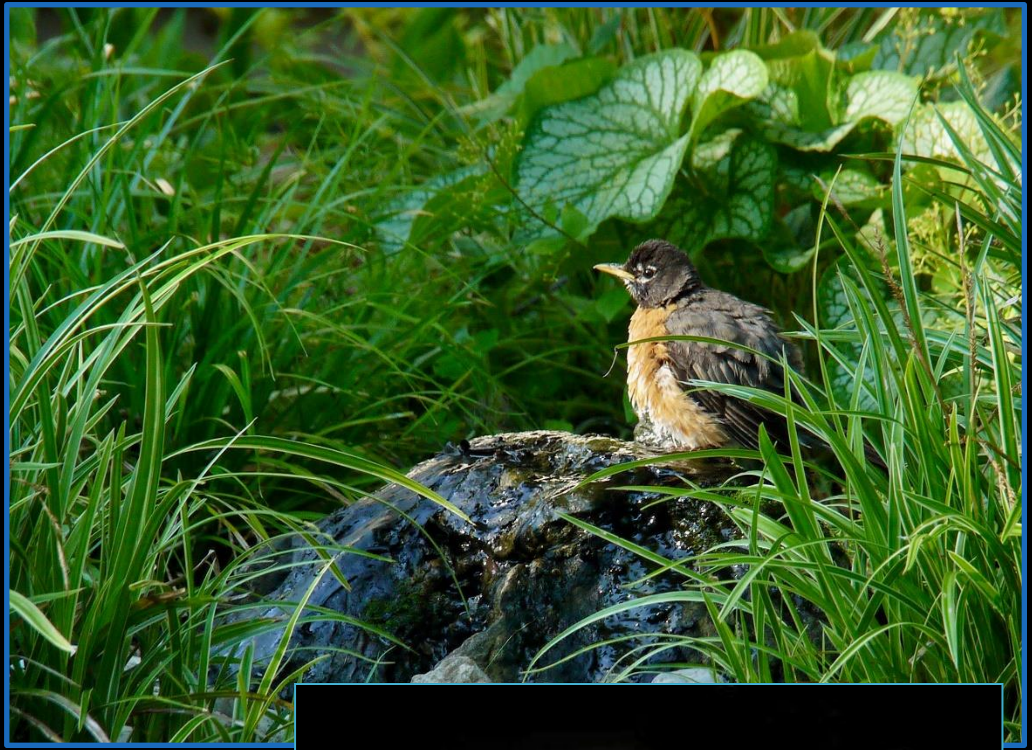


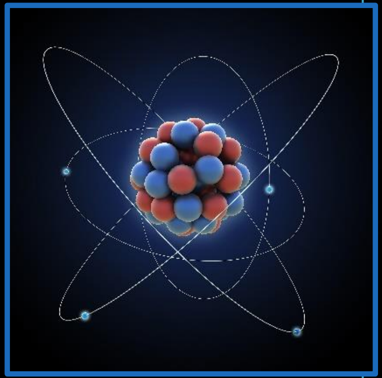
IMAGES: CB, GENEGEEK.CA, QUORA

EFT APPLICATIONS

- ELECTROWEAK PHYSICS
- SUBSTRUCTURE
- NREFT
- GRAVITY



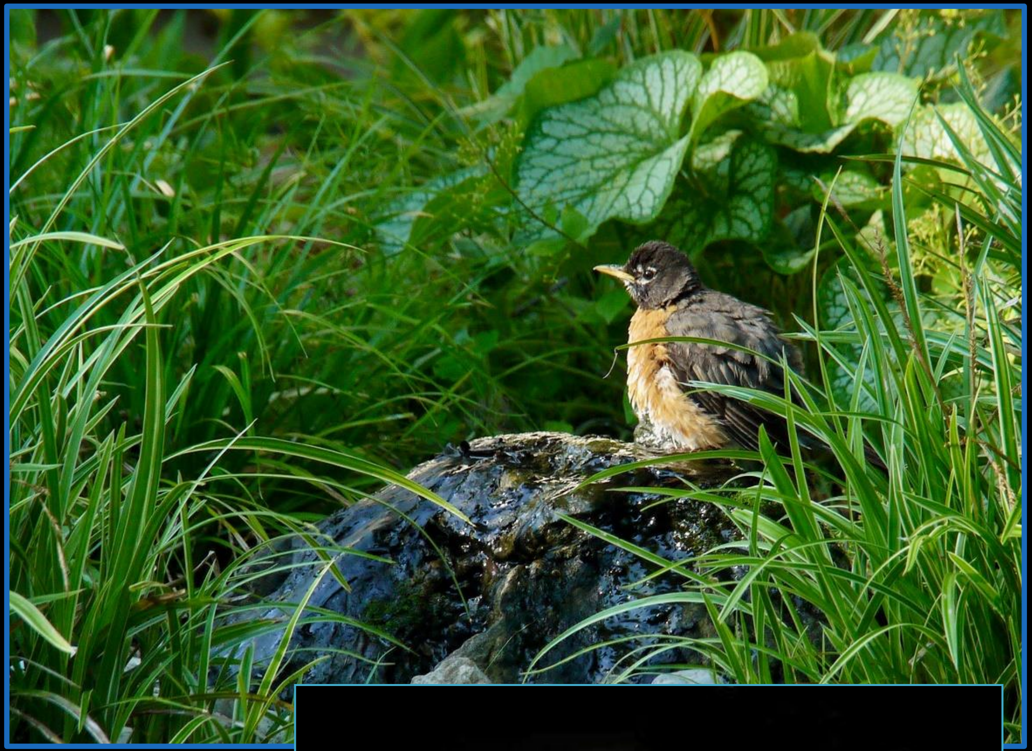


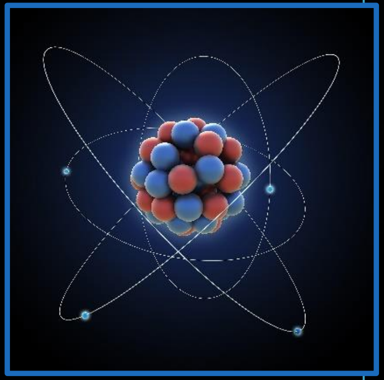


https://physics.mcmaster.ca/~cburgess/cburgess/?page_id=630 IMAGES: CB, GENEGEEK.CA, QUORA

GENERAL FRAMEWORK

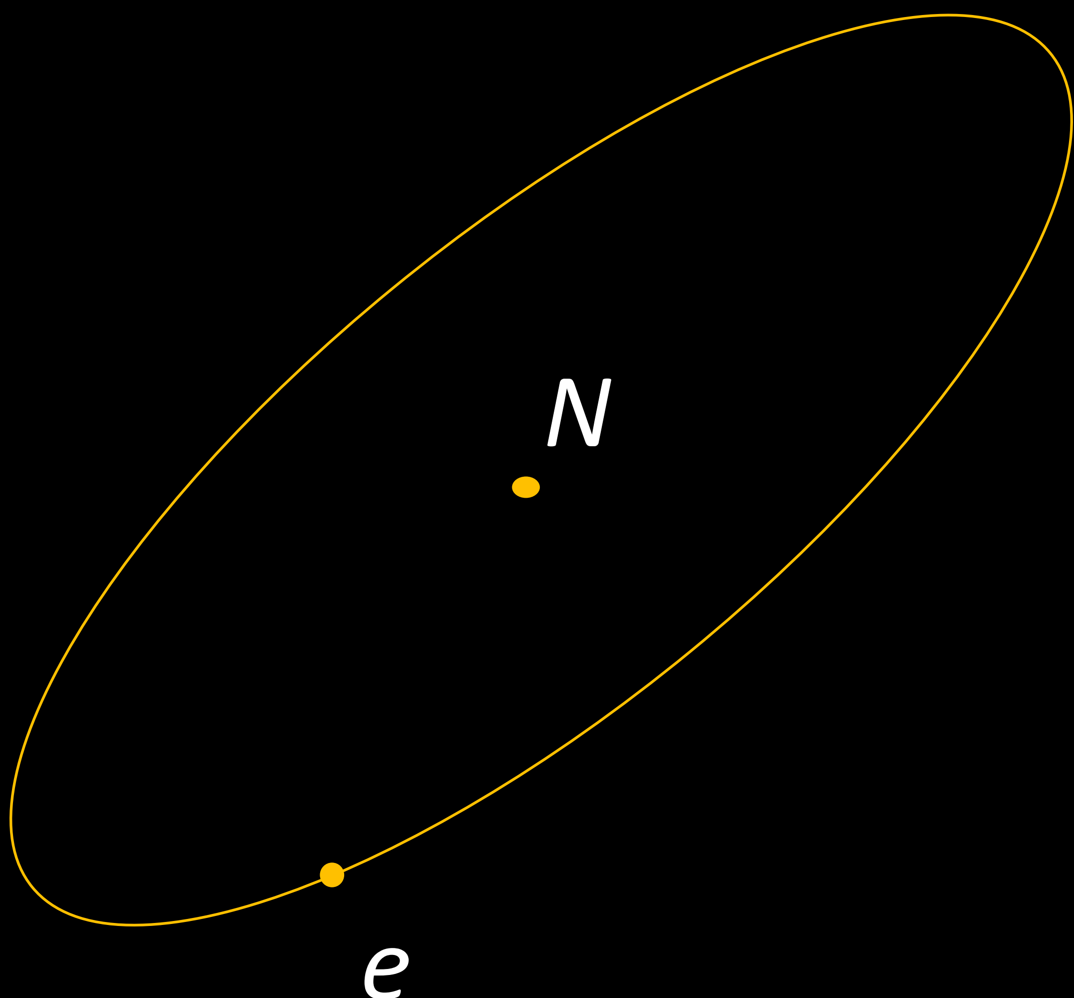
DECOUPLING



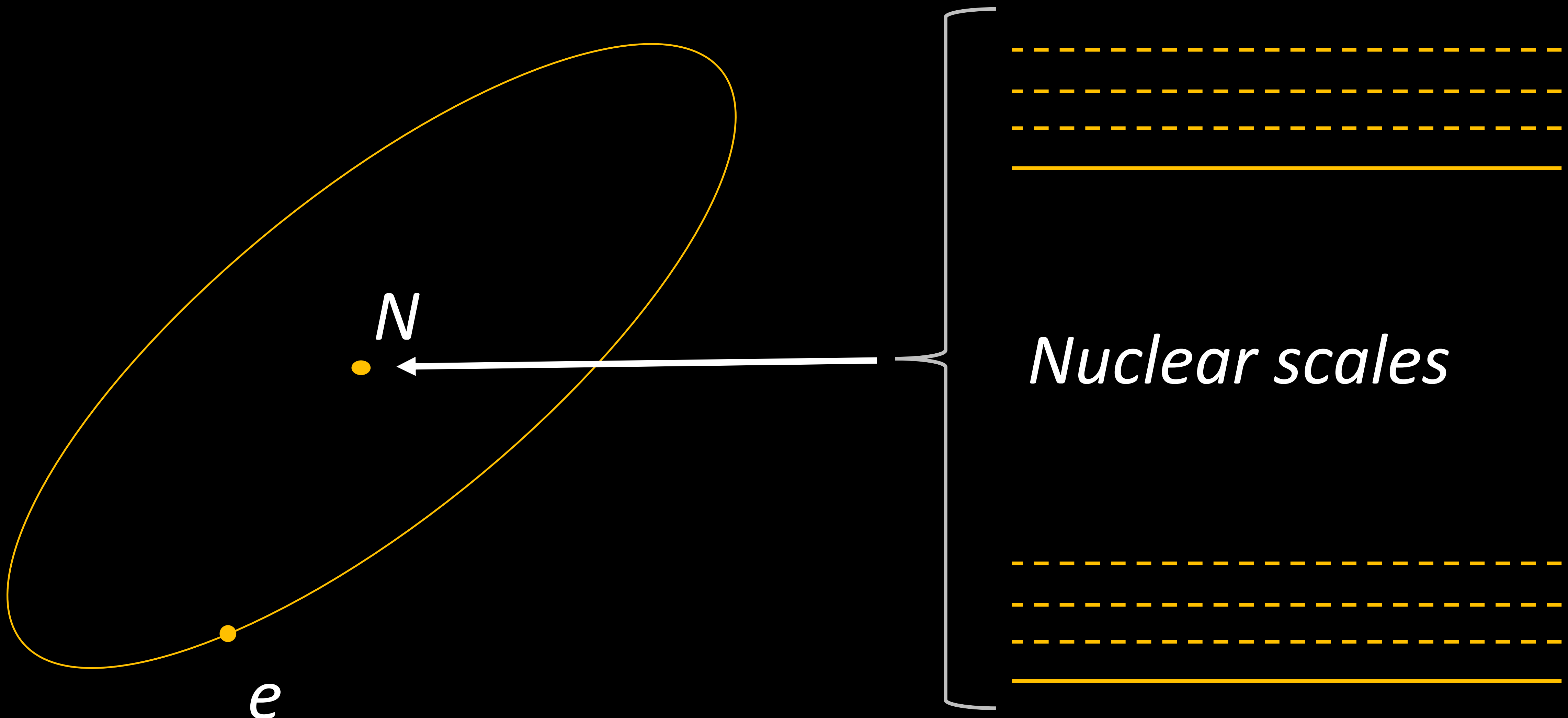


IMAGES: CB, GENEGEEK.CA, QUORA

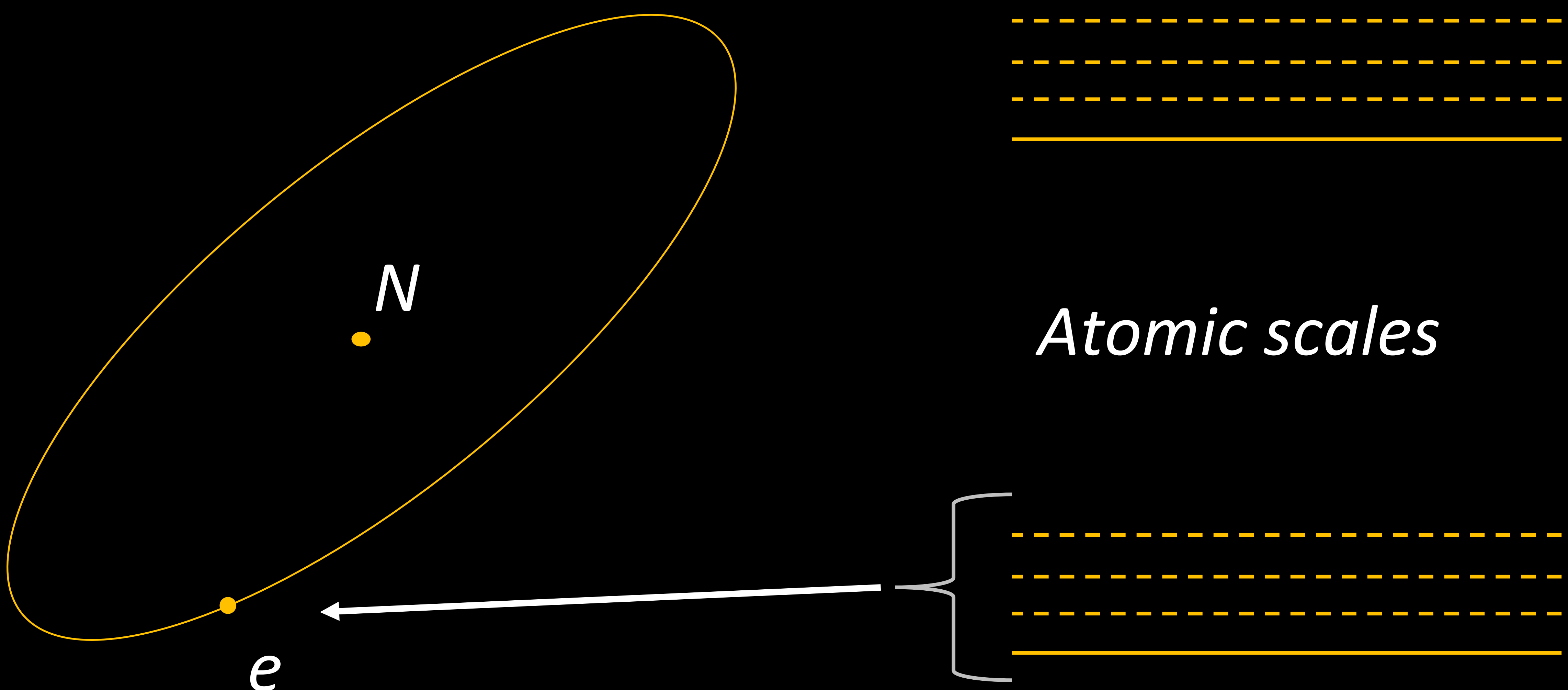
FACT: *Nature comes with many hierarchies of scale, and details of small distances are not needed to understand long distances*



FACT: *Nature comes with many hierarchies of scale, and details of small distances are not needed to understand long distances*



FACT: *Nature comes with many hierarchies of scale, and details of small distances are not needed to understand long distances*



FACT: *Quantum field theory shares this property that small distance physics drops out of long distance physics*

$$
\begin{aligned}
A(m, M, \theta) &= M^p \, f\left(\tfrac{m}{M}, \theta\right) \\
&\simeq M^p \, f(0, \theta)\Big[1 + O(m/M)\Big]
\end{aligned}
$$

(modulo logarithms)

Behooves us to exploit this simplicity as early as possible in a calculation: EFTs

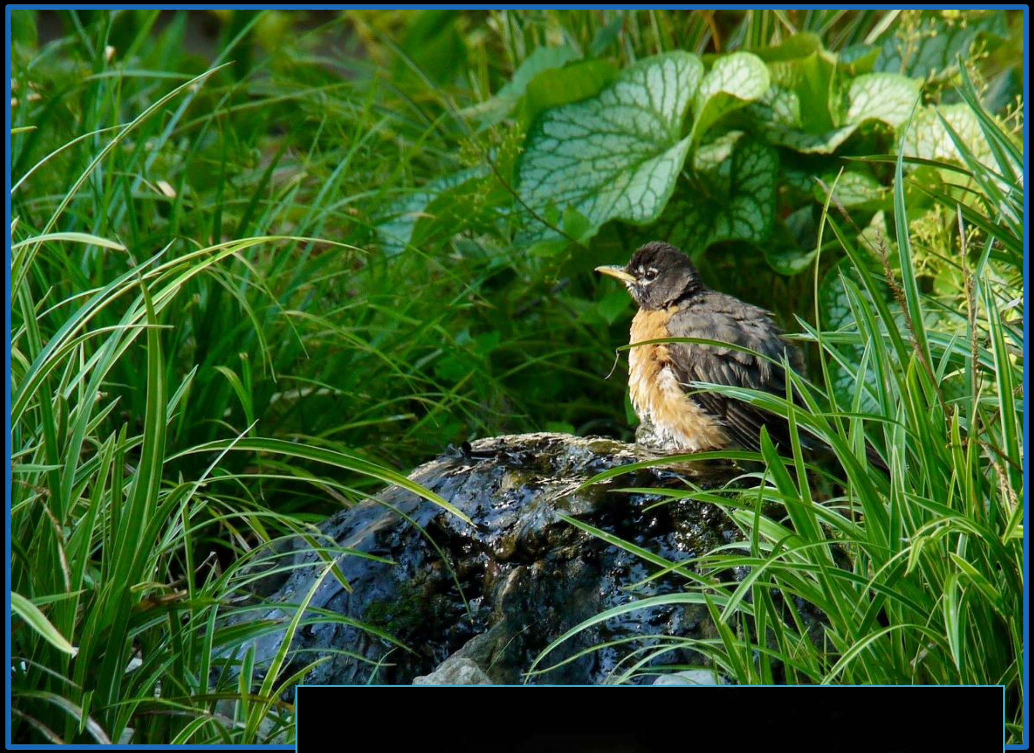
$$A(m, M, \theta) \simeq M^p\, f(0, \theta)\Big[1 + O(m/M)\Big]$$

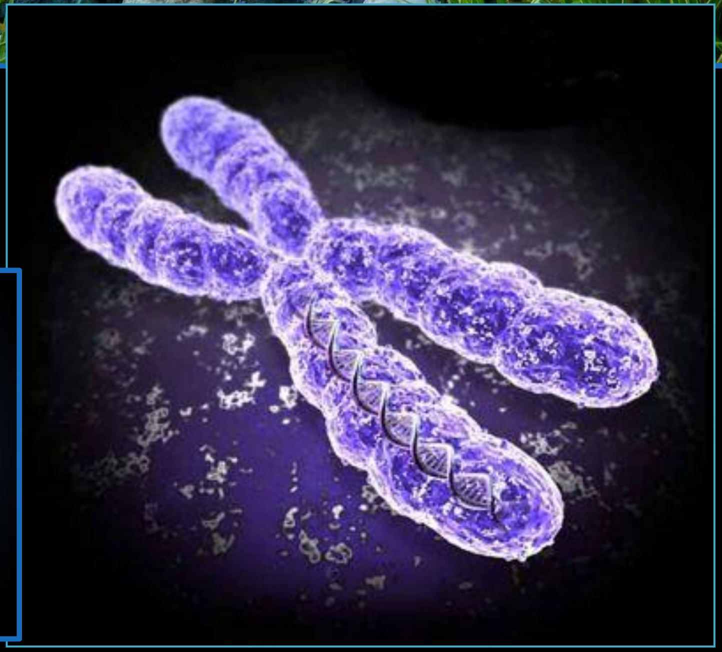
Applicable everywhere because in quantum physics we cannot help probing very short distances

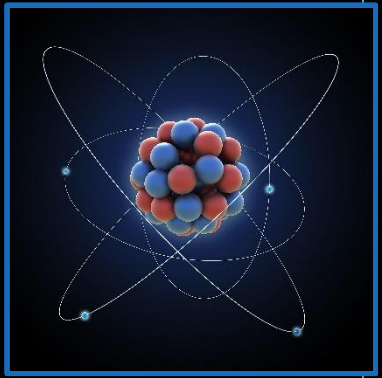
$$E_k \simeq \langle k|H_I|k\rangle + \sum_n \frac{|\langle n|H_I|k\rangle|^2}{E_k - E_n} + \cdots$$

GENERAL FRAMEWORK

EXPLOITING HIERARCHIES







IMAGES: CB, GENEGEEK.CA, QUORA

Simple example: two spinless fields

$$S := -\int \mathrm{d}^4x \left[\partial_\mu \phi^* \partial^\mu \phi + V(\phi^* \phi)\right]$$

$$V(\phi^*\phi) = \tfrac{\lambda}{4}\left(\phi^*\phi - v^2\right)^2$$

Perturbative treatment: $\phi = v + \frac{1}{\sqrt{2}}\Big(R + iI\Big)$

$$S_0 := -\tfrac{1}{2}\int \mathrm{d}^4x \left[\partial_\mu R\, \partial^\mu R + \partial_\mu I\, \partial^\mu I + \lambda\, v^2 R^2\right]$$

$$S_{\rm int} := -\int \mathrm{d}^4x \left[\tfrac{\lambda v}{2\sqrt{2}}\, R(R^2 + I^2) + \tfrac{\lambda}{16}(R^2 + I^2)^2\right]$$

EXPLOITING HIERARCHIES

Simple example: two spinless fields

Particle masses

$$m_R^2 = m^2 = \lambda v^2 \qquad m_I^2 = 0$$

Hierarchy $E \ll m$

$$\phi)\Big]$$

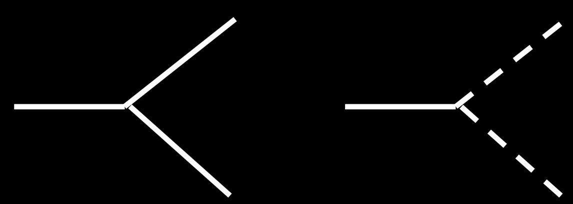
Perturbative treatment:

$$S_0 := -\tfrac{1}{2}\int \mathrm{d}^4x \left[\partial_\mu R\,\partial^\mu R + \partial_\mu I\,\partial^\mu I + \boxed{\lambda\, v^2 R^2}\right]$$

$$S_{\rm int} := -\int \mathrm{d}^4x \left[\frac{\lambda v}{2\sqrt{2}}\, R(R^2+I^2) + \frac{\lambda}{16}(R^2+I^2)^2\right]$$

Simple example: two spinless fields

Interactions



λv

$$\Big[\partial_\mu \phi^* \partial^\mu \phi + V(\phi^* \phi)\Big]$$

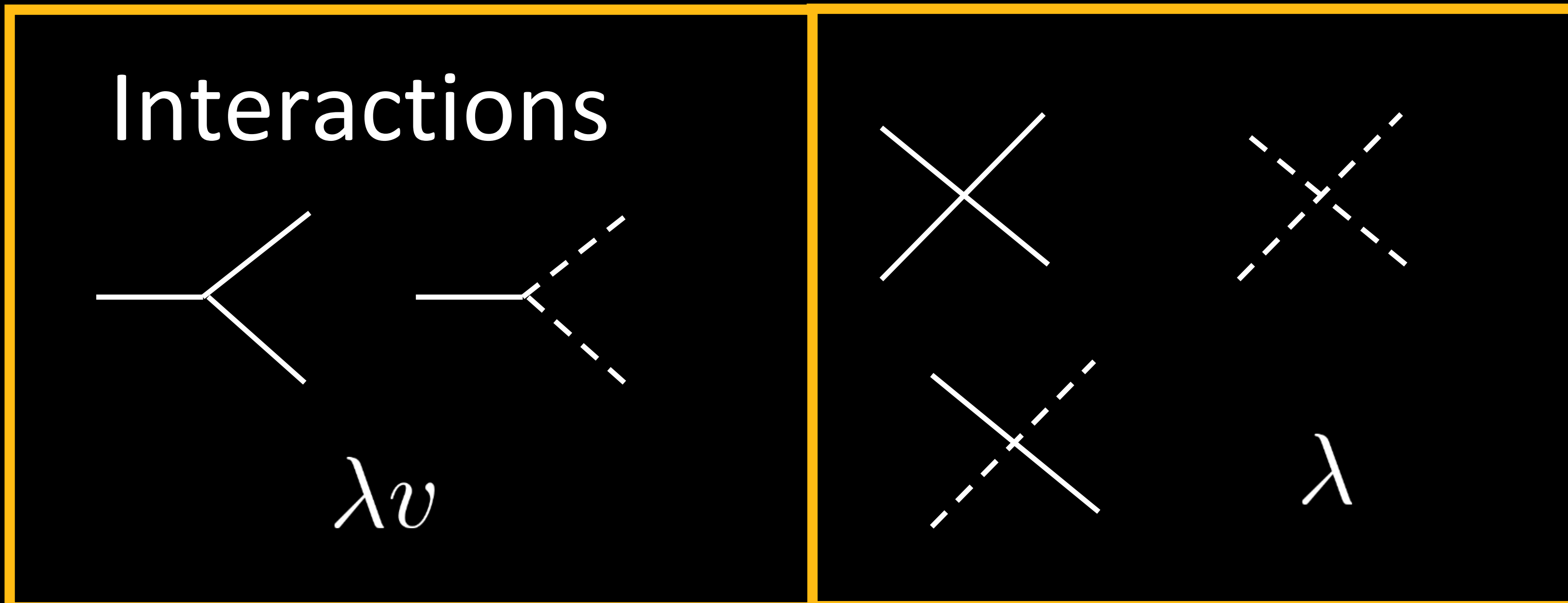
$$= \frac{\lambda}{4} \left(\phi^* \phi - v^2\right)^2$$

Perturbative treatment:

$$S_0 := -\frac{1}{2} \int \mathrm{d}^4 x \left[\partial_\mu R \, \partial^\mu R + \partial_\mu I \, \partial^\mu I + \lambda \, v^2 R^2\right]$$

$$S_{\text{int}} := - \int \mathrm{d}^4 x \left[\boxed{\frac{\lambda v}{2\sqrt{2}} R(R^2 + I^2)} + \frac{\lambda}{16} (R^2 + I^2)^2\right]$$

Simple example: two spinless fields

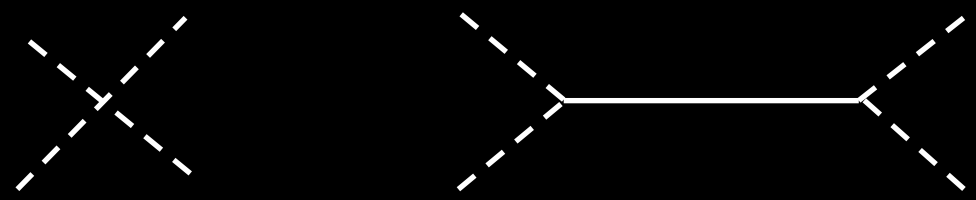


Perturbative treatment:

$$S_0 := -\frac{1}{2}\int \mathrm{d}^4x \left[\partial_\mu R\, \partial^\mu R + \partial_\mu I\, \partial^\mu I + \lambda\, v^2 R^2\right]$$

$$S_{\text{int}} := -\int \mathrm{d}^4x \left[\boxed{\frac{\lambda v}{2\sqrt{2}}\, R(R^2 + I^2)} + \boxed{\frac{\lambda}{16}(R^2 + I^2)^2}\right]$$

Low-energy scattering: $I(q)+I(p)\to I(q')+I(p')$



plus 'crossed' graphs

has (on shell) amplitude

$$\mathcal{A}=-\frac{3i\lambda}{2}+\frac{i(\lambda v)^2}{2}\left[\frac{1}{m^2+2p\cdot q}+\frac{1}{m^2-2q\cdot q'}+\frac{1}{m^2-2p\cdot q'}\right]$$

which at low energies becomes

$$\mathcal{A}\simeq 2i\lambda\left[\frac{(p\cdot q)^2+(p\cdot q')^2+(q\cdot q')^2}{m^4}\right]+O\left(m^{-6}\right)$$

Low-energy scattering: $I(q) + I(p) \to I(q') + I(p')$

$\boxed{\text{Weaker than expected at energies } E \ll m_R \quad \text{Turns out to persist to higher orders in } \lambda}$

has (on shell) amplitude

$$\mathcal{A} = -\frac{3i\lambda}{2} + \frac{i(\lambda v)^2}{2}\left[\frac{1}{m^2+2p\cdot q} + \frac{1}{m^2-2q\cdot q'} + \frac{1}{m^2-2p\cdot q'}\right]$$

which at low energies becomes

$$\mathcal{A} \simeq 2i\lambda\left[\frac{(p\cdot q)^2+(p\cdot q')^2+(q\cdot q')^2}{m^4}\right] + O\left(m^{-6}\right)$$

Low-energy scattering: $I(q) + I(p) \to I(q') + I(p')$

Weaker than expected at energies $E \ll m_R$

Turns out to persist to higher orders in λ

has (on shell) amplitude

$\mathcal{A} =$

Also true for low-energy R+I scattering:

$$\mathcal{A}(R + I \to R + I) \simeq 2i\lambda \left(\frac{q \cdot q'}{m^2}\right) + O\left(m^{-4}\right)$$

whic

$$\mathcal{A} \simeq 2i\lambda \left[\frac{(p\cdot q)^2 + (p\cdot q')^2 + (q\cdot q')^2}{m^4}\right] + O\left(m^{-6}\right)$$

Underlying symmetry $\phi \to e^{i\omega}\,\phi$

$$S := -\int \mathrm{d}^4x\,\left[\partial_\mu\phi^*\partial^\mu\phi + V(\phi^*\phi)\right]$$

Underlying symmetry $\phi \to e^{i\omega}\,\phi$

$$S := -\int \mathrm{d}^4x \left[\partial_\mu \phi^* \partial^\mu \phi + V(\phi^*\phi)\right]$$

Better low-E variables: $\phi = \left(v + \frac{\chi}{\sqrt{2}}\right)\, e^{i\xi/\sqrt{2}\,v}$

$$\chi \to \chi \qquad \xi \to \xi + \sqrt{2}\,\omega$$

Underlying symmetry $\phi \to e^{i\omega}\,\phi$

$$S := -\int \mathrm{d}^4x \left[\partial_\mu \phi^* \partial^\mu \phi + V(\phi^*\phi)\right]$$

Better low-E variables: $\phi = \left(v + \frac{\chi}{\sqrt{2}}\right) e^{i\xi/\sqrt{2}\,v}$

$$\chi \to \chi \qquad \xi \to \xi + \sqrt{2}\,\omega$$

$$S_0 := -\tfrac{1}{2}\int \mathrm{d}^4x \left[\partial_\mu \chi\, \partial^\mu \chi + \partial_\mu \xi\, \partial^\mu \xi + \lambda\, v^2 \chi^2\right]$$

$$S_{\rm int} = -\int \mathrm{d}^4x \,\left[\left(\frac{\chi}{\sqrt{2}\,v} + \frac{\chi^2}{4\,v^2}\right)\partial_\mu \xi \partial^\mu \xi + \frac{\lambda v}{2\sqrt{2}}\,\chi^3 + \frac{\lambda}{16}\,\chi^4\right]$$

Underlying symmetry $\phi \to e^{i\omega}\,\phi$

$$S := -\int \mathrm{d}^4x\,\Big[\partial_\mu\phi^*\partial^\mu\phi + V(\phi^*\phi)\Big]$$

Better low-E variables: $\phi = \left(v + \frac{\chi}{\sqrt{2}}\right)\, e^{i\xi/\sqrt{2}\,v}$

$$\chi \to \chi \qquad \boxed{\xi \to \xi + \sqrt{2}\,\omega}$$

$$S_0 := -\tfrac{1}{2}\int \mathrm{d}^4x\,\Big[\partial_\mu\chi\,\partial^\mu\chi + \boxed{\partial_\mu\xi\,\partial^\mu\xi} + \lambda\,v^2\chi^2\Big]$$

$$S_{\rm int} = -\int \mathrm{d}^4x\,\left[\left(\frac{\chi}{\sqrt{2}\,v} + \frac{\chi^2}{4\,v^2}\right)\boxed{\partial_\mu\xi\partial^\mu\xi} + \frac{\lambda v}{2\sqrt{2}}\,\chi^3 + \frac{\lambda}{16}\,\chi^4\right]$$

EXPLOITING HIERARCHIES - SYMMETRIES

Underlying symmetry $\phi \to e^{i\omega}\,\phi$

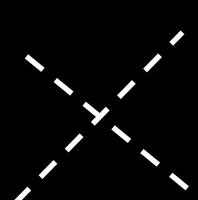
$$S := -\int \mathrm{d}^4x \left[\partial_\mu\phi^*\partial^\mu\phi + V(\phi^*\phi)\right]$$

Better low-E variables: $\phi = \left(v + \frac{\chi}{\sqrt{2}}\right)\, e^{i\xi/\sqrt{2}\,v}$

$$\chi \to \chi \qquad \xi \to \xi + \sqrt{2}\,\omega$$

$$S_0 := -\tfrac{1}{2}\int \mathrm{d}^4x \left[\partial_\mu\chi\,\partial^\mu\chi + \partial_\mu\xi\,\partial^\mu\xi + \lambda\, v^2\chi^2\right]$$

$$S_{\rm int} = -\int \mathrm{d}^4x \,\left[\left(\frac{\chi}{\sqrt{2}\,v} + \frac{\chi^2}{4\,v^2}\right)\partial_\mu\xi\partial^\mu\xi + \frac{\lambda v}{2\sqrt{2}}\,\chi^3 + \frac{\lambda}{16}\,\chi^4\right]$$



plus ‘crossed’ graphs

EXPLOITING HIERARCHIES - SYMMETRIES

Underlying symmetry $\phi \to e^{i\omega}\,\phi$

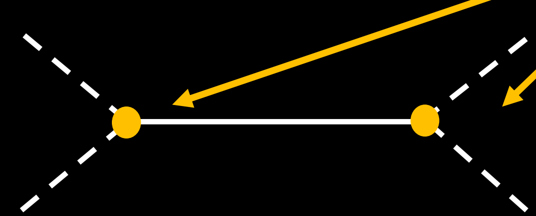
$$S := -\int \mathrm{d}^4x \left[\partial_\mu\phi^*\partial^\mu\phi + V(\phi^*\phi)\right]$$

Better low-E variables: $\phi = \left(v + \frac{\chi}{\sqrt{2}}\right)\, e^{i\xi/\sqrt{2}\,v}$

$$\chi \to \chi \qquad \xi \to \xi + \sqrt{2}\,\omega$$

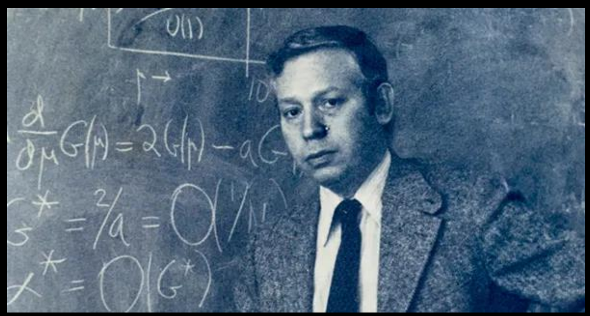
$$S_0 := -\tfrac{1}{2}\int \mathrm{d}^4x \left[\partial_\mu\chi\,\partial^\mu\chi + \partial_\mu\xi\,\partial^\mu\xi + \lambda\,v^2\chi^2\right]$$

$$S_{\rm int} = -\int \mathrm{d}^4x \,\left[\left(\frac{\chi}{\sqrt{2}\,v} + \frac{\chi^2}{4\,v^2}\right)\boxed{\partial_\mu\xi\partial^\mu\xi} + \frac{\lambda v}{2\sqrt{2}}\,\chi^3 + \frac{\lambda}{16}\,\chi^4\right]$$



plus ‘crossed’ graphs

Moral: make symmetries manifest $\phi \to e^{i\omega}\,\phi$



You can use any variables you like, but if you use the wrong ones you will be sorry.
(one of Weinberg's 3 Laws of Theoretical Physics)

Symmetries cannot always be realized linearly when restricted to low energy variables

$$\begin{pmatrix} R \\ I \end{pmatrix} \to \begin{pmatrix} \cos\omega & -\sin\omega \\ \sin\omega & \cos\omega \end{pmatrix} \begin{pmatrix} R \\ I \end{pmatrix}$$

vs

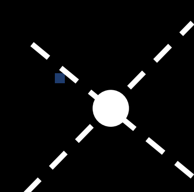
$$\chi \to \chi \qquad \xi \to \xi + \sqrt{2}\,\omega$$

Another Moral: the leading low-energy I-I scattering amplitude

$$\mathcal{A} \simeq 2i\lambda\left[\frac{(p\cdot q)^2+(p\cdot q')^2+(q\cdot q')^2}{m^4}\right]+O\left(m^{-6}\right)$$

is precisely as would have arisen from ‘effective’ interaction

$$S_{\text{eff}} = \frac{\lambda}{4m^2}\int \mathrm{d}^4x\,(\partial_\mu I\,\partial^\mu I)^2$$

through the Feynman graph  plus ‘crossed’ graphs

More remarkably: the order $(E/m)^4$ contributions to any low energy observable are captured by this same interaction, possibly with λ-corrected coefficient

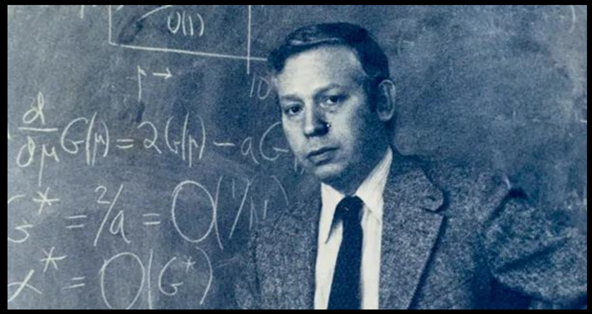
EXPLOITING HIERARCHIES - EFTS

Why does this work?

A low-energy lagrangian involving only the light field must exist because energy conservation ensures projecting onto low-energy is consistent with time-evolution

$$P_\Lambda e^{-iHt} P_\Lambda = e^{-iH_{\rm eff} t}$$

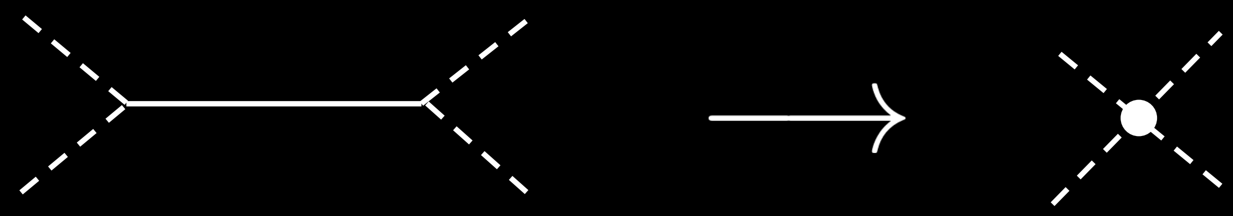
It can always be written in terms of the low energy field because this is a complete set of QFT operators at low energy



another of Weinberg's insights: QFT in itself contains no content beyond encoding things like special relativity, unitarity, cluster decomposition, etc in QM

But why is $H_{\rm eff}$ so simple? (eg why local? why no $1/m^2$ terms?)

Locality is a consequence of the uncertainty principle



Because energy conservation forbids actually producing them, heavy states can only influence light states as virtual particles.

Uncertainty relations allow temporary production of a state with energy *m* only for time intervals $\Delta t < 1/m$

$$G(x,y) = -i \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \, \frac{e^{ip(x-y)}}{p^2+m^2}$$

$$\simeq -\frac{i}{m^2} \sum_{k=0}^{\infty} \left(\frac{\partial^2}{m^2} \right)^k \, \delta^4(x-y)$$

More complicated interactions dimensionally cost more powers of 1/m, so should be less important at low energies

$$\mathcal{L}_{\rm eff} \simeq -\tfrac{1}{2}(\partial_\mu \xi \, \partial^\mu \xi) + \tfrac{\lambda}{m^4}(\partial_\mu \xi \, \partial^\mu \xi)^2 + \tfrac{a_8}{m^8}(\partial_\mu \xi \, \partial^\mu \xi)^3 + \cdots$$

so working to fixed order in E/m only involves a fixed number of interactions

Explains special role played by renormalizable interactions (unsuppressed by 1/m) in describing Nature

$$\mathcal{L}_{\rm ren} = g_3 m \, \xi^3 + g_4 \xi^4$$

(renormalizable intns forbidden for toy model by symmetry)

Why not start at *1/m²*?

$$\mathcal{L} = \frac{a_1}{m^2}(\partial_\mu\xi\partial^2\partial^\mu\xi) + \frac{a_2}{m^2}(\partial_\mu\partial_\nu\xi\,\partial^\mu\partial^\nu\xi)$$

$1/m^2$ terms (and other $1/m^4$ terms) are all *redundant*

$$\mathcal{L} = \frac{(a_1-a_2)}{m^2}(\partial_\mu\xi\partial^2\partial^\mu\xi) + \boxed{\frac{a_2}{m^2}\partial_\nu(\partial_\mu\xi\,\partial^\mu\partial^\nu\xi)}$$

ie a total derivative

Why not start at *1/m²*?

$$\mathcal{L} = \frac{a_1}{m^2}(\partial_\mu \xi \partial^2 \partial^\mu \xi) + \frac{a_2}{m^2}(\partial_\mu \partial_\nu \xi \, \partial^\mu \partial^\nu \xi)$$

$1/m^2$ terms (and other $1/m^4$ terms) are all *redundant*

$$\mathcal{L} = \boxed{\frac{(a_1 - a_2)}{m^2}(\partial_\mu \xi \partial^2 \partial^\mu \xi)} + \frac{a_2}{m^2}\partial_\nu(\partial_\mu \xi \, \partial^\mu \partial^\nu \xi)$$

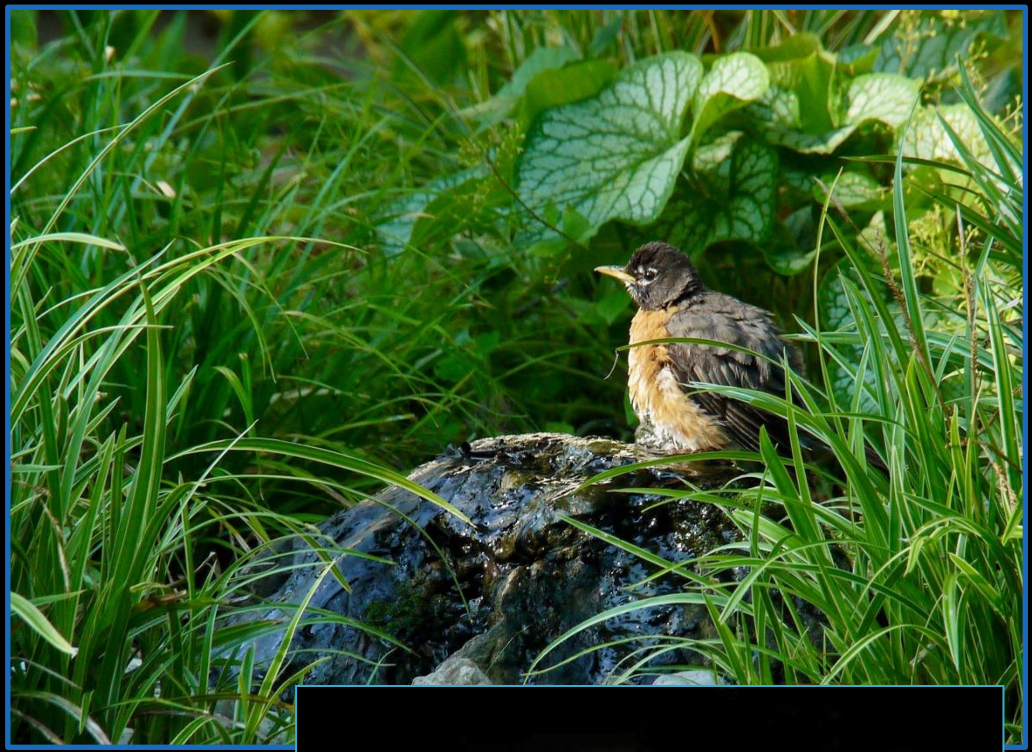
ie a total derivative or can be removed with a field redefinition

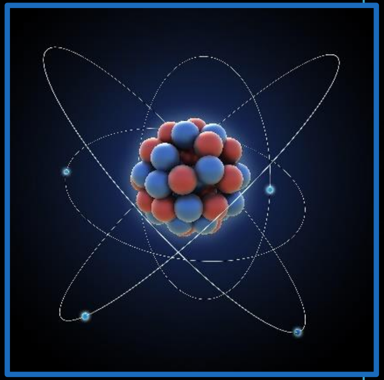
$$\xi \to \xi + \frac{a_2 - a_1}{m^2} \, \partial^2 \, \xi$$ for which

$$-\frac{1}{2}\,\partial_\mu \xi \partial^\mu \xi \to -\frac{1}{2}\,\partial_\mu \xi \partial^\mu \xi + \frac{a_2 - a_1}{m^2}\,(\partial_\mu \xi \partial^2 \partial^\mu \xi)$$

GENERAL FRAMEWORK

WHY RENORMALIZATION IS A GOOD THING





IMAGES: CB, GENEGEEK.CA, QUORA

Suppose heavy and light degrees of freedom exist but only the light ones are ever measured:

$$\langle O(\ell)\rangle = \int \mathcal{D}\ell\,\mathcal{D}h\; e^{iS[\ell,h]}\; O(\ell)$$

Most simple if m/M expansion is done as early as possible

Define low-energy by $E < \Lambda$ and the Wilson action by

$$e^{iS_\Lambda[\ell]} = \int_\Lambda \mathcal{D}h\; e^{iS[\ell,h]}$$

so

$$\langle O(\ell)\rangle = \int^\Lambda \mathcal{D}\ell\; e^{iS_\Lambda[\ell]}\; O(\ell)$$

Important Properties follow from the definition:

1. Same low-energy expansion for S_Λ applies to all low-energy observables

$$\langle O(\ell)\rangle = \int^\Lambda \mathcal{D}\ell \; e^{iS_\Lambda[\ell]} \; O(\ell)$$

2a. The precise form for S_Λ depends in detail on precisely how the high- and low-energy sectors get split up

$$e^{iS_\Lambda[\ell]} = \int_\Lambda \mathcal{D}h \; e^{iS[\ell,h]}$$

2b. The details in S_Λ precisely cancel their counterparts in the measure once physical quantities are computed

$$\langle O(\ell)\rangle = \int \mathcal{D}\ell \, \mathcal{D}h \; e^{iS[\ell,h]} \; O(\ell)$$

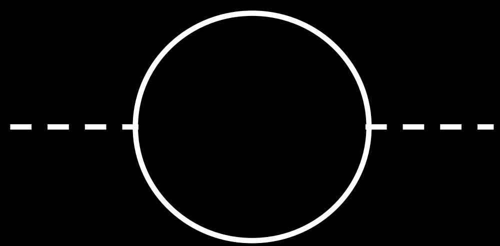
WILSON ACTION - RENORMALIZATION

Why renormalization is a good thing:

S_Λ appears in the path integral in the same way as does the traditional classical action

$$\langle O(\ell) \rangle = \int^{\Lambda} \mathcal{D}\ell \; e^{iS_\Lambda[\ell]} \; O(\ell)$$

The cancellation of Λ-dependence of integral with the dependence in S_Λ sounds exactly like traditional way of describing renormalization



$$\frac{1}{\alpha_{\rm phys}} = \frac{1}{\alpha_0(\Lambda)} + b \ln\left(\frac{\Lambda}{m}\right)$$

Surely the classical action is really just a Wilson action for a still higher UV completion?

EFTs & dimensional regularization: Can use freedom of definition to use dim-reg (rather than cutoffs like Λ) in the effective theory (with couplings fixed by 'matching')

Cutoff split

UV: ℓ, h

M

Λ

EFT: ℓ

m

Dim-reg split

UV: ℓ, h

EFT: ℓ

EFTs & dimensional regularization: Can use freedom of definition to use dim-reg (rather than cutoffs like Λ) in the effective theory (with couplings fixed by 'matching')

Cutoff Definition:

High-energy sector:
all modes of *h* fields
& E > Λ modes of ℓ

Low-energy sector:
E < Λ modes of ℓ

Dim-Reg Definition:

High-energy sector:
all modes of *h*

Low-energy sector:
all modes of ℓ

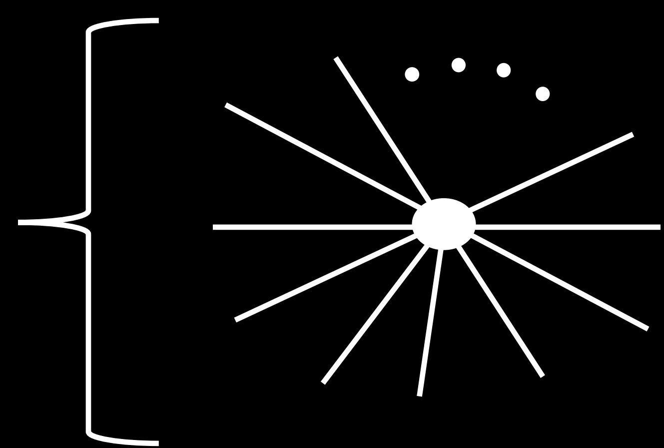
These differ only in how they treat high-energy modes (E > Λ modes of ℓ) and so diff. can be absorbed into eff. couplings.

WILSON ACTION – POWER COUNTING

Dimensional regularization allows more precise identification of which interactions contribute at each order in energy/m.

$$\mathcal{L}_{\rm int} = \mu^4 \sum_n M^{-d_n} v^{-f_n}\, O(\partial^{d_n}, \phi^{f_n})$$

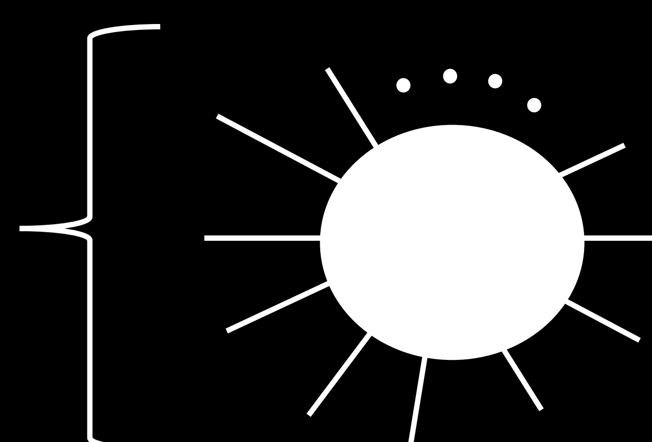
f_n fields



d_n derivatives

Use these to build a Feynman graph with E external lines, L loops and V_n vertices with f_n fields and d_n derivatives

E external lines



L loops, V_n vertices

Use relations amongst *E, I, L, V* coming from fact they connect together to make a graph

$$E + 2I = \sum_n f_n V_n \quad \text{(conservation of ends)}$$

$$L = 1 + I - \sum_n V_n \quad \text{(definition of \# of loops)}$$

Track factors of μ, *M* and *v* from vertices and use dimensional analysis to determine power of external energy scale *Q*

$$\mathcal{L}_{\text{int}} = \mu^4 \sum_n M^{-d_n} v^{-f_n}\, O(\partial^{d_n}, \phi^{f_n})$$

Use relations amongst *E, I, L, V* coming from fact they connect together to make a graph

$$E + 2I = \sum_n f_n V_n \quad \text{(conservation of ends)}$$

$$L = 1 + I - \sum_n V_n \quad \text{(definition of \# of loops)}$$

Track factors of μ, *M* and *v* from vertices and use dimensional analysis to determine power of external energy scale *Q*

$$\boxed{\mathcal{A}_E \sim \mu^4 \left(\frac{1}{v}\right)^E \left(\frac{MQ}{4\pi\mu^2}\right)^{2L} \prod_n \left(\frac{Q}{M}\right)^{2+(d_n-2)V_n}}$$

Only positive powers of external energy scale *Q* implies systematic low-energy expansion beyond leading order.

For example in the toy model we had $v = M = \mu = m$.

Amplitude with E external ξ particles depends on external energy scale Q by an amount

$$\boxed{\mathcal{A}_E \sim m^4 \left(\frac{1}{m}\right)^E \left(\frac{Q}{4\pi m}\right)^{2L} \prod_n \left(\frac{Q}{m}\right)^{2+(d_n-2)V_n}}$$

where all interactions satisfy $d_n \geq f_n \geq 4$

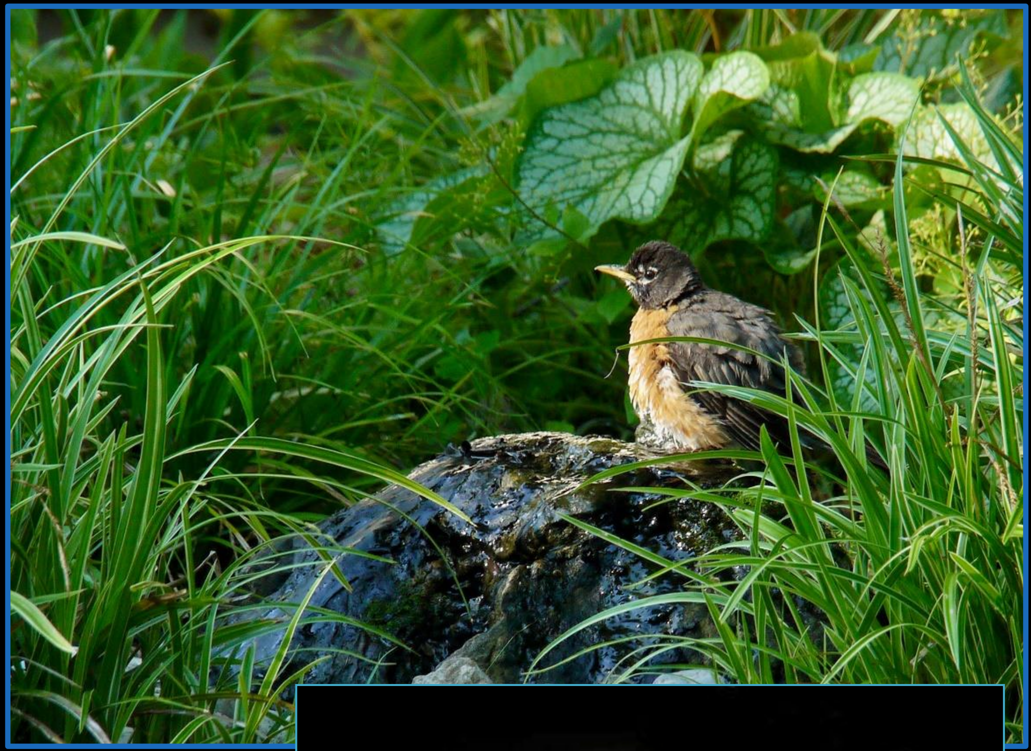
When $E = 4$ largest contribution has: $L = 0$ and $V_n = 1$ only if $d_n = f_n = 4$

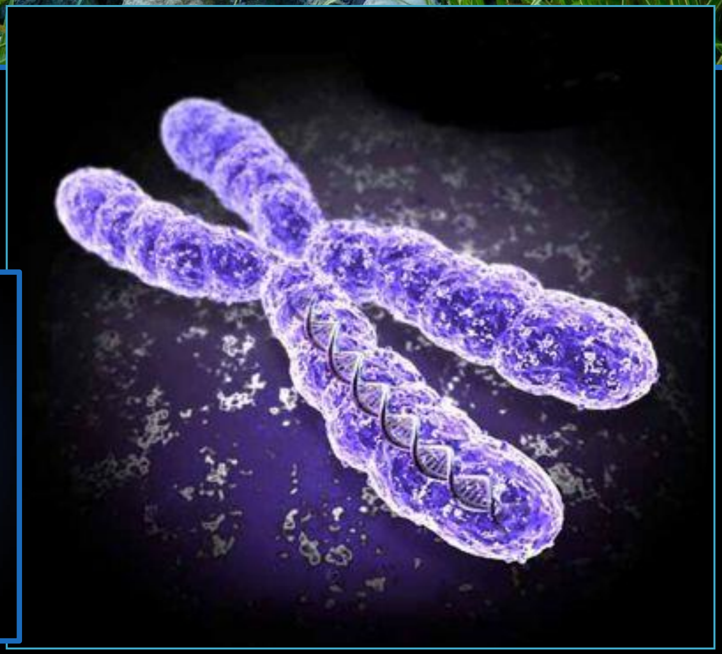
for which

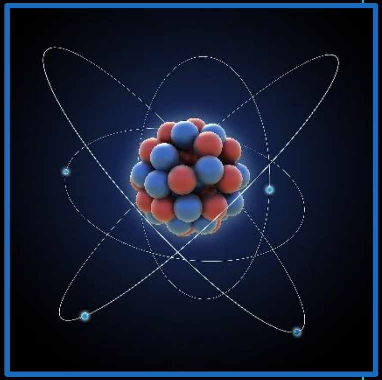
$$\boxed{\mathcal{A}_E \sim \left(\frac{Q}{m}\right)^4}$$

GENERAL FRAMEWORK

TIME-DEPENDENT FIELDS





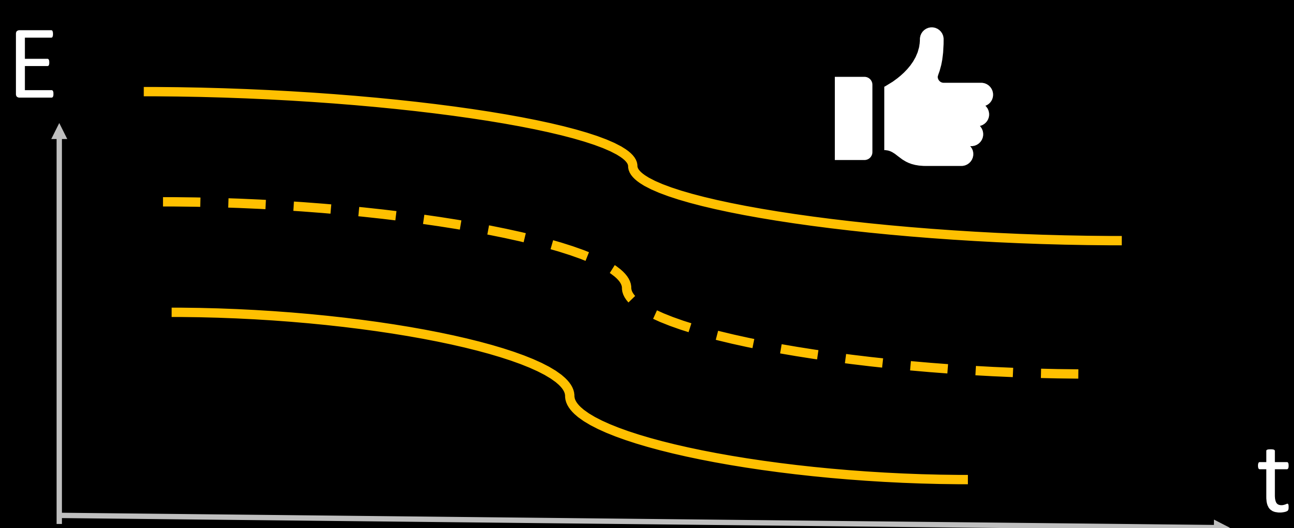


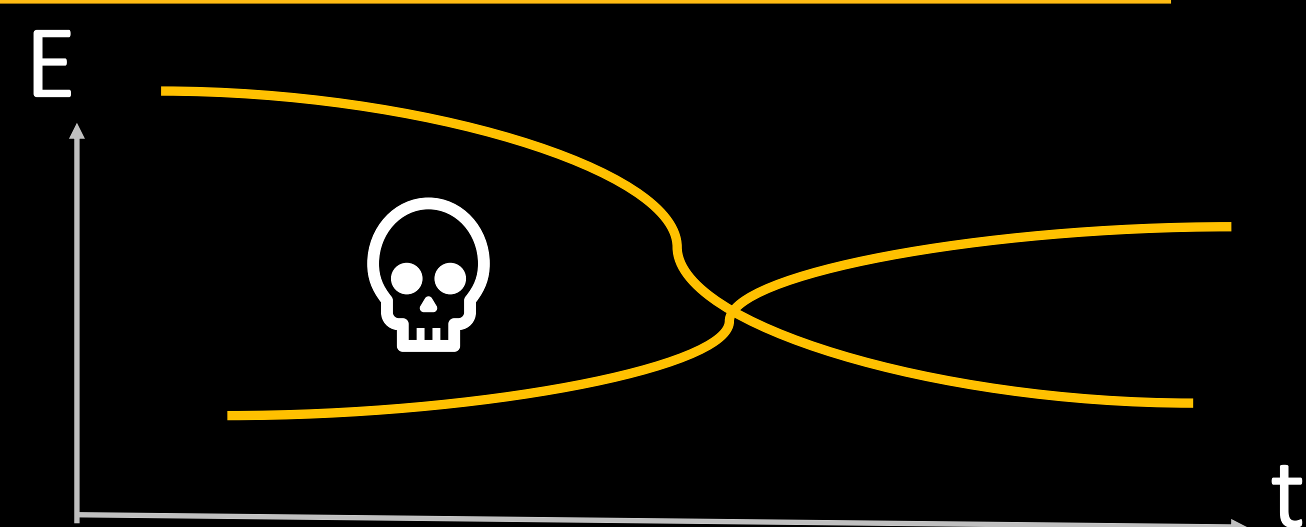
IMAGES: CB, GENEGEEK.CA, QUORA

Time-dependent background fields: Can EFTs be used with time-dependent backgrounds?

Naively not, because energy conservation is central while energy for fluctuations need not be conserved for time-dependent backgrounds.

They can if evolution is adiabatic, so $A^{-1}\, dA/dt \ll$ UV scales.
Then energy levels vary parametrically with time, $E_n = E_n(t)$.
Must also demand high and low energy levels do not cross.





Toy model of two spinless fields

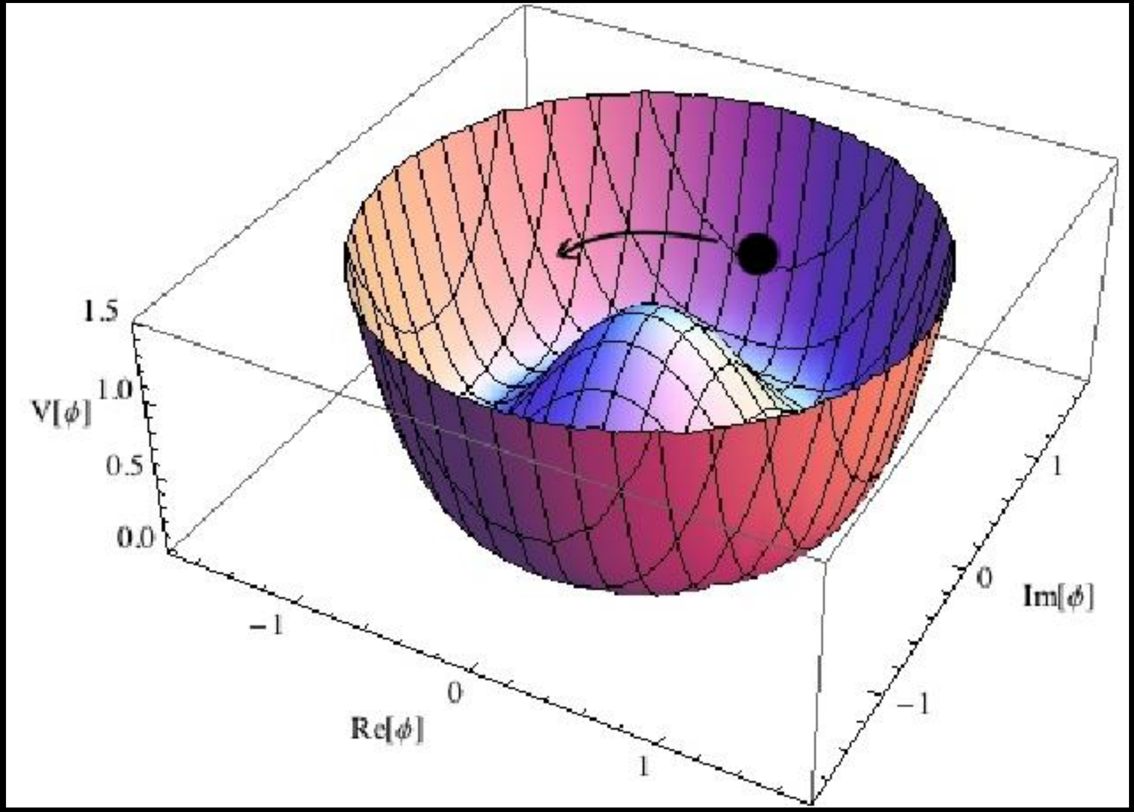
$$S := -\int \mathrm{d}^4x \left[\partial_\mu\phi^*\partial^\mu\phi + V(\phi^*\phi)\right]$$

$$V(\phi^*\phi) = \frac{\lambda}{4}\left(\phi^*\phi - v^2\right)^2$$

Time dependent classical solution:

$$\phi_{\mathrm{cl}} = \rho_0\, e^{i\omega t}$$

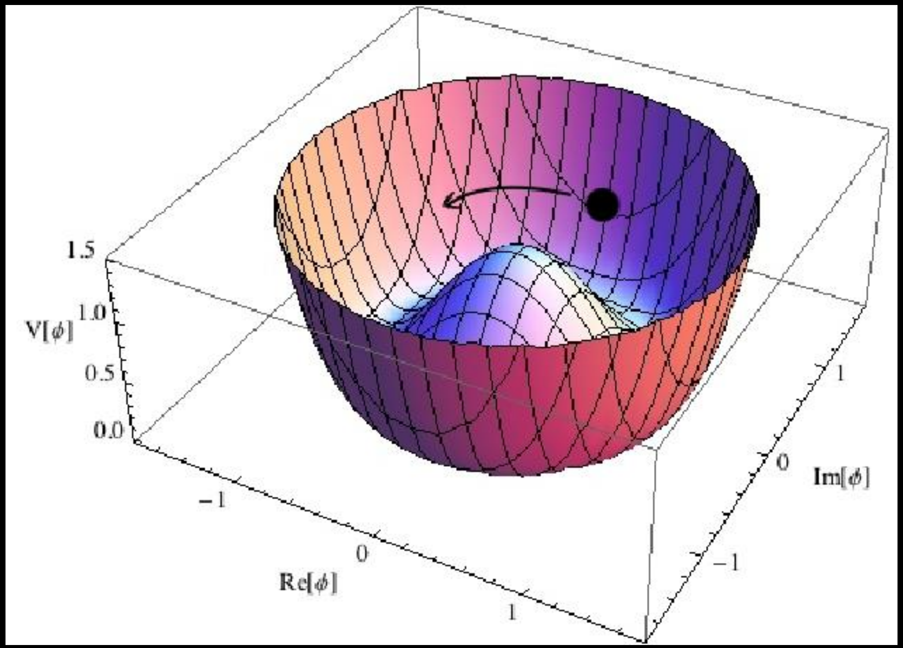
EOM: $\rho_0 = \sqrt{v^2 + \frac{2\omega^2}{\lambda}}$



Classical energy partly kinetic and partly due to field climbing the potential

$$\varepsilon = \dot{\phi}^* \dot{\phi} + V = \omega^2 \left(v^2 + \frac{3\omega^2}{\lambda} \right)$$

How does the EFT provide the energy due to V?



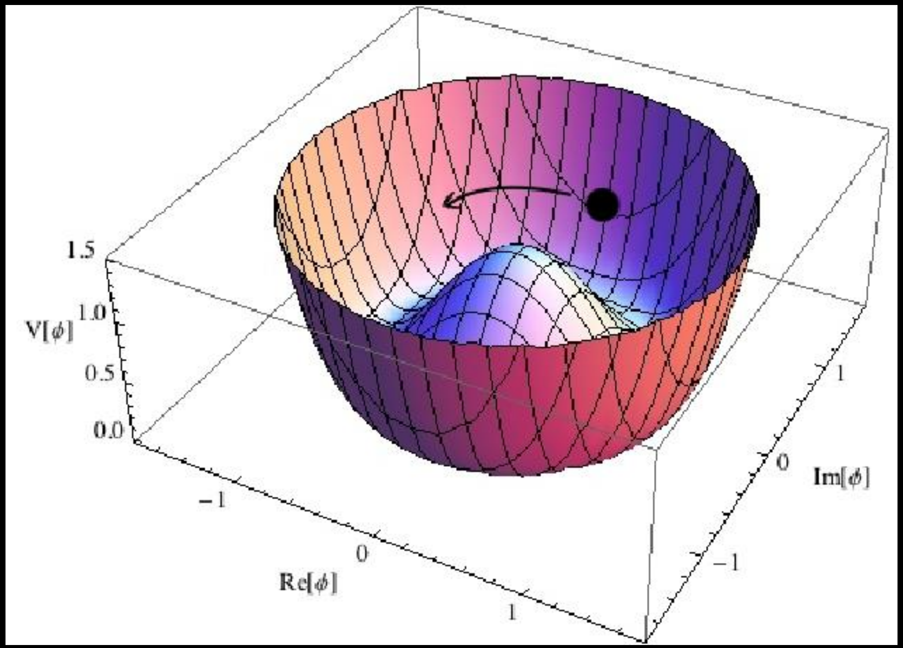
$$\varepsilon = \frac{1}{2}\dot{\xi}^2 + \frac{3\lambda}{4m^4}\dot{\xi}^4$$
$$= \omega^2 \left(v^2 + \frac{3\omega^2}{\lambda} \right)$$

Effective interaction (with same coupling as needed for scattering) provides precisely the required classical energy

Classical energy partly kinetic and partly due to field climbing the potential

$$\varepsilon = \dot{\phi}^* \dot{\phi} + V = \omega^2 \left(v^2 + \frac{3\omega^2}{\lambda} \right)$$

How does the EFT provide the energy due to V?

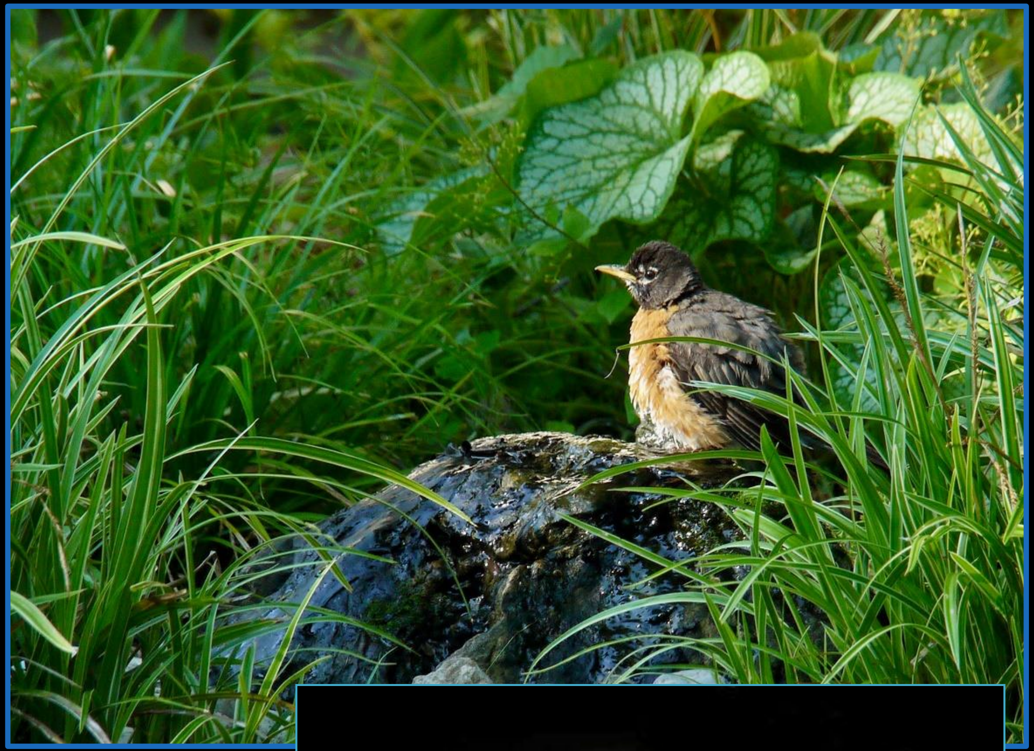


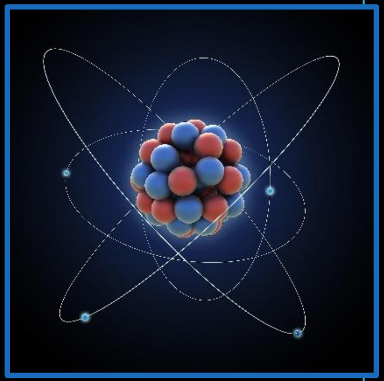
$$\varepsilon = \frac{1}{2}\dot{\xi}^2 + \frac{3\lambda}{4m^4}\dot{\xi}^4$$
$$= \omega^2 \left(v^2 + \frac{3\omega^2}{\lambda} \right)$$

This is why EFT methods can be applied eg in cosmological applications (which proves important for consistency in GR)

EFT EXAMPLES

ELECTROWEAK PHYSICS





IMAGES: CB, GENEGEEK.CA, QUORA