EFFECTIVE THEORIES FOR QUARK FLAVOUR PHYSICS

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- Introduction
- From effective theories to the SM
- The WET as a precision tool for NP searches
- NP, the SMEFT and flavour
- Conclusions

Mainly based on 1905.00798, where more details & refs can be found

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INTRODUCTION

- Quark flavour physics has played a key role in the development of the SM (bottom-up, i.e. from the effective theory to the fundamental theory)
- It now plays a key role in testing the SM (top-down, using the effective theory as a tool for precision studies in the fundamental theory)
- … and in searching for NP (bottom-up again)

FERMI THEORY & UNIVERSALITY

- Fermi Lagrangian: a local four-fermion interaction with vector currents

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} J^h_\mu J^{\mu\ell} \quad \text{with} \quad J^h_\mu = \bar{P}\gamma_\mu N \,, \quad J^\ell_\mu = \bar{e}\gamma_\mu \nu_e$$

- Feynman & Gell-Mann's CVC:

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} J^\dagger_\mu J^\mu \quad \text{with} \quad J_\mu = \bar{N}\gamma_\mu P + \bar{e}\gamma_\mu \nu_e + \bar{\mu}\gamma_\mu \nu_\mu$$

  - normalization of the hadronic current fixed by isospin
  - local effective interaction valid to a cutoff Λ such that $G_F\Lambda^2 \sim 1$, i.e. $\Lambda \sim 300$ GeV

THE CABIBBO ANGLE

- Two big puzzles:
  - small but non-negligible difference between G_F measured in muon and beta decays
  - G_F measured in decays of strange baryons is much smaller, ~ 0.2 G_F in muon decay
- However, hadronic SU(3) symmetry suggests universality should extend to strange hadrons…

THE CABIBBO ANGLE

- Cabibbo's solution: strong isospin → SU(3), but weak current keeps doublet structure with a linear combination of Λ and neutron:

$$J_\mu = \bar{P}\gamma_\mu(\cos\theta_C N + \sin\theta_C \Lambda) + \bar{\nu}_e\gamma_\mu e + \bar{\nu}_\mu\gamma_\mu\mu$$

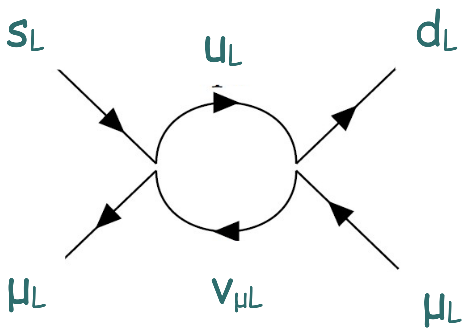
Cabibbo '63

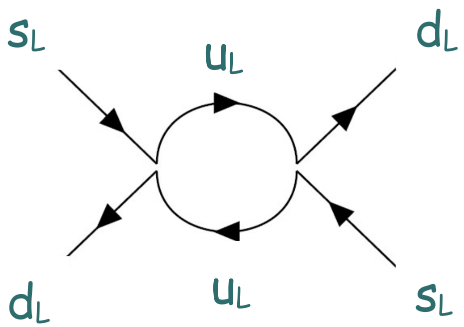
- In quark language, and introducing the V-A structure,

$$J_\mu = \bar{u}\gamma_\mu(1-\gamma_5)(\cos\theta_C d + \sin\theta_C s) + \bar{\nu}_e\gamma_\mu(1-\gamma_5)e + \bar{\nu}_\mu\gamma_\mu(1-\gamma_5)\mu$$

A FCNC PROBLEM

- Two insertions of the effective Lagrangian generate $K_L \to \mu^+\mu^-$ or $K\text{-}\bar{K}$ mixing:





- The loop is saturated at the cutoff, so it is of order $G_F^2\Lambda^2 \sim G_F(G_F\Lambda^2) \sim G_F$; however, experimentally it is much smaller, requiring $G_F\Lambda^2 \ll 1$, $\Lambda \sim 3$ GeV

THE GIM MECHANISM

- Complete the weak SU(2) doublet structure: introduce a partner also for the up quark and turn Cabibbo mixing into a matrix:

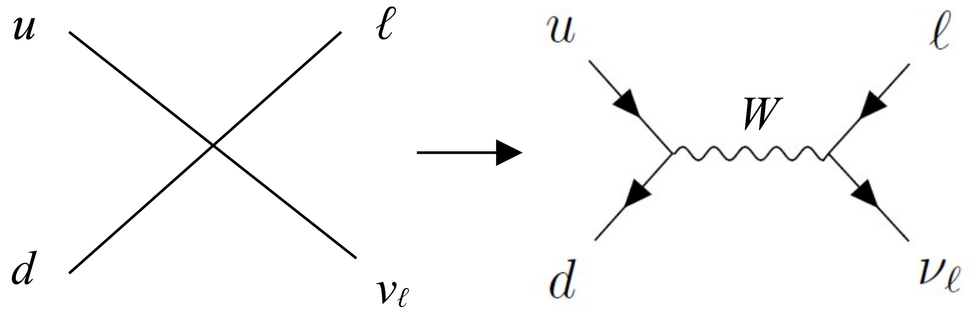
$$J_\mu = (\cos\theta_C \bar{u}_L - \sin\theta_C \bar{c}_L)\gamma_\mu(\cos\theta_C d_L + \sin\theta_C s_L)$$

- The effective cutoff is now m_c^2, since for $q^2 >> m_c^2$ GIM cancellation is effective. Predict $m_c \sim 3$GeV!

Glashow, Iliopoulos & Maiani '70

THE ROAD TO THE SM

- GIM paved the road to the two-generation SM: Fermi theory valid up to a cutoff ~ 300 GeV, where new particles come into play:



- The SU(2) structure involves a neutral current, but GIM ensures it is flavour conserving; loop corrections are suppressed by quark masses.

HOW MANY GENERATIONS?

- CPV in K-$\bar{K}$ mixing discovered in 1963
- For two generations, Cabibbo mixing matrix can be made real by quark field redefinitions: no CPV in weak interactions with two generations only
- One observable phase remains in the case of three generations: CKM matrix described by three angles and one phase

Kobayashi & Maskawa '73

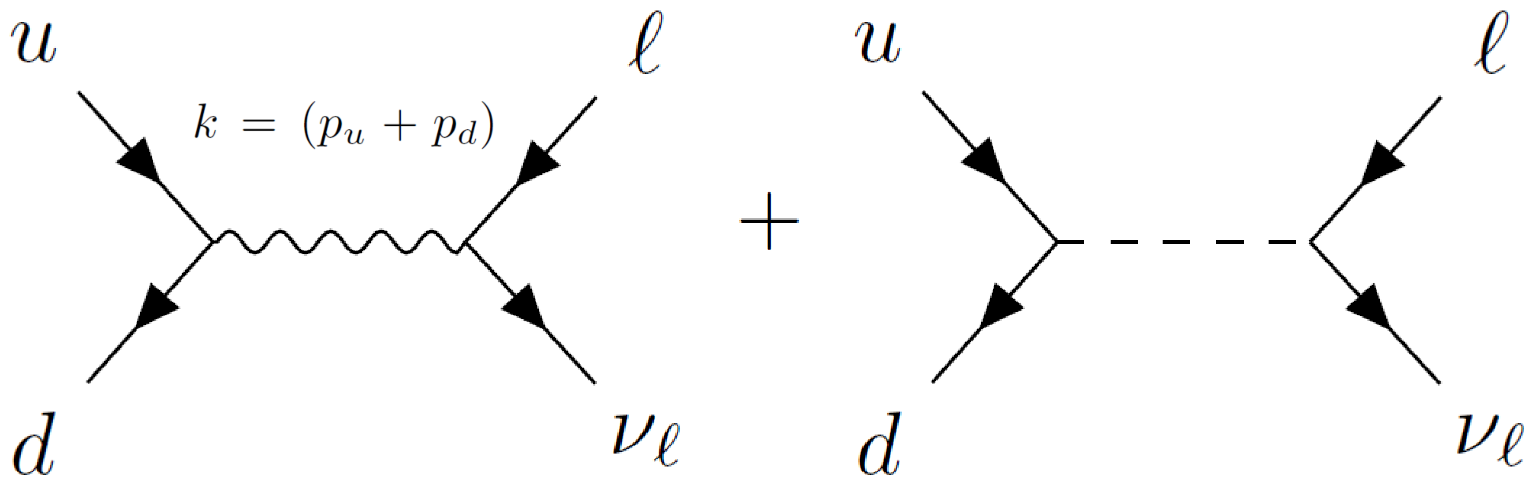
THE STANDARD MODEL

- The SM is the most general renormalizable theory built with three generations of quarks and leptons, $SU(3)_c \otimes SU(2)_L \otimes U(1)$ gauge symmetry and one Higgs doublet
- No tree-level FCNC, so all FCNC processes are finite and computable in terms of SM parameters (quark masses, CKM parameters and gauge couplings).

BACK TO EFFECTIVE THEORIES

- The SM contains all ingredients to compute FCNCs, but W, Z and top are much heavier than mesons and leptons
- W, Z and top can only contribute to meson FCNCs as virtual intermediate states
- Their effects can be described with an effective theory of local interactions of d>4 among light fermions, such as the Fermi Lagrangian

BACK TO EFFECTIVE THEORIES



$$i\mathcal{A}_W = \left(\frac{ig_2}{\sqrt{2}}\right)^2 V_{ud}^* \left(\overline{u}_{\nu_\ell}(p_{\nu_\ell})\gamma_\nu P_L v_\ell(p_\ell)\right)\left(\overline{v}_d(p_d)\gamma_\mu P_L u_u(p_u)\right)\frac{-ig^{\mu\nu}}{k^2 - M_W^2 + i\epsilon},$$

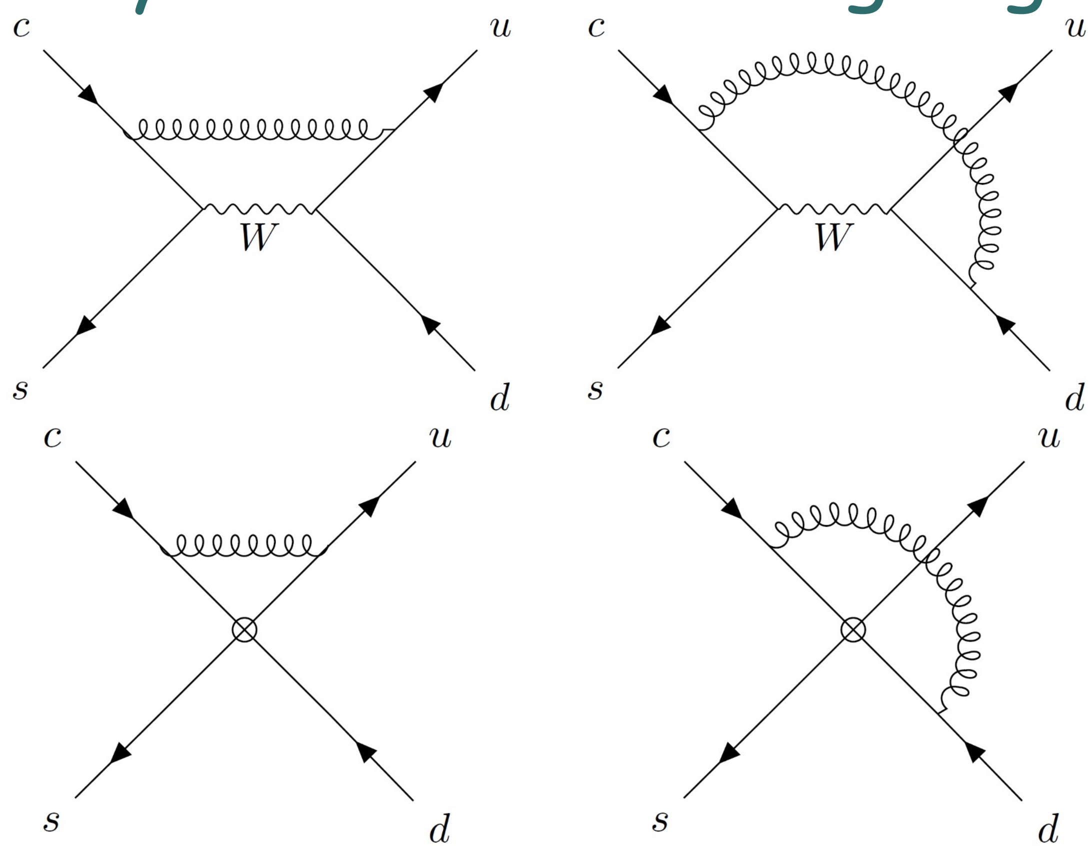
$$= -i\frac{V_{ud}^* g_2^2}{2M_W^2}\left(\overline{u}_{\nu_\ell}(p_{\nu_\ell})\gamma^\mu P_L v_\ell(p_\ell)\right)\left(\overline{v}_d(p_d)\gamma_\mu P_L u_u(p_u)\right)\sum_{n=0}^{\infty}\left(\frac{k^2}{M_W^2}\right)^n$$

$$\simeq -i\frac{4G_F}{\sqrt{2}} V_{ud}^* \left(\overline{u}_{\nu_\ell}(p_{\nu_\ell})\gamma^\mu P_L v_\ell(p_\ell)\right)\left(\overline{v}_d(p_d)\gamma_\mu P_L u_u(p_u)\right) + \mathcal{O}\left(\frac{k^2}{M_W^2}\right)$$

$$\mathcal{A}_W = \langle -\mathcal{H}_{\text{eff}}\rangle + \mathcal{O}\left(\frac{k^2}{M_W^2}\right), \qquad \mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ud}^* Q^{\overline{d}u\overline{\nu}_\ell \ell} \qquad Q^{\overline{d}u\overline{\nu}_\ell \ell} \equiv \overline{d}_L\gamma^\mu u_L \overline{\nu}_{\ell L}\gamma_\mu \ell_L$$

EFTs AS PRECISION TOOLS

- Why should I bother going through the EFT?



$$\alpha_s \log\left(\frac{M_W^2}{-p^2}\right)$$

spoils the convergence of the perturbative expansion

↕

$$\alpha_s \log\left(\frac{\Lambda^2}{-p^2}\right)$$

(with cutoff Λ)

$$\alpha_s \log\left(\frac{\mu^2}{-p^2}\right)$$

(in dim. reg.)

Identical in the infrared, but very different in the UV

EFTs AS PRECISION TOOLS

- Cutoff or μ dependence is governed by the Renormalization Group Equations, which link Green functions computed at different values of the renormalization scale μ
- Use EFTs to resum large logs and improve the behaviour of perturbation theory

FLAVOUR OBSERVABLES IN THREE STEPS

1) Match the SM onto the WET renormalized at $\mu \sim M_W$ on off-shell quark states:

$$\mathcal{A}_{\rm full}^{c\to s\bar{d}u}(p_i) = \mathcal{A}_{\rm eff}^{c\to s\bar{d}u}(p_i) = C(\mu, M_W)\langle s\bar{d}u|Q(\mu)|c\rangle$$

$$i\mathcal{A}_{\rm full} = -i\frac{4G_F}{\sqrt{2}}V_{cs}^* V_{ud}\left(\overline{u}_u(p_u)\gamma_\nu P_L v_d(p_d)\right)\left(\overline{u}_s(p_s)\gamma_\mu P_L u_c(p_c)\right)$$
$$\left(1+\gamma_Q\frac{\alpha_s}{4\pi}\log\frac{M_W^2}{-p^2}+\mathcal{O}(\frac{\alpha_s}{4\pi})\right)$$

$$i\mathcal{A}_{\rm eff} = -iC\left(\overline{u}_u(p_u)\gamma_\nu P_L v_d(p_d)\right)\left(\overline{u}_s(p_s)\gamma_\mu P_L u_c(p_c)\right)\left(1+\gamma_Q\frac{\alpha_s}{4\pi}\log\frac{\mu^2}{-p^2}+\mathcal{O}(\frac{\alpha_s}{4\pi})\right)$$

$$\Rightarrow C(M_W) = \frac{4G_F}{\sqrt{2}}V_{cs}^* V_{ud}(1+\mathcal{O}(\frac{\alpha_s}{4\pi}))$$

Infrared physics cancels out in matching

FLAVOUR OBSERVABLES IN THREE STEPS

2) Evolve the EFT from $\mu_W \sim M_W$ to $\mu_{had} \sim M_D$ using the RGE's :

$$\vec{C}(\mu_h) = \hat{U}(\mu_h, \mu_W)\vec{C}(\mu_W)$$

$$\hat{U}(\mu_h, \mu_W) = \left(\frac{\alpha_s(\mu_W)}{\alpha_s(\mu_h)}\right)^{\frac{\gamma_0}{2\beta_0}} + \mathcal{O}(\alpha_s)$$

in general, C is a vector and the ADM a matrix, resulting in operator mixing.

FLAVOUR OBSERVABLES IN THREE STEPS

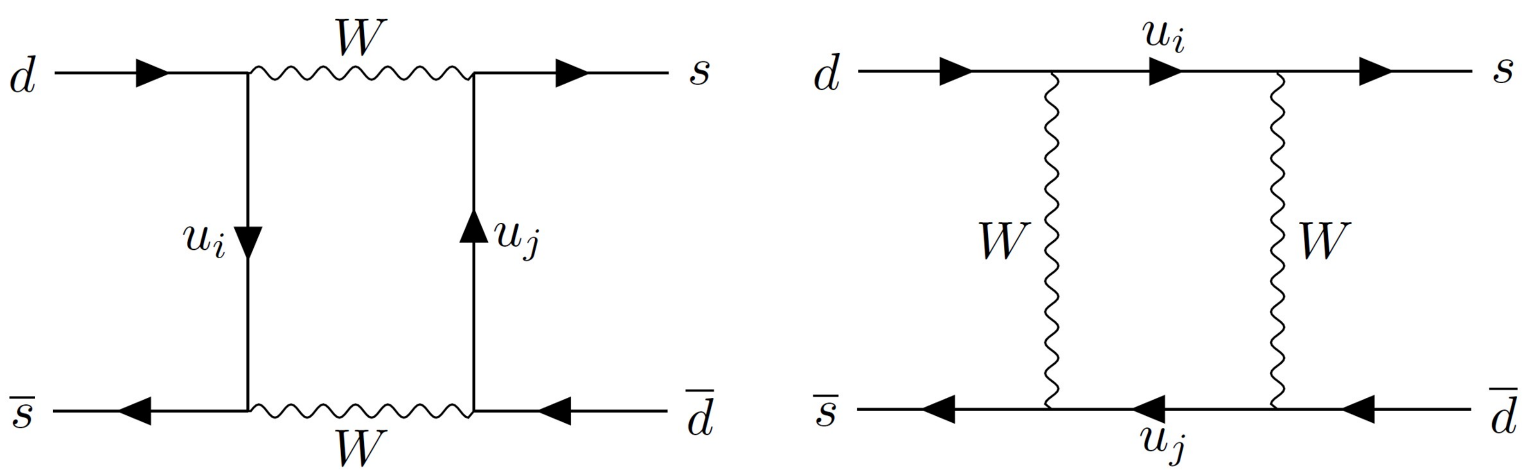
3) Compute the matrix element of the EFT operators for the process of interest

- nonperturbative methods needed for hadronic matrix elements
- nonleptonic decays particularly difficult due to final state interactions
- nonleptonic B decays tractable in the infinite m_b limit, but corrections numerically important

BBNS '99

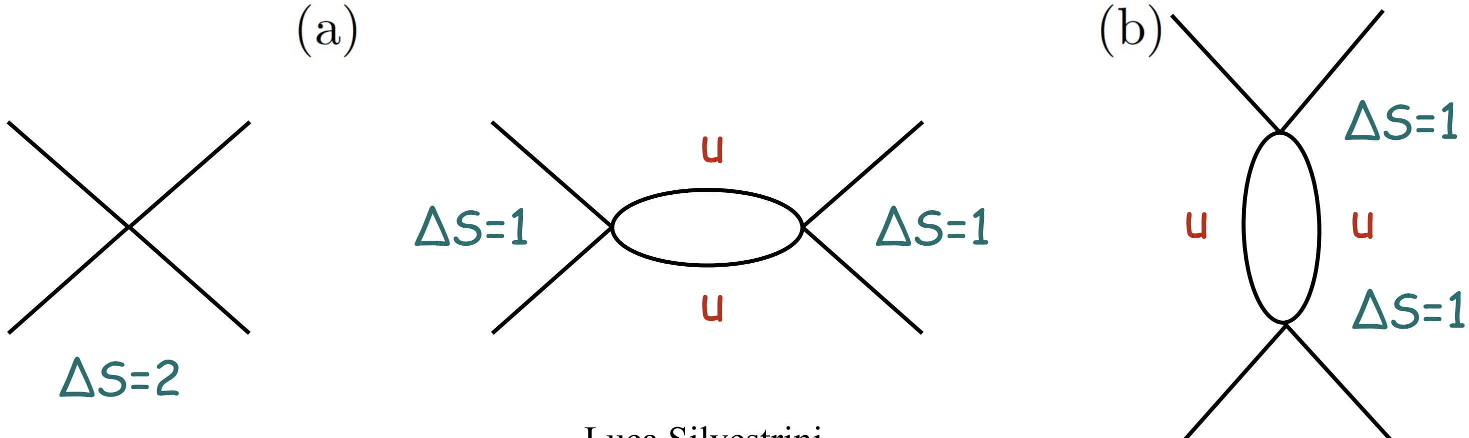
AN EXAMPLE: ΔS=2

- Relevant diagrams for matching: (+ φ's)



(a) (b)

one c or t quark enough to ensure locality



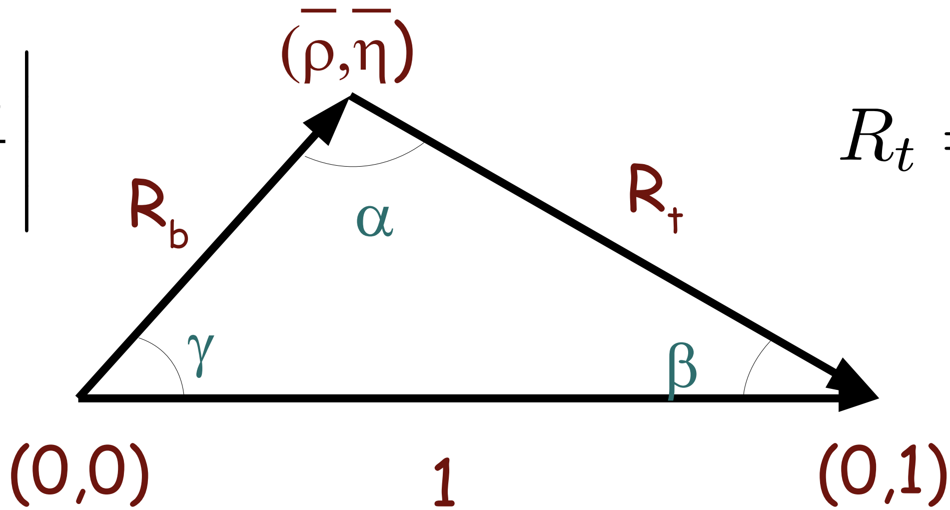
AN EXAMPLE: $\Delta S=2$

- K-$\bar{K}$ mixing dominated by long-distance matrix elements of two $\Delta S=1$ effective Hamiltonians with up quarks
- However, u-quark contributions are CP-even (up to $O(\alpha_s)$), so the CPV parameter ε_K is dominated by charm and top
- $\langle K^0|\bar{s}_L\gamma_\mu d_L \bar{s}_L\gamma^\mu d_L|\bar{K}^0\rangle$ computable w. lattice QCD
- $\varepsilon_K = (1.99 \pm 0.14)\ 10^{-3}$ vs $\varepsilon_K^{exp} = (2.228 \pm 0.011)\ 10^{-3}$

THE UNITARITY TRIANGLE

- CKM unitarity implies triangular relations:
  $(V^\dagger V)_{bd}=0=V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^*$

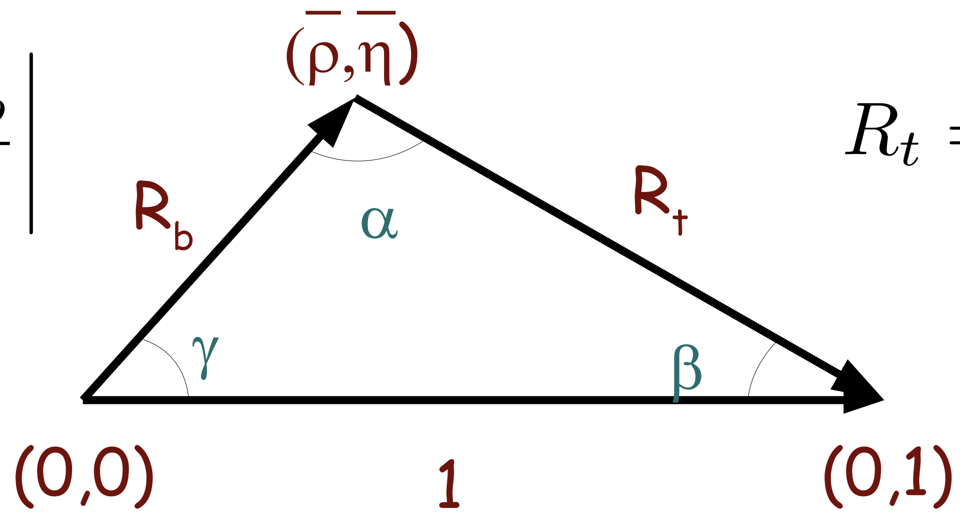
$$R_b = \left|\frac{V_{ud}V_{ub}}{V_{cd}V_{cb}}\right| \qquad R_t = \left|\frac{V_{td}V_{tb}}{V_{cd}V_{cb}}\right|$$



$$\alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \quad \beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \quad \gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

THE UNITARITY TRIANGLE

$$R_b = \left| \frac{V_{ud} V_{ub}}{V_{cd} V_{cb}} \right|$$

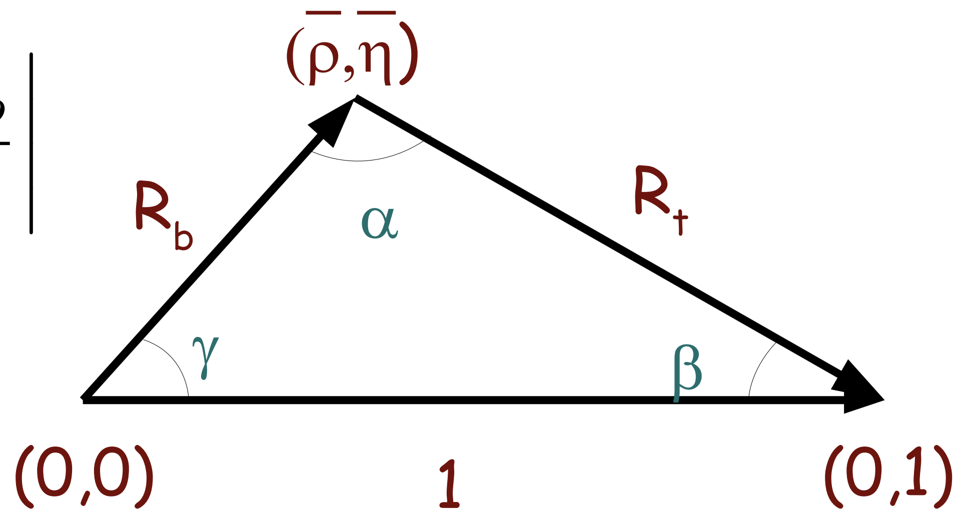


$$R_t = \left| \frac{V_{td} V_{tb}}{V_{cd} V_{cb}} \right|$$

- $|V_{cd}|$ and $|V_{cb}|$ can be extracted from β decays and from semileptonic K and b→c decays, using form factors and decay constants from lattice QCD

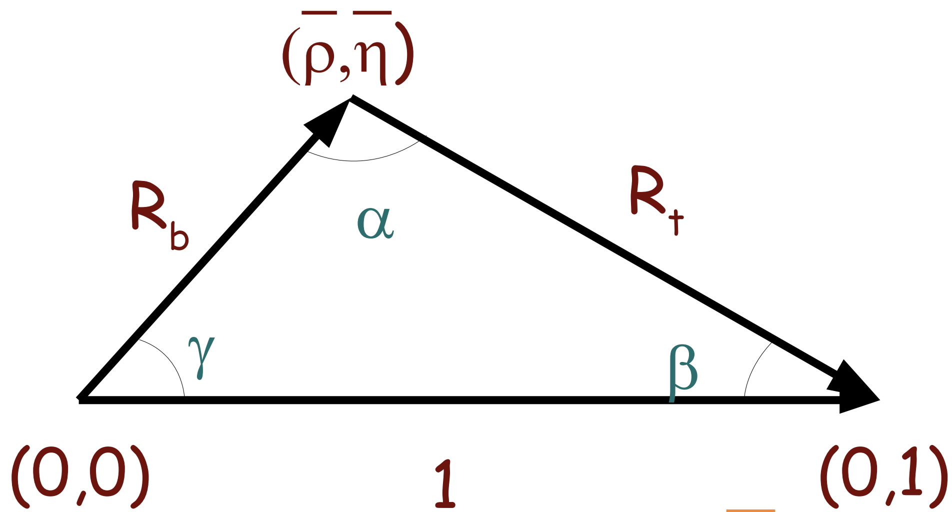
THE UNITARITY TRIANGLE

$$R_b = \left| \frac{V_{ud} V_{ub}}{V_{cd} V_{cb}} \right|$$



- R_b can be extracted from semileptonic $b \to u$ decays, using form factors and decay constants from lattice QCD

THE UNITARITY TRIANGLE

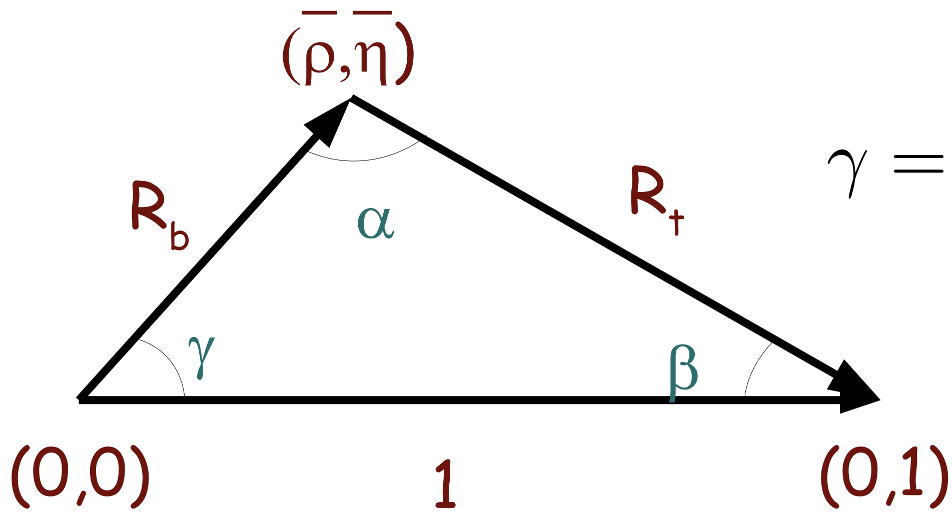


$$R_t = \left| \frac{V_{td}V_{tb}}{V_{cd}V_{cb}} \right|$$

$$\beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right)$$

- R_t can be extracted from B_d-$\bar{B}_d$ mixing using matrix elements from lattice QCD
- β can be extracted from time-dependent CP asymmetry in $B \to J/\Psi K_S$: B_d-$\bar{B}_d$ mixing involves $V_{td}V_{tb}^*$, $B_d \to J/\Psi K^0$ $V_{cs}V_{cb}^*$ and K^0-K^0 mixing $V_{cd}V_{cs}^*$

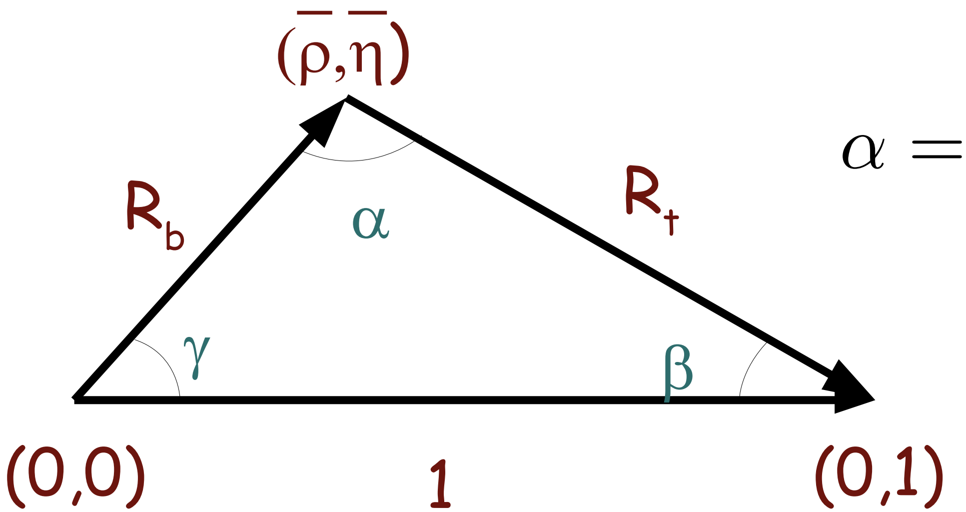
THE UNITARITY TRIANGLE



$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

- γ can be extracted from $b \to cdu$ transitions, using interference between $V_{cd}V_{ub}^*$ and $V_{ud}V_{cb}^*$ contributions (possibly including D-$\bar{D}$ mixing)

THE UNITARITY TRIANGLE

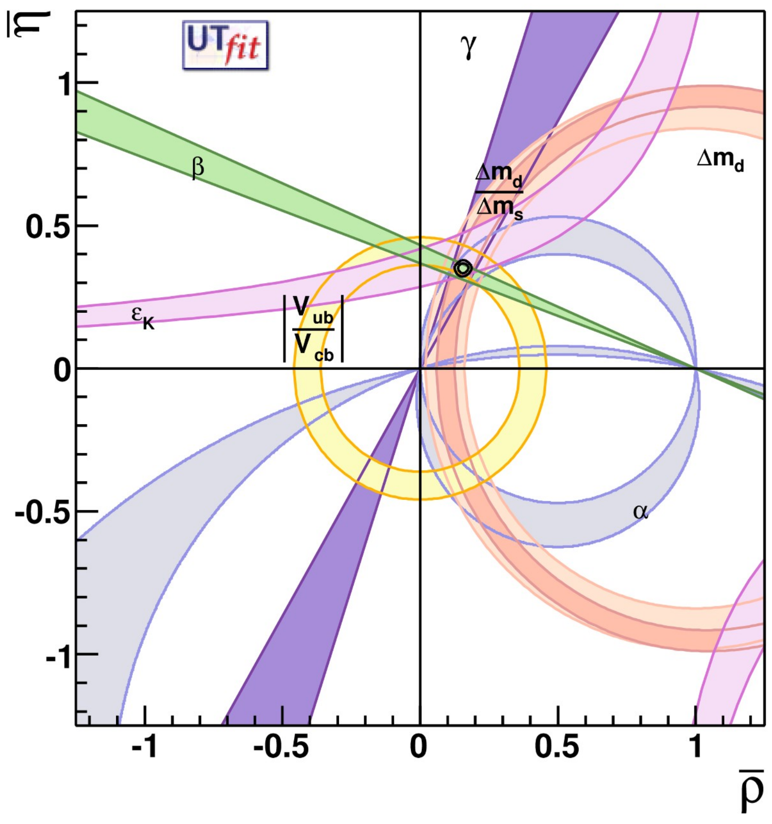


$$\alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

- α can be extracted from $B\to\pi\pi$, $\rho\pi$ and $\rho\rho$ decays in the form of $\pi-\beta-\gamma$, using isospin symmetry to isolate the I=3/2 decay amplitude, which goes as $e^{i\alpha}$

THE UNITARITY TRIANGLE

- Additional constraints come from ε_K, ΔM_{Bs}, CP violation in $B_s \to J/\Psi\phi$, $BR(K\to\pi\nu\bar{\nu})$ and $BR(B\to\tau\nu)$



NEW PHYSICS IN $\Delta F=2$

- Generalize the UTA allowing for NP in loop-mediated processes:
  - V_{us}, V_{cb}, V_{ub}, γ and α obtained from tree-level decays and thus unaffected (provided no huge NP effect in EWP)
  - NP allowed in $\Delta F=2$ processes
- Extract both CKM parameters and NP contributions

NP ANALYSIS: RESULTS

$\bar{\rho} = 0.174 \pm 0.026$

$\bar{\eta} = 0.380 \pm 0.025$

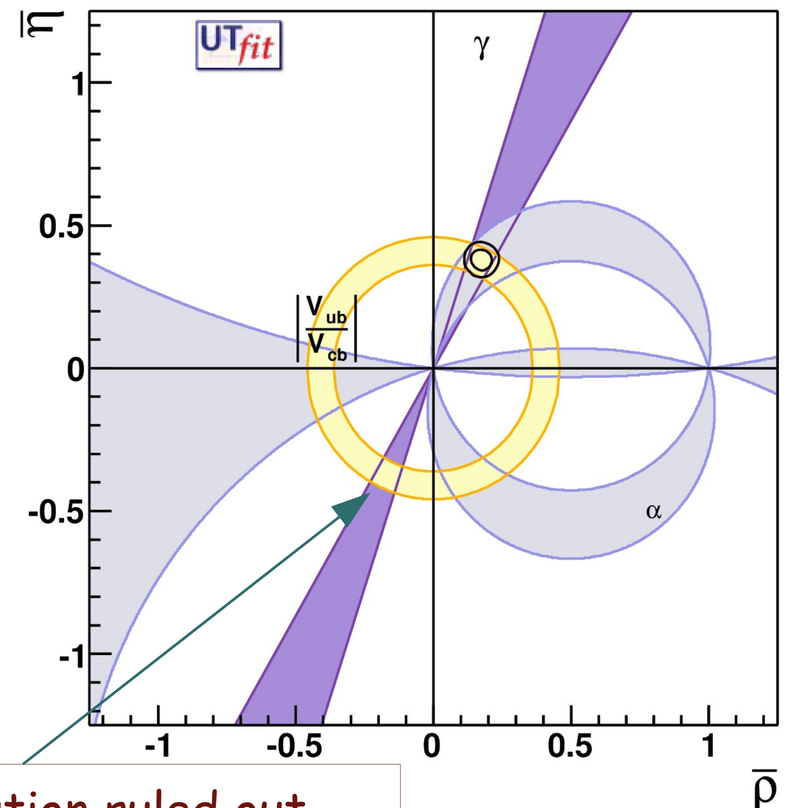
to be compared w.

$\bar{\rho} = 0.155 \pm 0.011$

$\bar{\eta} = 0.350 \pm 0.010$

in the SM

All UTfit NP results here



Second solution ruled out by semileptonic asymmetries

Laplace et al '02

NP CONTRIBUTIONS TO ΔF=2

- Phenomenological parameterization:

$$C_{B_q} e^{2i\phi_{B_q}} = \frac{\langle B_q | H_{\rm eff}^{\rm full} | \bar{B}_q \rangle}{\langle B_q | H_{\rm eff}^{\rm SM} | \bar{B}_q \rangle} = 1 + \frac{A_q^{\rm NP}}{A_q^{\rm SM}} e^{2i\phi_q^{\rm NP}}$$

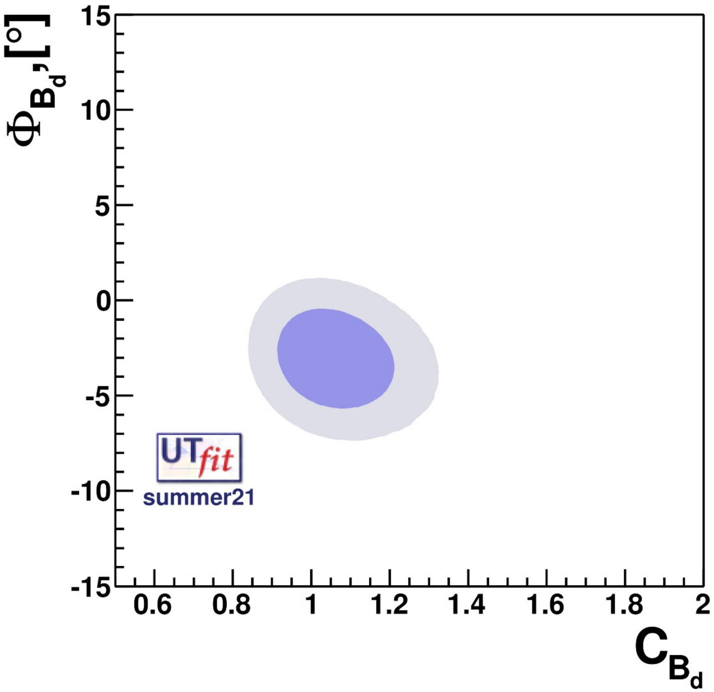
$$\Delta m_{d,s}^{\rm exp} = C_{B_{d,s}} \Delta m_{d,s}^{\rm SM}$$

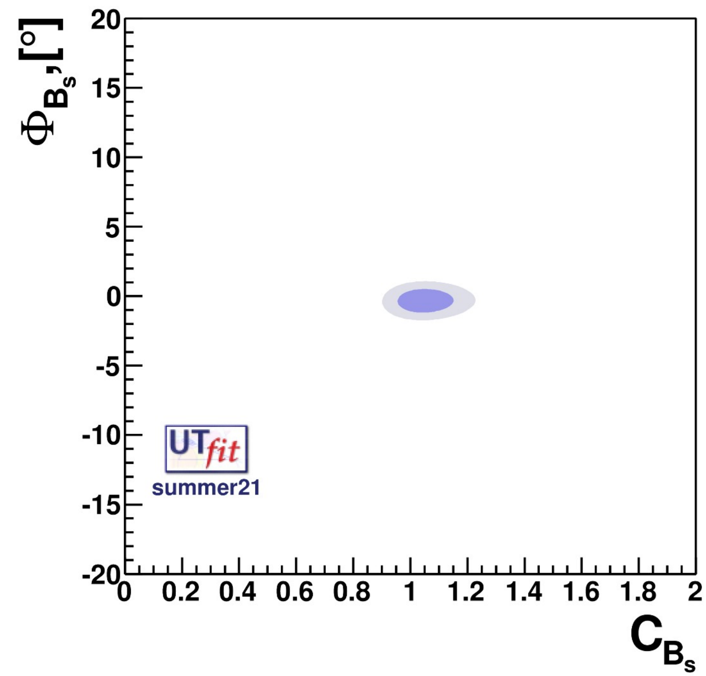
$$\sin 2\beta^{\rm exp} = \sin(2\beta + 2\phi_{B_d}) \qquad \phi_s^{\rm exp} = \beta_s - \phi_{B_s}$$

$$A_{\rm SL}^{d,s;\rm exp} = {\rm Im}\left(\frac{\Gamma_{12}^{\rm SM}}{M_{12}^{\rm SM}}\right)\frac{\cos 2\phi_{B_{d,s}}}{C_{B_{d,s}}} - {\rm Re}\left(\frac{\Gamma_{12}^{\rm SM}}{M_{12}^{\rm SM}}\right)\frac{\sin 2\phi_{B_{d,s}}}{C_{B_{d,s}}}$$

$$C_{\varepsilon_K} = \frac{\varepsilon_K^{exp}}{\varepsilon_K^{\rm SM}} = \frac{{\rm Im}\langle K^0 | \mathcal{H}_{\rm eff}^{\rm full} | \bar{K}^0 \rangle}{{\rm Im}\langle K^0 | \mathcal{H}_{\rm eff}^{\rm SM} | \bar{K}^0 \rangle} \qquad C_{\Delta m_K} = \frac{\Delta m_K^{exp}}{\Delta m_K^{\rm SM}} = \frac{{\rm Re}\langle K^0 | \mathcal{H}_{\rm eff}^{\rm full} | \bar{K}^0 \rangle}{{\rm Re}\langle K^0 | \mathcal{H}_{\rm eff}^{\rm SM} | \bar{K}^0 \rangle}$$

RESULTS ON NP PARAMETERS





$C_{\varepsilon K} = 1.08 \pm 0.10$, $C_{Bd} = 1.07 \pm 0.10$, $\phi_{Bd} = (-3.0 \pm 1.7)^\circ$,

$C_{Bs} = 1.05 \pm 0.07$, $\phi_{Bs} = (-0.3 \pm 0.6)^\circ$

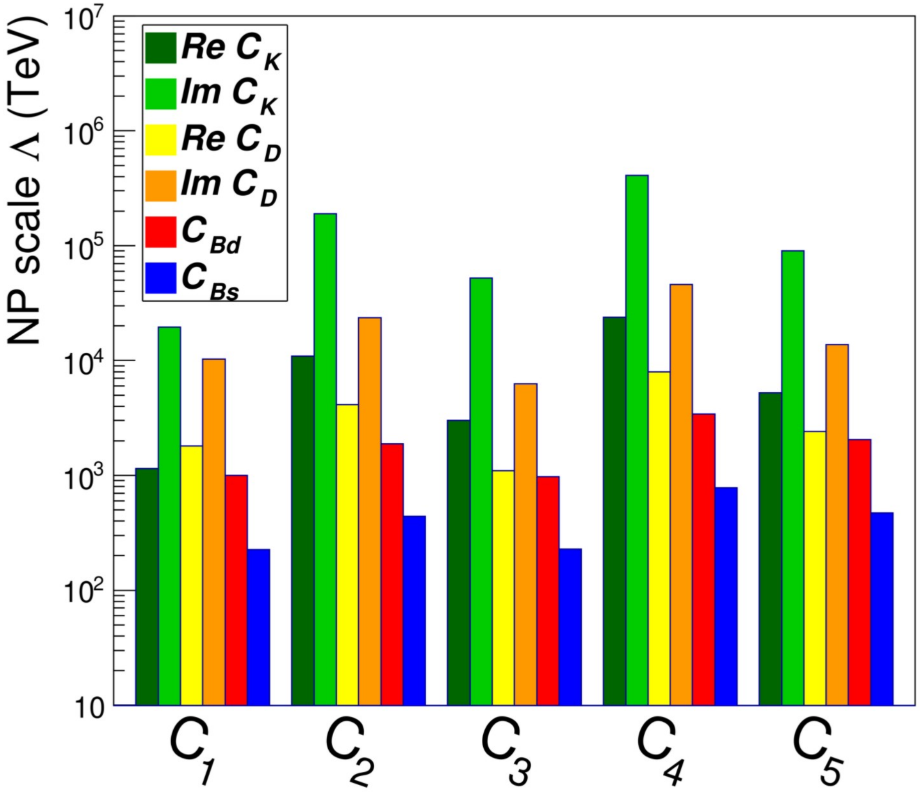
FROM ΔF=2 TO THE NP SCALE

- $H_{eff}^{\Delta F=2} = \sum_{i=1}^{5} C_i\, O_i + \sum_{i=1}^{3} C_i'\, O_i'$
- In the SM only O_1 (left-handed)
- Operators with i>1 are RG- and chirally-enhanced
- In general, $C_i \sim F_i L_i / \Lambda^2$
- Take L_i=1 and F_i = 1 (generic) or $F_i \sim F_1^{SM}$ (next-to-minimal flavour violation)

UTfit '07, ...

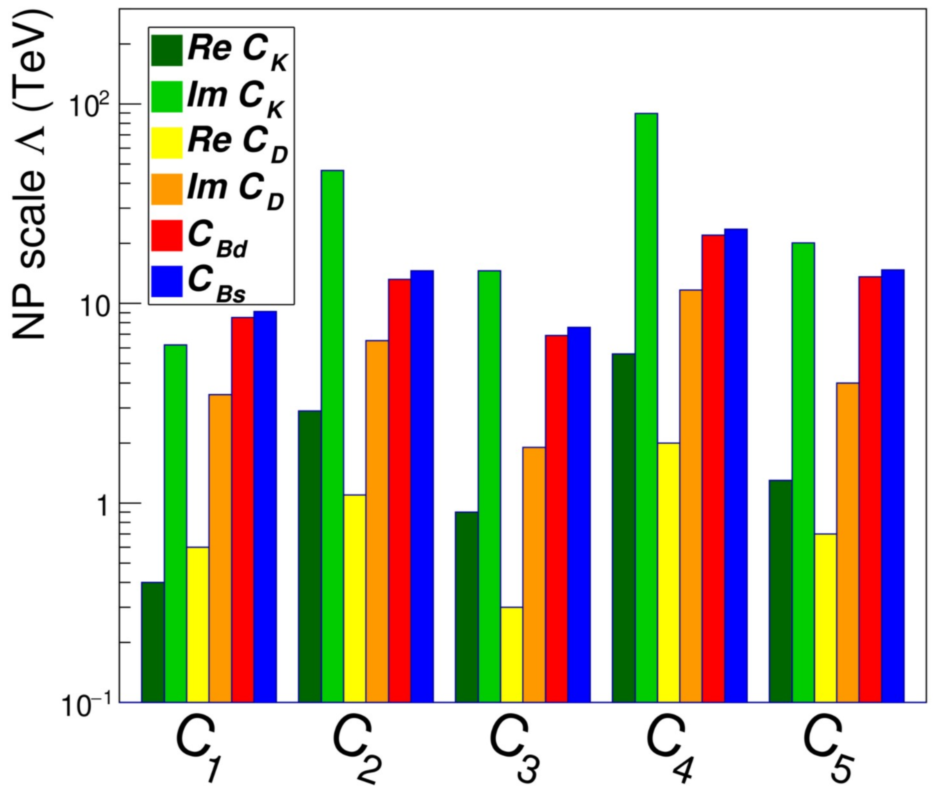
GENERIC STRONGLY-INTERACTING NP

- Best bound from ε_K, dominated by CKM error
- CPV in charm mixing follows, exp error dominant
- B_d and B_s behind, error from both CKM and B-params
- Non-perturbative NP:
  - $\Lambda > 4\ 10^5$ TeV
- Weakly interacting:
  - $\Lambda > 10^4$ TeV



NMFV STRONGLY-INTERACTING NP

- If new chiral structures present, ε_K still gives best constraint
- B_d and B_s most powerful if no new operators arise
- Non-perturbative NMFV NP (e.g. composite Higgs)
  - $\Lambda > 94$ TeV
- Weakly interacting:
  - $\Lambda > 3$ TeV



THE SMEFT

- Any NP much heavier than the EW scale must be invariant under the SM gauge group
- Define the SMEFT as the most general SU(3)xSU(2)xU(1)-invariant Lagrangian built with SM fields including operators up to dimension D>4
- Accidental symmetries of the SM are broken at D=6: Baryon & Lepton number conservation, absence of tree-level FCNCs

ΔF=2 IN SMEFT

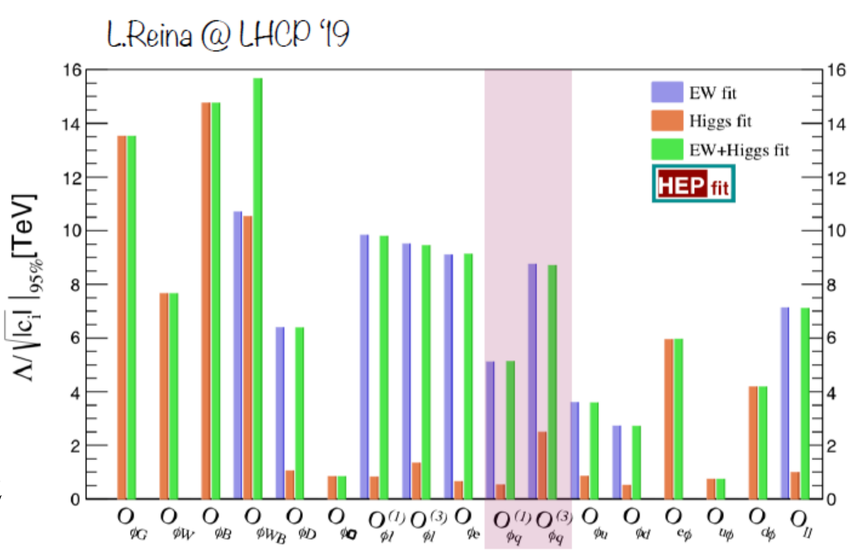
- Four out of the eight ΔF=2 operators (Q_1, Q_1', Q_4, Q_5) can be written in an SU(2)xU(1) invariant way as D=6 SMEFT operators, so the stringent bounds from ΔF=2 also apply to the SMEFT
- Between the NP and EW scales, the SM acts as a generator of flavour violation, due to Yukawa couplings
- ΔF=1 or even ΔF=0 operators at the NP scale mix into ΔF=2 operators at the EW scale via SMEFT RGEs

BOUNDS FROM ΔF=2 IN SMEFT

- New flavour structures must be defined with respect to SM Yukawa couplings. Two extreme possibilities: Y_u or Y_d diagonal basis

$$O_{jk}^{HQ(1[3])}$$

$$(H^\dagger i \overleftrightarrow{D}_\mu^{[A]} H)\,(\bar{Q}_j \gamma^\mu \left[\tau^A\right] Q_k)$$



L.S. & M.V. '19

ij	$C_{ij}^{HQ(1,3)}$ [TeV^{-2}] Y_D diag	Y_U diag
11	$\varnothing$	$4.1^{\Box}\ 10^{-3}$
12	$(8.9^{\Box}, 3.8^{\Box})\ 10^{-4}$	$(9.9^{\Box}, 3.8^{\Box})\ 10^{-4}$
13	$(7.4^{\triangle}, 6.3^{\triangle})\ 10^{-3}$	$(7.6^{\triangle}, 6.4^{\triangle})\ 10^{-3}$
22	$\varnothing$	$4.1^{\Box}\ 10^{-3}$
23	$(3.0^{\triangledown}, 1.0^{\triangledown})\ 10^{-2}$	$(3.1^{\triangledown}, 1.0^{\triangledown})\ 10^{-2}$
33	$\varnothing$	$7.3^{\triangle}\ 10^{-1}$

$$\Lambda_{\rm NP} \gtrsim 15\ \text{TeV}$$

CONCLUSIONS

- EFTs and flavour have played a key role in the development of the SM, from Fermi theory to GIM to the SM
- The WET is a very powerful tool to test the SM in the quark flavour sector
- Bounds from $\Delta F=2$ transitions among the most powerful, reaching beyond 10^5 TeV

CONCLUSIONS

- The coming decades will not witness a remarkable increase in the scales directly probed at colliders, but a very remarkable increase in luminosity and precision, also in the flavour sector
- Flavour continues playing a crucial role in indirect searches for NP, hopefully revealing new phenomena and/or shaping our expectations for NP models (see Gino's talk)