Higgs Self-coupling in EFT framework

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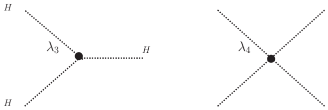
SM Higgs potential & New Physics

Higgs potential & EWSB in the SM,

$$V^{SM}(\Phi) = -\mu^2(\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2$$

$$\text{EWSB} \Rightarrow V(H) = \frac{1}{2}m_H^2 H^2 + \lambda_3 v H^3 + \frac{1}{4}\lambda_4 H^4.$$



The mass and the self-couplings of the Higgs boson depend only on λ and $v = (\sqrt{2}G_\mu)^{-1/2}$,

$$m_H^2 = 2\lambda v^2; \; \lambda_3^{SM} = \lambda_4^{SM} = \lambda.$$

$$m_H = 125 \text{ GeV and } v \sim 246 \text{ GeV}, \Rightarrow \boxed{\lambda \simeq 0.13}.$$

Presence of new physics at higher energy scales can contribute to the Higgs potential and modify the Higgs self-couplings.

Independent measurements of λ_3 and λ_4 are crucial.

New Physics Parametrization

Model independent parametrization of new physics such that Higgs mass (m_H) and vev (v) remain unchanged,

$$\lambda_3 = \kappa_3 \lambda_3^{\rm SM}, \;\; \lambda_4 = \kappa_4 \lambda_4^{\rm SM}.$$

In an EFT framework, these deviations can be captured by higher dim operators.

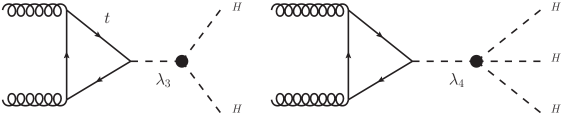
$$\begin{aligned} V^8(\Phi) &= V^{\rm SM}(\Phi) + \frac{C_6}{v^2}(\Phi^\dagger \Phi)^3 + \frac{C_8}{v^4}(\Phi^\dagger \Phi)^4 \\ \Rightarrow \kappa_3 &= 1 + (2C_6 + 4C_8)\frac{v^2}{m_H^2}, \\ \kappa_4 &= 1 + (12C_6 + 32C_8)\frac{v^2}{m_H^2} \end{aligned}$$

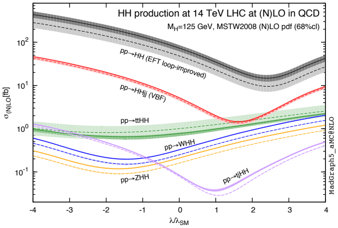
@ dim-6 : there is a one-to-one correspondence between κ_3 and C_6

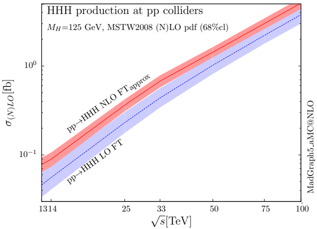
@ dim-8 : κ_3 and κ_4 are no more correlated !

Direct determination of Higgs self-couplings

Information on λ_3 and λ_4 can be extracted by studying multi-Higgs production processes.







[Frederix et al. '14, 1408.6542]

Very challenging due to small cross sections: ~33 fb (HH), ~ 0.1 fb (HHH)

Compare it with the single Higgs production ($gg \to H$) cross section: ~ 50 pb

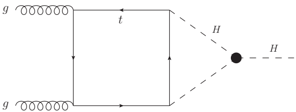
Indirect determination of λ_3

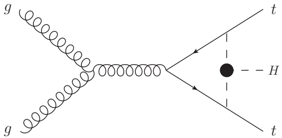
A complementary strategy of probing λ_3 in single Higgs processes via quantum effects (NLO EW)

Proposed for the first time by *McCullough 1312.3322* in ZH production at e^+e^- collider.

Production	Loops
ggF	2
VBF, VH	1
ttH, tHj	1

Decay	Loops
$\gamma\gamma$, gg	2
ZZ,WW	1
fermions	1





Gorbahn, Haisch 1607.03773; Degrassi, Giardino, Maltoni, Pagani 1607.04251; Bizon, Gorbahn, Haisch, Zanderighi 1610.05771; Di Vita, Grojean, Panico, Riembau, Vantalon 1704.01953; Maltoni, Pagani, AS, Zhao 1709.08649

λ_3 in single Higgs : Calculation

Degrassi, Giardino, Maltoni, Pagani '16; Maltoni, Pagani, AS, Zhao '17

Master formula: *Anomalous trilinear coupling* ($\kappa_3 = \lambda_3/\lambda_3^{\mathrm{SM}}$)

$$\Sigma_{\mathrm{NLO}}^{\mathrm{BSM}} = Z_H^{\mathrm{BSM}}[\Sigma_{\mathrm{LO}}(1 + \kappa_3 C_1 + \delta Z_H + \delta_{\mathrm{EW}}|_{\lambda_3=0})]$$

$$Z_H^{\mathrm{BSM}} = \frac{1}{1-(\kappa_3^2-1)\delta Z_H}, \quad \delta Z_H = -1.536\times 10^{-3}$$

Z_H^{BSM} arises from wave function renormalization and it is *universal* to all processes.

C_1 arises from the interference between LO amplitude and λ_3-dependent virtual corrections. It is finite, *process dependent* and can have non-trivial kinematic dependence. For event-by-event calculation of C_1

$$C_1(\{p_i\}) \equiv \frac{2\mathcal{R}(\mathcal{M}^{0*}\mathcal{M}^1_{\lambda_3^{\mathrm{SM}}})}{|\mathcal{M}^0|^2}$$

Σ_{LO} includes any factorizable higher order correction like QCD.

$\delta_{\mathrm{EW}}|_{\lambda_3=0}$ includes contribution from virtual W, Z and γ as well as real emissions.

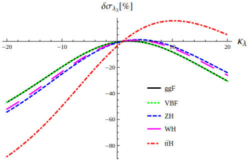
C_1 for cross section and decay

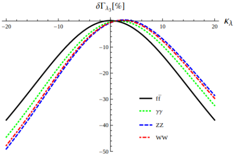
Degrassi, Giardino, Maltoni, Pagani '16; Maltoni, Pagani, AS, Zhao '17

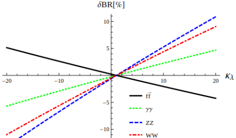
Channels	ggF	VBF	ZH	WH	$t\bar{t}H$	tHj	$H \to 4\ell$
$C_1(\%)$	0.66	0.63	1.19	1.03	3.52	0.91	0.82

$C_1^\Gamma[\%]$	$\gamma\gamma$	ZZ	WW	$f\bar{f}$	gg
on-shell H	0.49	0.83	0.73	0	0.66

$$\delta\Sigma_{\kappa_3} = \frac{\Sigma^{\rm BSM}_{\lambda_3} - \Sigma^{\rm SM}_{\lambda_3}}{\Sigma_{\rm LO}} = (Z_H^{\rm BSM} - 1)(1 + \delta Z_H) + (Z_H^{\rm BSM}\kappa_3 - 1)C_1,$$

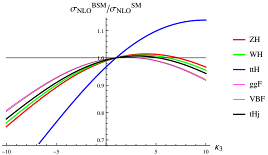






BR does not depend on $Z_H^{\rm BSM}$

The impact of full NLO EW corrections : more important for production channels



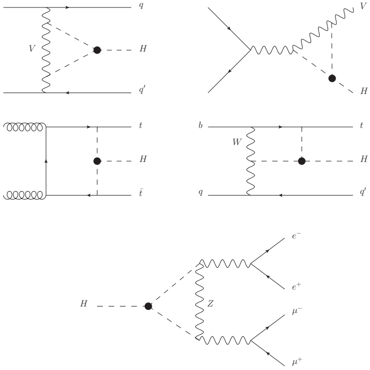
Channels	ggF	VBF	ZH	WH	ttH	tHj
$K_{\rm EW}$	1.049	0.932	0.947	0.93	1.014	0.95

The full EW effect at inclusive level in signal strength is neglegible.

Calculation of Differential C_1

Two MC public codes to calculate C_1 at differential level:
1. `trilinear-FF` 2. `trilinear-RW` (Recommended)

https://cp3.irmp.ucl.ac.be/projects/madgraph/wiki/HiggsSelfCoupling

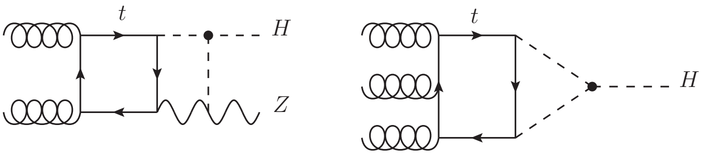


Relevant for processes with non-trivial final state kinematics
Production: VBF, VH, ttH and tHj; Decay: $H \to 4\ell$

Calculation of Differential C_1

Although C_1 for ggF is small at inclusive level, the process gives dominant contribution to single H production.

Differential C_1 not available for ggF channel. For that one needs to compute λ_3 induced effect in gg → H + g, H + Z processes. Very challenging calculations.



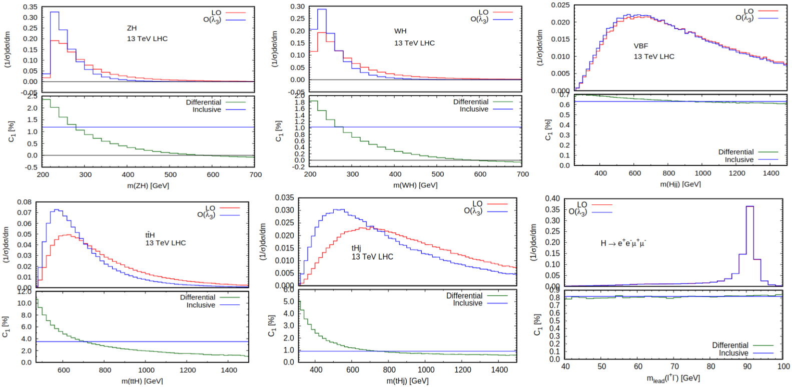
Calculation of relevant two-loop H + g amplitude available in heavy top quark limit (valid for $p_T(H) << m_t$); *Gorbahn, Haisch* 1902.05480

Like for $gg \to H$, C_1 may not be large for $gg \to H + g$ and kinematic dependence is expected to be small.

Inclusive vs Differential

C_1 peaks in the threshold region due to Sommerfeld enhancement in VH, ttH and tHj.

The kinematic dependence of C_1 is most significant in ttH.



C_1 is small and flat for VBF and Hto4ℓ.

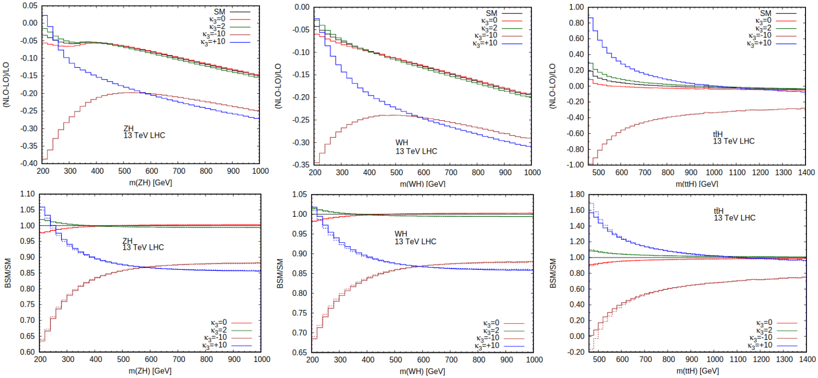
Since VBF events have VH contamination, they can be combined to have an effective C_1

Impact of NLO EW corrections

NLO EW corrections (except for ggF) can be computed using `MadGraph5_aMC@NLO`

In the SM, the EW corrections are large in the boosted regime.

$(K_{\rm EW} - 1)$ vs Signal Strength (with & without full NLO EW)

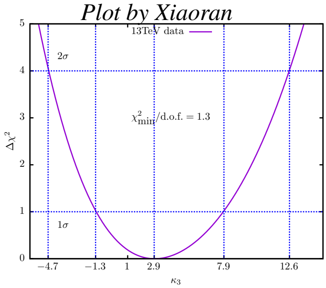


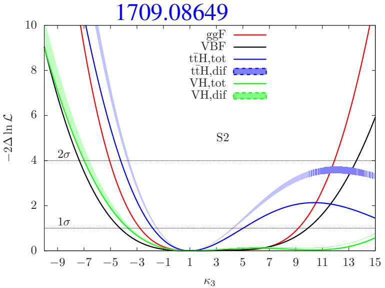
Shape in the threshold region is affected by C_1, $Z_H^{\rm BSM}$ responsible for overall shift.

Like in the inclusive case, the EW effects do not alter the signal strength significantly for small values of κ_3.

The global fit based on signal strength will not be affected by the NLO EW corrections.

Current and future reach at the LHC





13 TeV:

$$-4.7 < \kappa_3 < 12.6$$

HL-LHC:

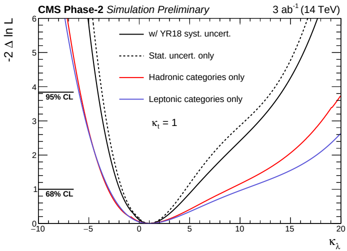
$$-2 \lesssim \kappa_3 \lesssim 8$$

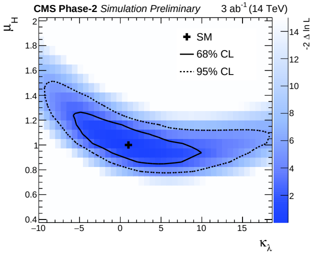
1709.08649 : *for further discussion on κ_3 in presence of κ_V and κ_t*

CMS Projections: HL-LHC

tH + ttH: using the calculation of Maltoni, Pagani, AS, Zhao: 1709.08649

[CMS-PAS-FTR-18-020]





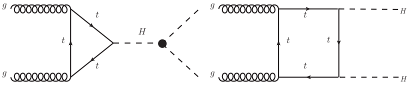
$$-3 \lesssim \kappa_\lambda = \kappa_3 \lesssim 13$$

Can we extend this strategy to double Higgs production ?

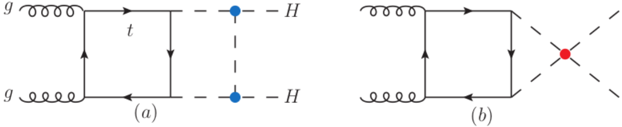
Maltoni, Pagani, Zhao: 1802.07616; Bizon, Haisch, Rottoli: 1810.04665; Borowka, Duhr, Maltoni, Pagani, AS, Zhao: 1811.12366

Indirect determination of λ_4 in double Higgs

Recall : at LO, the $gg \to HH$ amplitude is sensitive to only λ_3.



λ_4 affects $gg \to HH$ amplitude at two-loop level via NLO EW corrections.



EFT framework is necessary in order to vary cubic and quartic couplings independently in a consistent way.

$$V^{\mathrm{NP}}(\Phi) \equiv \sum_{n=3}^{4} \frac{c_{2n}}{\Lambda^{2n-4}} \left(\Phi^\dagger \Phi - \frac{1}{2} v^2\right)^n .$$

This parametrization also ensures gauge invariance and UV finiteness in calculation.

NP Paramterization

In this parametrization, κ_3 depends on c_6 only.

$$V(H) = \frac{1}{2} m_H^2 H^2 + \lambda_3 v H^3 + \frac{1}{4} \lambda_4 H^4,$$

$$\kappa_3 \equiv \frac{\lambda_3}{\lambda_3^{\mathrm{SM}}} = 1 + \frac{c_6 v^2}{\lambda \Lambda^2} \quad \equiv \quad 1 + \bar{c}_6,$$

$$\kappa_4 \equiv \frac{\lambda_4}{\lambda_4^{\mathrm{SM}}} = 1 + \frac{6 c_6 v^2}{\lambda \Lambda^2} + \frac{4 c_8 v^4}{\lambda \Lambda^4} \quad \equiv \quad 1 + 6\bar{c}_6 + \bar{c}_8 \,.$$

We can trade κ_3 and κ_4 with parameters $\bar{c}_6$ and $\bar{c}_8$.

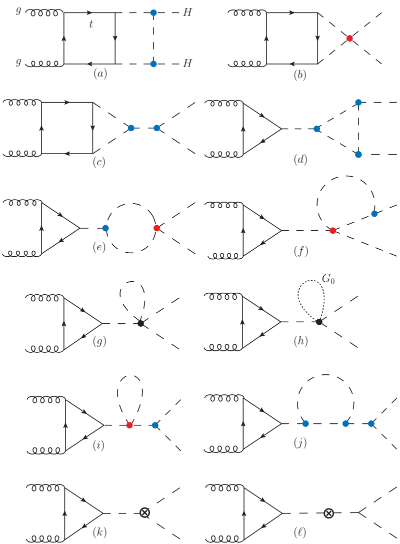
$$\bar{c}_6 \quad \equiv \quad \frac{c_6 v^2}{\lambda \Lambda^2} = \kappa_3 - 1\,,$$

$$\bar{c}_8 \quad \equiv \quad \frac{4 c_8 v^4}{\lambda \Lambda^4} = \kappa_4 - 1 - 6(\kappa_3 - 1)\,.$$

For pheno predictions, we need to compute $|M_{1\mathrm{L}}|^2 + 2\mathcal{R}(M_{1\mathrm{L}}^* M_{2\mathrm{L}})$ organised in powers of $\bar{c}_6$ and $\bar{c}_8$

Relevant two-loop topologies

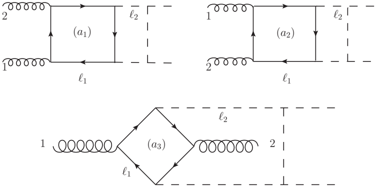
Non-factorizable, factorizable and counterterms:



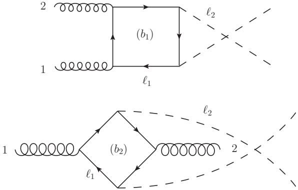
This set will give a finite amplitude as a function of $\bar{c}_6$ *and* $\bar{c}_8$

Non-factorizable contributions

The most challenging part of the calculation:



Double Box



Box-Triangle

$$
\begin{aligned}
\mathcal{M}_a &= 2(\mathcal{M}_{a_1} + \mathcal{M}_{a_2} + \mathcal{M}_{a_3}), \\
\mathcal{M}_b &= 2\mathcal{M}_{b_1} + \mathcal{M}_{b_2}, \\
\mathcal{M}_c &= \mathcal{M}_b \times \frac{6v^2}{\lambda_4} \frac{\lambda_3^2}{s - m_H^2},
\end{aligned}
$$

Projection to spin-0 and spin-2 form factors

For both one-loop and two-loop $gg \to HH$ amplitudes:

$$\mathcal{M}^{\mu_1\mu_2}\epsilon_{1,\mu_1}\epsilon_{2,\mu_2} = \delta^{c_1c_2}\mathcal{A}_0^{\mu_1\mu_2}\epsilon_{1,\mu_1}\epsilon_{2,\mu_2}F_0 + \delta^{c_1c_2}\mathcal{A}_2^{\mu_1\mu_2}\epsilon_{1,\mu_1}\epsilon_{2,\mu_2}F_2\,.$$

$$\mathcal{A}_0^{\mu_1\mu_2} = \sqrt{\frac{2}{d-2}}\left(g^{\mu_1\mu_2} - \frac{p_1^{\mu_2}p_2^{\mu_1}}{p_1\cdot p_2}\right),$$

$$\mathcal{A}_2^{\mu_1\mu_2} = \sqrt{\frac{d-2}{2(d-3)}}\Bigg(-\frac{d-4}{d-2}\left[g^{\mu_1\mu_2} - \frac{p_1^{\mu_2}p_2^{\mu_1}}{p_1\cdot p_2}\right] + g^{\mu_1\mu_2} + \frac{(p_3\cdot p_3)p_1^{\mu_2}p_2^{\mu_1} + (2p_1\cdot p_2)p_3^{\mu_1}p_3^{\mu_2} - (2p_1\cdot p_3)p_2^{\mu_1}p_3^{\mu_2} - (2p_2\cdot p_3)p_3^{\mu_1}p_1^{\mu_2}}{p_T^2(p_1\cdot p_2)}\Bigg),$$

$$\mathcal{A}_i.\mathcal{A}_i = 1; \quad \mathcal{A}_0.\mathcal{A}_2 = 0$$

$\to F_{0,a}$, $F_{0,b}$, $F_{0,c}$ and $F_{2,a}$

The box-triangle amplitudes depend only on the spin-0 form factor.

Tools: `QGRAF`, `FORM`

Numerical evaluation of form factors

The form factors contain two-loop integrals. They are computed using pySecDec.
Borowka, Heinrich, Jahn, Jones, Kerner, Schlenk: 1703.09692, 1712.05755

Correctness of the calculation is ensured by various checks:

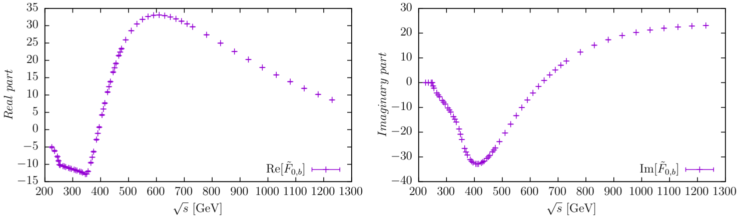
- UV finiteness of the form factors
- The large m_t limit of box-triangle amplitude
- Reduction of double-box into box triangle in heavy propagator limit

For phenomenological predictions at colliders, the form factors are required to be computed for many phase space points which can become very time consuming.

We build grids for form factors which can be interpolated for an efficient phase space integration.

Numerical evaluation of form factors

One-dimensional grid is sufficient for box-triangle spin-0 form factor.



The double box spin-0 form factor, depends on $\sqrt{s}$ as well as on θ. The θ dependence is found to be very weak below top pair threshold.

The double box spin-2 form factor displays a large θ dependence, however, this form factor is suppressed wrt the spin-0 form factor.

Once we know the form factors, we can compute the interference of two-loop amplitudes with the LO amplitude for phenomenological predictions.

The Phenomenological quantity of interest

Disclaimer : *Not a precision study*

Inclusive/differential cross section in terms of c_6 and c_8

$$\sigma_{\text{NLO}}^{\text{pheno}} = \sigma_{\text{LO}} + \Delta\sigma_{\bar{c}_6} + \Delta\sigma_{\bar{c}_8} \,,$$

EFT insertion at one-loop :

$$\sigma_{\text{LO}} = \sigma_0 + \sigma_1 \bar{c}_6 + \sigma_2 \bar{c}_6^2 \,,$$

EFT insertions at two-loop :

$$\Delta\sigma_{\bar{c}_6} = \bar{c}_6^2 \left[\sigma_{30} \bar{c}_6 + \sigma_{40} \bar{c}_6^2 \right] + \tilde{\sigma}_{20} \bar{c}_6^2 \,,$$

$$\Delta\sigma_{\bar{c}_8} = \left[\sigma_{01} + \sigma_{11} \bar{c}_6 + \sigma_{21} \bar{c}_6^2 \right] \bar{c}_8 \,,$$

Taking an agnostic view on possible values of κ_3 and κ_4, we have ignored the SM EW corrections, and have kept highest powers of $\bar{c}_6$ in $\Delta\sigma_{\bar{c}_6}$. *perturbativity requirement* : $|\bar{c}_6| < 5$, $|\bar{c}_8| < 31$

The quantity $\Delta\sigma_{\bar{c}_8}$ is the most relevant part of computation and it solely induces the sensitivity on λ_4.

We assume that higher order QCD corrections factorize from two-loop EW effects.

Effect on inclusive cross section

Input parameters :

m_t = 173.2 GeV, m_W = 80.385 GeV, m_Z = 91.1876 GeV, m_H = 125.09 GeV

One-loop:

$\sqrt{s}$ [TeV]	σ_0 [fb]	σ_1 [fb]	σ_2 [fb]
14	19.49 -	-15.59 (-80.0%)	5.414 (27.8%)
100	790.8 -	-556.8 (-70.5%)	170.8 (21.6%)

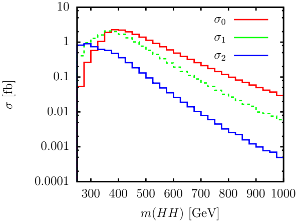
Two-loop:

$\sqrt{s}$ [TeV]	$\tilde{\sigma}_{20}$ [fb]	σ_{30} [fb]	σ_{40} [fb]	σ_{01} [fb]	σ_{11} [fb]	σ_{21} [fb]
14	0.7112 (3.6%)	-0.5427 (-2.8%)	0.0620 (0.3%)	0.3514 (1.8%)	-0.0464 (-0.2%)	-0.1433 (-0.7%)
100	24.55 (3.1%)	-16.53 (-2.1%)	1.663 (0.2%)	12.932 (1.6%)	-0.88 (-0.1%)	-4.411 (-0.6%)

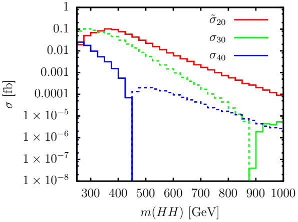
Cross sections grow considerably with energy. The contributions (numbers in brackets) from $\bar{c}_6$ and $\bar{c}_8$ slowly decrease wrt the SM LO prediction.

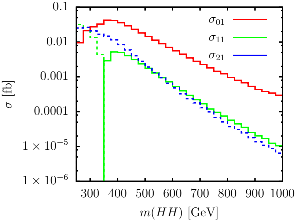
Effect on differential cross section : $m(HH)$

One-loop :



Two-loop :



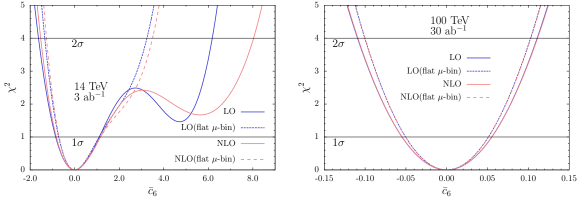


The dashed lines show absolute values of -ve contributions.

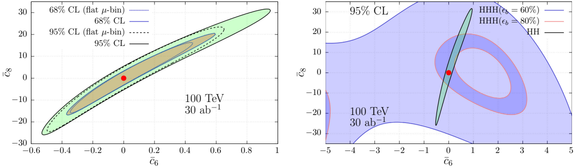
Like in the case of single Higgs, the $\bar{c}_6$ and $\bar{c}_8$ incluced effects are important in the threshold region.

Projections for κ_3 and κ_4

Scenarios for HH $(2b2\gamma)$



For $\bar{c}_8 = 0$: $-0.5 < \kappa_3 < 8$ at 14 TeV, $0.9 < \kappa_3 < 1.1$ at 100 TeV



For $\bar{c}_6 = 0$: $-6 \lesssim \kappa_4 \lesssim 18$ from $HHH(4b2\gamma)$ $-4.2 \lesssim \kappa_4 \lesssim 6.7$ from $HH(2b2\gamma)$

Summary

- The determination of Higgs potential is one of the most important goals of HL-LHC and future colliders.
- Due to low rates for multi-Higgs production, it is very challenging to measure Higgs self-couplings.
- Alternative strategies are needed to improve sensitivity on Higgs self-couplings. Higher order EW effects in single and double Higgs production are indirectly sensitive to cubic and quartic couplings respectively.
- An EFT framework is required for a systematic calculation of EW effects in presence of non-standard Higgs self-couplings.
- Our studies indicate that constraints on cubic coupling from single H are complementary to those from double Higgs at HL-LHC. At FCC-hh, the HH channel would be more sensitive to independent variations in self-couplings than HHH channel.

Thank You.