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Dark Matter Freeze-in and Freeze-out via Effective Operators: A couple of Illustrations

Subhaditya Bhattacharya
IIT Guwahati

Why Effective theory ?

- Physics Beyond the SM (BSM) is required to explain Dark Matter (DM) etc..
- But we have little idea what kind of BSM/DM
- Effective theory provides a way to gauge the effect of New Physics just with low energy fields/particles
- Effective theory is *sufficiently* model independent

What is Dark Matter ?

It can be constituted of **fundamental particle(s)** with the following broad properties:

EM charge neutral: Dark
Stable : don't decay
Massive: gravitational effect (cold/warm/hot)

Weakly Interacting massive particle: WIMP

Strongly Interacting massive particle: SIMP

Thermal Freeze out

Feebly Interacting Massive Particle: FIMP

Non-thermal freeze in

Main two mechanisms to address relic density of Dark Matter

$$\Omega h^2 \simeq 0.1199 \pm 0.0022$$

Many many unknown issues

- Spin of Dark Matter: scalar, fermion, Vector Boson
- Single or Multi component
- Mass of Dark Matter and coupling with visible sector

Other dark objects like **Massive Compact Halo Objects (MACHOS)**, **Primordial Black Holes (PBH)** or even **Modified gravity (MOND)** can account for Dark Matter (partially).

Building DM Effective theory

$$\mathcal{L}_{EFF} = \mathcal{L}_{SM} + \sum_n c_n \frac{\mathcal{O}_{SM-DM}}{\Lambda^{(n-4)}}$$

$\mathcal{O}_{SM-DM}$ is invariant under $\mathcal{G}_{SM} \times \mathcal{G}_{DM}$

$$\mathcal{O}_{SM-DM} \sim \mathcal{O}_{DM}\mathcal{O}_{SM}$$

$\mathcal{O}_{SM}$ is constituted of SM fields,
$\mathcal{O}_{DM}$ is constituted with DM fields.

Factorisation assumes that DM do not possess SM charge and vice versa

- We are guided by the principle that the Lagrangian should be **Lorentz Invariant**.
- χ, the BSM particle, is odd under an imposed $\mathcal{Z}_2$ symmetry which stabilizes the dark matter.
- Thus any effective operator would contain at least two dark matter particles.

Simplest DM-SM Operator

$$\mathcal{L}_{\rm DM-SM} \sim \frac{1}{\Lambda^{(n-4)}}\mathcal{O}_{\rm DM}\mathcal{O}_{\rm SM}$$

Renormalisable interactions (n=4)

$$\mathcal{O}_{\rm SM}: \quad H^\dagger H, \;\; B_{\mu\nu} \quad [M]:2$$

$$\mathcal{L}_{\rm DM-SM} \sim H^\dagger H \phi^2$$

Higgs Portal Interaction

ϕ is a DM !

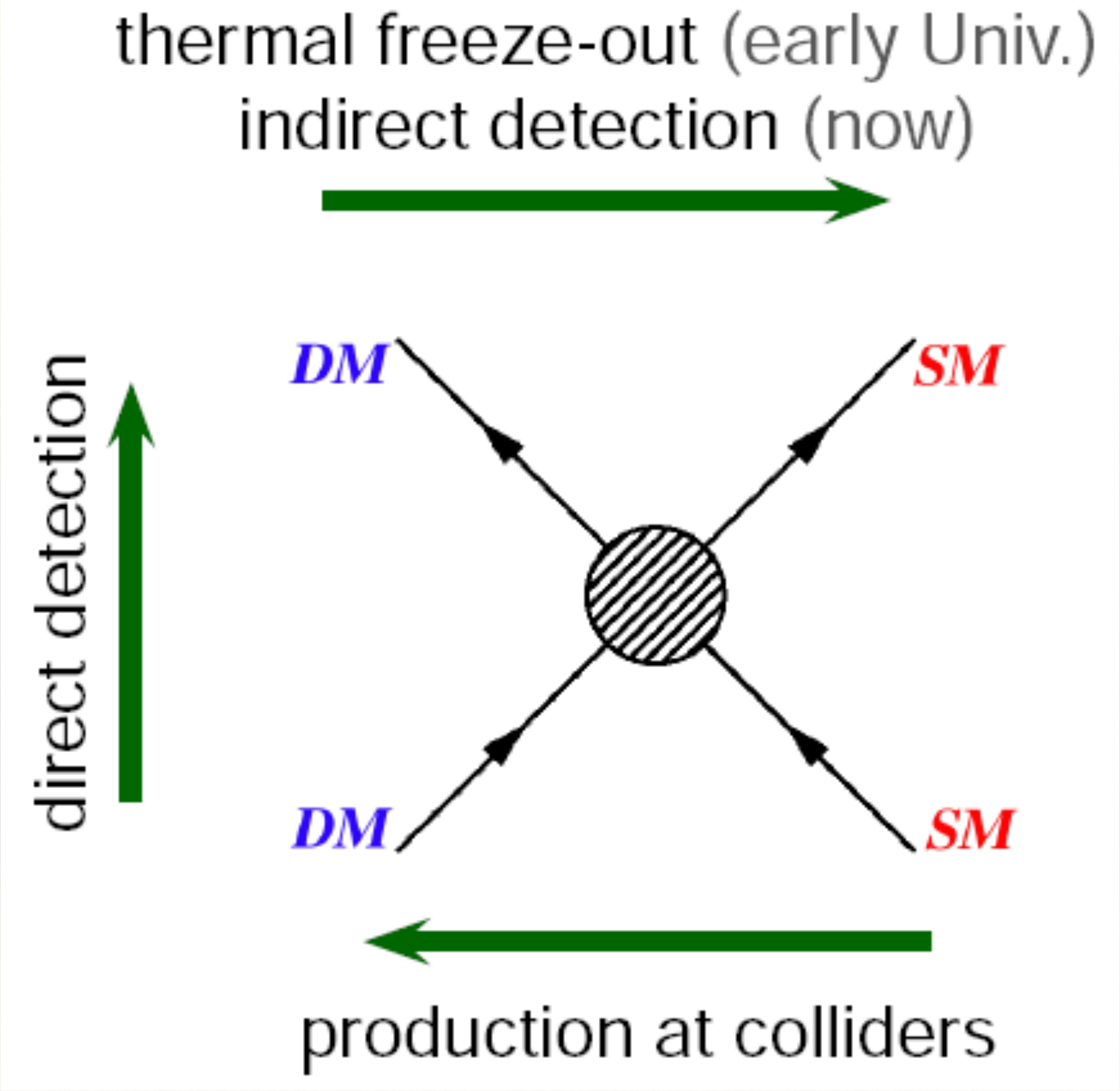
provided it possess no EM charge

Stability of DM is ensured by a Z_2 symmetry under which

$\phi \to -\phi$ ➡ $\lambda_1 \phi^2 H^\dagger H$ $\quad \cancel{\phi H^\dagger H}$

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}(\partial^\mu \phi)(\partial_\mu \phi) - V(H,\phi)$$

$$V(H,\phi) = -\mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + \frac{1}{2}\mu_\phi^2 \phi^2 + \frac{1}{4!}\lambda_\phi \phi^4 + \frac{1}{2}\lambda_1 H^\dagger H \phi^2$$



Effective DM operators upto dimension six

dim.	category	operators	
4	I	$\lvert\phi\rvert^2(\Phi^\dagger\Phi)$	
5	II	$\lvert\phi\rvert^2\bar{\Psi}\Psi$	$\lvert\phi\rvert^2\Phi^3$
	III	$(\bar{\Psi}\Phi)(\phi^T\epsilon\ell)$	
	IV	$B_{\mu\nu}X^{\mu\nu}\Phi$	$B_{\mu\nu}\bar{\Psi}\sigma^{\mu\nu}\Psi$
6	V	$\lvert\phi\rvert^2\mathcal{O}^{(4)}_{\texttt{dark}}$	$\Phi^2\mathcal{O}^{(4)}_{\texttt{SM}}$
	VI	$(\bar{\Psi}\Phi^2)(\phi^T\epsilon\ell)$	$(\bar{\Psi}\Phi)\not{\partial}(\phi^T\epsilon\ell)$
	VII	$\mathcal{J}^{\mu}_{\texttt{SM}}\mathcal{J}_{\texttt{dark}\,\mu}$	
	VIII	$B_{\mu\nu}\mathcal{O}^{(4)\mu\nu}_{\texttt{dark}}$	

$$\mathcal{G}_{SM} : SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$\mathcal{O}^{(4)}_{\texttt{SM}} \in \{\lvert\phi\rvert^4,\ \Box\lvert\phi^2\rvert,\ \bar{\psi}\phi\psi',\ B^2_{\mu\nu},\ (W^I_{\mu\nu})^2,\ (G^A_{\mu\nu})^2\}\,,$$

$$\mathcal{O}^{(4)}_{\texttt{dark}} \in \{\lvert\Phi\rvert^4,\ \Box\lvert\Phi^2\rvert,\ \Phi\bar{\Psi}P_{L,R}\Psi,\ X^2_{\mu\nu}\}\,,$$

$$\mathcal{O}^{(4)}_{\texttt{dark}\ \mu\nu} \in \{\lvert\Phi\rvert^\dagger X_{\mu\nu}\Phi,\ \Phi\bar{\Psi}\sigma_{\mu\nu}P_{L,R}\Psi,\ \bar{\Psi}(\gamma_\mu\mathcal{D}_\nu - \gamma_\nu\mathcal{D}_\mu)P_{L,R}\Psi\}\,;$$

$$\mathcal{J}^{(\psi)\mu}_{\texttt{SM}} = \bar{\psi}\gamma^\mu\psi, \qquad \mathcal{J}^{(\phi)\mu}_{\texttt{SM}} = \frac{1}{2i}\phi^\dagger \overleftrightarrow{D}^\mu\phi,$$

$$\mathcal{J}^{(L,R)\mu}_{\texttt{dark}} = \bar{\Psi}\gamma^\mu P_{L,R}\Psi, \qquad \mathcal{J}^{(\Phi)\mu}_{\texttt{dark}} = \frac{1}{2i}\Phi^\dagger \overleftrightarrow{\mathcal{D}}^\mu\Phi.$$

M. Duch, B. Grzadkowski, and J. Wudka, JHEP **05**, 116 (2015), arXiv:1412.0520 [hep-ph].

V. Gonzalez Macias and J. Wudka, JHEP **07**, 161 (2015), arXiv:1506.03825 [hep-ph].

SB, Jose Wudka, *Int.J.Mod.Phys.D* 30 (2021) 13, 2130004 • e-Print: 2104.01788 [hep-ph]

There are many many more references where DMEFT is discussed.

Fermion DM Effective operators

$$\mathcal{O} = (\bar{\chi}\Gamma\chi)(\bar{f}\Gamma f) \quad \Gamma = \{1, \gamma^5, \gamma^\mu, \gamma^\mu\gamma^5, \sigma^{\mu\nu}\}$$

$$\mathcal{G}_{SM} : SU(3)_C \times U(1)_{EM}$$

Name	Operator	Coefficient
D1	$\bar{\chi}\chi\bar{q}q$	m_q/Λ^3
D2	$\bar{\chi}\gamma^5\chi\bar{q}q$	im_q/Λ^3
D3	$\bar{\chi}\chi\bar{q}\gamma^5 q$	im_q/Λ^3
D4	$\bar{\chi}\gamma^5\chi\bar{q}\gamma^5 q$	m_q/Λ^3
D5	$\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu q$	$1/\Lambda^2$
D6	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu q$	$1/\Lambda^2$
D7	$\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu\gamma^5 q$	$1/\Lambda^2$

D8	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu\gamma^5 q$	$1/\Lambda^2$
D9	$\bar{\chi}\sigma^{\mu\nu}\chi\bar{q}\sigma_{\mu\nu}q$	$1/\Lambda^2$
D10	$\bar{\chi}\sigma^{\mu\nu}\gamma^5\chi\bar{q}\sigma_{\mu\nu}q$	$1/\Lambda^2$
D11	$\bar{\chi}\sigma^{\mu\nu}\gamma^5\chi\bar{q}\sigma_{\mu\nu}\gamma^5 q$	$1/\Lambda^2$
D12	$\bar{\chi}\sigma^{\mu\nu}\chi\bar{q}\sigma_{\mu\nu}\gamma^5 q$	i/Λ^2

J.~Goodman, M.~Ibe, A.~Rajaraman, W.~Shepherd, T.~M.~P.~Tait and H.~B.~Yu, Phys.Rev.D82, 116010 (2010)

WIMP: Thermal Freeze out

DM is assumed to be in equilibrium with thermal bath: Evolution of DM number **Boltzmann Equations**

$$\dot{n}_\psi + 3Hn_\psi = -\langle \sigma_{\psi\bar{\psi}\to X\bar{X}} |v| \rangle (n_\psi n_{\bar{\psi}} - {n_\psi}^{EQ} {n_{\bar{\psi}}}^{EQ})$$

One may recast the number density in co moving volume by Yield

$$Y = \frac{n}{s}, \quad x = \frac{m}{T} \longrightarrow \frac{dY}{dx} = -\frac{x \langle \sigma_{\psi\bar{\psi}\leftrightarrow X\bar{X}} |v| \rangle \, s}{H(m)} \left[Y^2 - (Y_{EQ})^2\right]$$

Recast the BEQ in terms of $\Gamma = n^{EQ}\langle \sigma v \rangle$.

$$\frac{x}{Y_{EQ}} \frac{dY}{dx} = -\frac{\Gamma}{H}\left[\left(\frac{Y}{Y_{EQ}}\right)^2 - 1\right]$$

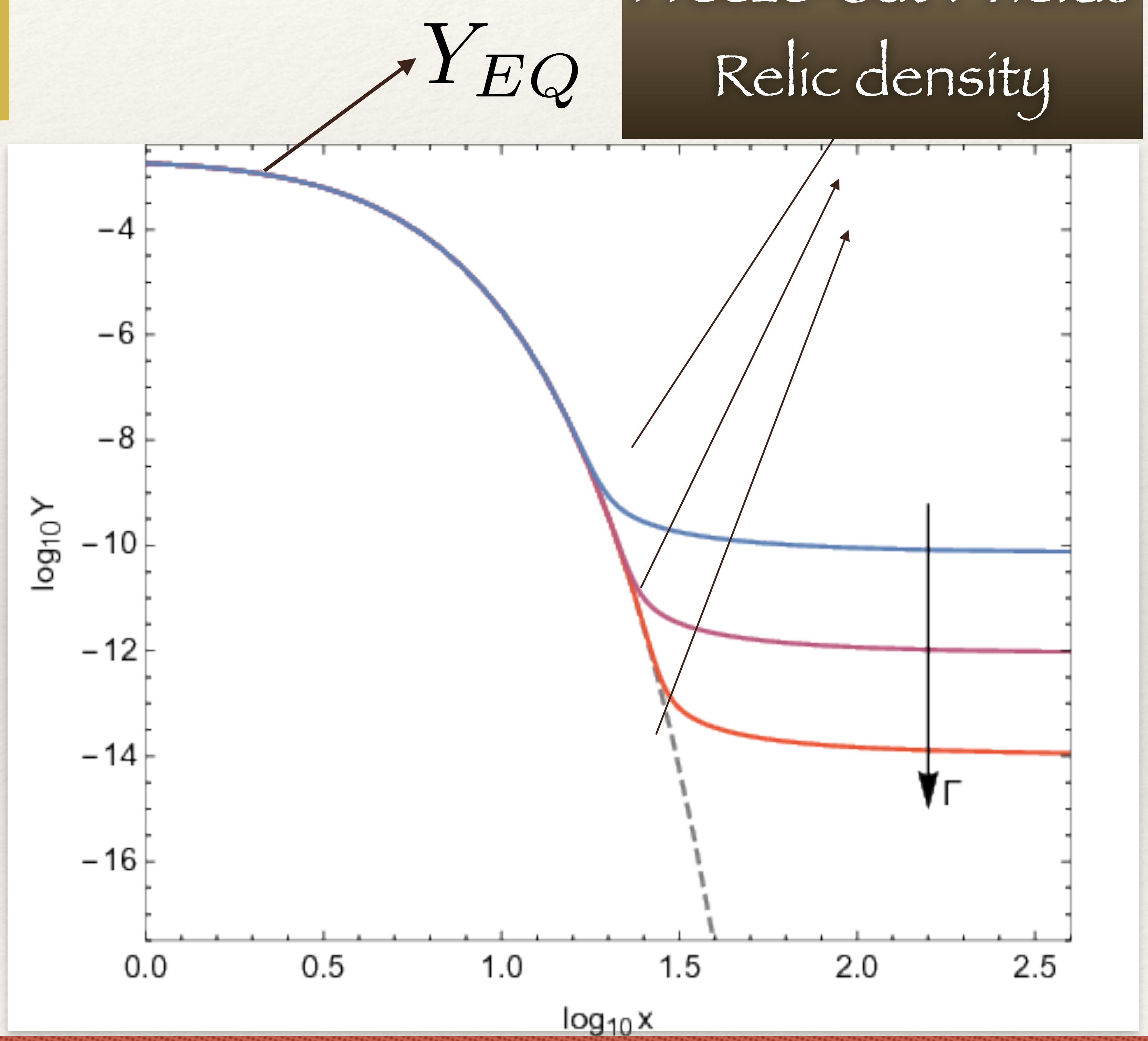
For $\Gamma >> H$: Particle is in Equilibrium $Y(x < x_{fo}) = Y_{EQ}$.
For $\Gamma << H$: Particle freezes out: $Y(x > x_{fo}) = Y(x = x_{fo})$.

Solving BEQ

$$\Omega h^2 \simeq \frac{2.4 \times 10^{-10} \mathrm{GeV}^{-2}}{(\sigma \ \mathrm{v})_{X_1 X_1 \to SM\ SM}}$$

Relic density is inversely proportional to annihilation cross-section

Freeze-out ! Yields Relic density



Correct relic density, when the interaction cross-section is of weak interaction strength, hence WIMP

FIMP: Non-thermal freeze-in

DM is assumed **out-of-equilibrium** due to feeble interaction, produced from particles in thermal bath

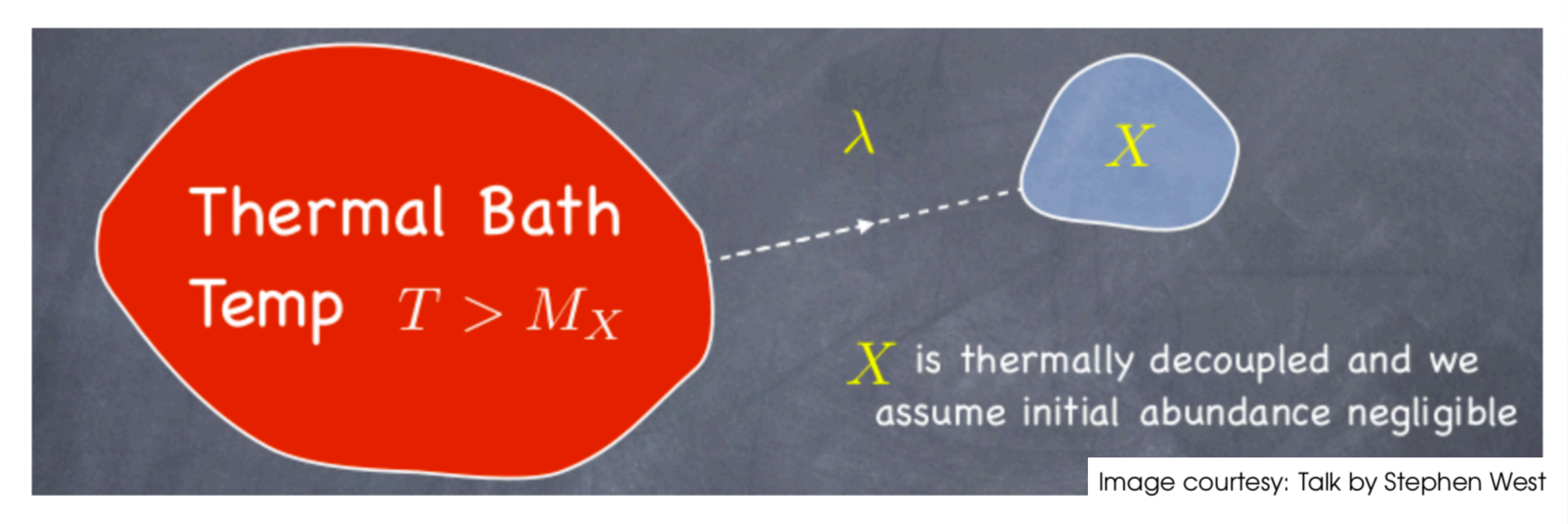
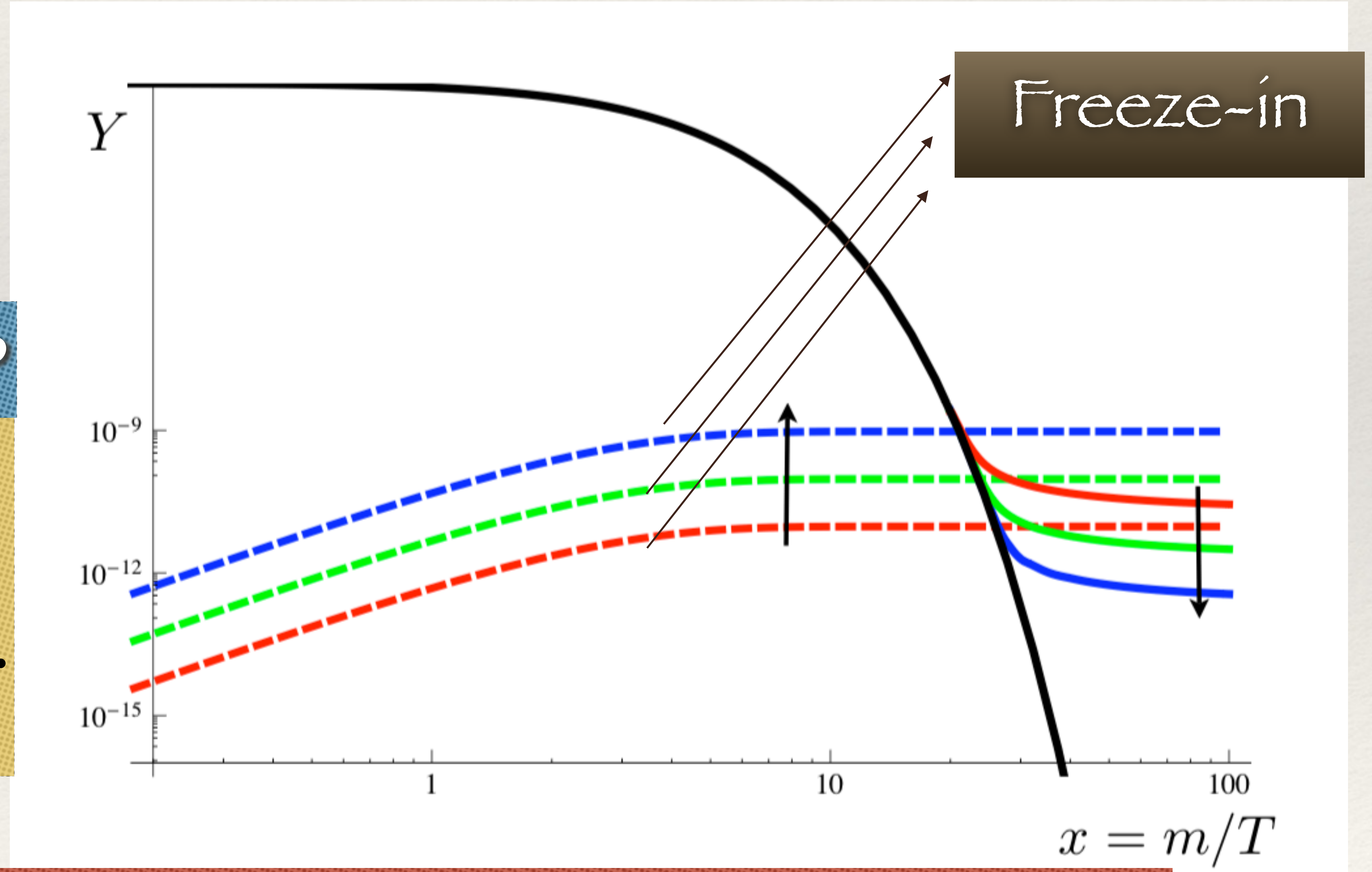
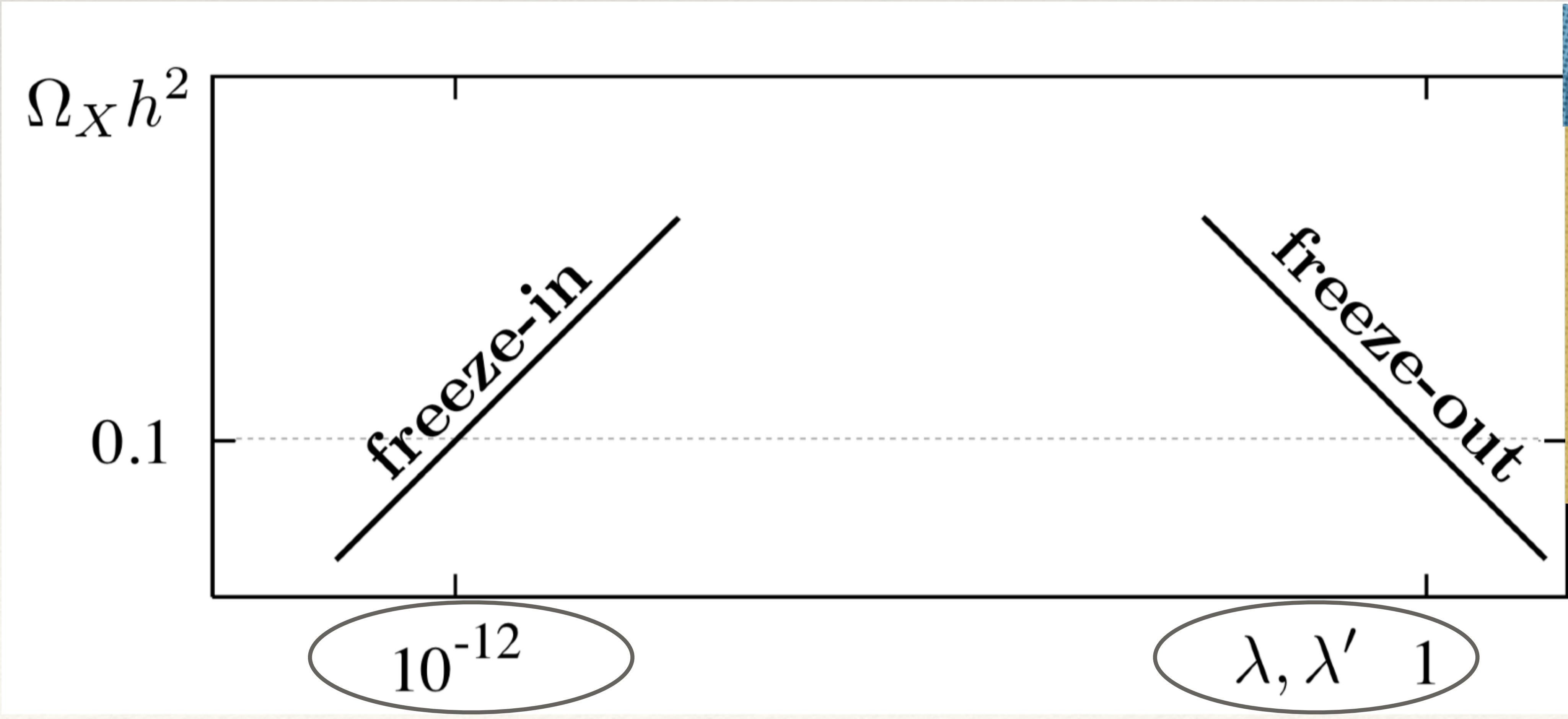


Image courtesy: Talk by Stephen West

Boltzmann Equation for FIMP (Initial DM density is zero):

$$\frac{dY_\psi}{dx} = \frac{4\pi^2 \mathrm{M_{Pl}}}{45 \times 1.66} \frac{g_*^s(x)}{\sqrt{g_*^\rho(x)}} \frac{m_\psi}{x^2} \left[\langle \sigma v \rangle_{\chi\bar{\chi}\to\psi\bar{\psi}} (Y_\chi^{eq})^2\right].$$





Compare: WIMP-FIMP

$$Y_{\mathrm{FI}} \sim \langle \sigma v \rangle \sim \lambda^2,$$

$$Y_{\mathrm{FO}} \sim \frac{1}{\langle \sigma v \rangle} \sim \frac{1}{\lambda'^2}.$$

Possibility of Direct and collider search is limited for FIMP!

Lawrence J. Hall, Karsten Jedamzik, John March-Russell and Stephen M. West, JHEP03(2010)080

Example 1: Vector DM freeze-in

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Feebly coupled vector boson dark matter in effective theory

Basabendu Barman,[a] Subhaditya Bhattacharya[a] and Bohdan Grzadkowski[b]

[a] *Department of Physics, Indian Institute of Technology Guwahati, North Guwahati, Assam-781039, India*
[b] *Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland*

E-mail: bb1988@iitg.ac.in, subhab@iitg.ac.in, bohdan.grzadkowski@fuw.edu.pl

• *JHEP* 12 (2020) 162 • e-Print: 2009.07438 [hep-ph]

Vector Boson Dark Matter Model

Symmetry: $\mathcal{G} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X \times Z_2$

Fields	SU(3)$_c$	SU(2)$_L$	U(1)$_Y$	Z_2
Φ	1	1	0	$-\Phi$
X	1	1	0	$-X$
S	1	1	0	S^*

S transforms under $U(1)_X$, acquires VEV (v_s) to break $SM \times U(1)_X \to SM$ spontaneously.

$$m_X = g_X v_s$$

$v_\Phi = 0$ and Z_2 remains unbroken.

$$\mathcal{L}_{\rm tot} = -\frac{1}{4}X_{\mu\nu}X^{\mu\nu} + |D_\mu^X S|^2 + (D_\mu^{SM} H)^\dagger (D^{SM\,\mu} H) + \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - V(H,S,\Phi) + \mathcal{L}_{\rm SM}\,,$$

$$X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu; \quad D_\mu^X S = \partial_\mu S - i g_X S X_\mu;$$

$$\begin{aligned} V(H,S,\Phi) &= -\mu_H^2|H|^2 - \mu_S^2|S|^2 + \mu_\Phi^2\Phi^2 + \lambda_H|H|^4 + \lambda_S|S|^4 + \lambda_\Phi\Phi^4 \\ &+ \lambda_{H\Phi}|H|^2\Phi^2 + \lambda_{S\Phi}|S|^2\Phi^2 + \lambda_{SH}|H|^2|S|^2\,. \end{aligned}$$

$s - h$ mixes after EWSB to yield h_1, h_2, where h_1 is SM like Higgs ($m_{h_1} = 125$ GeV).

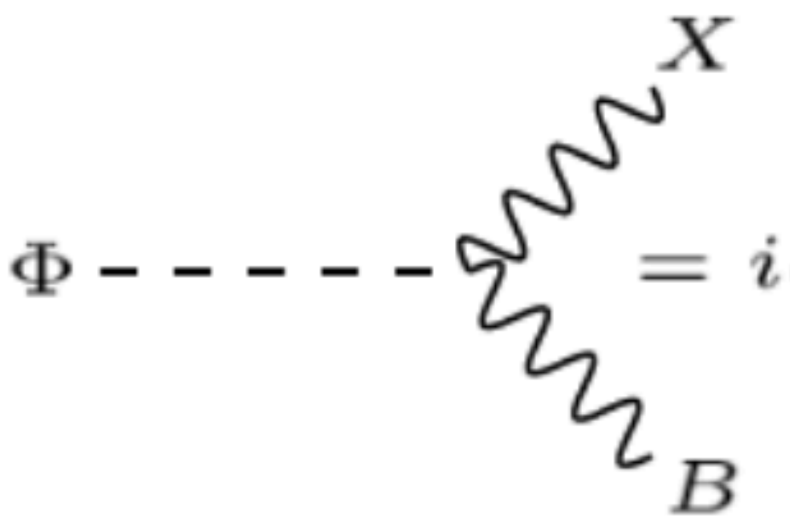
We are interested in the limit $m_{h_2} >> m_{h_1}$, i.e. when h_2 is decoupled.

Effective DM-SM interaction limit

In the limit when the second scalar is very very heavy (decoupling limit), the leading dark sector SM interaction is given by dimension five EFT operator:

$$\mathcal{L}_{\text{dim-5}} = \frac{c}{\Lambda} B_{\mu\nu} X^{\mu\nu} \Phi + \frac{\tilde{c}}{\Lambda} B_{\mu\nu} \tilde{X}^{\mu\nu} \Phi .$$

Generates a trilinear vertex suppressed by NP scale



$$= i\tilde{\alpha}\varepsilon_{\mu\nu\rho\sigma} p_1^{\rho} p_2^{\sigma} + i\alpha \left(\eta_{\mu\nu} p_1 \cdot p_2 - p_{1_\nu} p_{2_\mu} \right)$$

$$\left(\alpha = \frac{c}{\Lambda},\ \tilde{\alpha} = \frac{\tilde{c}}{\Lambda}\right)$$

$$\beta = \frac{\tilde{\alpha}}{\alpha}$$

In the limit $\Lambda \gtrsim m_2 \gtrsim T_{RH} > m_\Phi > m_X ,$

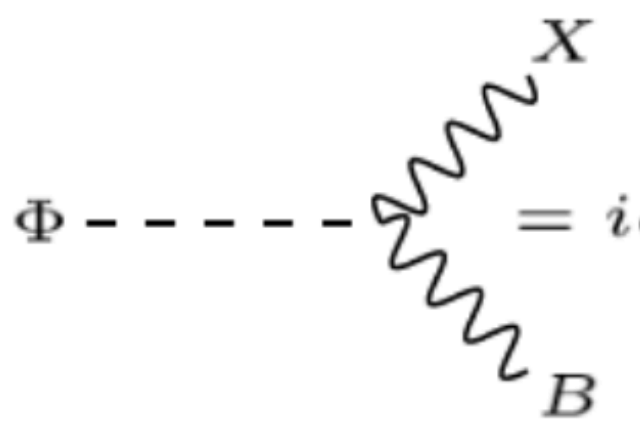
X is DM

In the limit of large Λ, it is natural to assume that X is a FIMP !

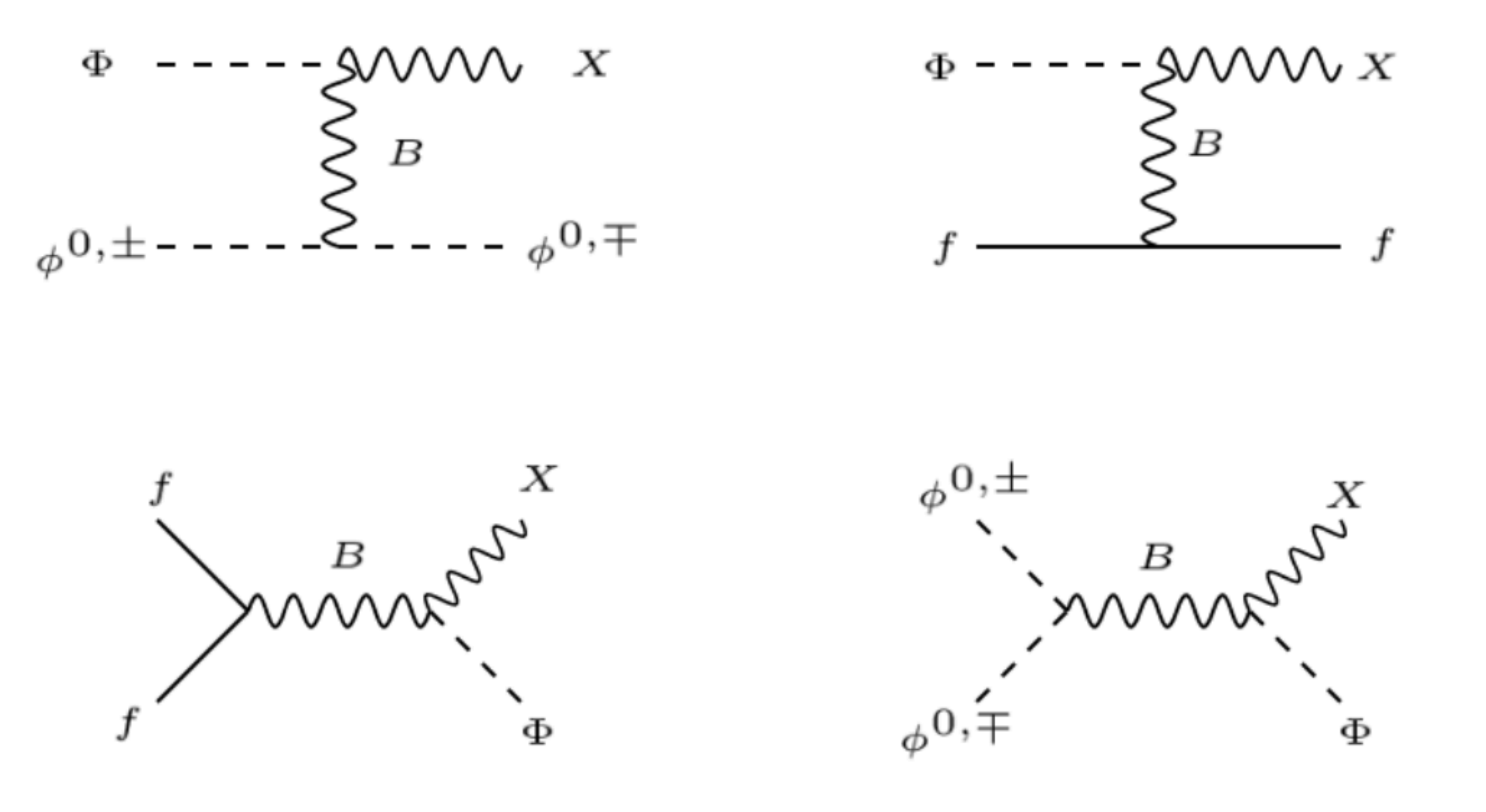
Parameters: $m_X , m_\Phi , T_{RH} , \Lambda$

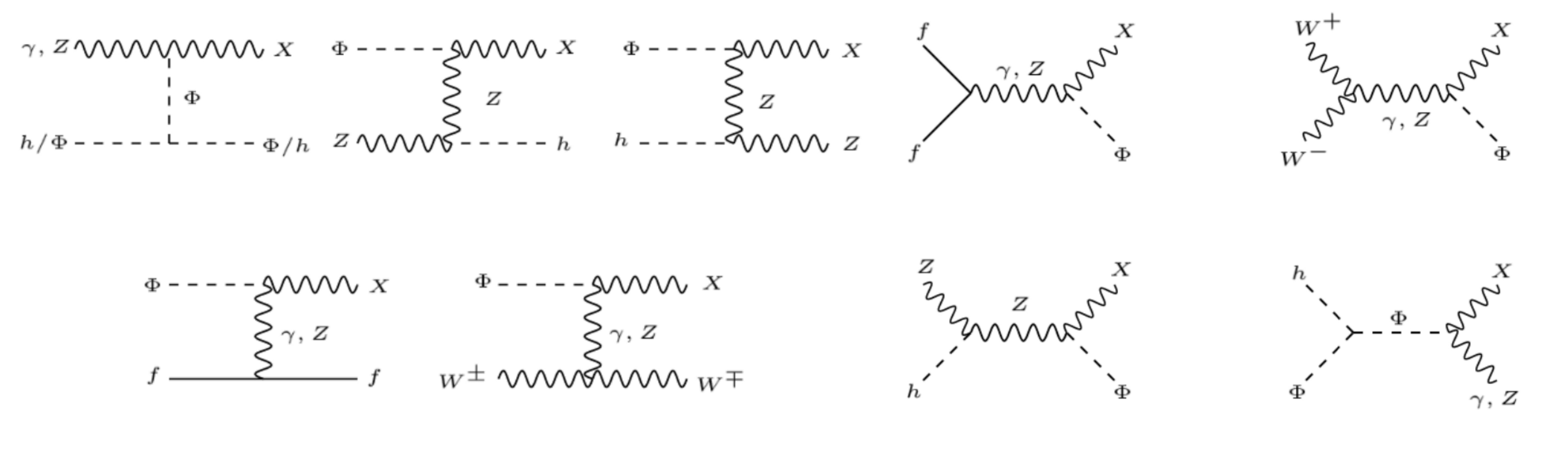
DM production

Before EWSB:



$$= i\tilde{\alpha}\varepsilon_{\mu\nu\rho\sigma} p_1^{\rho} p_2^{\sigma} + i\alpha\left(\eta_{\mu\nu}\, p_1\cdot p_2 - p_{1_\nu} p_{2_\mu}\right)$$





After EWSB:

Solving BEQ, the DM yield:

$$xHs\frac{dY_X}{dx} = \gamma_{\rm ann} + \gamma_{\rm D}.$$

γ : reaction density

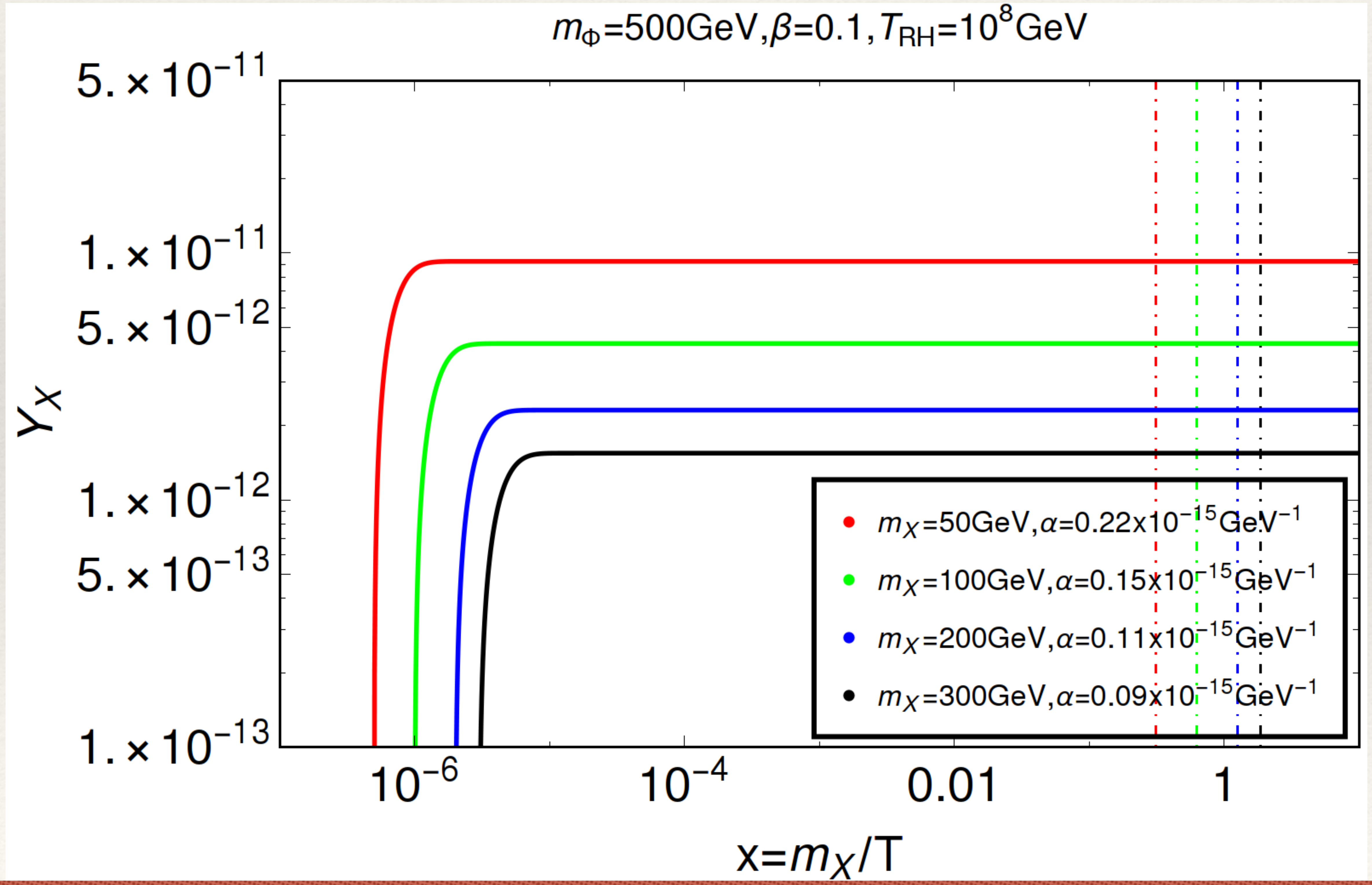
$$\begin{aligned}
Y_X^{\rm total}(T_0) = &\left\{\int_{T_{\rm EW}}^{T_{\rm RH}} dT \frac{m_\Phi^2 \Gamma_{\Phi\to X,B}}{2\pi^2}\frac{K_1(m_\Phi/T)}{s(T)H(T)}\right.\\
&\left. + \frac{1}{512\pi^6}\sum_{i,j,k}\int_{T_{\rm EW}}^{T_{\rm RH}}\frac{dT}{s(T)H(T)}\int_0^\infty ds d\Omega\left(\frac{\sqrt{s}}{2}\right)^2 |\mathcal{M}^{\rm bEWSB}|^2_{i,j\to X,k}\frac{1}{\sqrt{s}}K_1\left(\frac{\sqrt{s}}{T}\right)\right\}\\
&+\left\{\int_{T_0}^{T_{\rm EW}} dT \frac{m_\Phi^2 \Gamma_{\Phi\to X,\gamma(Z)}}{2\pi^2}\frac{K_1(m_\Phi/T)}{s(T)H(T)}\right.\\
&\left. + \frac{1}{512\pi^6}\sum_{i,j,k}\int_{T_0}^{T_{\rm EW}}\frac{dT}{s(T)H(T)}\int_0^\infty ds d\Omega\left(\frac{\sqrt{s}}{2}\right)^2 |\mathcal{M}^{\rm aEWSB}|^2_{i,j\to X,k}\frac{1}{\sqrt{s}}K_1\left(\frac{\sqrt{s}}{T}\right)\right\},
\end{aligned} \tag{3.14}$$

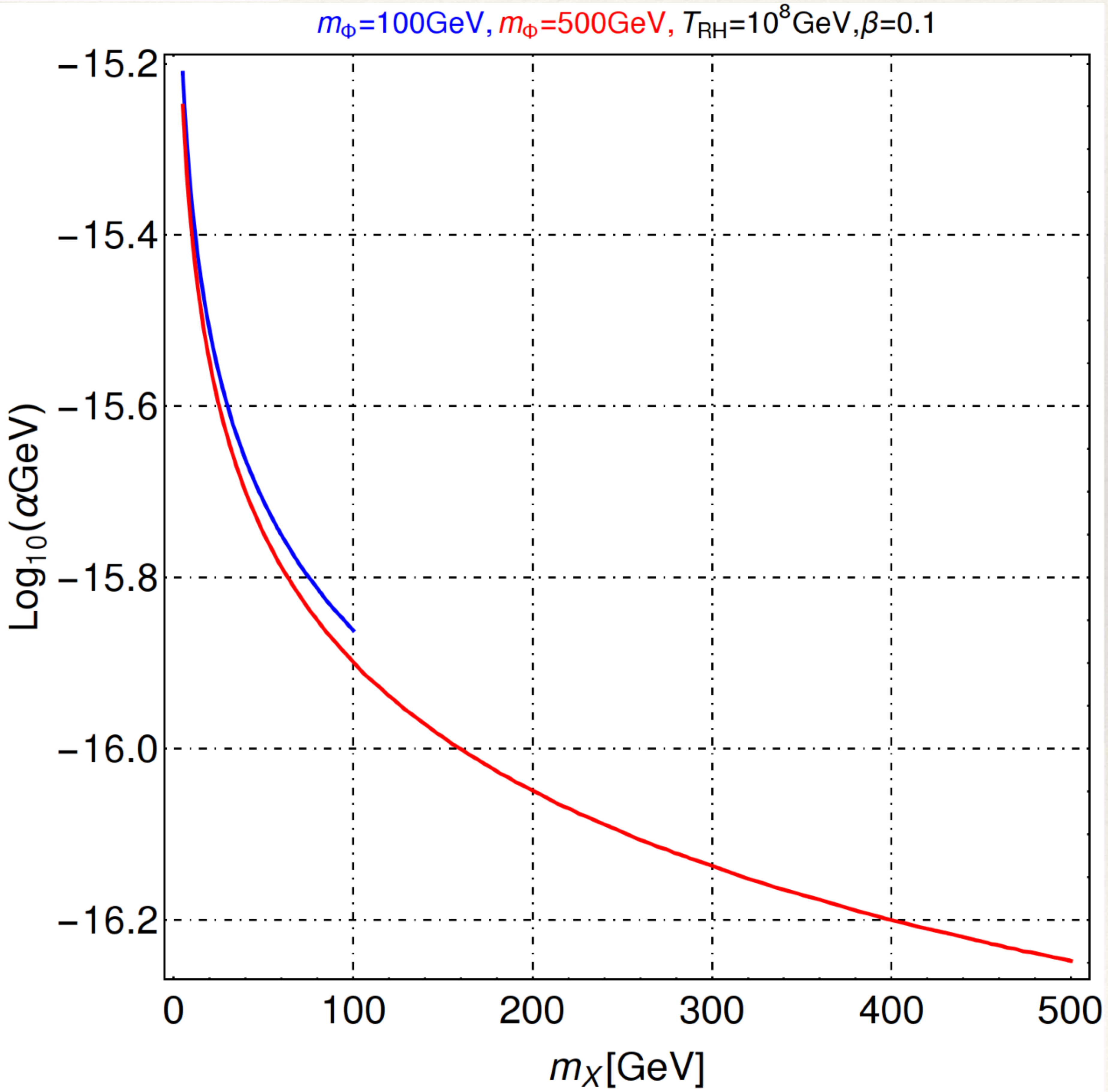
Large Reheat temperature: UV limit

$T_{\mathrm{RH}} >> m$

$$\frac{Y_X^{ann}}{Y_X^{\mathrm{D}}} \sim \frac{\sigma M_{pl} T_{FI}}{\Gamma_\Phi M_{pl}/T_{FI}^2} \sim \frac{\alpha^2 M_{pl} T_{RH}}{\left(\alpha^2 m_\Phi^3\right) M_{pl}/m_\Phi^2} \sim \frac{T_{RH}}{m_\Phi},$$

Annihilation wins over decay contribution





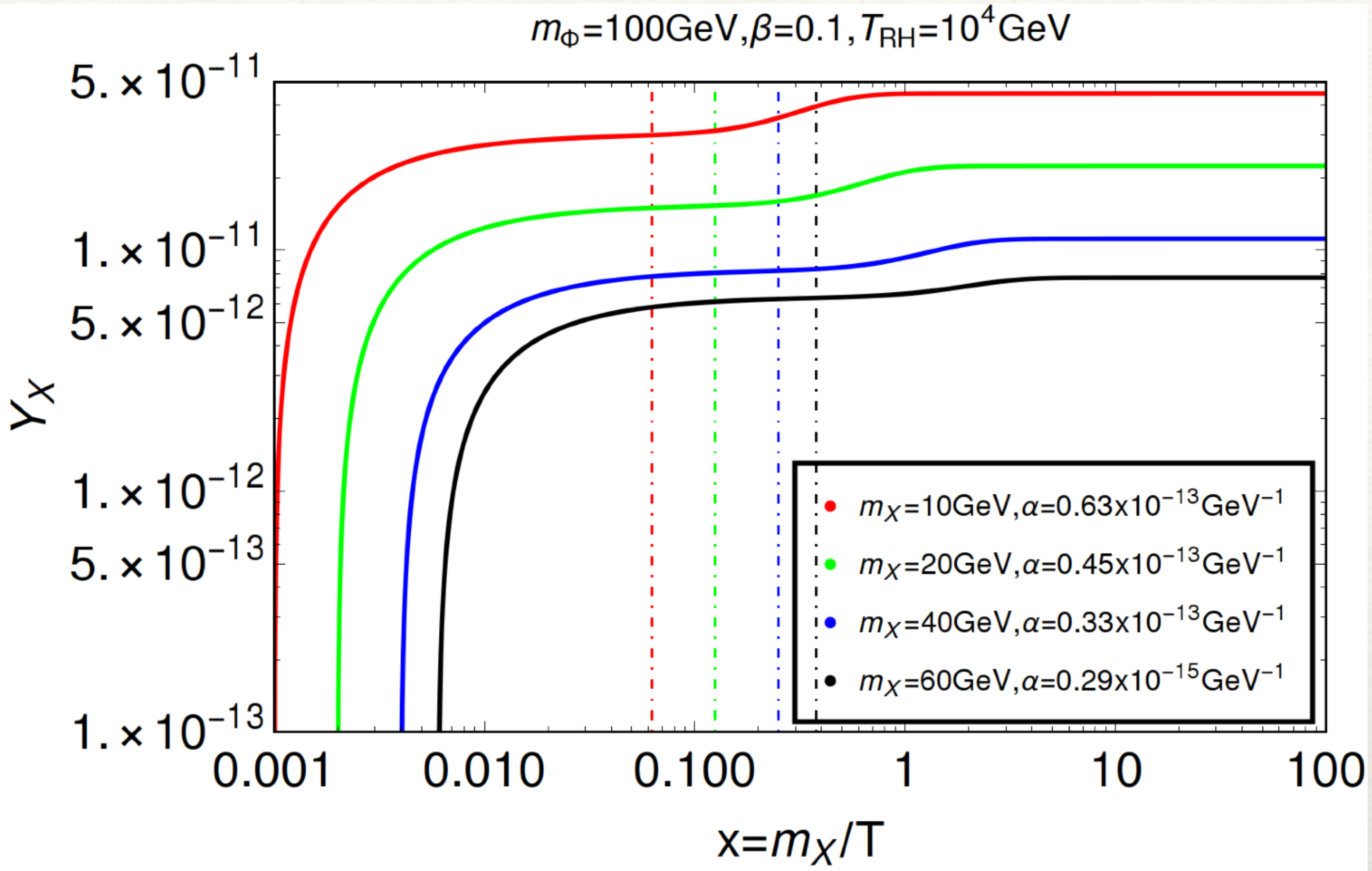
DM yield saturates at high temperature: UV freeze in

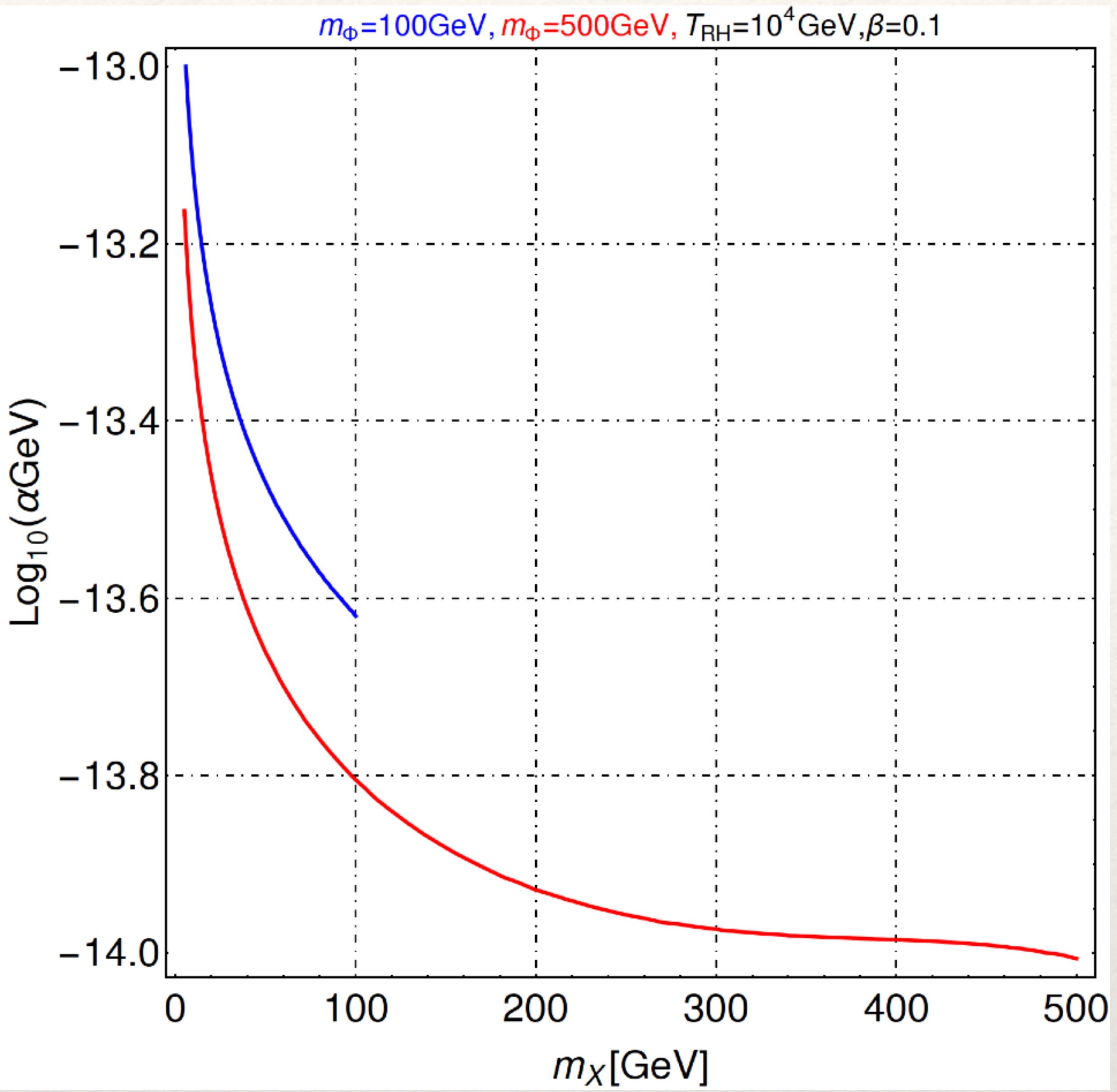
Small Reheat temperature: IR limit

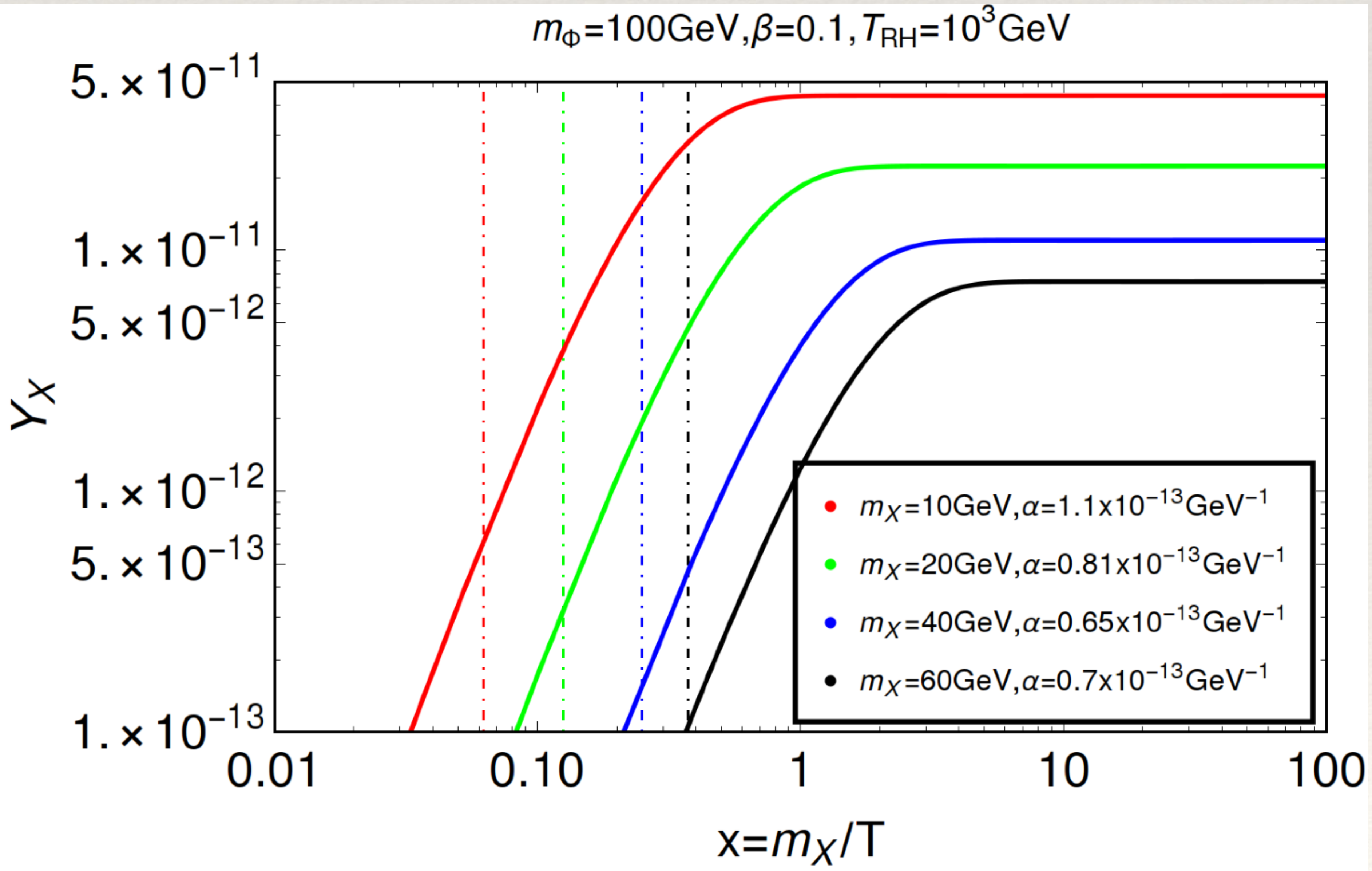
$$T_{\rm RH} \gtrsim m$$

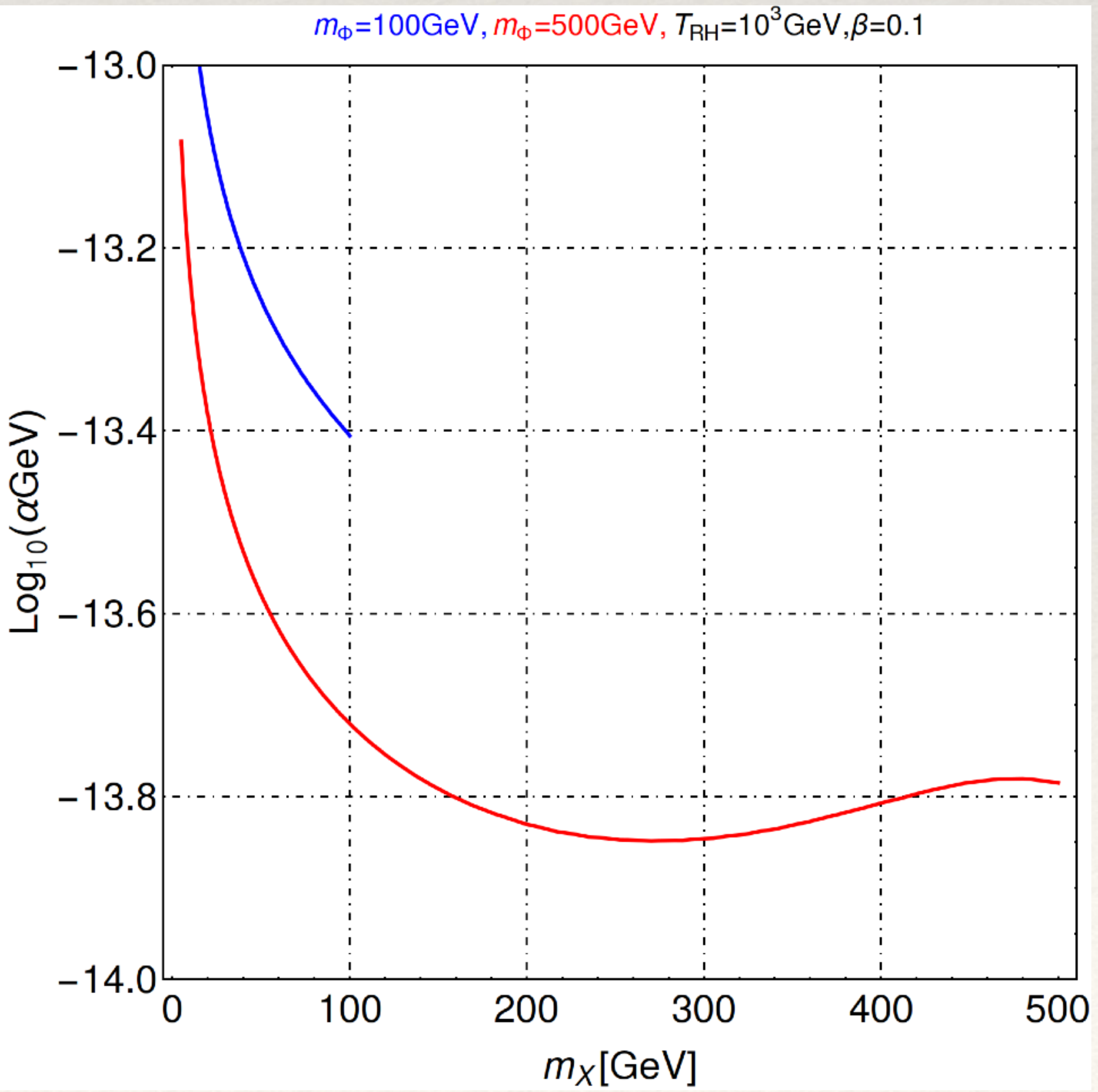
$$\frac{Y_X^{ann}}{Y_X^{\rm D}} \sim \frac{\sigma M_{pl} T_{FI}}{\Gamma_\Phi M_{pl}/T_{FI}^2} \sim \frac{\alpha^2 M_{pl} T_{RH}}{\left(\alpha^2 m_\Phi^3\right) M_{pl}/m_\Phi^2} \sim \frac{T_{RH}}{m_\Phi} \sim 1.$$

Decay contribution plays an equally important role, so the mass effect comes into the picture and DM density accumulates at small temperature also and provides IR freeze- in pattern



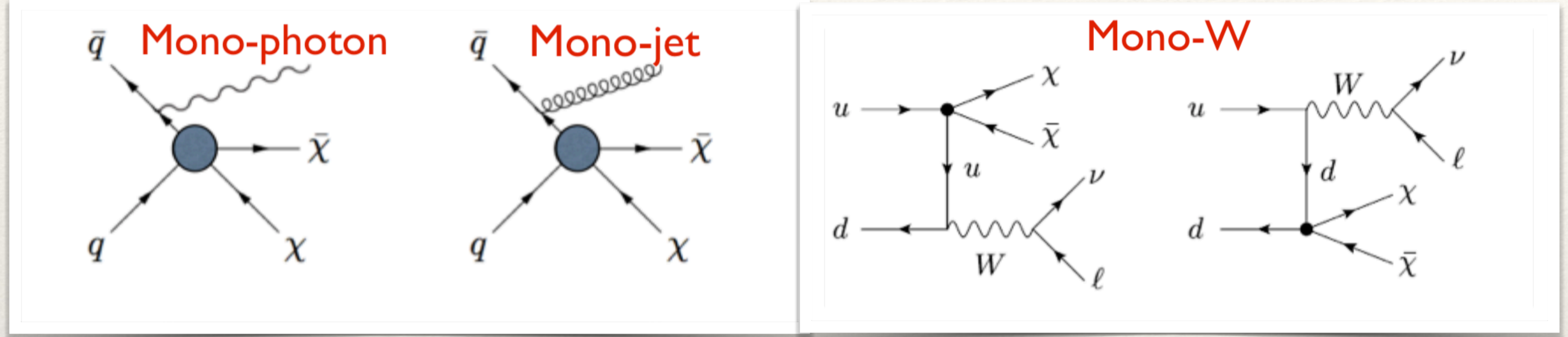






Dark Matter signals at Collider

Mono-X Signals



Signature: X+Missing Energy X=photon, jet, W, Z, H

What is Missing energy / Missing Transverse Momentum ?

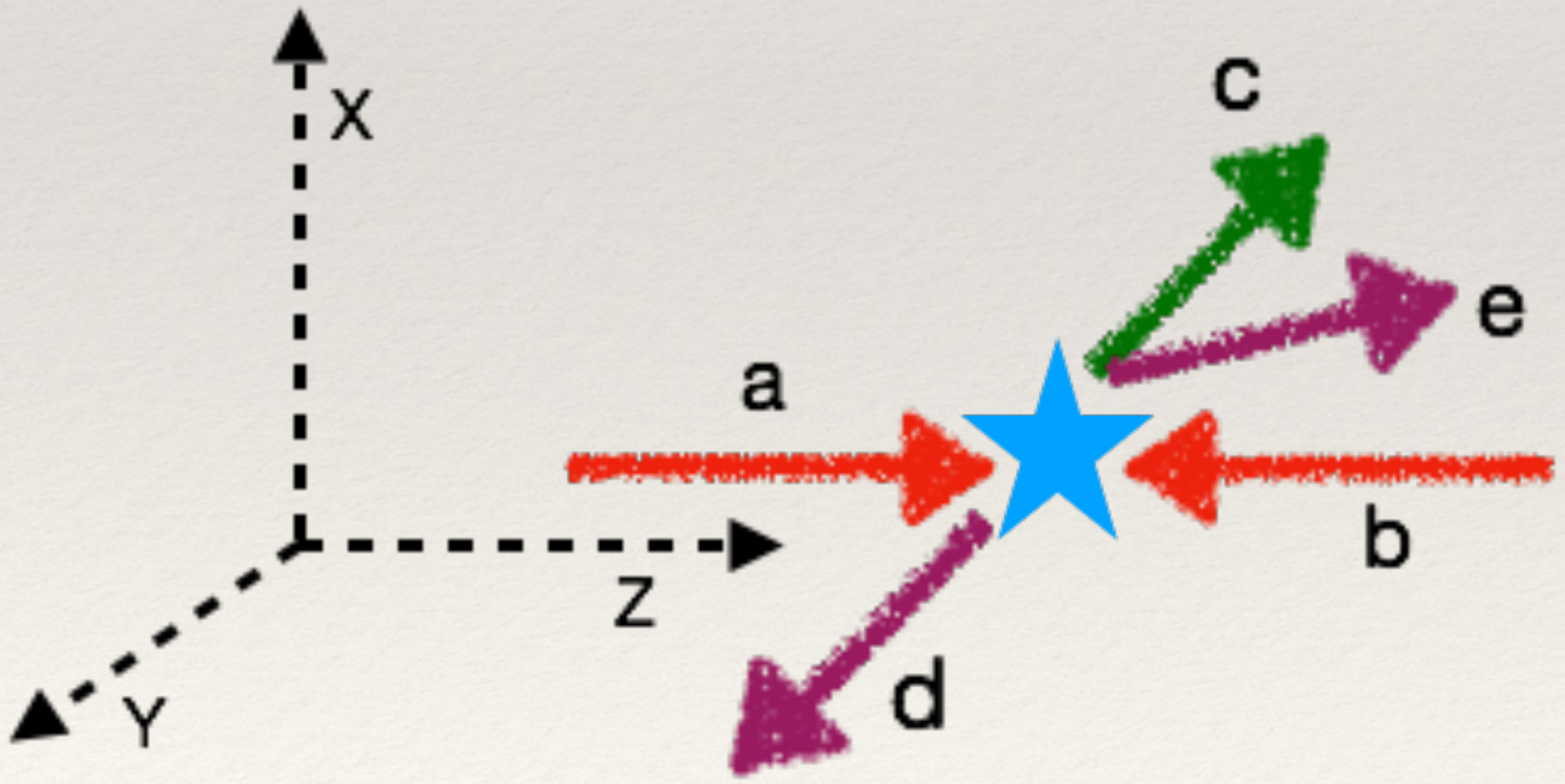
Consider: a+b -> c+d+e → Dark matter

$$\not{E}_T = \sqrt{(p_{x_d} + p_{x_e})^2 + (p_{y_d} + p_{y_e})^2}$$

$$= \sqrt{(p_{x_c})^2 + (p_{y_c})^2} = (P_T)_{vis}$$

Missing transverse momentum

But, you can't measure them !



- The collision occurs along Z direction.

$$p_{x_c} + p_{x_d} + p_{x_e} = 0 \rightarrow p_{x_d} + p_{x_e} = -p_{x_c}$$

$$p_{y_c} + p_{y_d} + p_{y_e} = 0 \rightarrow p_{y_d} + p_{y_e} = -p_{y_c}$$

$$\not{E}_T = (p_T)_{mis} = -(p_T)_{vis}, \ (p_T)_{vis} = \sqrt{(\sum_{\ell,j} p_x)^2 + (\sum_{\ell,j} p_y)^2}\,.$$

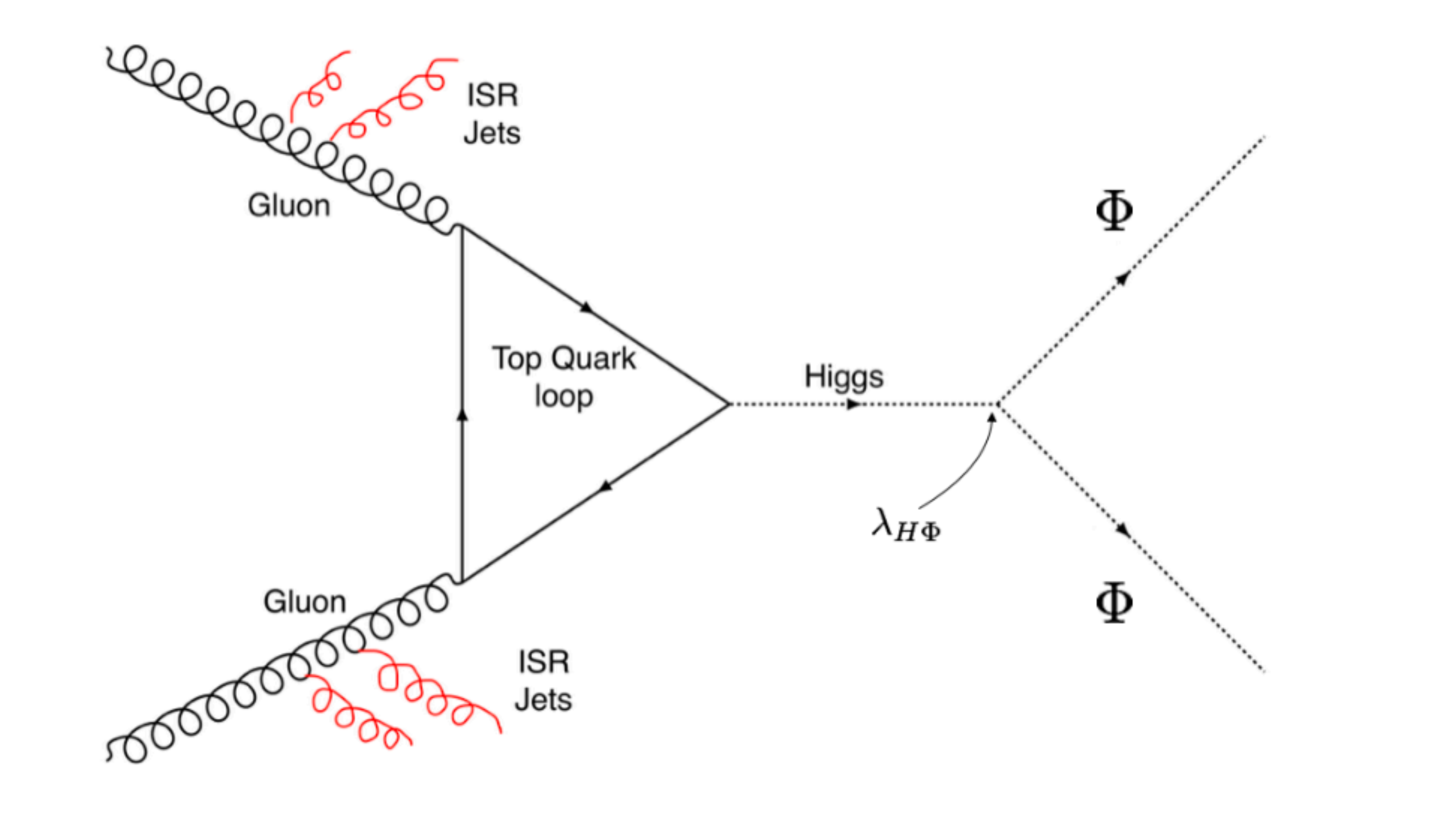
Hence, unless we have some visible particles to recoil from , missing energy makes no sense.

Richer Signal: n-leptons+ m-jets+Missing Energy

Collider phenomenology for the model

FIMP can't be produced at collider

Unsuppressed $\lambda_{\Phi H}$ coupling can cause Φ production



Decay of Φ to DM plus Z/γ is delayed beyond the detector scale

For $\frac{1}{\Lambda} = 10^{-14} - 10^{-13}\mathrm{GeV}^{-1}$ the average decay length of Φ is $L_\Phi = c\tau_\Phi \gtrsim 100$ km.

This will produce mono X plus missing transverse momentum (MET) signal

However, the decay can not be infinitely long. This is restricted by Big Bang Nucleosynthesis (BBN)

$\tau_\Phi \leq \tau_{\mathrm{BBN}} \sim 1$ sec

Leptophilic DM in EFT

Catch ’em all: Effective Leptophilic WIMPs at the e^+e^- Collider

Basabendu Barman,[a] **Subhaditya Bhattacharya,**[b] **Sudhakantha Girmohanta,**[c] **Sahabub Jahedi**[b]

[a] *Centro de Investigaciones, Universidad Antonio Nariño*
Carrera 3 este # 47A-15, Bogotá, Colombia

[b] *Department of Physics, Indian Institute of Technology Guwahati, Assam 781039, India*

[c] *C. N. Yang Institute for Theoretical Physics and Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794, USA*

E-mail: basabendu88barman@gmail.com, subhab@iitg.ac.in, sudhakantha.girmohanta@stonybrook.edu, sahabub@iitg.ac.in

Abstract: We consider higher-dimensional effective operators consisting of fermion dark matter (DM) connecting to Standard Model (SM) leptons upto dimension six. Considering all operators together and assuming the DM to undergo thermal freeze-out, we find out relic density allowed parameter space in terms of DM mass and New Physics (NP) scale with one loop direct search constraints from XENON1T experiment. Allowed parameter space of the model is probed at the proposed International Linear Collider (ILC) via monophoton signal. We also find out the precision with which the Wilson coefficients of the effective operators can be extracted via optimal observable technique (OOT) and comment on the separability of one model from another, assuming the presence of few selected operators for each case.

Keywords: Beyond the Standard Model, Dark Matter, e^+e^- Experiments

arXiv:2109.10936v2 [hep-ph] 1 Oct 2021

Operators for Fermion DM

Symmetry: $\underbrace{SU(3)_C \times SU(2)_L \times U(1)_Y}_{SM} \times Z_2$

Fields: SM particles that do not transform under Z_2 and DM fermion χ do not transform under $\mathcal{G}_{SM}$.

Without assuming a specific form of NP, all operators of same dimension should be considered on equal footing.

Dim 5:

$$\mathcal{O}^5_{D1} = \frac{g}{8\pi^2\Lambda}\left(\overline{\chi}\sigma_{\mu\nu}\chi\right)B^{\mu\nu}\,, \quad \mathcal{O}^5_{D2} = \frac{g}{8\pi^2\Lambda}\left(\overline{\chi}i\sigma_{\mu\nu}\gamma^5\chi\right)B^{\mu\nu}\,,$$
$$\mathcal{O}^5_{3} = \frac{c_{H_1}}{\Lambda}\left(\overline{\chi}\chi\right)\left(H^\dagger H\right)\,, \quad \mathcal{O}^5_{4} = \frac{c_{H_2}}{\Lambda}\left(\overline{\chi}i\gamma^5\chi\right)\left(H^\dagger H\right).$$

- Leptophilic: $c_{\ell_i} = 1, c_{H_i} = c_{q_i} = 0$;

Dim 6:

$$\mathcal{O}^6_{DQ} = \frac{c_{q1}}{\Lambda^2}\left(\overline{\chi}\gamma_\mu\chi\right)\left(\bar{\mathfrak{q}}\gamma^\mu\mathfrak{q}\right)\,, \qquad \mathcal{O}^6_{DL} = \frac{c_{\ell_1}}{\Lambda^2}\left(\overline{\chi}\gamma_\mu\chi\right)\left(\bar{\ell}\gamma^\mu\ell\right)\,,$$
$$\mathcal{O}^6_{Q} = \frac{c_{q2}}{\Lambda^2}\left(\overline{\chi}\gamma_\mu\gamma^5\chi\right)\left(\bar{\mathfrak{q}}\gamma^\mu\mathfrak{q}\right)\,, \qquad \mathcal{O}^6_{L} = \frac{c_{\ell_2}}{\Lambda^2}\left(\overline{\chi}\gamma_\mu\gamma^5\chi\right)\left(\bar{\ell}\gamma^\mu\ell\right)\,,$$
$$\mathcal{O}^6_{Q1} = \frac{c_{q3}}{\Lambda^2}\left(\overline{\chi}\gamma_\mu\gamma^5\chi\right)\left(\bar{\mathfrak{q}}\gamma^\mu\gamma_5\mathfrak{q}\right)\,, \qquad \mathcal{O}^6_{L1} = \frac{c_{\ell_3}}{\Lambda^2}\left(\overline{\chi}\gamma_\mu\gamma^5\chi\right)\left(\bar{\ell}\gamma^\mu\gamma_5\ell\right)$$
$$\mathcal{O}^6_{DQ1} = \frac{c_{q4}}{\Lambda^2}\left(\overline{\chi}\gamma_\mu\chi\right)\left(\bar{\mathfrak{q}}\gamma^\mu\gamma_5\mathfrak{q}\right)\,, \qquad \mathcal{O}^6_{DL1} = \frac{c_{\ell_4}}{\Lambda^2}\left(\overline{\chi}\gamma_\mu\chi\right)\left(\bar{\ell}\gamma^\mu\gamma_5\ell\right)\,,$$
$$\mathcal{O}^6_{DH} = \frac{c_{H_3}}{\Lambda^2}\left(\overline{\chi}\gamma^\mu\chi\right)\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\,, \quad \mathcal{O}^6_{H} = \frac{c_{H_4}}{\Lambda^2}\left(\overline{\chi}\gamma^\mu\gamma^5\chi\right)\left(H^\dagger i\overleftrightarrow{D}_\mu H\right).$$

Majorana DM

Relic Density

Consider the DM to be WIMP

Boltzmann Equation for WIMP (DM is in thermal bath):

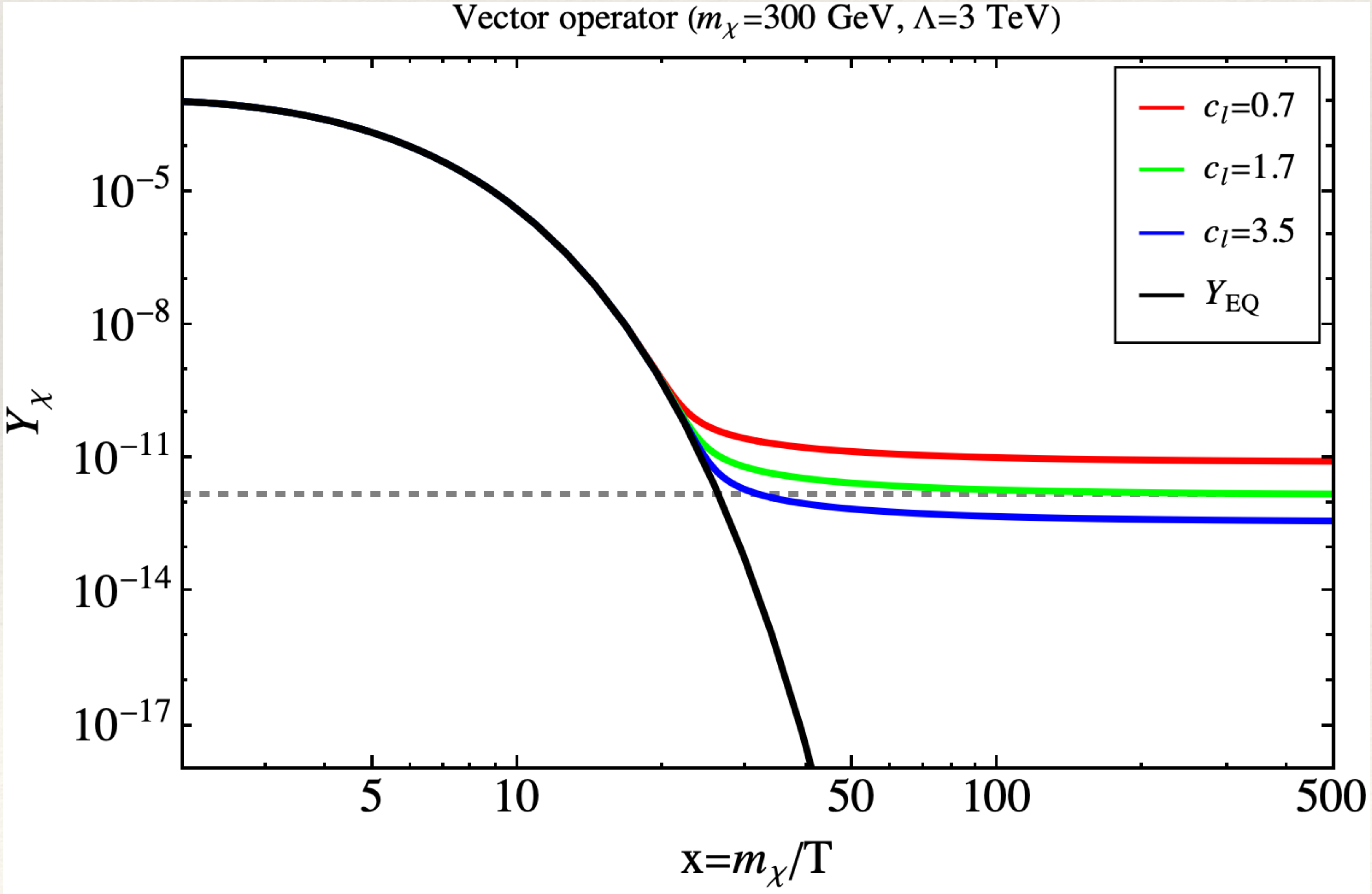
$$\frac{dY_\chi}{dx} = -1.32\,\sqrt{g_*}\;M_{\mathtt{pl}}\;\frac{m_\chi}{x^2}\;\langle\sigma v\rangle_{2\mathrm{DM}\to2\mathrm{SM}}\;\left({Y_\chi}^2 - Y_{\mathtt{eq}}^2\right)\,,$$

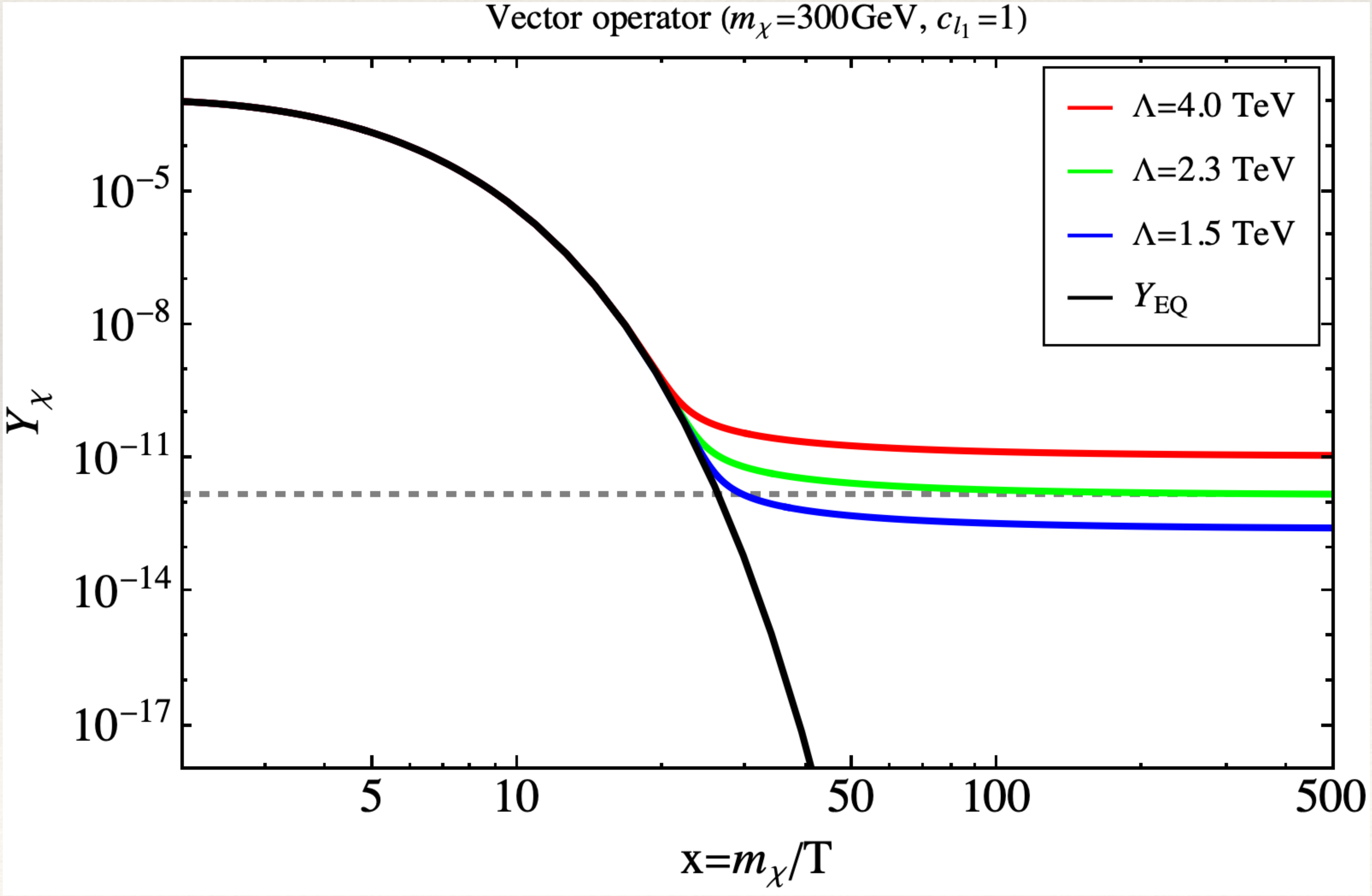
For purely vector like mediator:

$$\langle\sigma v\rangle_{2\mathrm{DM}\to2\mathrm{SM}} = \frac{m_\chi^2}{2\pi\Lambda^4}\sum_\ell (c_{\ell_1})^2\left\{2+\frac{m_\ell^2}{m_\chi^2}+\left[\frac{8m_\chi^4-4m_\ell^2m_\chi^2+5m_\ell^4}{24m_\chi^2(m_\chi^2-m_\ell^2)}\right]v^2\right\}\sqrt{1-\frac{m_\ell^2}{m_\chi^2}}\,,$$

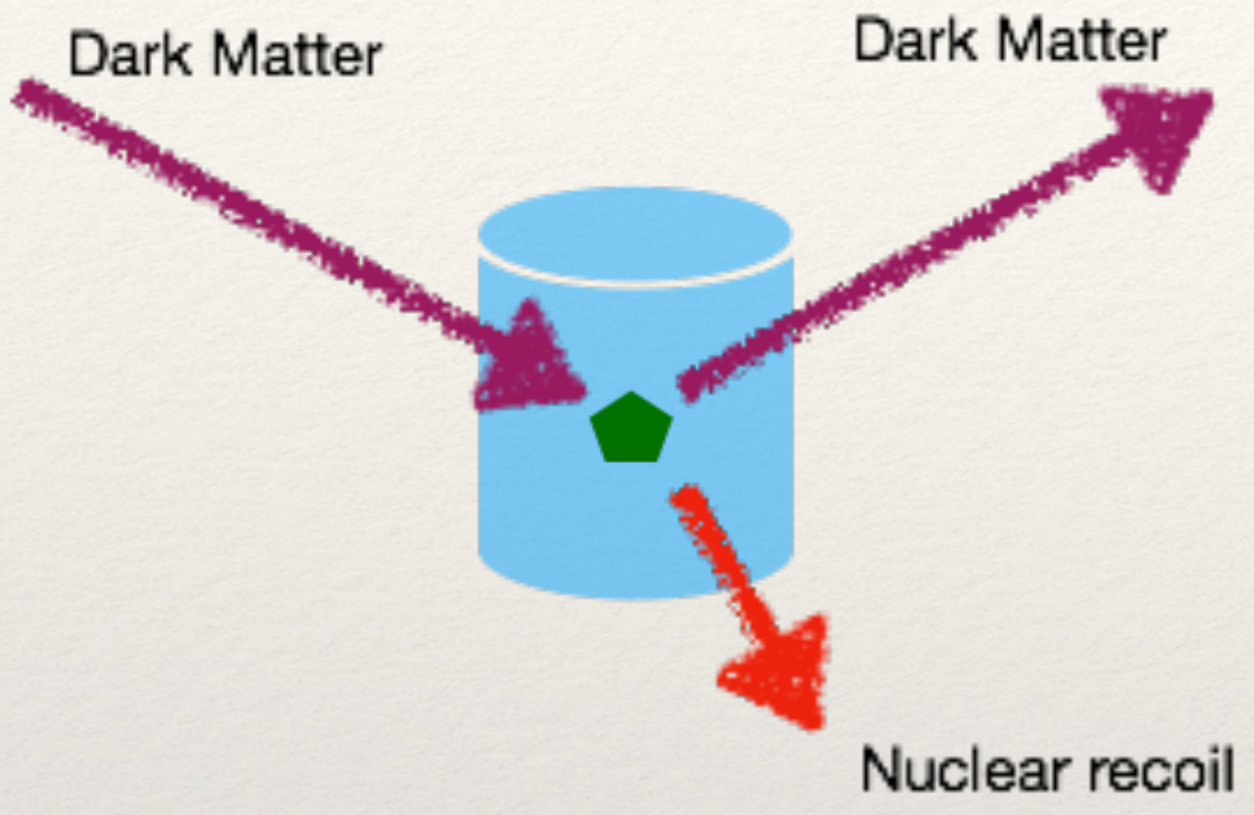
For all operators:

$$\langle\sigma v\rangle_{\mathrm{tot}} \sim \left|\sum_{\mathcal{O}_i}\mathcal{M}_{\mathcal{O}_i}\right|^2,$$

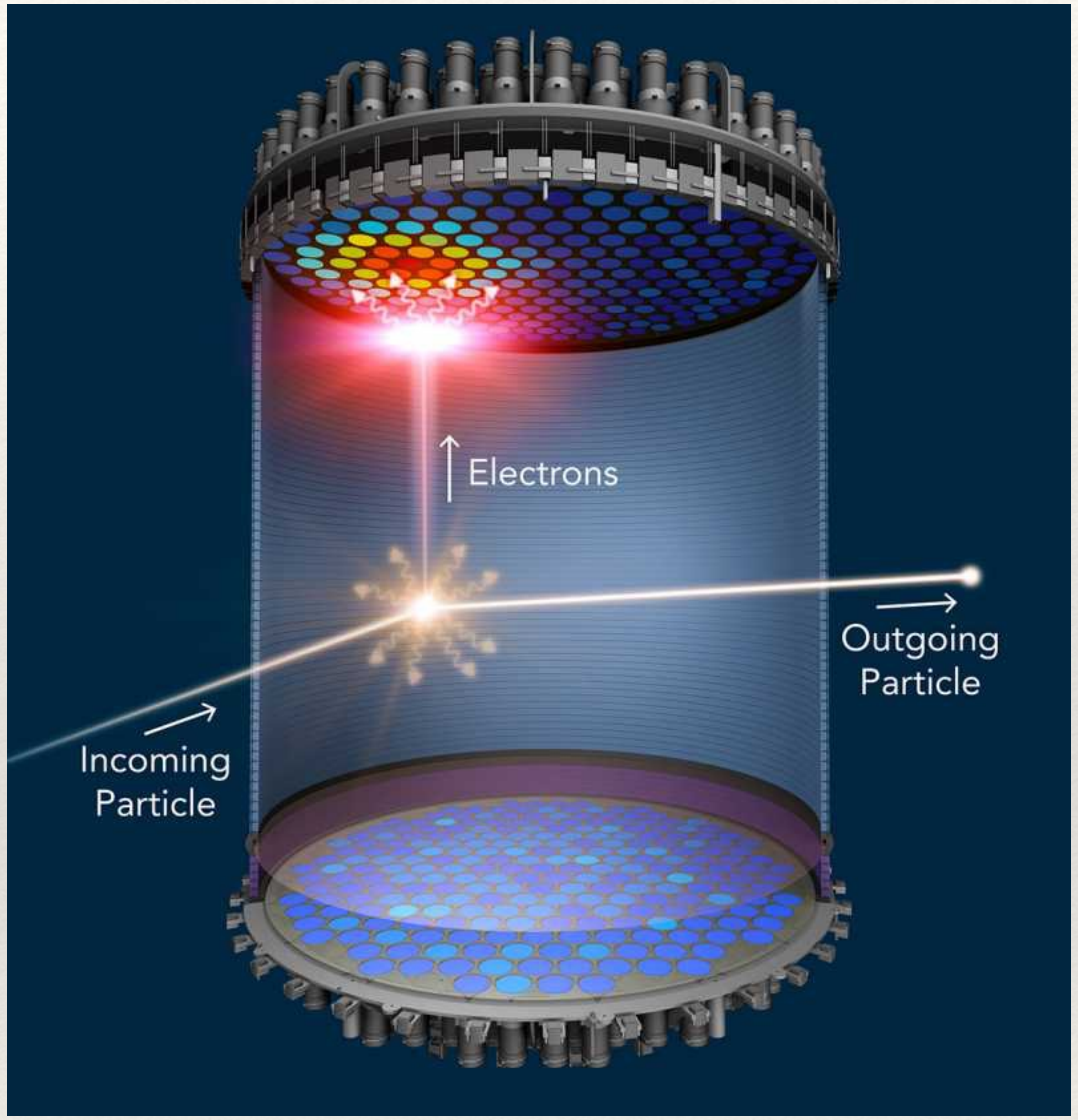




Dark matter direct search



- Our galaxy is immersed in a Dark matter halo.
- Dark Matter in the halo has a velocity distribution.
- Elastic scattering of Dark matter occurs with detector nuclei.
- Nuclear recoil should be observed if such events detected.



Liquid Xenon detector

Recoil rate of nuclei

$$\frac{dR}{dE_R}(E,t) = \frac{\rho_0}{m_\chi \cdot m_A} \cdot \int v \cdot f(\mathbf{v},t) \boxed{\frac{d\sigma}{dE_R}(E,v)} \mathrm{d}^3 v$$

$$E_R = \frac{q^2}{2m_N} = \frac{\mu^2 \nu^2}{m_N}(1 - cos\theta)$$

$$f(v) = \frac{1}{\sqrt{2\pi\sigma}} \exp(-\frac{|v|^2}{2\sigma^2})$$

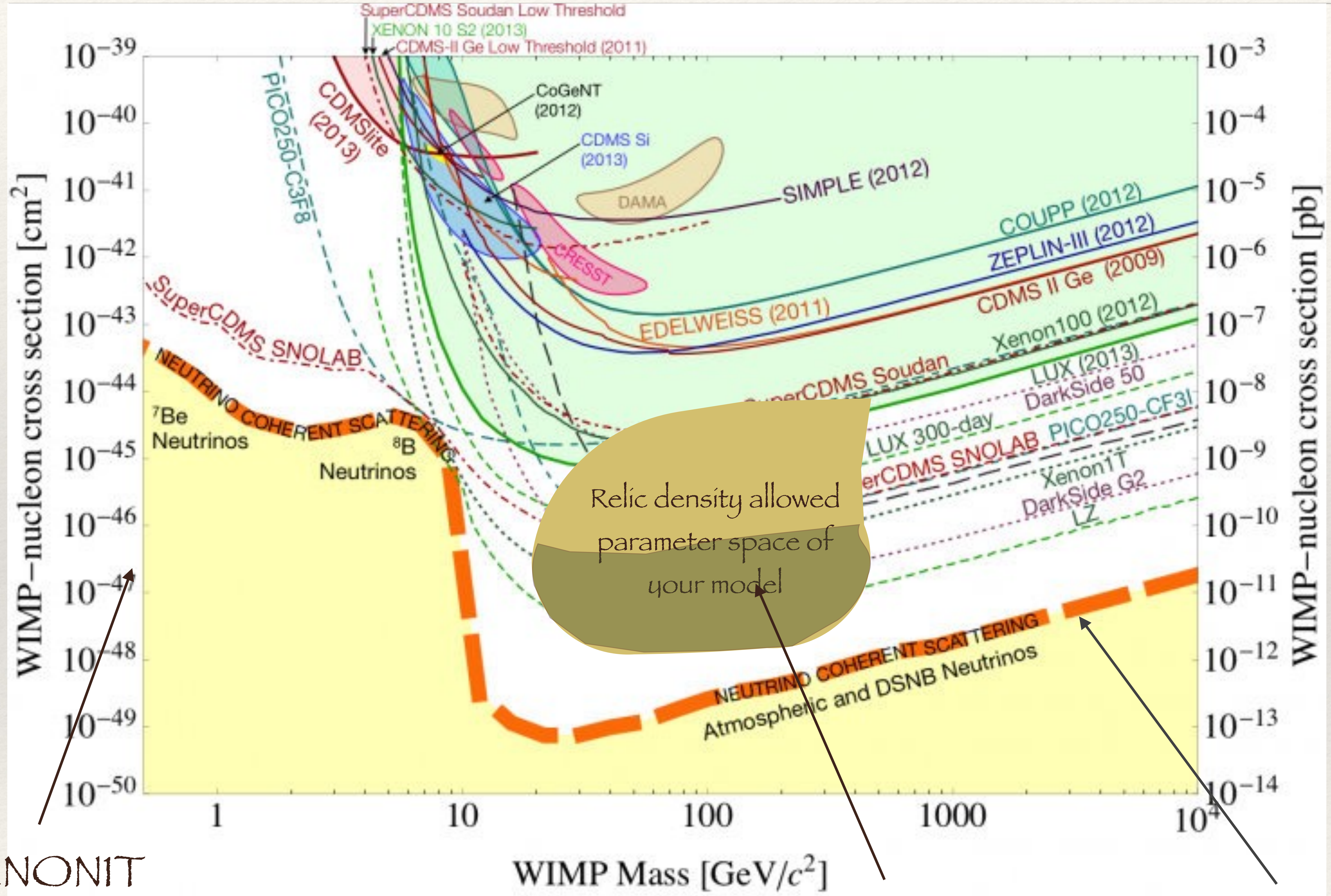
- q = momentum transfer
- m_N = target nucleus mass
- μ = reduced mass
- ν = mean WIMP-velocity on respect to the target
- θ = scattering angle in the center of mass

$$\frac{d\sigma}{dE_R} = \frac{m_A}{2m_\chi^2 v^2}\left[\sigma_{SI} F_{SI}^2(E_R) + \sigma_{SD} F_{SD}^2(E_R)\right]$$

SI: Spin Independent, SD: Spin dependent

First proposed by Goodman and Witten in 1985

Direct search results/bounds



XENON1T

Allowed !

Neutrino floor

This is spin independent bound!

Spin dependent bound is much lighter !

Recently, June 2020

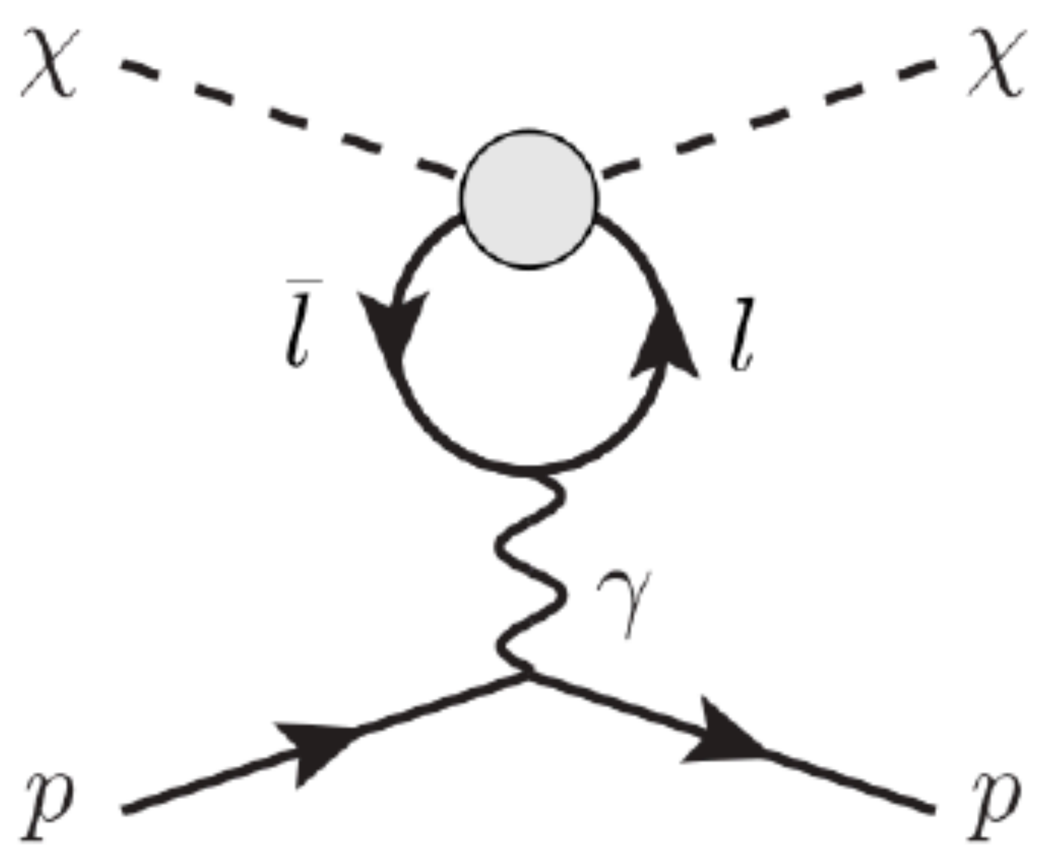
Some excess observed in XENON 1T at very low energy of the order of KeV

Strong bound on dark matter models from null direct search !

Leptophilic Dark Matter: Direct Search

Direct search is a loop induced process

$$\mathcal{L} = \frac{1}{\Lambda^2}\left(\overline{\chi}\Gamma_\chi \chi\right)\left(\overline{\ell}\Gamma_\ell \ell\right)$$



$$\mathcal{M} = \left(\frac{\alpha_{\rm em}}{3\pi}\frac{c_\ell}{\Lambda^2}\log\left[\frac{m_\ell^2}{\mu^2}\right] Z F(q)\right)\left(\overline{u_\chi}\Gamma^\mu_\chi u_\chi\right)\left(\overline{u_N}\gamma_\mu u_N\right).$$

$$\sigma_{\chi N\to\chi N} \approx \frac{\mu_N^2 c_\ell^2}{9\pi}\left(\frac{\alpha_{\rm em} Z}{\pi\Lambda^2}\right)^2\left[\log\left(\frac{m_\ell^2}{\mu^2}\right)\right]^2.$$

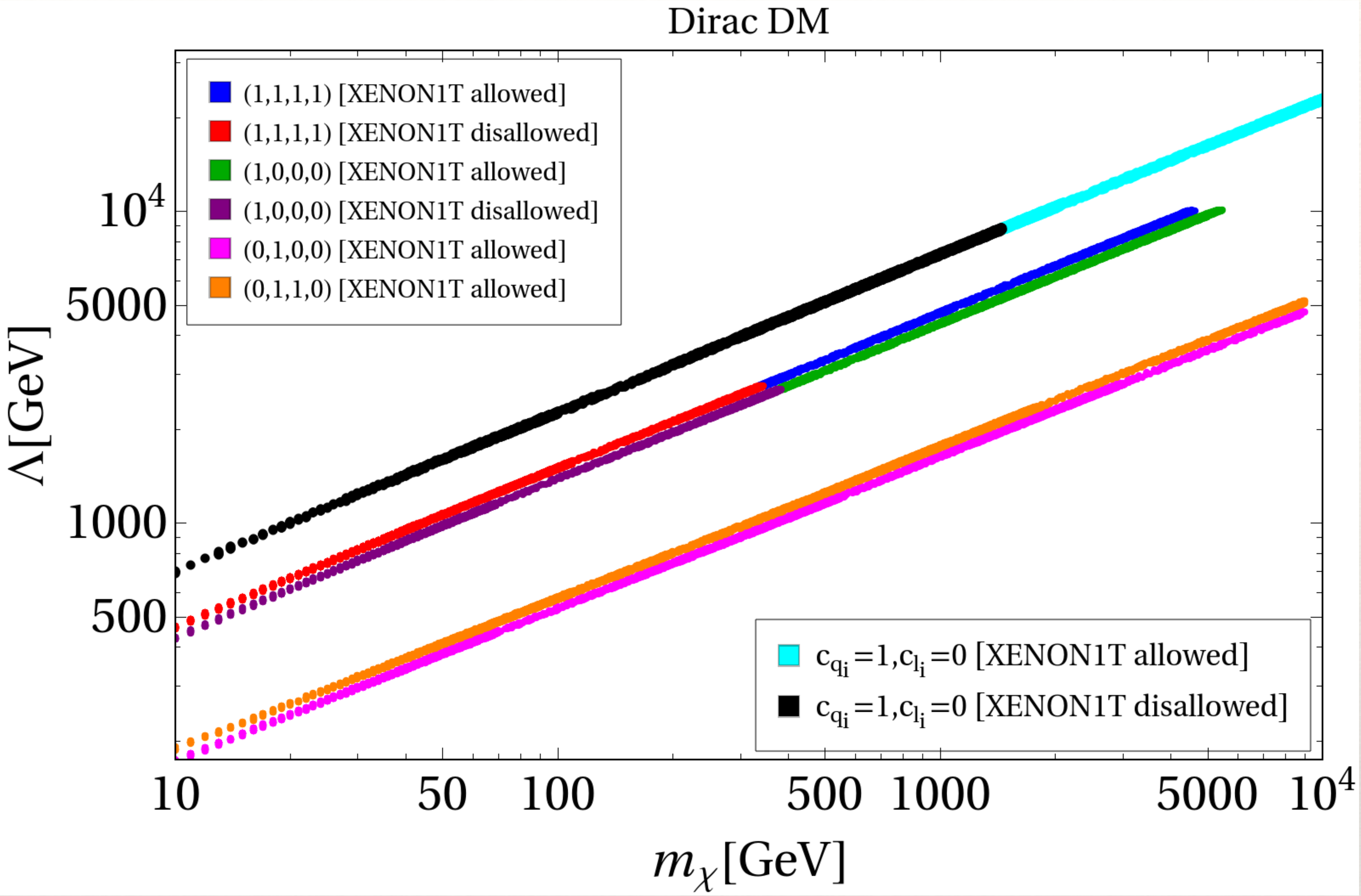
Renormalisation group running of Wilson coefficients to direct search scale:

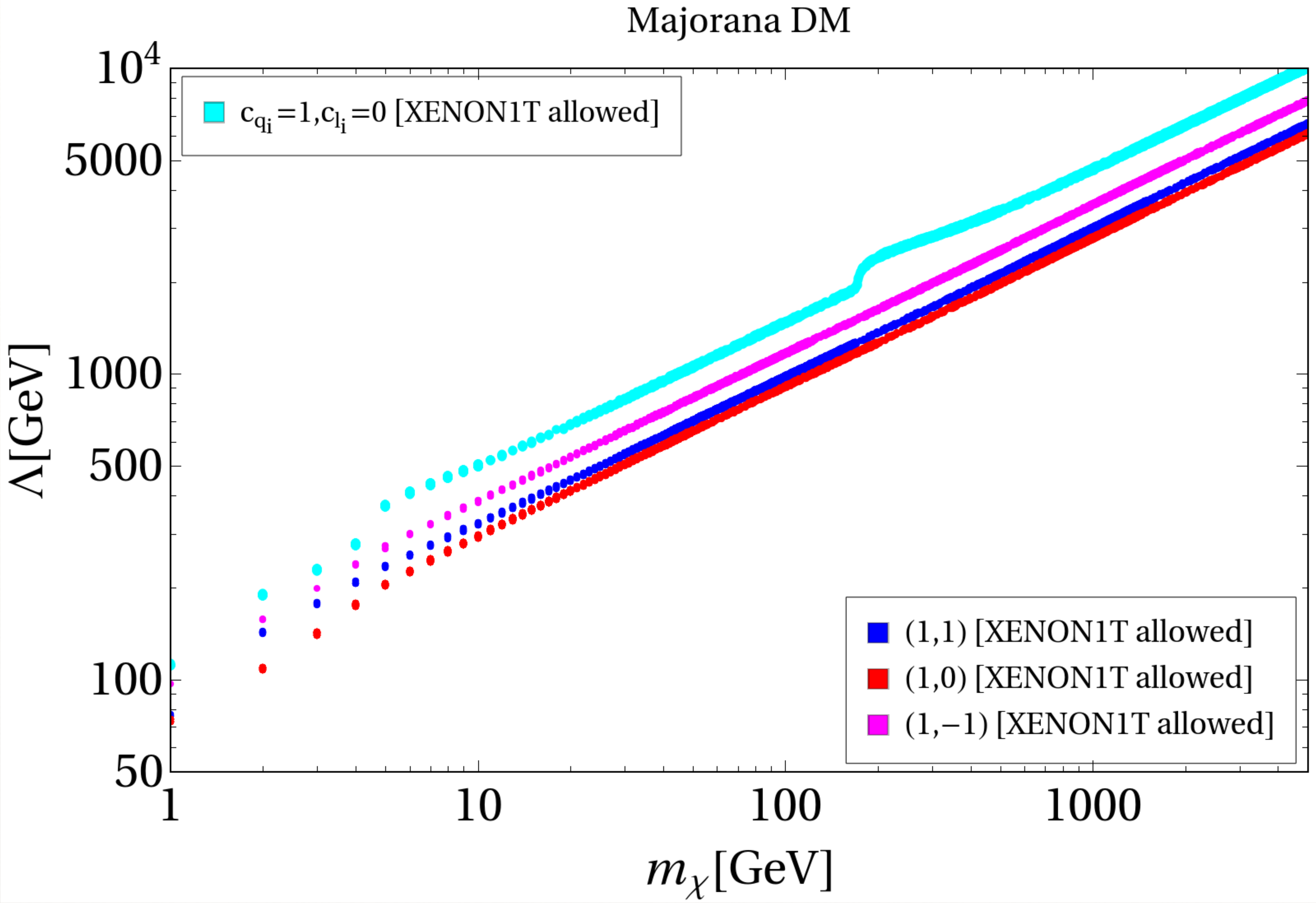
$$c_{q1}^{(q_j)}(\Lambda_f) \approx Q_j \frac{\alpha_{\rm em}}{3\pi}\frac{c_{\ell_1}}{\Lambda^2}\ln\left(\frac{\Lambda^2}{\Lambda_f^2}\right)$$

The loop factor brings the direct search x-sec down !

Most of the hadrophilic operators are strongly constrained by the present Direct search data

Relic and Direct Search Allowed region





Hadrophilic Dirac fermion DM is ruled out upto DM mass 6.4 TeV!

Leptophilic DM: (1,1,1,1) case is ruled out upto $(m_\chi, \Lambda) \sim (345,\ 2780)$ GeV.

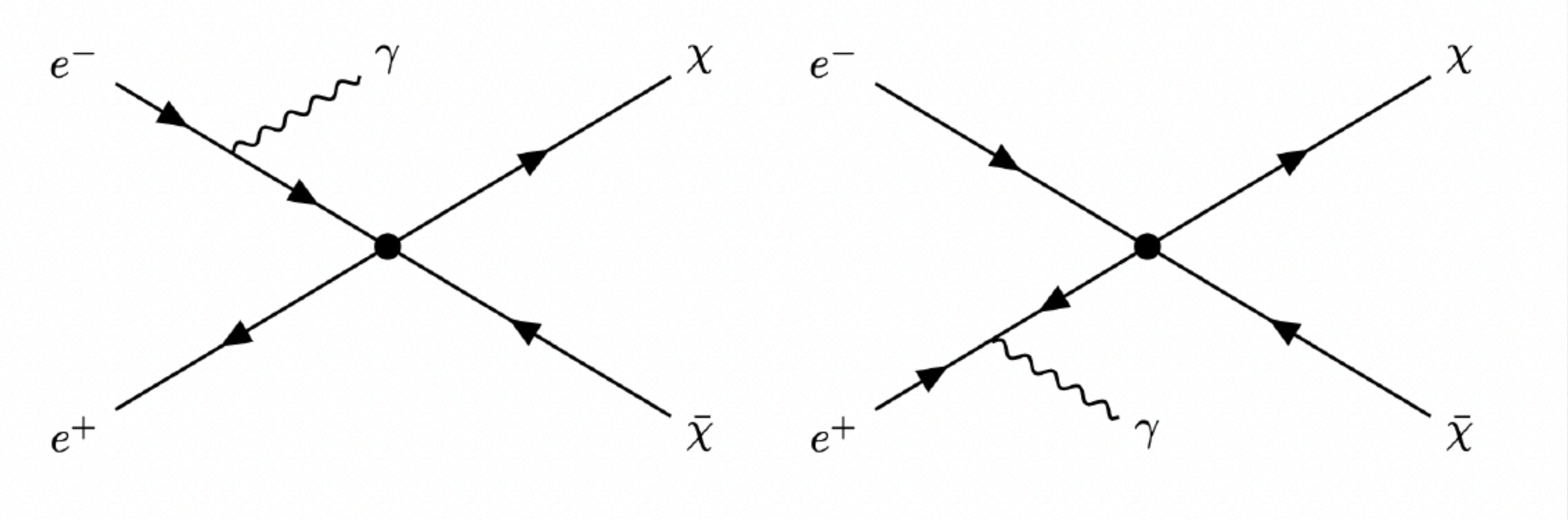
For Majorana DM, SI direct search cross-section identically vanish and hence much lower DM mass is allowed.

Leptophilic DM can be probed in lepton collider

Collider search at electron positron collider

DM scenario	Benchmark Points	Model	m_χ (GeV)	Λ (GeV)
Majorana	BP1	$(1,1)$	70	816
	BP2	$(1,1)$	112	1025
	BP3	$(1,-1)$	70	990
Dirac	BP4	$(1,1,1,1)$	350	2800
	BP5	$(1,1,1,1)$	400	3000
	BP6	$(1,1,-1,-1)$	350	3300

Benchmark points



Mono-photon signal out of ISR

Differential production cross-section

$$\frac{d\sigma(P_{e^-},P_{e^+})}{d\Omega} = \frac{(1+P_{e^-})(1-P_{e^+})}{4}\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{RL}} + \frac{(1-P_{e^-})(1+P_{e^+})}{4}\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{LR}}$$

$P_{e^\pm} = \frac{n_R - n_L}{n_R + n_L}$ Longitudinal beam polarisation

$$\left(\frac{d\sigma}{d\Omega}\right)^{\mathtt{dirac}}_{\mathrm{RL}} = \frac{s\sqrt{1-\beta_\chi^2}}{32\pi^2\Lambda^4}\Big[\Big\{(1+\beta_\chi^2)(c_{\ell_1}+c_{\ell_4})^2 + (1-\beta_\chi^2)(c_{\ell_2}+c_{\ell_3})^2\Big\} + 4\sqrt{1-\beta_\chi^2}$$
$$\Big\{(c_{\ell_1}+c_{\ell_4})(c_{\ell_2}+c_{\ell_3})\Big\}\cos\theta + (1-\beta_\chi^2)\Big\{(c_{\ell_1}+c_{\ell_4})^2 + (c_{\ell_2}+c_{\ell_3})^2\Big\}\cos^2\theta\Big].$$

$$\left(\frac{d\sigma}{d\Omega}\right)^{\mathtt{dirac}}_{\mathrm{LR}} = \frac{s\sqrt{1-\beta_\chi^2}}{32\pi^2\Lambda^4}\Big[\Big\{(1+\beta_\chi^2)(c_{\ell_1}-c_{\ell_4})^2 + (1-\beta_\chi^2)(c_{\ell_2}-c_{\ell_3})^2\Big\} + 4\sqrt{1-\beta_\chi^2}$$
$$\Big\{(c_{\ell_4}-c_{\ell_1})(c_{\ell_2}-c_{\ell_3})\Big\}\cos\theta + (1-\beta_\chi^2)\Big\{(c_{\ell_1}-c_{\ell_4})^2 + (c_{\ell_2}-c_{\ell_3})^2\Big\}\cos^2\theta\Big].$$

Total production cross-section

$$\sigma_0^{\mathtt{dirac}} = \frac{s}{12\pi\Lambda^4}\sqrt{1-\beta_\chi^2}\Big[\Big(c_{\ell_1}^2+c_{\ell_4}^2\Big)\Big(1+\frac{\beta_\chi^2}{2}\Big) + \Big(c_{\ell_2}^2+c_{\ell_3}^2\Big)\Big(1-\beta_\chi^2\Big)\Big],$$
$$= \frac{s}{3\pi\Lambda^4}\left(1-\frac{m_\chi^2}{s}\right)\sqrt{1-\frac{4m_\chi^2}{s}}\,;\ \text{where } c_{\ell_i} = 1\ (i = 1\text{-}4)\,.$$

$$\beta_\chi = \frac{2m_\chi}{\sqrt{s}}$$

Effect of beam polarisation

In the limit of $\{c_{\ell_i}\} = 1$

$$\mathcal{L}_{\rm eff} = \sum_i \mathcal{O}_i \sim \begin{cases} (\bar{\chi}\gamma^\mu\gamma^5\chi)\,\bar{e}\gamma_\mu\left(1+\gamma^5\right)e & \text{Majorana DM} \\ (\bar{\chi}\gamma^\mu\gamma^5\chi + \bar{\chi}\gamma^\mu\chi)\,\bar{e}\gamma_\mu\left(1+\gamma^5\right)e & \text{Dirac DM} \end{cases}$$

Signal cross-section is maximum for fully right handed electron and left handed positron

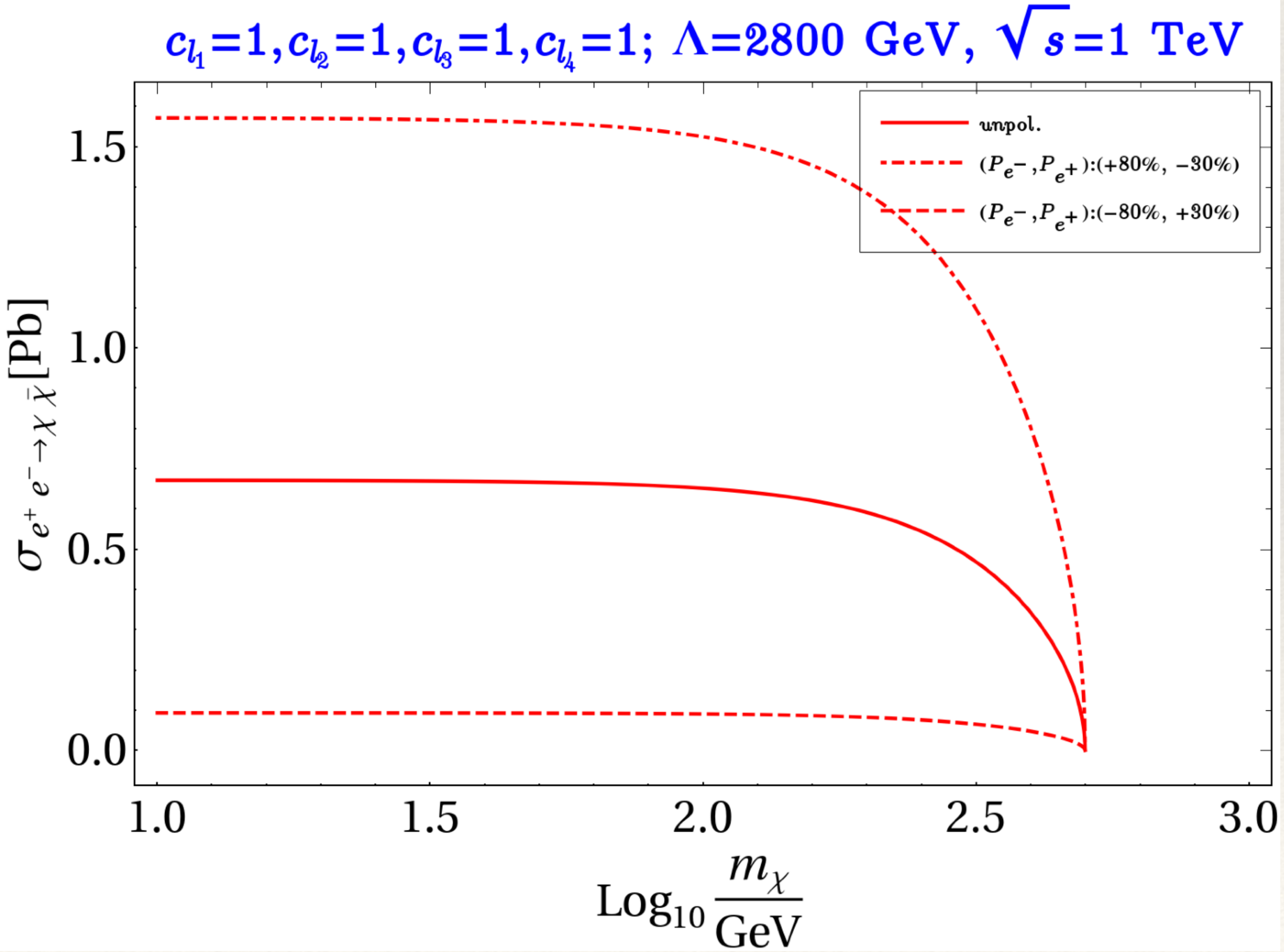
$c_{\ell_i} \equiv \{1, 1, -1, -1\}$ for Dirac DM and $c_{\ell_i} \equiv \{1, -1\}$ for Majorana DM

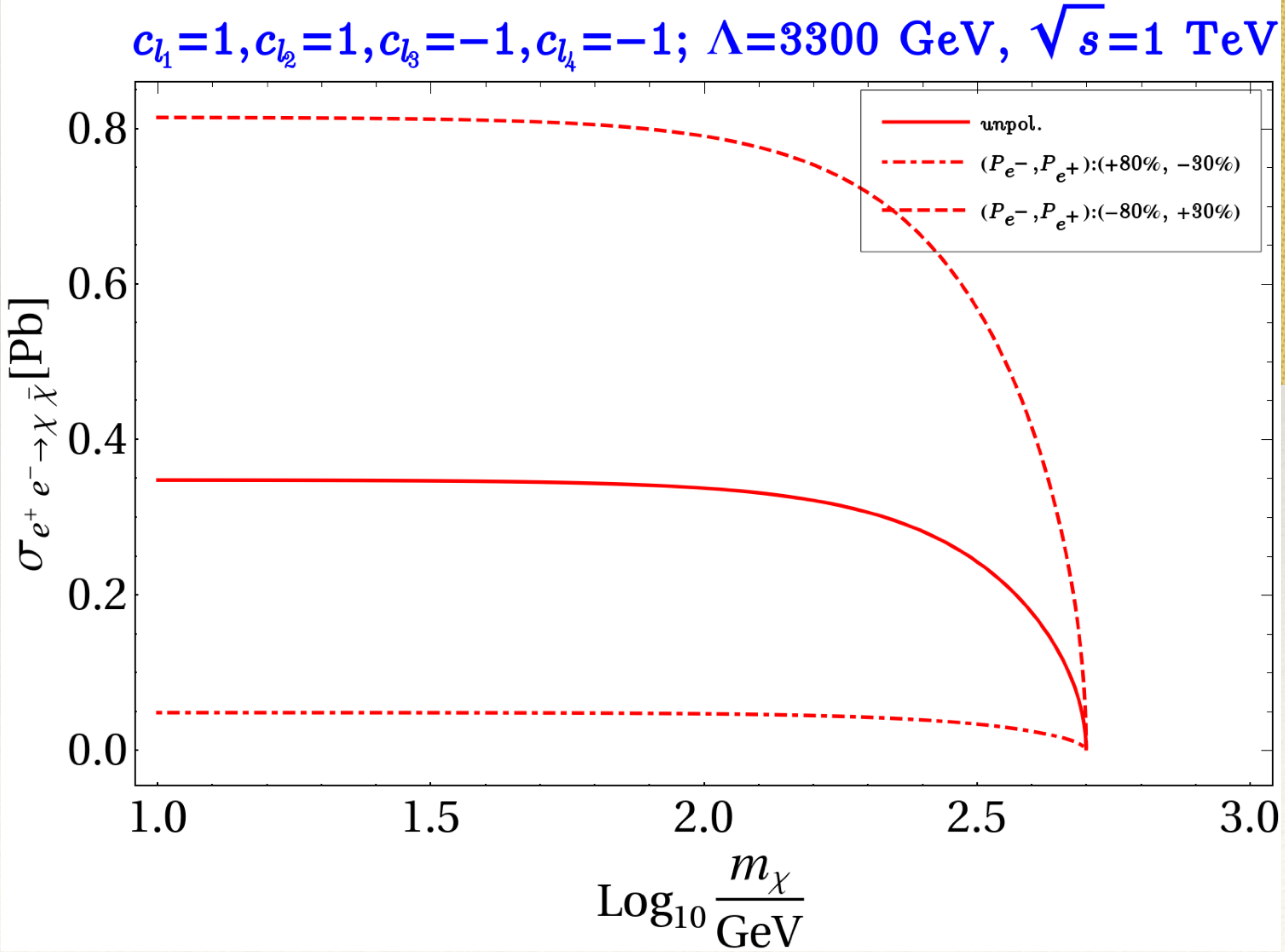
$$\mathcal{L}_{\rm eff} = \begin{cases} (\bar{\chi}\gamma^\mu\gamma^5\chi + \bar{\chi}\gamma^\mu\chi)\;\bar{e}\gamma_\mu\left(1-\gamma^5\right)e & \text{Dirac DM} \\ (\bar{\chi}\gamma^\mu\gamma^5\chi)\;\bar{e}\gamma_\mu\left(1-\gamma^5\right)e & \text{Majorana DM} \end{cases}$$

Left-polarized electron and right-polarized positron maximizes the signal cross-section

Production cross-section at Benchmark points

Beam polarization		Production cross-section ($\sigma_{e^+e^-\to\chi\bar{\chi}}$) (pb)					
		Majorana DM ($\sqrt{s}$ = 250 GeV)			Dirac DM ($\sqrt{s}$ = 1 TeV)		
P_{e^-}	P_{e^+}	BP1	BP2	BP3	BP4	BP5	BP6
-0.8	+0.3	0.40	0.02	0.78	0.05	0.03	0.42
+0.8	-0.3	6.73	0.34	0.05	0.81	0.47	0.03
0.0	0.0	2.87	0.14	0.33	0.35	0.20	0.18





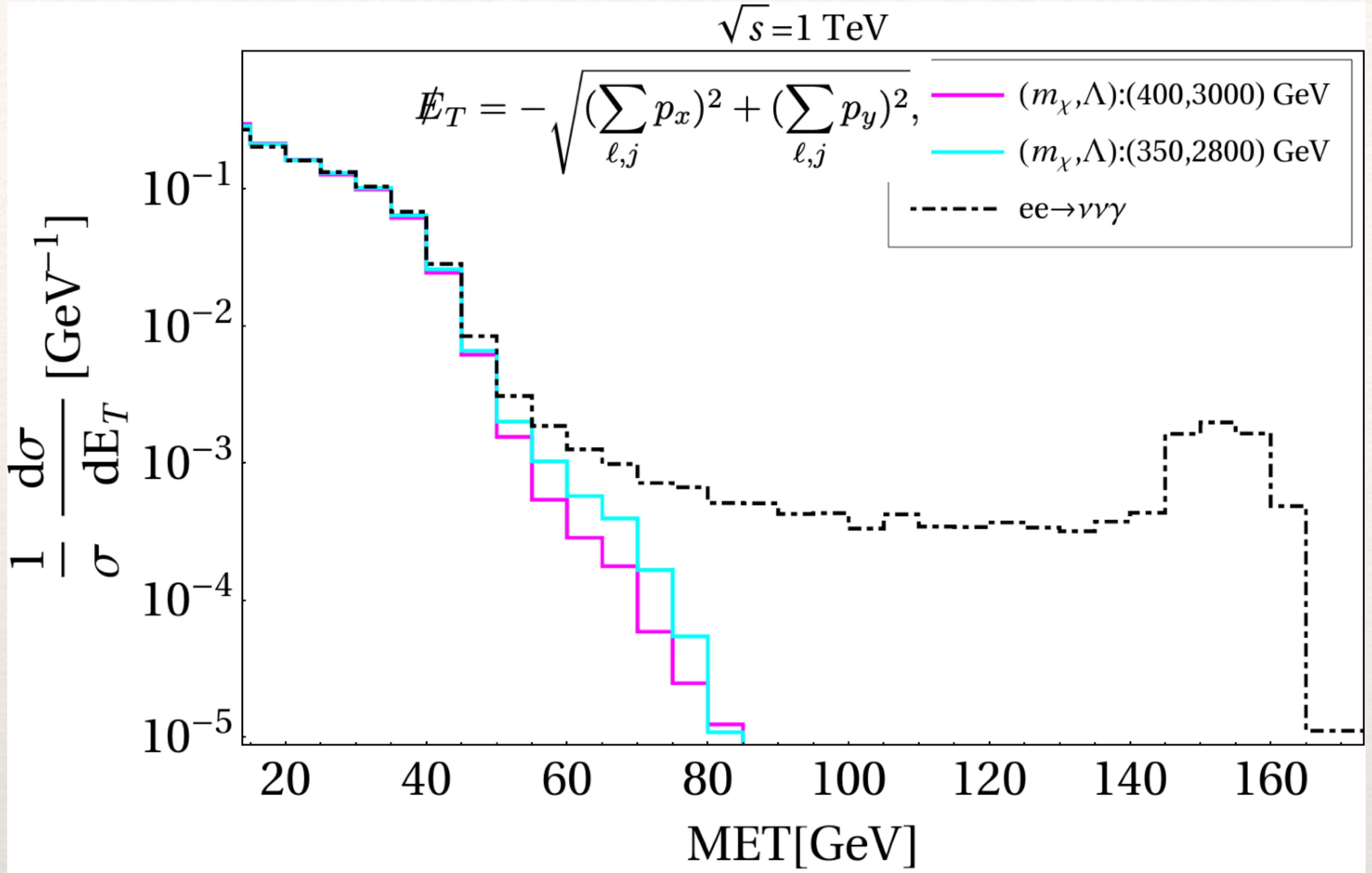
Three important points

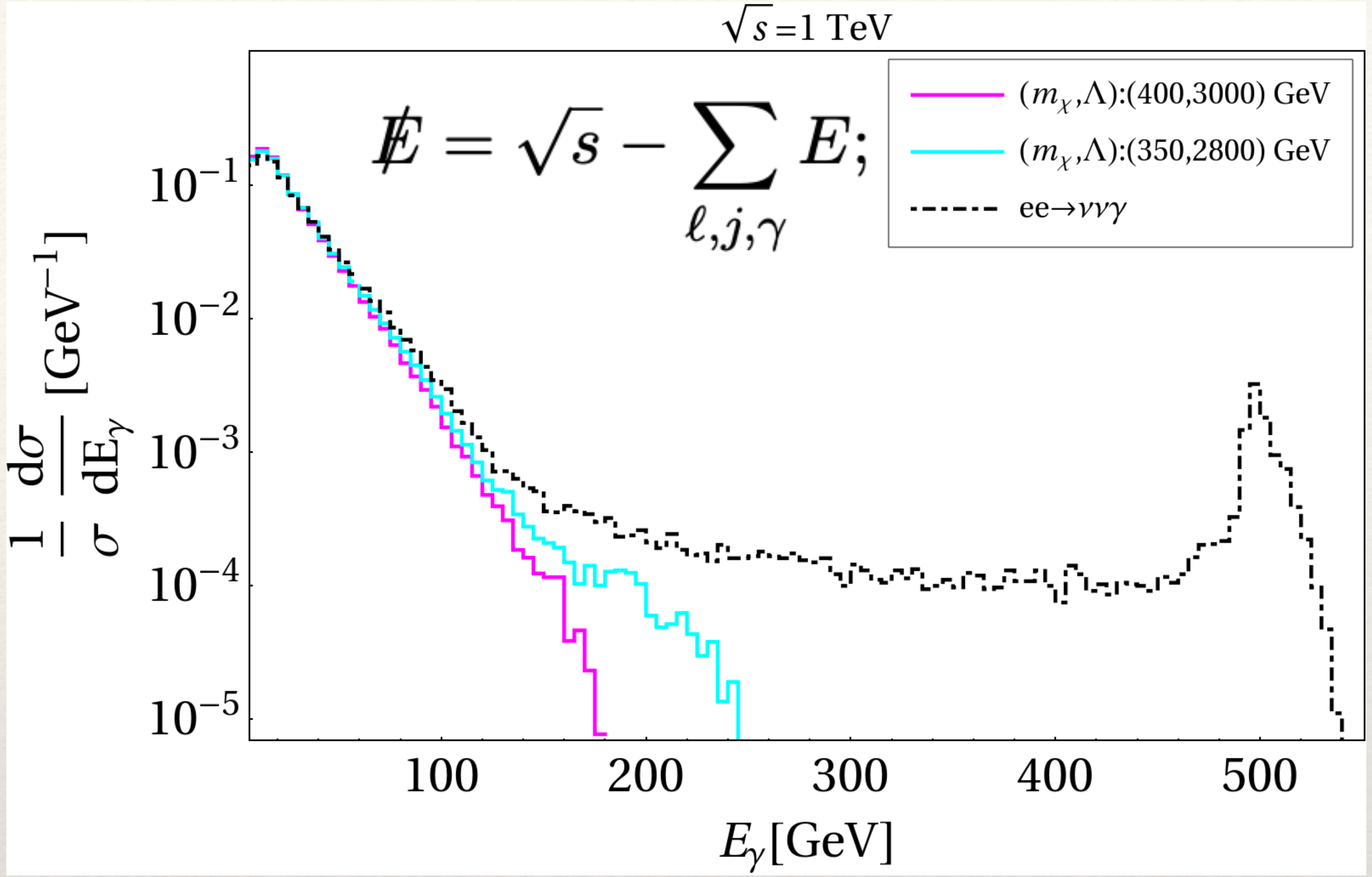
It is much easier to ensure EFT at lepton collider by ensuring $\Lambda > \sqrt{s}$

Contrary to LHC where $\sqrt{\hat{s}}$ is not known and DM invariant mass is impossible to calculate.

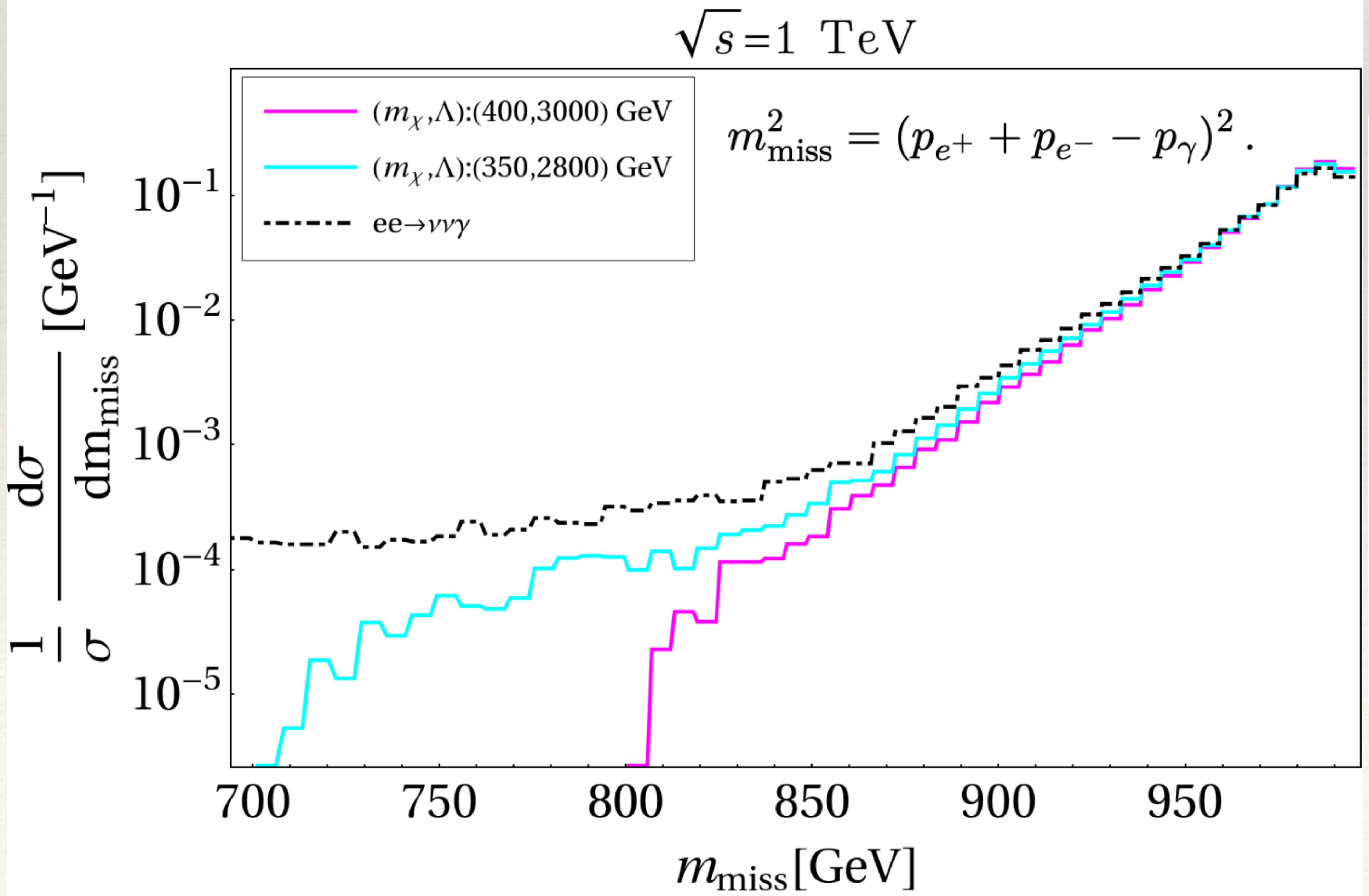
Beam polarisation can distinguish models with different signs of Wilson coefficient

Event simulation for monophoton signal





Missing Energy and Missing Mass can only be constructed at ILC



Cuts	BP4 (fb)	BP5 (fb)	$\nu\bar{\nu}\gamma$ (fb)	$\epsilon^{\rm BP4}$	$\epsilon^{\rm BP5}$	$\sigma^{\rm BP4} = \frac{S}{\sqrt{S+B}}$ ($\mathcal{L} = 100$ fb^{-1})	$\sigma^{\rm BP5} = \frac{S}{\sqrt{S+B}}$ ($\mathcal{L} = 100$ fb^{-1})
$\mathcal{C}_1$	28.20	14.17	281.36	0.035	0.030	16.02	8.24
$\mathcal{C}_2$	28.10	14.16	275.05	0.035	0.030	16.13	8.32
$\mathcal{C}_3$	27.81	14.02	268.15	0.034	0.030	16.16	8.33
$\mathcal{C}_4$	27.81	14.02	265.42	0.034	0.030	16.24	8.40

$\mathcal{C}_1$: zero lepton, zero jet veto, $\mathcal{C}_2$: Missing energy cut,
$\mathcal{C}_3$: MET cut, $\mathcal{C}_4$: Missing mass cut

Summary

- DM EFT is applicable to the cases where no explicit New Physics is assumed excepting for the DM field and the symmetry that it transforms under. DM-SM EFT interactions can also appear in UV complete models absent a renormalisable interaction term.

- Most of DM EFT analysis is done in operator specific way. But, in EFT framework, as we are not supposed to know the high scale physics, so we should instead consider all operators of same dimension given the nature of DM.

- For freeze-in of DM via EFT interaction, UV freeze-in is limited to very high reheat temperature, when it drops close to DM mass, it may also show IR freeze pattern.

- DM EFT Direct search: The Wilson coefficients (c) are assumed at the EWSB scale. However direct search interaction occurs at much lower energy, so it is often suggested to use RGE to run them down appropriately.

- Validity of EFT strictly depends on the assumption that the scale of the experiment must be below the NP scale. At LHC, the parton level CM energy is unknown, it can only be known from the final state DM invariant mass square, but they remain undetected, so doing DM EFT at LHC is difficult. At ILC this is more reliable.

- Beam polarisation, Missing energy and Missing mass help elucidating DM signal at electron positron machine.

The search for BSM is on….

Thank you

Additional slides

Scalar DM EFT operators

$\mathcal{G}_{SM} : SU(3)_C \times U(1)_{EM}$

$$\mathcal{O} = \overline{\chi}\Gamma\chi\overline{q}\Gamma q$$

$$\Gamma = \{\mathcal{I}, \gamma^5, \gamma^\mu, \gamma^\mu\gamma^5, \sigma^{\mu\nu}\}$$

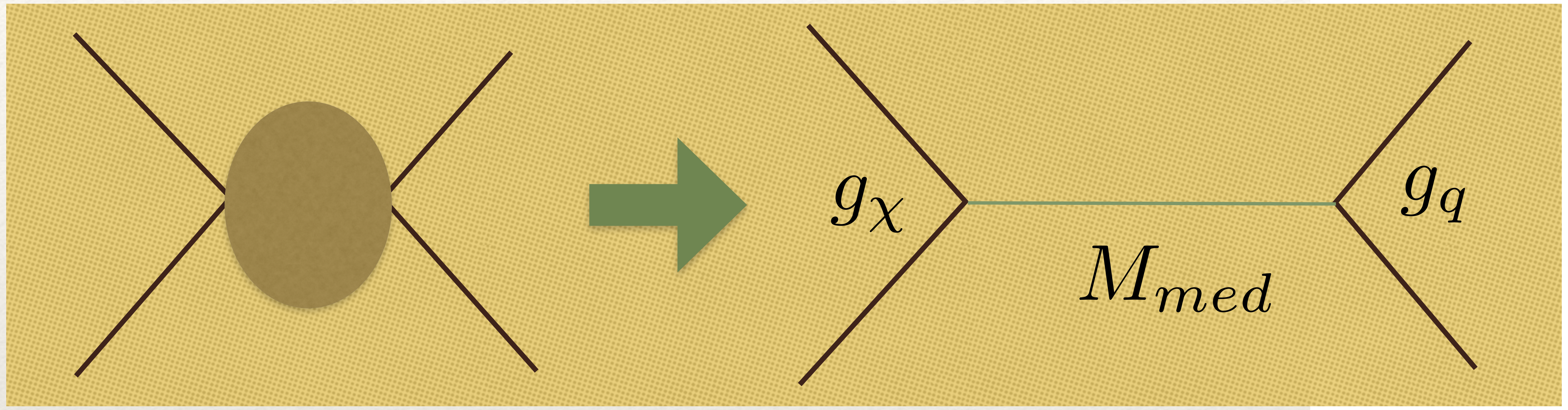
Name	Operator	Coefficient
C1	$\chi^\dagger\chi\bar{q}q$	m_q/Λ^2
C2	$\chi^\dagger\chi\bar{q}\gamma^5 q$	im_q/Λ^2
R1	$\chi^2\bar{q}q$	$m_q/2\Lambda^2$
R2	$\chi^2\bar{q}\gamma^5 q$	$im_q/2\Lambda^2$

Name	Operator	Coefficient
C3	$\chi^\dagger\partial_\mu\chi\bar{q}\gamma^\mu q$	$1/\Lambda^2$
C4	$\chi^\dagger\partial_\mu\chi\bar{q}\gamma^\mu\gamma^5 q$	$1/\Lambda^2$
C5	$\chi^\dagger\chi G_{\mu\nu}G^{\mu\nu}$	$\alpha_s/4\Lambda^2$
C6	$\chi^\dagger\chi G_{\mu\nu}\tilde{G}^{\mu\nu}$	$\alpha_s/4\Lambda^2$

Name	Operator	Coefficient
R3	$\chi^2 G_{\mu\nu}G^{\mu\nu}$	$\alpha_s/8\Lambda^2$
R4	$\chi^2 G_{\mu\nu}\tilde{G}^{\mu\nu}$	$i\alpha_s/8\Lambda^2$

J.~Goodman, M.~Ibe, A.~Rajaraman, W.~Shepherd, T.~M.~P.~Tait and H.~B.~Yu, Phys.Rev.D82, 116010 (2010)

EFT to Simplified Model



$$\frac{1}{\Lambda^2} = \frac{g_\chi g_q}{M_{med}^2}$$

$$\Lambda = \frac{M_{med}}{\sqrt{g_\chi g_q}}$$

For example,

$$\mathcal{L}_{F,V} = \frac{1}{\Lambda^2}(\bar{\chi}\gamma^\mu\chi)(\bar{q}\gamma_\mu q)$$

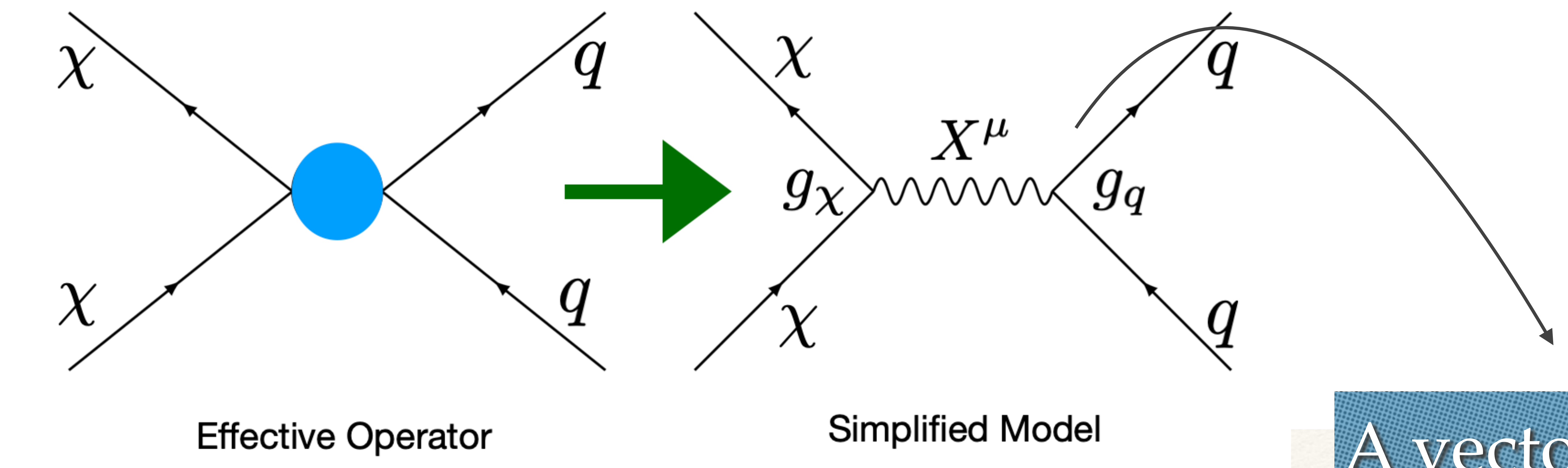
Now for the validity of the perturbative theory:

$$g_\chi g_q \lesssim (4\pi)^2$$

Now for the mediator to remain off-shell we have:

$$M_{med} \gtrsim 2m_\chi$$

$$\Lambda \gtrsim \frac{m_\chi}{2\pi}$$

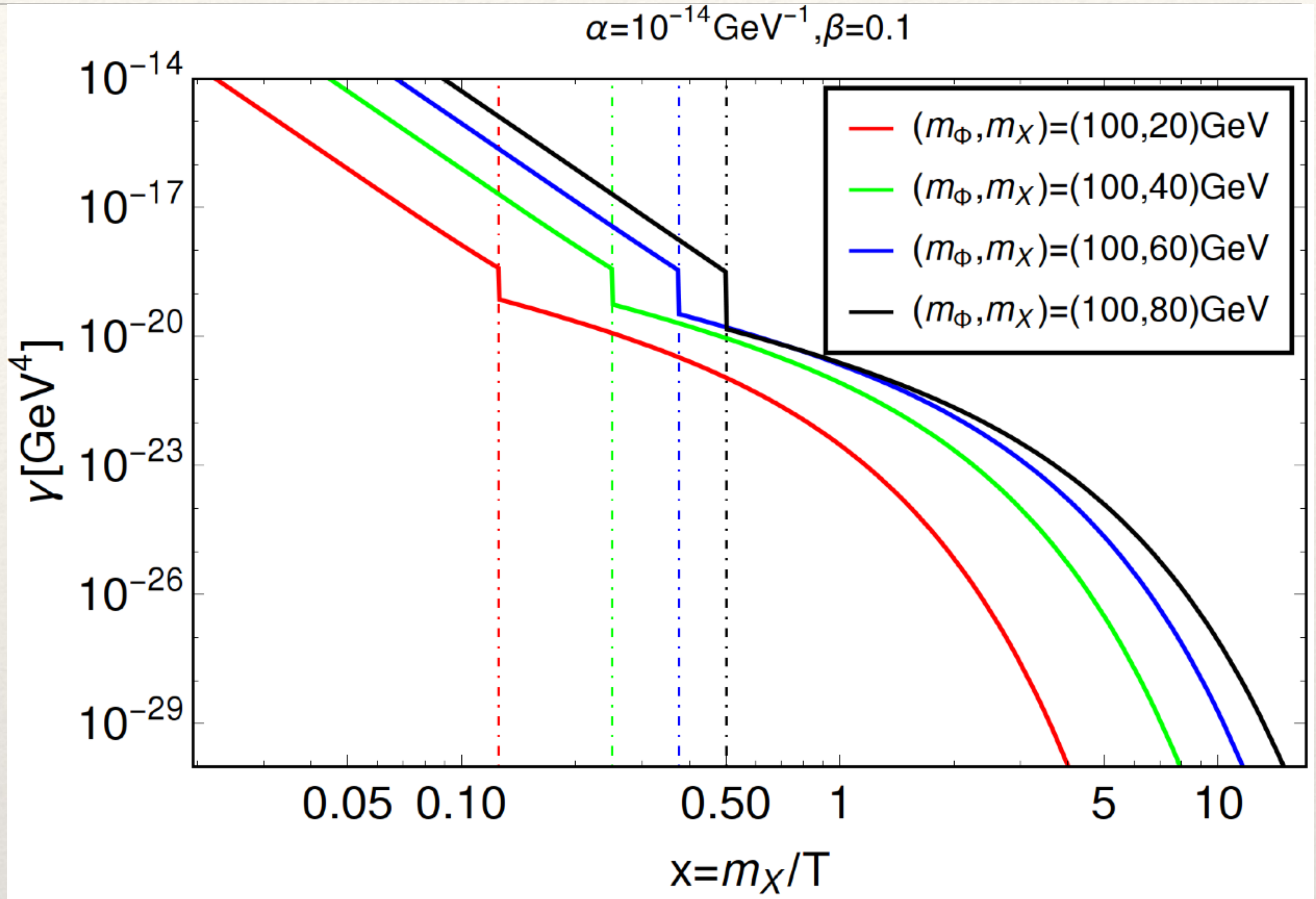


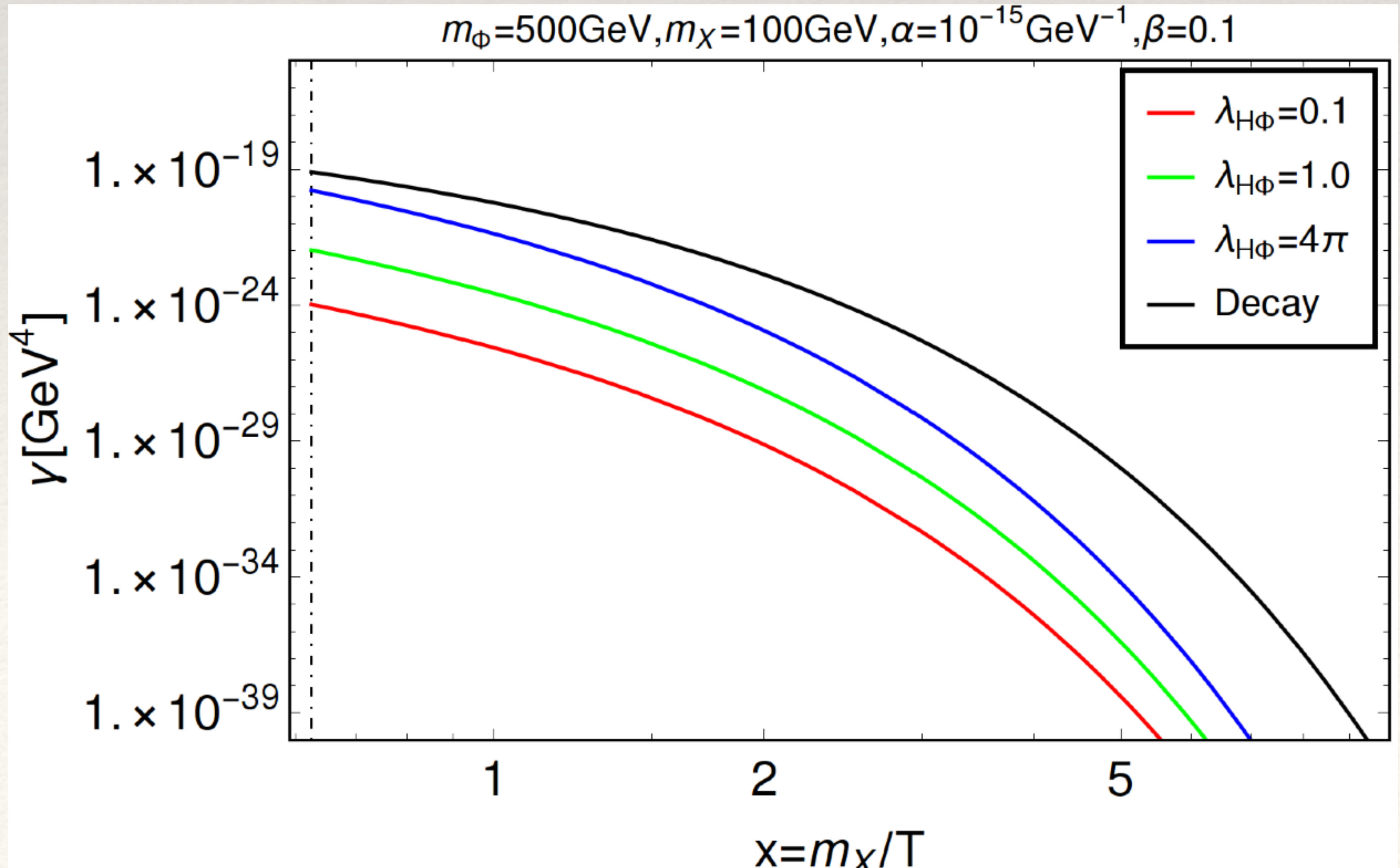
A vector boson mediator

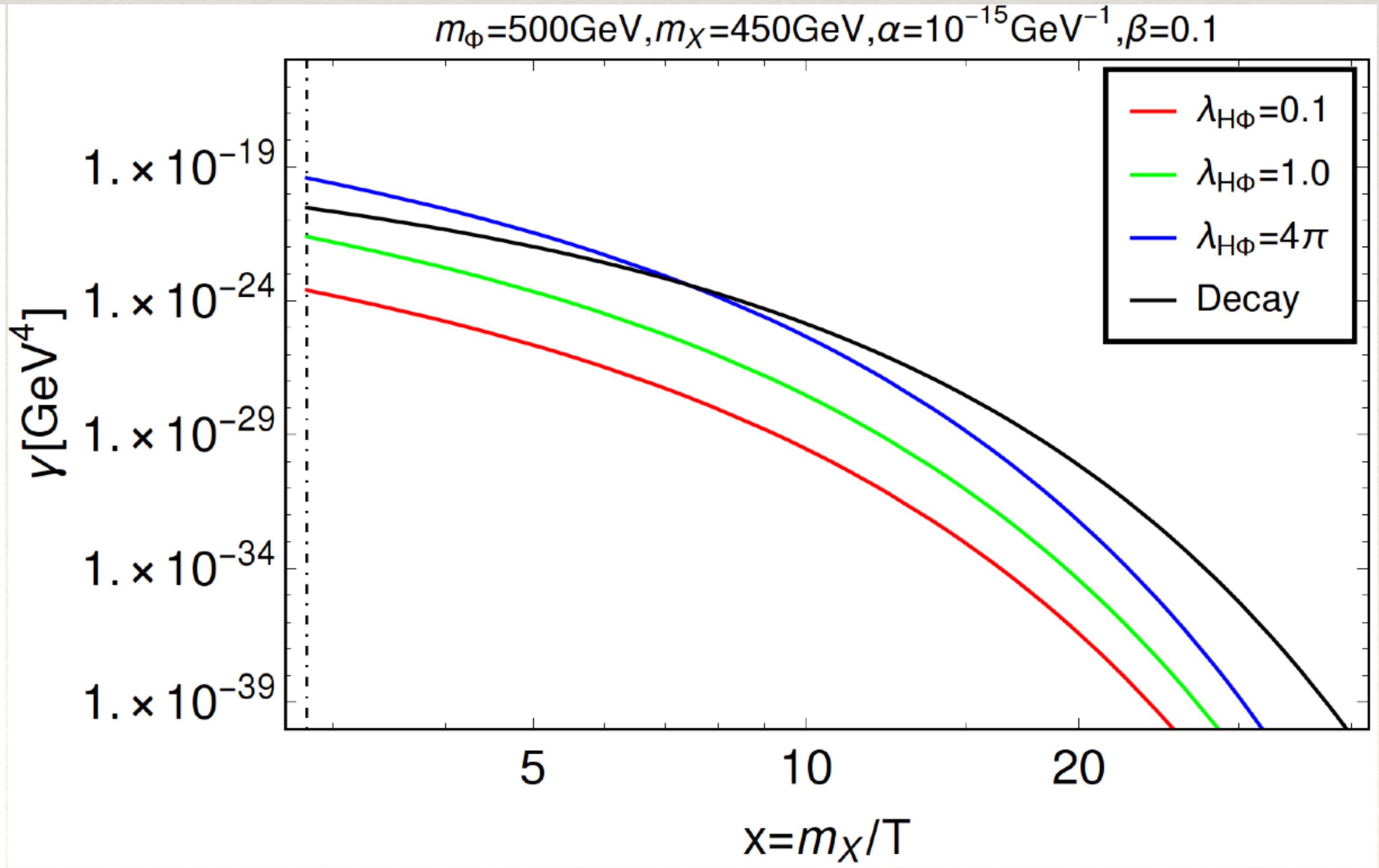
Reaction densities

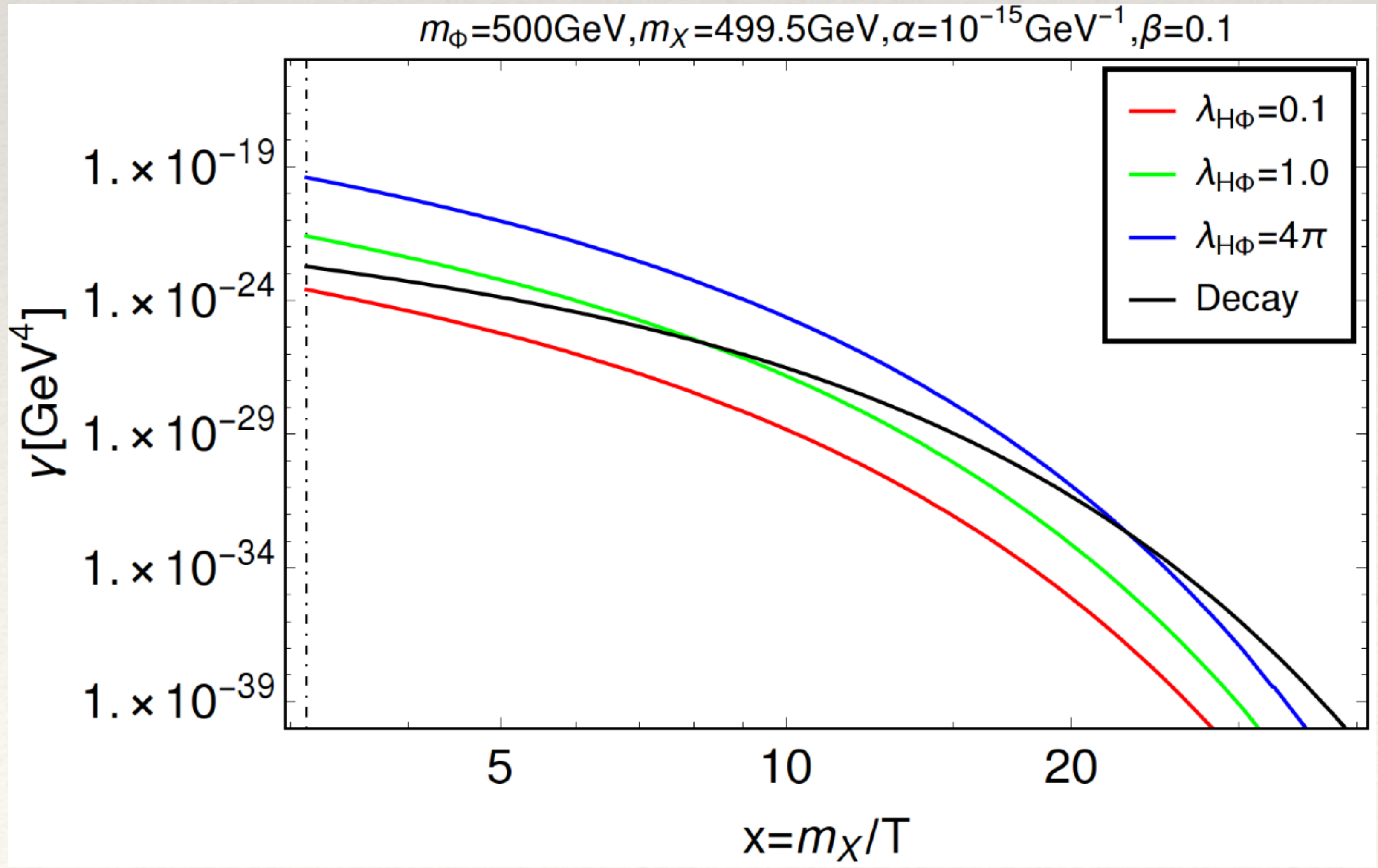
$$\gamma(a,b\to 1,2) = \int \prod_{i=1}^{4} d\Pi_i (2\pi)^4 \delta^{(4)}\left(p_a + p_b - p_1 - p_2\right) f_a^{eq} f_b^{eq} |\mathcal{M}_{a,b\to 1,2}|^2$$
$$= \frac{T}{32\pi^4} g_a g_b \int_{s_{min}}^{\infty} ds \frac{\left[\left(s - m_a^2 - m_b^2\right)^2 - 4m_a^2 m_b^2\right]}{\sqrt{s}} \sigma(s)_{a,b\to 1,2} K_1\left(\frac{\sqrt{s}}{T}\right), \quad \text{(C.1)}$$

$$\gamma(a\to 1,2) = \int \sum_{i=1}^{3} d\Pi_i (2\pi)^4 \delta^{(4)}\left(p_a - p_1 - p_2\right) f_a^{eq} |\mathcal{M}_{a\to 1,2}|^2$$
$$= \frac{g_a}{2\pi^2} m_a^2 \Gamma_{a\to 1,2} T K_1\left(\frac{m_a}{T}\right).$$







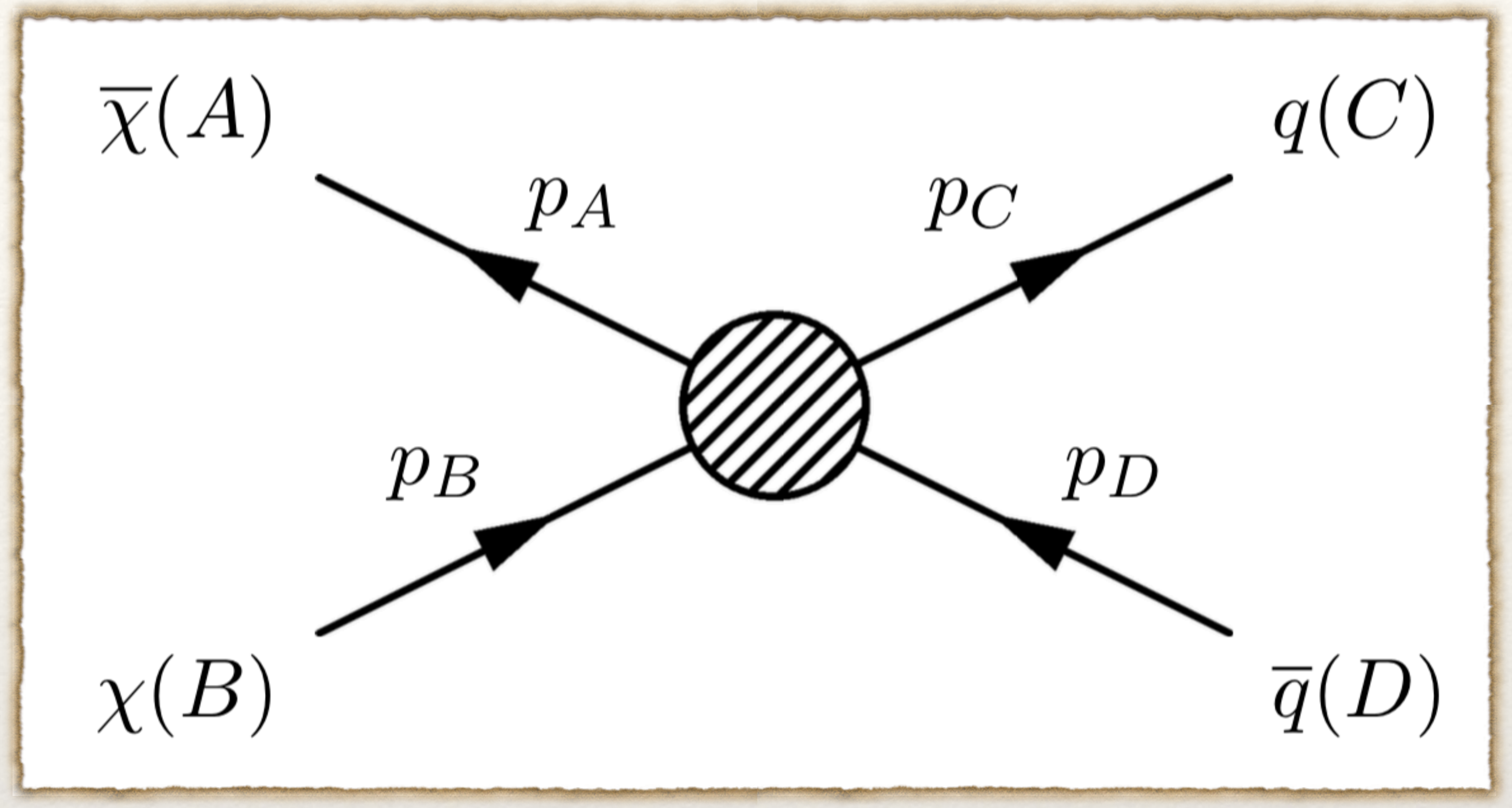


Annihilation cross-section

Consider vector interaction operator

$$\mathcal{L}_{F,V} = \frac{1}{\Lambda^2}(\bar{\chi}\gamma^\mu\chi)(\bar{q}\gamma_\mu q)$$

Annihilation to SM quarks



Lorentz invariant Feynman amplitude

$$-i\mathcal{M} = \frac{1}{\Lambda^2}(\overline{v}_B\gamma^\mu u_A)(\overline{u}_C\gamma_\mu v_D)$$

Matrix element squared

$$|\mathcal{M}|^2_{av} = \frac{8}{\Lambda^4}\left[\frac{s^2}{8}(1+cos^2\theta) + \frac{s}{2}(m_\chi^2 + m_q^2)sin^2\theta + 2m_\chi^2 m_q^2 cos^2\theta\right]$$

$$\sigma|\vec{v}| = \frac{1}{32\pi^2 s}\sqrt{1-\frac{4m_q^2}{s}}\int |\mathcal{M}|^2_{av} d\Omega$$

Where

$$s = (p_A + p_B)^2 = \frac{4m_\chi^2}{1-\frac{v^2}{4}}$$

$$(\sigma|v|)_{F,V} = \frac{3m_\chi^2}{2\pi\Lambda^4}\sum_q \sqrt{1-\frac{m_q^2}{m_\chi^2}} \times \left[(2+\frac{m_q^2}{m_\chi^2}) + (\frac{8m_\chi^4 - 4m_q^2 m_\chi^2 + 5m_q^4}{24m_\chi^2(m_\chi^2 - m_q^2)})v^2\right]$$

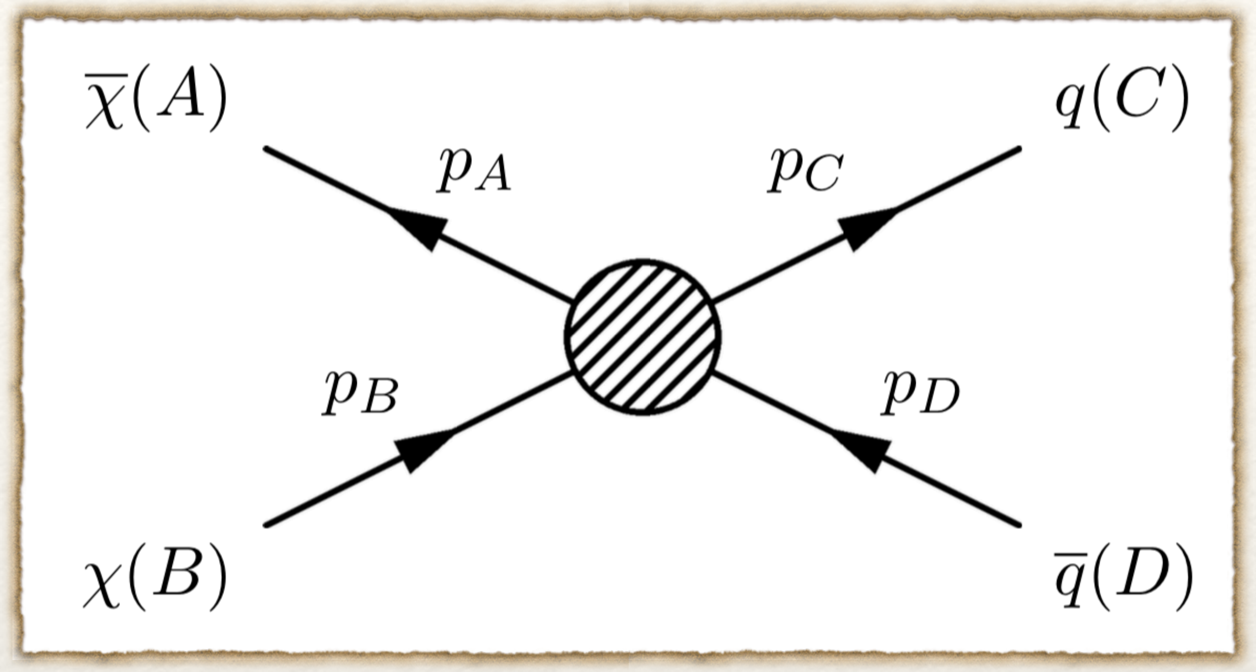
$$\Omega h^2 \sim \frac{1}{\langle\sigma v\rangle_{eff}}$$

$$\Omega h^2 \simeq 0.1199 \pm 0.0022$$

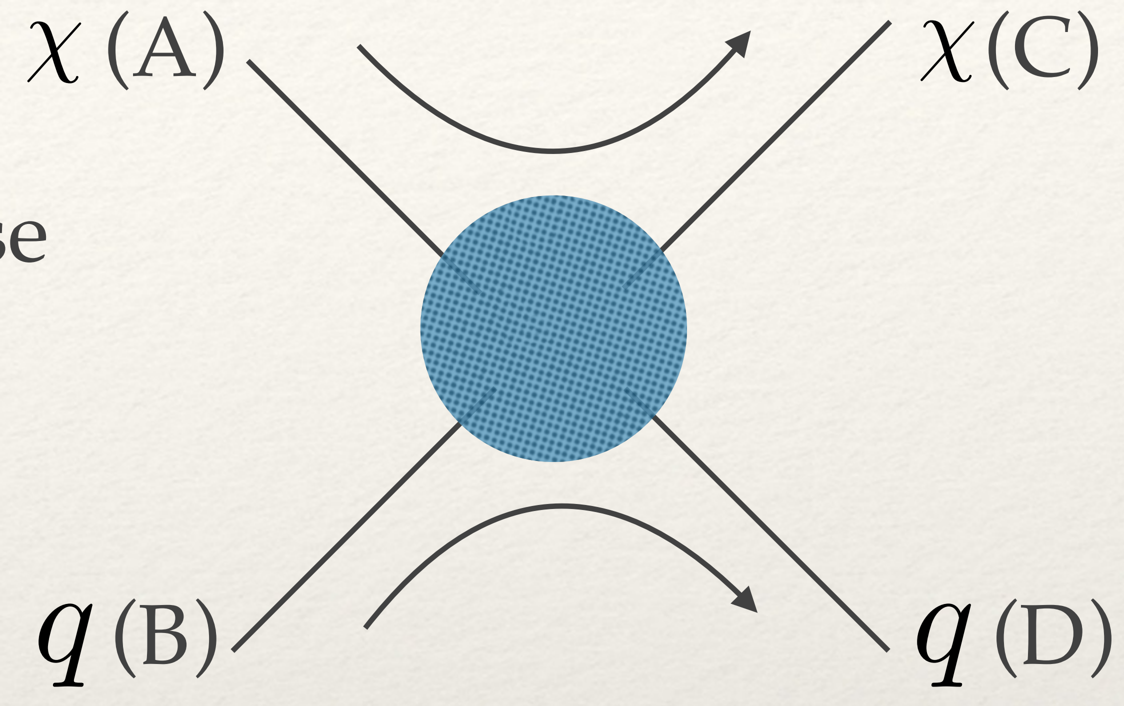
How to compute Direct Search X-sec ?

Given an operator

$$\mathcal{L}_{F,V} = \frac{1}{\Lambda^2}(\bar{\chi}\gamma^\mu\chi)(\bar{q}\gamma_\mu q)$$



Rotate clockwise



Replace

$$p_A \to -p_A, p_B \to p_C, p_C \to p_B, p_D \to -p_D$$

Annihilation $\to$ Direct Detection Cross Section in Quark Level

Now typically, the DM hits the detector with velocity ~100 km/s; very non-relativistic. So, we can then take p ->0 limit.

Spin independent cross-section:

$$\sigma_{\chi N} = \frac{\kappa^2}{\pi}\mu_N^2[Zf_p + (A-Z)f_n]^2 \rightarrow \sigma_{\chi n} = \frac{1}{A^2}\sigma_{\chi N}$$

$$\frac{1}{\Lambda^2}$$

$$\frac{f_{p,n}}{m_n} = \sum_{q=u,d,s} f_{T_q}^{(p,n)}\frac{d_q}{m_q} + \frac{2}{27}\sum_{Q=c,b,t} f_{TG}^{(p,n)}\frac{d_Q}{m_Q} - \frac{8\pi}{9\alpha_s}d_G f_{TG}^{(p,n)}$$

$$f_{TG}^{(n)} = 1 - \sum_{q=u,d,s} f_{T_q}^{(n)}$$

Spin Independent vs Spin Dependent

In general, we have four types of interaction: S, P, V, A

- **scalar interaction** (scalar and vector parts of L, spin independent)

$$\sigma_{SI} = \frac{m_N^2}{4\pi(m_\chi + m_N)^2}\left[Zf_p + (A-Z)f_n\right]^2 \rightarrow \boxed{\sigma_{\chi N} \simeq \sigma_{\chi n} A^2}$$

- **axial interaction** (axial part of L, spin dependent)

$$\sigma_{SD} = \frac{32}{\pi} G_F \frac{m_\chi m_N^2}{4\pi(m_\chi + m_N)^2} \frac{J_N + 1}{J_N} \left(a_p\langle S_p\rangle + a_n\langle S_n\rangle\right)^2$$

a_p, a_n: effective couplings to p and n; $\langle S_p\rangle$ and $\langle S_n\rangle$ expectation values of the p and n spins within the nucleus

In the SI case Interaction is coherent over nucleus, since de Broglie wavelength of WIMP is of nuclear dimensions

Leptophilic Majorana DM Production at ILC

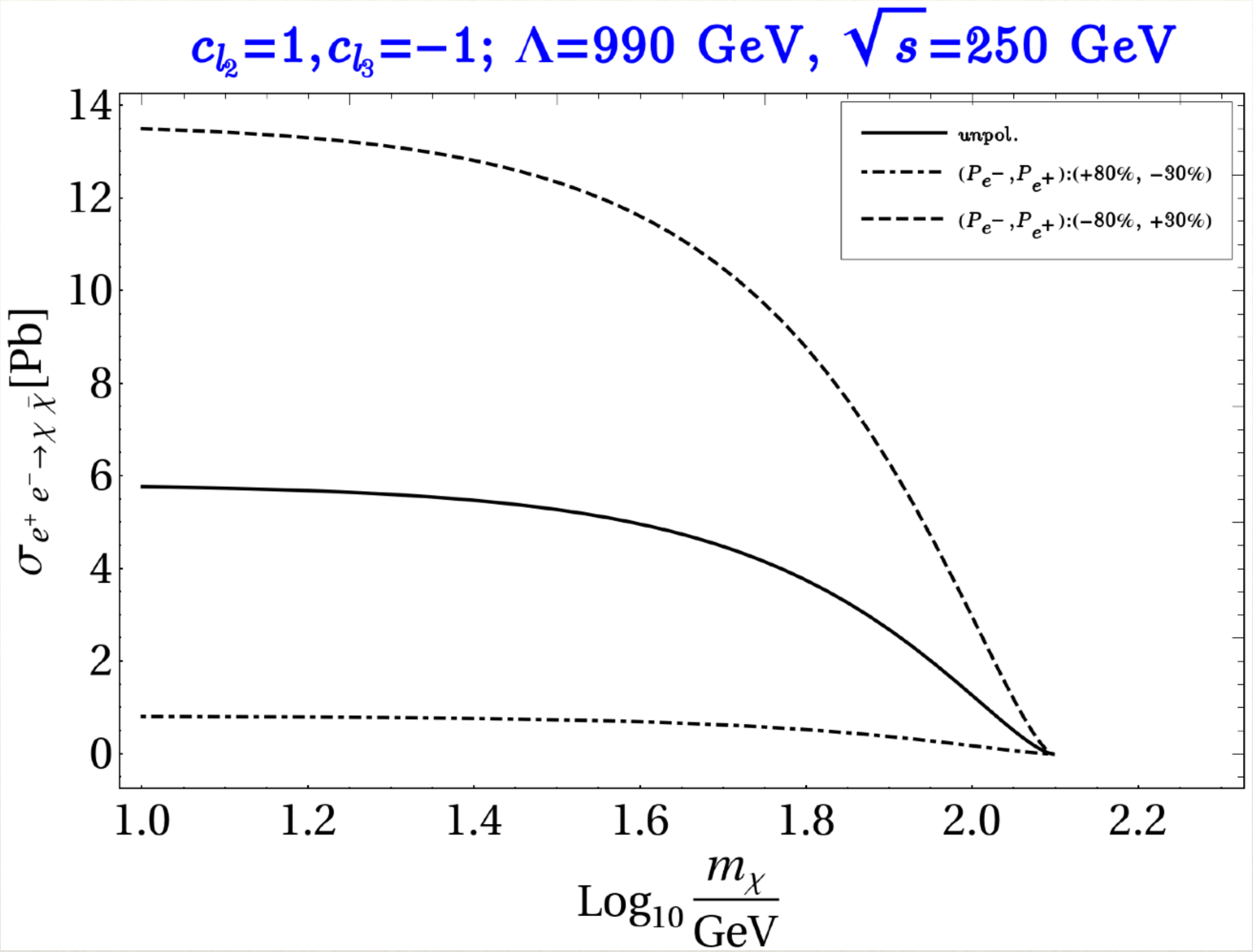
$$\left(\frac{d\sigma}{d\Omega}\right)^{\texttt{majorana}}_{\mathrm{RL}} = \frac{s}{16\pi^2\Lambda^4}\left[\left(c_{\ell_2}+c_{\ell_3}\right)^2\left(1-\beta_\chi^2\right)^{3/2}\left(1+\cos^2\theta\right)\right] ,$$

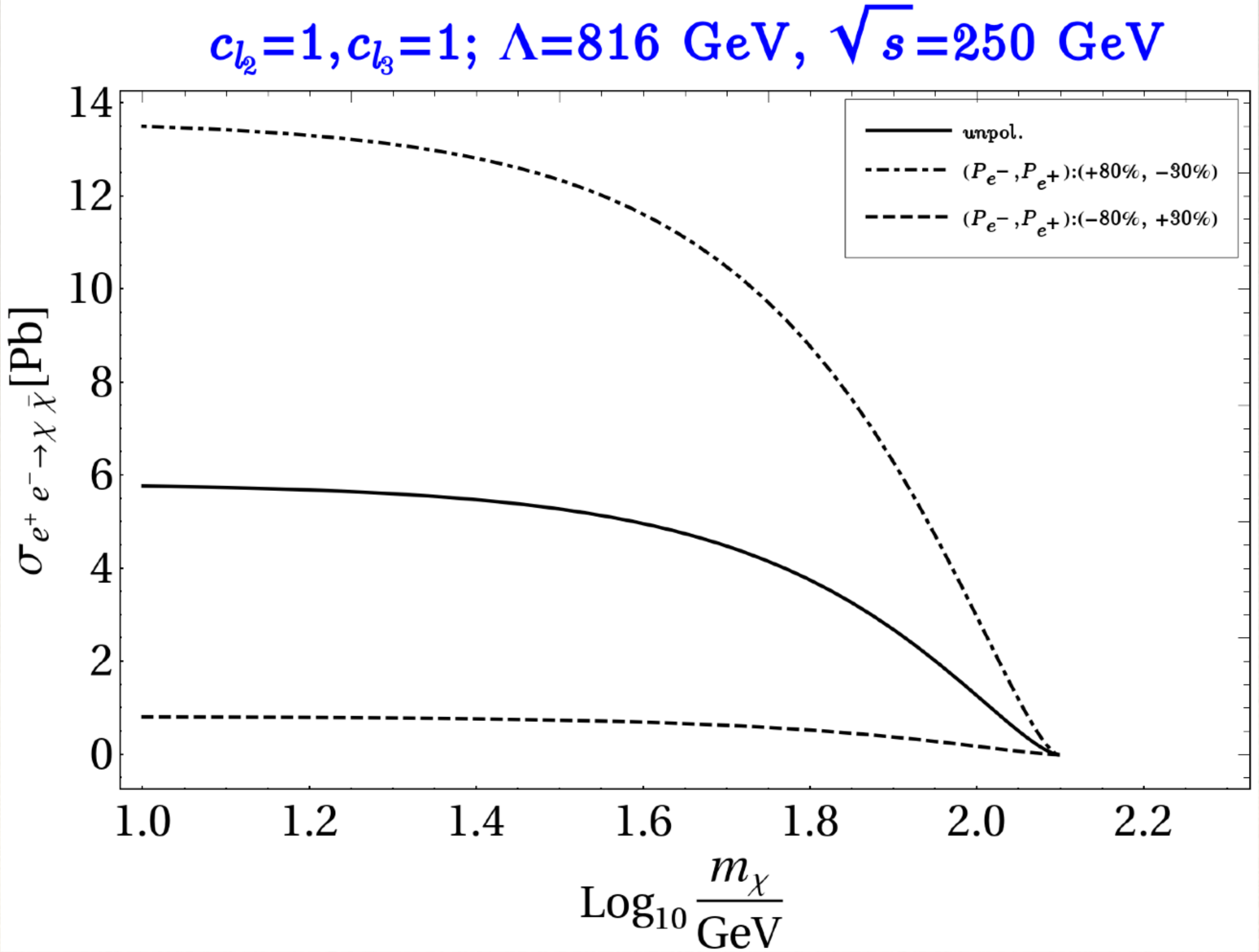
$$\left(\frac{d\sigma}{d\Omega}\right)^{\texttt{majorana}}_{\mathrm{LR}} = \frac{s}{16\pi^2\Lambda^4}\left[\left(c_{\ell_2}-c_{\ell_3}\right)^2\left(1-\beta_\chi^2\right)^{3/2}\left(1+\cos^2\theta\right)\right] .$$

$$\begin{aligned}\sigma_0^{\texttt{majorana}} &= \frac{s}{6\pi\Lambda^4}\left(c_{\ell_2}^2+c_{\ell_3}^2\right)\left(1-\beta_\chi^2\right)^{3/2} ,\\ &= \frac{s}{3\pi\Lambda^4}\left(1-\frac{4m_\chi^2}{s}\right)^{3/2} ; \quad \text{where } c_{\ell_i}=1\ (i=2,3) .\end{aligned}$$

$$\sigma\left(P_{e^-},P_{e^+}\right) = 2\sigma_0\left(\mathcal{L}_{\rm eff}/\mathcal{L}\right)\left[1-\mathcal{A}_{LR}\,P_{\rm eff}\right] ;$$

$$\sigma_0 = \frac{1}{4}\left[\sigma_{\rm RR}+\sigma_{\rm LL}+\sigma_{\rm RL}+\sigma_{\rm LR}\right] ; \quad \mathcal{L}_{\rm eff} = \frac{1}{2}\left(1-P_{e^-}P_{e^+}\right)\mathcal{L} ;$$





Cut flow

- Cut1 ($\mathcal{C}_1$) : Events with zero lepton and jet-veto with exactly one photon in the final state.

- Cut2 ($\mathcal{C}_2$): We choose photons with energy lying within the window $0.1 \leq E_\gamma < 60$ (130) GeV or Majorana (Dirac) DM scenario. This helps to avoid the background events around the Z-mass window by retaining majority of the signal events.

- Cut3 ($\mathcal{C}_3$): We apply a cut on the missing transverse energy $\not{E}_T \leq 33$ (43) GeV for for Majorana (Dirac) DM scenario.

- Cut4 ($\mathcal{C}_4$): Finally, we employ the missing mass cut $m_{\text{miss}} \geq 140$ (220) GeV for BP1 (BP2), Majorana DM scenario and $m_{\text{miss}} \geq 700$ (800) GeV for BP4 (BP5), Dirac DM scenario.