Automatisation of Integrating out Heavy Fields at 1-loop

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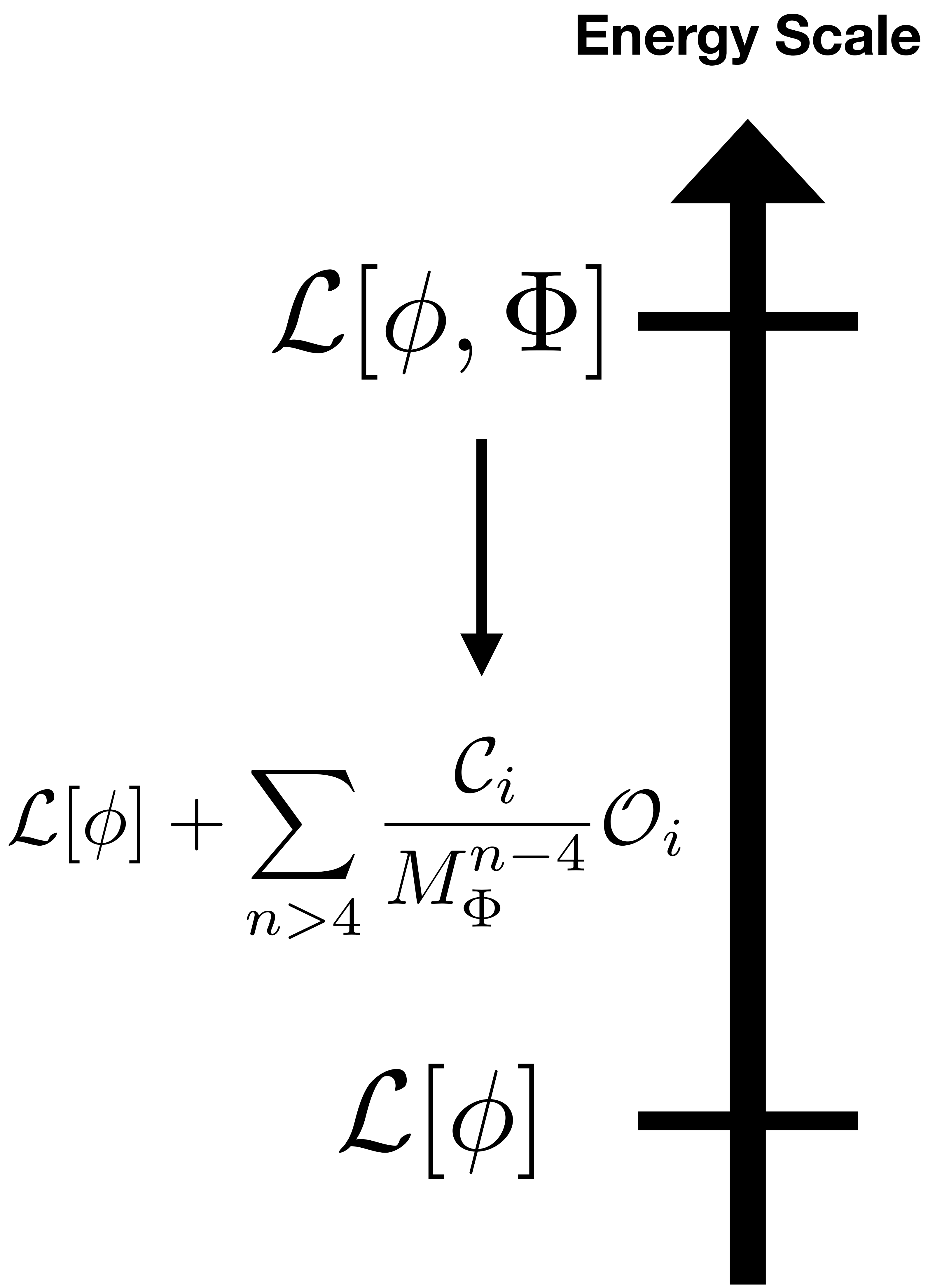
IMEPNP 2022, IOP Bhubaneswar
10 Feb’22

Top-Down EFT Approach

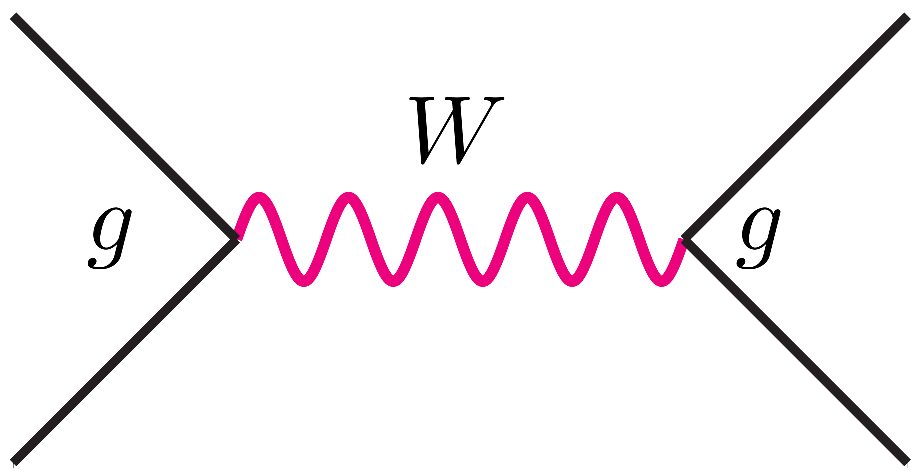
- Starts from a theory which resides at high energy.
- Identify heavy degrees of freedom.
- Integrate out the heavy degrees of freedom.
- Generates effective operators and coefficients functions of the parameters of the UV theory - Wilson coefficients.



Matching: Diagrammatic Approach

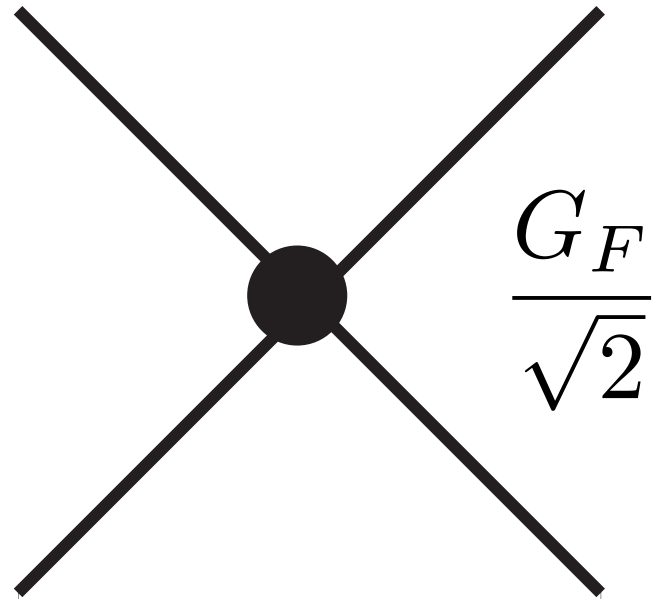
- This requires computing the same processes with light particles in external legs from the renormalisable theory at high scale (M) theory and the low energy effective theory at scale $\mu = M$.

Renormalisable Theory



$$\mu^- \to e^- \, \bar{\nu}_e \, \nu_\mu$$

EFT



Example: Fermi's theory of four fermion

$$\frac{g^2}{8M_W^2} \equiv \frac{G_F}{\sqrt{2}}$$

Matching condition

Functional Approach: Covariant Derivative Expansion

- Although the previous approach works fine, it could be really cumbersome sometimes to calculate all the diagrams to capture the total effect of the UV theory.

- Alternative - Covariant Derivative Expansion (CDE).

- Expansion in terms of the Covariant Derivative $\mathcal{D}_\mu$ without breaking it into the derivative and the gauge part.

- Example: $\dfrac{1}{-\mathcal{D}^2 - m^2} = \dfrac{-1}{M^2} + \dfrac{\mathcal{D}^2}{M^4} + \cdots$

O. Cheyette (1985)
M. K. Gaillard (1986)
Henning, Lu, Murayma (2014) [1412.18837]

- The basic idea is to integrate over the heavy modes. Φ denotes the heavy particles, and ϕ denotes the lighter degrees of freedom.

$$e^{iS_{\text{eff}}[\phi](\mu)} = \int \mathcal{D}\Phi \, e^{iS[\phi,\Phi](\mu)} \tag{1}$$

- Expanding the action around this minimum,

$$S[\phi, \Phi_c + \eta] = S[\Phi_c] + \frac{1}{2} \frac{\delta^2 S}{\delta \Phi^2}\bigg|_{\Phi_c} \eta^2 + \mathcal{O}(\eta^3)$$

Where, $\dfrac{\delta S[\phi, \Phi]}{\delta \Phi} = 0 \Rightarrow \Phi_c[\phi]$

- The integral is computes as,

$$S_{\text{eff}} \approx \underbrace{\boxed{S[\Phi_c]}}_{\text{Tree level}} + \underbrace{\boxed{\frac{i}{2} \mathrm{Tr} \log \left(- \frac{\delta^2 S}{\delta \Phi^2}\bigg|_{\Phi_c} \right)}}_{\text{1-Loop}} \tag{2}$$

Henning, Lu, Murayma (2014) [1412.18837]

- The Lagrangian is arranged in the power of the heavy fields:

$$\mathcal{L}_{renrom.} = \Phi^\dagger B + h.c. + \Phi^\dagger(-\mathcal{D}^2 - M^2 - U)\Phi + \mathcal{O}(\Phi^3)$$

- The classical solution for the heavy fields is given by: $\Phi_c = -\frac{1}{P^2 - m^2 - U(x)} B(x)$

$$\begin{aligned}\Phi_c &= \left[1 - \frac{1}{m^2}(P^2 - U)\right]^{-1} \frac{1}{m^2} B \\ &= \frac{1}{m^2}B + \frac{1}{m^2}(P^2 - U)\frac{1}{m^2}B + \frac{1}{m^2}(P^2 - U)\frac{1}{m^2}(P^2 - U)\frac{1}{m^2}B + \ldots\end{aligned}$$

- The effective operators generated by tree level diagrams is given by

$$\mathcal{L}_{\text{eff,tree}} = B^\dagger \frac{1}{m^2} B + B^\dagger \frac{1}{m^2}(P^2 - U)\frac{1}{m^2} B + \cdots + \mathcal{O}(\Phi_c^3)$$

- where, $B(\phi), U(\phi)$ are functions of ϕ, and $P_\mu = i\mathcal{D}_\mu$

Henning, Lu, Murayma (2014) [1412.18837]

- This is the effective action formula for 1-Loop.
- $P_\mu U \equiv [P_\mu, U]$
- $G'_{\mu\nu} \equiv [\mathcal{D}_\mu, \mathcal{D}_\nu]$
- c_s depends on the types of the heavy particle, for real scalar 1/2, complex scalar 1, fermions -1/2 etc.

$$\Delta S_{\text{eff}} = i c_s \mathrm{Tr} \log \Big(- P^2 + m^2 + U(x) \Big)$$

$$\begin{aligned}
\Delta \mathcal{L}_{\text{eff,1-loop}} = \frac{c_s}{(4\pi)^2} \mathrm{tr} \Bigg\{ & \\
& + m^4 \left[-\frac{1}{2} \left(\log \frac{m^2}{\mu^2} - \frac{3}{2} \right) \right] \\
& + m^2 \left[-\left(\log \frac{m^2}{\mu^2} - 1 \right) U \right] \\
& + m^0 \left[-\frac{1}{12} \left(\log \frac{m^2}{\mu^2} - 1 \right) G'^2_{\mu\nu} - \frac{1}{2} \log \frac{m^2}{\mu^2} U^2 \right] \\
& + \frac{1}{m^2} \left[-\frac{1}{60} \left(P_\mu G'_{\mu\nu} \right)^2 - \frac{1}{90} G'_{\mu\nu} G'_{\nu\sigma} G'_{\sigma\mu} - \frac{1}{12} (P_\mu U)^2 - \frac{1}{6} U^3 - \frac{1}{12} U G'_{\mu\nu} G'_{\mu\nu} \right] \\
& + \frac{1}{m^4} \left[\frac{1}{24} U^4 + \frac{1}{12} U \left(P_\mu U \right)^2 + \frac{1}{120} \left(P^2 U \right)^2 + \frac{1}{24} \left(U^2 G'_{\mu\nu} G'_{\mu\nu} \right) \right. \\
& \qquad \left. - \frac{1}{120} \left[(P_\mu U), (P_\nu U) \right] G'_{\mu\nu} - \frac{1}{120} \left[U [U, G'_{\mu\nu}] \right] G'_{\mu\nu} \right] \\
& + \frac{1}{m^6} \left[-\frac{1}{60} U^5 - \frac{1}{20} U^2 \left(P_\mu U \right)^2 - \frac{1}{30} \left(U P_\mu U \right)^2 \right] \\
& + \frac{1}{m^8} \left[\frac{1}{120} U^6 \right] \Bigg\}.
\end{aligned}$$

Henning, Lu, Murayma (2014) [1412.18837]

Summary of CDE

- It is a framework that integrates out heavy degrees of freedom.
- It is manifestly gauge invariant - all the terms appeared in the formula were in commutator.
- It’s applicability is universal - the form of the Lagrangian it started with had a very general structure.
- After the realisation of its merit many works have been done and the master formula shown in previous slide have been modified to incorporate the effect from light particles and the formula has been dubbed as the **Universal One Loop Effective Action** - the **UOLEA**.

Henning, Lu, Murayama [1604.01019]
Aguila, Kunszt, Santiago [1602.00126]
Ellis, Quevillon, You, Zhang [1604.02445]
Zhang [1610.00710]
Ellis, Quevillon, You, Zhang [1706.07765]
Krämer, Summ, Voigt [1908.04798]

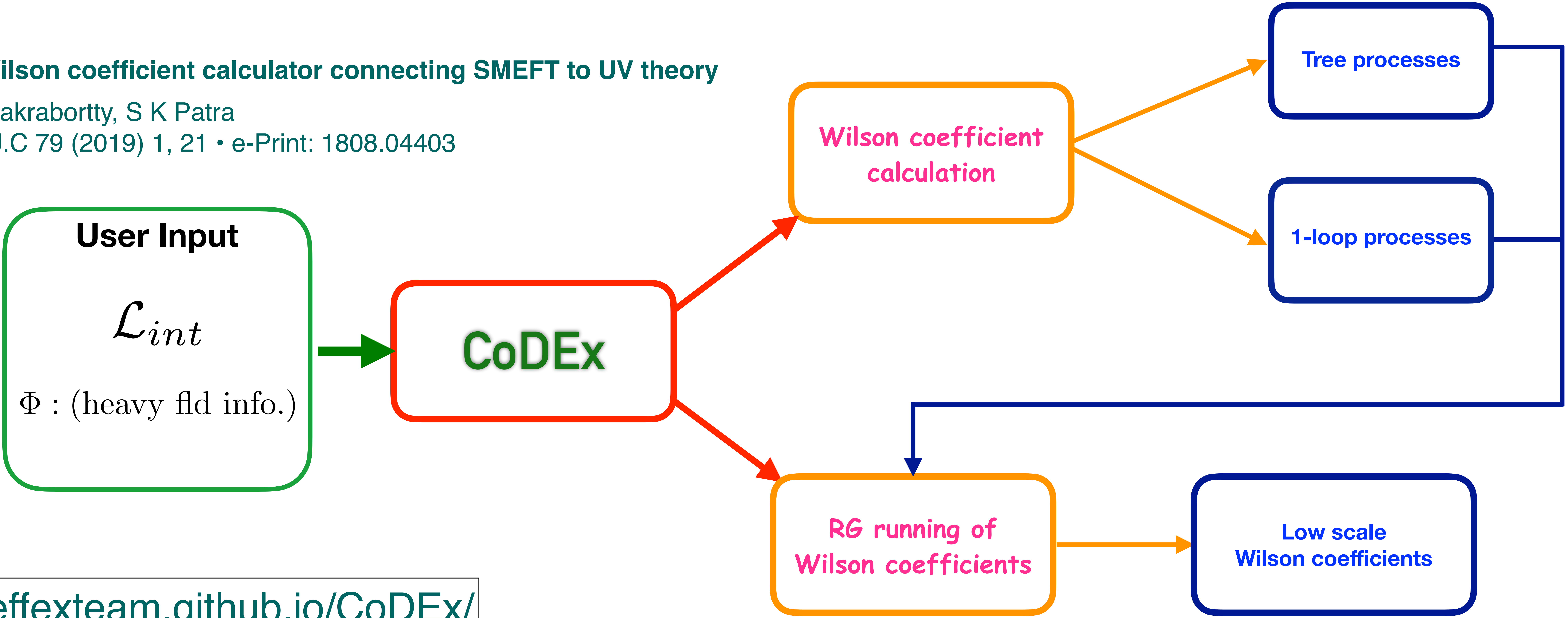
CoDEx

A Package to compute Wilson coefficients up to mass dimension-6 SMEFT ops

CoDEx: Wilson coefficient calculator connecting SMEFT to UV theory

SDB, J Chakrabortty, S K Patra

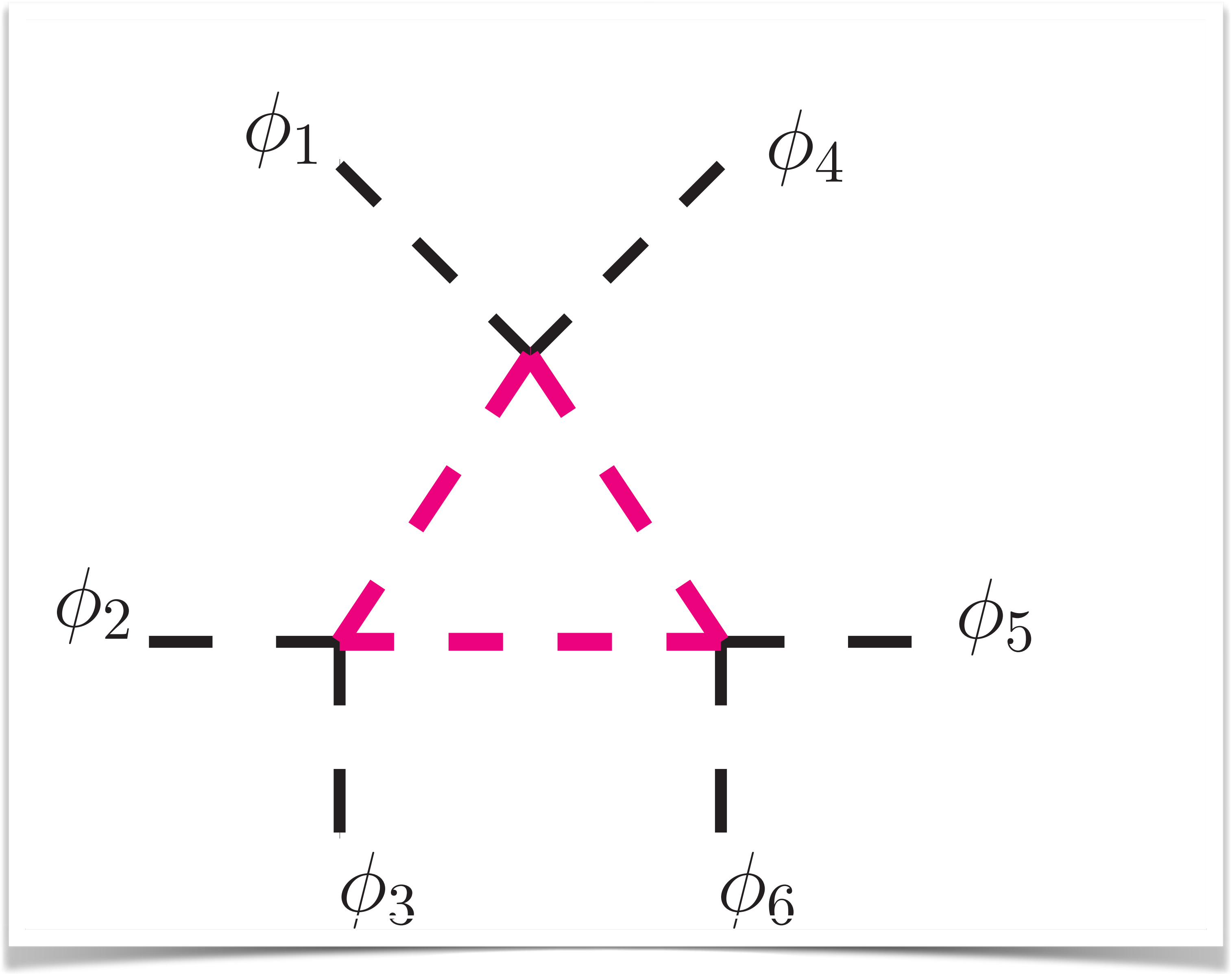
Eur.Phys.J.C 79 (2019) 1, 21 • e-Print: 1808.04403



https://effexteam.github.io/CoDEx/

CoDEx(v1.0)

- Can generate effective operators for only heavy scalar particles in loop.
- This version is publicly available.
- Example: $\Phi_{(1,3,0)}$
- Tree level: $\{Q_{HD}, Q_{H\Box}, Q_H\}$
- Only heavy loop: $\{Q_{ll}, Q_{Hl}^{(3)}, Q_{Hq}^{(3)}, Q_{HW}, Q_{qq}^{(3)}, Q_{lq}^{(3)}\}$



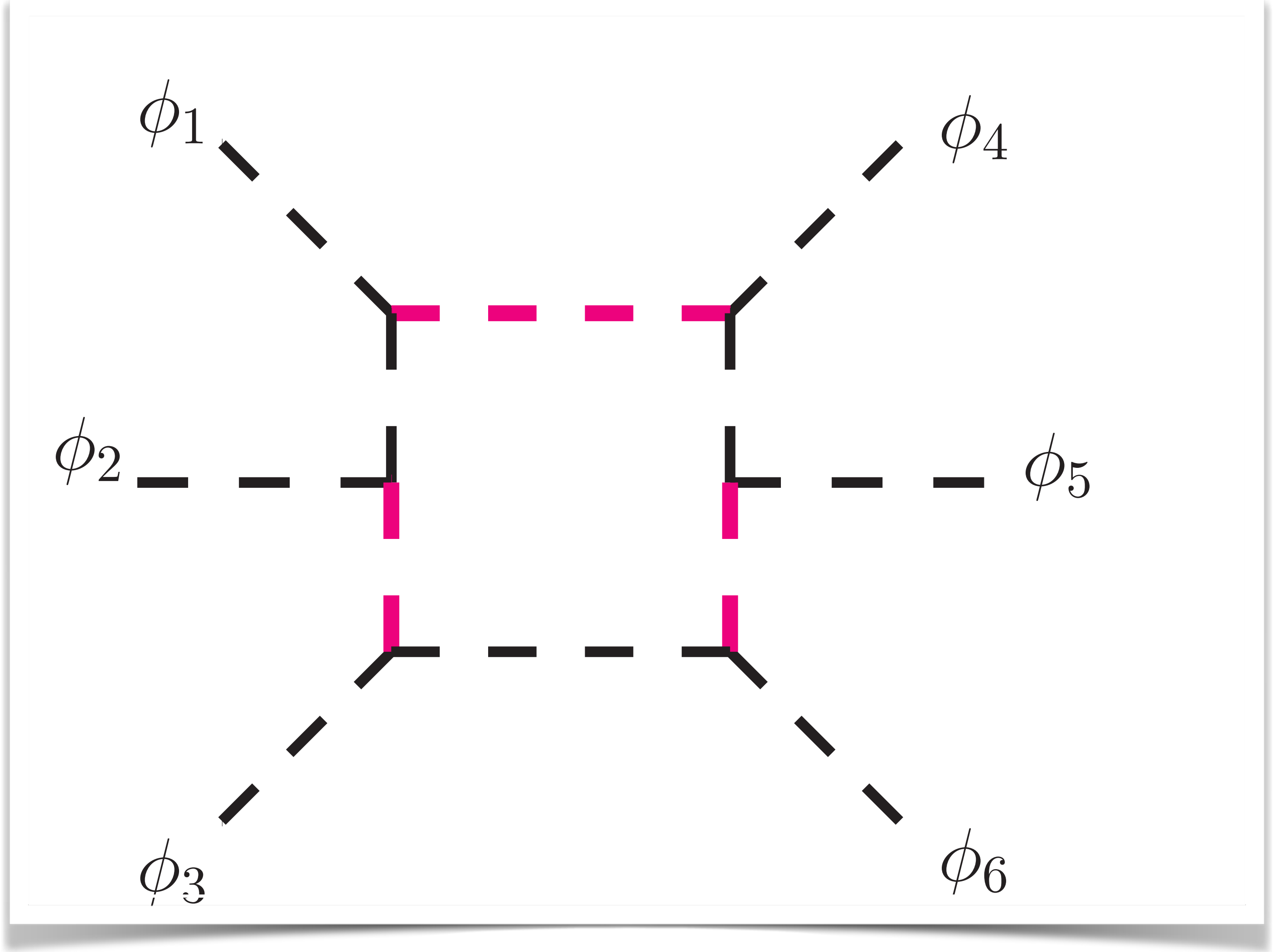
Bakshi, Chakrabortty, Spannowsky [2012.03839]
Bakshi, Chakrabortty, Prakash, SUR, Spannowsky [2103.11593]

Beta-version

- Loops containing heavy as well as light particles will also contribute.
- This version can further generates these operators for the same extension.

$$\{Q_{HWB}, Q_{HB}, Q_{eH}, Q_{uH}, Q_{dH}\}$$

- This version of CoDEx is not publicly available but the results have been used and verified.



Bakshi, Chakrabortty, Spannowsky [2012.03839]
Anisha, Bakshi, Banerjee, Biekotter, Chakrabortty [2111.05876]
Anisha, Banerjee, Chakrabortty, Englert, Spannowsky [2108.07683]
Bakshi, Chakrabortty, Prakash, SUR, Spannowsky [2103.11593]

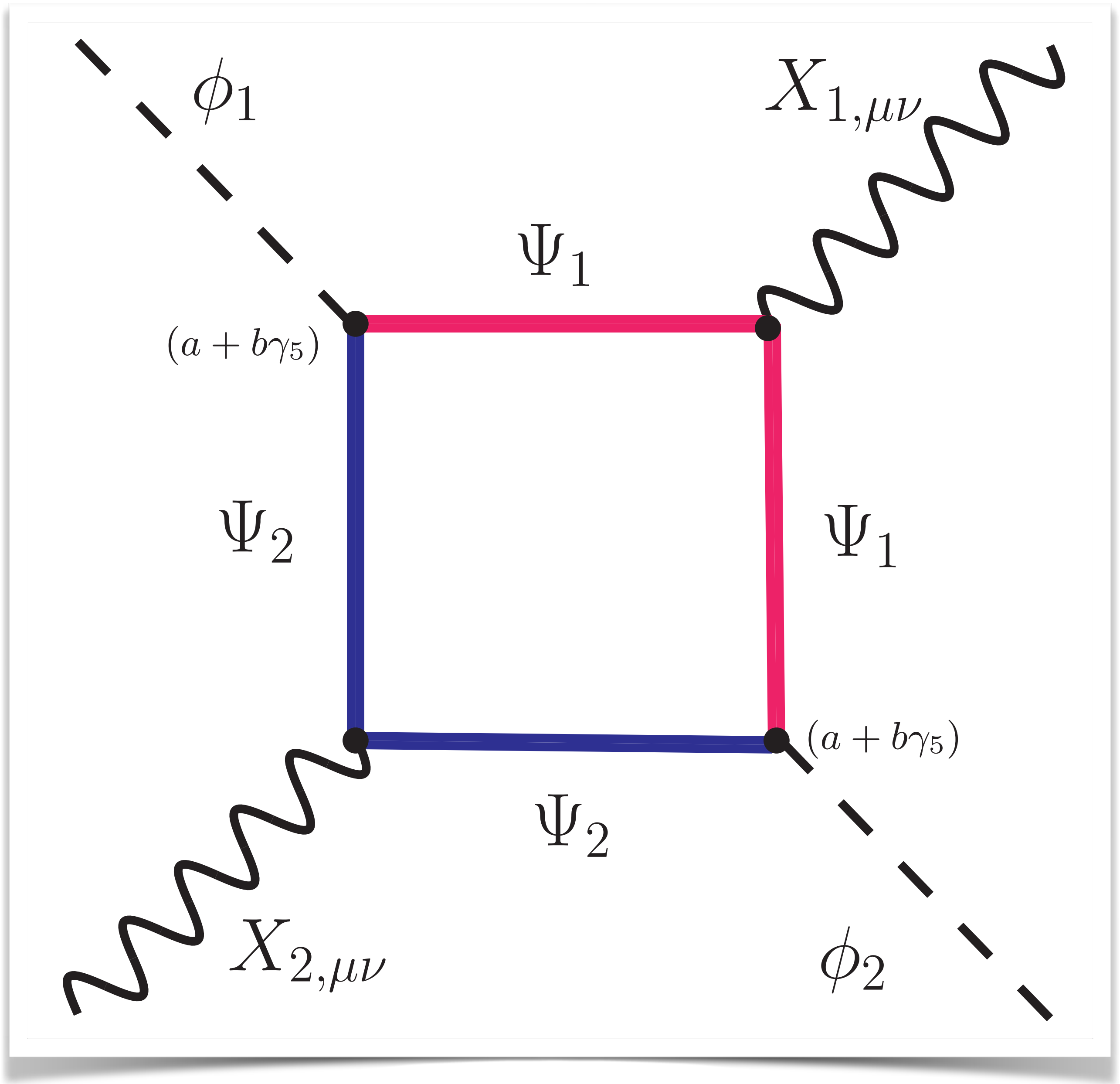
Alpha-version

- Considering only scalars in loops cannot generate the CP-violating operators.
- In case of only heavy fermions in loop, multiple heavy fermions are required to generate CPV operators.
- Example: vector like fermion extension of the SM.

$$L_{(1,2,-1/2)_{l,r}},\ E_{(1,1,-1)_{l,r}},\ N_{(1,1,0)_{l,r}}$$

- Following CP-violating operators are generated.

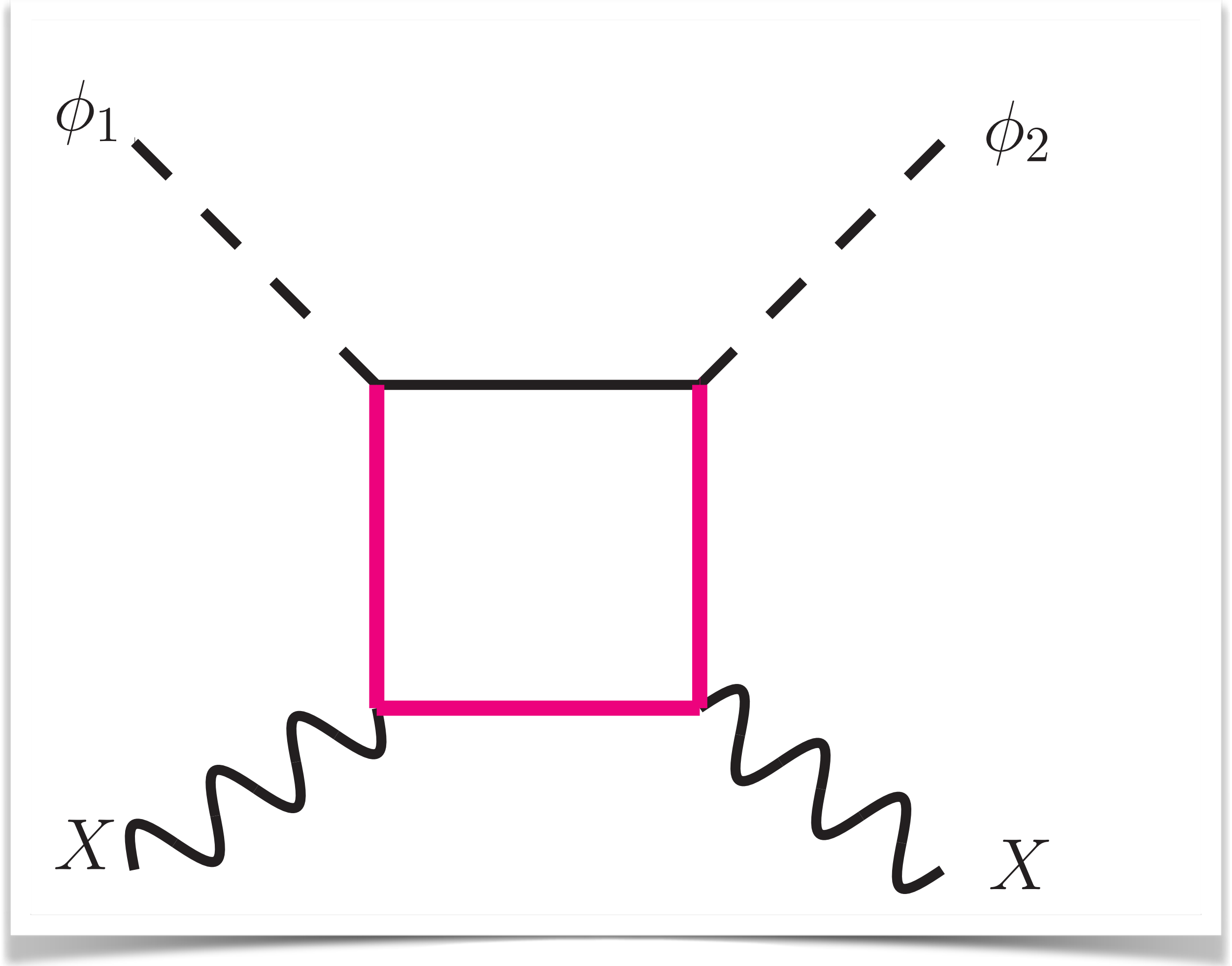
$$\{Q_{H\tilde{B}}, Q_{H\tilde{W}}, Q_{H\tilde{W}B}\}$$

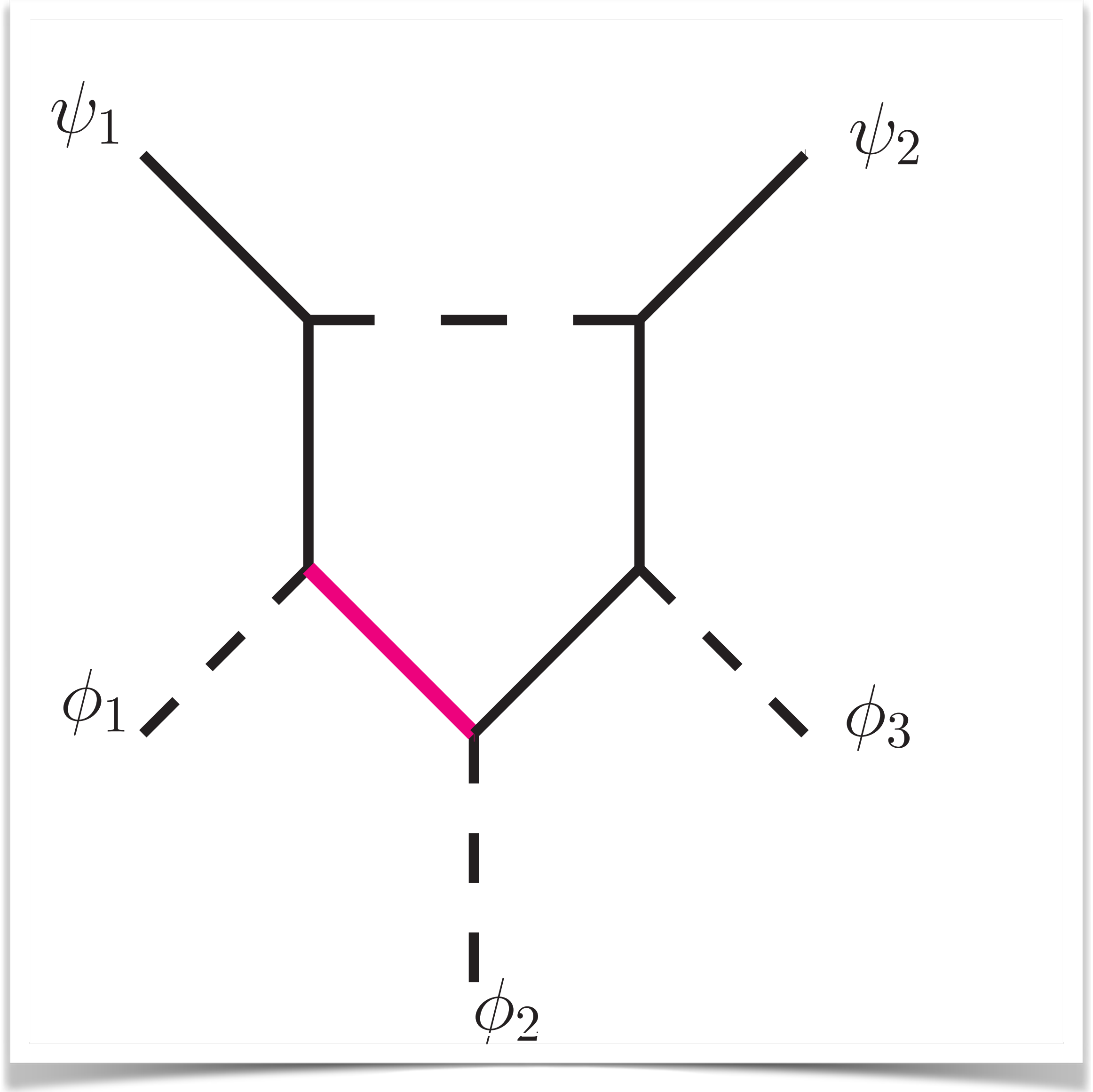


Angelescu, Huang [2006.16532]
Bakshi, Chakrabortty, Englert, Spannowsky[2009.13394]

Future Plans

- Incorporate light heavy mixing at loop level for fermions.
- Incorporate mixed spin at 1-loop (both scalar and fermion in same one loop).
- Generation of the Dimension-8 operators.
- Integrating multiple non-degenerate heavy particles.





Conclusion

- Complete matching in the diagrammatic approach can be difficult, so there is an alternative approach - covariant derivative expansion (CDE).
- CDE is a framework which integrates out heavy degrees of freedom and generate effective operator and their corresponding Wilson coefficients.
- CDE is manifestly gauge invariant and universal.
- CoDEx is based on CDE which automatises the integrating out process.
- There are few shortcomings of CoDEx: heavy-light mixing in the fermionic loops, mixed spin particles in 1-loop.

Thanks for your attention