CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

CP-odd flavour-invariants in SMEFT

Quentin Bonnefoy
(DESY Hamburg)

Workshop SM@LHC 2022 (CERN)
11/04/2022

arXiv:2112.03889 [hep-ph] + work in progress
with E. Gendy, C. Grojean and J. Ruderman

SMEFT

Assuming no new light d.o.f. beyond those of the SM, and a (parametrically) large gap to any heavy mode : use **SMEFT**

SMEFT

Assuming no new light d.o.f. beyond those of the SM, and a (parametrically) large gap to any heavy mode : use **SMEFT**

i.e. all **effective interactions of the SM d.o.f.s** (subject to gauge invariance, and additional symmetry requirements, e.g. B,L)

- $A_\mu^{a,i,Y} \quad \psi_{i,L/R} \quad H$
- $SU(3) \times SU(2) \times U(1)$
- $\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_i \frac{c_i}{\Lambda^{d_i - 4}} \mathcal{O}_i$

$$\mathcal{O}_i = C^{(1)}_{H\psi_L,ij} \overline{\psi}^i_L \gamma^\mu \psi^j_L \left(H^\dagger \overleftrightarrow{D}_\mu H \right)$$

[Buchmüller/Wyler '85, Grzadkowski et al '10]

SMEFT

Assuming no new light d.o.f. beyond those of the SM, and a (parametrically) large gap to any heavy mode : use **SMEFT**

i.e. all **effective interactions of the SM d.o.f.s** (subject to gauge invariance, and additional symmetry requirements, e.g. B,L)

- $A_\mu^{a,i,Y} \quad \psi_{i,L/R} \quad H$
- $SU(3) \times SU(2) \times U(1)$
- $\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_i \frac{c_i}{\Lambda^{d_i-4}} \mathcal{O}_i$

$$\mathcal{O}_i = C^{(1)}_{H\psi_L,ij} \bar{\psi}^i_L \gamma^\mu \psi^j_L \left(H^\dagger \overleftrightarrow{D}_\mu H \right)$$

[Buchmüller/Wyler '85, Grzadkowski et al '10]

Many parameters ! **What is the actual parameter space of observables ?**

CPV in SMEFT

Here : focus on **CP-odd observables.**

CPV in SMEFT

Here : focus on **CP-odd observables.**

CPV in the SM : **strongly suppressed**. Room for BSM effects…

CPV in SMEFT

Here : focus on **CP-odd observables.**

CPV in the SM : **strongly suppressed**. Room for BSM effects…

… already quite constrained :

$$\left|\frac{d_e}{e\cdot \mathrm{cm}}\right| < 10^{-29}$$

[ACME II '18]

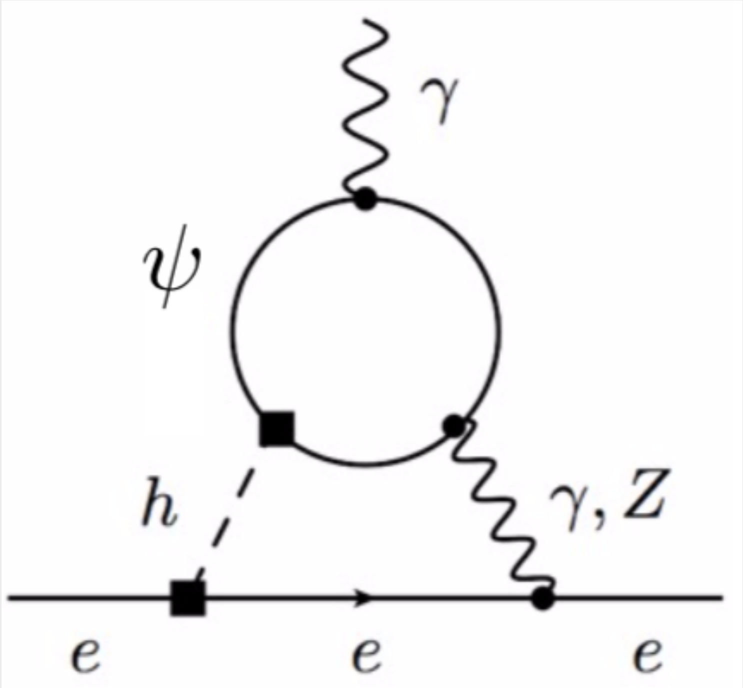
CPV in SMEFT

Here : focus on **CP-odd observables.**

CPV in the SM : **strongly suppressed**. Room for BSM effects…

… already quite constrained : $\left|\frac{d_e}{e \cdot \mathrm{cm}}\right| < 10^{-29}$ **[ACME II '18]**

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C_{uH}}{\Lambda^2}|H|^2\overline{Q_L}u_R\tilde{H} + h.c. \qquad \Longrightarrow \Lambda \gtrsim \mathrm{TeV}$$



[Barr/Zee '90, Brod/Haisch/Zupan '13]

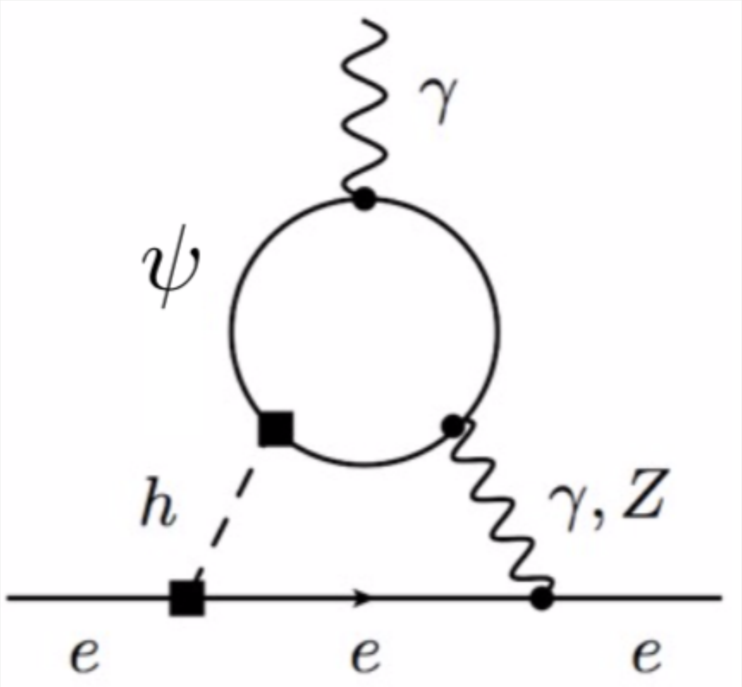
CPV in SMEFT

Here : focus on **CP-odd observables.**

CPV in the SM : **strongly suppressed**. Room for BSM effects…

… already quite constrained : $\left|\frac{d_e}{e \cdot \mathrm{cm}}\right| < 10^{-29}$ **[ACME II ’18]**

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C_{uH}}{\Lambda^2}|H|^2\overline{Q_L}u_R\tilde{H} + h.c. \quad \Longrightarrow \Lambda \gtrsim \mathrm{TeV}$$



Kaon mixing $\Longrightarrow \Lambda \gtrsim 10^{4-5}\ \mathrm{TeV}$

[Aebischer, Bobeth, Buras, Straub ’18]

[Barr/Zee ’90, Brod/Haisch/Zupan ‘13]

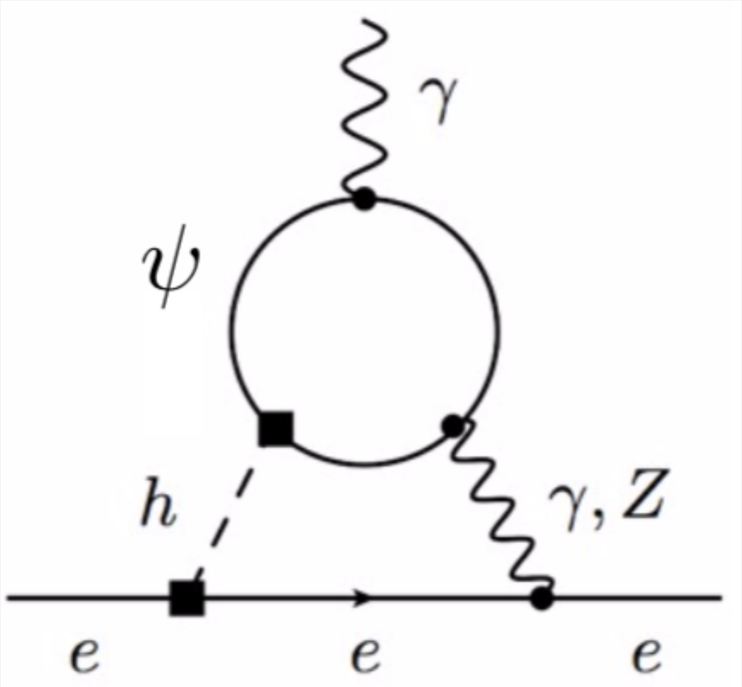
CPV in SMEFT

Here : focus on **CP-odd observables.**

CPV in the SM : **strongly suppressed**. Room for BSM effects…

… already quite constrained : $\left|\frac{d_e}{e\cdot \mathrm{cm}}\right| < 10^{-29}$ **[ACME II '18]**

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C_{uH}}{\Lambda^2}|H|^2\overline{Q_L}u_R\tilde{H} + h.c. \qquad \Longrightarrow \Lambda \gtrsim \mathrm{TeV}$$



[Barr/Zee '90, Brod/Haisch/Zupan '13]

Kaon mixing $\Longrightarrow \Lambda \gtrsim 10^{4-5}\ \mathrm{TeV}$

[Aebischer, Bobeth, Buras, Straub '18]

Need to understand the structure of CPV beyond the SM (here in the fermionic sector of SMEFT), its link to symmetries, UV assumptions…

CPV in SMEFT

Here : focus on **CP-odd observables. What is the CP-odd parameter space in the SMEFT ?**

CPV in SMEFT

Here : focus on **CP-odd observables. What is the CP-odd parameter space in the SMEFT ?**

Why asking ?

Class	N_{op}	CP-even n_g	1	3	CP-odd n_g	1	3
1	4	2	2	2	2	2	2
2	1	1	1	1	0	0	0
3	2	2	2	2	0	0	0
4	8	4	4	4	4	4	4
5	3	$3n_g^2$	3	27	$3n_g^2$	3	27
6	8	$8n_g^2$	8	72	$8n_g^2$	8	72
7	8	$\frac{1}{2}n_g(9n_g+7)$	8	51	$\frac{1}{2}n_g(9n_g-7)$	1	30
8 : $(\overline{L}L)(\overline{L}L)$	5	$\frac{1}{4}n_g^2(7n_g^2+13)$	5	171	$\frac{7}{4}n_g^2(n_g-1)(n_g+1)$	0	126
8 : $(\overline{R}R)(\overline{R}R)$	7	$\frac{1}{8}n_g(21n_g^3+2n_g^2+31n_g+2)$	7	255	$\frac{1}{8}n_g(21n_g+2)(n_g-1)(n_g+1)$	0	195
8 : $(\overline{L}L)(\overline{R}R)$	8	$4n_g^2(n_g^2+1)$	8	360	$4n_g^2(n_g-1)(n_g+1)$	0	288
8 : $(\overline{L}R)(\overline{R}L)$	1	n_g^4	1	81	n_g^4	1	81
8 : $(\overline{L}R)(\overline{L}R)$	4	$4n_g^4$	4	324	$4n_g^4$	4	324
8 : All	25	$\frac{1}{8}n_g(107n_g^3+2n_g^2+89n_g+2)$	25	1191	$\frac{1}{8}n_g(107n_g^3+2n_g^2-67n_g-2)$	5	1014
Total	59	$\frac{1}{8}(107n_g^4+2n_g^3+213n_g^2+30n_g+72)$	53	1350	$\frac{1}{8}(107n_g^4+2n_g^3+57n_g^2-30n_g+48)$	23	1149

[Grzadkowski/Iskrzynski/Misiak/Rosiek '10, Alonso/Jenkins/Manohar/Trott '13]

CPV in the SM

complex matrix !

When does the SM break CP ?

$$\mathcal{L} \supset \frac{i}{\sqrt{2}} \overline{u_L} \gamma^\mu W_\mu^+ V_{\rm CKM} d_L - \overline{u_L} \text{diag}(m_{u_i}) u_R + (d) + h.c.$$

CPV in the SM

complex matrix !

When does the SM break CP ?

$$\mathcal{L} \supset \frac{i}{\sqrt{2}} \overline{u_L} \gamma^\mu W_\mu^+ V_{\rm CKM} d_L - \overline{u_L} \mathrm{diag}(m_{u_i}) u_R + (d) + h.c.$$

$$\mathcal{J} = \mathrm{Im}(V_{\mathrm{CKM},us} V_{\mathrm{CKM},cb} V^*_{\mathrm{CKM},ub} V^*_{\mathrm{CKM},cs})$$

CPV in the SM

complex matrix !

When does the SM break CP ?

$$\mathcal{L} \supset \frac{i}{\sqrt{2}} \overline{u_L} \gamma^\mu W_\mu^+ V_{\rm CKM} d_L - \overline{u_L} \mathrm{diag}(m_{u_i}) u_R + (d) + h.c.$$

$$6(y_t^2 - y_c^2)(y_t^2 - y_u^2)(y_c^2 - y_u^2)(y_b^2 - y_s^2)(y_b^2 - y_d^2)(y_s^2 - y_d^2)\mathcal{J}$$

$$\mathcal{J} = \mathrm{Im}(V_{\mathrm{CKM},us} V_{\mathrm{CKM},cb} V^*_{\mathrm{CKM},ub} V^*_{\mathrm{CKM},cs})$$

CPV in the SM

complex matrix !

When does the SM break CP ?

$$\mathcal{L} \supset \frac{i}{\sqrt{2}} \overline{u_L} \gamma^\mu W_\mu^+ V_{\rm CKM} d_L - \overline{u_L} \text{diag}(m_{u_i}) u_R + (d) + h.c.$$

$$J_4 = \text{Im}\,\text{Tr}\left[Y_u Y_u^\dagger, Y_d Y_d^\dagger\right]^3 = 6(y_t^2 - y_c^2)(y_t^2 - y_u^2)(y_c^2 - y_u^2)(y_b^2 - y_s^2)(y_b^2 - y_d^2)(y_s^2 - y_d^2)\mathcal{J}$$

$$\mathcal{L} \supset -\overline{Q_L} Y_u u_R \tilde{H} + (d) + h.c.$$

$$Y_u = \text{diag}(y_u, y_c, y_t) \;,\; Y_d = V_{\rm CKM}\text{diag}(y_d, y_s, y_b)$$

CPV in the SM

complex matrix !

When does the SM break CP ?

$$\mathcal{L} \supset \frac{i}{\sqrt{2}} \overline{u_L} \gamma^\mu W_\mu^+ V_{\rm CKM} d_L - \overline{u_L} \mathrm{diag}(m_{u_i}) u_R + (d) + h.c.$$

$$J_4 = \mathrm{Im\,Tr}\left[Y_u Y_u^\dagger, Y_d Y_d^\dagger\right]^3$$

Flavour invariant

	$SU(3)_Q$	$SU(3)_u$	$SU(3)_d$	$SU(3)_L$	$SU(3)_e$
Q_L	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Y_u	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Y_d	$\mathbf{3}$	$\mathbf{1}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$
Y_e	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\bar{\mathbf{3}}$

CPV in the SM

complex matrix !

When does the SM break CP ?

$$\mathcal{L} \supset \frac{i}{\sqrt{2}} \overline{u_L} \gamma^\mu W_\mu^+ V_{\rm CKM} d_L - \overline{u_L} {\rm diag}(m_{u_i}) u_R + (d) + h.c.$$

$$J_4 = {\rm Im\,Tr}\left[Y_u Y_u^\dagger, Y_d Y_d^\dagger\right]^3$$

Flavour invariant

	$SU(3)_Q$	$SU(3)_u$	$SU(3)_d$	$SU(3)_L$	$SU(3)_e$
Q_L	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Y_u	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Y_d	$\mathbf{3}$	$\mathbf{1}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$
Y_e	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\bar{\mathbf{3}}$

CP is conserved in the SM
iff
J4=0

[Jarlskog '85]

CPV in the SM

complex matrix !

When does the SM break CP ?

$$\mathcal{L} \supset \frac{i}{\sqrt{2}} \overline{u_L} \gamma^\mu W_\mu^+ V_{\mathrm{CKM}} d_L - \overline{u_L} \mathrm{diag}(m_{u_i}) u_R + (d) + h.c.$$

$$J_4 = \mathrm{Im}\,\mathrm{Tr}\left[Y_u Y_u^\dagger, Y_d Y_d^\dagger\right]^3$$

Presence of Y_u and Y_d

$\Longrightarrow$ CP breaking is a **collective effect**

CP is conserved in the SM
iff
J4=0

[Jarlskog '85]

CPV in the SM

complex matrix !

When does the SM break CP ?

$$\mathcal{L} \supset \frac{i}{\sqrt{2}} \overline{u_L} \gamma^\mu W^+_\mu V_{\rm CKM} d_L - \overline{u_L} \mathrm{diag}(m_{u_i}) u_R + (d) + h.c.$$

$$J_4 = \mathrm{Im}\,\mathrm{Tr}\left[Y_u Y_u^\dagger, Y_d Y_d^\dagger\right]^3$$

Presence of Y_u and Y_d

$\Longrightarrow$ CP breaking is a **collective effect**

Explains suppressions

$$d_e^{\rm Fig.1a} \sim e\mathcal{J} \frac{m_e m_c^2 m_s^2}{m_W^6} \frac{\alpha_W^3 \alpha_s}{(4\pi)^4}$$

[Pospelov/ Ritz '13]

CP is conserved in the SM
iff
J4=0

[Jarlskog '85]

CPV BSM

One can include neutrino masses, more Higgs doublets, SUSY, vector-like fermions, etc

[Many authors]

CPV in SMEFT

When does the SMEFT break CP ?

CPV in SMEFT

When does the SMEFT break CP ?

Collective effects also in the SMEFT

CPV in SMEFT

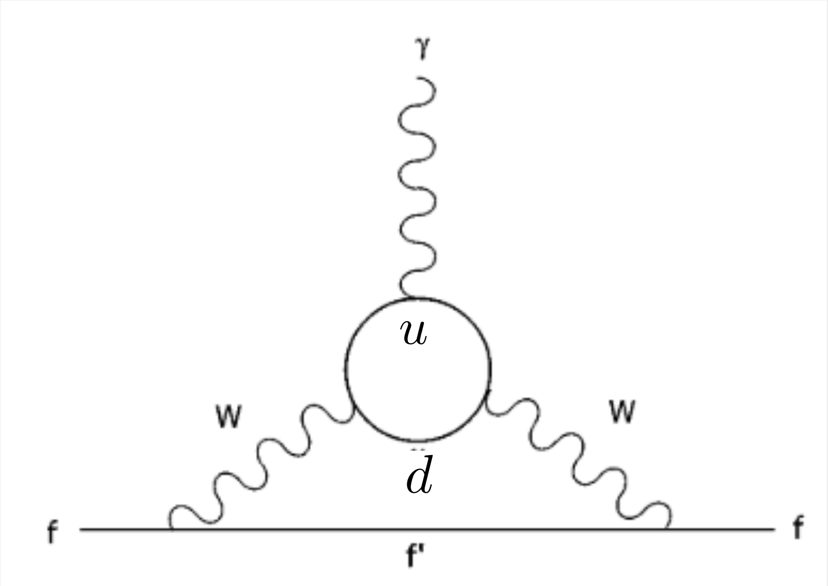
When does the SMEFT break CP ?

Collective effects also in the SMEFT

Example : two-loop contribution to the electron EDM

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{C_{Hud,mn}}{\Lambda^2} i\tilde{H}^\dagger D_\mu H \overline{u}_{R,m}\gamma^\mu d_{R,n}$$

$$\frac{d}{e} = \sum_{i,j} \mathrm{Im}\left(V_{CKM,ij} C^*_{Hud,ij}\right) F(m^2_{u_i}, m^2_{d_j})$$



[Kadoyoshi/Oshimo '97]

CPV in SMEFT

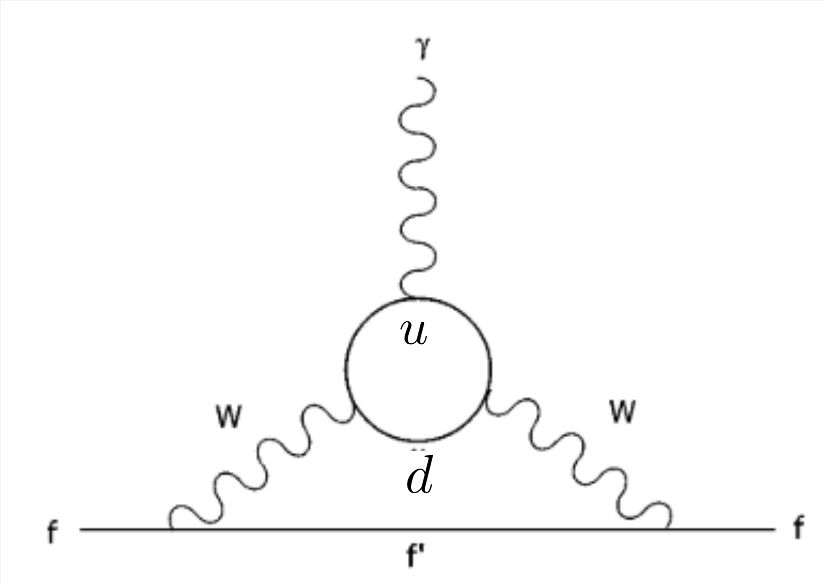
When does the SMEFT break CP ?

Collective effects also in the SMEFT

Example : two-loop contribution to the electron EDM

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{C_{Hud,mn}}{\Lambda^2} i\tilde{H}^\dagger D_\mu H \overline{u}_{R,m}\gamma^\mu d_{R,n}$$

$$\frac{d}{e} = \sum_{i,j} \mathrm{Im}\left(V_{CKM,ij} C^*_{Hud,ij}\right) F(m^2_{u_i}, m^2_{d_j})$$



[Kadoyoshi/Oshimo '97]

Can be studied using **CP-odd flavour invariants**

CPV in SMEFT

When does the SMEFT break CP **at order** $1/\Lambda^2$?

$$A_\mu^{a,i,Y} \quad \psi_{i,L/R} \quad H$$

$$SU(3) \times SU(2) \times U(1)$$

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_i \frac{c_i}{\Lambda^{d_i-4}} \mathcal{O}_i$$

$$\mathcal{O}_i = C^{(1)}_{H\psi_L,ij} \overline{\psi}^i_L \gamma^\mu \psi^j_L \left(H^\dagger \overleftrightarrow{D}_\mu H \right)$$

[Buchmüller/Wyler '85, Grzadkowski et al '10]

CPV in SMEFT

When does the SMEFT break CP **at order** $1/\Lambda^2$?

$$\mathcal{A} = \mathcal{A}^{(4)} + \mathcal{A}^{(6)} + \ldots \Longrightarrow |\mathcal{A}|^2 = |\mathcal{A}^{(4)}|^2 + 2\mathrm{Re}\left(\mathcal{A}^{(4)}\mathcal{A}^{(6)*}\right) + \ldots$$

$$A_\mu^{a,i,Y} \quad \psi_{i,L/R} \quad H$$

$$SU(3) \times SU(2) \times U(1)$$

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{c_i}{\Lambda^{d_i - 4}} \mathcal{O}_i$$

$$\mathcal{O}_i = C^{(1)}_{H\psi_L,ij} \overline{\psi}^i_L \gamma^\mu \psi^j_L \left(H^\dagger \overleftrightarrow{D}_\mu H\right)$$

[Buchmüller/Wyler ’85, Grzadkowski et al ’10]

CPV in SMEFT

The SMEFT breaks CP **at dimension-four** (J4 != 0)

$$\mathcal{A} = \mathcal{A}^{(4)} + \mathcal{A}^{(6)} + \ldots \Longrightarrow |\mathcal{A}|^2 = |\mathcal{A}^{(4)}|^2 + 2\mathrm{Re}\left(\mathcal{A}^{(4)}\mathcal{A}^{(6)*}\right) + \ldots$$

breaks CP

CPV in SMEFT

The SMEFT breaks CP **at dimension-four** (J4 != 0)

$$\mathcal{A} = \mathcal{A}^{(4)} + \mathcal{A}^{(6)} + \ldots \Longrightarrow |\mathcal{A}|^2 = |\mathcal{A}^{(4)}|^2 + 2\mathrm{Re}\left(\mathcal{A}^{(4)}\mathcal{A}^{(6)*}\right) + \ldots$$

breaks CP

?

CPV in SMEFT

The SMEFT breaks CP **at dimension-four** (J4 != 0)

$$\mathcal{A} = \mathcal{A}^{(4)} + \mathcal{A}^{(6)} + \ldots \Longrightarrow |\mathcal{A}|^2 = |\mathcal{A}^{(4)}|^2 + 2\mathrm{Re}\left(\mathcal{A}^{(4)}\mathcal{A}^{(6)*}\right) + \ldots$$

breaks CP

?

Question : what are the **new sources of CPV at order** $1/\Lambda^2$?

CPV in SMEFT

The SMEFT breaks CP **at dimension-four** (J4 != 0)

$$\mathcal{A} = \mathcal{A}^{(4)} + \mathcal{A}^{(6)} + \ldots \Longrightarrow |\mathcal{A}|^2 = |\mathcal{A}^{(4)}|^2 + 2\mathrm{Re}\left(\mathcal{A}^{(4)}\mathcal{A}^{(6)*}\right) + \ldots$$

breaks CP

?

Question : what are the **new sources of CPV at order** $1/\Lambda^2$?

(and : how are they related to CPV in UV models ? how large is the first SMEFT correction to CPV obs. ? etc)

CPV in SMEFT

The SMEFT breaks CP **at dimension-four** (J4 != 0)

$$\mathcal{A} = \mathcal{A}^{(4)} + \mathcal{A}^{(6)} + \ldots \Longrightarrow |\mathcal{A}|^2 = |\mathcal{A}^{(4)}|^2 + 2\mathrm{Re}\left(\mathcal{A}^{(4)}\mathcal{A}^{(6)*}\right) + \ldots$$

breaks CP

?

Question : what are the **new sources of CPV at order** $1/\Lambda^2$?

(and : how are they related to CPV in UV models ? how large is the first SMEFT correction to CPV obs. ? etc)

Expressed in terms of **CP-odd flavour invariants**

CP in SMEFT

CP is conserved in the SMEFT at order $1/\Lambda^2$
iff
? = 0

CP in SMEFT

CP is conserved in the SMEFT at order $1/\Lambda^2$
iff
J4 = 0 & ? = 0

CP conservation follows the EFT power counting

$\Longrightarrow$ CP should be conserved order by order

$$|\mathcal{A}|^2 = |\mathcal{A}^{(4)}|^2 + 2\mathrm{Re}\left(\mathcal{A}^{(4)}\mathcal{A}^{(6)*}\right) + \ldots$$

CP in SMEFT

CP is conserved in the SMEFT at order $1/\Lambda^2$
iff
J4 = 0 & ? = 0

CP conservation follows the EFT power counting

$\Longrightarrow$ CP should be conserved order by order

$$|\mathcal{A}|^2 = |\mathcal{A}^{(4)}|^2 + 2\mathrm{Re}\left(\mathcal{A}^{(4)}\mathcal{A}^{(6)*}\right) + \ldots$$

CP in SMEFT

CP is conserved in the SMEFT at order $1/\Lambda^2$
iff
J4 = 0 & L_{1,2,...,N} = 0

How many conditions ?

$$|\mathcal{A}|^2 = |\mathcal{A}^{(4)}|^2 + 2\mathrm{Re}\left(\mathcal{A}^{(4)}\mathcal{A}^{(6)*}\right) + \ldots$$

CP in SMEFT

CP is conserved in the SMEFT at order $1/\Lambda^2$
iff
J4 = 0 & L_{1,2,...,N} = 0

How many conditions ?

Some imaginary parts cannot interfere at order $1/\Lambda^2$

$$|\mathcal{A}|^2 = |\mathcal{A}^{(4)}|^2 + 2\mathrm{Re}\left(\mathcal{A}^{(4)}\mathcal{A}^{(6)*}\right) + ...$$

CP in SMEFT

CP is conserved in the SMEFT at order $1/\Lambda^2$
iff
J4 = 0 & L_{1,2,...,N} = 0

How many conditions ?

Some imaginary parts cannot interfere at order $1/\Lambda^2$

$$|\mathcal{A}|^2 = |\mathcal{A}^{(4)}|^2 + 2\mathrm{Re}\left(\mathcal{A}^{(4)}\mathcal{A}^{(6)*}\right) + \dots$$

does not depend on 2484 SMEFT coefficients, but on 1596, among which 699 CP-odd

CP in SMEFT

CP is conserved in the SMEFT at order $1/\Lambda^2$
iff
J4 = 0 & L_{1,2,...,N} = 0

How many conditions ?

Some imaginary parts cannot interfere at order $1/\Lambda^2$

$$|\mathcal{A}|^2 = |\mathcal{A}^{(4)}|^2 + 2\text{Re}\left(\mathcal{A}^{(4)}\mathcal{A}^{(6)*}\right) + \ldots$$

does not depend on 2484 SMEFT coefficients, but on 1596, among which 699 CP-odd

CP in SMEFT

CP is conserved in the SMEFT at order $1/\Lambda^2$
iff
J4 = 0 & L_{1,2,...,N} = 0

There exist sets of flavour invariants which fulfil this !

CP in SMEFT

CP is conserved in the SMEFT at order $1/\Lambda^2$
iff
J4 = 0 & L_{1,2,...,N} = 0

There exist sets of flavour invariants which fulfil this !

Ex :

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{C^{(1)}_{HQ,ij}}{\Lambda^2}\left(iH^\dagger \overleftrightarrow{D}_\mu H\right)\overline{Q_{i,L}}\gamma^\mu Q_{j,L}$$

preserves CP at order $1/\Lambda^2$ iff

$$J_4 = 0$$

$$L_1 = {\rm ImTr}\left(H_u H_d C^{(1)}_{HQ}\right) = 0$$

$$L_2 = {\rm ImTr}\left(H_u^2 H_d^2 C^{(1)}_{HQ}\right) = 0$$

$$L_3 = {\rm ImTr}\left(H_u H_d H_u^2 H_d^2 C^{(1)}_{HQ}\right) = 0$$

$$H_u \equiv Y_u Y_u^\dagger$$
$$H_d \equiv Y_d Y_d^\dagger$$

CP in SMEFT

CP is conserved in the SMEFT at order $1/\Lambda^2$
iff
J4 = 0 & L_{1,2,…,N} = 0

There exist sets of flavour invariants which fulfil this !

Ex :

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{C^{(1)}_{HQ,ij}}{\Lambda^2}\left(iH^\dagger \overleftrightarrow{D}_\mu H\right)\overline{Q_{i,L}}\gamma^\mu Q_{j,L}$$

preserves CP at order $1/\Lambda^2$ iff

$$J_4 = 0$$
$$L_1 = \mathrm{Im}\mathrm{Tr}\left(H_u H_d C^{(1)}_{HQ}\right) = 0$$
$$L_2 = \mathrm{Im}\mathrm{Tr}\left(H_u^2 H_d^2 C^{(1)}_{HQ}\right) = 0$$
$$L_3 = \mathrm{Im}\mathrm{Tr}\left(H_u H_d H_u^2 H_d^2 C^{(1)}_{HQ}\right) = 0$$

One + three conditions, down to one + one when mt=mc & ms=md, …

CP in SMEFT

CP is conserved in the SMEFT at order $1/\Lambda^2$
iff
J4 = 0 & L_{1,2,…,N} = 0

Easily generalized to all SMEFT operators

Need a **finite set of algebraically-independent structures**

[Jenkins/Manohar '09]

For fermion bilinears : $\mathrm{Im}\,\mathrm{Tr}(H_u^a H_d^b H_u^c H_d^d M)$ with $a, b, c, d = 0, 1, 2$
$a \neq c, b \neq d$

For 4-Fermi operators : « A-type » $\mathrm{Im}\left([H^{(abcd)}]_{ji} [H^{(efgh)}]_{lk}\ M_{ijkl} \right)$

« B-type » $\mathrm{Im}\left([H^{(abcd)}]_{jk} [H^{(efgh)}]_{li}\ M_{ijkl} \right)$

CPV in SMEFT

CPV in SMEFT

Must be **proportional to invariants**. **Which ones?**

CPV in SMEFT

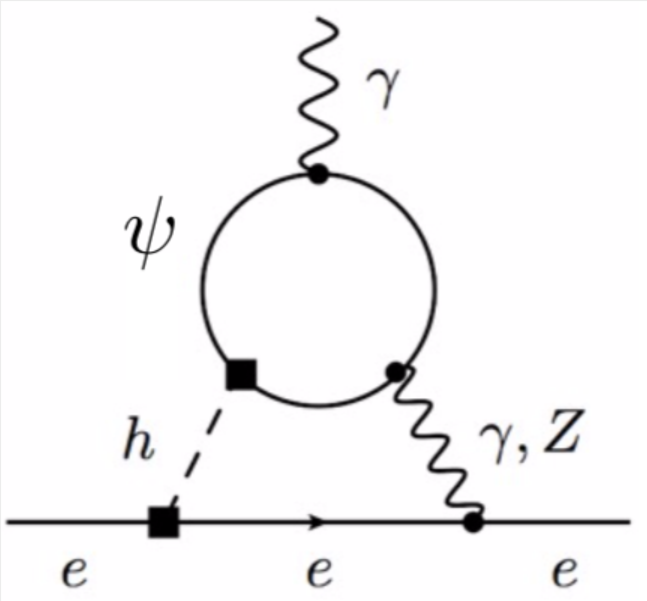
Must be **proportional to invariants**. **Which ones?**

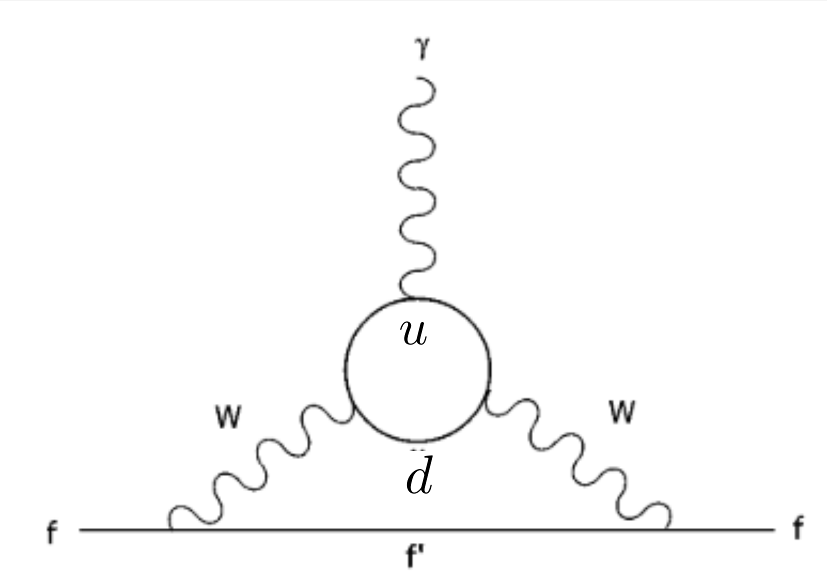
« Simple » graphs map to new sources of CPV (for invariant observables, e.g. EDMs)

CPV in SMEFT

Must be **proportional to invariants**. **Which ones?**

« Simple » graphs map to new sources of CPV (for invariant observables, e.g. EDMs)





CPV in SMEFT

Must be **proportional to invariants**. **Which ones?**

« Simple » graphs map to new sources of CPV (for invariant observables, e.g. EDMs)

For a given SMEFT coefficient : algebraic approach to the **suppression** associated to **collective effects**

CPV in SMEFT

Must be **proportional to invariants**. **Which ones?**

« Simple » graphs map to new sources of CPV (for invariant observables, e.g. EDMs)

For a given SMEFT coefficient : algebraic approach to the **suppression** associated to **collective effects**

À la Wolfenstein :

$$Y_u = \text{diag}\left(a_u\lambda^8, a_c\lambda^4, a_t\right)$$

$$Y_d = V_{\text{CKM}}\text{diag}\left(a_d\lambda^7, a_s\lambda^5, a_b\lambda^3\right)$$

$$V_{\text{CKM}} = \begin{pmatrix} 1 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

CPV in SMEFT

Must be **proportional to invariants**. **Which ones?**

« Simple » graphs map to new sources of CPV (for invariant observables, e.g. EDMs)

For a given SMEFT coefficient : algebraic approach to the **suppression** associated to **collective effects**

À la Wolfenstein :

$$Y_u = \text{diag}\left(a_u\lambda^8, a_c\lambda^4, a_t\right)$$

$$Y_d = V_{\text{CKM}}\text{diag}\left(a_d\lambda^7, a_s\lambda^5, a_b\lambda^3\right)$$

$$V_{\text{CKM}} = \begin{pmatrix} 1 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix} = \begin{pmatrix} Aa_b^2 a_t^2 \text{Im}C^{(1)}_{HQ,23}\lambda^8 \\ 0 \\ 0 \end{pmatrix} + \mathcal{O}(\lambda^9)$$

Rank increase at $\mathcal{O}(\lambda^{19})$

CPV in SMEFT

Must be **proportional to invariants**. **Which ones?**

« Simple » graphs map to new sources of CPV (for invariant observables, e.g. EDMs)

For a given SMEFT coefficient : algebraic approach to the **suppression** associated to **collective effects**

À la Wolfenstein :

$$Y_u = \mathrm{diag}\left(a_u\lambda^8, a_c\lambda^4, a_t\right)$$

$$Y_d = V_{\mathrm{CKM}}\mathrm{diag}\left(a_d\lambda^7, a_s\lambda^5, a_b\lambda^3\right)$$

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

16 with MFV

$$\begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix} = \begin{pmatrix} A a_b^2 a_t^2 \mathrm{Im} C^{(1)}_{HQ,23}\lambda^8 \\ 0 \\ 0 \end{pmatrix} + \mathcal{O}(\lambda^9)$$

Rank increase at $\mathcal{O}(\lambda^{19})$

Outlook

We built **flavour invariants** which capture all the **CP-odd** physical parameters in the fermionic sector of the SMEFT at order $1/\Lambda^2$

They can be used to identify collective aspects of CPV, evaluate its level of suppression, study how CPV is transferred from UV models to the SMEFT… and maybe more ?

THANK YOU

Warsaw basis

Bilinears

Modified Yukawa	
Q_{eH}	$(H^\dagger H)(\bar{L}_i e_j H)$
Q_{uH}	$(H^\dagger H)(\bar{Q}_i u_j \widetilde{H})$
Q_{dH}	$(H^\dagger H)(\bar{Q}_i d_j H)$

Dipole	
Q_{eW}	$(\bar{L}_i \sigma^{\mu\nu} e_j)\tau^I H W^I_{\mu\nu}$
Q_{eB}	$(\bar{L}_i \sigma^{\mu\nu} e_j) H B_{\mu\nu}$
Q_{uG}	$(\bar{Q}_i \sigma^{\mu\nu} T^A u_j)\widetilde{H}\, G^A_{\mu\nu}$
Q_{uW}	$(\bar{Q}_i \sigma^{\mu\nu} u_j)\tau^I \widetilde{H}\, W^I_{\mu\nu}$
Q_{uB}	$(\bar{Q}_i \sigma^{\mu\nu} u_j)\widetilde{H}\, B_{\mu\nu}$
Q_{dG}	$(\bar{Q}_i \sigma^{\mu\nu} T^A d_j) H\, G^A_{\mu\nu}$
Q_{dW}	$(\bar{Q}_i \sigma^{\mu\nu} d_j)\tau^I H\, W^I_{\mu\nu}$
Q_{dB}	$(\bar{Q}_i \sigma^{\mu\nu} d_j) H\, B_{\mu\nu}$

Current-current	
$Q^{(1)}_{HL}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{L}_i \gamma^\mu L_j)$
$Q^{(3)}_{HL}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{L}_i \tau^I \gamma^\mu L_j)$
Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_i \gamma^\mu e_j)$
$Q^{(1)}_{HQ}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{Q}_i \gamma^\mu Q_j)$
$Q^{(3)}_{HQ}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{Q}_i \tau^I \gamma^\mu Q_j)$
Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}_i \gamma^\mu u_j)$
Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}_i \gamma^\mu d_j)$
Q_{Hud} + h.c.	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_i \gamma^\mu d_j)$

4-Fermi

$(\bar{L}L)(\bar{L}L)$	
Q_{LL}	$(\bar{L}_i \gamma_\mu L_j)(\bar{L}_k \gamma^\mu L_l)$
$Q^{(1)}_{QQ}$	$(\bar{Q}_i \gamma_\mu Q_j)(\bar{Q}_k \gamma^\mu Q_l)$
$Q^{(3)}_{QQ}$	$(\bar{Q}_i \gamma_\mu \tau^I Q_j)(\bar{Q}_k \gamma^\mu \tau^I Q_l)$
$Q^{(1)}_{LQ}$	$(\bar{L}_i \gamma_\mu L_j)(\bar{Q}_k \gamma^\mu Q_l)$
$Q^{(3)}_{LQ}$	$(\bar{L}_i \gamma_\mu \tau^I L_j)(\bar{Q}_k \gamma^\mu \tau^I Q_l)$

$(\bar{R}R)(\bar{R}R)$	
Q_{ee}	$(\bar{e}_i \gamma_\mu e_j)(\bar{e}_k \gamma^\mu e_l)$
Q_{uu}	$(\bar{u}_i \gamma_\mu u_j)(\bar{u}_k \gamma^\mu u_l)$
Q_{dd}	$(\bar{d}_i \gamma_\mu d_j)(\bar{d}_k \gamma^\mu d_l)$
Q_{eu}	$(\bar{e}_i \gamma_\mu e_j)(\bar{u}_k \gamma^\mu u_l)$
Q_{ed}	$(\bar{e}_i \gamma_\mu e_j)(\bar{d}_k \gamma^\mu d_l)$
$Q^{(1)}_{ud}$	$(\bar{u}_i \gamma_\mu u_j)(\bar{d}_k \gamma^\mu d_l)$
$Q^{(8)}_{ud}$	$(\bar{u}_i \gamma_\mu T^A u_j)(\bar{d}_k \gamma^\mu T^A d_l)$

$(\bar{L}L)(\bar{R}R)$	
Q_{Le}	$(\bar{L}_i \gamma_\mu L_j)(\bar{e}_k \gamma^\mu e_l)$
Q_{Lu}	$(\bar{L}_i \gamma_\mu L_j)(\bar{u}_k \gamma^\mu u_l)$
Q_{Ld}	$(\bar{L}_i \gamma_\mu L_j)(\bar{d}_k \gamma^\mu d_l)$
Q_{Qe}	$(\bar{Q}_i \gamma_\mu Q_j)(\bar{e}_k \gamma^\mu e_l)$
$Q^{(1)}_{Qu}$	$(\bar{Q}_i \gamma_\mu Q_j)(\bar{u}_k \gamma^\mu u_l)$
$Q^{(8)}_{Qu}$	$(\bar{Q}_i \gamma_\mu T^A Q_j)(\bar{u}_k \gamma^\mu T^A u_l)$
$Q^{(1)}_{Qd}$	$(\bar{Q}_i \gamma_\mu Q_j)(\bar{d}_k \gamma^\mu d_l)$
$Q^{(8)}_{Qd}$	$(\bar{Q}_i \gamma_\mu T^A Q_j)(\bar{d}_k \gamma^\mu T^A d_l)$

$(\bar{L}R)(\bar{R}L)$ + h.c.	
Q_{LedQ}	$(\bar{L}^a_i e_j)(\bar{d}_k Q_{la})$

$(\bar{L}R)(\bar{L}R)$ + h.c.	
$Q^{(1)}_{QuQd}$	$(\bar{Q}^a_i u_j)\epsilon_{ab}(\bar{Q}^b_k d_l)$
$Q^{(8)}_{QuQd}$	$(\bar{Q}^a_i T^A u_j)\epsilon_{ab}(\bar{Q}^b_k T^A d_l)$
$Q^{(1)}_{LeQu}$	$(\bar{L}^a_i e_j)\epsilon_{ab}(\bar{Q}^b_k u_l)$
$Q^{(3)}_{LeQu}$	$(\bar{L}^a_i \sigma_{\mu\nu} e_j)\epsilon_{ab}(\bar{Q}^k_s \sigma^{\mu\nu} u_t)$

Flavour symmetries and CPV quantities

Non-generic values			Flavor symmetries of the SM Lagrangian
Generic case: $\delta_{\mathrm{CKM}} = 0$			$U(1)_B$
$s_{12} = 0$			$U(1)_B$
$c_{12} = 0$			$U(1)_B$
$s_{23} = 0$	-		$U(1)_B$
	$s_{12} = 0$		$U(1)^2$
	$c_{12} = 0$		$U(1)^2$
$c_{23} = 0$	-		$U(1)_B$
	$s_{12} = 0$		$U(1)^2$
	$c_{12} = 0$		$U(1)^2$
$s_{13} = 0$	-		$U(1)_B$
	$s_{12} = 0$	-	$U(1)^2$
		$s_{23} = 0$	$U(1)^3$
		$c_{23} = 0$	$U(1)^3$
	$c_{12} = 0$	-	$U(1)^2$
		$s_{23} = 0$	$U(1)^3$
		$c_{23} = 0$	$U(1)^3$
$c_{13} = 0$			$U(1)^2$
$m_{u_i} = m_{u_j}$	-		$U(1)_B$
	$s_{12} = 0$		$U(1)^2$
	$c_{12} = 0$		$U(1)^2$
	$s_{13} = 0$	-	$U(1)^2$
		$s_{12} = 0$	$U(1)^3$
		$c_{12} = 0$	$U(1)^3$
	$c_{13} = 0$	-	$U(1)^3$
	$m_{d_k} = m_{d_l}$	-	$U(1)^2$
		$c_{13} = 0$	$U(2) \times U(1)$
		$s_{13} = 0$	$U(1)^3$
All m_{u_i} equal	-		$U(1)^3$
	$m_{d_k} = m_{d_l}$		$U(2) \times U(1)$
	all m_{d_k} equal		$U(3)$

Flavour symmetries and CPV quantities

Flavour symmetries of the SM	C_{eH} C_{eW} C_{eB}	C_{uH} C_{dH} C_{uG} C_{uW} C_{uB} C_{dG} C_{dW} C_{dB} C_{Hud}	$C_{HL}^{1,3}$ C_{He}	$C_{HQ}^{1,3}$ C_{Hu} C_{Hd}	C_{LL} C_{ee}	C_{Le}	$C_{QQ}^{1,3}$ C_{uu} C_{dd}	$C_{LQ}^{1,3}$ C_{eu} C_{ed} C_{Lu} C_{Ld} C_{Qe}	$C_{ud}^{1,8}$ $C_{Qu}^{1,8}$ $C_{Qd}^{1,8}$	C_{LedQ} $C_{LeQu}^{1,3}$	$C_{QuQd}^{1,8}$
$U(1)_B$	3	9	0	3	0	3	18	9	36	27	81
$U(1)^2$	3	5	0	1	0	3	5	3	12	15	33
$U(1)^3$	3	3	0	0	0	3	0	0	3	9	15
$U(2) \times U(1)$	3	2	0	0	0	3	0	0	2	6	6
$U(3)$	3	1	0	0	0	3	0	0	1	3	2
Two degenerate leptons	$\times\frac{2}{3}$	$\times 1$	$\times\frac{2}{3}$	$\times 1$	$\times\frac{2}{3}$	$\times\frac{2}{3}$	$\times 1$	$\times\frac{2}{3}$	$\times 1$	$\times\frac{2}{3}$	$\times 1$
All leptons degenerate	$\times\frac{1}{3}$	$\times 1$	$\times\frac{1}{3}$	$\times 1$	$\times\frac{1}{3}$	$\times\frac{1}{3}$	$\times 1$	$\times\frac{1}{3}$	$\times 1$	$\times\frac{1}{3}$	$\times 1$

Physical parameters at order $1/\Lambda^2$

	Type of op.	# of ops	# real	# im.	
bilinears	Yuk.	3	$3N^2$	$3N^2$	$\longleftarrow$ # physical parameters
			$2N^2+N$	$2N^2+N$	$\longleftarrow$ # physical at $\mathcal{O}(1/\Lambda^2)$
	Dipole	8	$8N^2$	$8N^2$	
			$6N^2+2N$	$6N^2+2N$	
	curr-curr	8	$\frac{1}{2}N(9N+7)$	$\frac{1}{2}N(9N-7)$	
			$N(3N+5)$	$N(3N-2)$	
	all bilinears	19	$\frac{1}{2}N(31N+7)$	$\frac{1}{2}N(31N-7)$	
			$N(11N+8)$	$N(11N+1)$	
4-Fermi	LLLL	5	$\frac{1}{4}N^2\left(7N^2+13\right)$	$\frac{7}{4}N^2\left(N^2-1\right)$	
			$\frac{1}{2}N^2\left(N^2+2N+7\right)$	$\frac{1}{2}N^2\left(N^2+2N-3\right)$	
	RRRR	7	$\frac{1}{8}N\left(21N^3+2N^2+31N+2\right)$	$\frac{1}{8}N(21N+2)(N^2-1)$	
			$\frac{1}{2}N\left(3N^3+2N^2+8N+1\right)$	$\frac{1}{2}N^2\left(3N^2+2N-5\right)$	
	LLRR	8	$4\ N^2\left(N^2+1\right)$	$4N^2\left(N^2-1\right)$	
			$\frac{1}{2}N\left(4N^3+3N^2+9N+2\right)$	$\frac{1}{2}N\left(4N^3+3N^2-6N-1\right)$	
	LRRL	1	N^4	N^4	
			N^3	N^3	
	LRLR	4	$4N^4$	$4N^4$	
			$2N^3(N+1)$	$2N^3(N+1)$	
	all 4-Fermi	25	$\frac{1}{8}N\left(107N^3+2N^2+89N+2\right)$	$\frac{1}{8}N\left(107N^3+2N^2-67N-2\right)$	
			$\frac{1}{2}N\left(12N^3+13N^2+24N+3\right)$	$\frac{1}{2}N\left(12N^3+13N^2-14N-1\right)$	
	all	44	$\frac{1}{8}N\left(107N^3+2N^2+213N+30\right)$	$\frac{1}{8}N\left(107N^3+2N^2+57N-30\right)$	
			$\frac{1}{2}N\left(12N^3+13N^2+46N+19\right)$	$\frac{1}{2}N\left(12N^3+13N^2+8N+1\right)$	

More invariants : fermion bilinears

Wilson coefficient	Number of phases	Minimal set
$C_e \equiv \begin{cases} C_{eH} \\ C_{eW} \\ C_{eB} \end{cases}$	3	$\left\{ \begin{array}{c} L_0\left(C_e Y_e^\dagger\right) \\ L_1\left(C_e Y_e^\dagger\right) \\ L_2\left(C_e Y_e^\dagger\right) \end{array} \right\}$
$C_u \equiv \begin{cases} C_{uH} \\ C_{uG} \\ C_{uW} \\ C_{uB} \end{cases}$	9	$\left\{ \begin{array}{c} L_{0000}\left(C_u Y_u^\dagger\right) \\ L_{1000}\left(C_u Y_u^\dagger\right) \\ L_{0100}\left(C_u Y_u^\dagger\right) \\ L_{1100}\left(C_u Y_u^\dagger\right) \\ L_{0110}\left(C_u Y_u^\dagger\right) \\ L_{2200}\left(C_u Y_u^\dagger\right) \\ L_{0220}\left(C_u Y_u^\dagger\right) \\ L_{1220}\left(C_u Y_u^\dagger\right) \\ L_{0122}\left(C_u Y_u^\dagger\right) \end{array} \right\}$
$C_d \equiv \begin{cases} C_{dH} \\ C_{dG} \\ C_{dW} \\ C_{dB} \end{cases}$		Same with $C_u Y_u^\dagger \to C_d Y_d^\dagger$
C_{Hud}		Same with $C_u Y_u^\dagger \to Y_u C_{Hud} Y_d^\dagger$
$C_{HL}^{1,3}, C_{He}$	0	$\varnothing$
$C_{HQ}^{(1,3)}$	3	$\left\{ \begin{array}{c} L_{1100}\left(C_{HQ}^{(1,3)}\right) \\ L_{2200}\left(C_{HQ}^{(1,3)}\right) \\ L_{1122}\left(C_{HQ}^{(1,3)}\right) \end{array} \right\}$
C_{Hu}		Same with $C_{HQ}^{(1,3)} \to Y_u C_{Hu} Y_u^\dagger$
C_{Hd}		Same with $C_{HQ}^{(1,3)} \to Y_d C_{Hd} Y_d^\dagger$

More invariants : 4-Fermi

Wilson coefficient	Number of phases	Minimal set
C_{LL}, C_{ee}	0	$\varnothing$
C_{Le}	3	$\left\{ B_0^0\left(C_{LL\tilde{e}\tilde{e}}\right),\ B_0^1\left(C_{LL\tilde{e}\tilde{e}}\right),\ B_0^2\left(C_{LL\tilde{e}\tilde{e}}\right) \right\}$
C_{Qe}	9	$\left\{ A_0^{1100}\left(C_{QQee}\right),\ A_0^{1100}\left(C_{QQ\tilde{e}\tilde{e}}\right),\ A_0^{2200}\left(C_{QQee}\right),\ A_0^{2200}\left(C_{QQ\tilde{e}\tilde{e}}\right),\ A_0^{1122}\left(C_{QQee}\right),\ A_0^{1122}\left(C_{QQ\tilde{e}\tilde{e}}\right),\ A_1^{1100}\left(C_{QQ\tilde{e}\tilde{e}}\right),\ A_1^{2200}\left(C_{QQ\tilde{e}\tilde{e}}\right),\ A_1^{1122}\left(C_{QQ\tilde{e}\tilde{e}}\right) \right\}$
C_{ed}		Same with $C_{QQee} \to C_{ee\tilde{d}\tilde{d}}$ (as well as $\tilde{e}$ versions)
C_{eu}		Same with $C_{QQee} \to C_{ee\tilde{u}\tilde{u}}$ (as well as $\tilde{e}$ versions)
$C_{LQ}^{(1,3)}$		$\left\{ A_{1100}^0\left(C_{LQ}^{(1,3)}\right)\ A_{1100}^1\left(C_{LQ}^{(1,3)}\right)\ A_{1100}^2\left(C_{LQ}^{(1,3)}\right)$ $A_{2200}^0\left(C_{LQ}^{(1,3)}\right)\ A_{2200}^1\left(C_{LQ}^{(1,3)}\right)\ A_{2200}^2\left(C_{LQ}^{(1,3)}\right)$ $A_{1122}^0\left(C_{LQ}^{(1,3)}\right)\ A_{1122}^1\left(C_{LQ}^{(1,3)}\right)\ A_{1122}^2\left(C_{LQ}^{(1,3)}\right) \right\}$
C_{Ld}		Same with $C_{LQ}^{(1,3)} \to C_{LL\tilde{d}\tilde{d}}$
C_{Lu}		Same with $C_{LQ}^{(1,3)} \to C_{LL\tilde{u}\tilde{u}}$
C_{LeQu}	27	$\left\{ A_{0000}^0\left(C_{L\tilde{e}Q\tilde{u}}\right)\ A_{0000}^1\left(C_{L\tilde{e}Q\tilde{u}}\right)\ A_{0000}^2\left(C_{L\tilde{e}Q\tilde{u}}\right)$ $A_{1000}^0\left(C_{L\tilde{e}Q\tilde{u}}\right)\ A_{1000}^1\left(C_{L\tilde{e}Q\tilde{u}}\right)\ A_{1000}^2\left(C_{L\tilde{e}Q\tilde{u}}\right)$ $A_{0100}^0\left(C_{L\tilde{e}Q\tilde{u}}\right)\ A_{0100}^1\left(C_{L\tilde{e}Q\tilde{u}}\right)\ A_{0100}^2\left(C_{L\tilde{e}Q\tilde{u}}\right)$ $A_{1100}^0\left(C_{L\tilde{e}Q\tilde{u}}\right)\ A_{1100}^1\left(C_{L\tilde{e}Q\tilde{u}}\right)\ A_{1100}^2\left(C_{L\tilde{e}Q\tilde{u}}\right)$ $A_{0110}^0\left(C_{L\tilde{e}Q\tilde{u}}\right)\ A_{0110}^1\left(C_{L\tilde{e}Q\tilde{u}}\right)\ A_{0110}^2\left(C_{L\tilde{e}Q\tilde{u}}\right)$ $A_{2200}^0\left(C_{L\tilde{e}Q\tilde{u}}\right)\ A_{2200}^1\left(C_{L\tilde{e}Q\tilde{u}}\right)\ A_{2200}^2\left(C_{L\tilde{e}Q\tilde{u}}\right)$ $A_{0220}^0\left(C_{L\tilde{e}Q\tilde{u}}\right)\ A_{0220}^1\left(C_{L\tilde{e}Q\tilde{u}}\right)\ A_{0220}^2\left(C_{L\tilde{e}Q\tilde{u}}\right)$ $A_{1220}^0\left(C_{L\tilde{e}Q\tilde{u}}\right)\ A_{1220}^1\left(C_{L\tilde{e}Q\tilde{u}}\right)\ A_{1220}^2\left(C_{L\tilde{e}Q\tilde{u}}\right)$ $A_{0122}^0\left(C_{L\tilde{e}Q\tilde{u}}\right)\ A_{0122}^1\left(C_{L\tilde{e}Q\tilde{u}}\right)\ A_{0122}^2\left(C_{L\tilde{e}Q\tilde{u}}\right) \right\}$
C_{LedQ}		Same with $C_{L\tilde{e}Q\tilde{u}} \to C_{L\tilde{e}\tilde{d}Q}$ and $A_{bcde}^a \to A_{edcb}^a$

Wilson coefficient	Number of phases	Minimal set
$C_{QQ}^{(1,3)}$	18	$\left\{ A_{1100}^0\left(C_{QQQQ}\right)\ A_{1100}^{1000}\left(C_{QQQQ}\right)\ A_{1100}^{0100}\left(C_{QQQQ}\right)$ $A_{0200}^0\left(C_{QQQQ}\right)\ A_{1100}^{1100}\left(C_{QQQQ}\right)\ A_{2200}^{1000}\left(C_{QQQQ}\right)$ $A_{2200}^{0100}\left(C_{QQQQ}\right)\ A_{1122}^0\left(C_{QQQQ}\right)\ A_{2200}^{1100}\left(C_{QQQQ}\right)$ $A_{2100}^{1200}\left(C_{QQQQ}\right)\ A_{1122}^{1000}\left(C_{QQQQ}\right)\ A_{1122}^{0100}\left(C_{QQQQ}\right)$ $A_{1122}^{1100}\left(C_{QQQQ}\right)\ A_{2200}^{2200}\left(C_{QQQQ}\right)\ B_{1100}^0\left(C_{QQQQ}\right)$ $B_{2200}^0\left(C_{QQQQ}\right)\ B_{1122}^0\left(C_{QQQQ}\right)\ B_{1122}^{2200}\left(C_{QQQQ}\right) \right\}$
C_{uu}	18	$\left\{ A_{1100}^0\left(C_{uu\tilde{u}\tilde{u}}\right)\ A_{1100}^{1000}\left(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}\right)\ A_{1100}^{0100}\left(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}\right)$ $A_{2200}^0\left(C_{uu\tilde{u}\tilde{u}}\right)\ A_{1100}^{1100}\left(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}\right)\ A_{1100}^{0200}\left(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}\right)$ $A_{2200}^{0100}\left(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}\right)\ A_{1122}^0\left(C_{uu\tilde{u}\tilde{u}}\right)\ A_{2200}^{1100}\left(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}\right)$ $A_{1100}^{1100}\left(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}\right)\ A_{1122}^{0100}\left(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}\right)\ A_{0122}^{1100}\left(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}\right)$ $A_{2200}^{1200}\left(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}\right)\ B_{1100}^0\left(C_{u\tilde{u}\tilde{u}u}\right)\ B_{1100}^{0100}\left(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}\right)$ $B_{2100}^{0200}\left(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}\right)\ B_{1122}^{1200}\left(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}\right)\ B_{1200}^{1000}\left(C_{\tilde{u}\tilde{u}\tilde{u}\tilde{u}}\right) \right\}$
C_{dd}	18	$\left\{ A_{1100}^0\left(C_{dd\tilde{d}\tilde{d}}\right)\ A_{1100}^{1000}\left(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}\right)\ A_{2200}^0\left(C_{dd\tilde{d}\tilde{d}}\right)$ $A_{2000}^{1100}\left(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}\right)\ A_{1100}^{0100}\left(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}\right)\ A_{1100}^{1100}\left(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}\right)$ $A_{2200}^{1000}\left(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}\right)\ A_{1122}^0\left(C_{dd\tilde{d}\tilde{d}}\right)\ A_{2200}^{1100}\left(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}\right)$ $A_{1122}^{1000}\left(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}\right)\ A_{1122}^{1100}\left(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}\right)\ A_{2110}^{1200}\left(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}\right)$ $A_{0122}^{2100}\left(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}\right)\ A_{1220}^{2100}\left(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}\right)\ B_{1100}^0\left(C_{d\tilde{d}\tilde{d}d}\right)$ $B_{2100}^{0100}\left(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}\right)\ B_{1100}^{1000}\left(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}\right)\ B_{2000}^{1200}\left(C_{\tilde{d}\tilde{d}\tilde{d}\tilde{d}}\right) \right\}$
$C_{Qu}^{(1,8)}$	36	$\left\{ A_0^{1100}\left(C_{QQuu}\right)\ A_{1100}^0\left(C_{QQ\tilde{u}\tilde{u}}\right)\ A_{1100}^{1000}\left(C_{QQ\tilde{u}\tilde{u}}\right)$ $A_{0100}^{1100}\left(C_{QQ\tilde{u}\tilde{u}}\right)\ A_{1100}^{1100}\left(C_{QQ\tilde{u}\tilde{u}}\right)\ A_{0110}^{1100}\left(C_{QQ\tilde{u}\tilde{u}}\right)$ $A_{1000}^{1200}\left(C_{QQ\tilde{u}\tilde{u}}\right)\ A_0^{2200}\left(C_{QQuu}\right)\ A_{2200}^{1100}\left(C_{QQ\tilde{u}\tilde{u}}\right)$ $A_{0220}^{1100}\left(C_{QQ\tilde{u}\tilde{u}}\right)\ A_{0120}^{2200}\left(C_{QQ\tilde{u}\tilde{u}}\right)\ A_{1122}^{1100}\left(C_{QQ\tilde{u}\tilde{u}}\right)$ $A_{1220}^{1200}\left(C_{QQ\tilde{u}\tilde{u}}\right)\ A_{1122}^{2200}\left(C_{QQ\tilde{u}\tilde{u}}\right)\ B_{0100}^0\left(C_{QQ\tilde{u}\tilde{u}}\right)$ $B_{1000}^0\left(C_{QQ\tilde{u}\tilde{u}}\right)\ B_{0110}^0\left(C_{QQ\tilde{u}\tilde{u}}\right)\ B_{0220}^0\left(C_{QQ\tilde{u}\tilde{u}}\right)$ $B_{1100}^0\left(C_{QQ\tilde{u}\tilde{u}}\right)\ B_{0221}^0\left(C_{QQ\tilde{u}\tilde{u}}\right)\ B_{1000}^{0100}\left(C_{QQ\tilde{u}\tilde{u}}\right)$ $B_{1100}^{0100}\left(C_{QQ\tilde{u}\tilde{u}}\right)\ B_{2200}^{0100}\left(C_{QQ\tilde{u}\tilde{u}}\right)\ B_{2110}^{0100}\left(C_{QQ\tilde{u}\tilde{u}}\right)$ $B_{2000}^{0200}\left(C_{QQ\tilde{u}\tilde{u}}\right)\ B_{2100}^{0200}\left(C_{QQ\tilde{u}\tilde{u}}\right)\ B_{2110}^{0200}\left(C_{QQ\tilde{u}\tilde{u}}\right)$ $B_{0110}^{1000}\left(C_{QQ\tilde{u}\tilde{u}}\right)\ B_{0220}^{1000}\left(C_{QQ\tilde{u}\tilde{u}}\right)\ B_{0221}^{1000}\left(C_{QQ\tilde{u}\tilde{u}}\right)$ $B_{1100}^{1100}\left(C_{QQ\tilde{u}\tilde{u}}\right)\ B_{2200}^{1100}\left(C_{QQ\tilde{u}\tilde{u}}\right)\ B_{2100}^{1200}\left(C_{QQ\tilde{u}\tilde{u}}\right)$ $B_{2210}^{1200}\left(C_{QQ\tilde{u}\tilde{u}}\right)\ B_{1100}^{2100}\left(C_{QQ\tilde{u}\tilde{u}}\right)\ B_{1210}^{1100}\left(C_{QQ\tilde{u}\tilde{u}}\right) \right\}$
$C_{Qd}^{(1,8)}$	36	$\left\{ A_0^{1100}\left(C_{QQdd}\right)\ A_{1100}^0\left(C_{QQ\tilde{d}\tilde{d}}\right)\ A_{1100}^{1000}\left(C_{QQ\tilde{d}\tilde{d}}\right)$ $A_{1000}^{1100}\left(C_{QQ\tilde{d}\tilde{d}}\right)\ A_0^{1200}\left(C_{QQdd}\right)\ A_{1100}^{0100}\left(C_{QQ\tilde{d}\tilde{d}}\right)$ $A_{2200}^0\left(C_{QQ\tilde{d}\tilde{d}}\right)\ A_{1100}^{1100}\left(C_{QQ\tilde{d}\tilde{d}}\right)\ A_{2100}^{1100}\left(C_{QQ\tilde{d}\tilde{d}}\right)$ $A_0^{1122}\left(C_{QQdd}\right)\ A_{1122}^0\left(C_{QQ\tilde{d}\tilde{d}}\right)\ A_{2200}^{1100}\left(C_{QQ\tilde{d}\tilde{d}}\right)$ $A_{0220}^{1100}\left(C_{QQ\tilde{d}\tilde{d}}\right)\ A_{1122}^{1000}\left(C_{QQ\tilde{d}\tilde{d}}\right)\ A_{1122}^{1100}\left(C_{QQ\tilde{d}\tilde{d}}\right)$ $A_{0122}^{2100}\left(C_{QQ\tilde{d}\tilde{d}}\right)\ B_{0100}^0\left(C_{QQ\tilde{d}\tilde{d}}\right)\ B_{1000}^0\left(C_{QQ\tilde{d}\tilde{d}}\right)$ $B_{0110}^0\left(C_{QQ\tilde{d}\tilde{d}}\right)\ B_{0220}^0\left(C_{QQ\tilde{d}\tilde{d}}\right)\ B_{1100}^0\left(C_{QQ\tilde{d}\tilde{d}}\right)$ $B_{0221}^0\left(C_{QQ\tilde{d}\tilde{d}}\right)\ B_{2200}^0\left(C_{QQ\tilde{d}\tilde{d}}\right)\ B_{2210}^0\left(C_{QQ\tilde{d}\tilde{d}}\right)$ $B_{1000}^{0100}\left(C_{QQ\tilde{d}\tilde{d}}\right)\ B_{0120}^{0100}\left(C_{QQ\tilde{d}\tilde{d}}\right)\ B_{1100}^{0100}\left(C_{QQ\tilde{d}\tilde{d}}\right)$ $B_{2210}^{0100}\left(C_{QQ\tilde{d}\tilde{d}}\right)\ B_{0110}^{1000}\left(C_{QQ\tilde{d}\tilde{d}}\right)\ B_{0220}^{1000}\left(C_{QQ\tilde{d}\tilde{d}}\right)$ $B_{2210}^{1000}\left(C_{QQ\tilde{d}\tilde{d}}\right)\ B_{1200}^{1000}\left(C_{QQ\tilde{d}\tilde{d}}\right)\ B_{2200}^{1100}\left(C_{QQ\tilde{d}\tilde{d}}\right)$ $B_{2210}^{1100}\left(C_{QQ\tilde{d}\tilde{d}}\right)\ B_{2100}^{1200}\left(C_{QQ\tilde{d}\tilde{d}}\right)\ B_{2211}^{1200}\left(C_{QQ\tilde{d}\tilde{d}}\right) \right\}$

More invariants : 4-Fermi

Wilson coefficient	Number of phases	Minimal set
$C_{ud}^{(1,8)}$	36	$\left\{\begin{array}{ccc} A_0^{1100}\left(C_{\tilde u\tilde u dd}\right) & A_{1100}^{0}\left(C_{uu\tilde d\tilde d}\right) & A_{1100}^{1000}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) \\ A_{1000}^{1100}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) & A_0^{2200}\left(C_{\tilde u\tilde u dd}\right) & A_{1100}^{0100}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) \\ A_{2200}^{0}\left(C_{uu\tilde d\tilde d}\right) & A_{1100}^{1100}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) & A_{0110}^{1100}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) \\ A_{2200}^{1000}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) & A_{2100}^{1100}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) & A_0^{1122}\left(C_{\tilde u\tilde u dd}\right) \\ A_{2200}^{0100}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) & A_{1122}^{0}\left(C_{uu\tilde d\tilde d}\right) & A_{2200}^{1100}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) \\ A_{1122}^{1000}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) & A_{1122}^{0100}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) & A_{1122}^{1100}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) \\ B_{0100}^{0}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) & B_{1000}^{0}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) & B_{0110}^{0}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) \\ B_{1100}^{0}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) & B_{0221}^{0}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) & B_{2200}^{0}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) \\ B_{1000}^{0100}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) & B_{0110}^{0100}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) & B_{2110}^{0100}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) \\ B_{2000}^{0200}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) & B_{2110}^{0200}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) & B_{0110}^{1000}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) \\ B_{0221}^{1000}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) & B_{1200}^{1000}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) & B_{2200}^{1100}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) \\ B_{2211}^{1100}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) & B_{2100}^{1200}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) & B_{1200}^{2100}\left(C_{\tilde u\tilde u\tilde d\tilde d}\right) \end{array}\right\}$
$C_{QuQd}^{(1,8)}$	81	$\left\{\begin{array}{ccc} A_0^{0}\left(C_{Q\tilde uQ\tilde d}\right) & A_{1000}^{0}\left(C_{Q\tilde uQ\tilde d}\right) & A_0^{1000}\left(C_{Q\tilde uQ\tilde d}\right) \\ A_{1000}^{1000}\left(C_{Q\tilde uQ\tilde d}\right) & A_{0100}^{0}\left(C_{Q\tilde uQ\tilde d}\right) & A_0^{0100}\left(C_{Q\tilde uQ\tilde d}\right) \\ A_{1100}^{0}\left(C_{Q\tilde uQ\tilde d}\right) & A_{0110}^{0}\left(C_{Q\tilde uQ\tilde d}\right) & A_{1000}^{100}\left(C_{Q\tilde uQ\tilde d}\right) \\ A_{0100}^{1000}\left(C_{Q\tilde uQ\tilde d}\right) & A_0^{1100}\left(C_{Q\tilde uQ\tilde d}\right) & A_0^{0110}\left(C_{Q\tilde uQ\tilde d}\right) \\ A_{1100}^{1000}\left(C_{Q\tilde uQ\tilde d}\right) & A_{0110}^{1000}\left(C_{Q\tilde uQ\tilde d}\right) & A_{1000}^{1100}\left(C_{Q\tilde uQ\tilde d}\right) \\ A_{0100}^{0100}\left(C_{Q\tilde uQ\tilde d}\right) & A_{1100}^{0100}\left(C_{Q\tilde uQ\tilde d}\right) & A_{0110}^{0100}\left(C_{Q\tilde uQ\tilde d}\right) \\ A_{0100}^{0110}\left(C_{Q\tilde uQ\tilde d}\right) & A_{2200}^{0}\left(C_{Q\tilde uQ\tilde d}\right) & A_{0220}^{0}\left(C_{Q\tilde uQ\tilde d}\right) \\ A_{2000}^{0200}\left(C_{Q\tilde uQ\tilde d}\right) & A_{1100}^{1100}\left(C_{Q\tilde uQ\tilde d}\right) & A_{0110}^{1100}\left(C_{Q\tilde uQ\tilde d}\right) \\ A_{0200}^{2000}\left(C_{Q\tilde uQ\tilde d}\right) & A_{0100}^{2100}\left(C_{Q\tilde uQ\tilde d}\right) & A_{1100}^{0110}\left(C_{Q\tilde uQ\tilde d}\right) \\ A_{0110}^{0110}\left(C_{Q\tilde uQ\tilde d}\right) & A_{1000}^{0210}\left(C_{Q\tilde uQ\tilde d}\right) & A_{1220}^{0}\left(C_{Q\tilde uQ\tilde d}\right) \\ A_{2000}^{1200}\left(C_{Q\tilde uQ\tilde d}\right) & A_{0122}^{0}\left(C_{Q\tilde uQ\tilde d}\right) & A_{1220}^{0100}\left(C_{Q\tilde uQ\tilde d}\right) \\ A_{0122}^{1000}\left(C_{Q\tilde uQ\tilde d}\right) & A_{2200}^{1100}\left(C_{Q\tilde uQ\tilde d}\right) & A_{0220}^{1100}\left(C_{Q\tilde uQ\tilde d}\right) \\ A_{2100}^{1200}\left(C_{Q\tilde uQ\tilde d}\right) & A_{1200}^{2100}\left(C_{Q\tilde uQ\tilde d}\right) & A_{0210}^{2100}\left(C_{Q\tilde uQ\tilde d}\right) \\ A_{0110}^{2200}\left(C_{Q\tilde uQ\tilde d}\right) & A_{2200}^{0110}\left(C_{Q\tilde uQ\tilde d}\right) & A_{0220}^{0110}\left(C_{Q\tilde uQ\tilde d}\right) \\ A_{2000}^{0112}\left(C_{Q\tilde uQ\tilde d}\right) & A_{1220}^{1100}\left(C_{Q\tilde uQ\tilde d}\right) & A_{0112}^{2100}\left(C_{Q\tilde uQ\tilde d}\right) \\ A_{1220}^{1200}\left(C_{Q\tilde uQ\tilde d}\right) & A_{2200}^{2200}\left(C_{Q\tilde uQ\tilde d}\right) & A_{1122}^{0110}\left(C_{Q\tilde uQ\tilde d}\right) \\ A_{2100}^{0122}\left(C_{Q\tilde uQ\tilde d}\right) & A_{0220}^{0220}\left(C_{Q\tilde uQ\tilde d}\right) & B_0^{0}\left(C_{Q\tilde uQ\tilde d}\right) \\ B_{0100}^{0}\left(C_{Q\tilde uQ\tilde d}\right) & B_{1000}^{0}\left(C_{Q\tilde uQ\tilde d}\right) & B_{1100}^{0}\left(C_{Q\tilde uQ\tilde d}\right) \\ B_{2200}^{0}\left(C_{Q\tilde uQ\tilde d}\right) & B_{0110}^{0}\left(C_{Q\tilde uQ\tilde d}\right) & B_{0122}^{0}\left(C_{Q\tilde uQ\tilde d}\right) \\ B_{0220}^{0}\left(C_{Q\tilde uQ\tilde d}\right) & B_0^{0100}\left(C_{Q\tilde uQ\tilde d}\right) & B_{1000}^{0100}\left(C_{Q\tilde uQ\tilde d}\right) \\ B_{1100}^{0100}\left(C_{Q\tilde uQ\tilde d}\right) & B_{2100}^{0100}\left(C_{Q\tilde uQ\tilde d}\right) & B_{0120}^{0100}\left(C_{Q\tilde uQ\tilde d}\right) \\ B_{0122}^{0100}\left(C_{Q\tilde uQ\tilde d}\right) & B_{1120}^{0200}\left(C_{Q\tilde uQ\tilde d}\right) & B_0^{1000}\left(C_{Q\tilde uQ\tilde d}\right) \\ B_{0100}^{1000}\left(C_{Q\tilde uQ\tilde d}\right) & B_{1200}^{1000}\left(C_{Q\tilde uQ\tilde d}\right) & B_{0110}^{1000}\left(C_{Q\tilde uQ\tilde d}\right) \\ B_{0122}^{1000}\left(C_{Q\tilde uQ\tilde d}\right) & B_{2100}^{1000}\left(C_{Q\tilde uQ\tilde d}\right) & B_0^{1100}\left(C_{Q\tilde uQ\tilde d}\right) \\ B_{1100}^{1100}\left(C_{Q\tilde uQ\tilde d}\right) & B_{2200}^{1100}\left(C_{Q\tilde uQ\tilde d}\right) & B_{0110}^{1100}\left(C_{Q\tilde uQ\tilde d}\right) \\ B_{0220}^{1100}\left(C_{Q\tilde uQ\tilde d}\right) & B_{1122}^{1100}\left(C_{Q\tilde uQ\tilde d}\right) & B_{2100}^{1200}\left(C_{Q\tilde uQ\tilde d}\right) \\ B_{0122}^{2100}\left(C_{Q\tilde uQ\tilde d}\right) & B_0^{2200}\left(C_{Q\tilde uQ\tilde d}\right) & A_{1122}^{2200}\left(C_{Q\tilde uQ\tilde d}\right) \end{array}\right\}$

CPV in SMEFT

Must be **proportional to invariants**. **Which ones?**

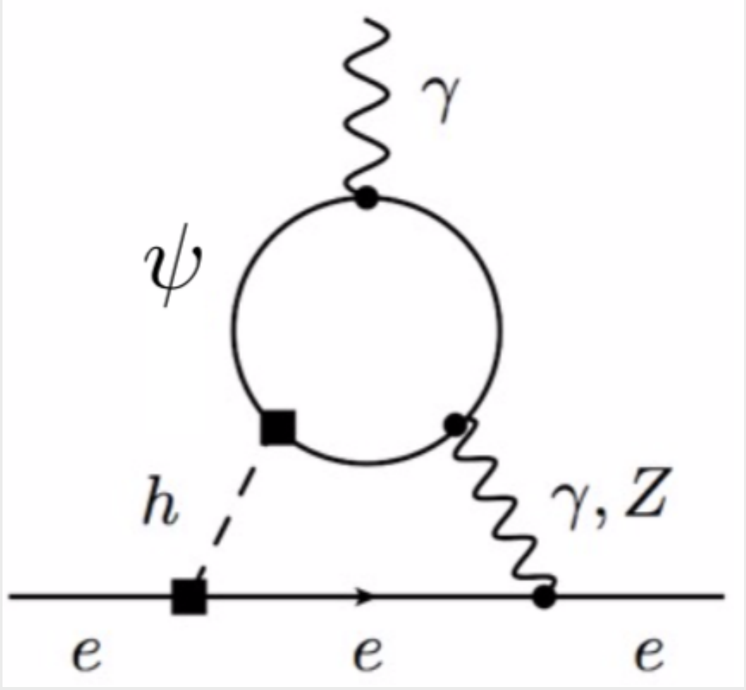
SM CPV could interfere with SMEFT coefficients. Ex : loops over three generations, electroweak and SMEFT vertices.

$\Longrightarrow$ « Simple » graphs map to new sources of CPV (for invariant observables, e.g. EDMs)

Example : Barr-Zee two-loop contribution to the electron EDM

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C_{uH}}{\Lambda^2} |H|^2 \overline{Q_L} u_R \tilde{H} + h.c.$$

$$\frac{d_e}{e} = -\frac{1}{48\pi^2} \frac{v m_e m_u}{m_h^2} \frac{\mathrm{Im}(C_{uH})}{\Lambda^2} F_1 \left(\frac{m_u^2}{m_h^2}, 0 \right)$$



[Barr/Zee '90, Brod/Haisch/Zupan '13]

one-to-one correspondance with invariants

CPV in SMEFT

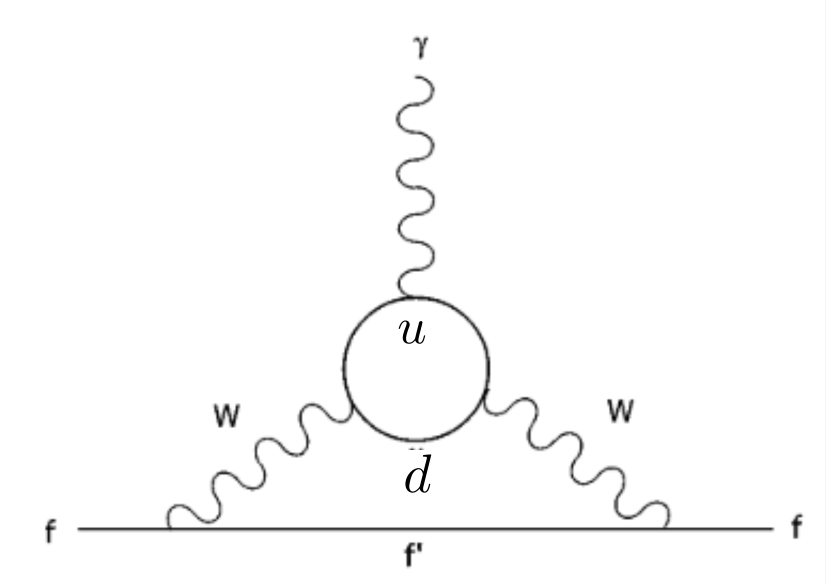
Must be **proportional to invariants**. **Which ones?**

SM CPV could interfere with SMEFT coefficients. Ex : loops over three generations, electroweak and SMEFT vertices.

$\Longrightarrow$ « Simple » graphs map to new sources of CPV (for invariant observables, e.g. EDMs)

Example : Barr-Zee two-loop contribution to the electron EDM

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{C_{Hud,mn}}{\Lambda^2} i\tilde{H}^\dagger D_\mu H \overline{u}_{R,m}\gamma^\mu d_{R,n}$$



[Kadoyoshi/Oshimo '97]

$$\frac{d}{e} = \sum_{i,j} \text{Im}\left(V_{CKM,ij} C^*_{Hud,ij}\right) F(m^2_{u_i}, m^2_{d_j})$$

one-to-one correspondance with invariants

CPV in SMEFT

Must be **proportional to invariants**. **Which ones?**

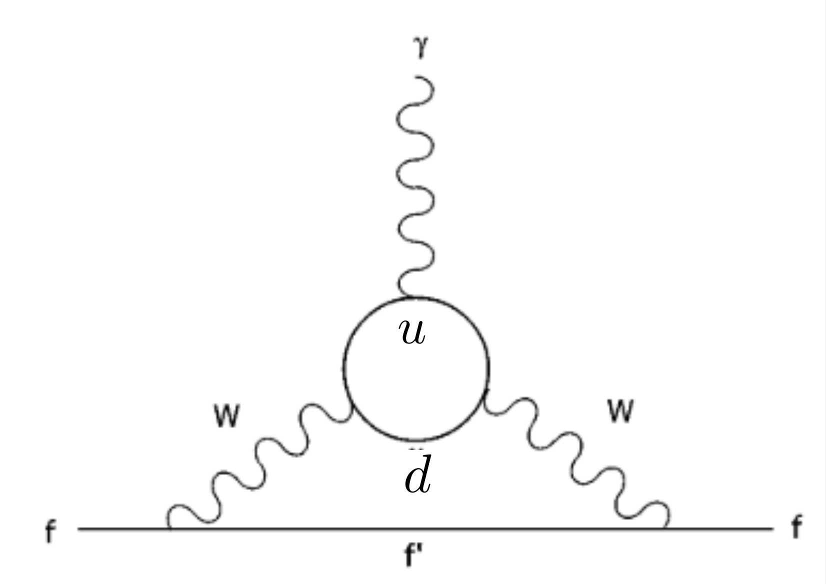
SM CPV could interfere with SMEFT coefficients. Ex : loops over three generations, electroweak and SMEFT vertices.

$\Longrightarrow$ « Simple » graphs map to new sources of CPV (for invariant observables, e.g. EDMs)

Example : Barr-Zee two-loop contribution to the electron EDM

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{C_{Hud,mn}}{\Lambda^2} i\tilde{H}^\dagger D_\mu H \overline{u}_{R,m}\gamma^\mu d_{R,n}$$

$$\frac{d}{e} = \sum_{i,j} \mathrm{Im}\left(V_{CKM,ij} C^*_{Hud,ij}\right) F(m^2_{u_i}, m^2_{d_j})$$



[Kadoyoshi/Oshimo '97]

$$\frac{d_e}{e} \sim \# \times \mathrm{Im}\, C_{Hud,23} + \#' \times (\cos\delta_{\rm CKM}\, \mathrm{Im}\, C_{Hud,13} + \sin\delta_{\rm CKM}\, \mathrm{Re}\, C_{Hud,13}) + \ldots$$

CPV in SMEFT

Invariants easily track CPV

Ex : spontaneous CP breaking model

$\langle S \rangle = V e^{i\alpha}$

$$\mathcal{L} \supset -\overline{Q}_L Y_u \tilde{H} u_R - \begin{pmatrix} \overline{Q}_L & \overline{D}_L \end{pmatrix} \begin{pmatrix} H\Gamma & 0 \\ FS + F'S^\dagger & \mu \end{pmatrix} \begin{pmatrix} d_R \\ D_R \end{pmatrix} + h.c.$$

$$\mathcal{L}_{\text{SMEFT}} \supset -\overline{Q}_L Y_u \tilde{H} u_R - \overline{Q}_L Y_d H d_R + \frac{\left(\lambda_D^\dagger \lambda_D Y_d\right)_{ij}}{2M^2} \mathcal{O}_{dH,ij} - \frac{\left(\lambda_D^\dagger \lambda_D\right)_{ij}}{4M^2} \left(\mathcal{O}^{(1)}_{HQ,ij} + \mathcal{O}^{(3)}_{HQ,ij}\right)$$

$$\lambda_D = \frac{V}{M} \mathcal{F} \Gamma^T \qquad Y_d Y_d^\dagger = \Gamma \Gamma^T - \frac{V^2}{M^2} \Gamma \mathcal{F}^\dagger \mathcal{F} \Gamma^T \qquad \left(\mathcal{F} = e^{i\alpha} F + e^{-i\alpha} F'\right)$$

What makes CP conserved, at dim-4, 6…? **Algebraic answers**

Ex : can get J4=0 but not the L's, or vice-versa, etc

Theta QCD

$$\mathcal{L}_{\rm QCD} \supset -\theta_{QCD} \frac{g_s^2}{16\pi^2} {\rm Tr}(G\tilde{G})$$

	$SU(3)_{Q_L}$	$U(1)_{Q_L}$	$SU(3)_{u_R}$	$U(1)_{u_R}$	$SU(3)_{d_R}$	$U(1)_{d_R}$
Q_L	$\mathbf{3}$	1	$\mathbf{1}$	0	$\mathbf{1}$	0
u_R	$\mathbf{1}$	0	$\mathbf{3}$	1	$\mathbf{1}$	0
d_R	$\mathbf{1}$	0	$\mathbf{1}$	0	$\mathbf{3}$	1
Y_u	$\mathbf{3}$	1	$\bar{\mathbf{3}}$	-1	$\mathbf{1}$	0
Y_d	$\mathbf{3}$	1	$\mathbf{1}$	0	$\bar{\mathbf{3}}$	-1
$e^{i\theta_{QCD}}$	$\mathbf{1}$	6	$\mathbf{1}$	-3	$\mathbf{1}$	-3

$$\bar{\theta} \equiv \theta_{QCD} - \arg\det\left(Y_u Y_d\right)$$

$${\rm Im}\left(e^{-i\theta_{QCD}} \epsilon^{ABC} \epsilon^{abc} \epsilon^{DEF} \epsilon^{def} Y_{u,Aa} Y_{u,Bb} C_{QuQd,CcDd} Y_{d,Ee} Y_{d,Ff}\right)$$

In the UV, suppressed by $e^{-\frac{8\pi^2}{g_s^2}} \approx \lambda^{37-38}$. Relevant in the IR ?