

# Small-x resummation and differential cross sections in heavy-quark pair production

Federico Silvetti

INFN, Rome 1 unit

Sapienza University of Rome

SM@LHC2022 conference: 13 April 2022  
work in collaboration with Marco Bonvini (INFN Rome 1)



Istituto Nazionale di Fisica Nucleare  
Sezione di ROMA



SAPIENZA  
UNIVERSITÀ DI ROMA

## Small- $x$ resummation

Perturbative coefficient functions and splitting functions contain logarithms of  $x$  that are single-logarithmically enhanced:

$$\begin{aligned} P(x, \alpha_s) \text{ or } C(x, \alpha_s) = & \quad a_0 \\ & + \alpha_s \left[ a_1 \ln \frac{1}{x} + b_1 \right] \\ & + \alpha_s^2 \left[ a_2 \ln^2 \frac{1}{x} + b_2 \ln \frac{1}{x} + c_2 \right] \\ & + \alpha_s^3 \left[ a_3 \ln^3 \frac{1}{x} + b_3 \ln^2 \frac{1}{x} + c_3 \ln \frac{1}{x} + d_3 \right] \\ & + \alpha_s^4 \left[ a_4 \ln^4 \frac{1}{x} + b_4 \ln^3 \frac{1}{x} + c_4 \ln^2 \frac{1}{x} + d_4 \ln \frac{1}{x} + e_4 \right] \\ & + \dots \end{aligned}$$

When  $\alpha_s \ln \frac{1}{x} \sim 1$  the fixed-order expansion is no longer predictive!

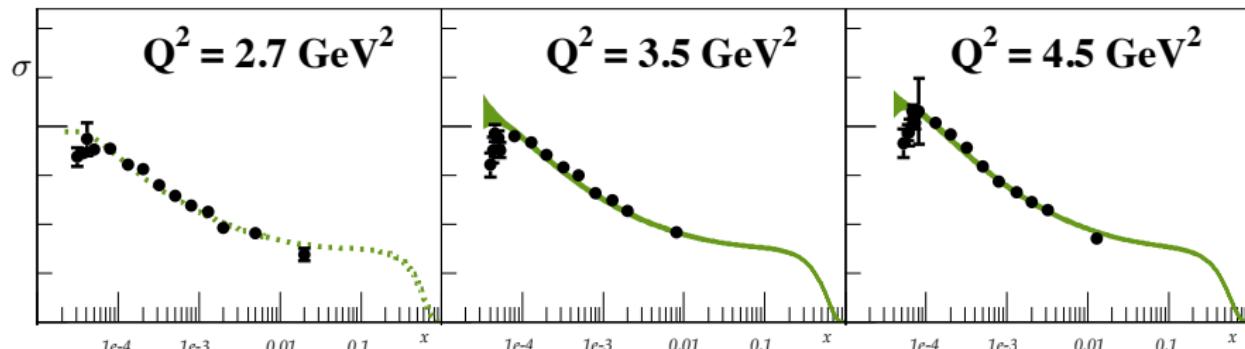


Resummation

## Low $x$ at HERA: importance of resummation in PDF fits

Deep-inelastic scattering (DIS) data from HERA extend down to  $x \sim 3 \times 10^{-5}$

Tension between HERA data at low  $Q^2$  and low  $x$  with fixed-order theory



Also leads to a deterioration of the  $\chi^2$  when including low- $Q^2$  data

Attempts to explain this deviation with higher twists, phenomenological models, ...

**Successful description of this region when including small- $x$  resummation!**

- NNPDF framework
- xFitter framework

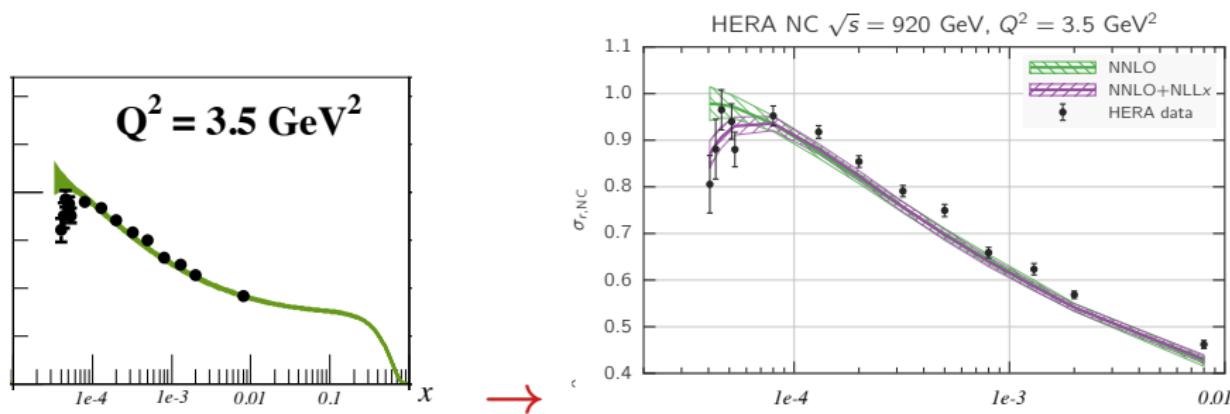
[Ball et al. 1710.05935]

[xFitterCollaboration 1802.00064] [Bonvini, Giuli 1902.11125]

# Significantly improved description of the HERA data

$\chi^2/N_{\text{dat}}$	NNLO	NNLO+NLL $x$
xFitter	<b>1.23</b>	<b>1.17</b>
NNPDF3.1sx	<b>1.130</b>	<b>1.100</b>

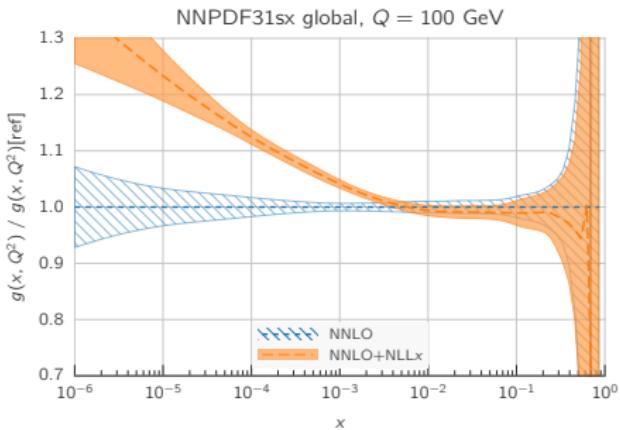
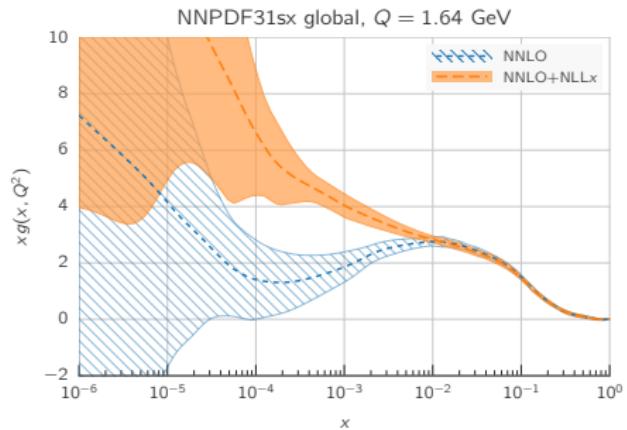
Substantial improvement due to better description of small- $x$  low- $Q^2$  HERA data



The better description mostly comes from a larger resummed  $F_L$

$$\sigma_{r,\text{NC}} = F_2(x_{\text{Bj}}, Q^2) - \frac{y^2}{1 + (1 - y)^2} F_L(x_{\text{Bj}}, Q^2) \quad y = \frac{Q^2}{x_{\text{Bj}} s}$$

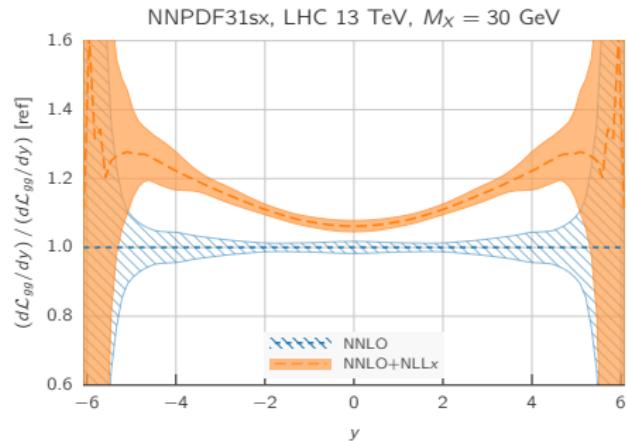
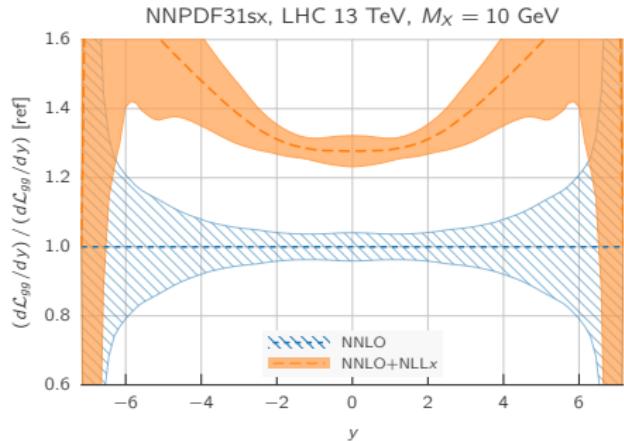
## Fit results: impact on gluon PDF



Note: future higher energy colliders will probe smaller values of  $x$  ( $x_{\min} \sim Q^2/s$ )  
→ small- $x$  resummation will be even more important in future!

# Impact on parton luminosities

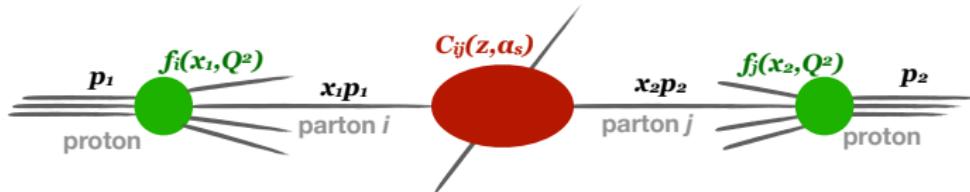
## Gluon-gluon parton luminosities



Significant impact for small invariant masses, especially at large rapidities

# LHC phenomenology

# Resummation of LHC observables



Challenges:

- two protons in the initial state ✓
- want to describe differential distributions ✓

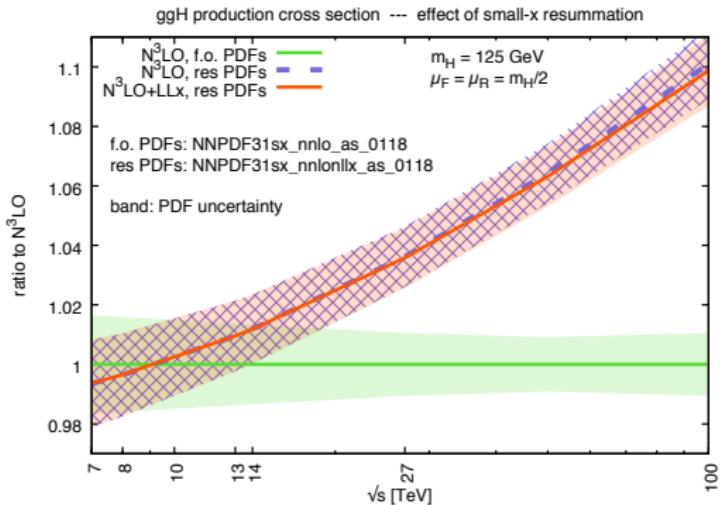
Processes considered so far in HELL:

- $gg \rightarrow H$  (inclusive cross section) [Bonvini 1805.08785]
- $c\bar{c}, b\bar{b}$  pair production (fully differential) (almost done)
- Drell-Yan (fully differential) (in progress)
- $gg \rightarrow H$  (fully differential) (*should be straightforward given the first two*)

# Impact of resummation in $ggH$

$gg \rightarrow H$  inclusive cross section

[Bonvini,Marzani 1802.07758] [Bonvini 1805.08785]



$ggH$  cross section at FCC-hh can be  $\sim 10\%$  larger than expected with NNLO PDFs!  
At LHC +1% effect

# LHC observables in collinear and $k_t$ factorization

Differential cross section in collinear factorization

$$\frac{d\sigma}{dQ^2 dY \dots} = \int_{\tau}^1 \frac{dz}{z} \int d\hat{y} \mathcal{L}_{ij} \left( \frac{\tau}{z}, \hat{y}, Q^2 \right) \frac{dC_{ij}}{dy \dots} (z, Y - \hat{y}, \dots, \alpha_s)$$

$$\mathcal{L}_{ij}(x, \hat{y}, Q^2) = f_i(\sqrt{x} e^{\hat{y}}, Q^2) f_j(x e^{-\hat{y}}, Q^2) \theta(e^{-2|\hat{y}|} - x)$$

and in  $k_t$  factorization

[Caola,Forte,Marzani 1010.2743] [Muselli 1710.09376]

$$\frac{d\sigma}{dQ^2 dY \dots} =$$

$$\int_{\tau}^1 \frac{dz}{z} \int d\hat{y} \int_0^\infty dk_1^2 \int_0^\infty dk_2^2 \mathcal{L}_{ij} \left( \frac{\tau}{z}, \hat{y}, k_1^2, k_2^2 \right) \frac{dC_{ij}}{dy \dots} (z, Y - \hat{y}, k_1^2, k_2^2, \dots, \alpha_s)$$

$$\mathcal{L}_{ij}(x, \hat{y}, k_1^2, k_2^2) = \mathcal{F}_i(\sqrt{x} e^{\hat{y}}, k_1^2) \mathcal{F}_j(\sqrt{x} e^{-\hat{y}}, k_2^2) \theta(e^{-2|\hat{y}|} - x)$$

$$\tau = Q^2/s, \quad y = Y - \frac{1}{2} \log \frac{x_1}{x_2}$$

$k_t$  factorisation allows to gather the small-x logarithms away from the coefficient function.

## small-x resummation: factorisation schemes

In Mellin-Fourier space, the **evoluter**  $U$  establishes a relation between collinear and off-shell PDFs

[Bonvini, Marzani, Peraro 1607.02153].

$$\mathcal{F}_g\left(N \pm \frac{ib}{2}, k_t^2\right) = U\left(N \pm \frac{ib}{2}, k_t^2, Q^2\right) f_g\left(N \pm \frac{ib}{2}, Q^2\right)$$

Then the resummed coefficient function is given as

$$\begin{aligned} \frac{d\tilde{\mathcal{C}}_{gg}}{dy\dots}(N, b, \dots) &= \int_0^\infty dk_1^2 \int_0^\infty dk_2^2 \frac{d}{dk_1^2} U\left(N + \frac{ib}{2}, k_1^2, Q^2\right) \frac{d}{dk_1^2} U\left(N - \frac{ib}{2}, k_2^2, Q^2\right) \\ &\quad \times \frac{d\tilde{\mathcal{C}}_{gg}}{dy\dots}(N, b, k_1^2, k_2^2, \dots, \alpha_s) \end{aligned}$$

Numerical integration → HELL public code

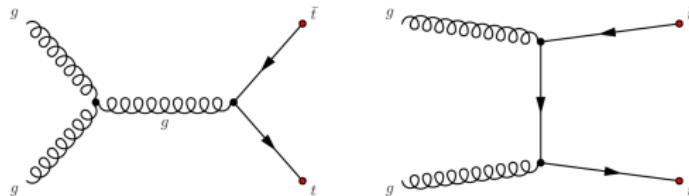
$$\begin{aligned} \frac{dC_{gg}}{dy\dots}(z, y, \dots) &= \int_0^\infty dk_1^2 \int_0^\infty dk_2^2 \int_z^1 \frac{dx}{x} \int d\hat{y} \frac{d\mathcal{C}_{gg}}{dy\dots}(x, y - \hat{y}, k_1^2, k_2^2, \dots, \alpha_s) \\ &\quad \times \frac{d}{dk_1^2} U\left(\sqrt{\frac{z}{x}} e^{\hat{y}}, k_1^2, Q^2\right) \frac{d}{dk_2^2} U\left(\sqrt{\frac{z}{x}} e^{-\hat{y}}, k_2^2, Q^2\right) \theta\left(e^{-2|\hat{y}|} - \frac{z}{x}\right) \end{aligned}$$

Transverse final-state variables do not play a role (except for kinematic constraints)

# Charm (and bottom) pair production at LHC

## Fully differential heavy-quark pair production

[Bonvini,FS (in preparation)]



We have resummed the cross section for different kinematics:

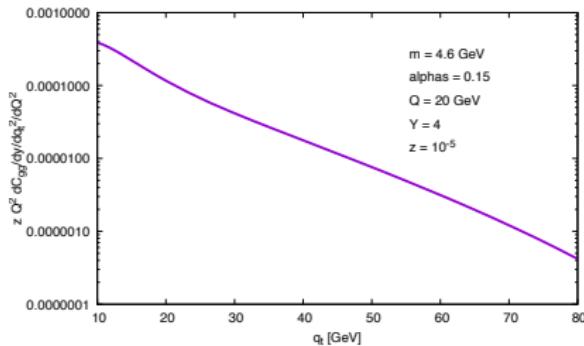
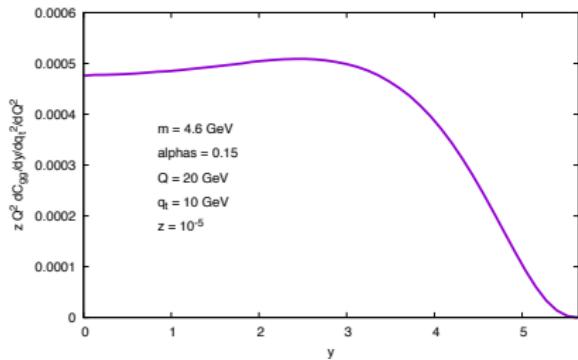
- heavy-quark pair:  $\frac{d\sigma}{dQ^2 dY dq_t}$  → quarkonium production
- single heavy quark:  $\frac{d\sigma}{dy dp_t}$  → D/B mesons production

Small- $x$  resummation crucial for **charm and bottom production**

- sensitive to very small  $x \rightarrow$  constrain the PDFs [Gauld, Rojo 1610.09373]
- key process at forward physics experiment e.g. FPF [Feng et al 2203.05090]

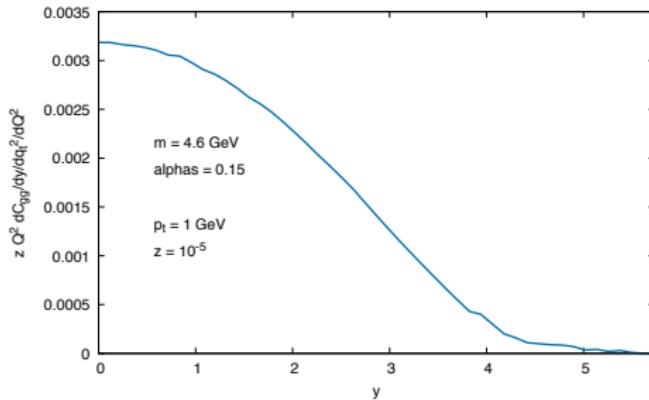
# Heavy-quark pair production at LHC: preliminary results

Results for  $\frac{d\sigma}{dQ^2 dY dq_t}$  in pair kinematics



- Pure resummed → match to fix order, and asses resummation impact
- Parton level → convolve with PDFs
- Open heavy quark final state → add fragmentation to compare with exp. data

Results for  $\frac{d\sigma}{dy dp_t}$  in **single-quark kinematics**

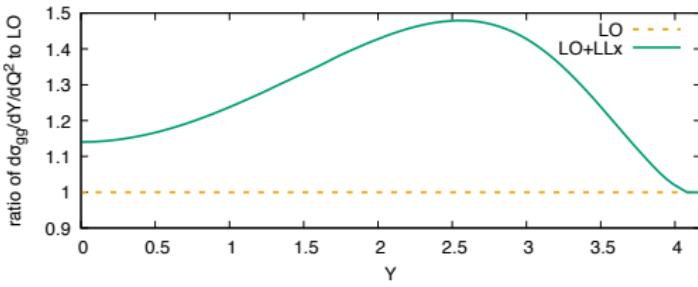
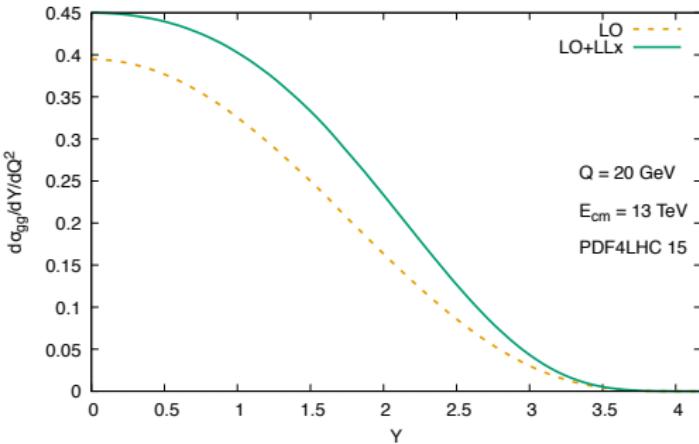


- Pure resummed → match to fix order, and asses resummation impact
- Parton level → convolve with PDFs
- Open heavy quark final state → add fragmentation to compare with exp. data
- Numerical instabilities still present → code optimization in progress

# Heavy-quark pair production at LHC: preliminary results

New (this morning) results for  $\frac{d\sigma}{dQ^2 dY}$  in **pair kinematics**

- Open heavy quark final state
- Parton level  $\rightarrow$  PDFs convolution included
- gluon channel only
- sizable impact from central to forward rapidity
- threshold effects in the right tail



# Concluding remarks

## Key messages:

- resummation is needed at small- $x$
- it stabilizes perturbative behaviour
- improved PDF fits
- large impact on gluon PDF for  $x \lesssim 10^{-3}$
- significant impact expected at LHC at low invariant mass and large rapidity in several processes

## completion → outlook:

- for  $c\bar{c}, b\bar{b}$  we considered only quark-level final states  
→ add hadronization and refit PDFs with LHCb data [Aaij et al. 1510.01707]
- at forward rapidities one parton is at large  $x$   
→ combine with threshold resummation
- low accuracy (two log orders known for  $P_{gg}$  only, for everything else just one)  
→ extension to the next logarithmic order

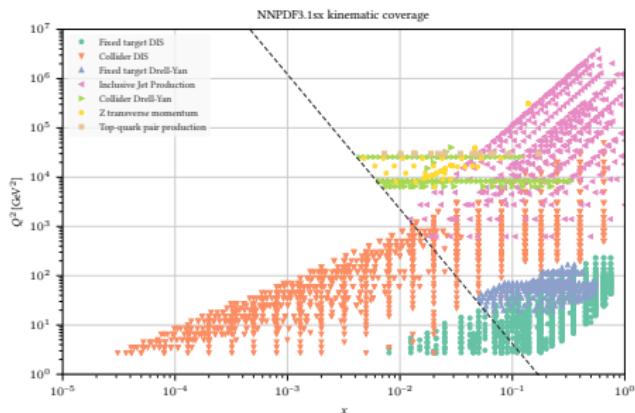
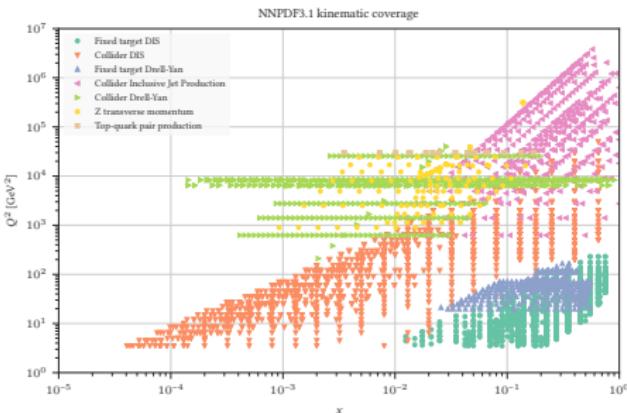
# Backup slides

# The global NNPDF fit in greater detail

Full resummation for DGLAP evolution and DIS structure functions, but not hadron-hadron collider observables (yet)

Cut those hadronic data potentially sensitive to small- $x$  resummation, i.e.

$$\alpha_s(Q^2) \log \frac{1}{x} > H_{\text{cut}} = 0.6 \quad (\text{default cut})$$



The most important missing observable is Drell-Yan

Would provide a very important validation of the fit (low  $x$  but high  $Q^2$ )

# NNPDF and xFitter fits with small- $x$ resummation

**APFEL+HELL** → make possible a PDF fit with small- $x$  resummation

NNPDF3.1sx [1710.05935]	xFitter [1802.00064]
NeuralNet parametrization of PDFs	polynomial parameterization
MonteCarlo uncertainty	Hessian uncertainty
VFNS: FONLL	VFNS: FONLL
charm PDF is fitted	charm PDF perturbatively generated
DIS+tevatron+LHC ( $\sim 4000$ datapoints)	only HERA data ( $\sim 1200$ datapoints)
NLO, NLO+NLL, NNLO, NNLO+NLL	NNLO, NNLO+NLL

One interesting difference in the HERA data we include:

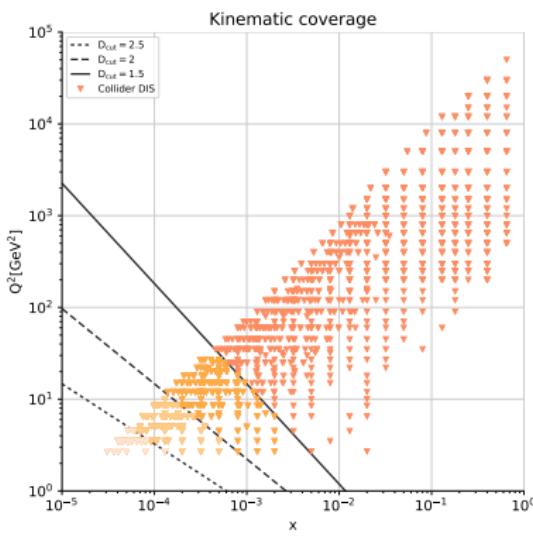
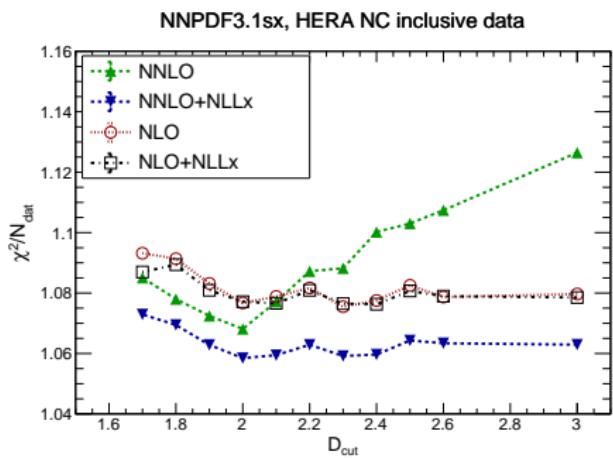
Lowest $Q^2$ HERA bins	NNPDF3.1/HERAPDF2.0	NNPDF3.1sx/xFitter
$Q^2 = 3.5 \text{ GeV}^2$	included	included
$Q^2 = 2.7 \text{ GeV}^2$	excluded	included
$Q^2 = 2.0 \text{ GeV}^2$	excluded	excluded
lower $Q^2 \rightarrow$ lower $x$		

# Fit results: $\chi^2$ as quality estimator and the onset of BFKL dynamics

$\chi^2/N_{\text{dat}}$	NLO	NLO+NLLx	NNLO	NNLO+NLLx
xFitter			<b>1.23</b>	<b>1.17</b>
NNPDF3.1sx	<b>1.117</b>	<b>1.120</b>	<b>1.130</b>	<b>1.100</b>
	these are similar		largest	smallest

Hierarchy as expected from splitting function behaviour!

Mostly due to HERA data: we study the  $\chi^2/N_{\text{dat}}$  profile as we cut out HERA data at small  $x$  small  $Q^2$



# Why is the effect of resummation mostly driven by the PDFs?

Let's consider again the collinear factorization formula

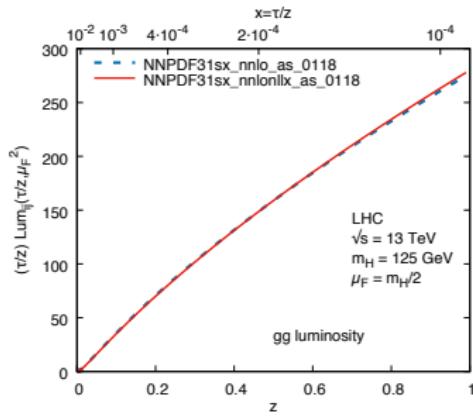
$$\frac{d\sigma}{dQ^2 dY \dots} = \int_{\tau}^1 \frac{dz}{z} \int d\hat{y} f_i\left(\sqrt{\frac{\tau}{z}} e^{\hat{y}}, Q^2\right) f_j\left(\sqrt{\frac{\tau}{z}} e^{-\hat{y}}, Q^2\right) \frac{dC_{ij}}{dy \dots}(z, Y - \hat{y}, \dots, \alpha_s)$$

The small  $z$  integration region, where logs in  $C$  are large, is weighted by the PDFs at large momentum fractions  $x = \sqrt{\frac{\tau}{z}} e^{\pm \hat{y}}$

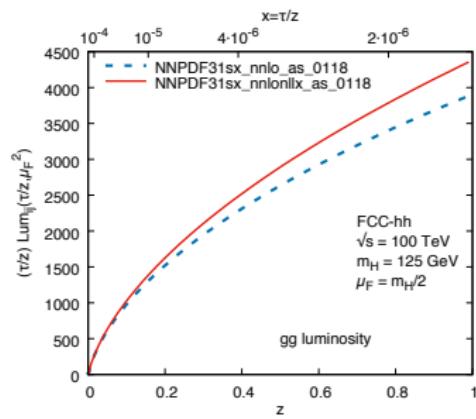
Since PDFs die fast at large  $x$ , especially the gluon, the small- $z$  region is suppressed!

Rather, the large  $z$  region is enhanced by the gluon-gluon luminosity

In that region, the difference between fixed-order and resummed PDFs is large

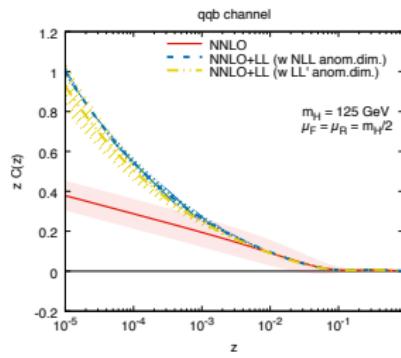
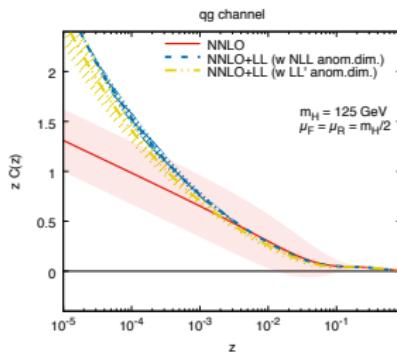
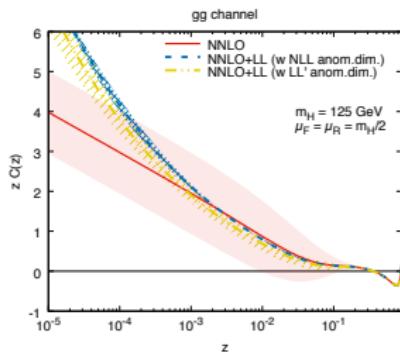
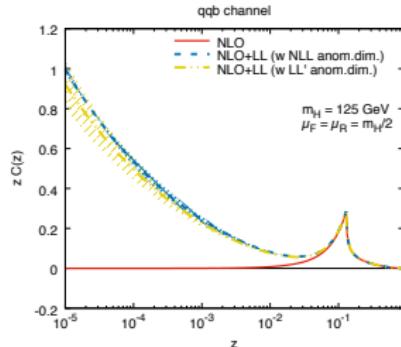
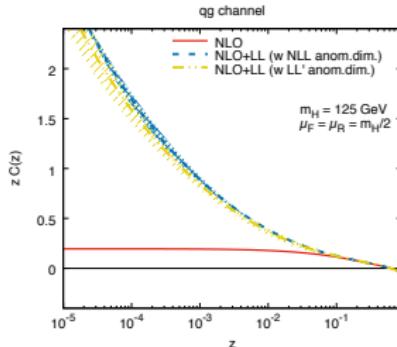
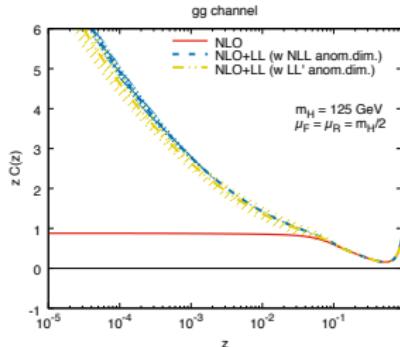


LHC:

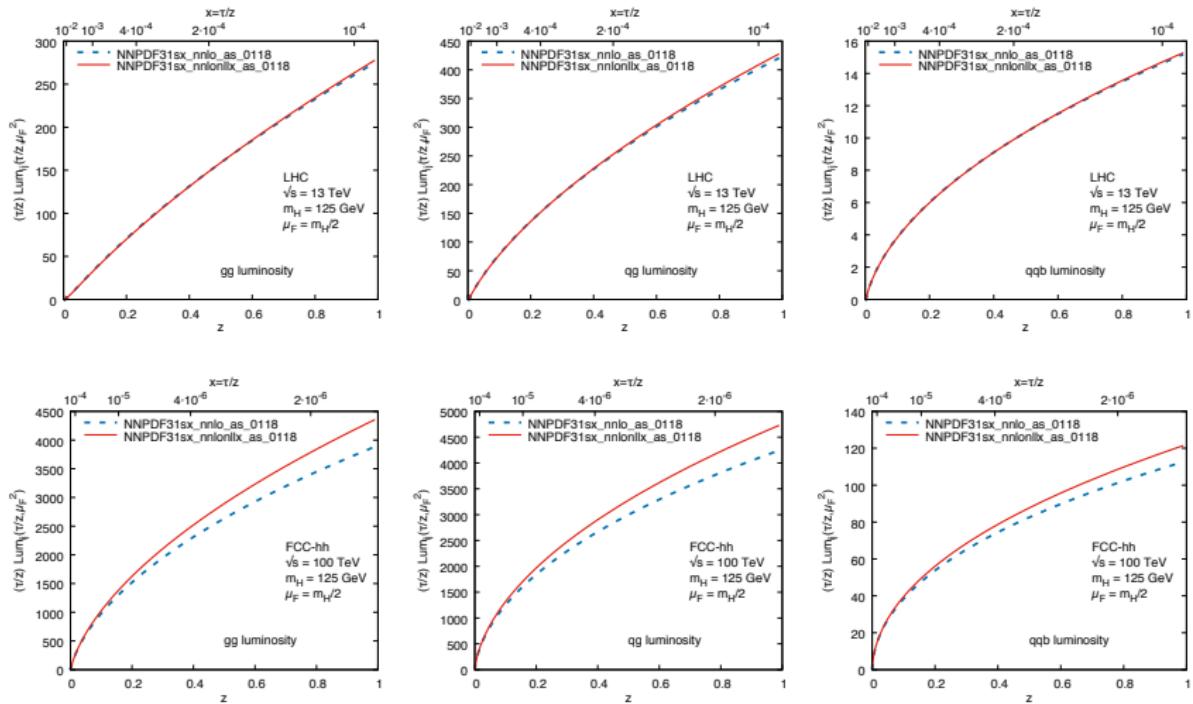


FCC:

# Higgs production: parton-level results



# Parton luminosities for $ggH$



$$\sigma(\tau, Q^2) = \sigma_0(Q^2) \sum_{i,j=g,q} \int_{\tau}^1 \frac{dz}{z} C_{ij}(z, \alpha_s) \mathcal{L}_{ij}\left(\frac{\tau}{z}, \mu_F^2\right) \quad \tau = \frac{m_H^2}{s}$$