

Status of (N)NNLO calculations

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Overview

[ttH production at NNLO: the flavour off-diagonal channels](#), Catani, Stefano and Fabre, Ignacio and Grazzini, Massimiliano and Kallweit, Stefan, 2102.03256

[Fully Differential Higgs Boson Production to Third Order in QCD](#), Chen, Gehrman, Glover, Huss, Mistlberger and Pelloni, 2102.07607

[B-hadron production in NNLO QCD: application to LHC tbar events with leptonic decays](#), Czakon, Generet, Mitov and Poncelet, 2102.08267

[Higgs pT Spectrum and Total Cross Section with Fiducial Cuts at Third Resummed and Fixed Order in QCD](#), Billis, Dehnadi, Ebert, Michel and Tackmann, 2102.08039

[Matching NNLO predictions to parton showers using N3LL color-singlet transverse momentum resummation in geneva](#), Alioli, Bauer, Broggio, Gavardi, Kallweit, Lim, Nagar, Napoletano, Rottoli, 2102.08390

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[NNLO QCD study of polarised W+W- production at the LHC](#), Poncelet and Popescu, 2102.13583

[Next-to-next-to-leading order event generation for \\$\\$Z\gamma\\$\\$ boson pair production matched to parton shower](#), Alioli, Broggio, Gavardi, Kallweit, Lim, Nagar, Napoletano, 2103.01214

[Estimating the impact of mixed QCD-electroweak corrections on the W-mass determination at the LHC](#), Behring, Buccioni, Caola, Delto, Jaquier, Melnikov and Rötsch, 2103.02671

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[Transverse momentum distributions in low-mass Drell-Yan lepton pair production at NNLO QCD](#), Gauld, Gehrman-De Ridder, Gehrman, Glover, Huss, Majer and Rodriguez, 2110.15839

[Fiducial cross sections for the lepton-pair-plus-photon decay mode in Higgs production up to NNLO QCD](#), Chen, Gehrman, Glover and Huss, 2111.02157

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N3LO computations

2→3 NNLO QCD

NNLO QCD + PS

Fragmentation

Mixed EW-QCD

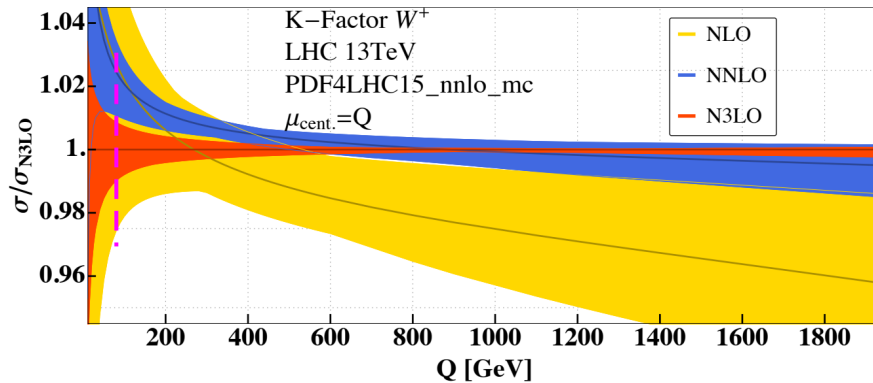
Differential N3LO QCD for Colour Singlets

First N3LO predictions become available

- Based on
 - Analytical integration (inclusive)
 - Projection-to-Born (differential)
 - qT-slicing+resummation (differential)

Processes:

- $pp \rightarrow H$ (+ diphoton decay with fiducial cuts)
- $pp \rightarrow W/Z/A$ (+leptonic decays)



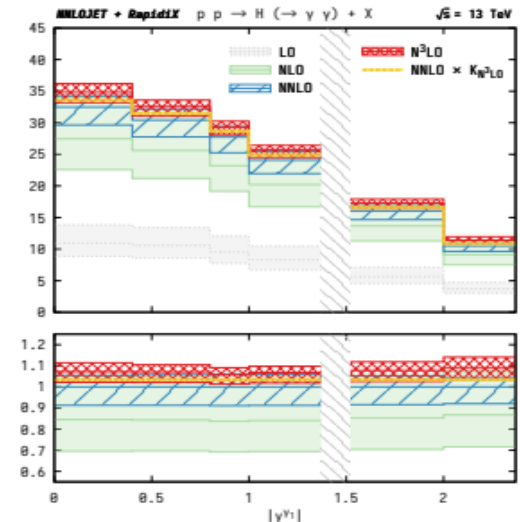
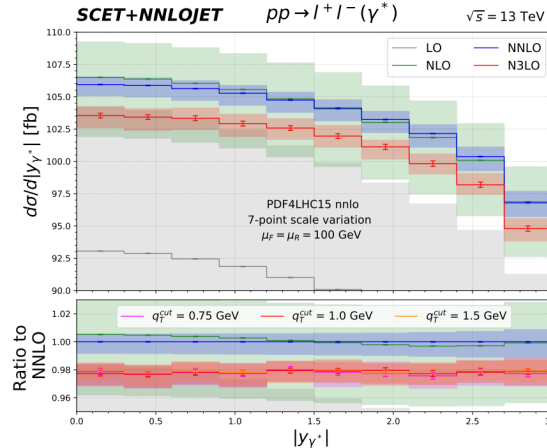
Higgs Boson Gluon-Fusion Production in QCD at Three Loops,
Anastasiou, Duhr, Dulat, Herzog and Mistlberger, 1503.06056
Charged current Drell-Yan production at N3LO,
Duhr, Dulat and Mistlberger, 2007.13313

Fully Differential Higgs Boson Production to Third Order in QCD,
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Dilepton Rapidity Distribution in Drell-Yan Production to Third Order in QCD,
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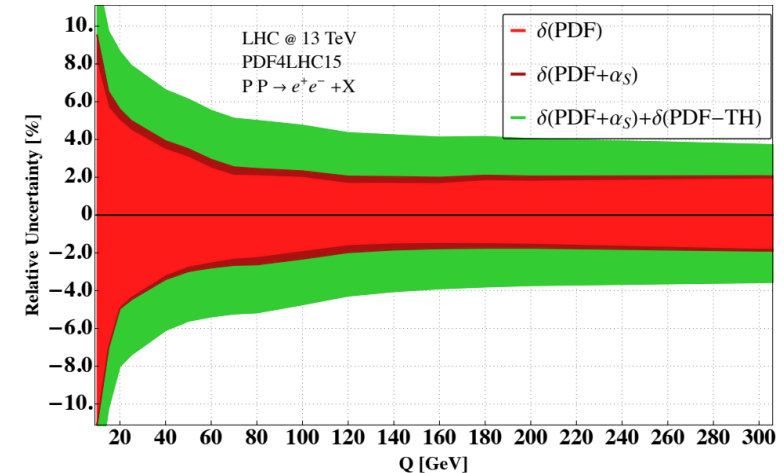
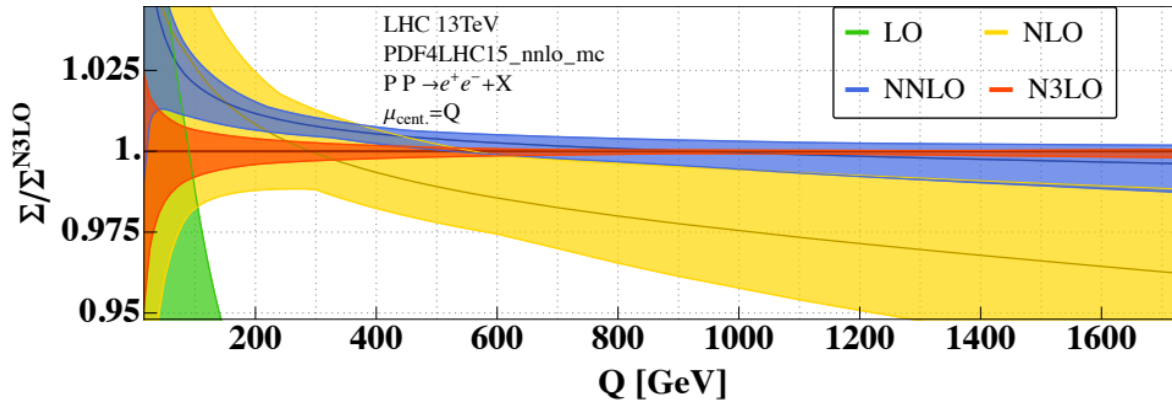
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Third order fiducial predictions for Drell-Yan at the LHC,
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More loops: N3LO QCD DrellYan II

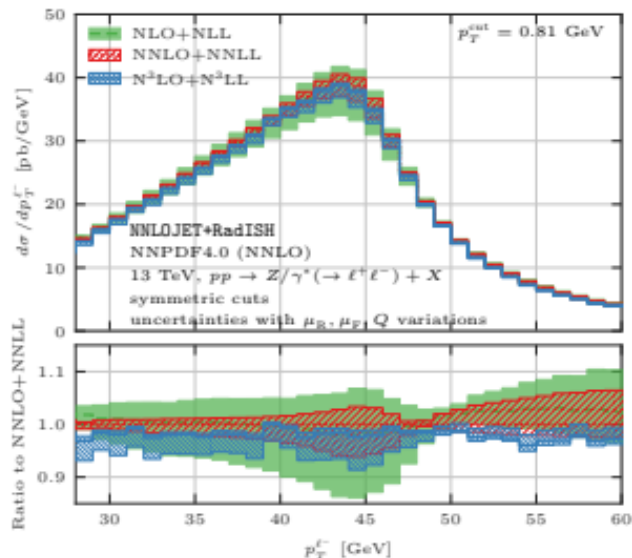
- Tiny scale dependence with respect to NNLO, in particular for large Q
- No overlap of scale bands for $50 < Q < 400$
- Maybe a PDF effect? No N3LO PDFs available
→ Estimated effect relatively large



Fiducial predictions at N3LO

Third order fiducial predictions for Drell-Yan at the LHC,

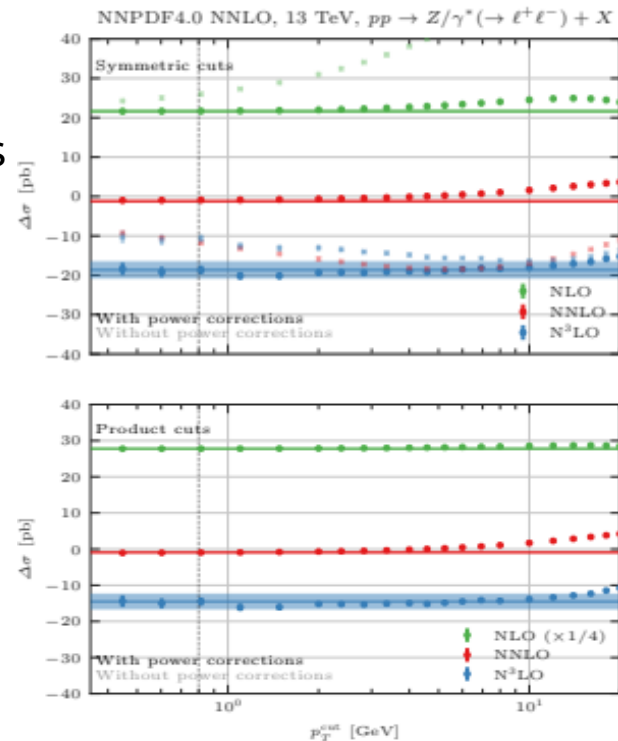
Chen, Gehrmann, Glover, Huss, Monni, Re, Rottoli, and Torrielli, 2203.01565



- N3LO + N3LL fiducial predictions
- Power corrections crucial for symmetric cuts → Rottoli's talk
- Symmetric vs. product cuts

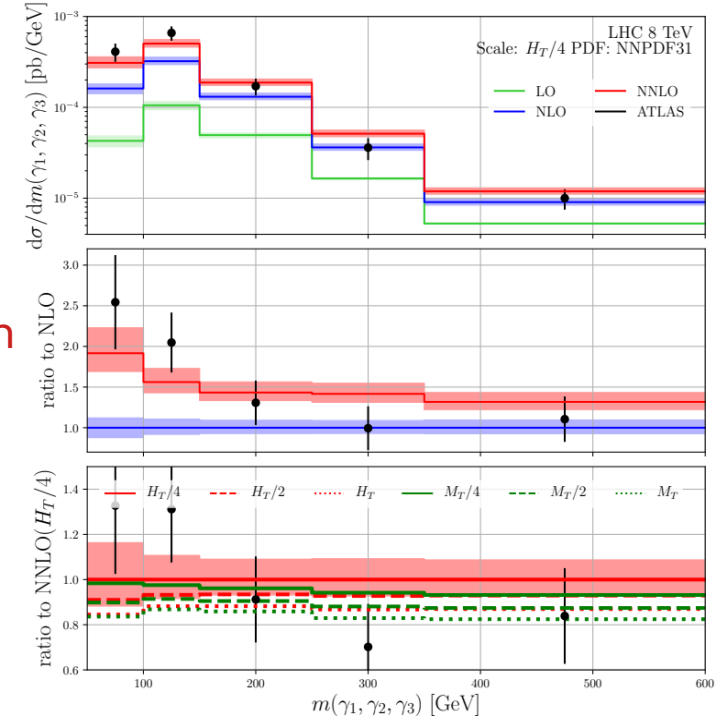
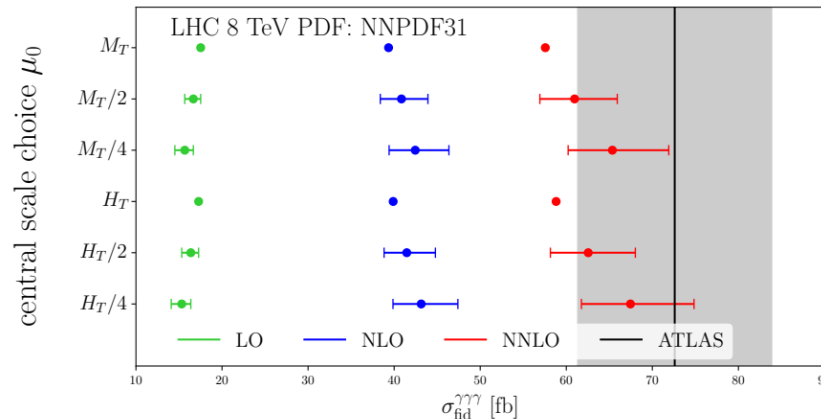
Cuts for two-body decays at colliders

Gavin P. Salam, Emma Slade, 2106.08329



Three photon production

- First NNLO QCD $2 \rightarrow 3$ cross sections:
 NNLO QCD corrections to three-photon production at the LHC, Chawdhry, Czakon, Mitov and Poncelet, 1911.00479
 Triphoton production at hadron colliders in NNLO QCD, Kallweit, Sotnikov and Wiesemann, 2010.04681
- Simplest among the $2 \rightarrow 3$ massless cases: colour singlet
- Approximation in two-loop virtuals: only planar diagrams
 → overall small contribution
- Large NNLO/NLO K-factors
- NNLO QCD corrections essential for theory/data comparison
 Here: ATLAS



Diphoton plus jet production

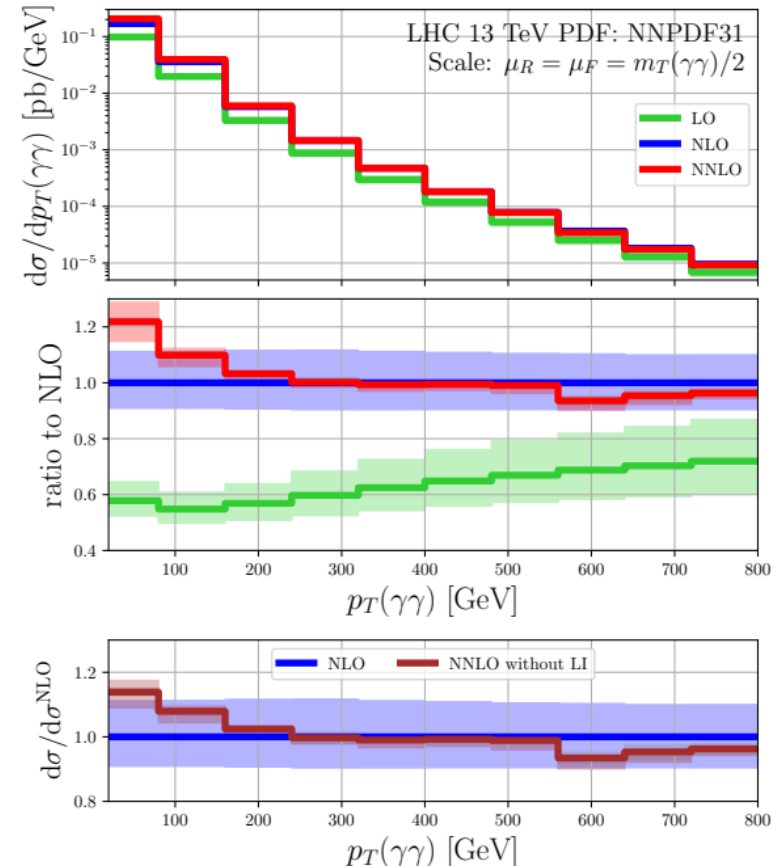
Photon pair production @ LHC is of particular interest:

- Main background to cleanest Higgs decay channel
- Large NNLO QCD corrections!
Perturbative convergence @ N3LO?
- Diphoton plus jet @ NNLO QCD ($p_T(\text{AA}) \rightarrow 0$ limit)
- $p_T(\gamma\gamma)$ spectrum itself interesting for Higgs $\rightarrow \gamma\gamma$

First NNLO QCD for $pp \rightarrow \text{AA}j$

NNLO QCD corrections to diphoton production with an additional jet at the LHC,
Chawdhry, Czakon, Mitov and Poncelet, 2105.06940

- Beautiful perturbative convergence
- Scale dependence: NLO: $\sim 10\%$ NNLO: $\sim 1-2\%$
- Low p_T region:
 - ? Resummation for $p_T(\gamma\gamma)/m(\gamma\gamma) \ll 1$
 - Strong effect from the loop induced!

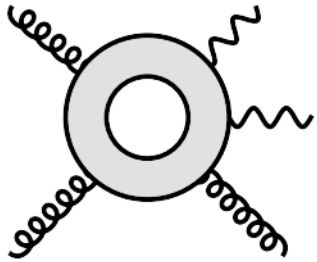


Diphoton plus jet – gg fusion

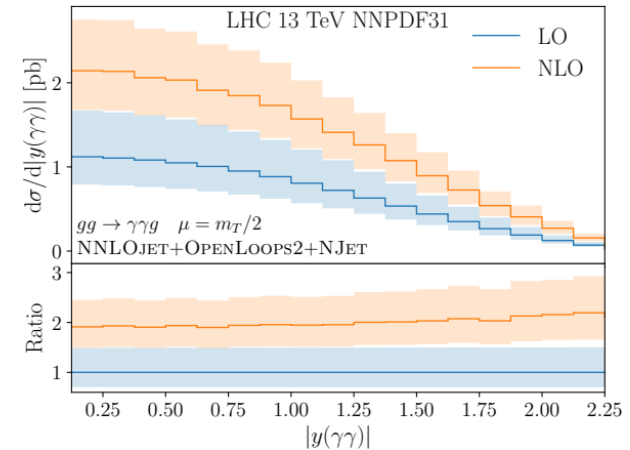
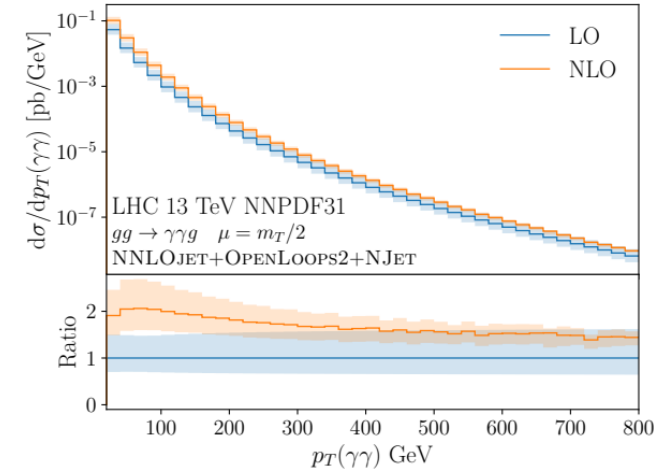
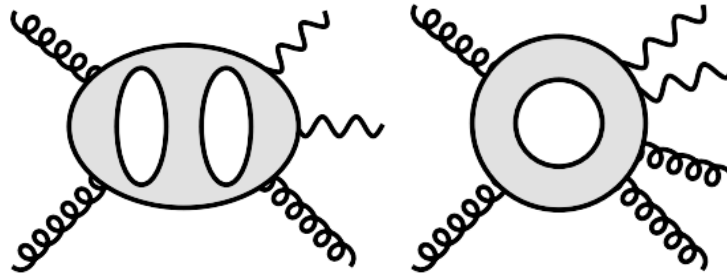
Next-to-leading order QCD corrections to diphoton-plus-jet production through gluon fusion at the LHC, Badger, Gehrmann, Marcoli and Moodie, 2109.12003

- NLO QCD to $gg \rightarrow \gamma\gamma$ (formally N3LO for $pp \rightarrow \gamma\gamma$)
- Challenging double virtual matrix element
- Large corrections of up to 100% \rightarrow relate to 5% in full $pp \rightarrow \gamma\gamma$
- Reduction of scale dependence at high transverse momentum

LO:



NLO:



Three-jet production

Next-to-Next-to-Leading Order Study of Three-Jet Production at the LHC, Czakon, Mitov and Poncelet, 2106.05331

Computational challenges:

- Sector-improved residue subtraction for real radiation
 - Efficient c++ implementation → STRIPPER
 - Highly automated to deal with enormous amount of channels in three-jet production
→ $O(1k)$ sectors → $O(1M)$ individual MC integrals
- Many-leg, IR stable one-loop amplitudes → OpenLoops 2
- Double virtual amplitudes in leading-colour approximation
 - Sub-leading colour corrections expected to be small
 - Analytical expressions challenging
 - Fast numerical evaluation → very small contribution to computational cost
- The pure gluonic process evaluated within the NNLOJet framework:

A novel subtraction scheme for double-real radiation at NNLO,
Czakon, 1005.0274

Four-dimensional formulation of the sector-improved residue subtraction scheme, Czakon and Heymes, 1408.2500

Single-jet inclusive rates with exact color at $O(\alpha_s^4)$
Czakon, van Hameren, Mitov and Poncelet, 1907.12911

OpenLoops 2, Buccioni, Lang, Lindert,
Maierhöfer, Pozzorini, Zhang, Zoller, 1907.13071

Leading-color two-loop QCD corrections for three-jet production at hadron colliders,
Abreu, Cordero, Ita, Klinkert, Page, Sotnikov, 2110.07541

**Automation of antenna subtraction in colour space:
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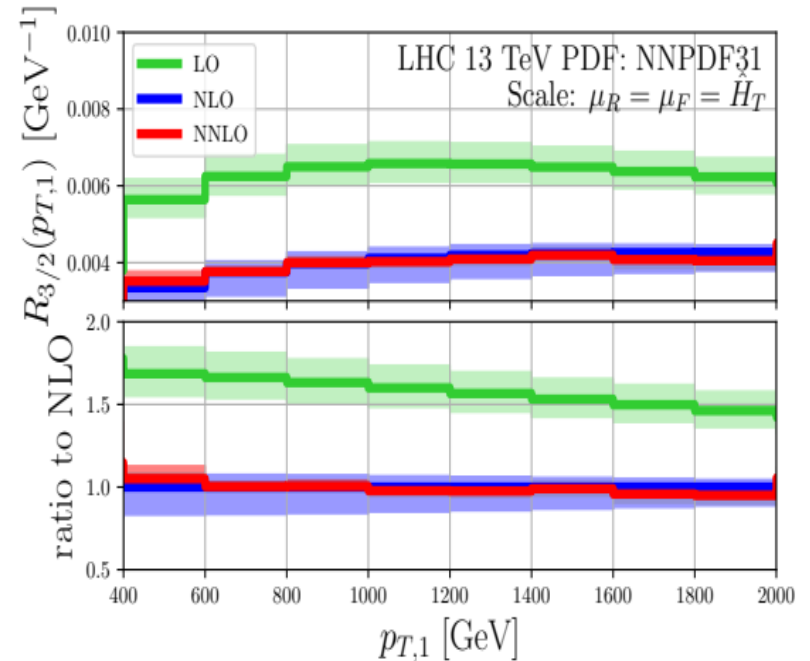
Three-jet production - R32

- LHC @ 13 TeV, NNPDF31
- Require at least three (two) jets:
 - $p_T(j) > 60$ GeV and $|y(j)| < 4.4$
 - $H_{T,2} = p_T(j_1) + p_T(j_2) > 250$ GeV
- Scales: $\mu_R = \mu_F = \hat{H}_T = \sum_{\text{partons}} p_T$

$$R_{3/2}(X, \mu_R, \mu_F) = \frac{d\sigma_3(\mu_R, \mu_F)/dX}{d\sigma_2(\mu_R, \mu_F)/dX} \sim \alpha_s$$

Interesting phenomenological applications:

- Extraction of α_s , tests of SM running and tests of QCD matrix elements
- R32, event-shapes, TEEC, azimuthal decorrelation



NNLO QCD meets parton showers

NNLO QCD + PS available (MiNNLO_PS, Geneva + Pythia)

- $pp \rightarrow H$
- $pp \rightarrow W/Z$
- $pp \rightarrow VH$
- $pp \rightarrow WW/ZZ/Z\gamma$
- $pp \rightarrow t\bar{t}$ (+PS decays)
→ Wiesemann's talk

Matching to LL accuracy → Prestel's talk

- keeping NNLO accuracy for inclusive observables
- but lifting fixed order kinematic constraints
→ example sub-leading top-quark p_T

Matching NNLO predictions to parton showers using N3LL color-singlet transverse momentum resummation in geneva, Alioli, Bauer, Broggio, Gavardi, Kallweit, Lim, Nagar, Napoletano, Rottoli, 2102.08390

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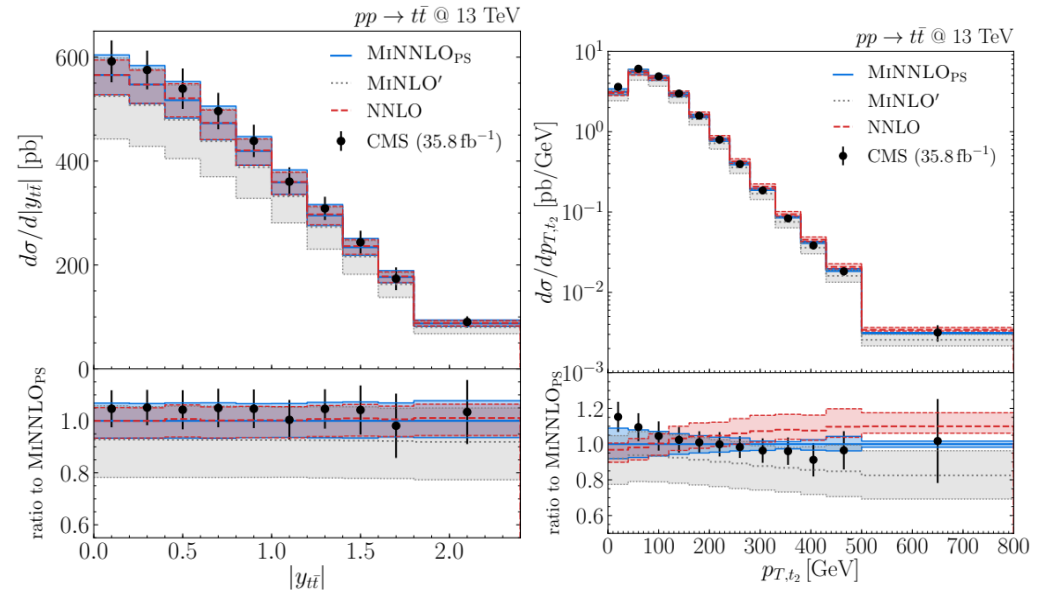
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NNLO+PS with MiNNLO_PS: status and prospects, Buonocore et al., 2203.07240

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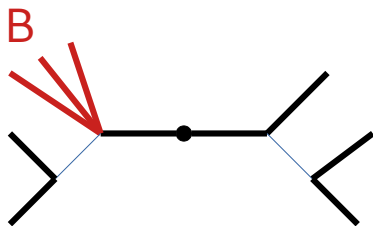


Fixed-order Fragmentation

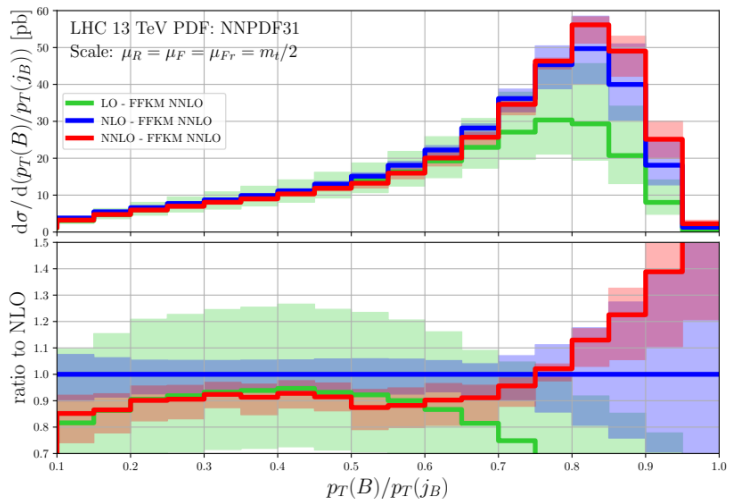
- Fixed order QCD predictions with a final state hadron
- Considering partonic computation + transition of parton to hadron (collinear fragmentation of massless partons)
- Advantage is that the hadrons momentum is measurable while the quark's is not
- Fragmentation function (similar to PDFs)
Probability to find a hadron with a fraction x of the quarks momentum: $D_{i \rightarrow h}(x)$
- No Parton-shower needed
- Implementation in the STRIPPER framework through NNLO QCD → **Czakon's talk**
B-hadron production in NNLO QCD: application to LHC $t\bar{t}$ events with leptonic decays,
Czakon, Generet, Mitov and Poncelet, 2102.08267
- Photon fragmentation in NNLOJet → **Höfer's talk**
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B-hadrons in ttbar production

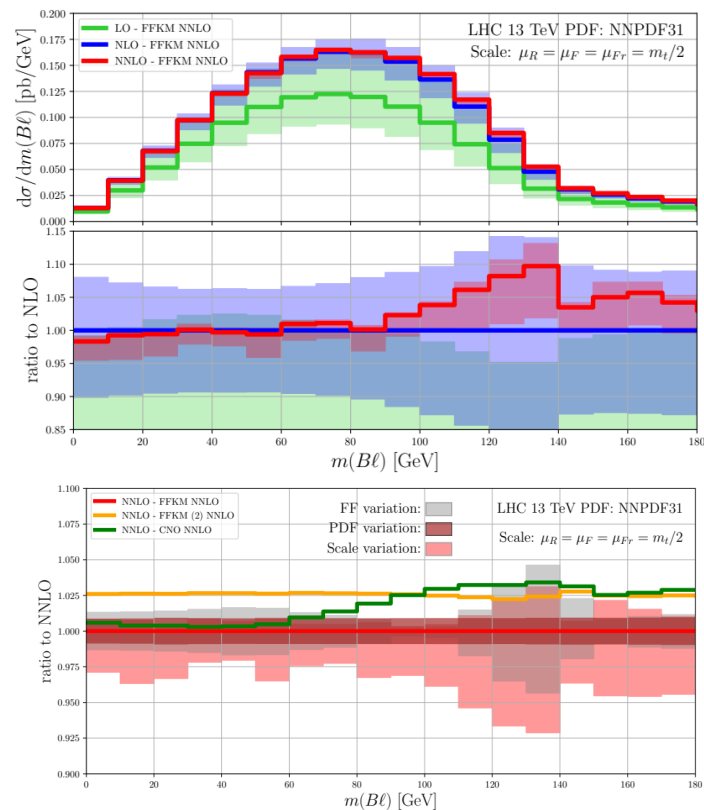
$$pp \rightarrow t\bar{t} \rightarrow B\ell\bar{\ell}\nu\bar{\nu}b + X$$



$p_T(B)/p_T(j_B)$: sensitive to B-hadron fraction x



$m(B\ell)$: sensitive to top-quark mass



Further interesting topics

- Mixed QCD-EW → Freitas', Vicini's, Bonetti's talk
- Flavoured jets → useful for PDF fits BUT: → Czakon's talk
 - Anti-kT is IR unsafe at NNLO QCD
 - Ways around by using the flavour kT algorithm
 - But: Measurements are performed with anti-kT algorithm
 - additional unfolding necessary
 - makes applications to PDF fits difficult
 - Room for improvement → flavour sensitive anti-kT algorithm
- New FastNLO tables @ NNLO by the NNLOJet group for DIS, single inclusive jets, pp → Zj
- SMEFT meets NNLO+PS
 - NNLO event generation for pp → Zh → l+l- bbar production in the SM effective field theory, Haisch, Scott, Wiesemann, Zanderighi, Zanoli, 2204.00663

What's next?

N3LO:

- Techniques based on q_T slicing+resummation do work for any colour singlet process
- Limitations:
 - Computational complexity:
 - Numerical stability of two-loop amplitudes
 - Stability of NNLO subtractions in $q_T \rightarrow 0$ limit
 - Power corrections in fiducial phase spaces
 - Three loop amplitudes $pp \rightarrow S$
 - Numerical stable two-loop amplitudes: $2 \rightarrow 3$ amplitudes

$pp \rightarrow AA+X$

Diphoton Amplitudes in Three-Loop Quantum Chromodynamics,

Caola, Von Manteuffel and Tancredi, 2011.13946

Two-Loop Helicity Amplitudes for Diphoton Plus Jet Production in Full Color,

Agarwal, Buccioni, von Manteuffel and Tancredi, 2105.04585

$pp \rightarrow WA+X ?$

Two-loop leading colour helicity amplitudes for WA_j production at the LHC,

Badger, Hartanto, Krys and Zoia, 2105.04585

What's next?

NNLO QCD:

- New processes → what two-loop amplitudes are or will be available soon?
 - $2 \rightarrow 3$ massless: $pp \rightarrow AAA$ (LC), $pp \rightarrow AAj$ (FC), $pp \rightarrow jjj$ (LC), $pp \rightarrow Ajj$?
→ All ingredients available for the complete set → at most technical challenges
 - $2 \rightarrow 3$ one-mass:
 - Progress on master integrals:
 - Analytic representation of all planar two-loop five-point Master Integrals with one off-shell leg**, Canko, Papadopoulos, Syrrakos, 2009.13917
 - Pentagon functions for one-mass planar scattering amplitudes**, Chicherin, Sotnikov and Zoia, 2110.10111
 - Two-loop hexa-box integrals for non-planar five-point one-mass processes**, Abreu, Ita, Page and Tschernow, 2107.14180
 - $pp \rightarrow Wjj$ (planar/LC):
 - Two-Loop QCD Corrections to Wbb Production at Hadron Colliders**, Badger, Hartanto and Zoia, 2102.02516
 - Leading-Color Two-Loop Amplitudes for Four Partons and a W Boson in QCD**, Abreu, Cordero, Ita, Klinkert, Page, Sotnikov, 2110.07541
 - $2 \rightarrow 3$ two-mass ???
- Matching to parton showers with final state jets:
 - Towards NNLO+PS Matching with Sector Showers**, Campbell, Höche, Li, Preuss and Skands, 2108.07133

Summary

- NNLO QCD covers more or less all $2 \rightarrow 1$ and $2 \rightarrow 2$ processes
- First $2 \rightarrow 3$ processes become available: $pp \rightarrow 3\gamma$, $pp \rightarrow yyj$, $pp \rightarrow jjj$
 - Double virtual amplitudes for $2 \rightarrow 3$ one-mass processes start to appear but virtual amplitudes are the main bottleneck for extending the portfolio \rightarrow automation of numerical two-loop amplitudes?
- Drell-Yan type processes at N3LO
 - Fully differential \rightarrow fiducial phase spaces
 - Need for N3LO PDFs
 - In foreseeable future: $pp \rightarrow yy$? \rightarrow many technical challenges
- NNLO QCD + parton showers: $pp \rightarrow S$, $pp \rightarrow t\bar{t}$
- Fragmentation at NNLO QCD
- And many other exciting applications!