Primordial black holes in a nutshell

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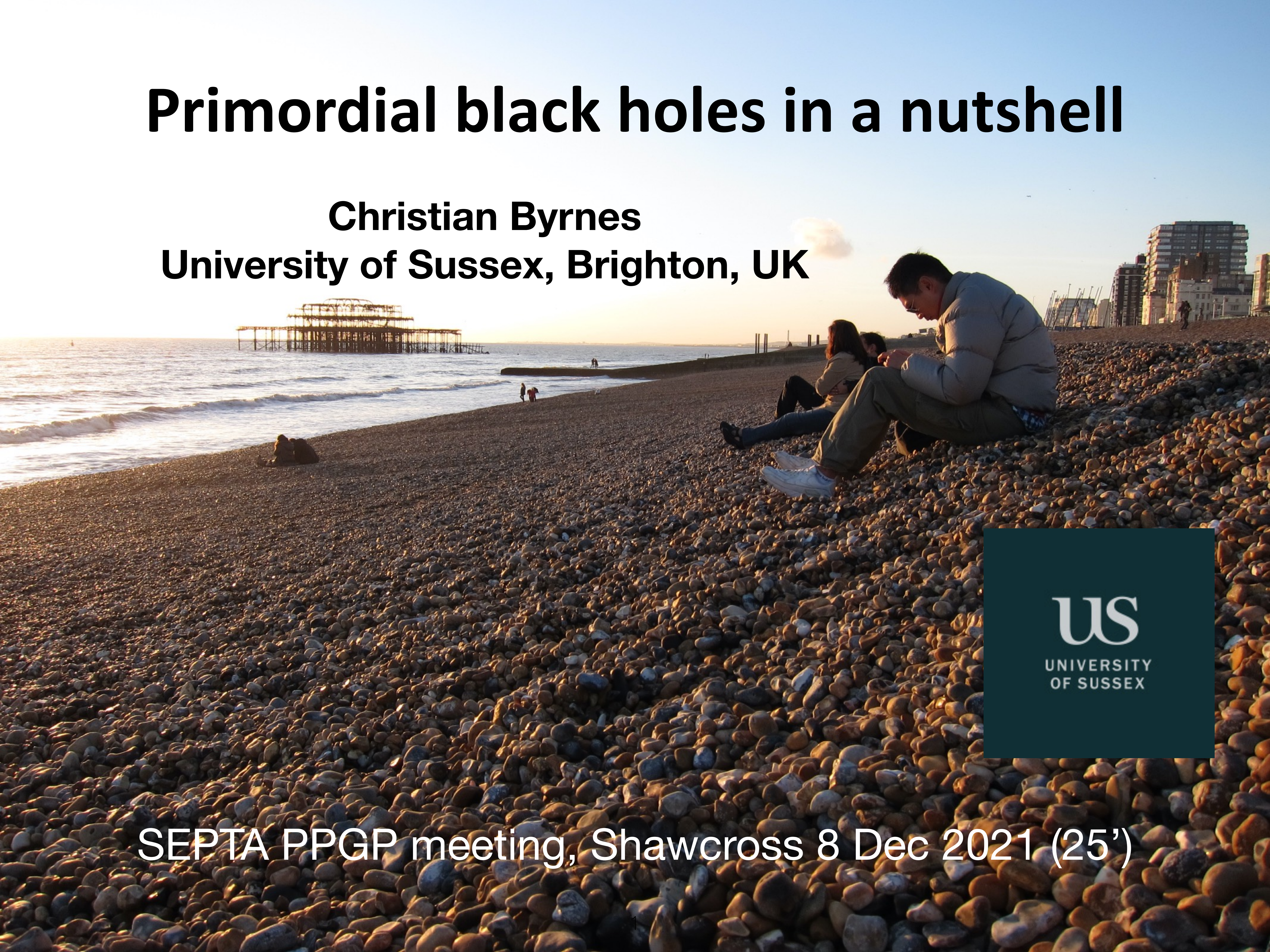
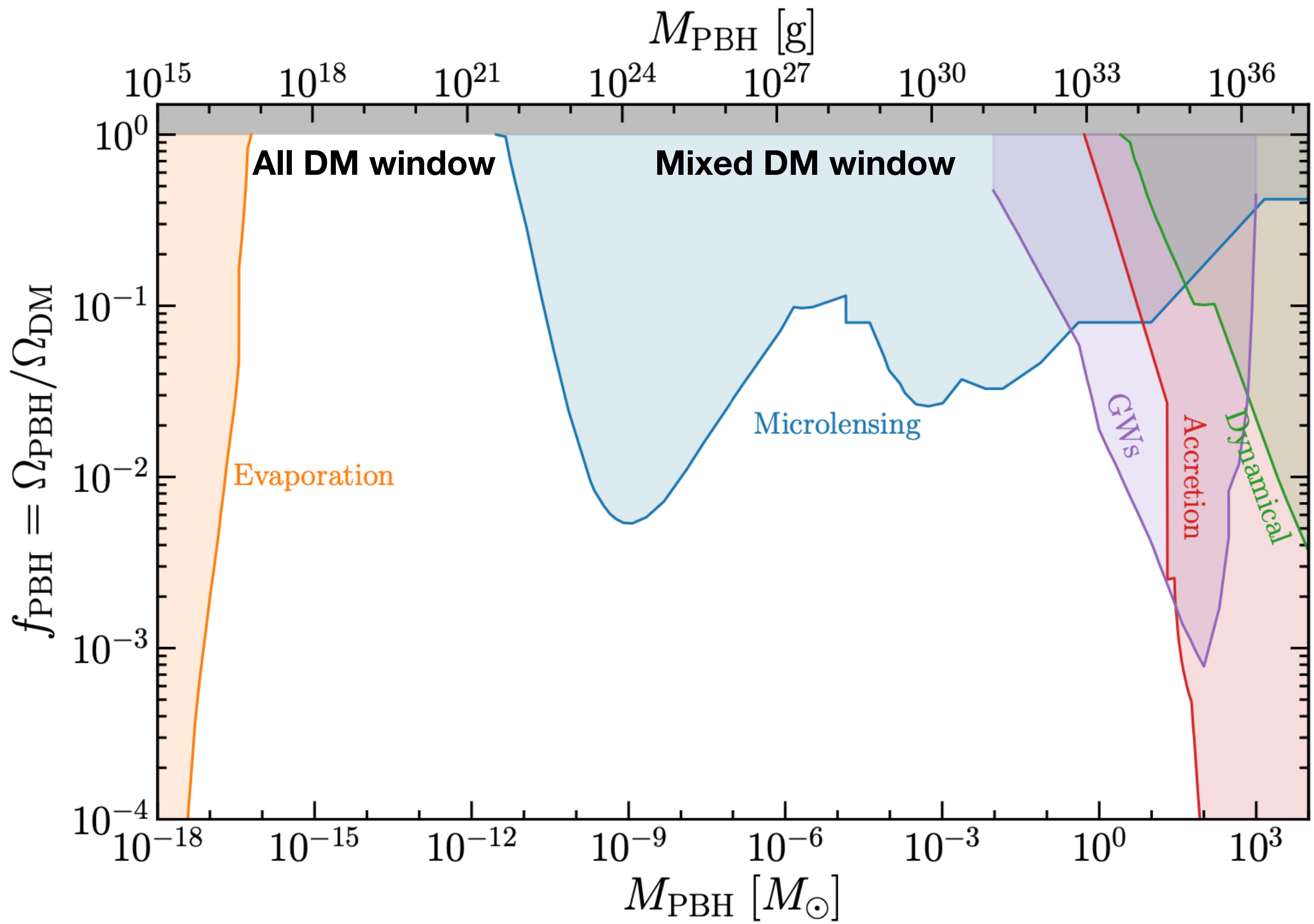


SEPTA PPGP meeting, Shawcross 8 Dec 2021 (25')

Cold dark matter

- The evidence includes galaxy rotation curves, galaxy cluster velocities and the bullet cluster
- Black holes are cold and dark
- However, the growth of structure from the CMB till today proves DM must have formed before the CMB.
- Primordial BHs are a candidate

Observational constraints



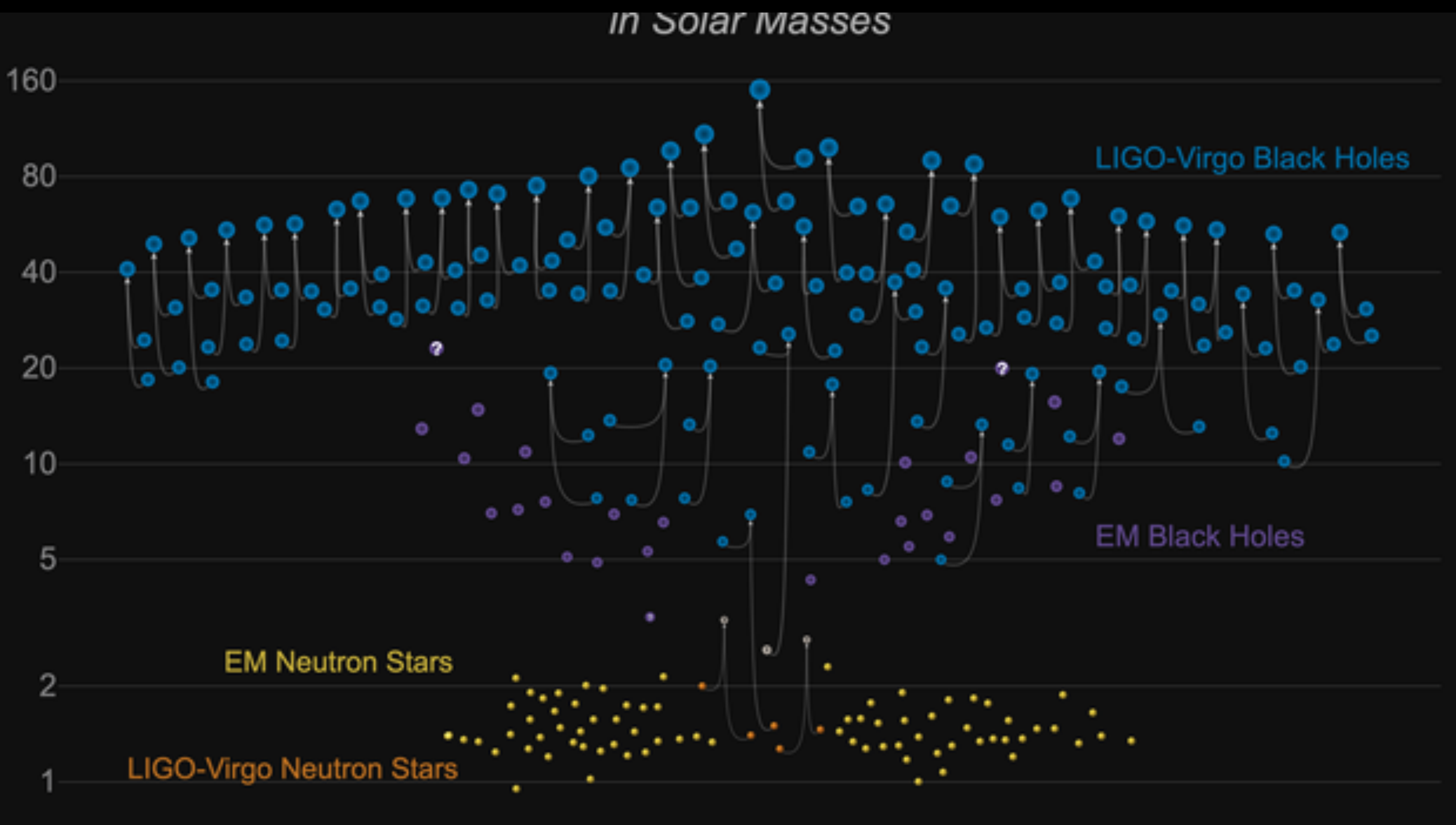
PBHs=All DM

PBHs=1% DM

Green and Kavanagh https://arxiv.org/pdf/2007.10722.pdf

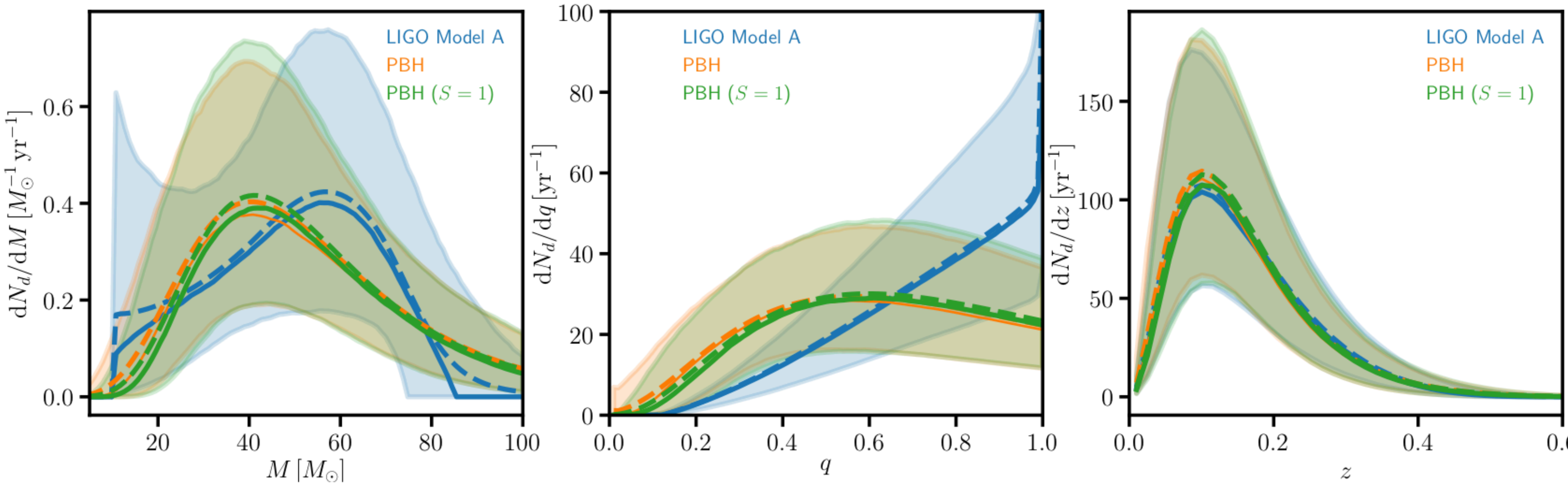
The LIGO-Virgo events

- It appears unlikely that more than 1% of the dark matter can be made out of LIGO mass PBHs
- But could the LIGO BHs be primordial?
- Black holes have no hair, so how can we know?
  Total/chirp mass
  Mass ratio
  (Spin, redshift distribution and location)



LIGO & Virgo collaboration

total mass **mass ratio**



Hall, Gow, CB, 2020: Bayesian comparison

PBHs are better at explaining events with a small mass ratio, but don't naturally explain the upper and lower mass gaps predicted by stellar models.
PBHs are more flexible at explaining individual events.

Overall, only PBHs is decisively disfavoured compared to only astrophysical BHs. Total/chirp mass information dominates the signal.

Spin in isolation favours PBHs: *Garcia-Bellido et al '20, Wong et al 2020*

Ligo-Virgo BH lesson

- PBHs alone strongly disfavoured: Even when attempting to fit arbitrarily tuned PBH mass functions: *Hall, Gow, CB, '20*
- However, some evidence that a subdominant PBH population improves the fit: e.g. *Franciolini et al 2021*
- This evidence depends on the astrophysical formation channels, which are highly uncertain and regularly updated
- Fitting into the mass gaps may be the best hope, but could be second generation compact objects?

Sub-solar mass compact objects

- Second generation compact objects can only be heavier
- A sub Chandrasekhar/solar mass compact object cannot form within standard model astrophysics

Search for subsolar-mass binaries in the first half of Advanced LIGO and Virgo's third observing run

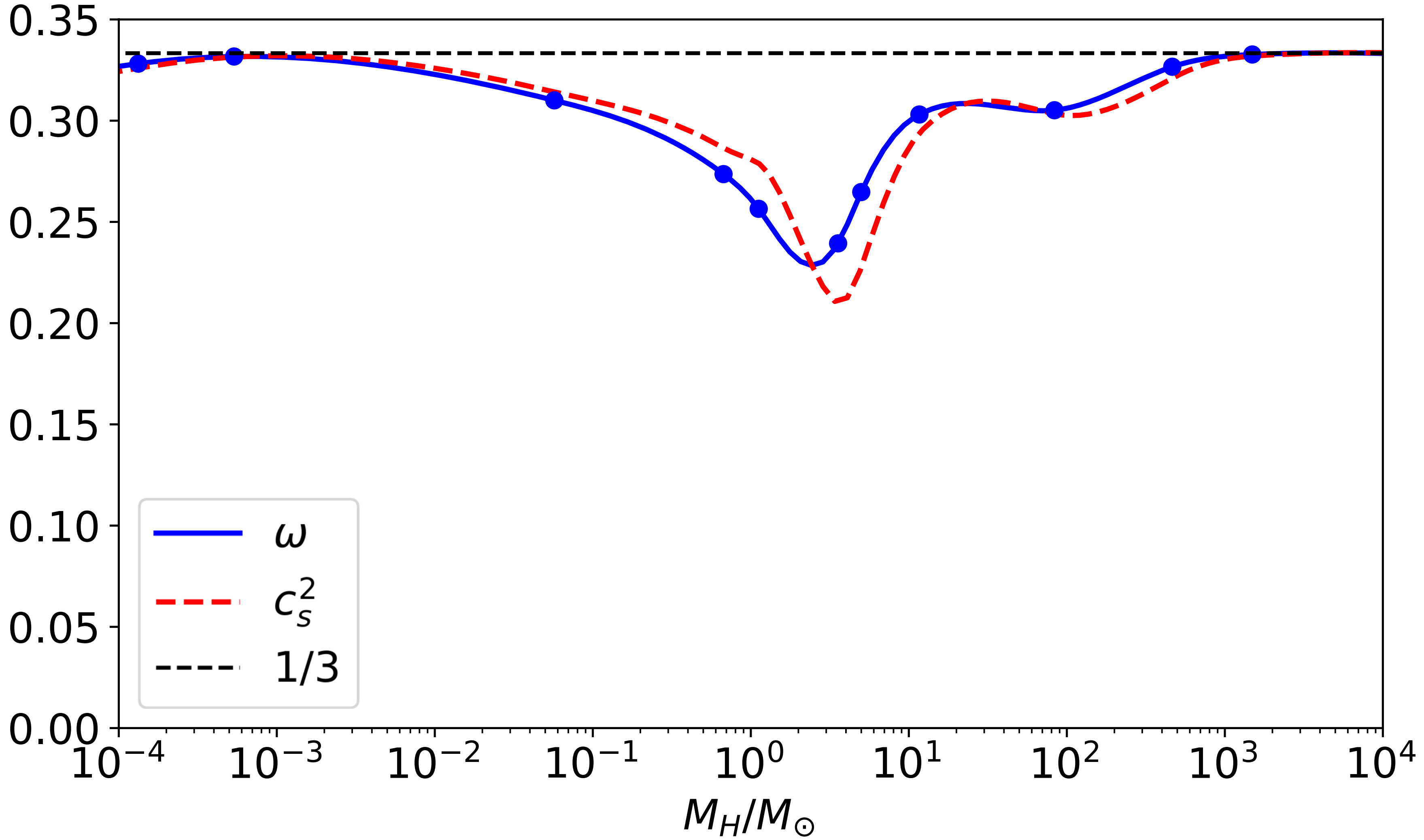
LIGO Scientific and VIRGO and KAGRA Collaborations • R. Abbott Show All(1634)
Sep 24, 2021

20 pages
e-Print: 2109.12197 [astro-ph.CO]

The QCD transition

Strong interactions confine quarks into hadrons and the equation-of-state parameter *w* decreases. *Crawford & Schramm `82, Jedamzik `98*

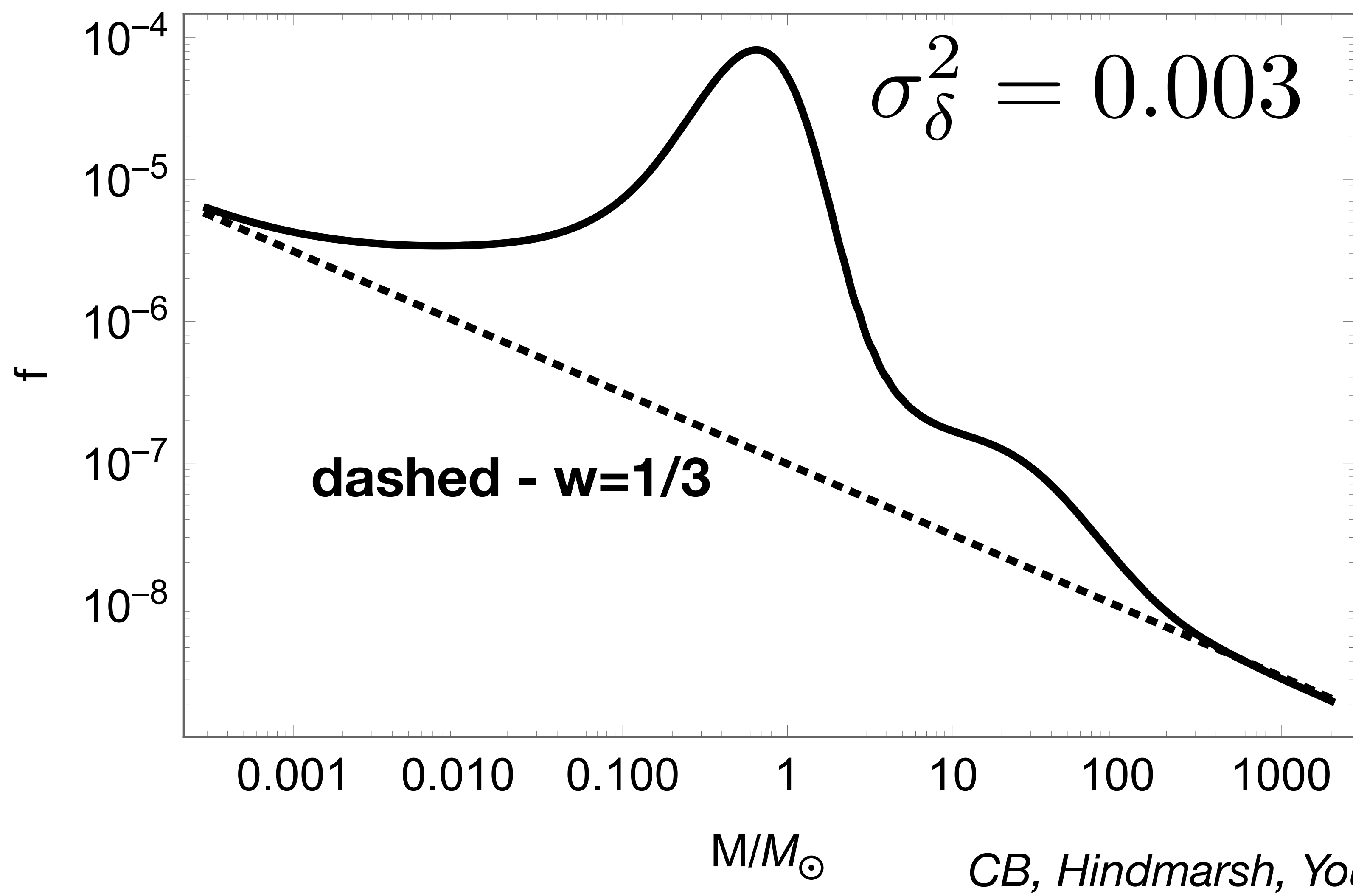
QCD transition: t~10^{-6} s, T~200 MeV, M~1 $M_\odot$, k~10^{7} Mpc^{-1}



CB, Hindmarsh, Young & Hawkins 2018 using Borsanyi et al 2016

The resultant PBH-QCD mass function

$$f(M) \propto M^{-1/2} e^{-\frac{\delta_c^2}{2\sigma_\delta^2}}$$



CB, Hindmarsh, Young & Hawkins 2018

The QCD phase transition took place during the time when LIGO mass PBHs would have formed. It boosts the formation rate of solar mass PBHs by 2 orders of magnitude These are below the Chandrasekhar mass - potential proof of PBHs

A huge boost in the perturbation amplitude is required, even with QCD

- At second order, scalar and tensor perturbations couple

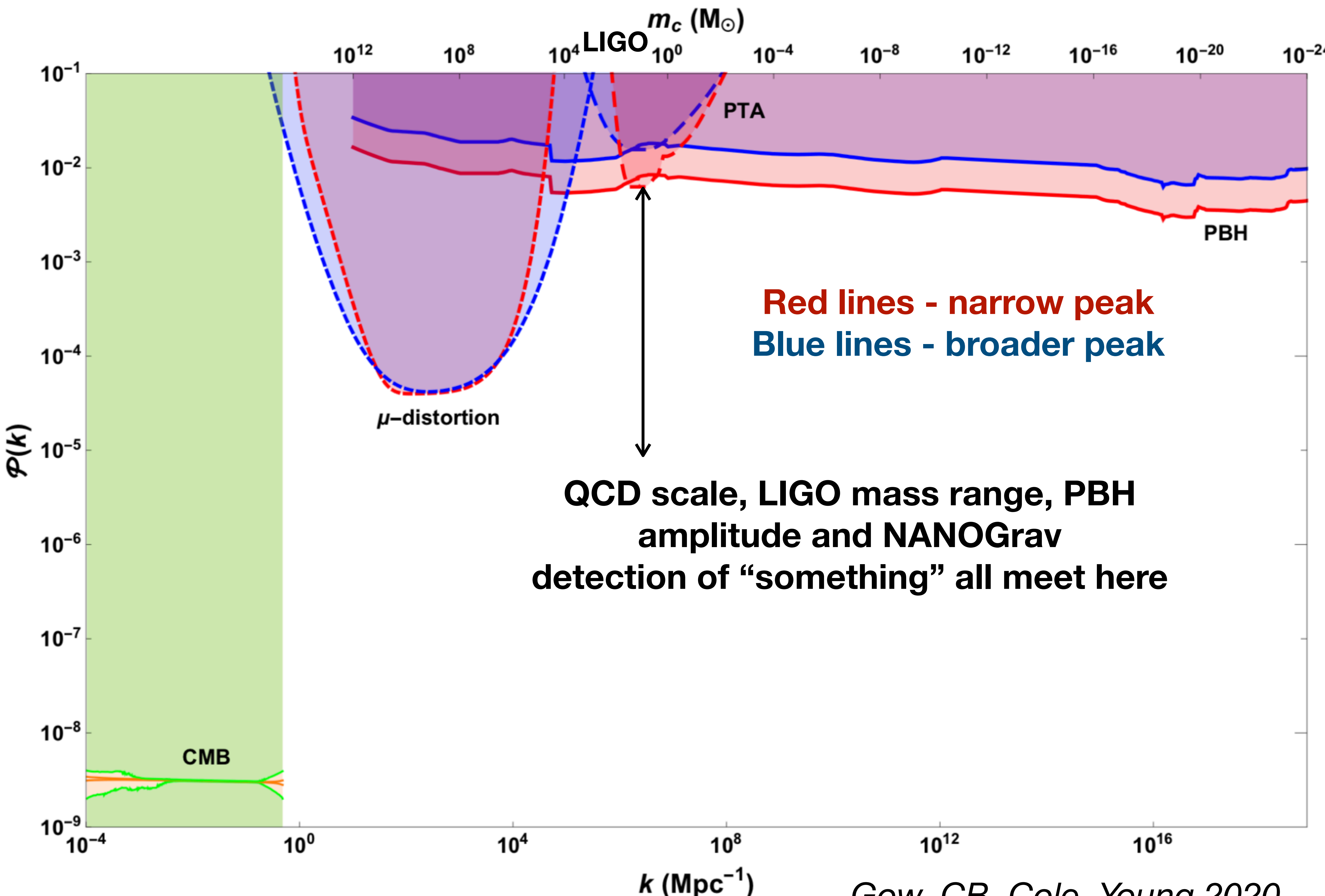
$$\Omega_{\rm GW}^{\rm induced} \sim \frac{1}{12}\Omega_{r,0}\mathscr{P}_{\mathscr{R}}^2 \sim 10^{-6}\mathscr{P}_{\mathscr{R}}^2(k \gg k_{\rm CMB})$$

- This stochastic GW background is determined by the spike in the power spectrum: relation between peak position, horizon mass and frequency + relation between peak height and GW amplitude

- The result is only logarithmically sensitive to f_{PBH}

- *Domenech induced GW review article 2021*

The initial conditions of the universe



Gow, CB, Cole, Young 2020

A collaboration of former PhD students

Andrew Gow (-> ICG, Portsmouth)

Pippa Cole (GRAPPA, Amsterdam)

Sam Young
Humboldt Fellow at the MPA, Munich
-> Marie Curie Fellow at Leiden Uni

Power spectrum messages

- Assuming Gaussian perturbations and that PBHs form from the direct collapse of large overdensities

    1. The formation of supermassive PBHs is completely ruled out

    2. LIGO-Virgo mass BHs produce a stochastic GW background which the PTA experiments should detect now/soon

    3. No competitive power spectrum constraints on even smaller scales, yet

Tentative PTA detection?

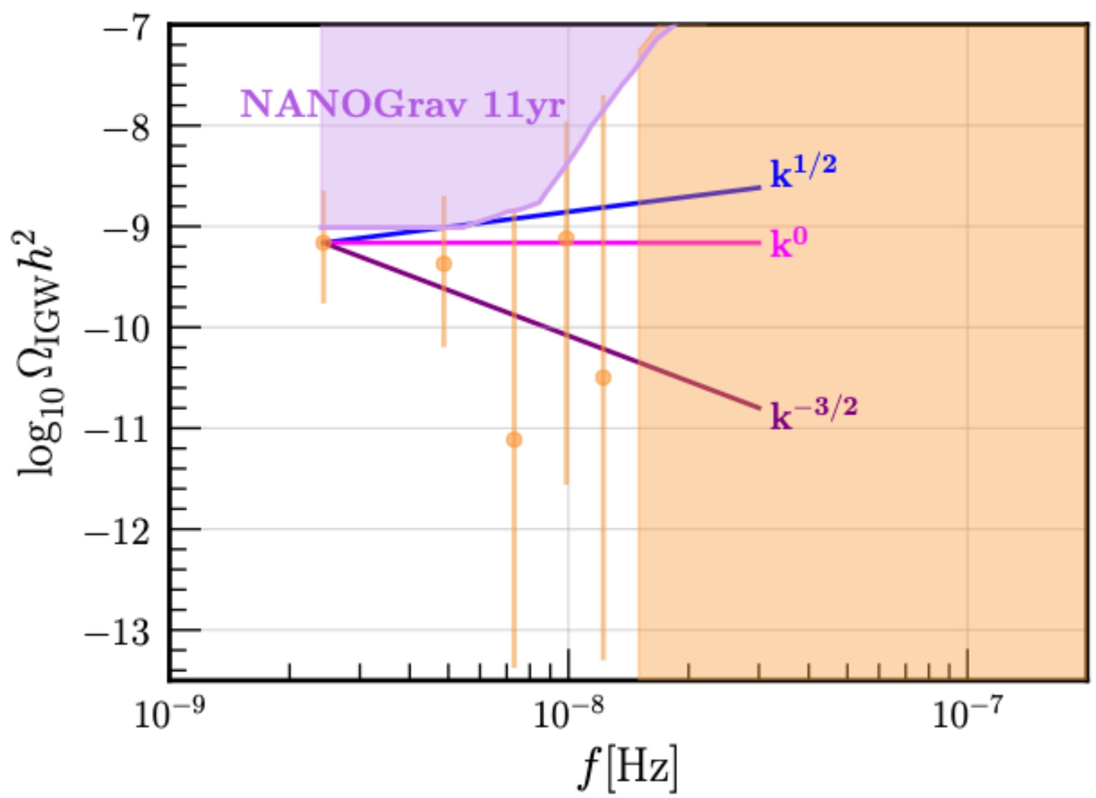
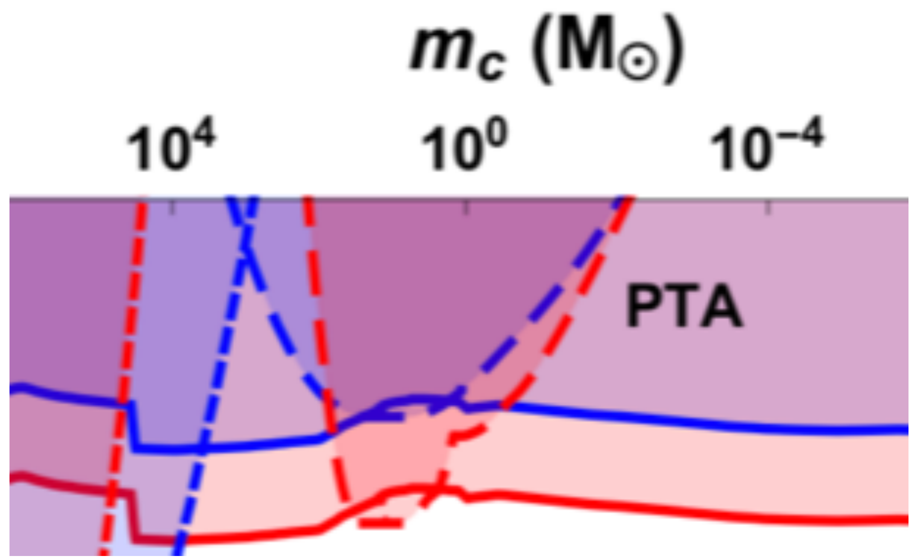


Figure 9: NANOGrav results on the possible SGWB [18] in terms of spectral density. The orange dots are the data points from the timing residuals with their error bars. We only show the first five data points which are the most relevant. The points in the orange shaded region were compatible with white noise and not considered for the fit. In light purple we also show the NANOGrav 11-yr sensitivity curve from [318]. In solid lines we show the allowed slopes within the 1-σ contours. In blue and purple we respectively show the upper and lower limits on the slope, $k^{1/2}$ and $k^{-3/2}$. In magenta we give the approximate best fit for the data, which is a scale invariant spectrum.

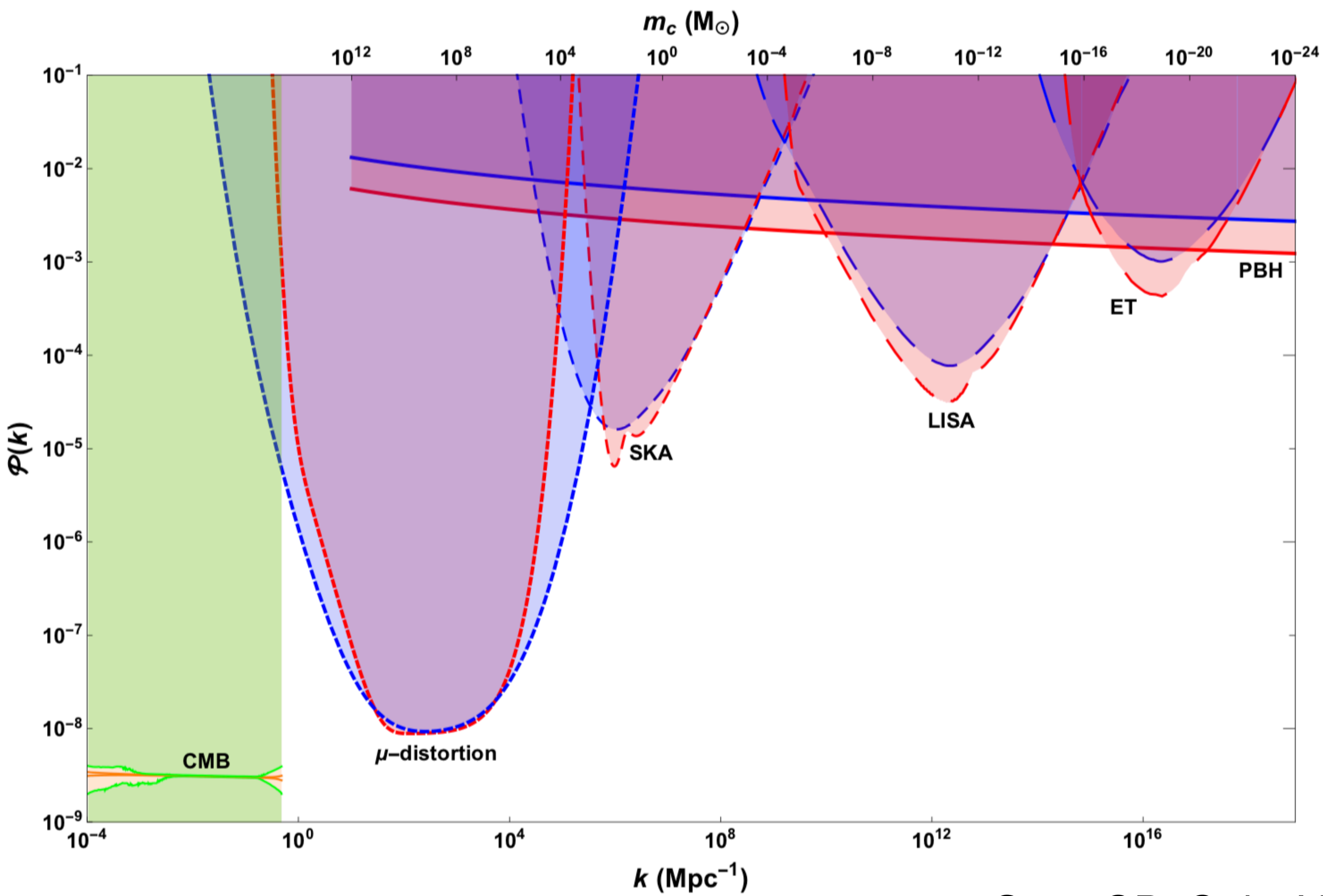
Domenech review https://arxiv.org/pdf/2109.01398.pdf

The hope



- Evidence for PBHs appears/strengthens
- This gives us a measurement of the (integrated) amplitude and position of a spike in the power spectrum
- The corresponding stochastic GW background is seen by PTA
- There is a consistency relation
- Non-Gaussianity is a key degeneracy - impact on the power spectrum amplitude, initial clustering, merger rates and stochastic GW spectrum
- The equation of state is a second degeneracy, but the SM predicts the QCD peak

Forecast constraints

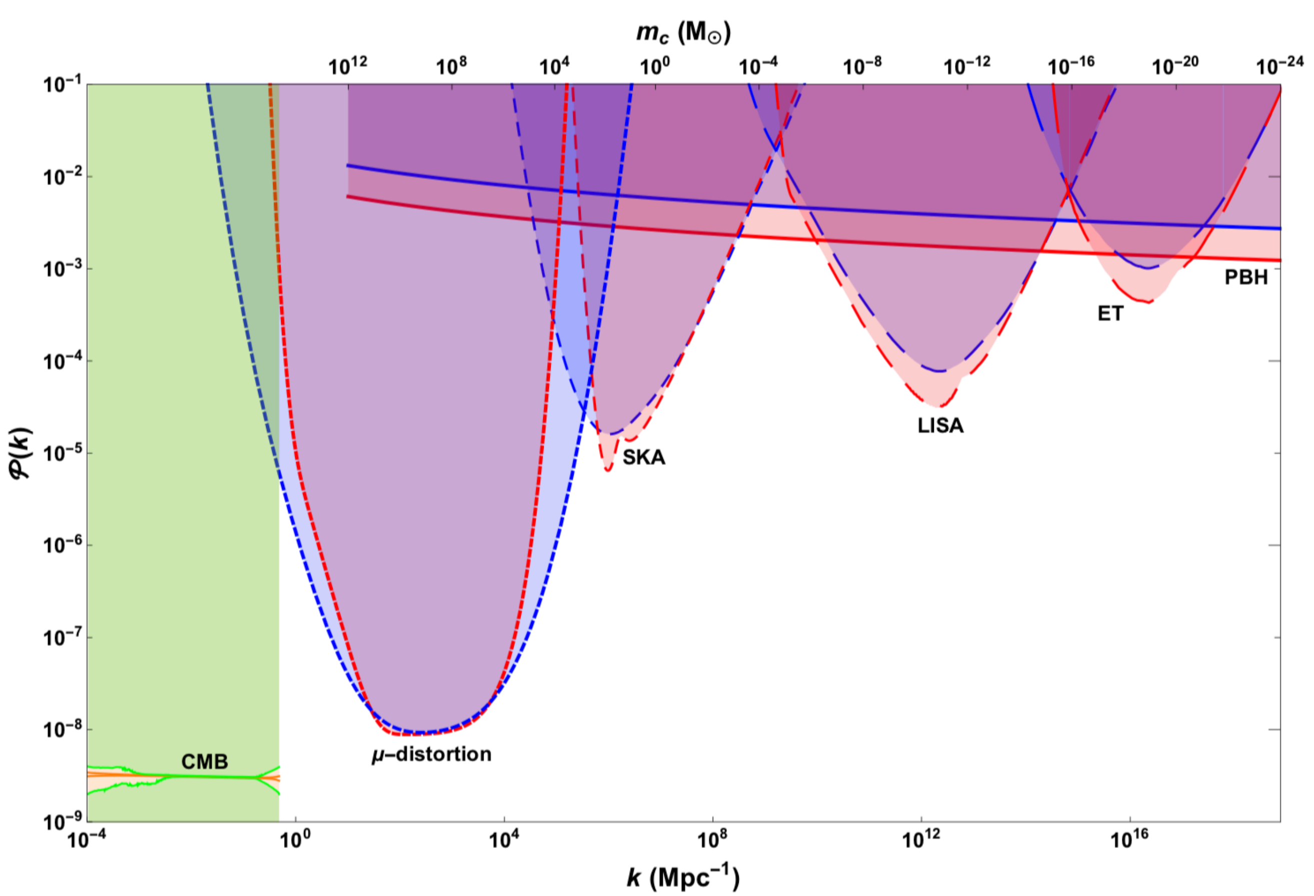


The PBH lines correspond to zero PBHs
Cole & CB '17

Gow, CB, Cole, Young 2020

Potential SEPTA research

- Check the secondary GW signal from alternative PBH generation mechanisms
- Use the Sussex TPP group expertise to study phase transitions or defect models
- PBH relics as a link to quantum nature of BHs/information paradox



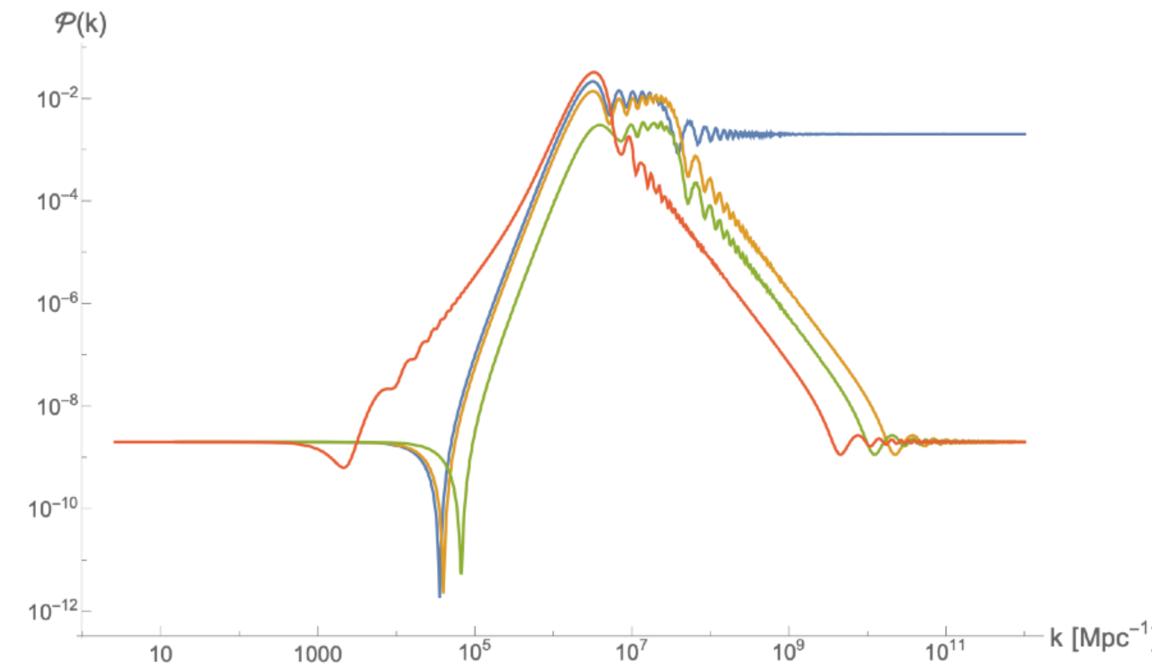
Ongoing research

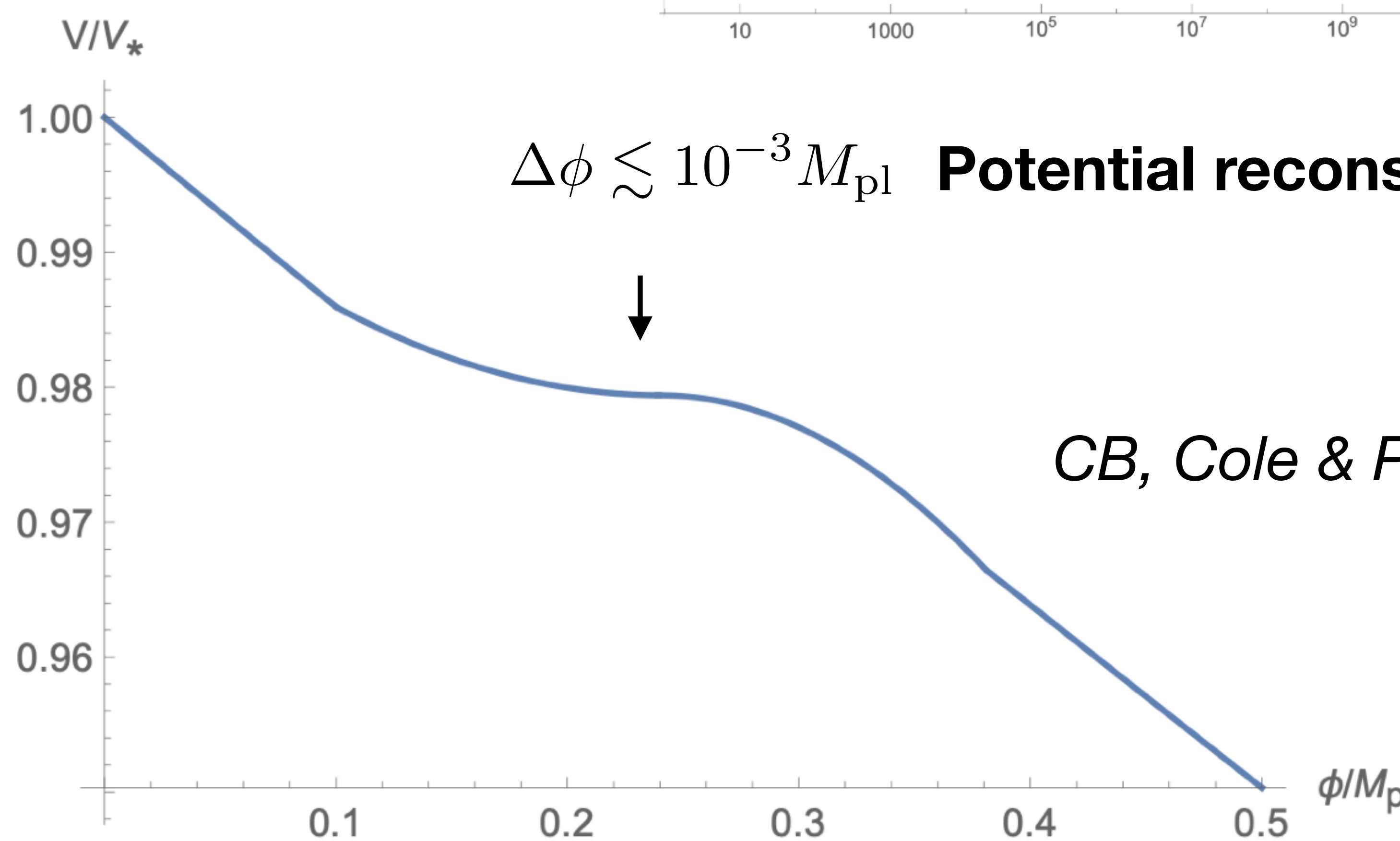
- With Itzi and David: PBHs in braneworld scenarios
  Small BHs are extra-dimensional
  Potential changes to accretion, spin, evaporation and early universe evolution

- With Pippa + David: Non-Gaussianity associated with single-field inflation generating PBHs

- With Pippa, Andrew + Subodh Patil: Quantifying the fine tuning of inflationary models that are capable of generating PBHs

Inflationary fine-tuning with an inflection point





$\Delta\phi \lesssim 10^{-3} M_{\rm pl}$ **Potential reconstruction**

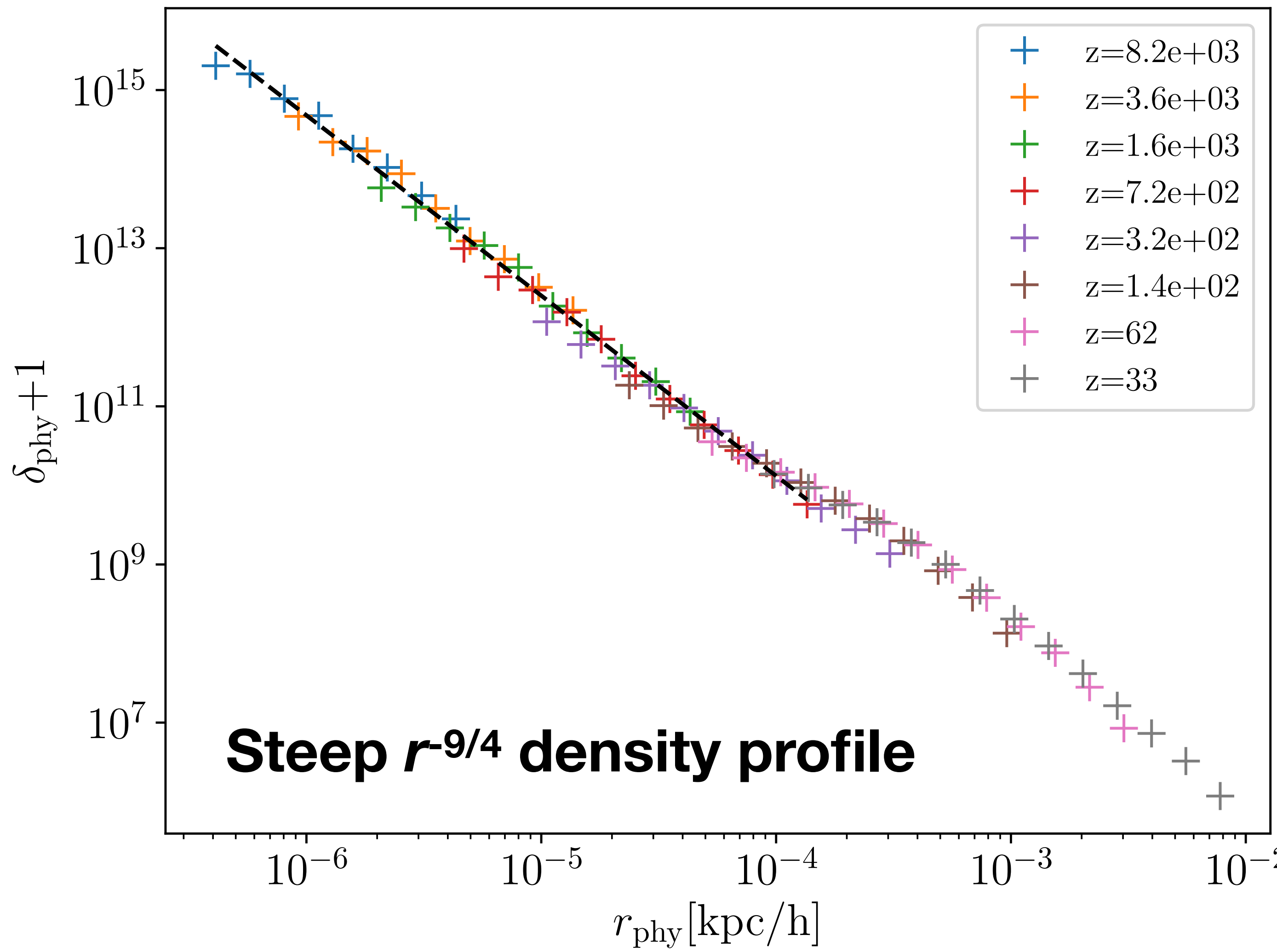
CB, Cole & Patil 2018

PBH fraction exponentially sensitive to power spectrum peak which is exponentially sensitive to duration of ultra-slow-roll - *Nakama and Wang 2019* + work in progress

WIMPs and PBHs are incompatible

- Assuming WIMPs have the standard, velocity independent cross section which gets the right abundance, and $M_{PBH}>10^{-6}\ M_{sun}$.
- If $f_{PBH}<1$, then another DM component is inevitable
- Steep and high density profiles form around PBHs (density~ $r^{-9/4}$). WIMPs would rapidly annihilate to gamma rays.
- In contrast to ultracompact minihalos without a PBH seed. *Gosenca et al '17, Delos et al '17*

- A detection of WIMPs or PBHs may effectively rule out the existence of the other



Adamek, CB, Gosenca & Hotchkiss 2019;

Lacki & Beacom 2010; Eroshenko 2016; Boucenna, Kühnel, Ohlsson & Visinelli 2017

The 3 papers above all find different profiles. We made the first simulations of this scenario

Summary

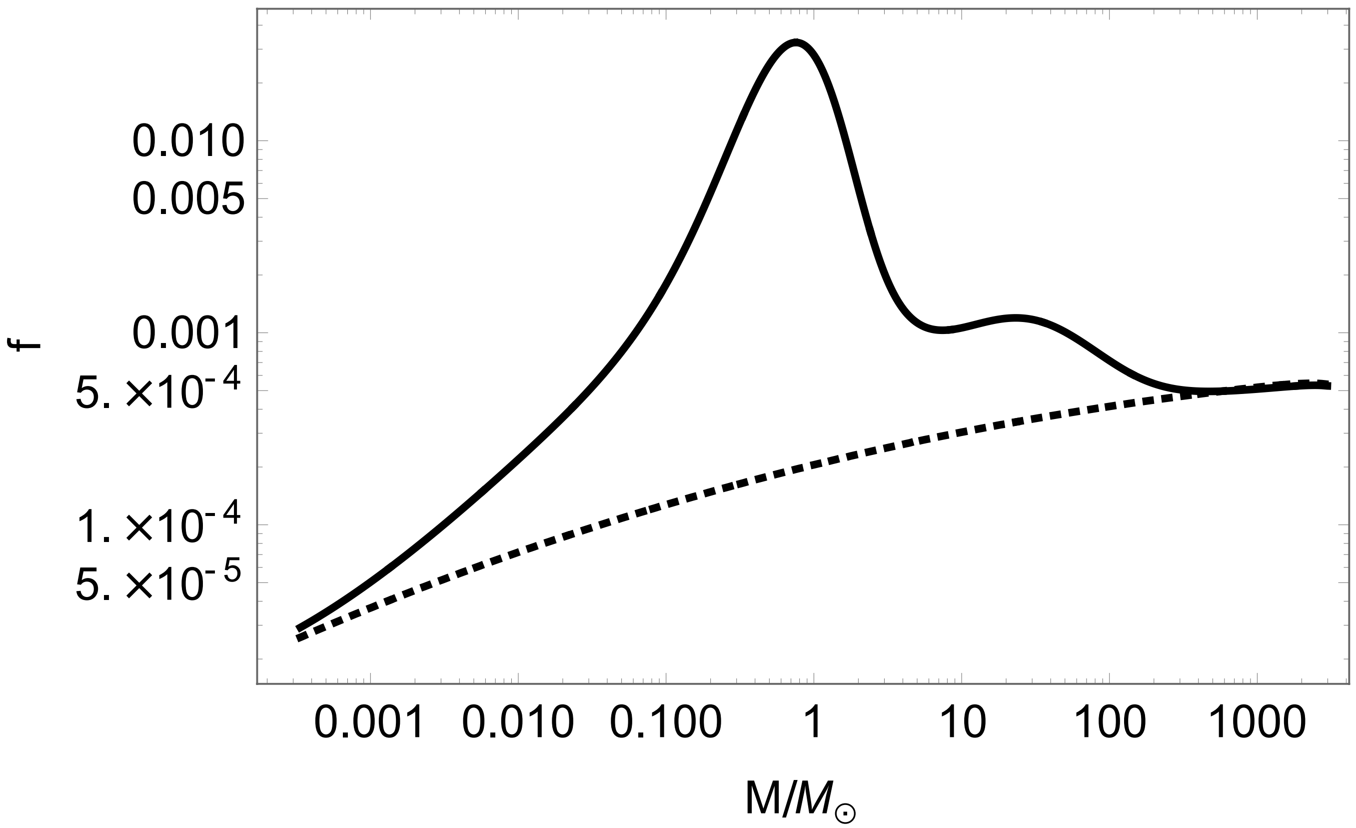
- Possible that some - not all - of the LIGO Virgo black holes were primordial
- PBHs are a special DM candidate
- Any PBH detection would be transformative

Backup slides

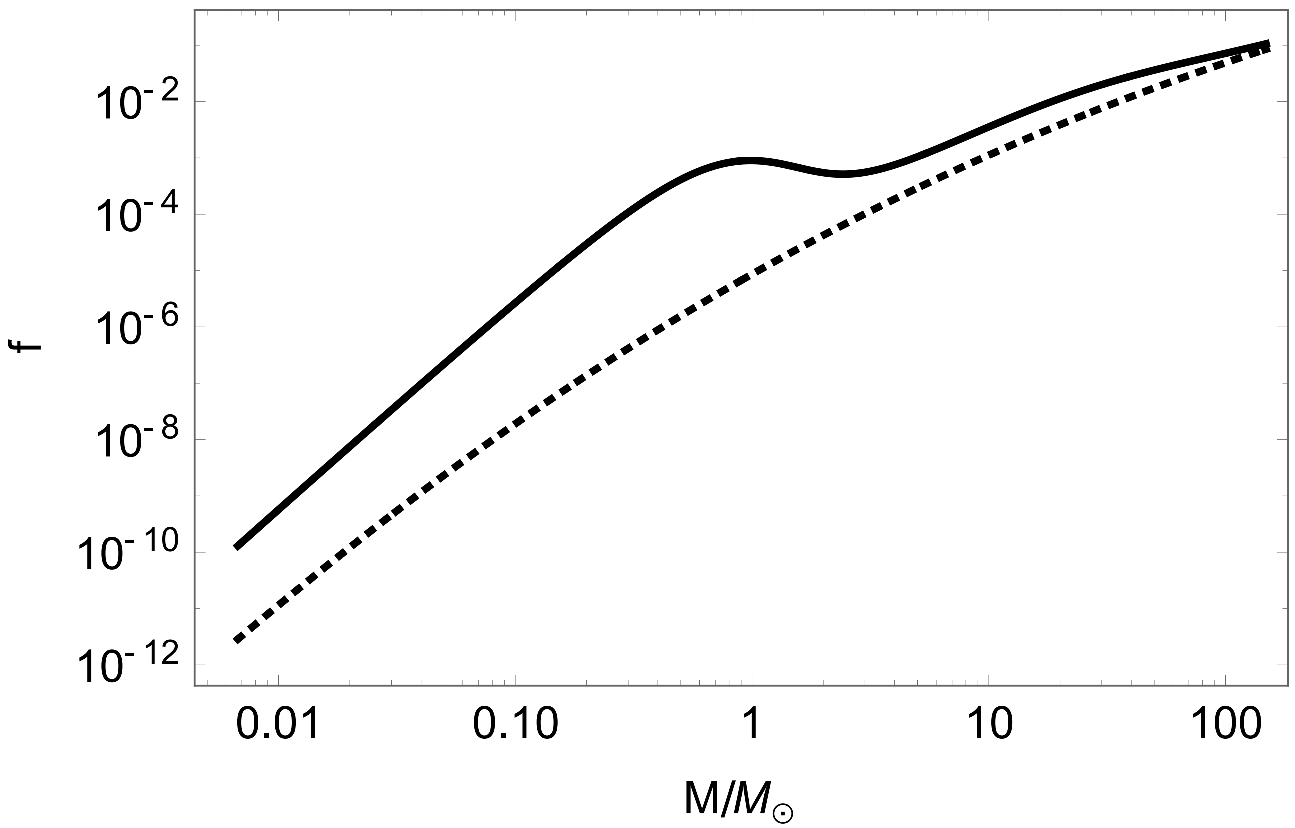
Varying the primordial perturbations

If the primordial power spectrum is not scale invariant on the relevant scales then the mass function changes, but a peak remains

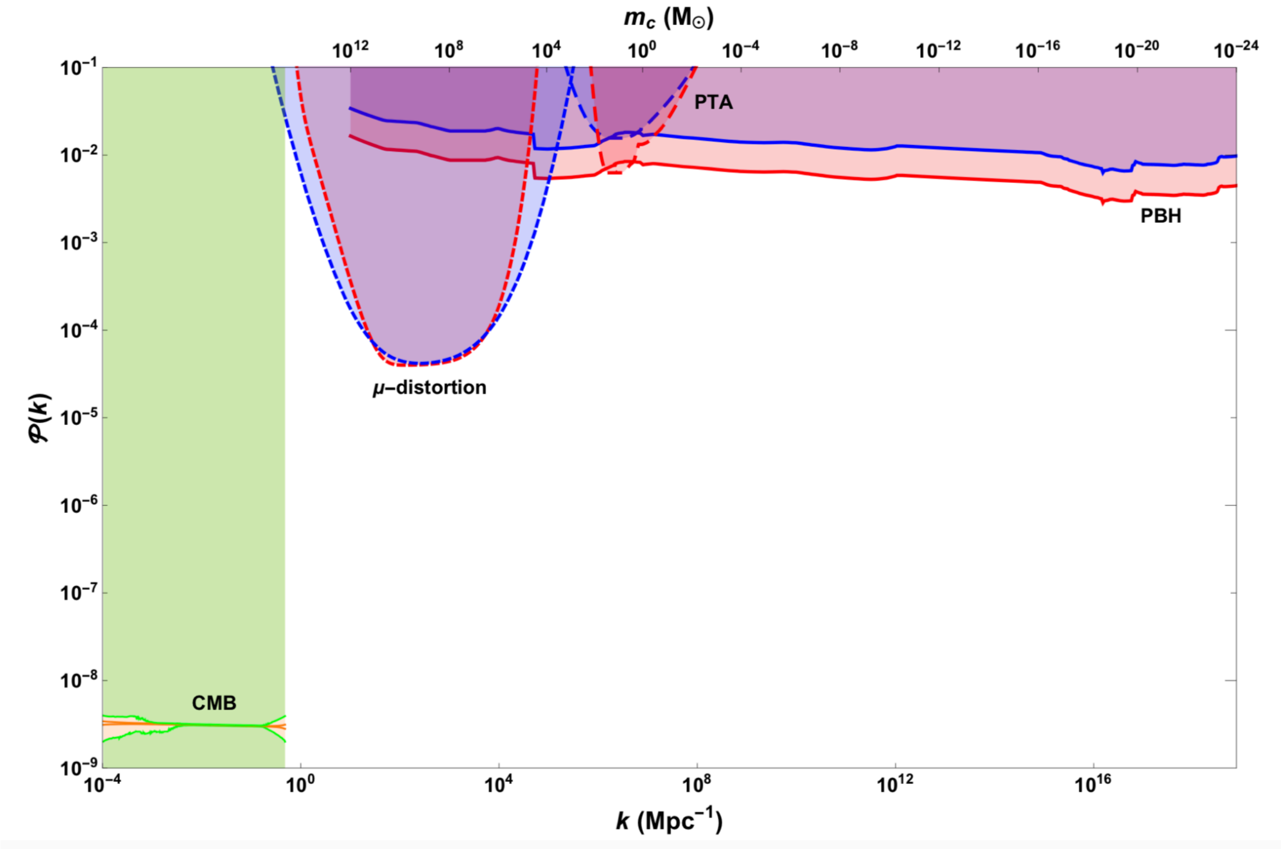
$n_s - 1 = -0.05$



$n_s - 1 = -0.2$



PBH constraints



- All constraints need to be made for a consistent choice of power spectrum peak
- Choice of Press-Schechter vs peaks not very important, likewise for the window function
- Beware the simple relations between horizon and PBH mass - we find an order-of-magnitude shift to heavier PBH masses for any given k value

Gow, CB, Cole, Young 2020

- Accurate calculations are (finally) required

Dependence on method and window function - *Gow, CB, Cole, Young 2020*

- **Method:** Normally people use Press-Schechter (PS) or peaks theory (TP) (*BBKS 1986*). We also consider a modified peaks theory by Young and Musso (2001)

- **Window function:** We consider a top hat (TH) in real space and Gaussian

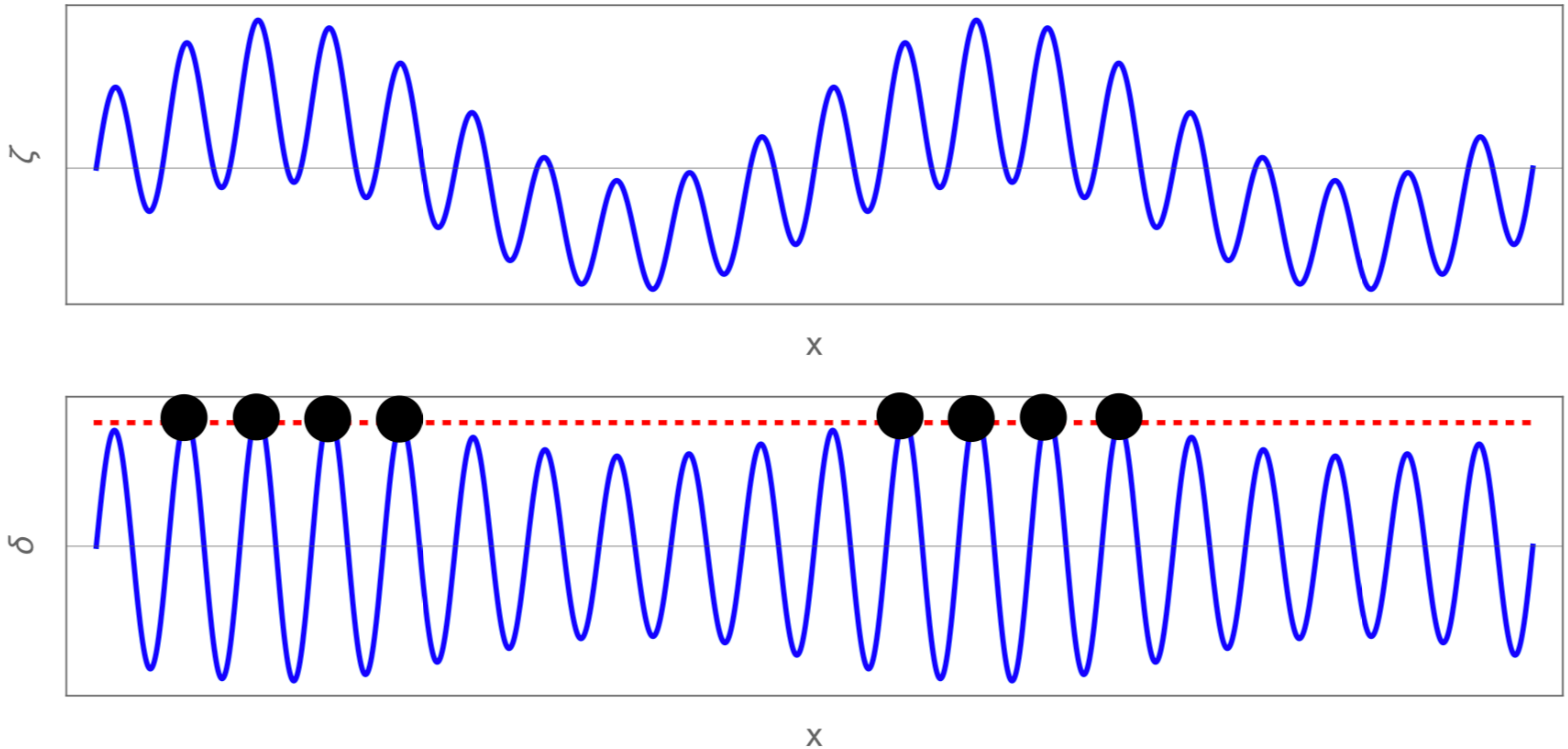
- Claims the window function has a huge impact on PBH formation (*Ando, Inomata and Kawasaki 2018*) - but one needs to calculate the collapse threshold and variance of density perturbations consistently (*Young 2019*)

- There is a genuine sensitivity to the shape of the power spectrum peak.
  Monochromatic power spectra and mass spectra are unphysical.

A noteworthy point here is that the typical mass of a PBH is actually significantly larger than the horizon mass corresponding to the scale at which the power spectrum peaks, $m_c/M_{H,\mathcal{P}} > 1$. At first glance, this statement may seem to be in disagreement with previous works where the expected PBH mass has been shown to be smaller than the horizon mass at re-entry. Physically, this apparent discrepancy is due to the fact that, if there is a narrow peak in the ζ power spectrum at a scale k_p, the resultant perturbations will, on average, have a significantly larger characteristic scale r_m. In the calculation presented here, this manifests itself in the fact that the variance $\sigma_0^2(R)$ peaks at a larger value of R than that corresponding to the scale k_p (as calculated in Ref. [32] for example). Thus, the final mass of PBHs is smaller than the horizon mass corresponding to r_m, but larger than the horizon mass corresponding to k_p. The important conclusion drawn from this is that constraints on the PBH abundance for a given mass of PBH correspond to constraints on the primordial power spectrum at a larger value of k than have previously been calculated.

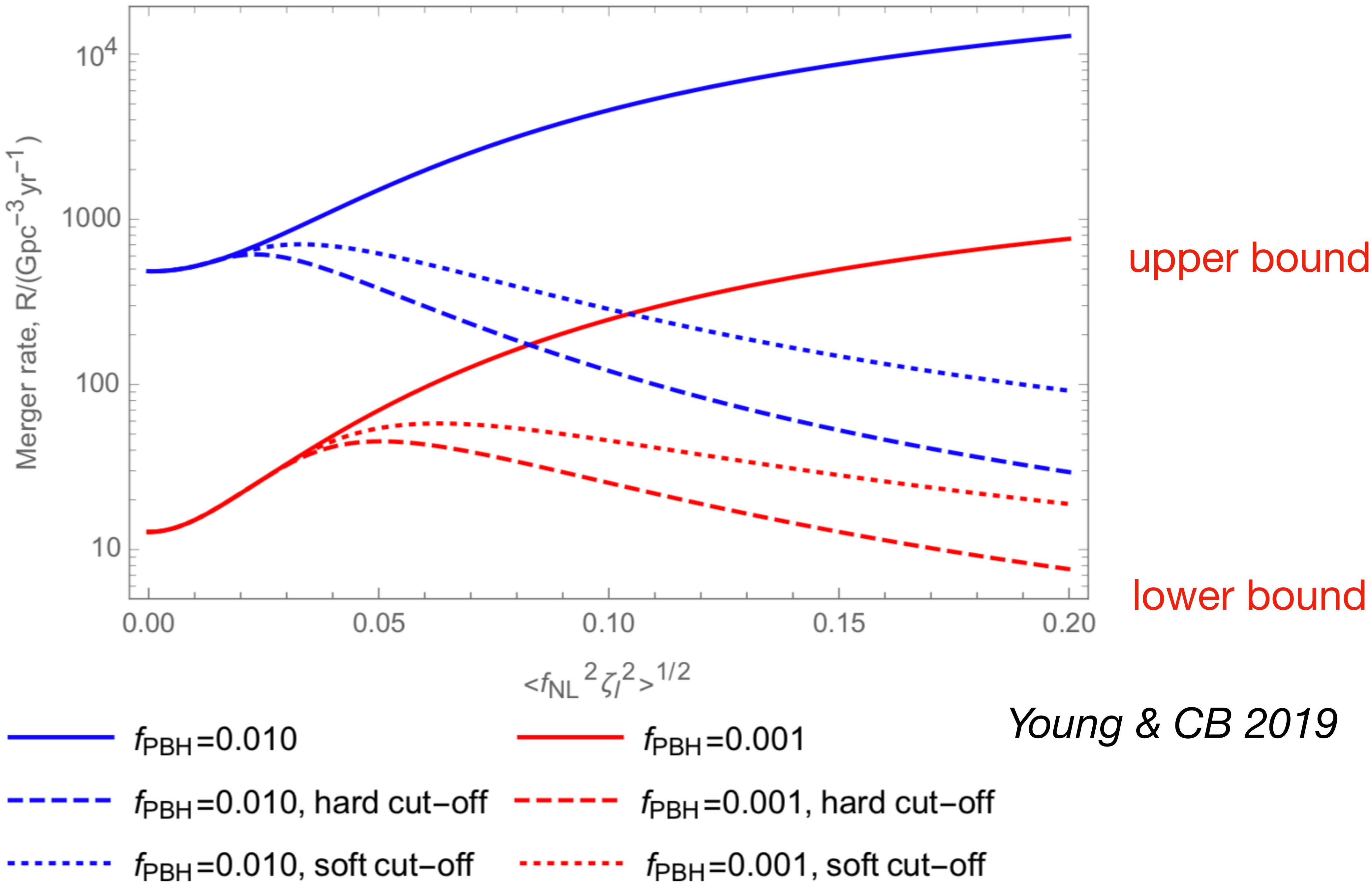
Gow, CB, Cole, Young 2020

The impact of non-Gaussianity



Local non-Gaussianity boosts the PBH fraction and creates an initial spatial clustering
Suyama & Yokoyama 2019
This (probably) increases the merger rate
It may also rule out the PBH scenario entirely, by generating a large DM-photon isocurvature perturbation - *Tada & Yokoyama 2015, Young & CB 2015*

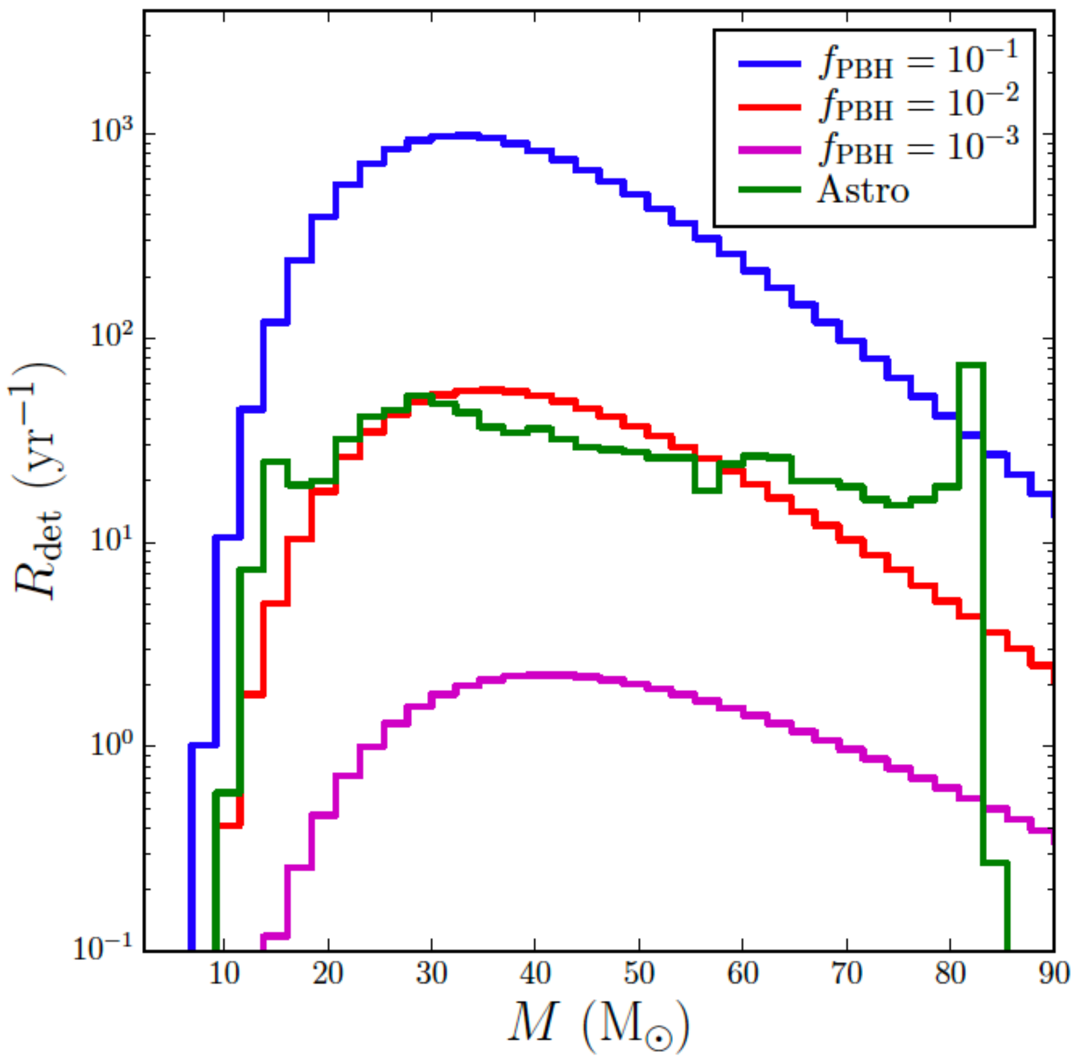
Clustering and the merger rate



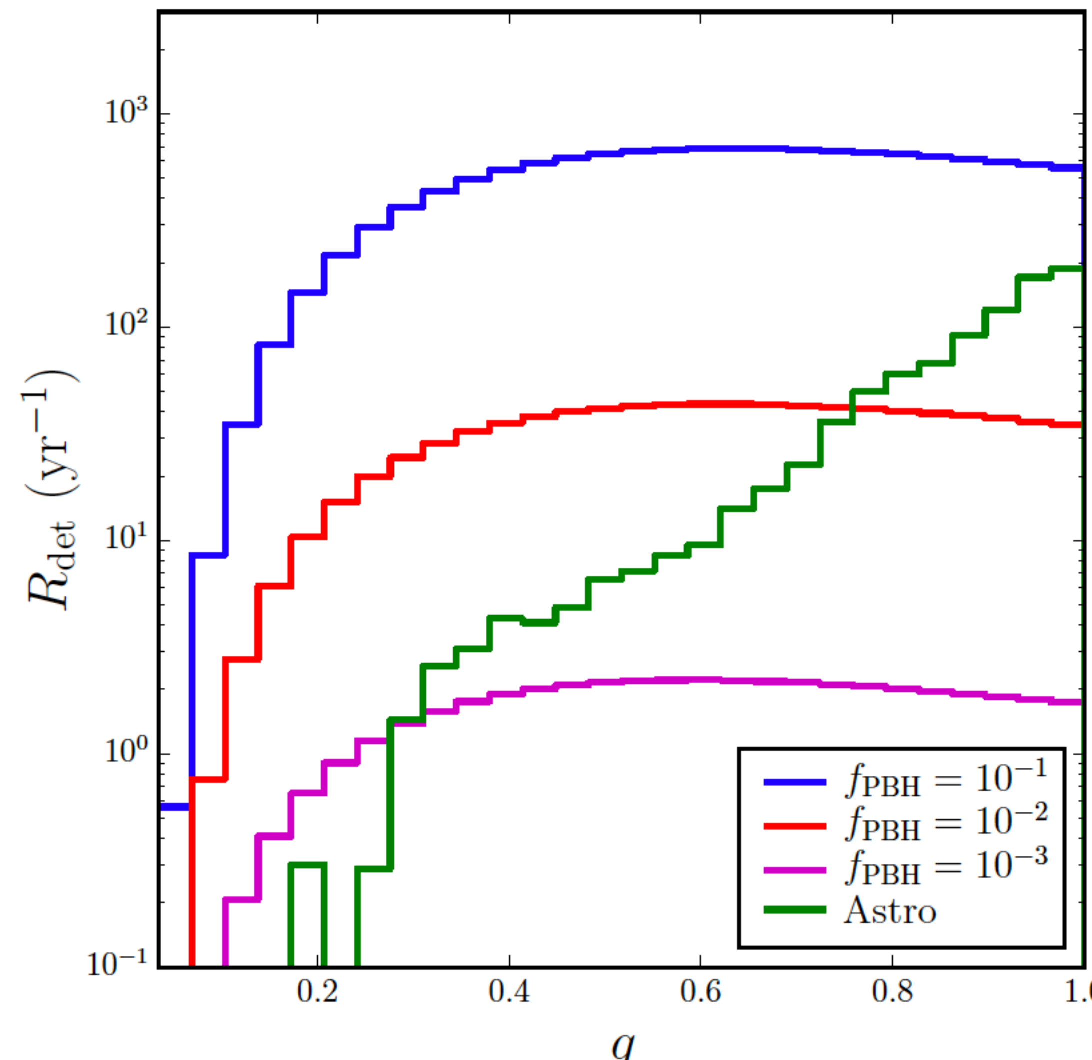
Young & CB 2019

With non-Gaussianity => spatial clustering => large local PBH densities
We don't know the merger rate in such cases - binaries are likely to be disrupted
One millionth of DM in PBHs may be large enough to explain the LIGO events

The total mass and mass ratio



total mass



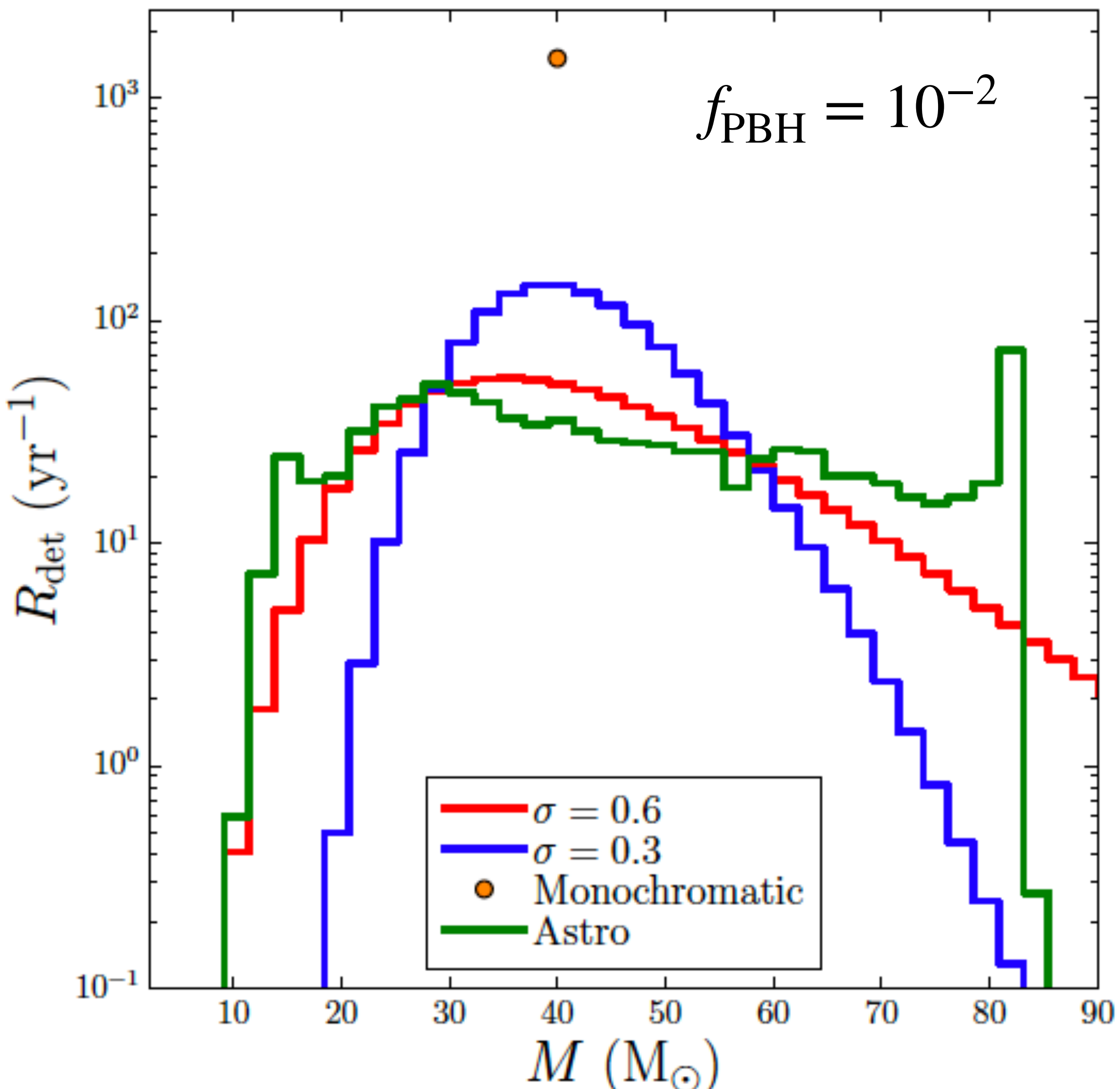
Gow, CB, Hall, Peacock 2019

mass ratio

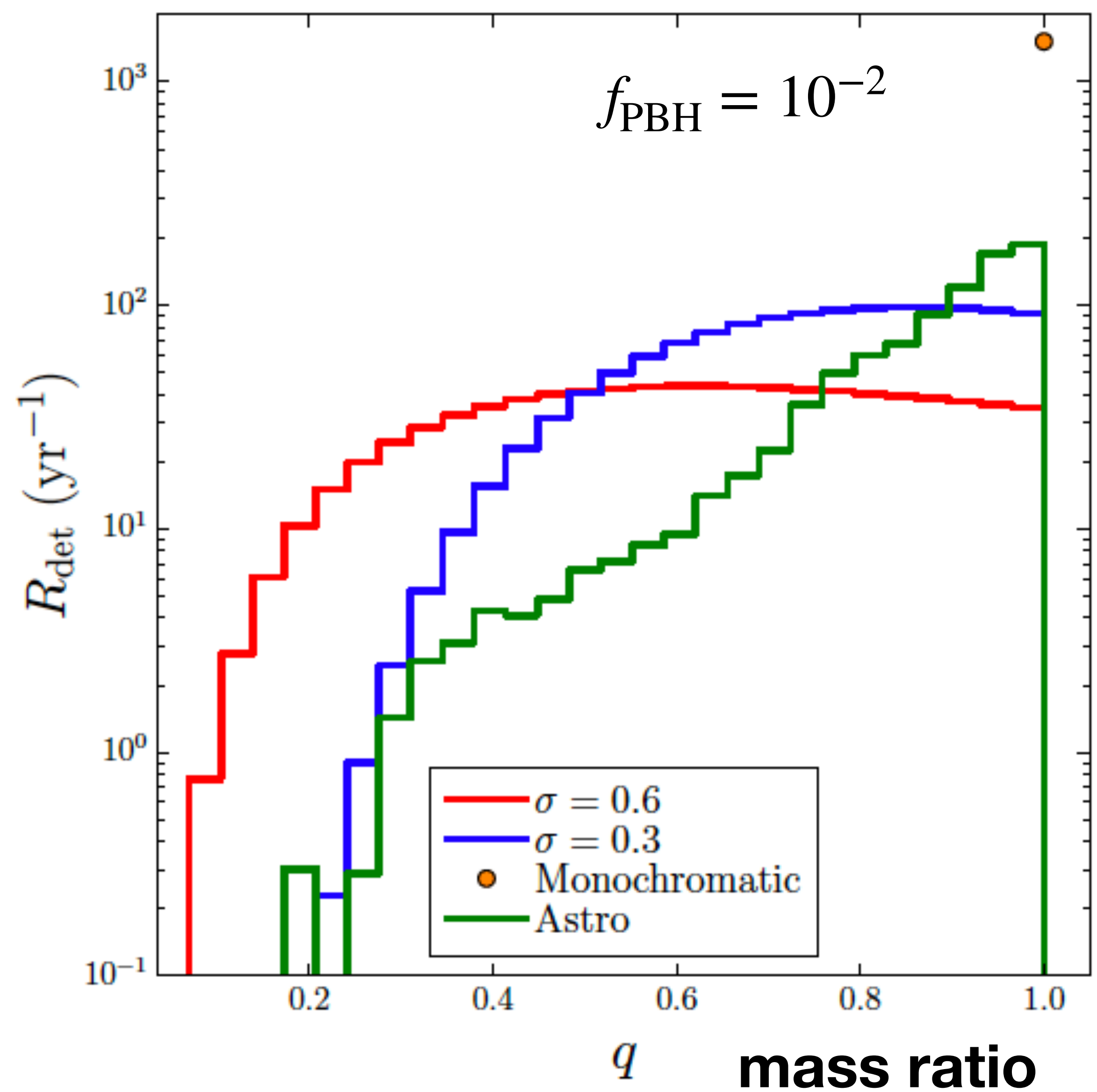
Notice how astrophysical black holes have an expected maximum and minimum mass
The mass ratio (q) looks like a promising discriminant between the two scenarios

Varying the PBH mass function width

Wide enough to fit the masses, yet not so wide to stop q~1



total mass



mass ratio

The “astro” distribution covers a broader range of total masses than sigma=0.3, but it still prefers the mass ratio q~1. A monochromatic mass function is ruled out.

Gow, CB, Hall, Peacock 2019

Has LIGO detected ~~DM~~ PBHs?

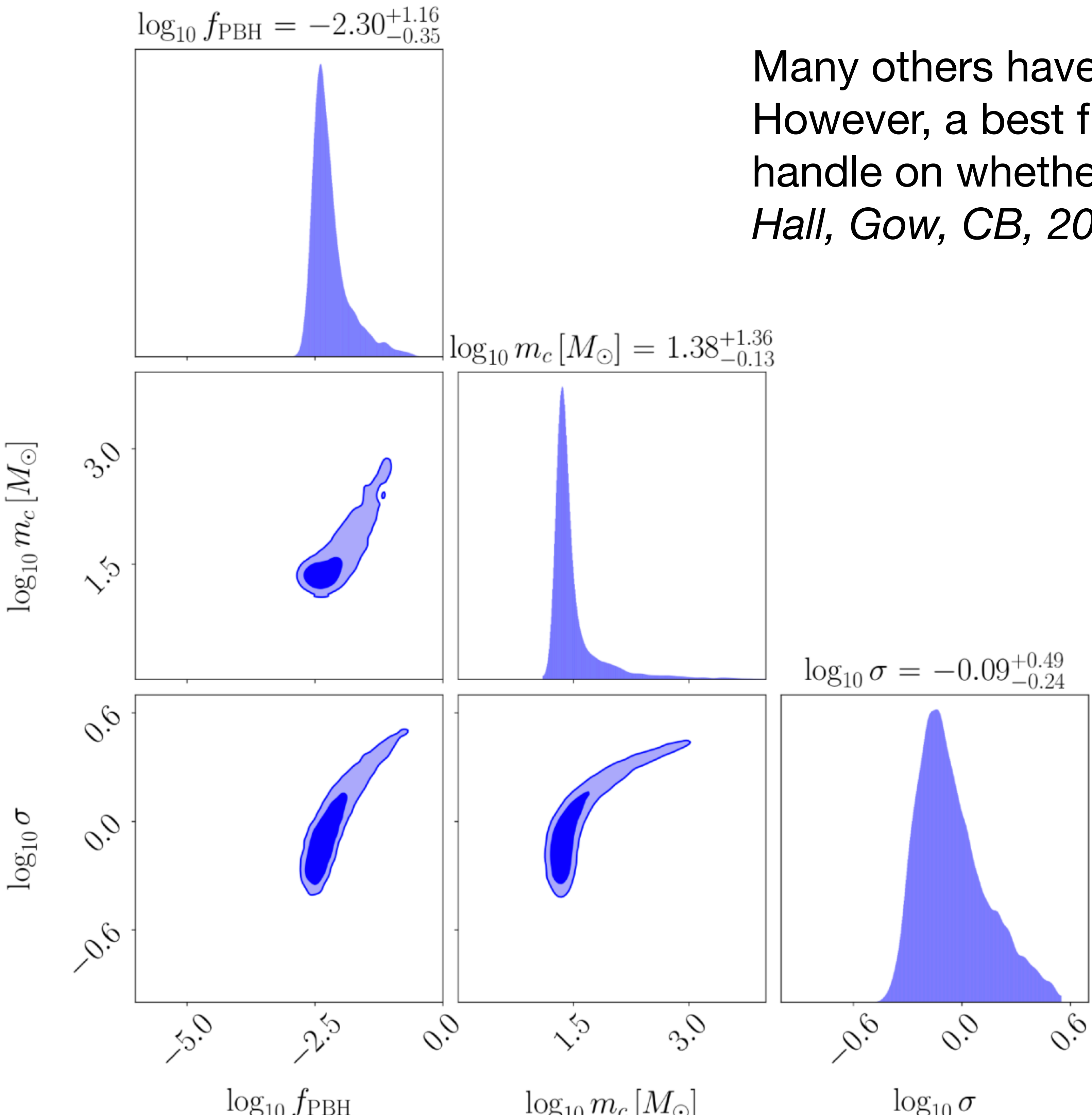
The Bayesian evidence ratio

$$\frac{Z_A}{Z_B} \equiv \frac{p(M_A|\mathbf{d})}{p(M_B|\mathbf{d})} = \frac{p(M_A)}{p(M_B)} \frac{\int p(\mathbf{d}|\boldsymbol{\theta}, M_A) p(\boldsymbol{\theta}|M_A) \mathrm{d}\boldsymbol{\theta}}{\int p(\mathbf{d}|\boldsymbol{\theta}', M_B) p(\boldsymbol{\theta}'|M_B) \mathrm{d}\boldsymbol{\theta}'}$$

Population parameters, i.e. mass function parameters, PBH abundance etc.

Hall, Gow, CB, 2020: Bayesian comparison

Fitting a lognormal mass function



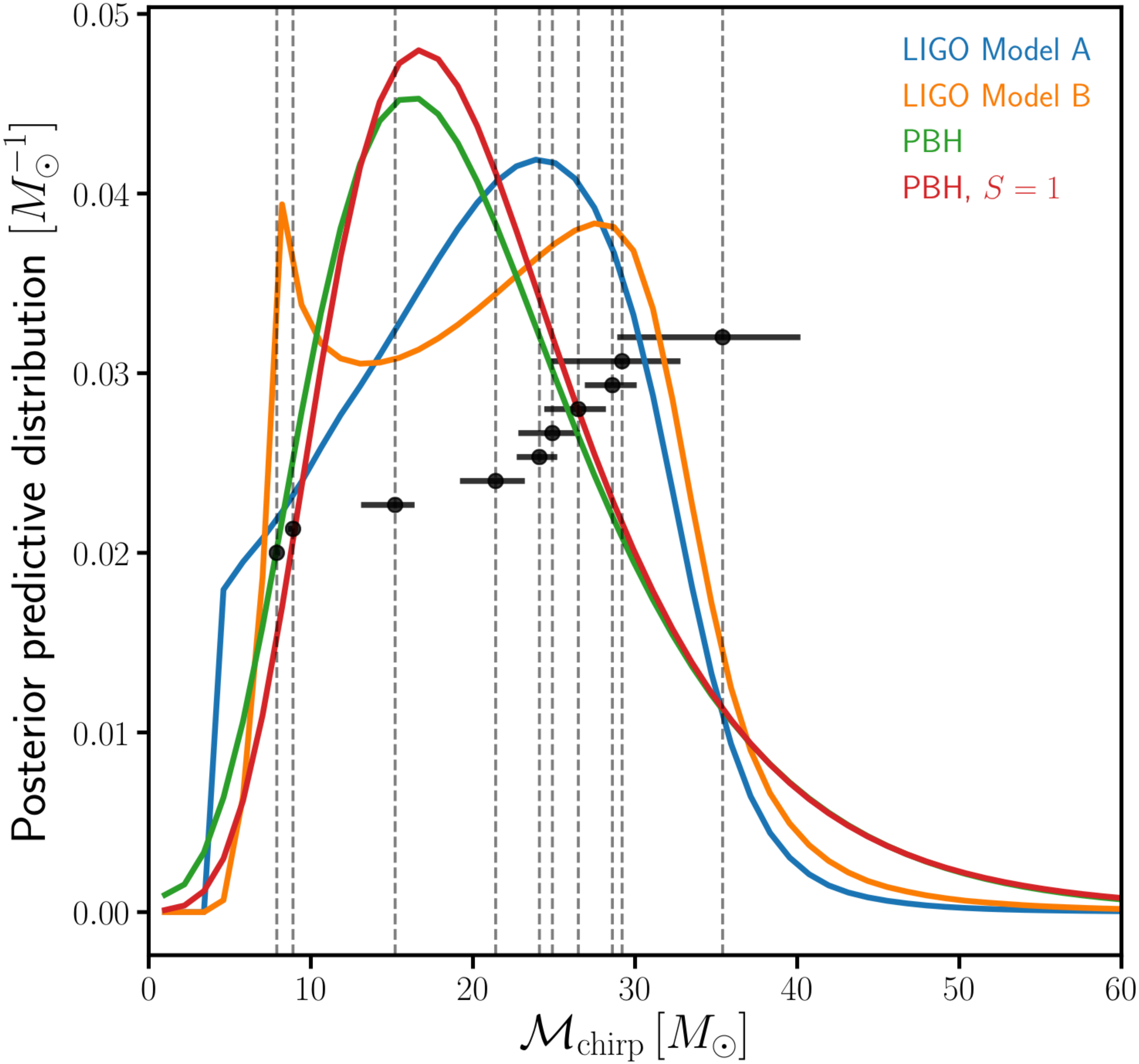
Many others have made similar fits
However, a best fit analysis does not give any handle on whether the best fit is also a good fit
Hall, Gow, CB, 2020

Bayesian results

- Our models are: All mergers are due to PBHs vs all due to stellar BHs
- We use 01/02 data only and carefully use the LIGO sensitivity curve.
- The Bayesian evidence can be approximated as the likelihood of the best fit model * the Occam factor
- Both are important but the Occam factor is prior dependent and more controversial
- PBHs are disfavoured by both terms - assuming the “normal” lognormal mass function

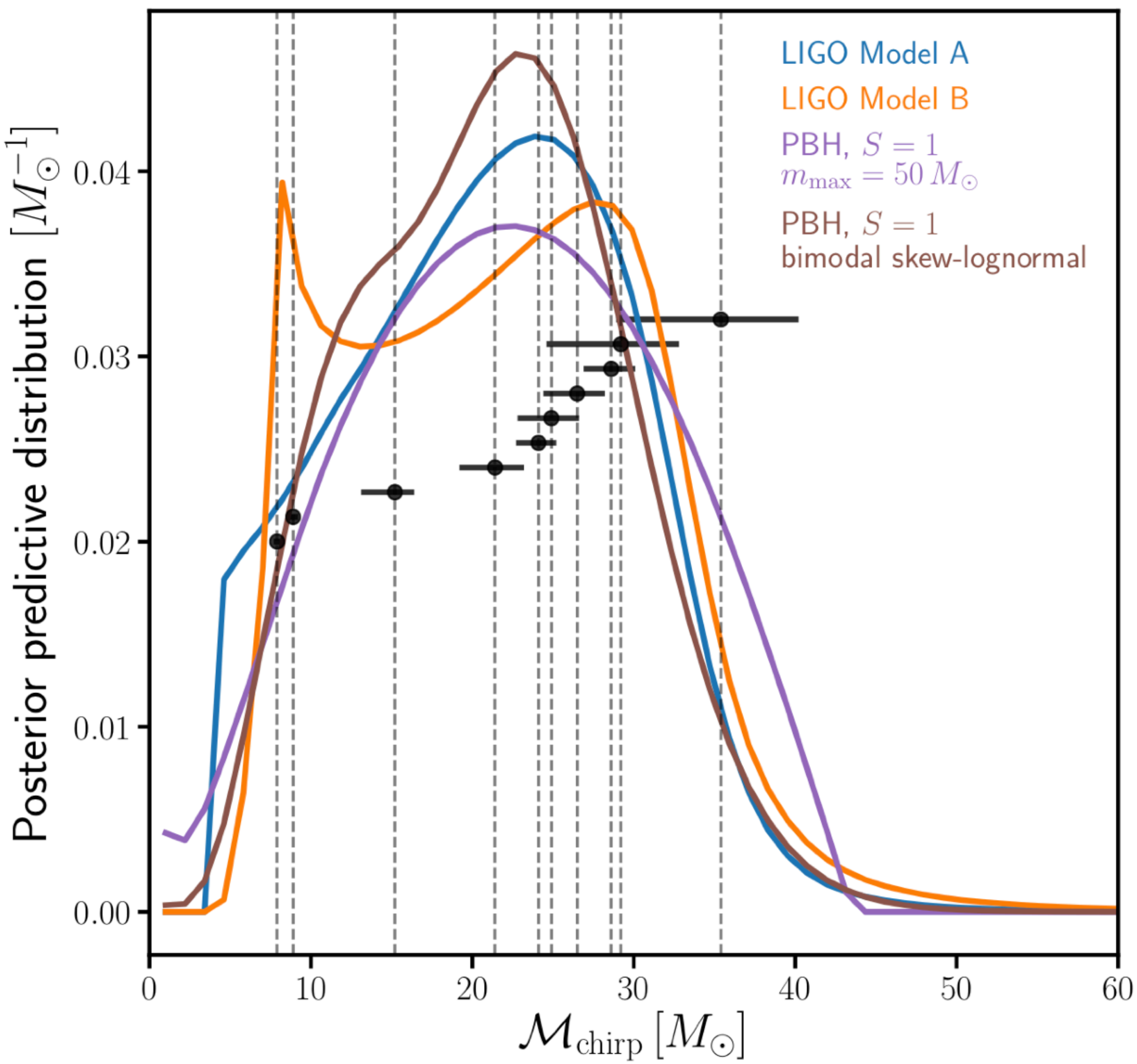
PBH models are disfavoured decisively

$$\ln Z_{\rm PBH}/Z_{\rm stellar} = -7.35 \pm 0.23$$



Hall, Gow, CB, 2020: Bayesian comparison

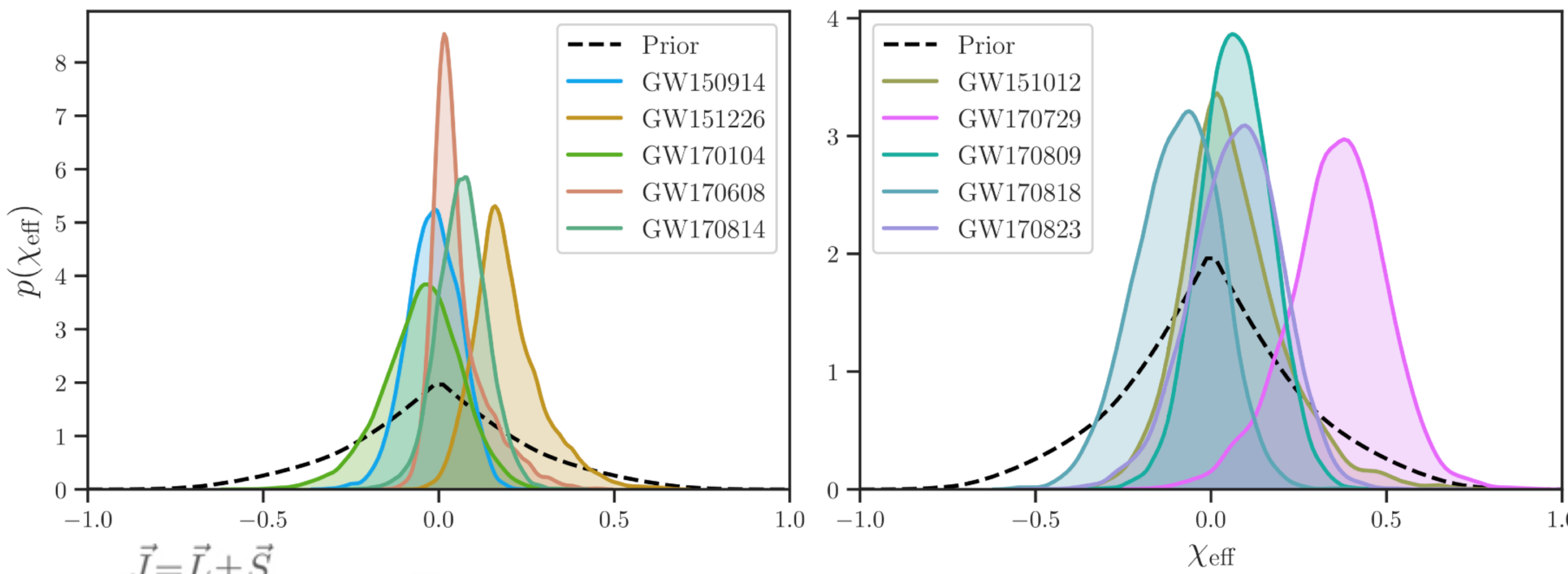
Trying hard to fit the data - cutoff or bimodal mass function

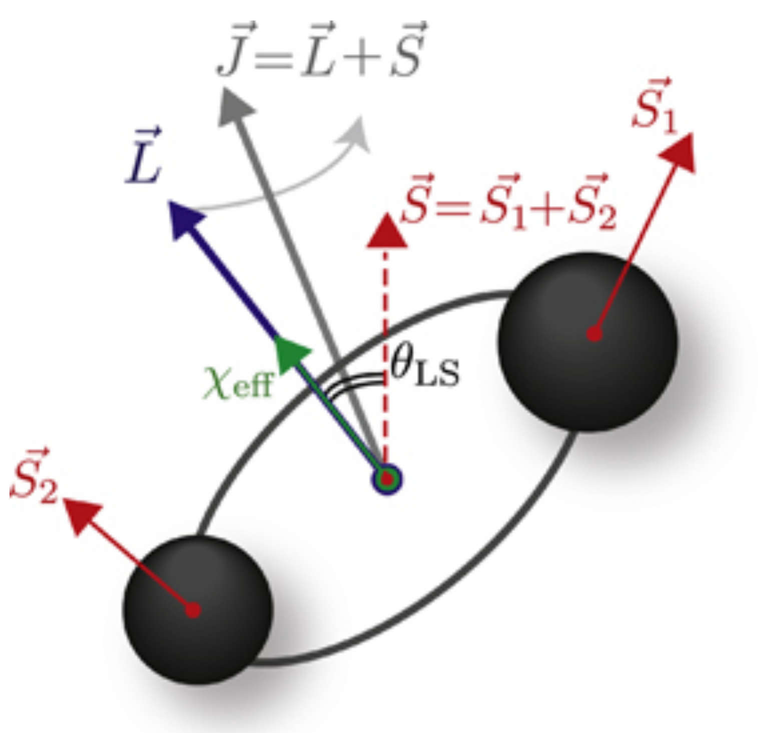


Hall, Gow, CB, 2020

These alternatives are a better fit, but still not a good fit compared to the stellar models
The late time PBH capture and merger rate is also a bad fit
Accretion broadens the mass function at large masses (*de Luca et al '20):* => worse fit

Black hole spin - in isolation favours PBHs





$$\chi_{\rm eff} = \frac{c}{G(m_1+m_2)}\left(\frac{\vec{S}_1}{m_1}+\frac{\vec{S}_2}{m_2}\right)\cdot\vec{L} \qquad a_* = \frac{c\,|\vec{S}\,|}{Gm^2} \leq 1$$

Model comparison based on spins
Fernandez & Profumo `19, Garcia-Bellido et al '20, Wong et al 2020 (uses 03a data)
Strongest constraining power comes from the masses

PBHs do not undergo much collapse before formation, small spin expected
Belczynski et al. `17; Mirbabyi et al `19; De Luca et al `19, Harada et al '20 + many more

Evidence for primordial black holes in LIGO/Virgo gravitational-wave data

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Kaze W. K. Wong,[2] Emanuele Berti,[2] Paolo Pani,[3,6] Antonio Riotto,[1] and Salvatore Vitale[4,5]

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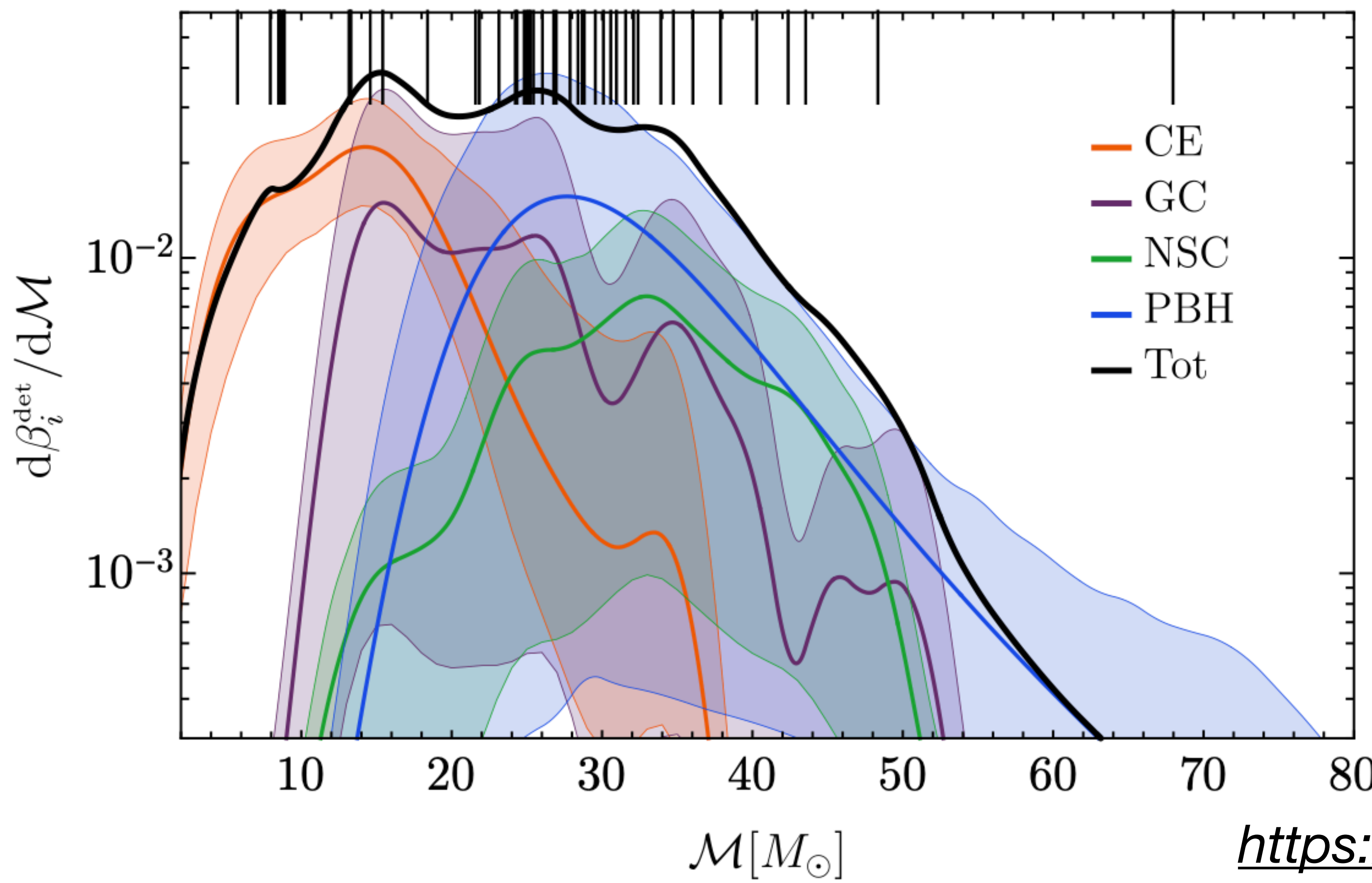
With approximately 50 binary black hole events detected by LIGO/Virgo to date and many more expected in the next few years, gravitational-wave astronomy is shifting from individual-event analyses to population studies aimed at understanding the formation scenarios of these sources. There is strong evidence that the black hole mergers detected so far belong to multiple formation channels. We perform a hierarchical Bayesian analysis on the GWTC-2 catalog using a combination of ab-initio astrophysical formation models (including common envelope, globular clusters, and nuclear star clusters) as well as a realistic population of primordial black holes formed in the early universe. The evidence for a primordial population is decisively favored compared to the null hypothesis and the inferred fraction of primordial black holes in the current data is estimated at $0.27^{+0.28}_{-0.24}$ (90% credible interval), a figure which is robust against different assumptions on the astrophysical populations. The primordial formation channel can explain events in the upper mass gap such as GW190521, which are in tension with astrophysical formation scenarios. Our results suggest the tantalizing possibility that LIGO/Virgo may have already detected black holes formed after inflation. This conclusion can ultimately be confirmed in the era of third-generation interferometers.

Lognormal mass function

$$\psi_{\rm L}(m) = \frac{1}{\sqrt{2\pi}\sigma m} \exp\left(-\frac{\ln^2(m/m_c)}{2\sigma^2}\right)$$



Several astrophysical binary formation channels are considered

https://arxiv.org/pdf/2105.03349.pdf

Their best fit width (assuming a lognormal mass distribution) is sigma=0.3
This is too narrow to be consistent with critical collapse, which requires sigma>0.37: *Gow, CB, Cole, Young 2020*

A lognormal mass function is anyway not very accurate in the limit of a narrow power spectrum peak: *Gow, CB, Hall 2020* 2009.03204 [astro-ph.CO]

$$\mathcal{P}_\zeta = A\frac{1}{\sqrt{2\pi}\Delta}\exp\left(-\frac{\ln^2(k/k_p)}{2\Delta^2}\right)$$

The 1/Delta normalisation ensures Delta->0 looks like a Dirac-delta distribution

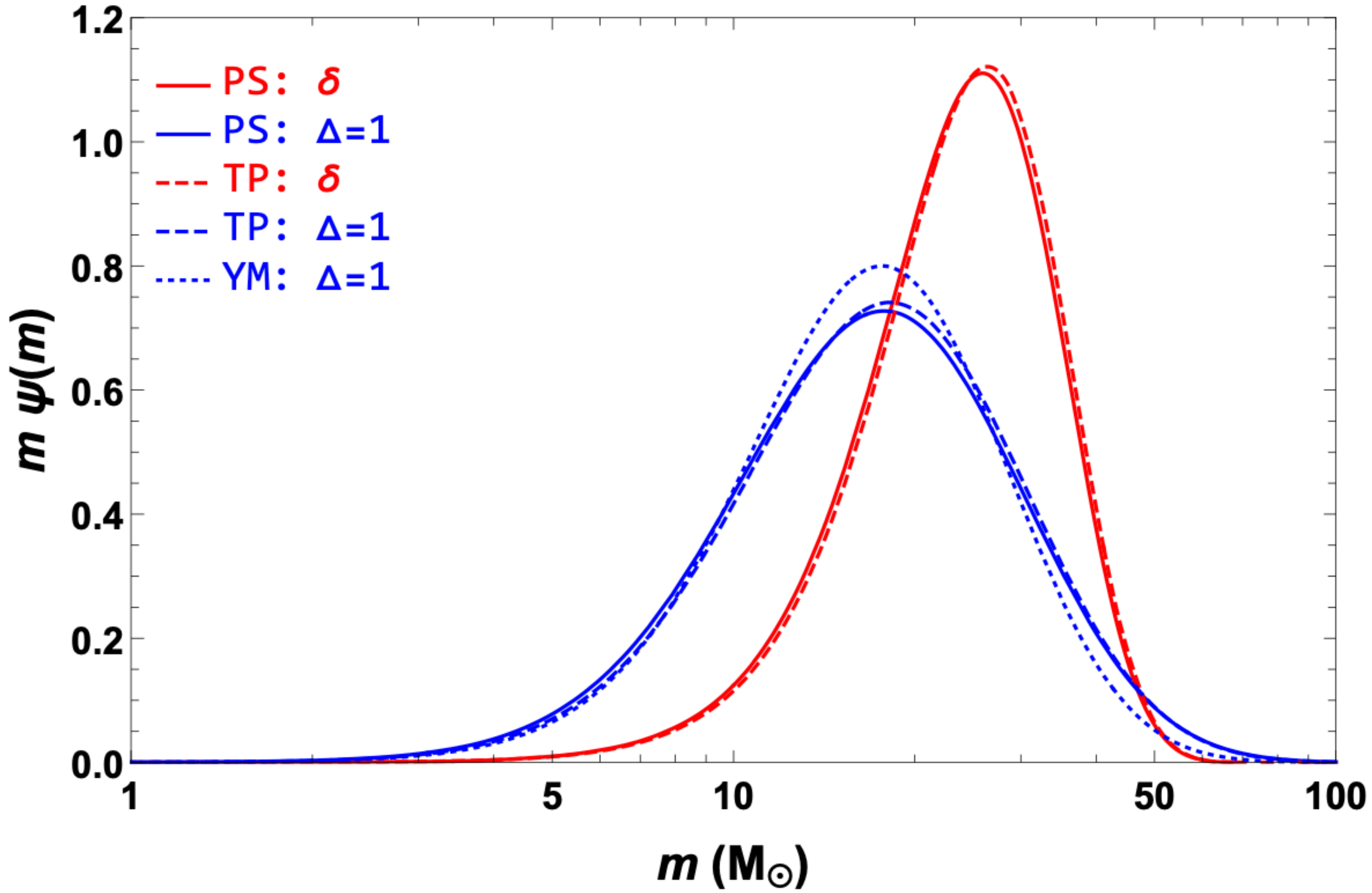
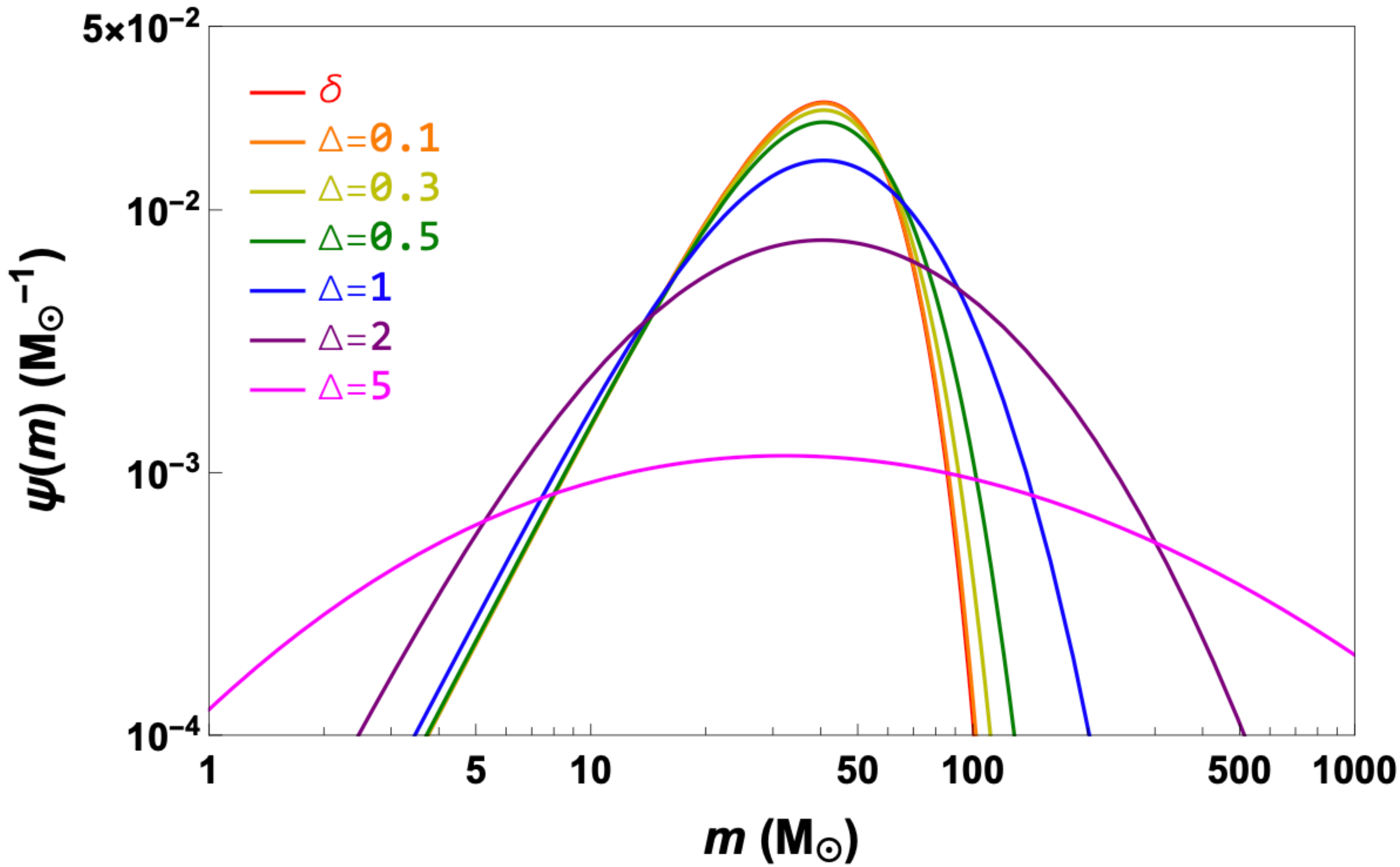


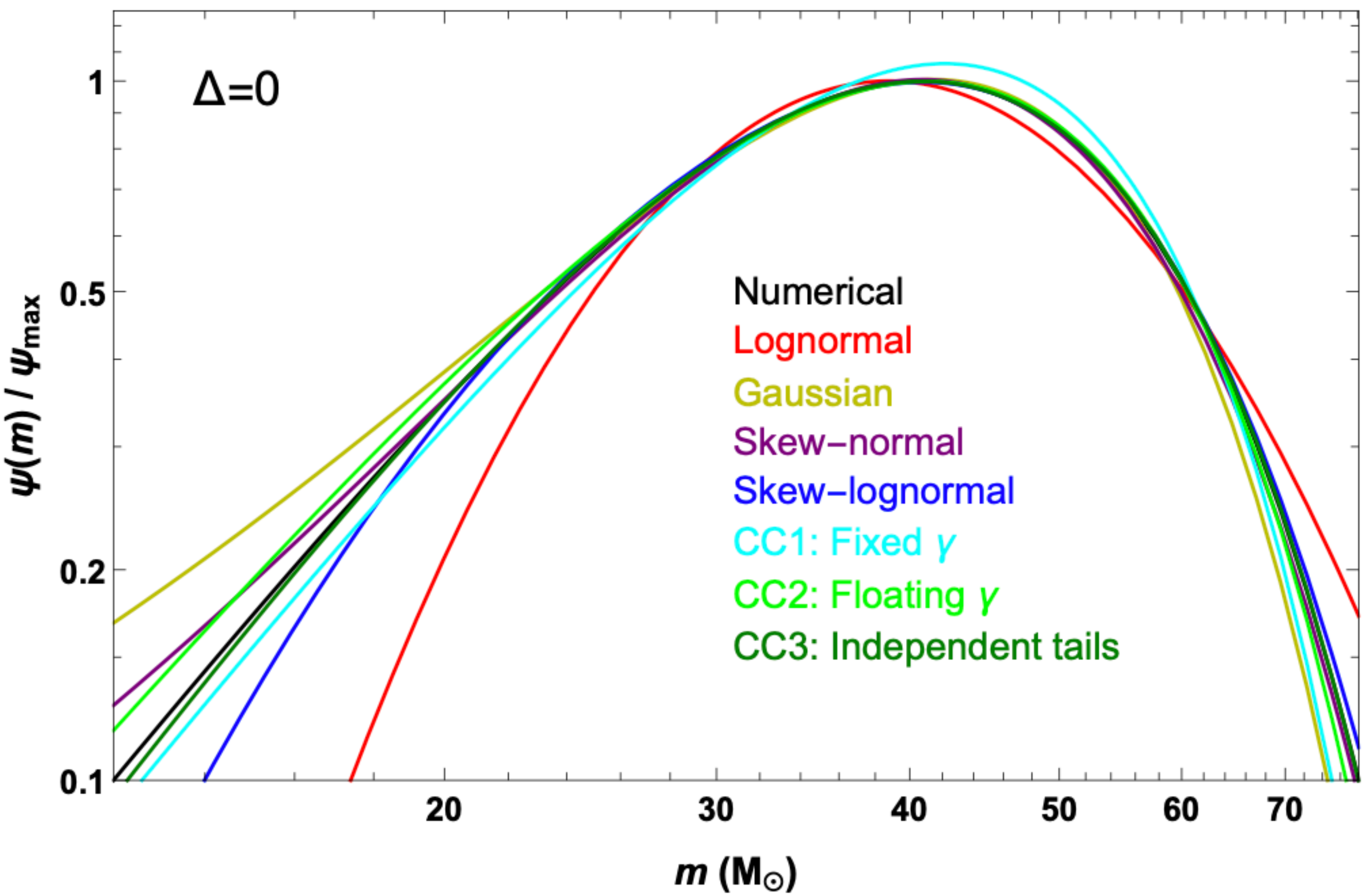
FIG. 1. Difference between PBH mass distributions calculated using different methods, while keeping the window function fixed. The Gaussian window function is used in every case. The red curves are for the delta function peak in the power spectrum, and the blue curves are for the lognormal peak with $\Delta = 1$. The Press-Schechter (PS), traditional peaks (TP), and modified peaks (YM) methods are shown with solid, dashed, and dotted lines respectively. All lines have $f_{\rm PBH} = 2 \times 10^{-3}$.

Gow, CB, Cole, Young 2020



For narrow peaks the mass function doesn't change
Hence, the power spectrum shape can't be reconstructed
Stochastic GWs can probe a much larger range of scales

Gow, CB, Hall



The “standard” lognormal mass function is a bad fit for narrow peaks

Gow, CB, Hall

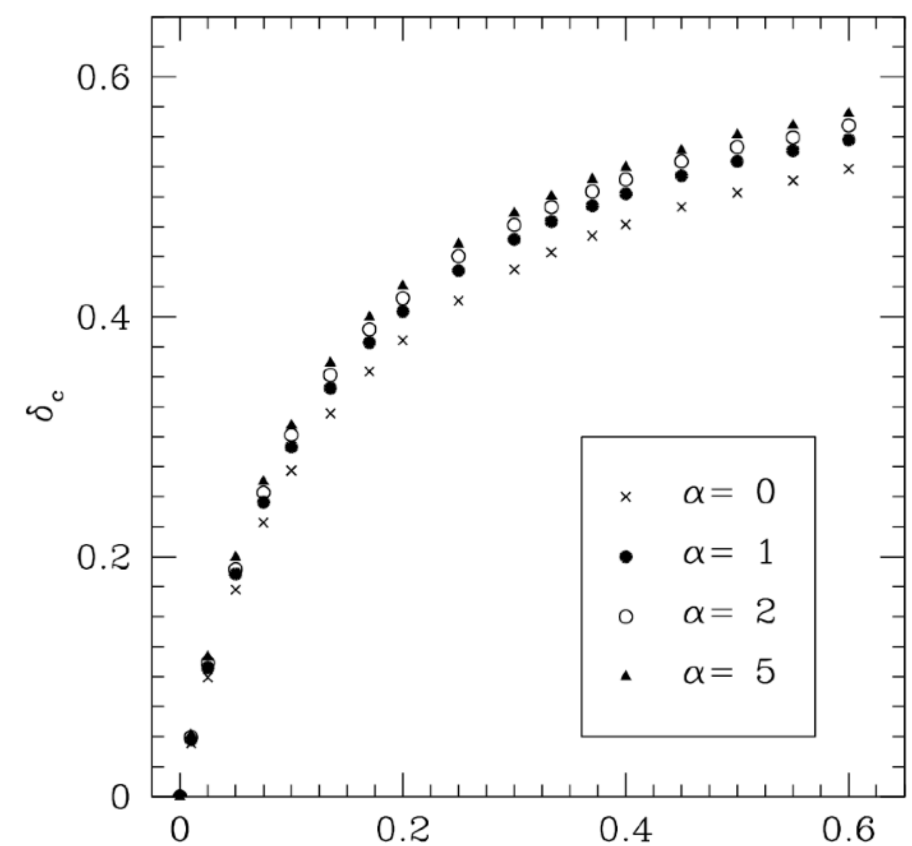
PBH formation

1. They could form from large amplitude density perturbations shortly after horizon entry

2. Causality prevents collapse before horizon entry

3. Approximate 1-to-1 relation between horizon entry time, horizon length and PBH mass

Collapse threshold



equation of state

Musco and Miller 2013

PBH formation comments

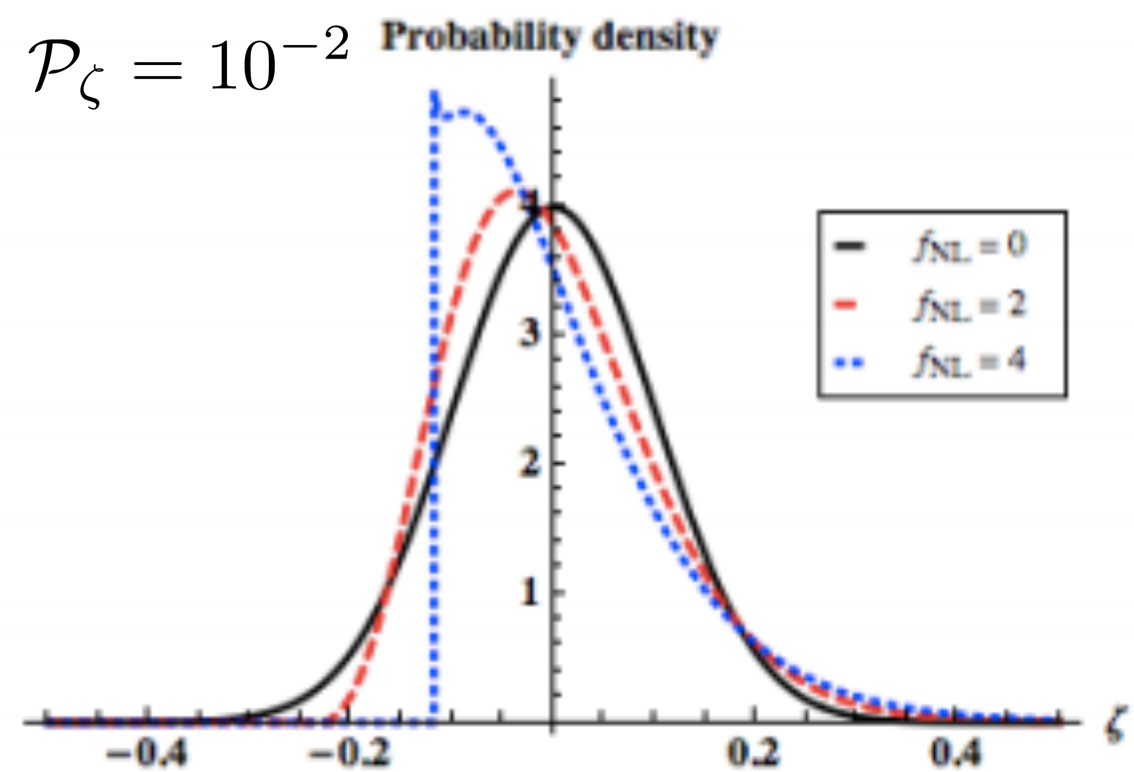
- The formation rate is exponentially sensitive to the amplitude of the power spectrum, and the collapse threshold
- Inflationary models posit an inflection point (ultra-slow-roll inflation) or other feature
- The power spectrum can't grow faster than about k4 (in canonical single-field inflation), impacts the constraints. *Byrnes, Cole & Patil '18; Carrilho, Malik & Mulryne '19*
- PBHs are very rare - very sensitive to non-Gaussianity
- The formation criteria depends on the density profile. Many spherically symmetric simulations exist, e.g. *Niemeyer & Jedamjik, Musco & Miller, Harada ++, Nakama ++…*
- Extensive recent analytic work has been done to relate the power spectrum to PBH formation rate at, but (at least) an order unity uncertainty remains (= tens of orders of magnitude in terms of the formation rate). *Germani & Musco '17, Yoo et al '17, Kawasaki & Nakatsuka '19, de Luca et al '19, Young et al '19, Young '19, Kalaja et al '19*

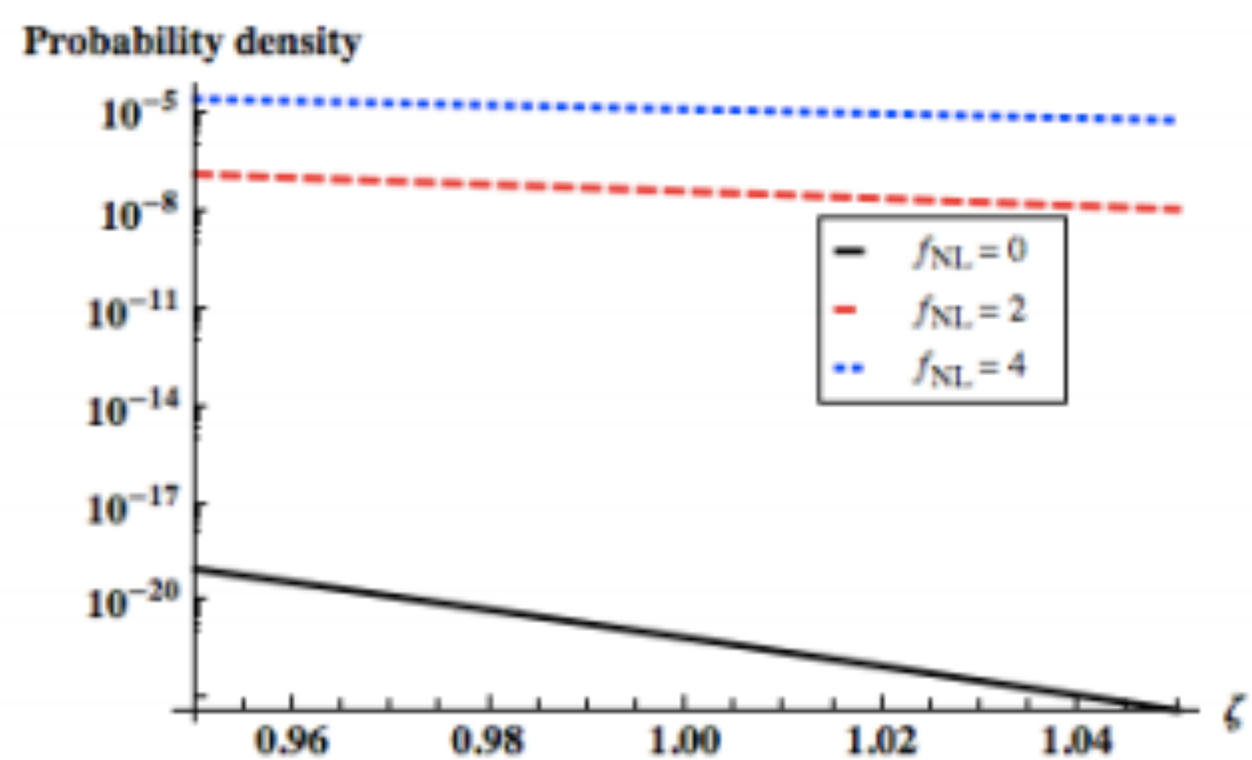
PBH abundance is exponentially sensitive to non-Gaussianity

Local non-Gaussianity

$$\zeta = \zeta_g + \frac{3}{5} f_{NL} \left(\zeta_g^2 - \sigma^2 \right)$$

$\mathcal{P}_\zeta = 10^{-2}$





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Non-Gaussianity take-away message

- The density contrast is non-Gaussian even if the curvature perturbation is Gaussian
- Beware of invoking non-Gaussianity to “evade” constraints, since it introduces new challenges. Approximations can be exponentially wrong.
- Stochastic effects during inflation also generate non-Gaussianity - *Pattison et al, Ballesteros et al, Figueroa et al, Ando & Vennin, Taoso & Urbano, recent*
- The PBH abundance, initial clustering, merger rate and isocurvature fraction are all very sensitive to primordial non-Gaussianity (depending on the type of non-Gaussianity)
- Small changes to the power spectrum or delta_c => exponential changes to the PBH fraction, but often unimportant