The graviton spectral function

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Bonanno, Denz, Pawlowski, MR: 2102.02217
Fehre, Litim, Pawlowski, MR: 2111.13232

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Perturbative quantum gravity

Einstein-Hilbert gravity

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_{\mathrm{N}}} \int_x \sqrt{\det g_{\mu\nu}} (2\Lambda - R(g_{\mu\nu}))$$

- Perturbatively non-renormalisable: $[G_{\mathrm{N}}] = -2$
- Need infinitely many counter terms: No predictivity

['t Hooft, Veltmann '74; Goroff, Sagnotti '85]

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Higher-derivative action

$$S_{\text{HD}} = \int_x \sqrt{\det\, g_{\mu\nu}} \left(\frac{1}{2\lambda}\, C^2_{\mu\nu\rho\sigma} - \frac{w}{3\lambda}\, R^2 \right) + S_{\text{EH}}$$

- Perturbatively renormalisable: $[w] = [\lambda] = 0$
- Perturbatively non-unitary [Stelle '74]

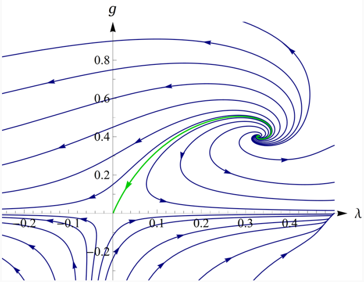
$$G_{\text{graviton}} \sim \frac{1}{p^2 + p^4/M_{\text{Pl}}^2} = \frac{1}{p^2} - \frac{1}{M_{\text{Pl}}^2 + p^2}$$

Asymptotically safe quantum gravity

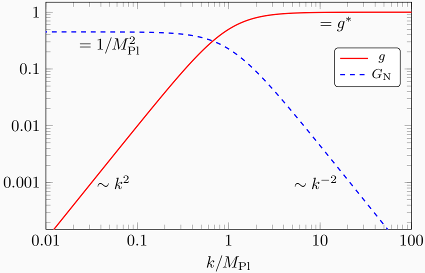
UV behaviour of quantum gravity could be governed by an interacting FP

[Weinberg '76]

$$S_{EH} = \frac{1}{16\pi G_N} \int_x \sqrt{g}\,(2\Lambda - R)$$



[Reuter '96; Reuter, Saueressig '01; Picture: Wikipedia]

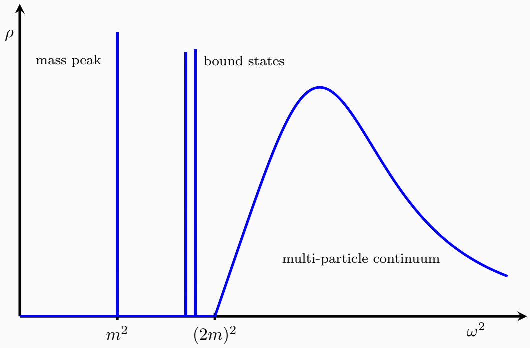


Predictivity ⇔ UV critical hypersurface is finite dimensional

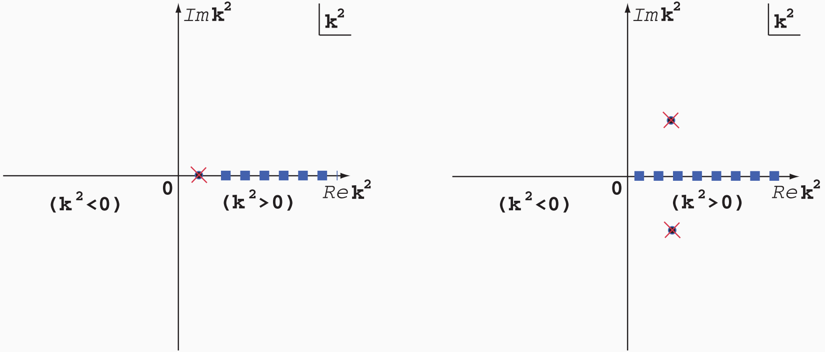
Unitarity ⇔ Properties of the spectral function

Källén-Lehmann spectral representation

$$\mathcal{G}(p^2) = \int_0^\infty \frac{d\lambda^2}{\pi} \frac{\rho(\lambda^2)}{p^2 - \lambda^2} \qquad \text{with} \qquad \rho(\omega^2) = -\lim_{\varepsilon \to 0} \operatorname{Im} \mathcal{G}(\omega^2 + i\varepsilon)$$



Propagator in the complex plane

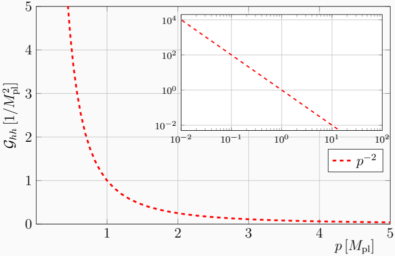


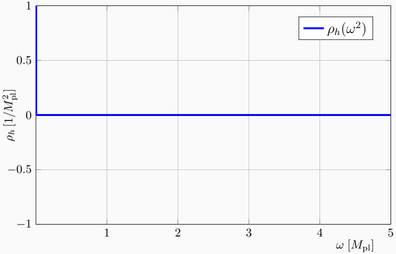
[Image: Kondo et al. '20]

Most non-perturbative methods only provide numerical data for $k^2 < 0$

Classical graviton spectral function

Einstein-Hilbert action $S_{\mathrm{EH}} = \frac{1}{16\pi G_{\mathrm{N}}} \int_x \sqrt{g}\,(2\Lambda - R)$



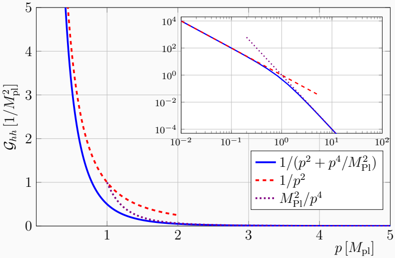


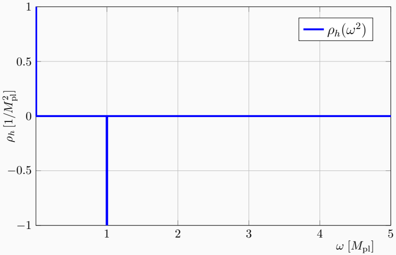
$$\mathcal{G}_{hh}(p^2) \sim \frac{1}{p^2}$$

$$\rho_h(\omega^2) \sim \delta(\omega^2)$$

Classical graviton spectral function

Higher-derivative action $S_{\text{HD}} = S_{\text{EH}} + \int_x \sqrt{g}\,(aR^2 + bC^2_{\mu\nu\rho\sigma})$



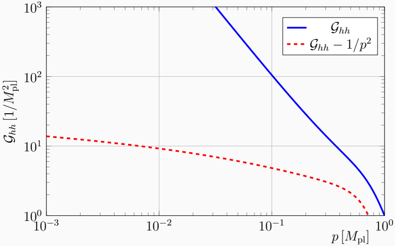


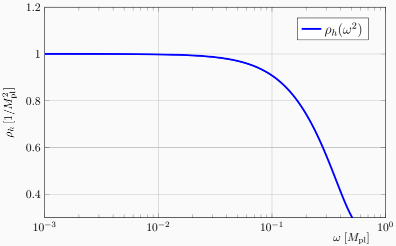
$$\mathcal{G}_{hh}(p^2) \sim \frac{1}{p^2} - \frac{1}{M_{\text{Pl}}^2 + p^2} \qquad \rho_h(\omega^2) \sim \delta(\omega^2) - \delta(\omega^2 - M_{\text{Pl}}^2)$$

EFT graviton spectral function

One-loop effective action:

$$\Gamma_{\text{1-loop}} = S_{\text{EH}} + \int_x \sqrt{g}\,(\alpha R \ln(\Box) R + \beta C \ln(\Box) C) + \dots$$





$$\mathcal{G}_{hh}(p^2) \sim \frac{1}{p^2 + \ln(p^2) p^4} \qquad \rho_h(\omega^2) \sim \delta(\omega^2) + 1 + 2w^2 \ln(w^2) + \dots$$

The functional renormalisation group

Non-perturbative renormalisation group equation [Wetterich '93]

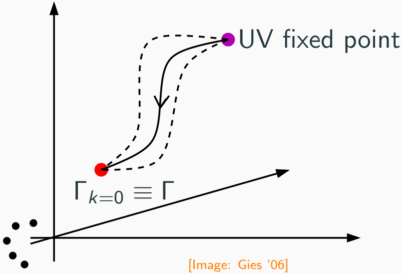
$$k\partial_k \Gamma_k = \frac{1}{2} \mathrm{Tr} \left[\frac{1}{\Gamma_k^{(2)} + R_k} k\partial_k R_k \right] =$$ 

R_k = regulator

Γ_k = scale-dependent effective action

Interpolation between

- bare action / UV FP
- quantum effective action Γ



[Image: Gies '06]

Direct Lorentzian computation

Standard Euclidean formulations

- Modified dispersion ($p^2 \to p^2 + R_k(p^2)$) introduces poles and cuts
- Can not use $\mathcal{G}_{hh}(p^2) = \int_0^\infty \frac{d\lambda}{\pi} \frac{\lambda \rho_h(\lambda^2)}{\lambda^2 + p^2}$ at finite k
- Analytic continuation only possible at $k = 0$ [Bonanno, Denz, Pawlowski, MR '21]

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New Lorentzian formulation [Fehre, Litim, Pawlowski, MR '21]

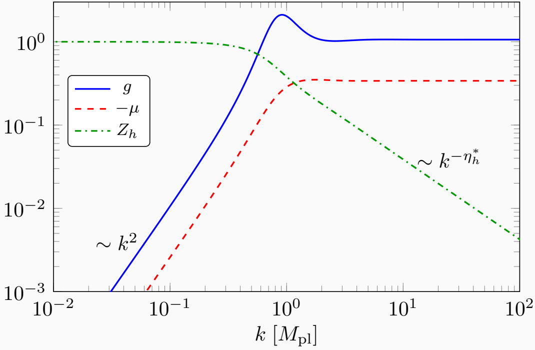
- Utilise Callan-Symanzik cutoff $R_k \sim k^2$
- UV divergences resurface $\to$ additional dimensional regularisation
- Use $\mathcal{G}_{hh}(p^2) = \int_0^\infty \frac{d\lambda}{\pi} \frac{\lambda \rho_h(\lambda^2)}{\lambda^2 + p^2}$ at finite k
- Directly compute flow of $\rho_h = \frac{1}{Z_h}\Big[2\pi\delta(\lambda^2 - m_h^2) + \theta(\lambda^2 - 4m_h^2) f_h(\lambda)\Big]$

UV-IR trajectories

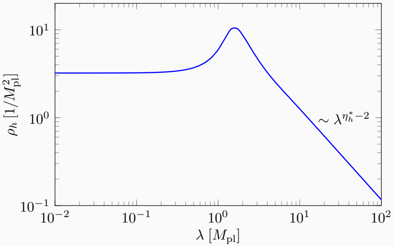
Einstein-Hilbert action with expansion about flat Minkowski background

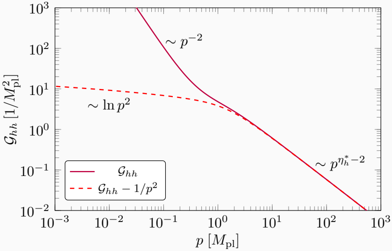
$$G_\mathrm{N}(k) = g(k)/k^2 \xrightarrow{k\to 0} G_\mathrm{N}$$

$$-2\Lambda(k) = k^2\mu(k) \xrightarrow{k\to 0} -2\Lambda = 0$$



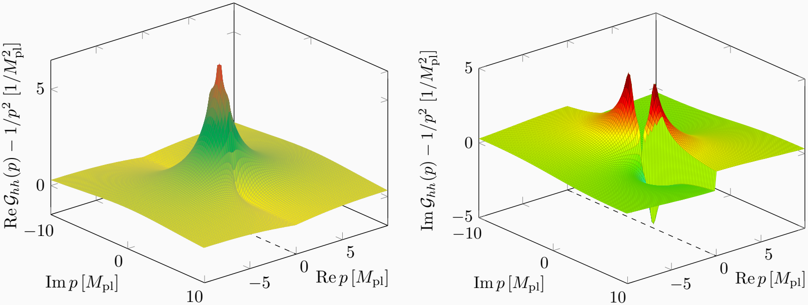
Graviton spectral function





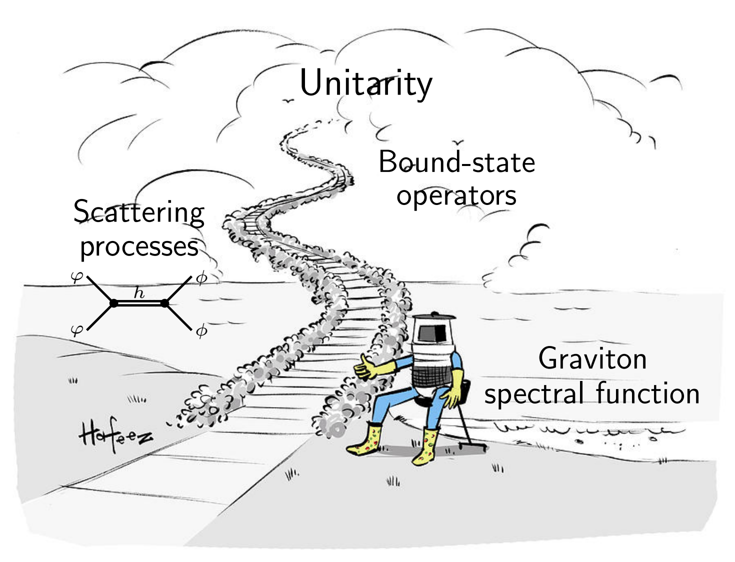
- Massless graviton delta-peak with positive multi-graviton continuum
- Matches effective field theory below $M_{\rm pl}$
- Asymptotically safe scaling above $M_{\rm pl}$
- Qualitative agreement to reconstruction results [Bonanno, Denz, Pawlowski, MR ’21]

Graviton propagator in the complex plane



No additional cuts and poles in the complex plane

Roadmap to unitarity



[Kaamran Hafeez]

Summary

- First direct computation of graviton spectral function
- Massless graviton delta-peak with postive multi-graviton continuum
- No additional cuts and poles in the complex plane
- Key step towards scattering processes and unitarity

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- First direct computation of graviton spectral function
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- Key step towards scattering processes and unitarity

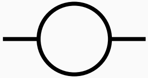
Thank you for your attention!

Back-up slides

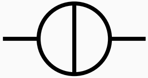
Perturbative quantum gravity: Einstein-Hilbert action

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_{\mathrm{N}}} \int_x \sqrt{\det\, g_{\mu\nu}}\,(2\Lambda - R(g_{\mu\nu}))$$

Perturbatively non-renormalisable: $[G_{\mathrm{N}}] = -2$



$$\sim p^4 \frac{1}{(p^2)^2}(p^2)^2 \sim p^4 \sim R^2, R^2_{\mu\nu}, R^2_{\mu\nu\rho\sigma}$$



$$\sim (p^4)^2 \frac{1}{(p^2)^5}(p^2)^4 \sim p^6 \sim R^3, R^3_{\mu\nu}, R^3_{\mu\nu\rho\sigma}, \ldots$$

First on-shell divergence: two-loop Goroff-Sagnotti counter term

['t Hooft, Veltmann '74; Goroff, Sagnotti '85]

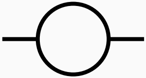
$$S_{\mathrm{GS}} \sim \int_x \sqrt{\det\, g_{\mu\nu}}\, C_{\mu\nu}{}^{\kappa\lambda} C_{\kappa\lambda}{}^{\rho\sigma} C_{\rho\sigma}{}^{\mu\nu}$$

Start of an infinite series of counter terms: No predictivity

Perturbative quantum gravity: Higher-derivative action

$$S_{\text{HD}} = \int_x \sqrt{\det g_{\mu\nu}} \left(\frac{1}{2\lambda} C^2_{\mu\nu\rho\sigma} - \frac{w}{3\lambda} R^2 \right) + S_{\text{EH}}$$

- Perturbatively renormalisable: $[w] = [\lambda] = 0$
- Asymptotically free: $\lambda^* = 0$



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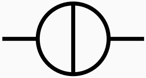

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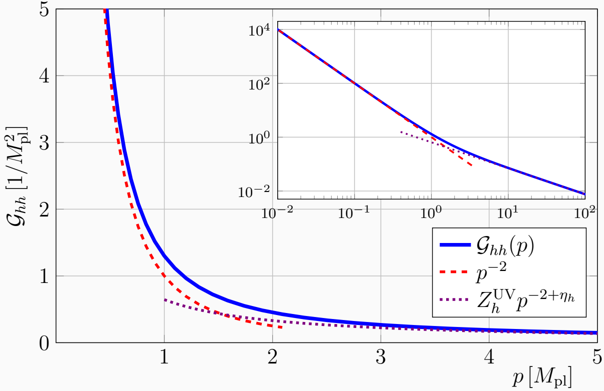
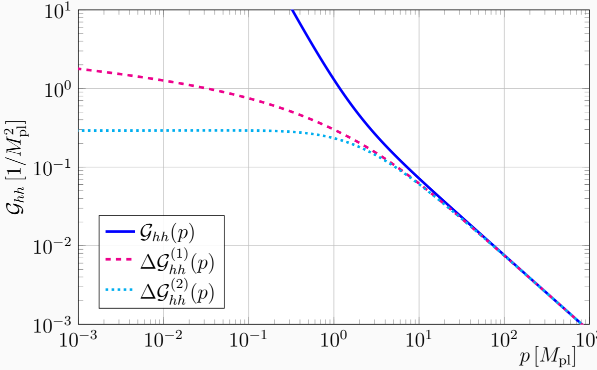
Euclidean fluctuation propagator



η_h = graviton fixed point anomalous dimension

Euclidean fluctuation propagator



Subleading behaviour fits to EFT results

['t Hooft, Veltman '74; Donoghue, El-Manoufi '14]

Analytic continutation

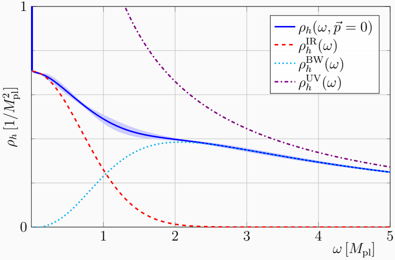
- Directly continuation of UV and IR asymptotics
- Breit-Wigner ansatz for rest (no poles in complex plane)

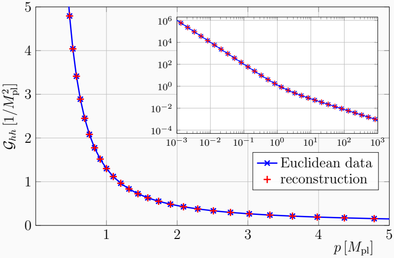
$$\mathcal{G}^{\mathrm{BW}}(p) \sim \sum_{i,j} \frac{\mathcal{N}_i}{\left(p + \Gamma_{i,j}\right)^2 + M_{i,j}^2}$$

- minimise error on reconstructed Euclidean data

$$E^{\mathrm{rel}} = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{\mathcal{G}(p_i) - \mathcal{G}^{\mathrm{rec}}(p_i)}{\mathcal{G}(p_i)} \right)^2 \quad \text{with} \quad \mathcal{G}^{\mathrm{rec}}(p) = \int_0^\infty \frac{d\lambda}{\pi} \frac{\lambda \rho(\lambda)}{\lambda^2 + p^2}$$

Graviton spectral function





- Positive spectral function (gauge dependent!)
- Very good reconstruction of Euclidean data $E^{\mathrm{rel}} < 10^{-6}$
- Based on assumption of no complex conjugate poles