Some Synchrotron Radiation Considerations in the LHeC design(s)
(certainly not exhaustive!)

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Based on Formula in [1]

- Radiated total power (integrated over 4π solid angle)

(A) $$P_0 = \frac{2\ r_e\ c\ m_0c^2\ \gamma^4}{3\ \rho^2} = \frac{2\ r_e\ c^3\ e^2 E_e^2\ B^2}{3\,(m_0c^2)^2}$$

with the classical electron radius $r_e = e^2/4\pi\varepsilon_0 m_0 c^2 = 2.818\cdot 10^{-15}$ m.

- 'critical' frequency (-> critical photon energy U_c, about at maximum energy flux)

(B) $$\omega_c = \frac{3\ c\ \gamma^3}{2\ \rho} \Rightarrow U_c = \hbar\omega_c = \frac{3\ \hbar\ c\ \gamma^3}{2\ \rho}$$

with h.bar = $1.054\cdot 10^{-34}$Js = $6.55\cdot 10^{-16}$ eVs.

For a machine that is a perfect circle, we get the energy loss per turn in multiplying with the revolution time T=c/(2πρ) for this case

(1) $$\Delta U / turn = \frac{4\ \pi\ r_e\ m_0c^2}{3}\frac{\gamma^4}{\rho}$$

reproducing the well-known proportionality with the 4th power of E (i.e. γ) and the inverse proportionality with the machine-radius.

In LEP and LHC there are straight sections, the radiating arcs having a length ratio η with respect to the total machine circumference C defined as

(2) $$\eta = 2\pi\rho / C \Rightarrow \rho = \eta\cdot C / 2\pi$$

hence the curvature seen by the beam is correspondingly stronger and the radiation power increases by $1/\eta^2$ compared to a circular machine of the same circumference C. On the other hand the length of the machine on which radiation takes place is reduced by the same factor η. Taking this into account we get for the energy loss per turn

(3) $$\Delta U / turn = \frac{8\ \pi^2\ r_e\ \ m_0c^2\ \gamma^4}{3\ \eta\ C}$$

and the critical photon energy

(4) $$U_c = \hbar\omega_c = \frac{3\ \pi\ \hbar\ c\ \gamma^3}{\eta\ C}$$

Check with LEP/LHC: The arc-factor η in LEP/LHC is about 2/3. Then for electrons at 100 GeV/c in LEP with a circumference of 27 km we get

(5) $$\Delta U_{e,LEP100} / turn = 3100\ \ MeV / turn$$

which agrees well with the findings in LEP of about 3 GeV/turn. Similarly we get for LHC with protons at 7 TeV

(6) $$\Delta U_{p,LHC7000} / turn = 6.5\ \ keV / turn$$

which again agrees well with the 6.7 keV from the LHC design report.

The critical energy in LEP at 100 GeV becomes from (B)

(8) $$\hbar\omega_{e,LEP100} = 770\ \ keV$$

and for LHC at 7 TeV

(9) $$\hbar\omega_{p,LHC7000} = 42.8\ \ eV$$

which perfectly agrees with the 43 eV from the LHC design report.

Application for LHeC, ring-option:

We assume an electron momentum of 70 GeV/c in the e-ring of LHeC. Then we can scale easily from (5) to

(10) $$\Delta U_{e,LHeC70} / turn \;=\; 740 \;\; MeV / turn$$

and the critical (about spectral peak) photon energy can be scaled from (8) to

(11) $$\hbar\omega_{e,LHeC70} = \; 260 \;\; keV$$

From (11) we see that gammas of this energy are not easily shielded and they will fly along the tunnel, part of them (and their scattered children) also hitting the LHC magnet's and cryogenic lines' cold mass. Also the peak of the spectrum is not sharp and at 4 times the critical energy the energy flux is still half of the peak flux (see Fig. 11 in [1]).

For a 'normalized' beam current of 100 mA from (10) we get a total energy loss of 74 MW. This power has to be replaced by the RF system with a 'from plug efficiency' not better 50% which means about 150 MW plug power.

Furthermore this radiation transforms into heat that has to be cooled away from shielding in the tunnel.

Finally, LHC allows only a total cryogenic loss of 5W/m in the arcs of which a good fraction will already be taken by p-beam image current heating, possible e-cloud heating and other effects. If we assume that the synchrotron radiation of LHeC might use 1 W/m, i.e. about 20 kW in the arcs, this means that only $3 \cdot 10^{-4}$ of this radiation could reach the cold mass, else cryogenics will break down. How to do that with 260 keV gammas ??

In this context it should also be kept in mind that at the end of the life of LEP in 2000, having encountered 'only' roughly 10 MW synchrotron radiation per beam for about 4 years, many plastic materials as super-insulation (turned brown in the sc. cavities) or electric insulations (magnets) had become brittle and were not considered reliable anymore.

Application for LHeC, linac-option:

A straight linac has no synchrotron radiation (apart in focusing quads) but recirculation is foreseen for some options. For any such race-track with double half-circle with a single linac arm we get immediately from (1)

(12) $$\Delta U / circle \;=\; \frac{4 \;\pi \;\; r_e \;\; m_0 c^2}{3} \frac{\gamma^4}{\rho} \;\;\Rightarrow\;\; \Delta U[eV] / circle = 6.03 \;\; 10^{-9} \frac{\gamma^4}{\rho[m]}$$

and (B) remains valid. Dividing (12) by (B) yields then (about) the expected number of photons per re-circulating passage (in the double half-circle)

(13) $$n / arc \;=\; \frac{8 \;\; \pi \;\; r_e \;\; m_0 c^2}{9 \;\; \hbar \;\; c} \gamma \;=\; 2 \cdot 10^{-2} \;\; \gamma$$

which is independent of the curvature radius and directly proportional to the beam energy (i.e. γ).

If we assume a 4-pass linac to 70 GeV, the last re-circulation and arc-passage takes place at 52.5 GeV. For simplicity we assume that all particles still have exactly the same energy before that arc. However, in the last arc at 52.5 GeV there are (in average) about 2000 photons with hence an rms scatter of about 2.2%. In a linac the beam rides on rising RF slope such that for non-relativistic particles lower energy (slower) particles are more accelerated than higher energy (faster) ones, compressing the momentum spread. However, electrons are already very relativistic at 50 GeV, hence in the last linac passage

there is no such momentum compression anymore. Therefore if we want to keep for the following beam steering – especially keeping the beam small enough at the interaction point – a momentum spread below $5 \cdot 10^{-5}$ (as is the case in LHC, but here at 70 GeV), the energy loss in the last circle should not exceed 0.16 GeV.
Resolving (12) for the (minimum) curvature radius we get

$$\rho[m] \;\geq\; 6 \;\; 10^{-9} \frac{\gamma^4}{\Delta U / arc}$$

or $\rho \geq 4200$ m, which is a size as LHC itself. !!
There are certainly tricks to contain a beam with a large momentum spread already at start of the arc. But here the momentum spread is created on-the-fly somewhere along the arc, each electron having ist own history, which changes the picture completely. To be followed up !

[1] A. Hofmann, "Synchrotron Radiation", Reviews of Accelerator Science and Technology, Vol. 1 (2008) 121-141.
(similar/same is also part of a CERN Yellow Report, don't remember which one).