Towards the spontaneous symmetry breaking: First order phase transition in QFT

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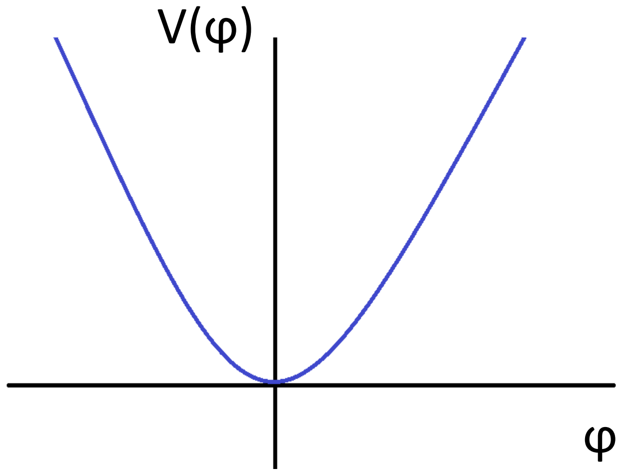
After the Big Bang - high temperature

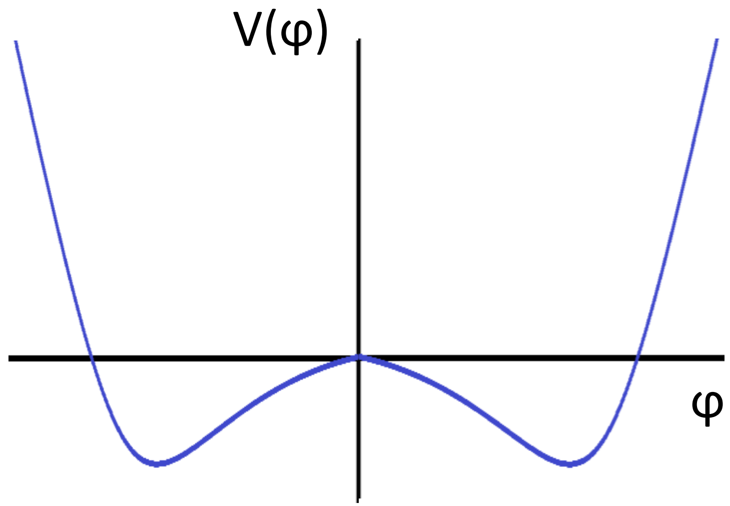
Only the symmetric vacuum in unbroken symmetry

Now - low temperature

$V(\varphi) \sim m^2\varphi^2+\lambda\varphi^4$

The symmetry is broken

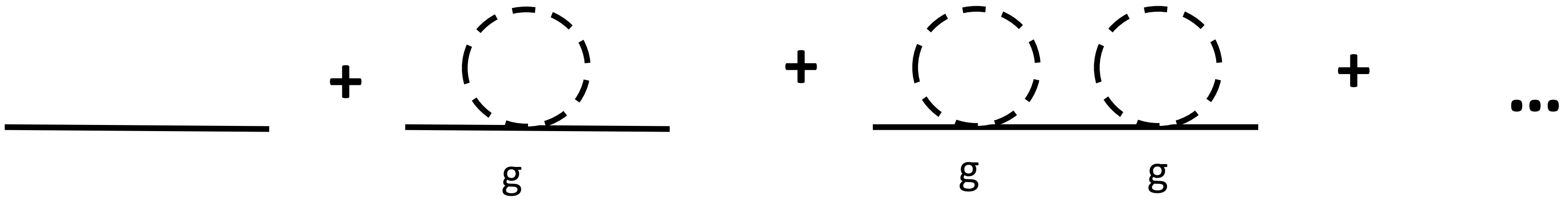




The goal: To get from high temperature to low temperature in a meaningfull way

From high temperature to low temperature

- Sum up the diagrams to get the effective masses
- This effect might be so strong, it generates the asymmetric vacuum even for the high temperature case



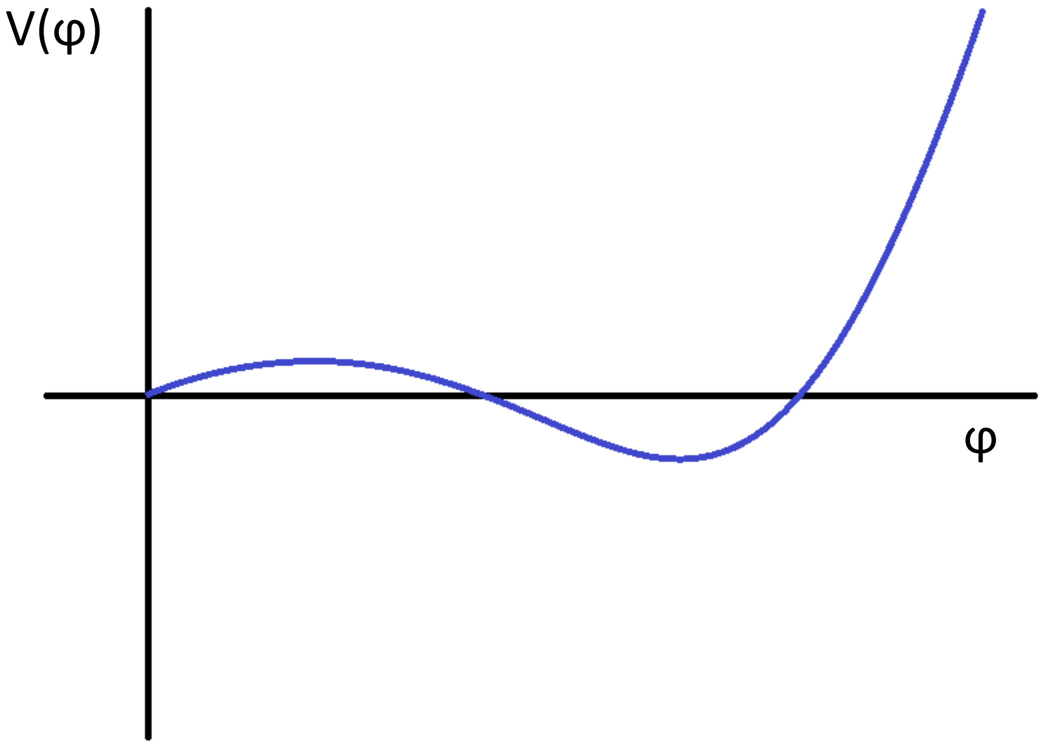
E.g. in theory with one vector boson and one SU(2) singlet with $L \ni g^2 AA\varphi\varphi$, we get in the lowest order (in 3 dim)

$$V_{eff,0}(\varphi) = \frac{\varphi^4 \; 4\pi\lambda - g^3\varphi^3 + 8\pi m^2\varphi^2}{16\pi}$$

This $g^3\varphi^3$ term can even create a barrier!

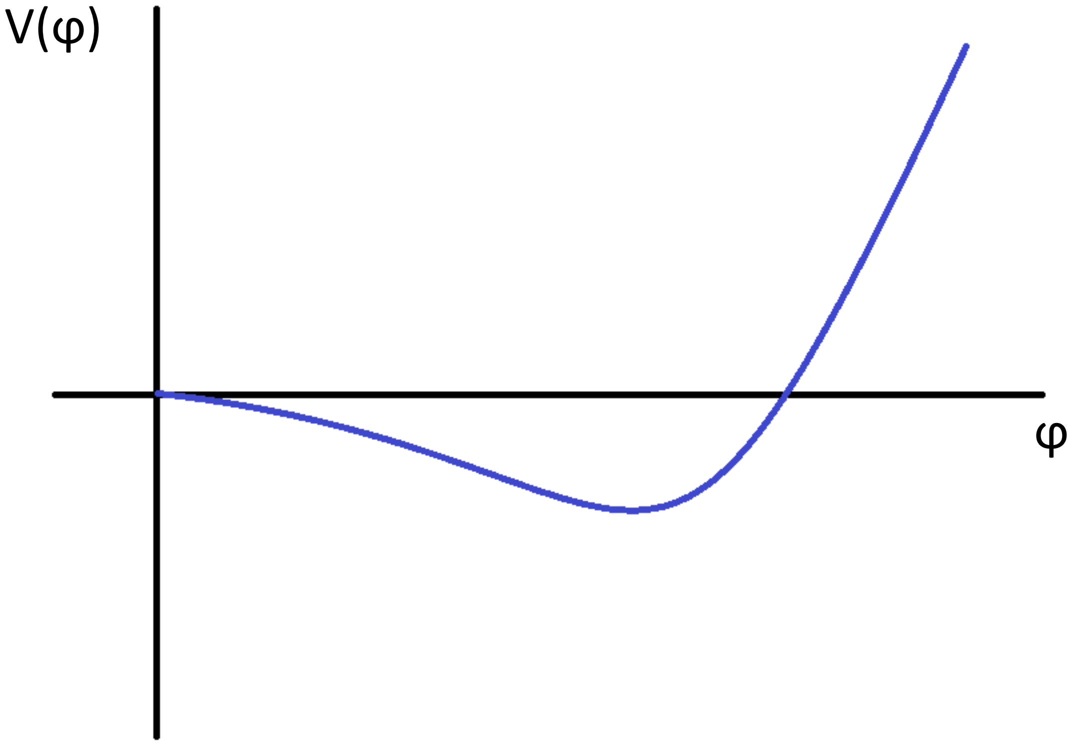
First order phase transition

- Tunneling creates bubbles
- The critical tepmerature is where the tunelling is posible for the first time



Second order phase transition

- Smooth process



Why first order phase transitions?

- To generate any matter/antimatter asymmetry, three condition are to be met
  - Baryon and lepton number violating processes – e.g. if the bubble refracts left-handed and right-handed particles differently (weak sphalerons), this is already in the SM
  - CP violation, not enough in the SM
  - Loss of thermal equilibrium – any asymmetry is washed out if there is equilibrium, there is no way we can loose thermal equilibrium in the SM
- The accelerating bubbles would have left traces in universe – mainly the gravitational wave background as it is a turbulent event.

Model: Scalar σ, doublet H and boson A, dim 3

$L \ni \mu_1^2\, HH^* + \mu_2^2\sigma^2 + \lambda_1(HH^*)^2 + \lambda_2\sigma^4 + \xi\,(HH^*)\sigma^2 + g^2 HH^* A^2$

Shift $H = \frac{1}{\sqrt{2}}\begin{pmatrix} \Psi_1 + i\,\Psi_2 \\ \varphi_1 + \Psi_3 + i\,\Psi_4 \end{pmatrix}$, $\sigma = \varphi + \varphi_2$

With backgroung fields φ_1, φ_2

Some effective masses after diagonalization to the first order:

$M_1^2 = \mu_2^2/2 + 1/2\,\lambda_1\,\varphi_1^2 + 1/2\,\xi\,\varphi_2^2$

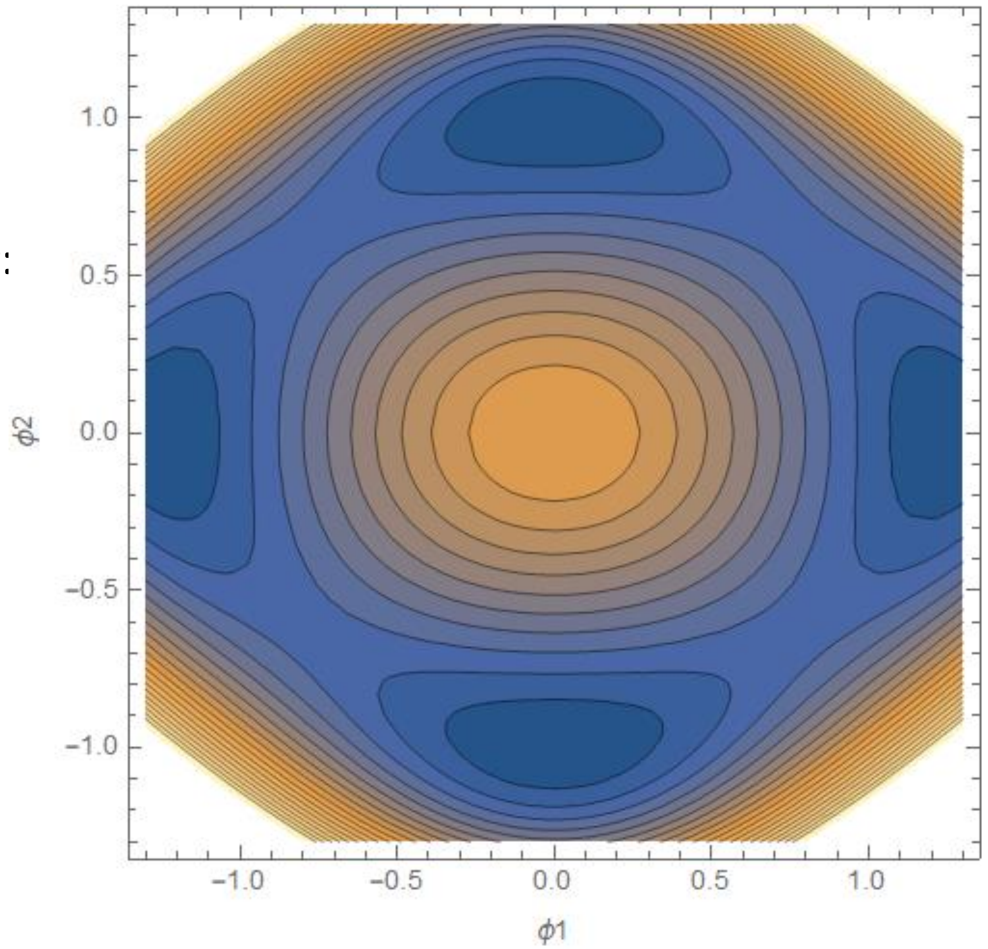
$M_\varphi^2 = \mu_2^2 + 1/2\,\xi\,\varphi_1^2 + 6\,\lambda_2\,\varphi_2^2$

$M_A^2 = 3/2\,g^2\,\varphi_1^2$

The potential:

$$V_0(\varphi_1,\varphi_2) = \frac{\mu_1^2\varphi_1^2}{2} + \frac{\lambda_1\varphi_1^4}{4} + \mu_2^2\,\varphi_2^2 + \frac{1}{2}\,\xi\,\varphi_1^2\,\varphi_2^2 + \lambda_2\,\varphi_2^4$$

$$V_1(\varphi_1,\varphi_2) = -\frac{1}{12\pi}\sum eigenvalues(M^2)$$



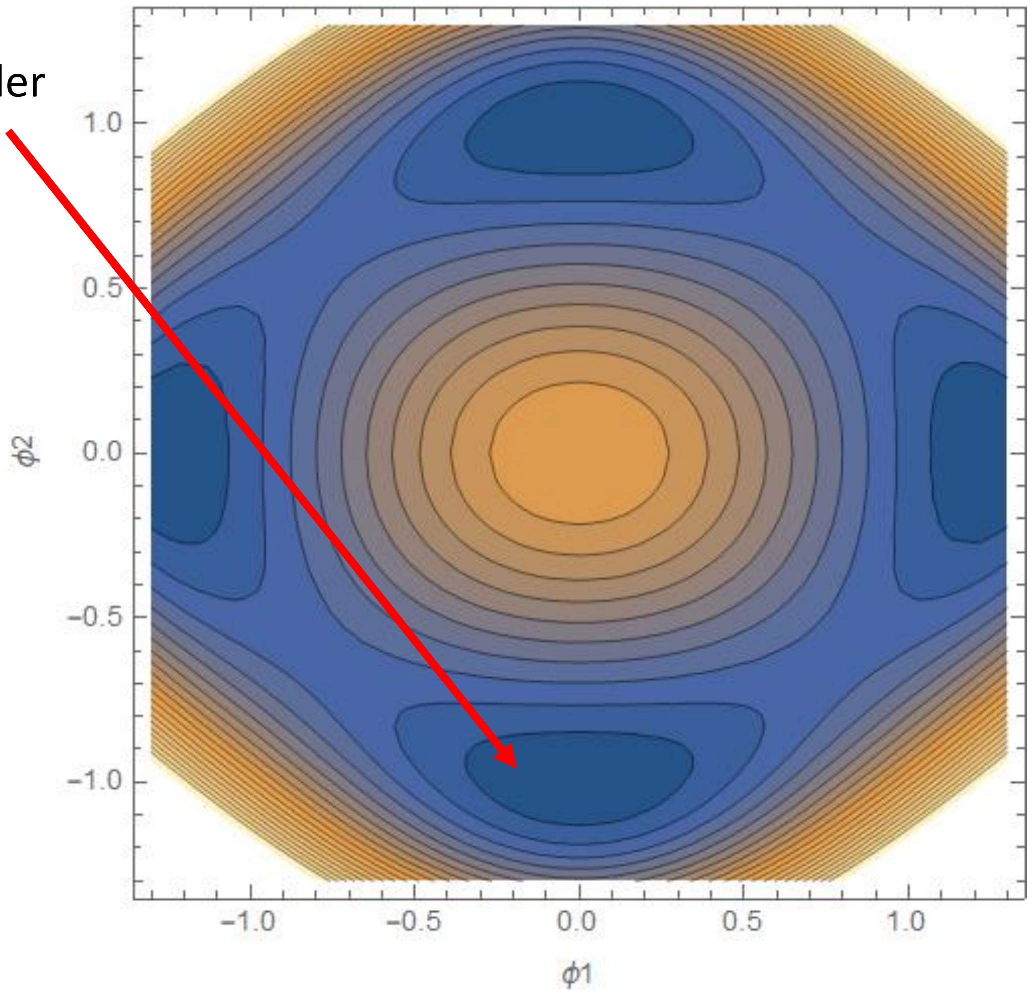
Model: Scalar σ, doublet H and boson A, dim 3

Some VEVs at the critical temperature at the lowest order

$$< H > = \frac{\mu_1^{\,2}}{2\sqrt{\lambda_1}}$$

$$< H^2 > = -\frac{\mu_1^{\,4}}{4\lambda_1}$$

$$< A > = 0$$



Some VEVs at the critical temperature at the first order

$< H > =$

$$-\frac{8\sqrt{-\frac{\sqrt{\lambda 1}\,\mu 22}{\sqrt{\lambda 2}}}-\frac{\sqrt{2}\,g^3\sqrt{-\frac{\sqrt{\lambda 1}\,\mu 22}{\sqrt{\lambda 2}}}}{\lambda 1^{3/2}}+\sqrt{2}\sqrt{\mu 22-\frac{\sqrt{\lambda 1}\,\mu 22}{\sqrt{\lambda 2}}}+\frac{\xi\sqrt{\mu 22\left(4-\frac{2\xi}{\sqrt{\lambda 1}\sqrt{\lambda 2}}\right)}}{\lambda 1}}{32\pi}-\frac{1}{1536\pi^2\sqrt{\lambda 2}}\left(-\frac{4}{\sqrt{-\frac{\sqrt{\lambda 1}\,\mu 22}{\sqrt{\lambda 2}}}}+\frac{g^3}{\sqrt{2}\,\lambda 1^{3/2}\sqrt{-\frac{\sqrt{\lambda 1}\,\mu 22}{\sqrt{\lambda 2}}}}-\frac{1}{\sqrt{2}\sqrt{\mu 22-\frac{\sqrt{\lambda 1}\,\mu 22}{\sqrt{\lambda 2}}}}-\frac{\xi^2}{\lambda 1^2\sqrt{\mu 22\left(4-\frac{2\xi}{\sqrt{\lambda 1}\sqrt{\lambda 2}}\right)}}\right)$$

$$\left(2\sqrt{2}\sqrt{\lambda 2}\left(g^2\sqrt{-\frac{g^2\mu 22}{\sqrt{\lambda 1}\sqrt{\lambda 2}}}+\xi\sqrt{-\frac{\mu 22\,\xi}{\sqrt{\lambda 1}\sqrt{\lambda 2}}}\right)-\sqrt{\lambda 1}\left(2\lambda 2\left(8\sqrt{2}\sqrt{-\mu 22}+\sqrt{2}\sqrt{\mu 22}+3\sqrt{2}\sqrt{-\frac{\mu 22\,\xi}{\sqrt{\lambda 1}\sqrt{\lambda 2}}}-\sqrt{\mu 22\left(2-\frac{\xi}{\lambda 2}\right)}\right)+\xi\left(3\sqrt{-\frac{\mu 22\,\xi}{\lambda 2}}+\sqrt{\mu 22\left(2-\frac{\xi}{\lambda 2}\right)}\right)\right)\right)$$

$< H^2 > =$

$$\frac{\sqrt{2}\,g^3\left(-\frac{\sqrt{\lambda 1}\,\mu 22}{\sqrt{\lambda 2}}\right)^{3/2}+\frac{\lambda 1\,\mu 22\left(12\lambda 1\sqrt{-\frac{\sqrt{\lambda 1}\,\mu 22}{\sqrt{\lambda 2}}}+\xi\sqrt{\mu 22\left(4-\frac{2\xi}{\sqrt{\lambda 1}\sqrt{\lambda 2}}\right)}\right)}{\sqrt{\lambda 2}}}{32\pi\lambda 1^{5/2}}+\frac{1}{1536\pi^2\lambda 1^{5/2}\sqrt{\lambda 2}}\left(-\frac{3g^3\sqrt{-\frac{\sqrt{\lambda 1}\,\mu 22}{\sqrt{\lambda 2}}}}{\sqrt{2}}+\frac{1}{2}\sqrt{\lambda 1}\left(36\lambda 1\sqrt{-\frac{\sqrt{\lambda 1}\,\mu 22}{\sqrt{\lambda 2}}}+\frac{\left(4\sqrt{\lambda 1}\sqrt{\lambda 2}-3\xi\right)\xi\sqrt{\mu 22\left(4-\frac{2\xi}{\sqrt{\lambda 1}\sqrt{\lambda 2}}\right)}}{2\sqrt{\lambda 1}\sqrt{\lambda 2}-\xi}\right)\right)$$

$$\left(2\sqrt{2}\sqrt{\lambda 2}\left(g^2\sqrt{-\frac{g^2\mu 22}{\sqrt{\lambda 1}\sqrt{\lambda 2}}}+\xi\sqrt{-\frac{\mu 22\,\xi}{\sqrt{\lambda 1}\sqrt{\lambda 2}}}\right)-\sqrt{\lambda 1}\left(2\lambda 2\left(8\sqrt{2}\sqrt{-\mu 22}+\sqrt{2}\sqrt{\mu 22}+3\sqrt{2}\sqrt{-\frac{\mu 22\,\xi}{\sqrt{\lambda 1}\sqrt{\lambda 2}}}-\sqrt{\mu 22\left(2-\frac{\xi}{\lambda 2}\right)}\right)+\xi\left(3\sqrt{-\frac{\mu 22\,\xi}{\lambda 2}}+\sqrt{\mu 22\left(2-\frac{\xi}{\lambda 2}\right)}\right)\right)\right)$$

$< A > =$

$$\frac{g^2\left(-\frac{\mu 22}{\sqrt{\lambda 1}\sqrt{\lambda 2}}\right)^{3/2}}{8\sqrt{2}\,\pi}+$$

$$\frac{\mu 22\left(4\sqrt{\lambda 1}\sqrt{\lambda 2}-3\xi\right)\left(-2\sqrt{2}\sqrt{\lambda 2}\left(g^2\sqrt{-\frac{g^2\mu 22}{\sqrt{\lambda 1}\sqrt{\lambda 2}}}+\xi\sqrt{-\frac{\mu 22\,\xi}{\sqrt{\lambda 1}\sqrt{\lambda 2}}}\right)+\sqrt{\lambda 1}\left(2\lambda 2\left(8\sqrt{2}\sqrt{-\mu 22}+\sqrt{2}\sqrt{\mu 22}+3\sqrt{2}\sqrt{-\frac{\mu 22\,\xi}{\sqrt{\lambda 1}\sqrt{\lambda 2}}}-\sqrt{\mu 22\left(2-\frac{\xi}{\lambda 2}\right)}\right)+\xi\left(3\sqrt{-\frac{\mu 22\,\xi}{\lambda 2}}+\sqrt{\mu 22\left(2-\frac{\xi}{\lambda 2}\right)}\right)\right)\right)}{1536\pi^2\lambda 1^{3/2}\lambda 2\sqrt{\mu 22\left(4-\frac{2\xi}{\sqrt{\lambda 1}\sqrt{\lambda 2}}\right)}}$$

What next?

- Rewritte the couplings constants from 3D to 4D. This has to be done for every model separately.
- Insert numbers
- Calculate higher order correction - still not clear how to incorporate them