Extended Dirac-Born-Infeld theory from soft theorems

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[Kampf-Novotny-PV: 2107.04587]

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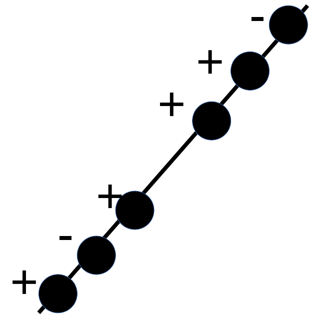
2003:

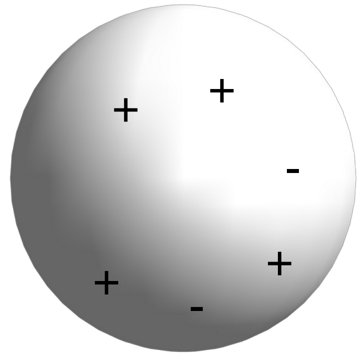
[hep-th/0312171]

Twistor string formalism



MHV tree gluon amplitudes supported on degree one, genus zero curves (i.e. lines) in twistor space

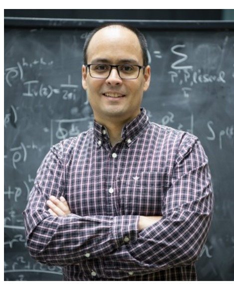
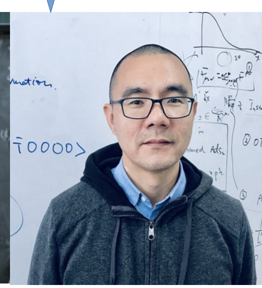
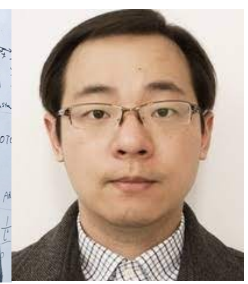
generalization that took 10 years



tree gluon amplitudes: correlation functions on genus zero curves (integral over the moduli space of such curves)

2013:

[1307.2199]

C H Y formalism

$$\mathcal{A}_n = \int \left[\prod_{i=1}^{n} d\sigma_i \delta\left(\frac{\partial F(\sigma)}{\partial \sigma_i} \right) \right] \mathcal{I}_n(p, \epsilon, \sigma) \left[\frac{\mathrm{tr}\left(T^{a_1} \cdots T^{a_n} \right)}{\sigma_{12} \cdots \sigma_{n1}} \right]$$

$$F(\sigma) = \sum_{i,j=1}^{n} s_{ij} \log |\sigma_{ij}|$$

theory independent: specifies kinematics

specifies theory

color factor: if needed

Theories with CHY representation

share one common feature: special **soft theorems**

Gravity

Yang-Mills

Galileons

NLSM (ChPT)

DBI

$\mathcal{I}_{\mathrm{eDBI}}$ interpolates between DBI and NLSM

↓

allows to compute tree amplitudes

↓

conjectured action for eDBI

by a series of (natural) operations modifying the integrands $\mathcal{I}_n(p, \epsilon, \sigma)$

extended DBI

[1412.3479]

$$S_{\mathrm{eDBI}} = \int_{\mathbb{R}^{1,3}} \mathrm{d}^4 x \left\{ \Lambda^4 \left[1 - \sqrt{-\det\left(\eta_{\mu\nu} - \Lambda^{-4} g_{\mu\nu} - \Lambda^{-2}(cW_{\mu\nu} + F_{\mu\nu})\right)} \right] \right\}$$

$$W_{\mu\nu} = \sum_{m=1}^{\infty} \sum_{k=0}^{m-1} \frac{2(m-k)}{2m+1} \lambda^{2m+1} \langle \partial_{[\mu}\phi\phi^{2k}\partial_{\nu]}\phi\phi^{2(m-k)-1} \rangle \qquad g_{\mu\nu} = \frac{1}{4\lambda^2} \langle \partial_\mu U^\dagger \partial_\nu U \rangle \qquad U = \frac{\mathbb{1} + \lambda\phi}{\mathbb{1} - \lambda\phi}, \qquad \phi \in \mathfrak{u}(N)$$

Outline of the talk

- Symmetries of the extended DBI action
- Soft theorems and constructibility of tree S-matrix
- Bottom-up approach and a missed parameter
- Search for a generalized theory

Symmetries of extended DBI action

- all other CHY theories have special soft theorems

  ?what about extended DBI? → study symmetries

- building blocks in action

$g_{\mu\nu}$: metric on coset space $\frac{\mathrm{U}(N)_\mathrm{L} \times \mathrm{U}(N)_\mathrm{R}}{\mathrm{U}(N)_\mathrm{diag}} \simeq \mathrm{U}(N)$

→ $\mathrm{U}(N)_\mathrm{L} \times \mathrm{U}(N)_\mathrm{R}$ symmetry (or $\mathrm{U}(N)_\mathrm{v} \times \mathrm{U}(N)_\mathrm{a}$)

$\mathcal{F}_{\mu\nu} = F_{\mu\nu} + cW_{\mu\nu}$ → (pull-back of) 2-form on $\mathrm{U}(N)$

$F = \mathrm{d}A$

trivially $\mathrm{U}(N)_\mathrm{v}$ invariant

not $\mathrm{U}(N)_\mathrm{a}$ invariant

$H^2_\mathrm{dR}(\mathrm{U}(N)) = 0$

if closed and hence exact under axial transformations

use a shift in A_μ to restore $\mathrm{U}(N)_\mathrm{a}$ invariance

Symmetries of extended DBI action

$$\mathcal{F} = \mathrm{d}A + cW \xmapsto{\text{axial}} dA + c\left(W + \mathrm{d}\beta\right) = cW + \mathrm{d}\left(A + c\beta\right)$$

define shift symmetry under axial transformations $A \xmapsto{\text{axial}} A - c\beta$

$\mathcal{F} \xmapsto{\text{axial}} \mathcal{F}$ is $\mathrm{U}(N)_\mathrm{v} \times \mathrm{U}(N)_\mathrm{a}$ invariant

Summary: $\left.\begin{array}{l} g_{\mu\nu} \\ \mathcal{F}_{\mu\nu} \end{array}\right\} \mathrm{U}(N)_\mathrm{v} \times \mathrm{U}(N)_\mathrm{a}$ invariant building blocks for extended DBI action (and others)

enhanced soft theorems

$$S = \int_{\mathbb{R}^{1,3}} \mathrm{d}^4 x \left\{ \Lambda^4 \left[1 - \sqrt{-\det\left(\eta - \Lambda^{-4} g - \Lambda^{-2}\mathcal{F}\right)_{\mu\nu}} \right] \right\}$$

Proof of symmetries

?is the axial transformation of the 2-form $W_{\mu\nu}$ really closed?

more elegantly

if so, then $\mathrm{d}W$ is bi-invariant (vector&axial or R&L)

YES, by brute force computation

harmonic 3-form

left-invariant Maurer-Cartan form

$\dim H^3_{\mathrm{dR}}(\mathrm{U}(N)) = 1$

unique Cartan 3-form $\Omega = \mathrm{tr}\left(\sigma_{\mathrm{L}} \wedge \sigma_{\mathrm{L}} \wedge \sigma_{\mathrm{L}}\right)$

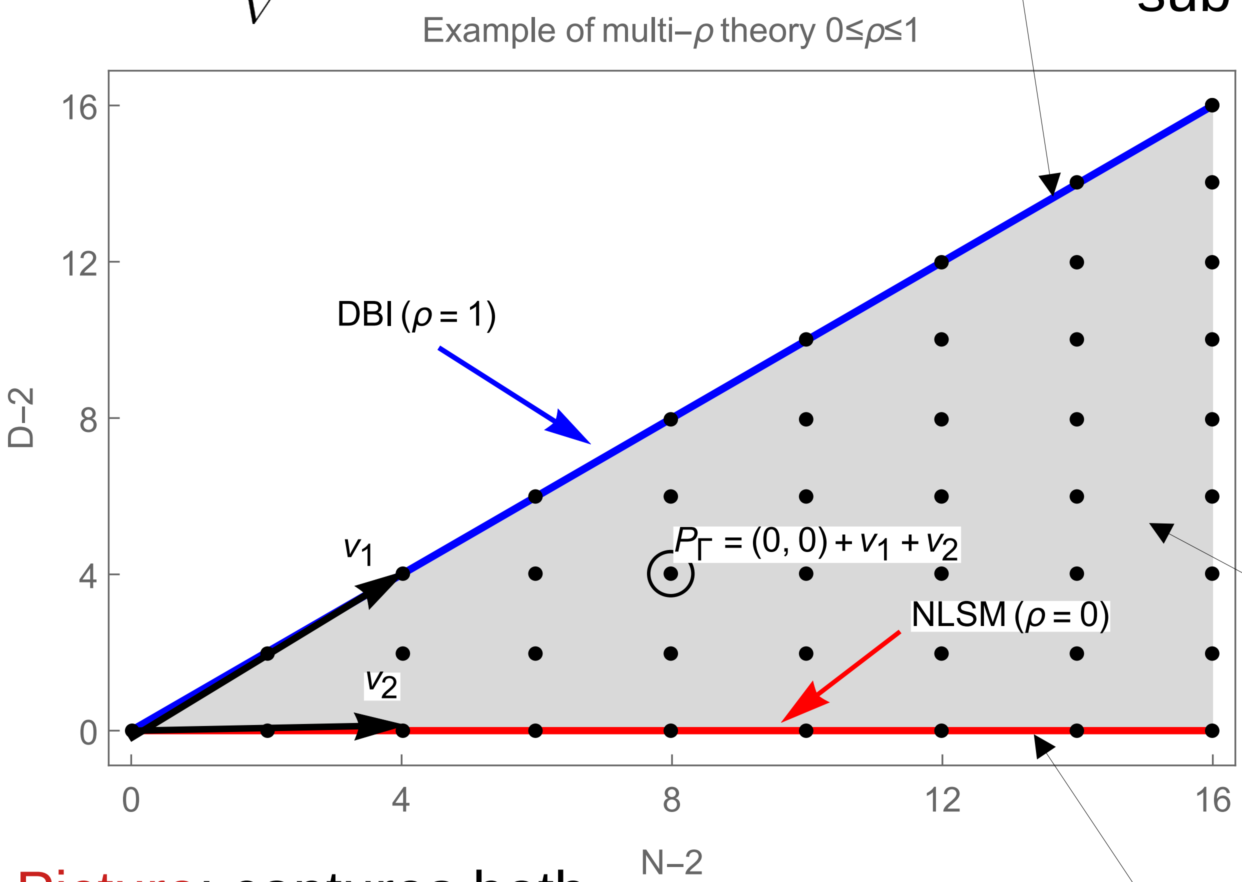
in local coordinates: $\Omega = \mathrm{d}W$

Group topology: $W_{\mu\nu}$ is not some ugly 2-form, in fact it is the only natural one

Soft theorems

Multi-ρ theories:

$$\mathcal{L} = \sum_V g_V \mathcal{L}_V$$



$\rho_{\max}$ represents consistent sub-theory

# of derivatives in vertex

$$\rho = \frac{D-2}{N-2}$$

characteristic of interaction vertices of a theory (or a given graph)

# of fields in vertex

not a sub-theory

$\rho_{\min}$ represents consistent sub-theory

Picture: captures both fundamental interaction vertices and Feynman graphs contributing to given amplitudes

Soft theorems

- **Single-ρ theories:**

enhanced soft behavior $A(p) = \mathcal{O}(p^\sigma)\,, \sigma > 1$

soft BCFW recursion
[1509.03309], [1611.03137]

if $\rho \leq \sigma$

on-shell constructible theories

NLSM (ρ=0, σ=1)
DBI scalar (ρ=1, σ=2)
Galileon (ρ=2, σ=2)
Special Galileon (ρ=2, σ=3)
Born-Infeld
Special scalar vector Galileon

- **Multi-ρ theories:**

soft behavior $A^{(\rho_{\min})} = \mathcal{O}\left(p^{\sigma_{\min}}\right)\,, \qquad A^{(\rho_{\max})} = \mathcal{O}\left(p^{\sigma_{\max}}\right)$

if

$\rho_{\min} \leq \sigma_{\min} = \rho_{\max} \leq \sigma_{\max}$

graded soft theorem

$A(p) - A^{(\rho_{\max})} = \mathcal{O}\left(p^{\sigma_{\min}}\right)$

$A^{(\rho_{\max})} = \mathcal{O}\left(p^{\sigma_{\max}}\right)$

on-shell constructible multi-ρ theories

extended DBI

Extended DBI tree-level on-shell constructibility

extended DBI theory $\rho_{\rm min} = 0 \leq \sigma_{\rm min} = 1 = \rho_{\rm max} \leq \sigma_{\rm max} = 2$

amplitudes with at least one scalar

constructed by **graded soft theorem**

$$A_{n_\phi n_\gamma}(p) - A^{(\rho=1)}_{n_\phi n_\gamma}(p) = \mathcal{O}(p)$$

$$A^{(\rho=1)}_{n_\phi n_\gamma}(p) = \mathcal{O}(p^2)$$

amplitudes with photons only (pure BI theory)

constructed using **multichiral soft limit**

tree S-matrix constructible by soft on-shell recursion

(seed amplitudes for recursion: 4-pt)

Bottom-up approach and a missed parameter

Point of view: postulate soft theorems + seed amplitudes + power-counting

? does this imply the extended DBI action, or is there space for generalization ?

Soft bootstrap (recursion):

impose soft theorems

generic seeds
(4-pt contact)

higher-pt amplitudes + generic contact terms

constraints on contact terms & perhaps seeds

Results (SU(2) theory):

one extra compared to extended DBI action

7+5
4-pt contact seeds

soft bootstrap

4 free parameters
(couplings)

?curse
or
blessing?

Search for a generalized extended DBI theory

Observation: original $\rho = \rho_{\max} = 1$ (i.e. DBI sub-theory) → **1 free parameter**

↓ soft bootstrap

$\rho = \rho_{\max} = 1$ sub-theory → **2 free parameters**

↓ is there a generalized DBI sub-theory?

2 energy scales

YES:

$$\mathcal{L}_{2\mathrm{DBI}} = -\Lambda^4 + (\Lambda^4 - M^4)\sqrt{\det\left(\eta_{\mu\nu} - \Lambda^{-4}\partial_\mu\phi\partial_\nu\phi\right)}$$
$$+M^4\sqrt{-\det\left(\eta_{\mu\nu} - \Lambda^{-4}\partial_\mu\phi\partial_\nu\phi - M^{-2}F_{\mu\nu}\right)}$$

2-scale DBI theory (ρ=1 sub-theory)

↓ **glue** this ρ=1 sub-theory **back to** the **extended DBI** action

The final theory

by gluing: promote $F_{\mu\nu} \to \mathcal{F}_{\mu\nu}$, $\delta_{ab} \to h_{ab}$

2-scale extended DBI theory:

$$\mathcal{L}_{2\mathrm{DBI}} = \Lambda^4 - (\Lambda^4 - M^4)\sqrt{\det\left(\eta_{\mu\nu} - \Lambda^{-4} h_{ab}\partial_\mu\phi^a\partial_\nu\phi^b\right)}$$
$$-M^4\sqrt{-\det\left(\eta_{\mu\nu} - \Lambda^{-4} h_{ab}\partial_\mu\phi^a\partial_\nu\phi^b - M^{-2}\mathcal{F}_{\mu\nu}\right)}$$

4 parameters (couplings): $\Lambda,\ M,\ \lambda,\ c$

as predicted by bottom-up **soft bootstrap**

from our analysis follows that its tree level S-matrix is on-shell constructible by soft bootstrap (a.k.a. soft BCFW recursion)

Do not get caught in the web

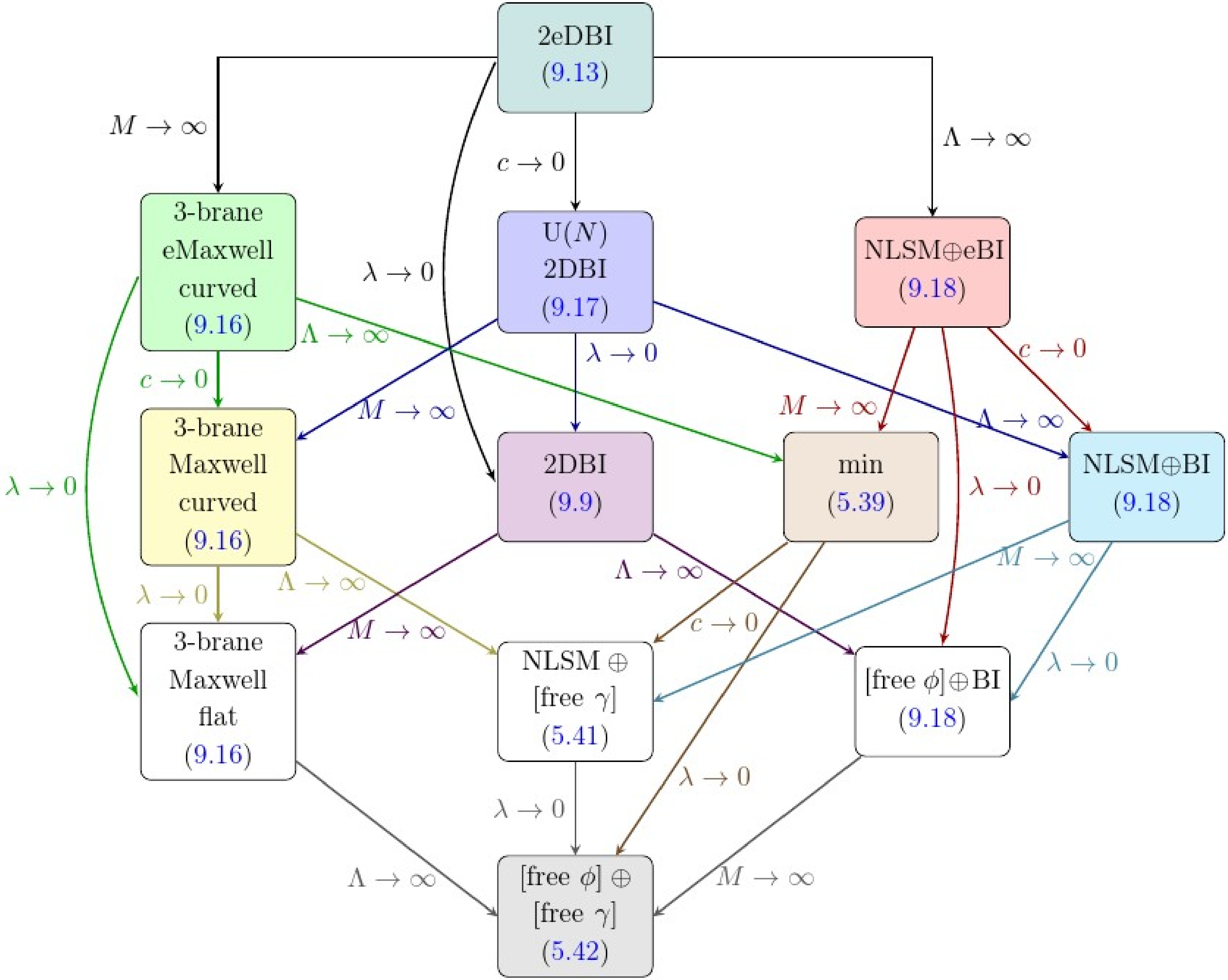


Figure 6. Web of limits for the 2-scale extended DBI theory.