Axion Fragmentation

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2022. 5. 14 @ Physics in LHC and beyond

Axion (-like) particle

Axion field : ϕ

- Shift symmetry (NG boson) + Chern-Simons coupling

$$\phi \rightarrow \phi + \delta\phi \qquad\qquad \frac{1}{f}\phi G_{\mu\nu}\widetilde{G^{\mu\nu}}$$

- Shift symmetry breaking by strong dynamics

$$V(\phi) = \Lambda_b^4 \cos\frac{\phi}{f}$$

- Theoretical motivation, interesting phenomenology, …
  - Strong CP problem, QCD axion
  - Naturalness of electroweak scale, Relaxion
  - Axion monodromy
  - Axion inflation
  - …

Axion (-like) particle & cosmology

Dynamics of axion field is interesting

- Axion dark mattter
- Relaxion : dynamical expanation of electroweak scale
- …

Solving EOM $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$ with some initial condition

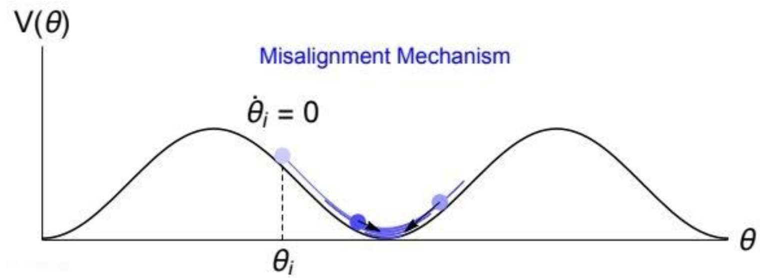
ex) Axion (-like) particle DM scenario

- **Misalignment mechanism**

[Preskill, Wise, Wilczek (1983)]
[Abbott, Sikivie (1983)]
[Dine, Fischler (1983)]

Initial condition $\phi = \phi_0 \neq 0$, $\dot{\phi} = 0$

EOM $\ddot{\phi} + 3H\dot{\phi} + \frac{\Lambda_b^4(T)}{f}\sin\frac{\phi}{f} = 0$



[taken from Co, Hall, Harigaya (2019)]

The axion starts to oscillate when $3H(T) \sim m(T)$

$$\rho_{DM} \sim m_a \times \left(\frac{a(T_{osc})}{a_0}\right)^3 \times \frac{\Lambda_b(T_{osc})^4\theta_i^2}{m_a(T_{osc})}$$

w/ $m_a(T_{osc}) \sim 3H(T_{osc})$

mass　　Dilution factor　　Number density at $T = T_{osc}$

ex) Axion (-like) particle DM scenario

- **Misalignment mechanism**

[Preskill, Wise, Wilczek (1983)]
[Abbott, Sikivie (1983)]
[Dine, Fischler (1983)]

Initial condition $\phi = \phi_0 \neq 0$, $\dot{\phi} = 0$

What happens if $\dot{\phi} > \Lambda_b^2$?

EOM $$\ddot{\phi} + 3H\dot{\phi} + \frac{\Lambda_b^4(T)}{f}\sin\frac{\phi}{f} = 0$$

[taken from Co, Hall, Harigaya (2019)]

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w/ $m_a(T_{osc}) \sim 3H(T_{osc})$

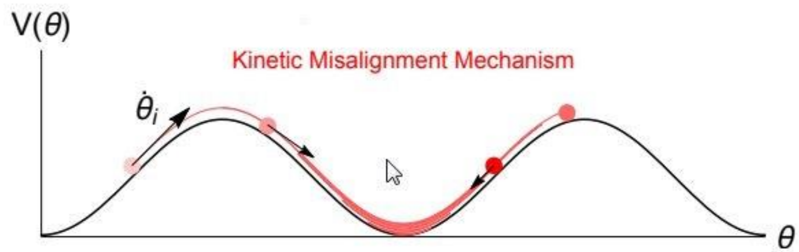
mass — Dilution factor — Number density at $T = T_{osc}$

ex) Axion (-like) particle DM scenario

- **Kinetic Misalignment mechanism** [Co, Hall, Harigaya (2019)] [Chang, Cui (2019)]

Initial condition $\dot{\phi} > \Lambda_b^2$

EOM $\ddot{\phi} + 3H\dot{\phi} + \frac{\Lambda_b^4(T)}{f} \sin \frac{\phi}{f} = 0$



[taken from Co, Hall, Harigaya (2019)]

The axion starts to oscillate when $\dot{\phi}^2(T) \sim \Lambda_b^4(T)$

$$\rho_{DM} \sim m_a \times \left(\frac{a(T_{osc})}{a_0}\right)^3 \times \frac{\Lambda_b(T_{osc})^4}{m_a(T_{osc})}$$

w/ $\dot{\phi}^2(T_{osc}) \sim \Lambda_b^4(T_{osc})$

mass — Dilution factor — Number density at $T = T_{osc}$

Delay of onset of oscillation → larger ρ_{DM}

Axion fluctuation?

What people usually do

Solving EOM for spatially homogeneous field : $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$

However...

Even we start from (almost) homogeneous field configuration,
fluctuations can grow later.

Velocity as U(1) charge

Velocity $\dot{\phi}$ is U(1) charge : $\rho_{\rm shift} = f \frac{\partial L}{\partial_0 \phi} = f \dot{\phi}$

$\phi \to \phi + f\,\delta$

Shift transf.

Explicit breaking of U(1) : $V(\phi) = \Lambda_b^4 \cos\frac{\phi}{f} + \cdots$

⇒ U(1) charge will be lose = energy dissipation

Axion fragmentation [Fonseca, Morgante, RS, Servant (2019)]

For related earlier works, see
[Green, Kofman, Starobinsky (1998)]
[Flauger, McAllister, Pajer, Westphal, Xu (2009)]
[Jaeckel, Mehta, Witkowski (2016)]
[Arvanitaki, Dimopoulos, Galanis, Lehner, Thompson, Van Tilburg (2019)]

1. Introduction
2. **Perturbative analysis**
3. Non-perturbative analysis

Axion fragmentation

Let us investigate the simplest case.

- $H = 0$ (no cosmic expansion)
- $V(\phi) = \Lambda_b^4 \cos(\phi/f)$

We have only three parameters :

$$\left\{ \begin{array}{ll} \dot{\phi}_0 & \text{: initial velocity} \\ f & \text{: decay constant} \\ \Lambda_b^4 & \text{: height of barrier} \end{array} \right.$$

EOM of axion :

$$\frac{d^2\phi}{dt^2} - \nabla^2\phi - \frac{\Lambda_b^4}{f}\sin\frac{\phi}{f} = 0$$

EOM of axion

We decompose $\phi(\vec{x},t) = \bar{\phi}(t) + \left[\int \frac{d^3k}{(2\pi)^3}\delta\phi_k(t)e^{ikx} + h.c.\right]$

$$\frac{d^2\phi}{dt^2} - \nabla^2\phi - \frac{\Lambda_b^4}{f}\sin\frac{\phi}{f} = \quad 0$$

EOM of axion

We decompose $\phi(\vec{x}, t) = \bar{\phi}(t) + \left[\int \frac{d^3k}{(2\pi)^3} \delta\phi_k(t) e^{ikx} + h.c. \right]$

At the leading order of $\delta\phi_k$,

$$\frac{d^2\bar{\phi}}{dt^2} - \frac{\Lambda_b^4}{f} \sin\frac{\bar{\phi}}{f} = \underbrace{\frac{1}{2}\frac{\Lambda_b^4}{f^3} \sin\frac{\bar{\phi}}{f} \int \frac{d^3x}{V_{vol}} \langle \delta\phi(x) \rangle^2}_{\text{Back reaction}}$$

$$\frac{d^2\delta\phi}{dt^2} - \nabla^2\delta\phi - \frac{\Lambda_b^4}{f^2} \cos\frac{\bar{\phi}}{f} \delta\phi = 0$$

EOM of axion

We decompose $\phi(\vec{x},t) = \bar{\phi}(t) + \left[\int \frac{d^3k}{(2\pi)^3}\delta\phi_k(t)e^{ikx} + h.c.\right]$

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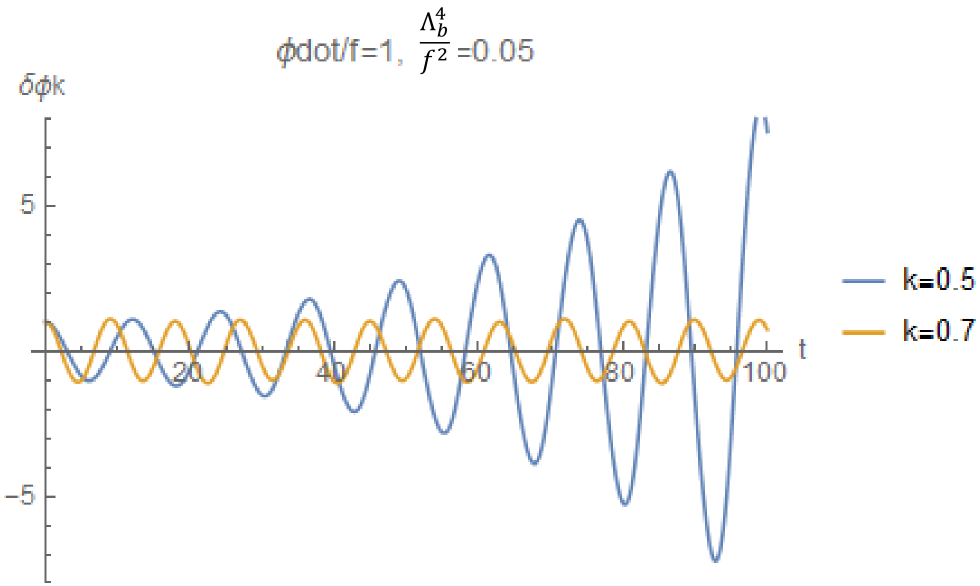
$$\frac{d^2\delta\phi_k}{dt^2} + \underbrace{\left(k^2 - \frac{\Lambda_b^4}{f^2}\cos\frac{\dot{\bar{\phi}}t}{f}\right)}_{\text{Mathieu equation}}\delta\phi_k = 0$$

EOM of axion

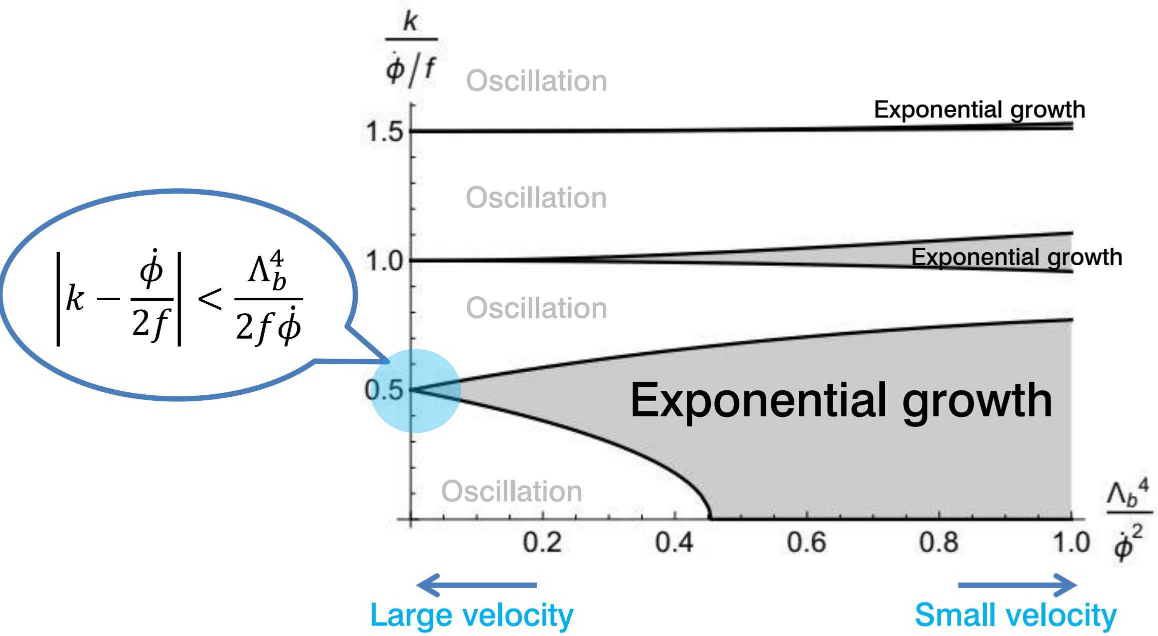


There exist resonant solutions for this.
It's like a swing!

$$\frac{d^2\delta\phi_k}{dt^2} + \left(k^2 - \frac{\Lambda_b^4}{f^2}\cos\frac{\dot{\phi}t}{f}\right)\delta\phi_k = 0$$

Mathieu equation

EOM of axion



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$$\frac{d^2\delta\phi_k}{dt^2} + \left(k^2 - \frac{\Lambda_b^4}{f^2}\cos\frac{\dot{\phi}t}{f}\right)\delta\phi_k = 0$$

Mathieu equation

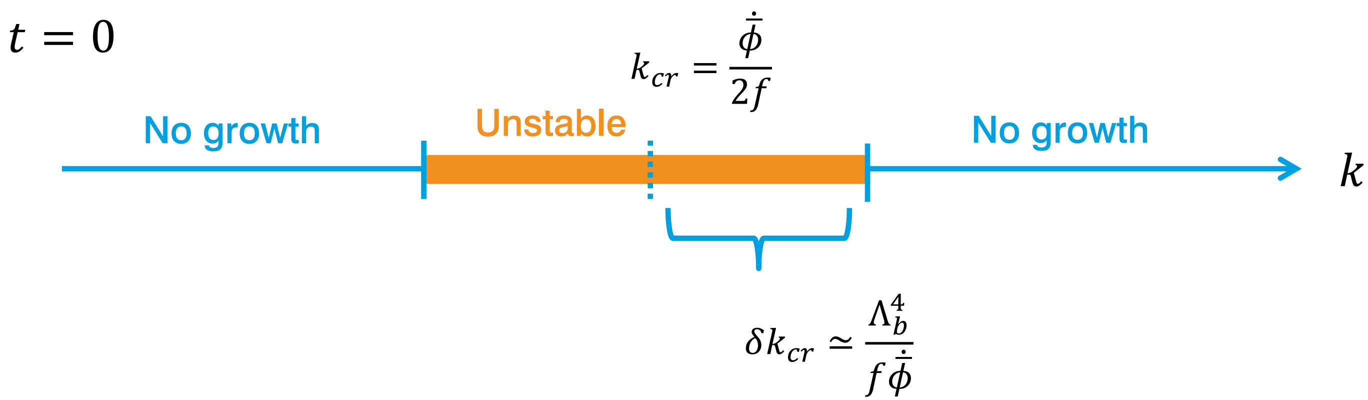
Growth of fluctuation

⇩

Back reaction to zeromode

Naïve estimation on back reaction

As long as $\dot{\phi}$ is constant, $\delta\phi_k \sim \exp\left(\frac{\Lambda_b^4 t}{f\dot{\phi}}\right)$ for $\left|k - \frac{\dot{\phi}}{2f}\right| < \frac{\Lambda_b^4}{2f\dot{\phi}}$



By using dimensional analysis

$$\rho_{fluc}(t) \sim k_{cr}^3 \delta k_{cr} \exp\left(\frac{\Lambda_b^4 t}{f\dot{\phi}}\right)$$

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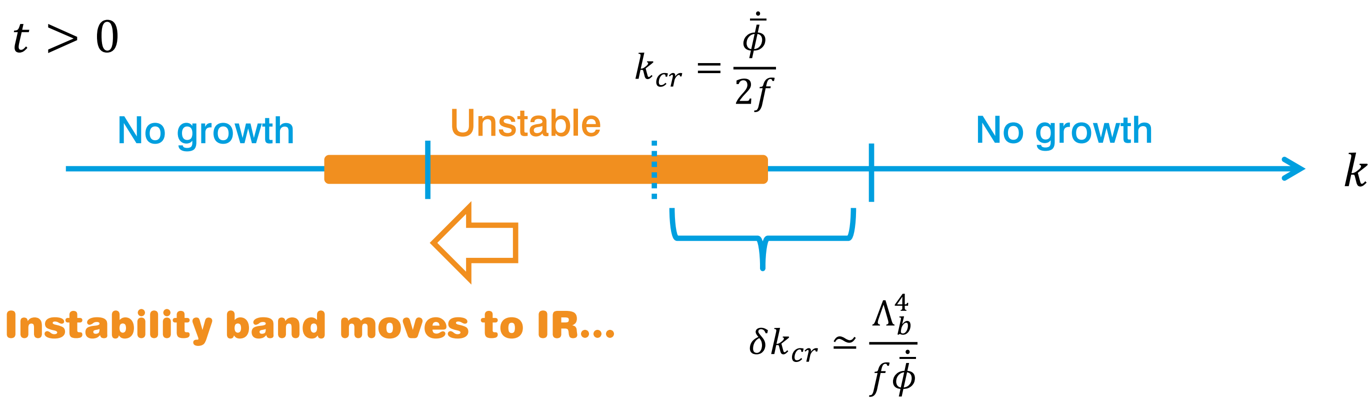

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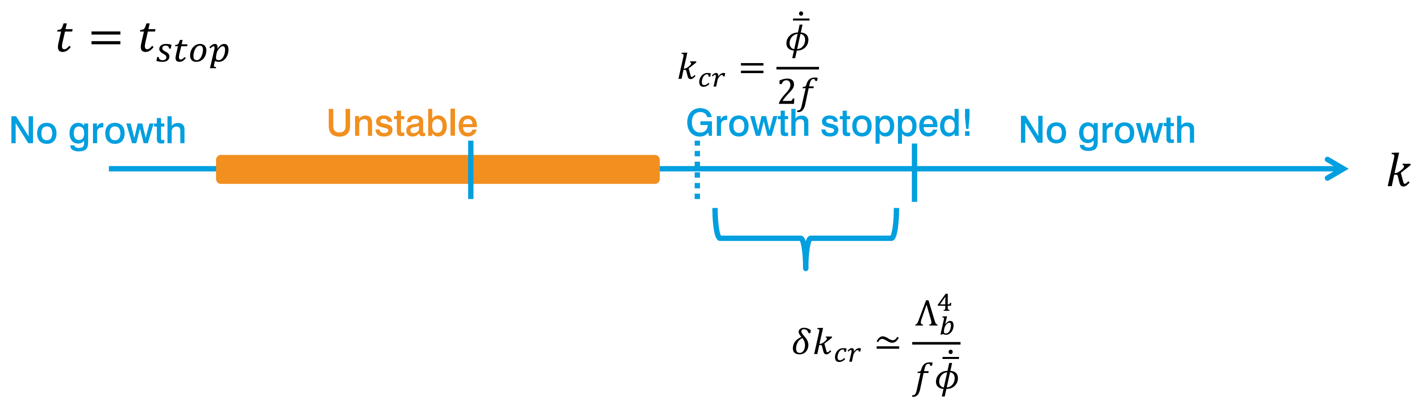


By using dimensional analysis

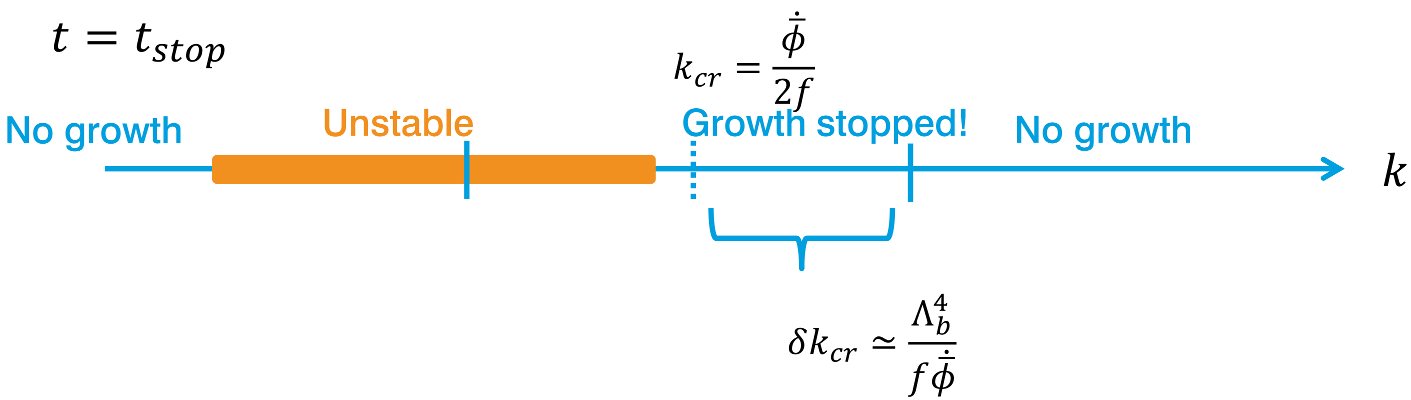
$$\rho_{fluc}(t) \sim k_{cr}^3 \delta k_{cr} \exp\left(\frac{\Lambda_b^4 t}{f\dot{\bar{\phi}}}\right)$$

The growth stops when (of mode with k=kcr)

$$\rho_{fluc}(t_{stop}) \sim \frac{1}{2}\dot{\bar{\phi}}^2 - \frac{1}{2}\left(\dot{\bar{\phi}} - 2f\delta k_{cr}\right)^2$$

Naïve estimation on back reaction

As long as $\dot{\phi}$ is constant, $\delta\phi_k \sim \exp\left(\frac{\Lambda_b^4 t}{f\dot{\phi}}\right)$ for $\left|k - \frac{\dot{\phi}}{2f}\right| < \frac{\Lambda_b^4}{2f\dot{\phi}}$



By using dimensional analysis

$$\rho_{fluc}(t) \sim k_{cr}^3 \delta k_{cr} \exp\left(\frac{\Lambda_b^4 t}{f\bar{\dot{\phi}}}\right)$$

of mode with k=kcr

The growth stops when

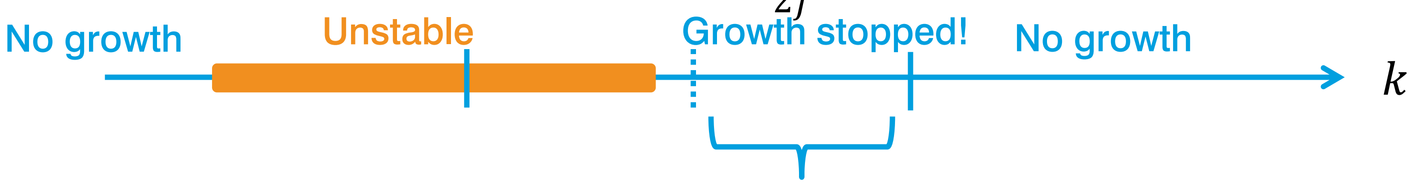
$$\rho_{fluc}(t_{stop}) \sim \bar{\dot{\phi}}^2 \times \frac{\delta k_{cr}}{k_{cr}}$$

Naïve estimation on back reaction

As long as $\dot{\phi}$ is constant, $\delta\phi_k \sim \exp\left(\frac{\Lambda_b^4 t}{f\dot{\phi}}\right)$ for $\left|k - \frac{\dot{\phi}}{2f}\right| < \frac{\Lambda_b^4}{2f\dot{\phi}}$

$t = t_{stop}$

$k_{cr} = \frac{\bar{\dot{\phi}}}{2f}$



$\delta k_{cr} \simeq \frac{\Lambda_b^4}{f\bar{\dot{\phi}}}$

By using dimensional analysis

$$\rho_{fluc}(t) \sim k_{cr}^3 \delta k_{cr} \exp\left(\frac{\Lambda_b^4 t}{f\bar{\dot{\phi}}}\right)$$

of mode with k=kcr

The growth stops when

$$\rho_{fluc}(t_{stop}) \sim \bar{\dot{\phi}}^2 \times \frac{\delta k_{cr}}{k_{cr}}$$

$$\Rightarrow \quad t_{stop} \sim \frac{f\bar{\dot{\phi}}}{\Lambda_b^4} \log \frac{f^4}{\bar{\dot{\phi}}^2}$$

Naïve estimation on back reaction

Time scale of growth of single mode :

$$t_{stop} \sim \frac{f\dot{\bar{\phi}}}{\Lambda_b^4}\log\frac{f^4}{\dot{\bar{\phi}}^2}$$

Energy stored in fluctuations :

$$\rho_{fluc}(t_{stop}) \sim \dot{\bar{\phi}}^2 \times \frac{\delta k_{cr}}{k_{cr}},$$

$$\Rightarrow \quad \frac{d}{dt}\dot{\phi}^2 \sim -\frac{\rho_{fluc}(t_{stop})}{t_{stop}} \sim -\frac{\Lambda_b^8}{f\dot{\phi}}\left(\log\frac{f^4}{\dot{\phi}^2}\right)^{-1}$$

$$\Rightarrow \quad \frac{d}{dt}\dot{\phi} \sim -\frac{\Lambda_b^8}{f\dot{\phi}^2}\left(\log\frac{f^4}{\dot{\phi}^2}\right)^{-1}$$

c.f.) WKB approx. with $\dot{\phi} \gg \Lambda_b^2$ gives $\frac{d\dot{\phi}}{dt} = -\frac{\pi}{2}\frac{\Lambda_b^8}{f\dot{\phi}^2}\left(\log\frac{32\pi^2 f^4}{\dot{\phi}^2}\right)^{-1}$

(see 1911.08472 for details)

Time scale of fragmentation :

$$\Delta t_{frag} \sim f\frac{\dot{\phi}_0^3}{\Lambda_b^8}\log\frac{f^4}{\dot{\phi}_0^2}$$

Field excursion:

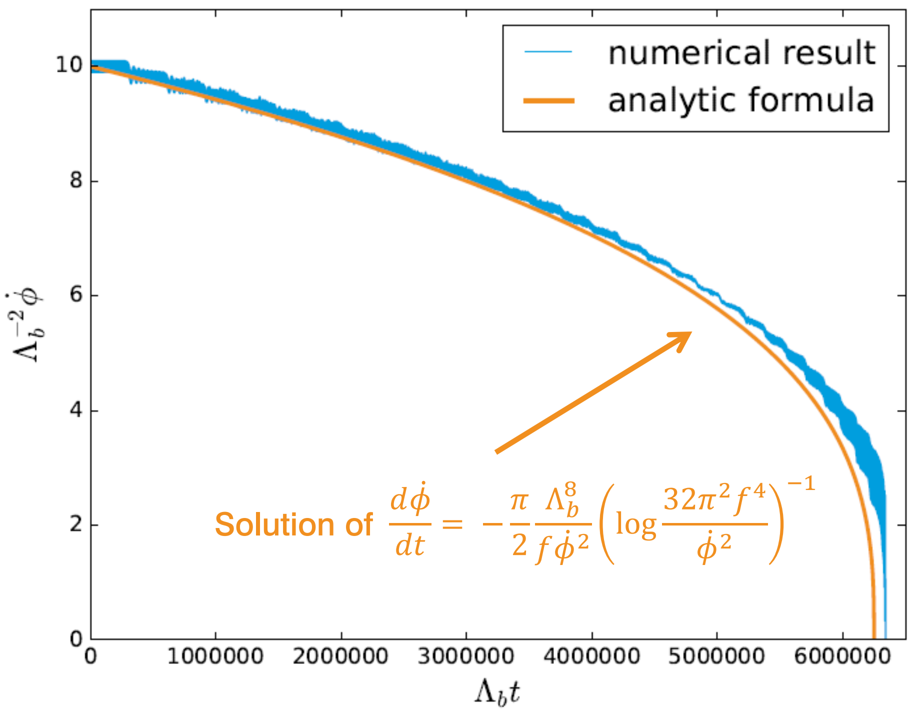
$$\Delta\phi_{frag} \sim \dot{\phi}_0\Delta t_{frag} \sim f\frac{\dot{\phi}_0^4}{\Lambda_b^8}\log\frac{f^4}{\dot{\phi}_0^2}$$

Numerical Example

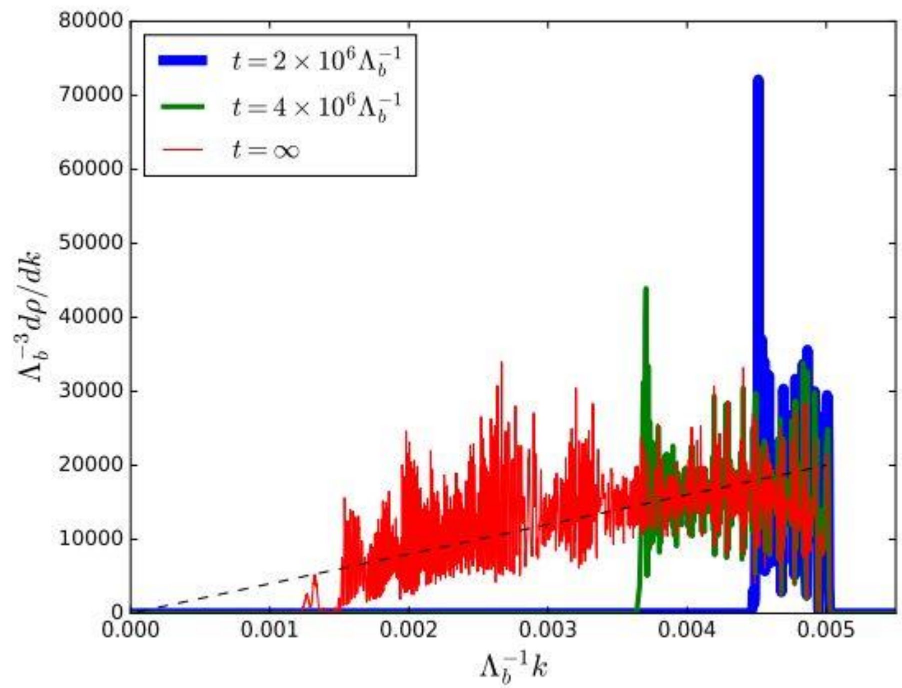
It works!

$$\frac{f}{\Lambda_b} = 1000$$

Time evolution of zeromode



Energy in fluctuation



[Fonseca, Morgante, RS, Servant (2019)]

Non-linear analysis

In perturbative analysis OK?

Inittial kinetic energy : $\dot{\phi}_0^2/2$

Typical wavenumber : $\dot{\phi}_0/f$

Energy conservation : $(\delta\phi)^2 \times \left(\dot{\phi}_0/f\right)^2 \sim \dot{\phi}_0{}^2$

⇨ Typical field variation : $\delta\phi \sim f$

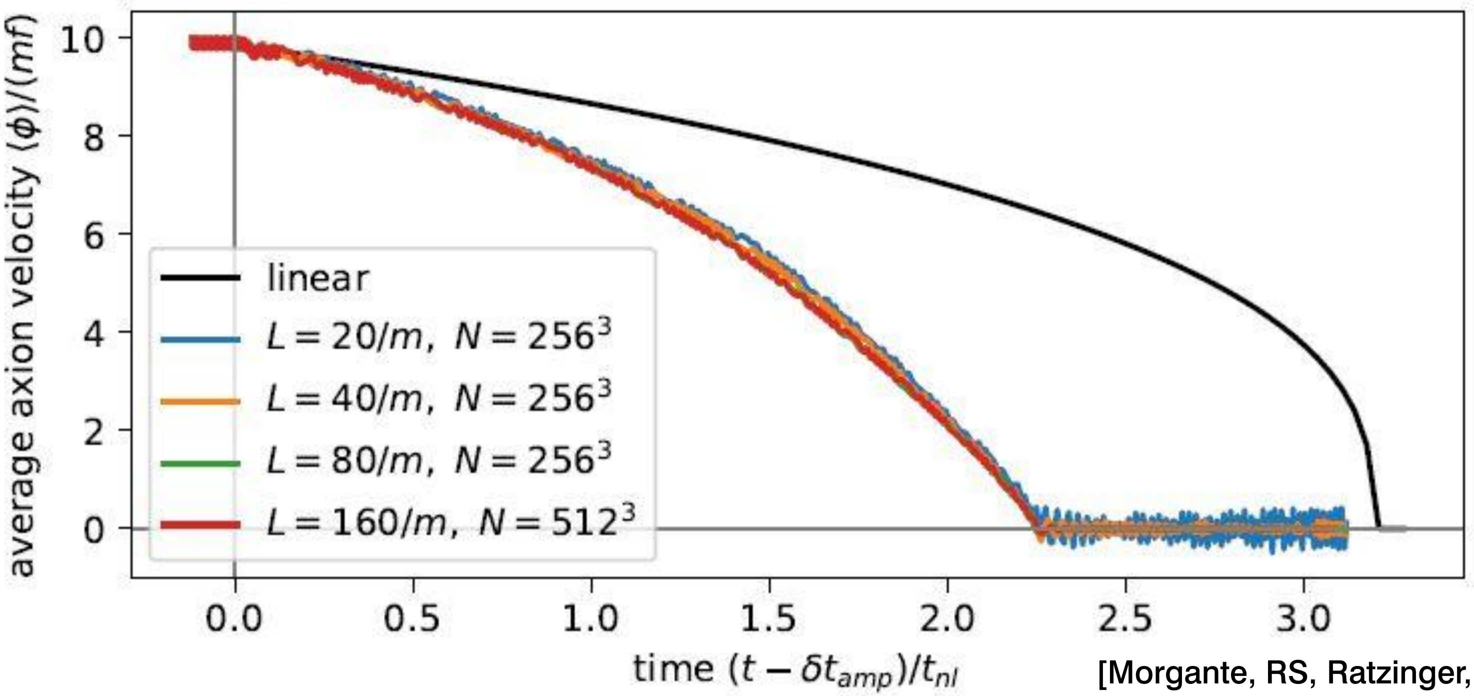
Classical lattice simulation

$$\ddot{\phi} = \nabla^2\phi + \frac{\Lambda_b^4}{f}\sin\frac{\phi}{f}$$

⇨

$$\begin{aligned}\frac{d^2\phi_{i,j,k}}{dt^2} =& \frac{1}{a^2}\left(\phi_{i+1,j,k} - 2\phi_{i,j,k} + \phi_{i-1,j,k}\right)\\ &+ \frac{1}{a^2}\left(\phi_{i,j+1,k} - 2\phi_{i,j,k} + \phi_{i,j-1,k}\right)\\ &+ \frac{1}{a^2}\left(\phi_{i,j,k+1} - 2\phi_{i,j,k} + \phi_{i,j,k-1}\right)\\ &+ \frac{\Lambda_b^4}{f}\sin\frac{\phi_{i,j,k}}{f}.\end{aligned}$$

Velocity of zeromode

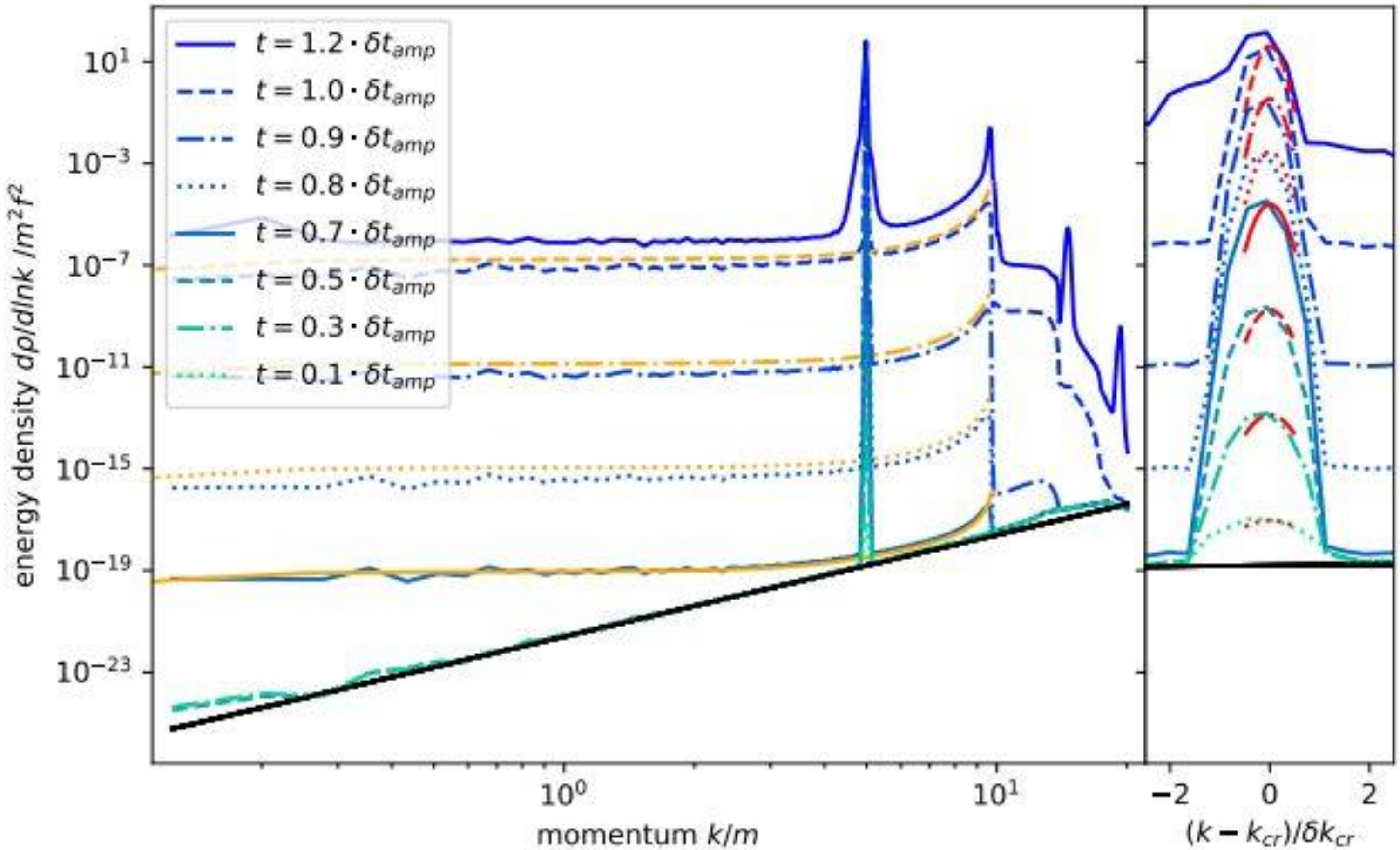


[Morgante, RS, Ratzinger, Stefanek (2021)]

$$\left(t_{nl} = \frac{f\dot{\phi}_0^3}{\Lambda_b^8}\right)$$

- Confirmed energy dissipation in non-linear calculation.
- Dissipation effect is stronger than linear analysis.

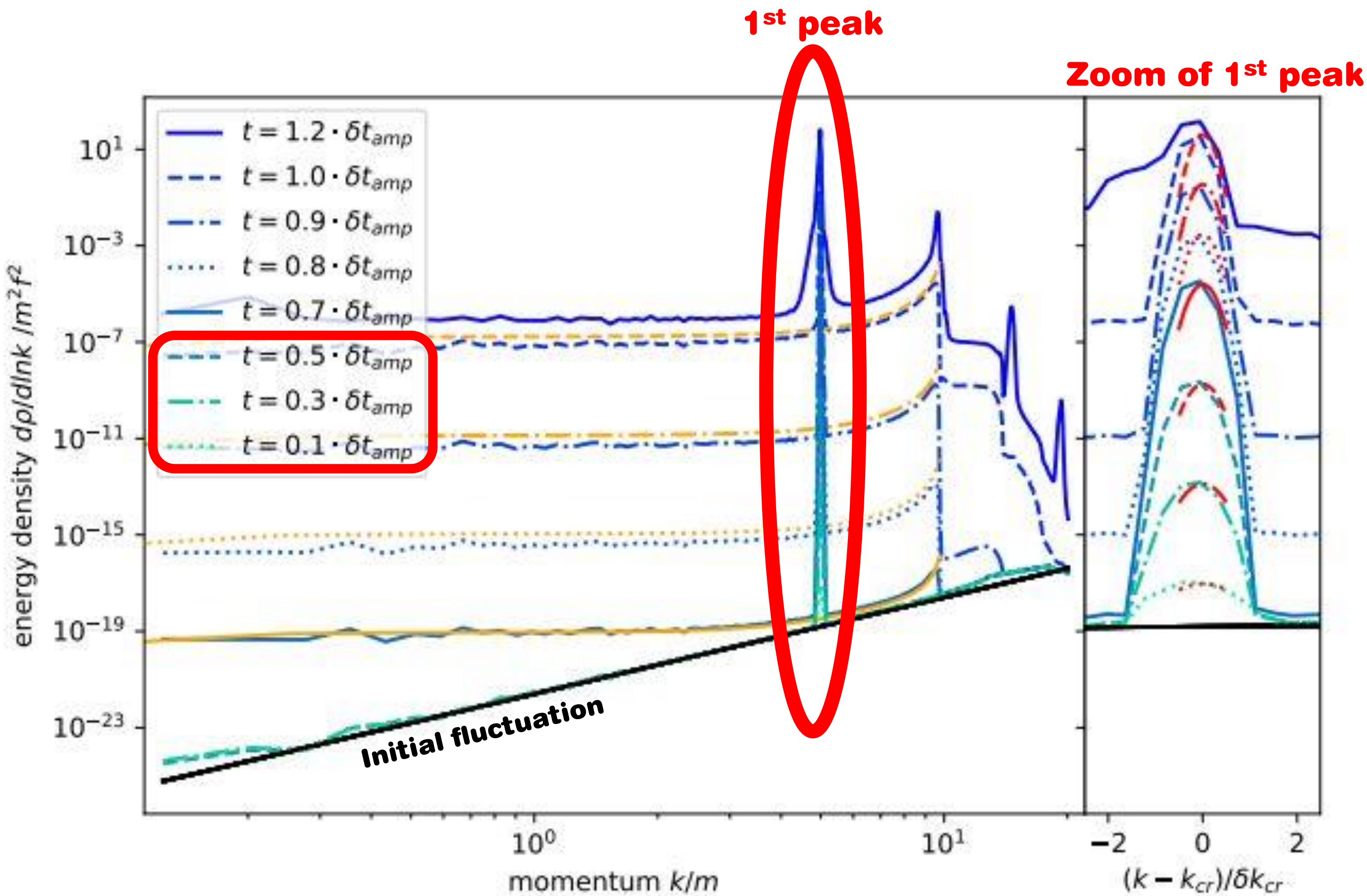
Growth of spectrum (early stage)



[Morgante, RS, Ratzinger, Stefanek (2021)]

$$\delta t_{amp} \equiv \frac{f\dot{\phi}}{\Lambda_b^4}\log\frac{16 f^4}{\dot{\phi}^2}$$

Growth of spectrum (early stage)



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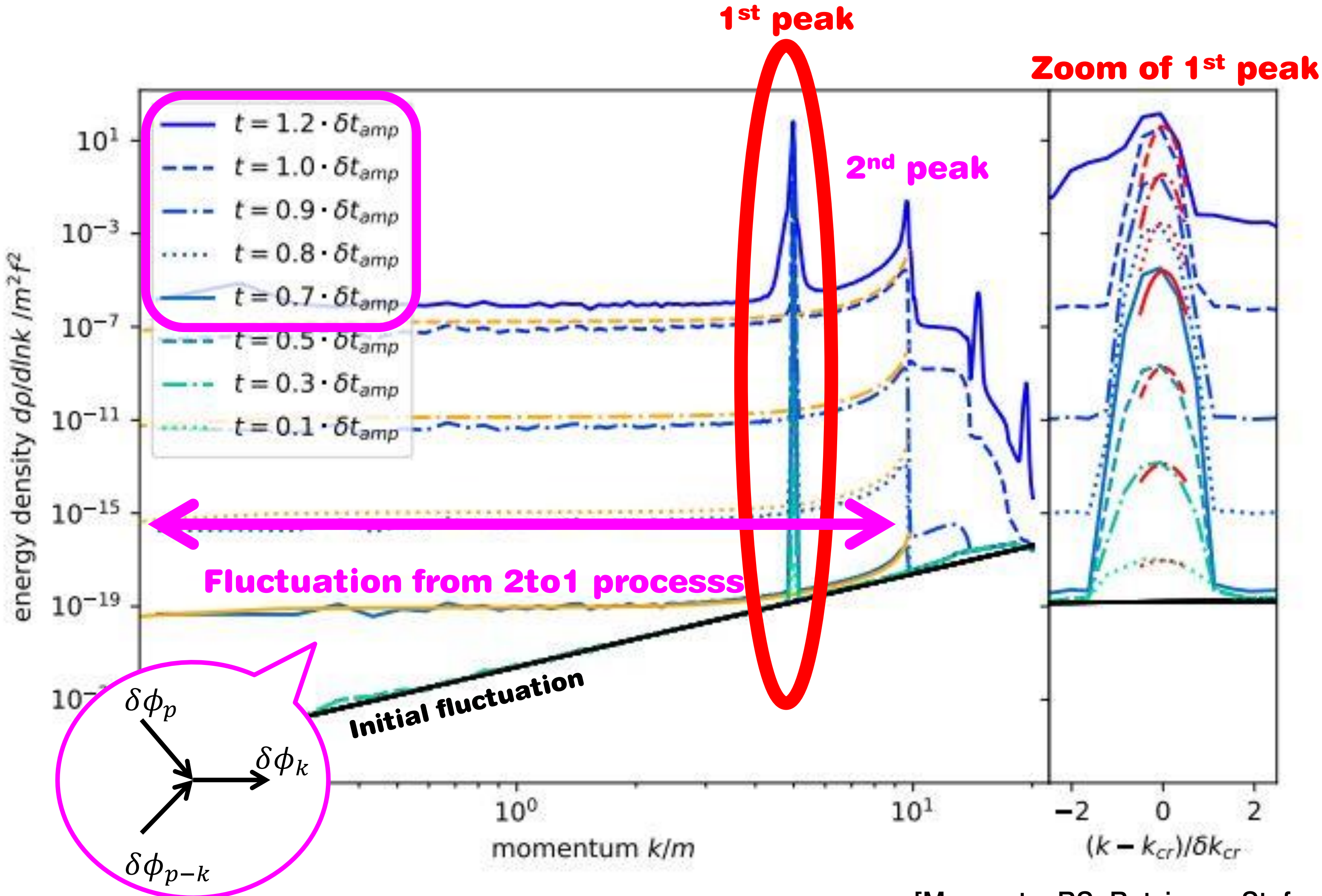

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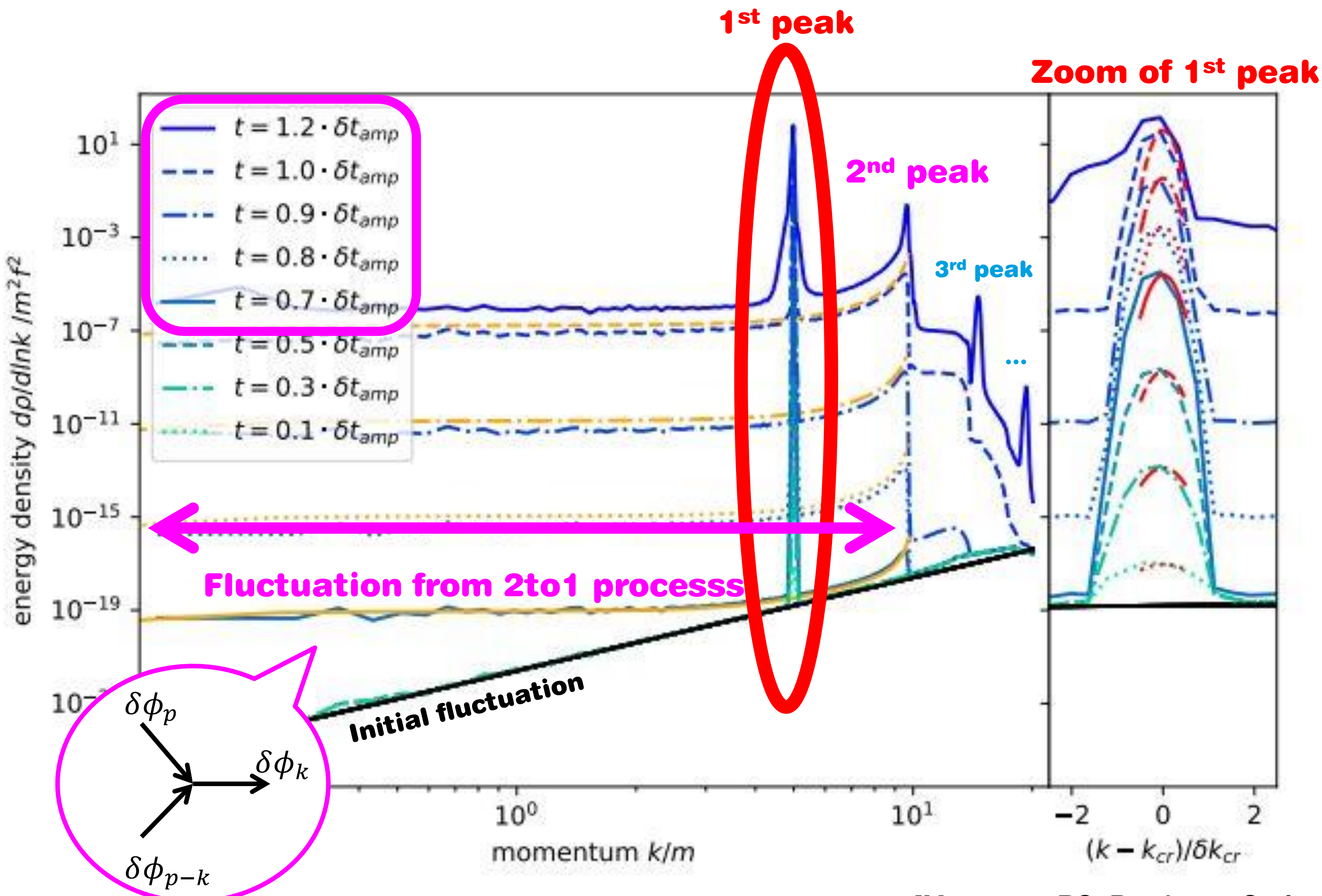
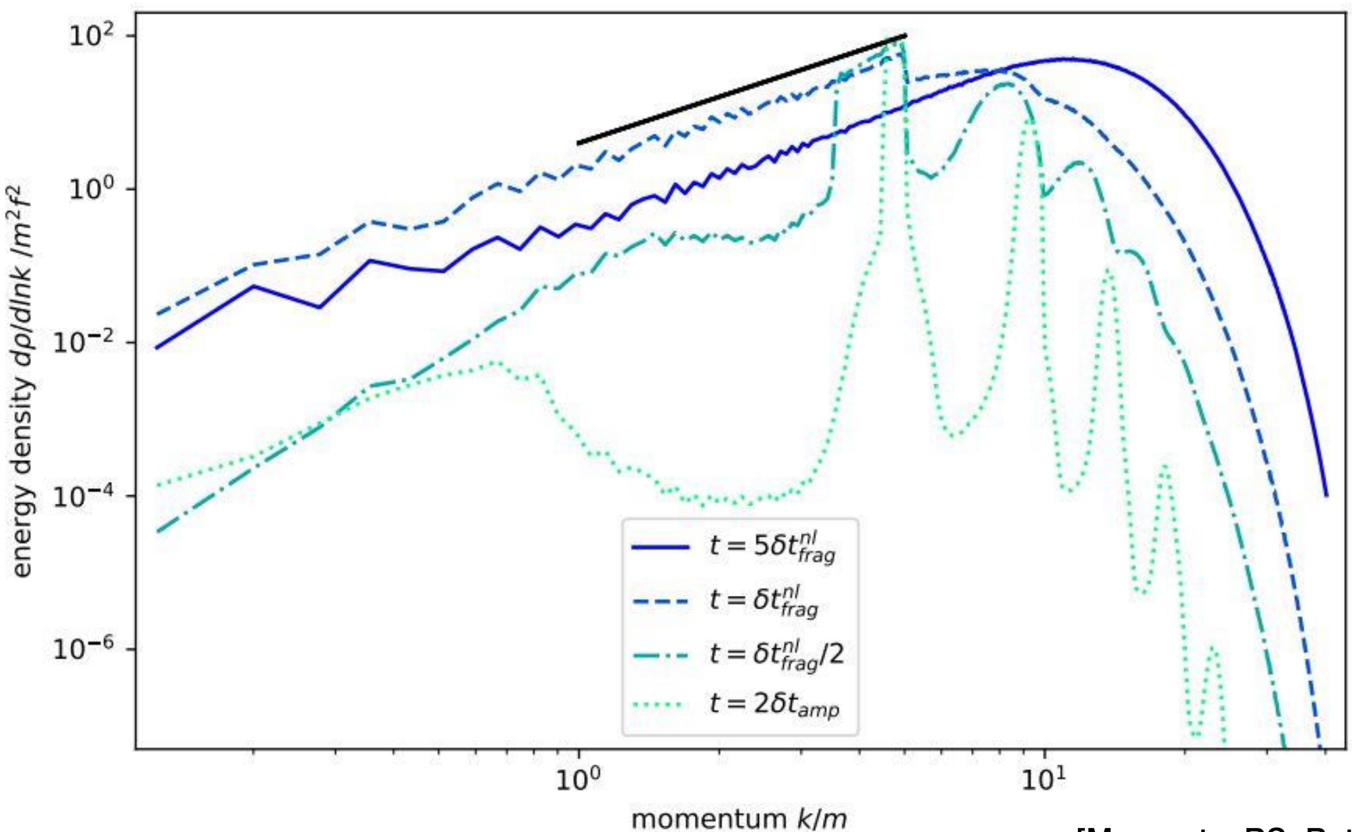


[Morgante, RS, Ratzinger, Stefanek (2021)]

$$\delta t_{amp} \equiv \frac{f\dot{\phi}}{\Lambda_b^4} \log \frac{16 f^4}{\dot{\phi}^2}$$

Growth of spectrum (late stage)



$$\delta t_{amp} \equiv \frac{f\dot{\phi}}{\Lambda_b^4}\log\frac{16f^4}{\dot{\phi}^2}$$

[Morgante, RS, Ratzinger, Stefanek (2021)]

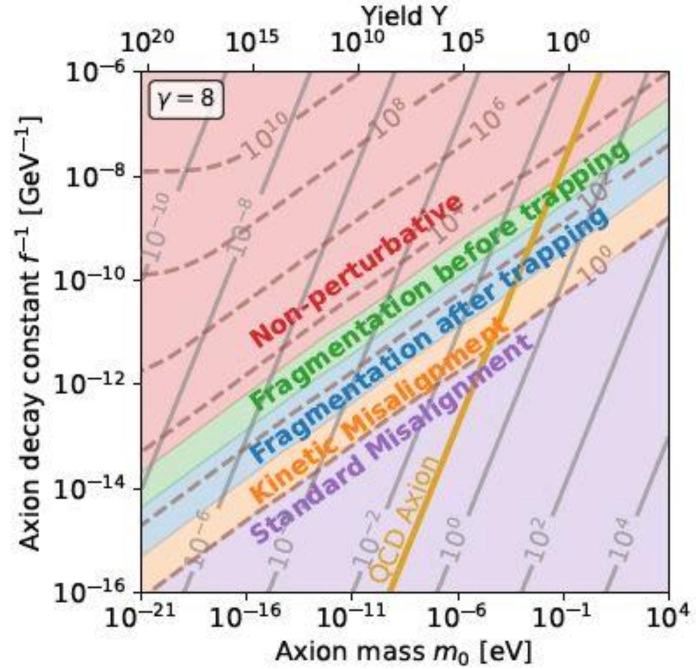
- We can see peak-like structure in the early stage
- The spectrum becomes broad
- Cascading towards UV (early stage of thermalization)

Physical implication?

ALP dark matter (Work in progress)

Fragmentation could happen
before axion starts to oscillate

- Fluctuation → axion minicluster?
- …



[Eröncel, RS, Sørensen, Servant, in preparation]

Relaxion scenario (1911.08473, Fonseca-Morgante-Sato-Servant)

Relaxion fragmentation can be a source of friction to stop relaxion.

Any other application?

Generic phenomena
- Periodic potential
- Large kinetic energy

Summary

- Large axion velocity $\rightarrow$ growth of fluctuation
- Zeromode kinetic energy dissipates into fluctuations

- Generic phenomena w/ periodic potential and large velocity.
- Applications

  - Relaxion
  - Axion dark matter scenario
  - …

Backup

References

Green, Kofman, Starobinsky, hep-ph/9808477

Parametric resonance from large amplitude

Flauger, McAllister, Pajer, Westphal, Xu, 0907.2916

Cosine + linear term, monodromy infl.

Jaeckel, Mehta, Witkowski, 1605.01367

Cosine + quadratic term, linear

Berges, Chatrchyan, Jaeckel, 1903.03116

Cosine + quadratic term, non-perturbative

Arvanitaki, Dimopoulos, Galanis, Lehner, Thompson, Van Tilburg, 1909.11665

Parametric resonance from large amplitude

Non-zero slope & Hubble expansion

What happens for non-zero μ^3 & non-zero H?

- Fragmentation $\ddot{\phi}_{frag} = -\frac{\pi\Lambda_b^8}{2f\dot{\phi}^2}\left(\log\frac{32\pi^2 f^4}{\dot{\phi}^2}\right)^{-1}$
- Acceleration by slope μ^3
- Hubble expansion $3H\dot{\phi}$

Fragmentation works if

- During inflation ($3H\dot{\phi} \sim \mu^3$)

  $3H\dot{\phi} < \sim|\ddot{\phi}_{frag}|$ If not, axion keeps rolling with slow-roll velocity

- Not during inflation ($3H\dot{\phi} \ll \mu^3$)

  $\mu^3 < \sim|\ddot{\phi}_{frag}|$ If not, axion is just accelerated by slope

Details on back reaction (1).

$$\phi(x,t) = \bar{\phi}(t) + \delta\phi(x,t) \qquad \delta\phi(x,t) = \int \frac{d^3k}{(2\pi)^3} a_k u_k(t) e^{ikx} + h.c.$$

Creation annihilation op : $\left[a_k, a^*_{k'}\right] = (2\pi)^3 \delta^{(3)}(k - k')$

Boundary condition for Wave function : $t \to -\infty \qquad u_k \to \frac{e^{-ikt}}{\sqrt{2k}}$

Bunch-Davies vacuum: $a|0\rangle = 0$

$$\Rightarrow \int \frac{dx^3}{V_{vol}} \langle \delta\phi(x)^2 \rangle = \int \frac{d^3k}{(2\pi)^3} |u_k|^2$$

Details on back reaction (2).

Asymptotic behavior of wave function u_k :

$$t \to -\infty \qquad u_k \to \frac{e^{-ikt}}{\sqrt{2k}}$$

$$t \to +\infty \qquad u_k \to \frac{1}{\sqrt{2k}} 2 \exp\left(-\frac{\pi}{4}\frac{\Lambda_b^8}{\dot{\phi}^2\ddot{\phi}f^4}\right) \times \cos kt$$

1) $d\rho/dt$ from u_k

$$\frac{d\rho}{dt} = -\left(\frac{d}{dt}\frac{\dot{\phi}}{2f}\right) \times \frac{4\pi k^2}{(2\pi)^3}\left(\frac{1}{2}|\dot{u}_k|^2 + \frac{1}{2}k^2|u_k|^2\right)$$

$$= -\frac{\dot{\phi}^3\ddot{\phi}}{32\pi^2 f^4}\exp\left(-\frac{\pi}{2}\frac{\Lambda_b^8}{\dot{\phi}^2\ddot{\phi}f^4}\right)$$

2) $d\rho/dt$ from definition

$$\frac{d\rho}{dt} = \dot{\phi}\ddot{\phi}$$

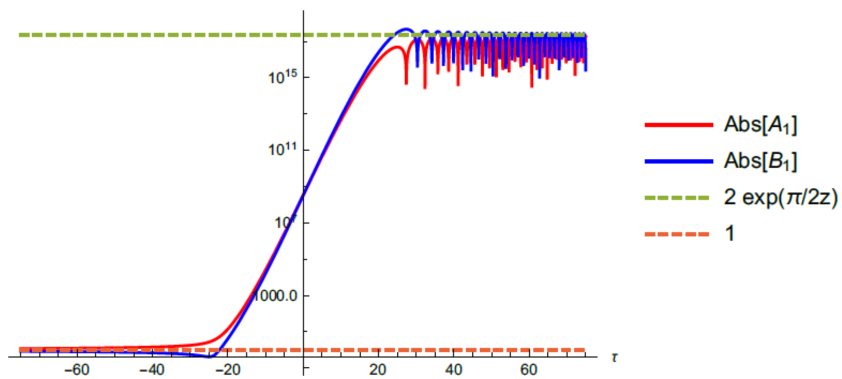
Consistency between 1) and 2) gives $\ddot{\phi} = -\frac{\pi\Lambda_b^8}{2\dot{\phi}^2 f}\left(\log\frac{32\pi^2 f^4}{\dot{\phi}^2}\right)^{-1}$

“Modified” Mathieu equation.

$$\frac{d^2u_k}{dt^2} + \left(k^2 + m^2\cos\frac{\bar\phi}{f}\right)u_k = 0 \qquad \text{with} \quad \dot{\bar\phi} = \dot{\bar\phi}_0 + \ddot{\bar\phi}t$$

Boundary condition at $t \to -\infty$: $\quad u_k \to \frac{e^{-ikt}}{\sqrt{2k}}$

Behavior at $\dot{\bar\phi}/f \simeq 2k$: $\quad u_k \simeq \frac{1}{\sqrt{2k}}\left(A(t)\cos\frac{\bar\phi}{2f} + B(t)\sin\frac{\bar\phi}{2f}\right)$



$$(1 + z\tau)A + \frac{dB}{d\tau} = 0$$
$$(1 - z\tau)B + \frac{dA}{d\tau} = 0$$

$$\tau = \frac{f m^2 t}{2\dot{\bar\phi}}$$
$$z = -\frac{2\dot{\bar\phi}_0\ddot{\bar\phi}}{f^3 m^4}$$

Asymptotic behavior at $t \to +\infty$: $\quad u_k \to \frac{1}{\sqrt{2k}} 2\exp\left(-\frac{\pi}{4}\frac{\Lambda_b^8}{\dot{\bar\phi}_0^2\ddot{\bar\phi}f^4}\right) \times \cos kt$

Setup.

> The original GKR (non-QCD) model [Graham, Kaplan, Rajendran (2015)]

$$V = -(\Lambda^2 - g'\Lambda\phi)H^2 + \lambda H^4 \quad + \quad g\Lambda^3\phi \quad + \quad \Lambda_b^4(H)\cos\frac{\phi}{f}$$

Higgs potential — slope — Wiggles

New strong dynamics gives wiggle

$$L_{eff} = m_N\bar{N}N + m_L\bar{L}L + yH\bar{N}L + \tilde{y}H^*\bar{L}N + \frac{\phi}{f}G'\widetilde{G'}$$

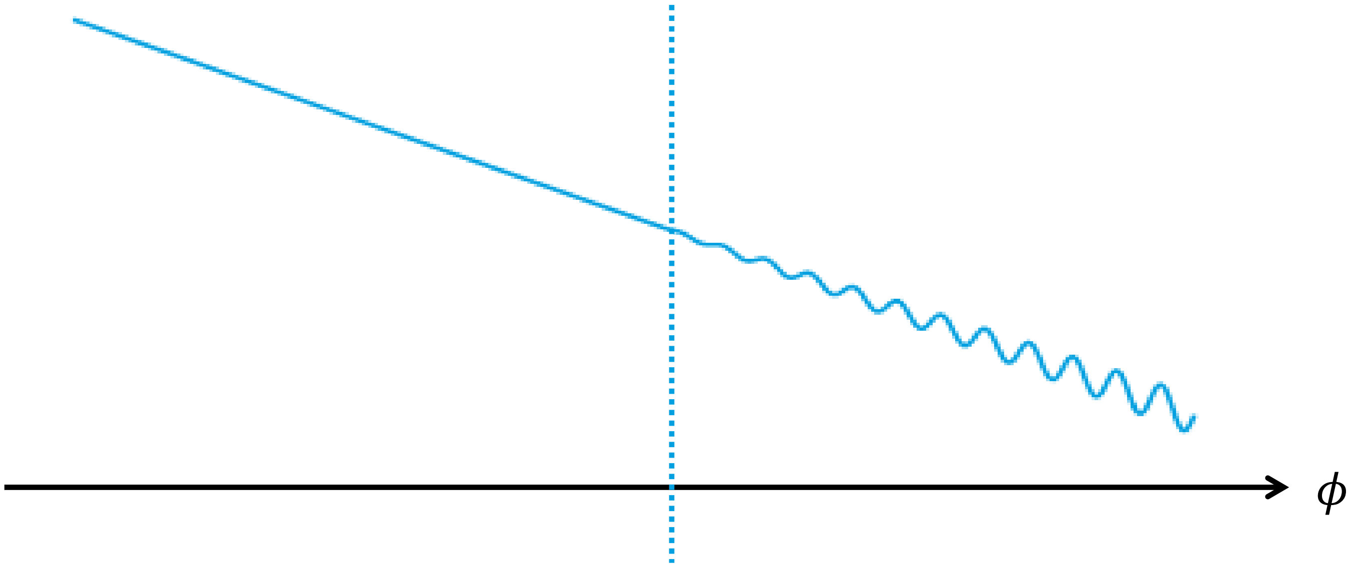
$$\Rightarrow \qquad V \simeq \frac{y\tilde{y}\Lambda_s^3}{m_L}|H|^2\cos\frac{\phi}{f}$$

$(m_L > \Lambda_s > m_N)$

Graham-Kaplan-Rajendran model

$$V = -(\Lambda^2 - g'\Lambda\phi)H^2 + \lambda H^4 \quad + \quad g\Lambda^3\phi \quad + \quad \Lambda_b^4(H)\cos\frac{\phi}{f}$$

Higgs potential | slope | Wiggles



$\Lambda^2 - g'\Lambda\phi < 0$	$\Lambda^2 - g'\Lambda\phi > 0$
$\Lambda_b(H) = 0$	$\Lambda_b(H) > 0$
$H = 0$	$\lvert H \rvert > 0$

Graham-Kaplan-Rajendran model

Friction is required to stop relaxion scanning (because of energy conservation)

Why relaxion stops?	when/where relaxion stops?
Hubble friction (original GKR model) Velocity $\dot{\phi}$ always tracks $V'/3H$	$g\Lambda^3 \sim \Lambda_b^4(\phi)/f$
Large barrier (mild Hubble friction) NEW Velocity $\dot{\phi}$ does not track $V'/3H$ But the average velocity is maintained as $\dot{\phi}_{SR} = g\Lambda^3/3H$	$\dot{\phi}_{SR}^2 \sim \Lambda_b^4(\phi)$
Axion Fragmentation NEW	$\Delta\phi_{frag} \sim f\,\frac{\dot{\phi}_0^4}{\Lambda_b^8}\log\frac{f^4}{\dot{\phi}_0^2}$

Parameter space.

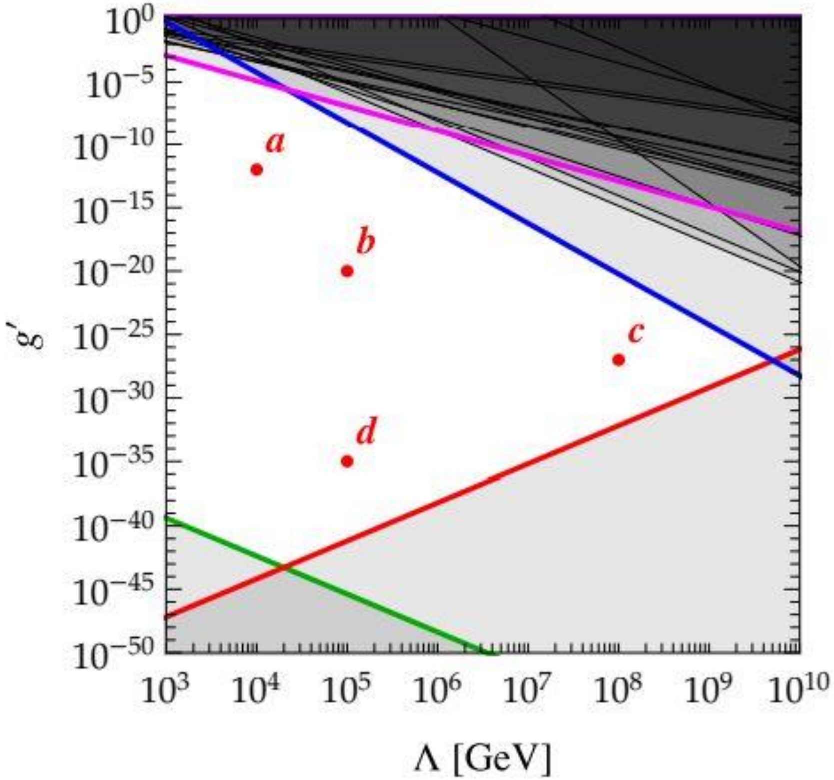
f is set to give correct EW scale :

$$f \simeq \frac{2\pi\lambda\Lambda_b^8 v_{ew}}{g'\Lambda\dot{\phi}_0^4 \times O(10)}$$

We also impose the following consistency conditions:

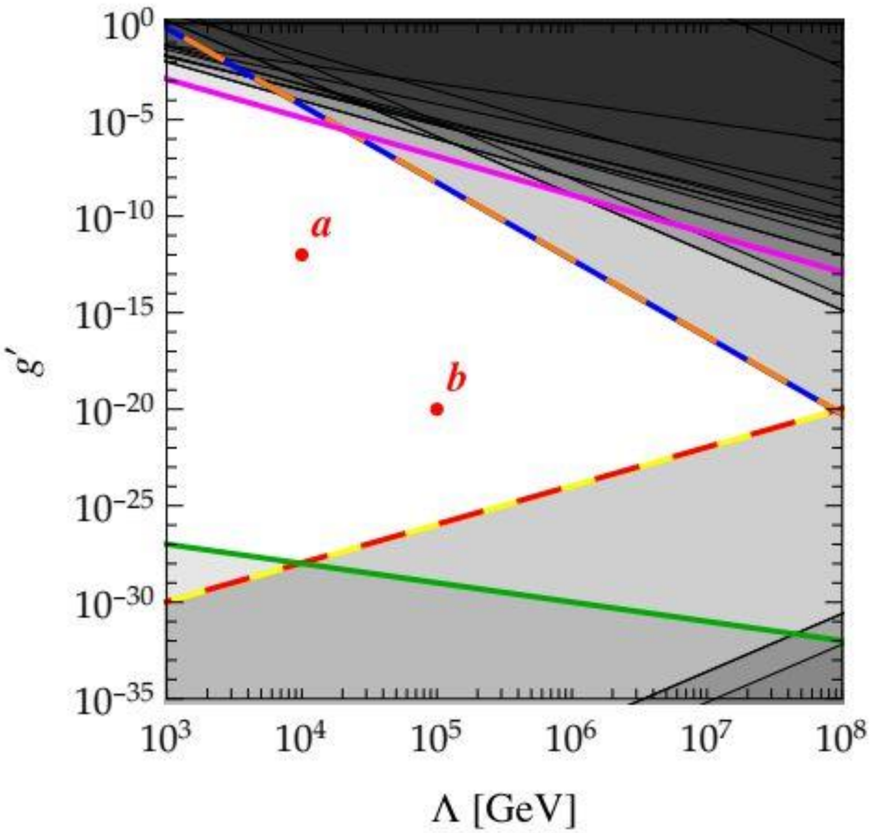
- Wiggle makes local minima $\qquad g\Lambda^3 < \sim \frac{\Lambda_b^4}{f}$
- Potential stability against rad. corr. $\qquad \Lambda_b < \sim v_{ew}$
- Consistency of EFT $\qquad f > \sim \Lambda$
- Initial kinetic energy is large enough $\qquad \frac{\dot{\phi}_0}{2} > \sim \Lambda_b^2$
- Particle production is fast enough $\qquad \Delta t_{pp} < \sim H^{-1}$

GKR (Hubble friction).



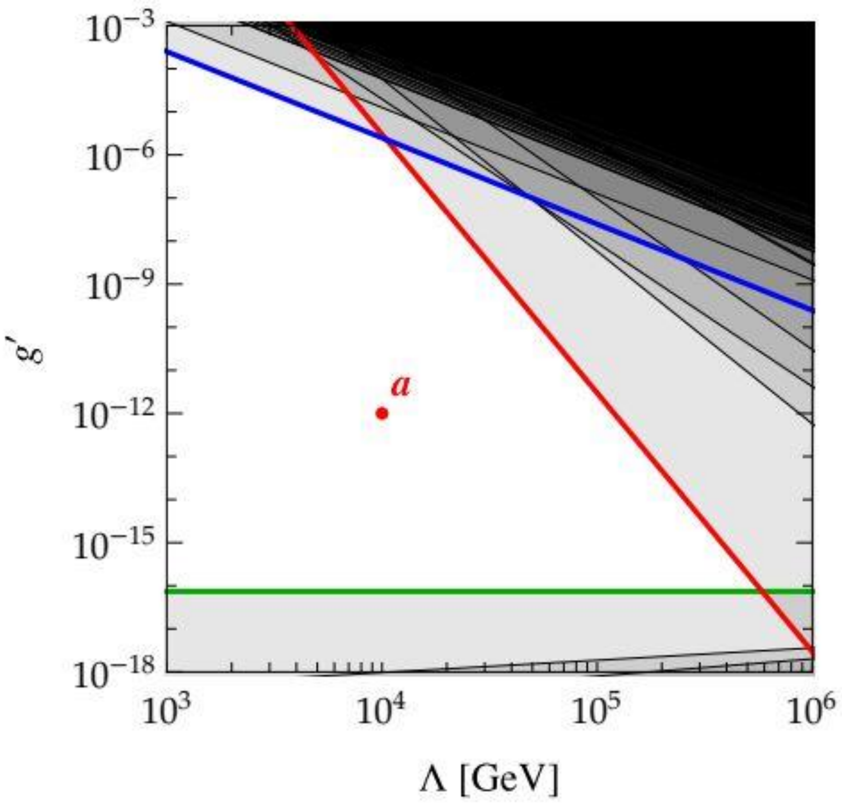
- Symmetry breaking pattern Eq. (3.5) and microscopic origin of the barriers Eq. (3.6)
- Classical rolling Eq. (3.16) and relaxion subdominant with respect to the inflaton Eq. (3.13)
- Reheating Eq. (3.9) and sub-Planckian decay constant Eq. (3.8)
- Eq. (3.5) and precision of the Higgs mass scanning Eq. (3.3)

GKR (large barrier).



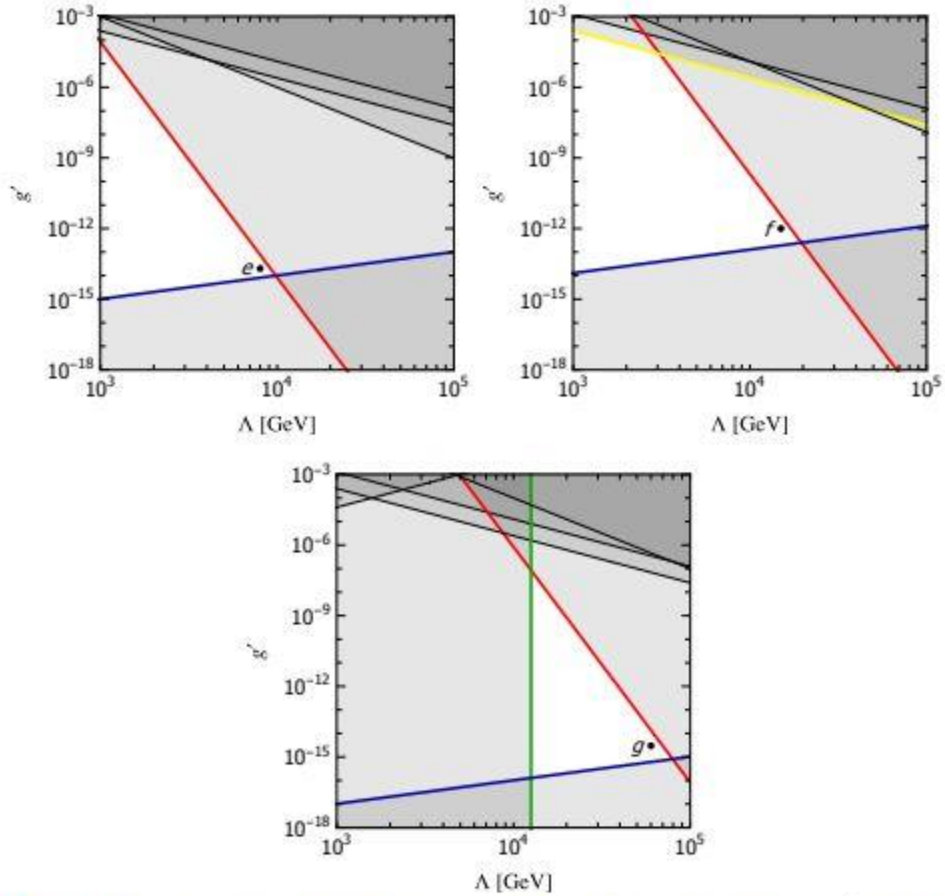
- (blue) Symmetry breaking pattern Eq. (3.5), going over 1 wiggle in less than 1 Hubble time Eq. (3.19) and microscopic origin of the barriers Eq. (3.6)
- (orange) Eq. (3.5), large barriers Eq. (3.4) and Eq. (3.6)
- (red) Eq. (3.5), Eq. (3.19) and relaxion subdominant with respect to inflaton Eq. (3.13)
- (yellow) Eq. (3.5), large barriers Eq. (3.4) and Eq. (3.13)
- (magenta) Eq. (3.5) and precision of the Higgs mass scanning Eq. (3.3)
- (green) Reheating Eq. (3.9) and Eq. (3.13)

GKR (relaxion fragmentation w/ inflation).



- Symmetry breaking pattern Eq. (3.5) and large velocity Eq. (3.22)
- Eq. (3.5), efficient fragmentation Eq. (2.16) and microscopic origin of the barriers Eq. (3.6)
- Relaxion subdominant with respect to inflaton Eq. (3.13) and Eq. (2.16)

GKR (relaxion fragmentation w/o inflation).



Microscopic origin of the barriers Eq. (3.6) and Symmetry breaking pattern Eq. (3.5)

Velocity larger than Λ_b^2 and Symmetry breaking pattern Eq. (3.5)

No slow-roll Eq. (3.26)

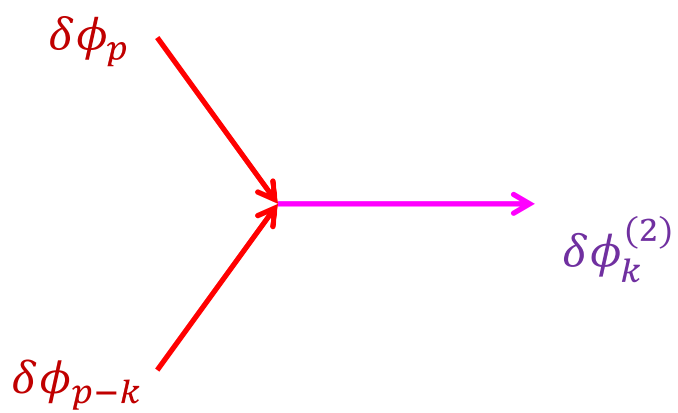
Slope can be neglected Eq. (2.17)

2 to 1 process

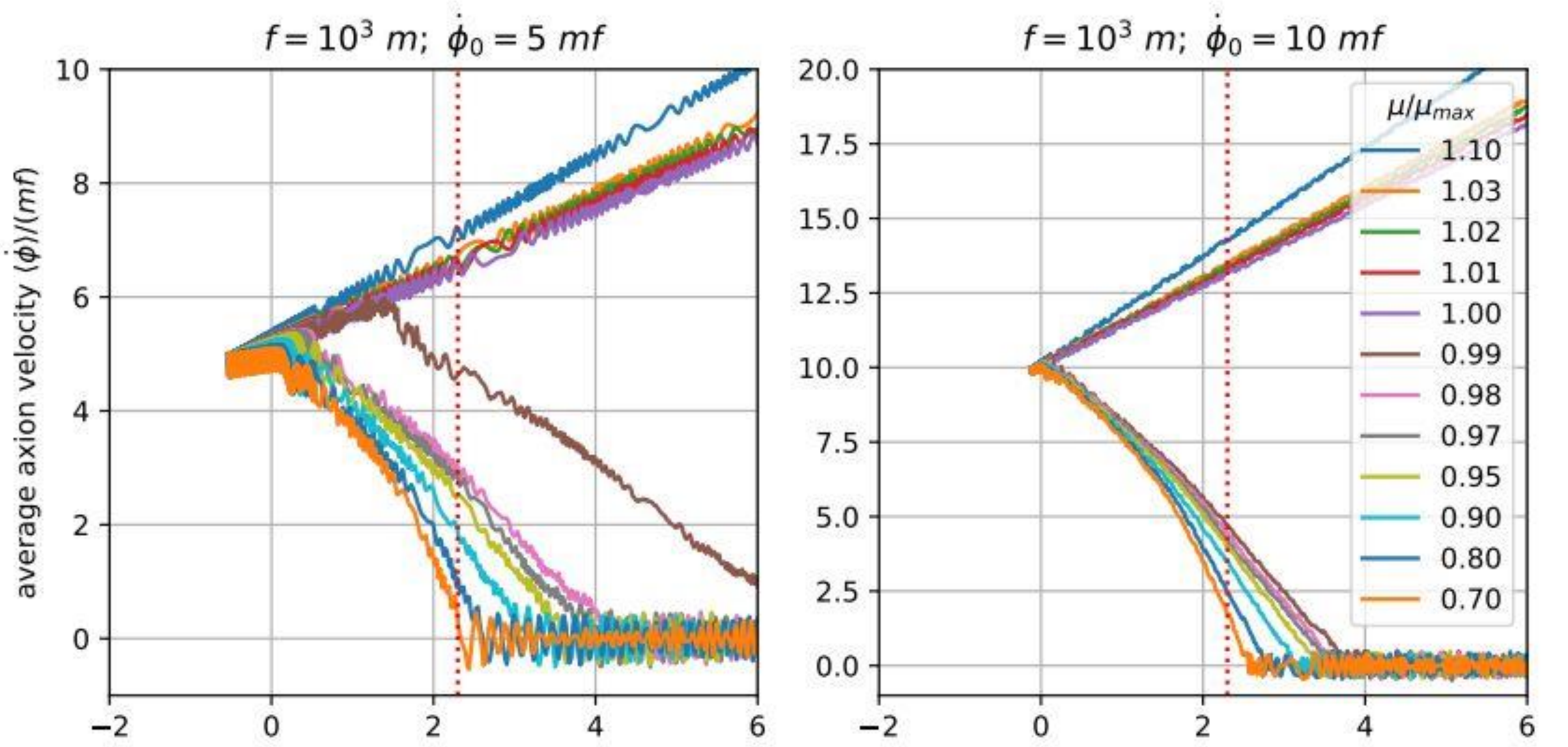
$$\phi(x,t) = \phi(t) + \delta\phi(x,t) + \delta\phi^{(2)}(x,t) + \ \ldots$$

$$\ddot{\phi} - \nabla^2\phi = V'(\phi) \quad \Rightarrow \quad \delta\ddot{\phi}^{(2)} + (k^2 + V'')\delta\phi^{(2)} = -\frac{1}{2}V'''\int d^3p \; \delta\phi_p \delta\phi_{k-p}$$

- $\delta\phi_p$ with $|p| = \dot{\phi}/2f$ is amplified by resonance
- $\delta\phi$ becomes source term for $\delta\phi^{(2)}$



Lattice calc. w/ slope term

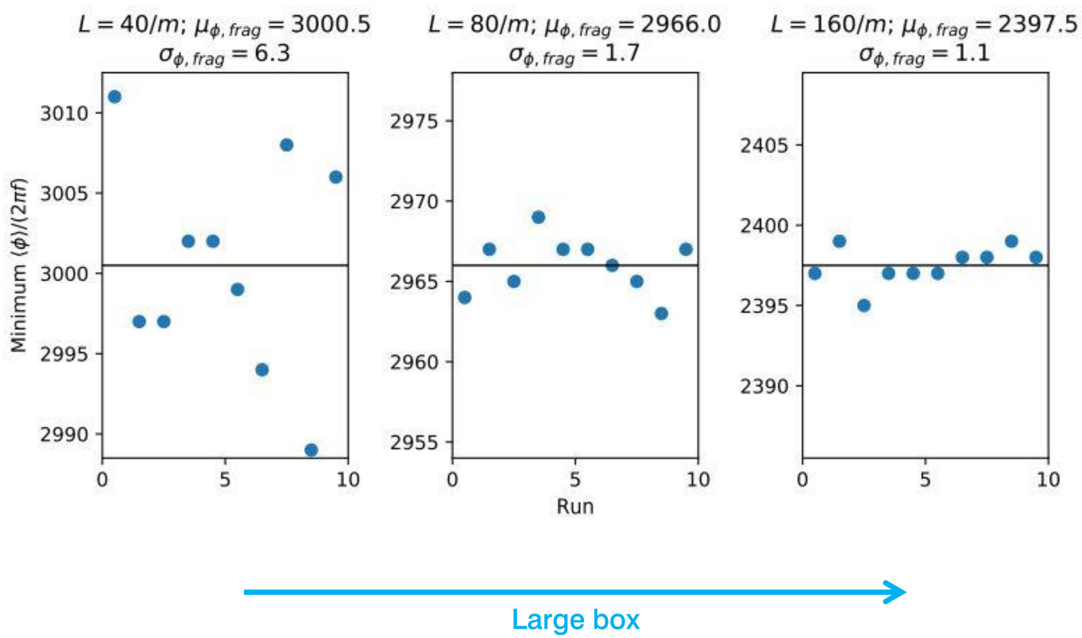


Domain wall?

Field variance after fragmentation is not so small : $\delta\phi \sim f$

Multiple run with finite size box

- $\delta\phi$ in multiple run = $\delta\phi$ of causally disconnected area
- Extrapolation to $V^{1/3} \approx \delta t_{\text{frag}}$



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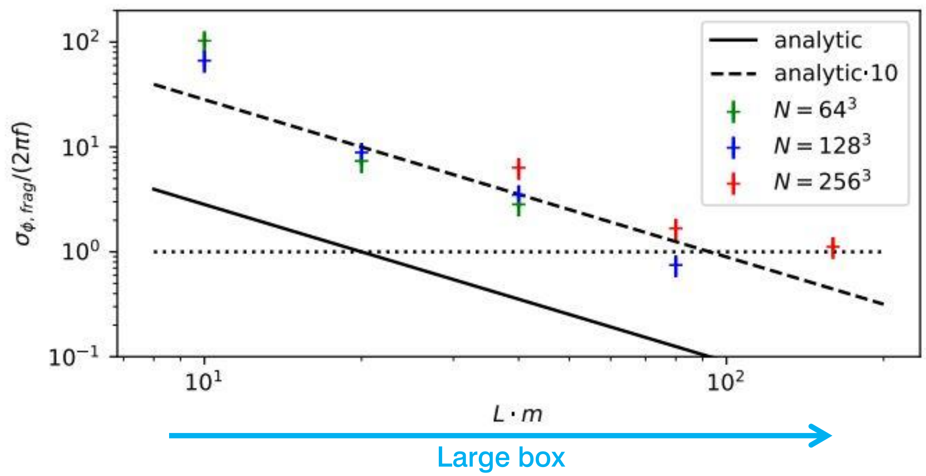


Empirical formula of variance:

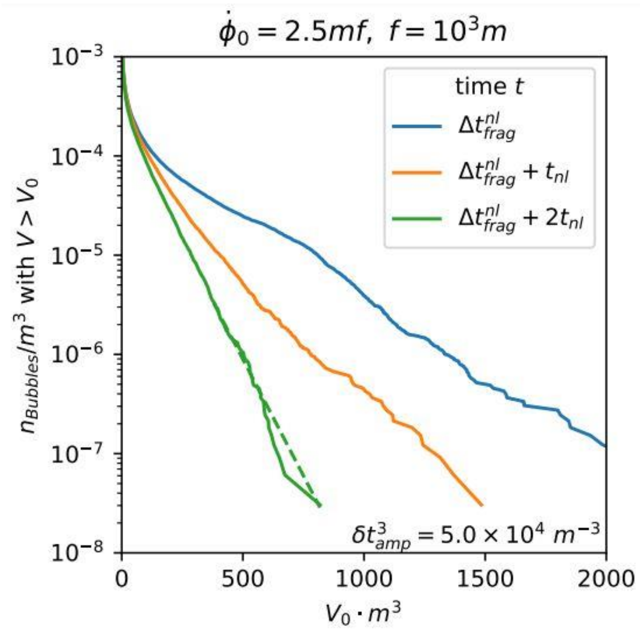
$$\frac{\delta\phi}{2\pi f} \sim O(10) \times V^{-1/2} \times \left(\frac{f\dot{\phi}_0}{\Lambda_b^2}\right)^{3/2}$$

Naïve extrapolation to $V^{1/3} \sim t_{amp}$: $\frac{\sigma}{2\pi f} \sim O(10) \times \left(\log \frac{8\pi f^2}{\dot{\phi}_0}\right)^{-\frac{3}{2}} \sim 0.01 - 0.1$

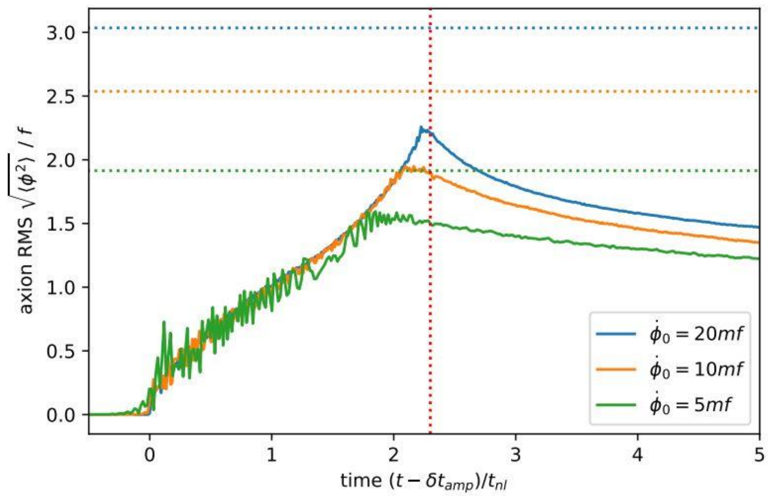
Domain wall formation probability is $\sim e^{-100} - e^{-10}$

Energy cascade into UV

Number counting of "bubble"



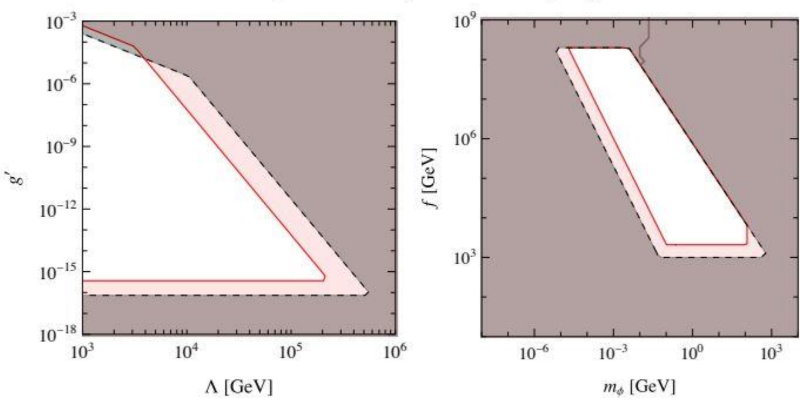
Time evolution of variance $\langle \delta\phi^2 \rangle$



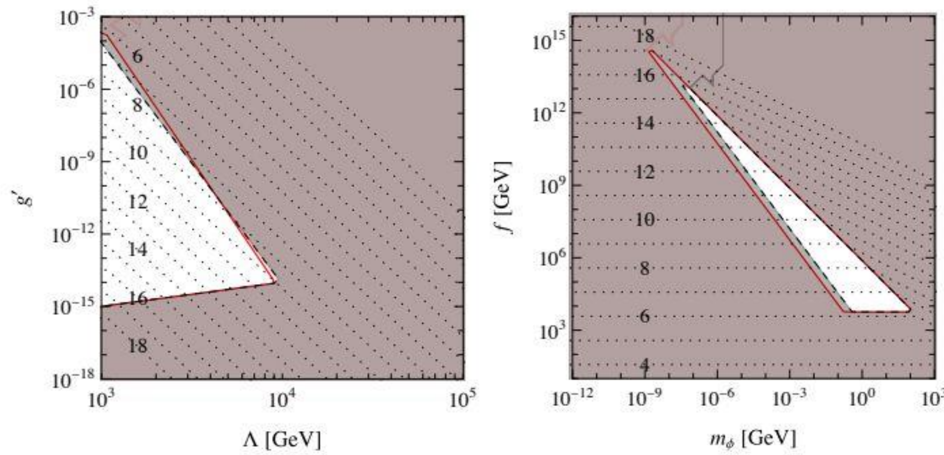
- Fluctuation with long wave-length is exponentially suppressed.
- The size of variance decreases in time.

How non-linear effect changes?

During inflation (Sec. 3.1 of [51])



After inflation, $g/g' = 1$ (Sec. 3.2 of [51])



After inflation, $g/g' = 1/(4\pi)^2$ (Sec. 3.2 of [51])