~~FAKE GUT based on SU(5) × SU(3)~~

More on Fake GUT

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Based on arXiv: 2205.01336

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Motivation

A mysterious fact in the SM

$$Q = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$L = \begin{pmatrix} \nu \\ e \end{pmatrix}$$

$$\bar{u} \quad \bar{d} \quad \bar{e}$$

→ **just enough!**

$$\bar{5} = \begin{pmatrix} \bar{d}_{\mathrm{R}} & \bar{d}_{\mathrm{G}} & \bar{d}_{\mathrm{B}} & e & -\nu \end{pmatrix},$$

$$10 = \begin{pmatrix} 0 & \bar{u}_{\mathrm{B}} & -\bar{u}_{\mathrm{G}} & u_{\mathrm{R}} & d_{\mathrm{R}} \\ -\bar{u}_{\mathrm{B}} & 0 & \bar{u}_{\mathrm{R}} & u_{\mathrm{G}} & d_{\mathrm{G}} \\ \bar{u}_{\mathrm{G}} & -\bar{u}_{\mathrm{R}} & 0 & u_{\mathrm{B}} & d_{\mathrm{B}} \\ -u_{\mathrm{R}} & -u_{\mathrm{G}} & -u_{\mathrm{B}} & 0 & \bar{e} \\ -d_{\mathrm{R}} & -d_{\mathrm{G}} & -d_{\mathrm{B}} & -\bar{e} & 0 \end{pmatrix}$$

This fact is quite remarkable!

In general, chiral fermions with SM gauge charges do not necessarily satisfy this property.

Motivation

An example of anomaly-free chiral fermion set

R. Foot et al, PRD 39 (1989) 3411-3424

	Q_1	Q_2	Q_3	Q_4	L_1	L_2	L_3	L_4
$SU(3)_c$	$\bar{3}$	3	3	$\bar{3}$	1	1	1	1
$SU(2)_L$	3	2	2	1	3	2	2	1
$U(1)_Y$	$-1/3$	$5/6$	$-1/6$	$-1/3$	1	$-3/2$	$-1/2$	1

Motivation

A mysterious fact in the SM

$$Q = \binom{u}{d}$$

$$L = \binom{\nu}{e}$$

$$\bar{u} \quad \bar{d} \quad \bar{e}$$

→ **just enough!**

$$\bar{5} = (\bar{d}_R \quad \bar{d}_G \quad \bar{d}_B \quad e \quad -\nu),$$

$$10 = \begin{pmatrix} 0 & \bar{u}_B & -\bar{u}_G & u_R & d_R \\ -\bar{u}_B & 0 & \bar{u}_R & u_G & d_G \\ \bar{u}_G & -\bar{u}_R & 0 & u_B & d_B \\ -u_R & -u_G & -u_B & 0 & \bar{e} \\ -d_R & -d_G & -d_B & -\bar{e} & 0 \end{pmatrix}$$

The SM cannot offer an explanation of this mystery.

→ **Fake GUT can do !**

M. Ibe et al. PRD100, 055024 (2019)

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1. Motivation
2. FAKE GUT
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2. FAKE GUT

Difference between SU(5) GUT and fake GUT

	group	fermions	SM matters
SU(5) GUT	SU(5) $\rightarrow$ SM	chiral $\bar{5}, 10$	embedded into chiral $\bar{5}, 10$
fake GUT	SU(5) $\times$ H $\rightarrow$ SM	chiral $\bar{5}, 10$ vector $\psi, \bar{\psi}$	**Not necessarily** embedded into chiral $\bar{5}, 10$ can be embedded into vector $\psi, \bar{\psi}$

2. FAKE GUT

The case where $\bar{\psi}$ has the same charge as L after SSB (1 generation case)

$$\mathcal{L}_{mass} = \psi_L (M_1 \quad M_2) \begin{pmatrix} \bar{5}_L \\ \bar{\psi}_L \end{pmatrix}$$

$$= \psi_L (M_1 \quad M_2) \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} L_M \\ L \end{pmatrix} = \psi_L (M_L \quad 0) \begin{pmatrix} L_M \\ L \end{pmatrix}$$

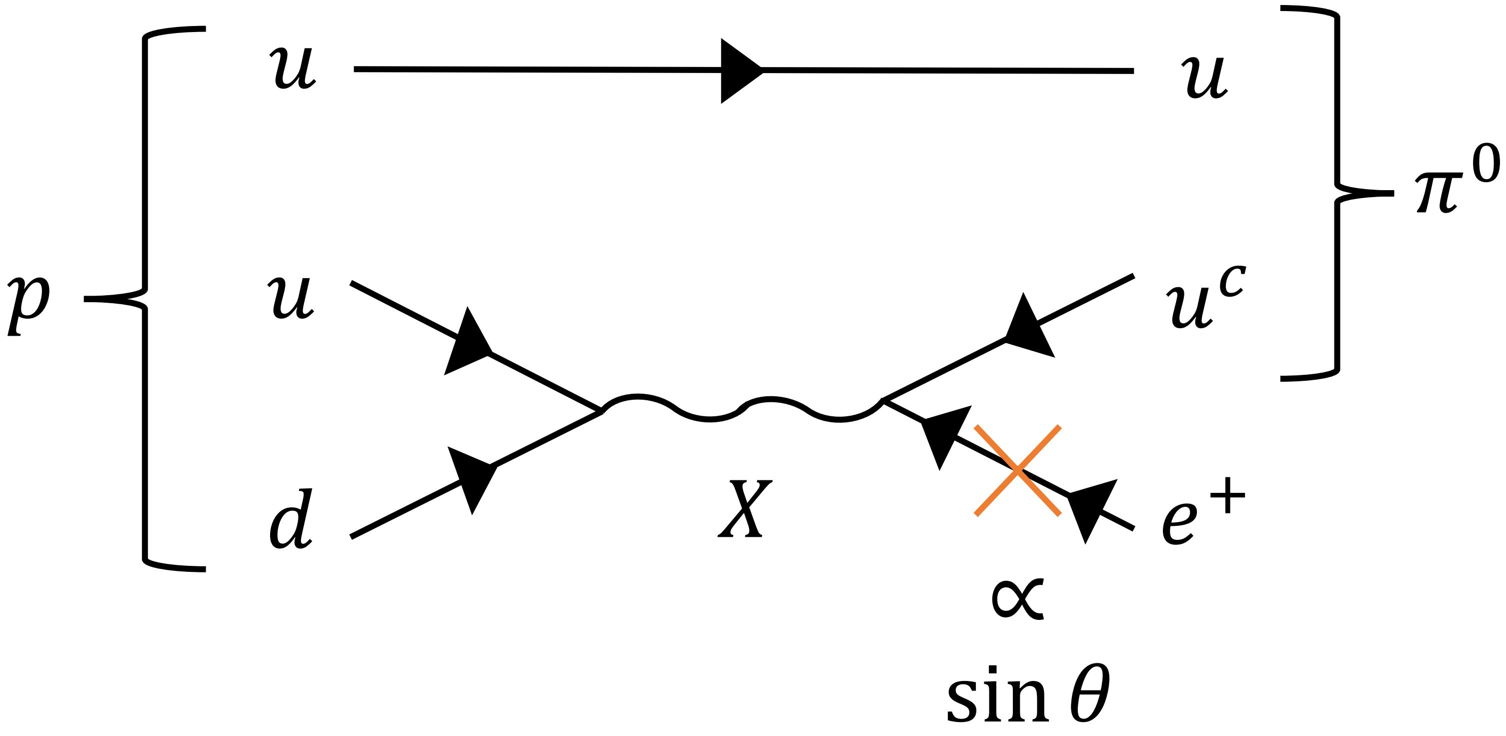
$\bar{5}_L$: lepton component in $\bar{5}$ $\quad$ L_M : heavy lepton

If $\bar{5}, 10, \psi$ and $\bar{\psi}$ exist at first, only SM fermions remain massless, even if they are not embedded into $\bar{5}$ and 10.

2. FAKE GUT

Diagram of proton decay ($p \to \pi^0 + e^+$)

If $\bar{5}_L$ and $10_{\bar{e}}$ contain $\sin\theta\, L$ and $\sin\theta\, \bar{e}$ respectively,



2. FAKE GUT

Proton lifetime ($\bar{5}_L$ and $10_{\bar{e}}$ contain $\sin\theta\, L$ and $\sin\theta\, \bar{e}$)

$$\tau(p \to \pi^0 + e^+) \cong 10^{26}\ \frac{1}{\sin^2\theta}\left(\frac{M_X/g_5}{10^{14}\ \mathrm{GeV}}\right)^4\ \mathrm{yrs}$$

$\longrightarrow$ $\sin\theta \lesssim 10^{-4}$ due to $\tau(p \to \pi^0 + e^+) > 2.4 \times 10^{34}\ \mathrm{yrs}$

A. Takenaka et al. (SK collaboration) PRD102, 112011 (2020)

Actually, mass terms of fermions have generation dependence.

2. FAKE GUT

An example of mixing of the 1st and 2nd generations

(consider SM lepton doublets case)

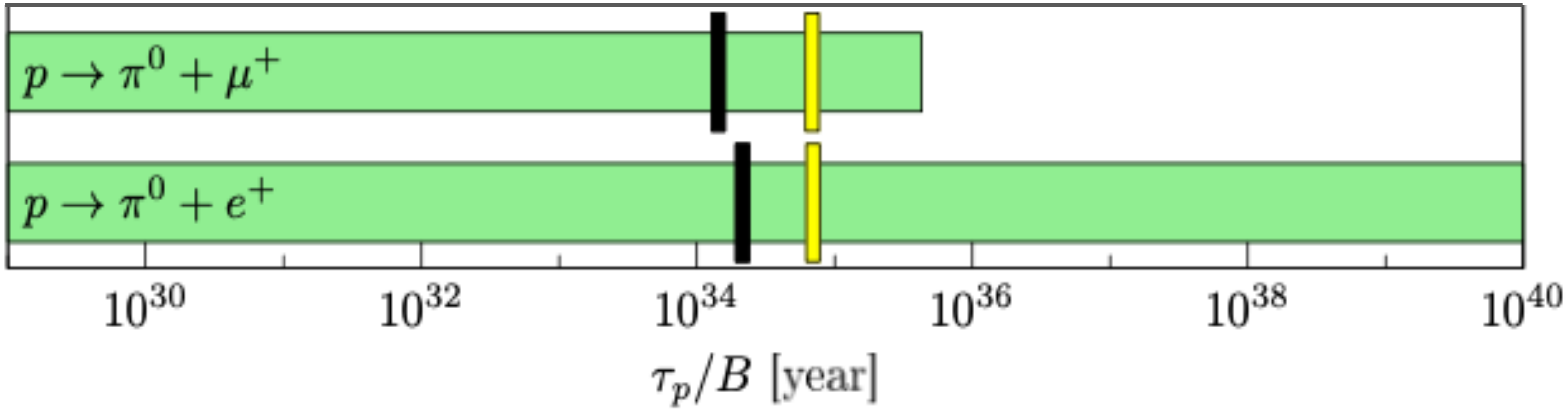
$$\mathcal{L}_{mass} = (\psi_{L1} \ \ \psi_{L2}) \begin{pmatrix} M & 0 & 0 & \delta \\ 0 & M & 0 & 0 \end{pmatrix} (\bar{5}_{1L} \ \ \bar{5}_{2L} \ \ \bar{\psi}_{L1} \ \ \bar{\psi}_{L2})^T$$
$$= \psi_{L1}(M \ \ \delta)(\bar{5}_{1L} \ \ \bar{\psi}_{L2})^T + M\psi_{L2}\bar{5}_{2L}$$

→ $\bar{\psi}_{L1} = L_e$. $\bar{5}_{2L}$ is a heavy fermion.

$\bar{5}_{1L}$ contains a little component of L_μ. $(M \gg \delta)$

2. FAKE GUT

Proton lifetime ($\bar{\mathbf{5}}_{1L}$ contains a little L_μ, $\bar{\mathbf{5}}_{2L}$ is heavy.
$\mathbf{10}_{1\bar{e}}$ contains a little $\bar{e}_\mu$, $\mathbf{10}_{2\bar{e}}$ is heavy.)



black : SK
yellow : HK

$$\tau(p \to \pi^0 + \mu^+) < \tau(p \to \pi^0 + e^+)$$

The fake GUT can make the different prediction than the conventional GUT.

Contents

3. Conclusion

- In the fake GUT, the SM fermions form $\bar{\mathbf{5}}$ and $\mathbf{10}$ of SU(5) at the low energy even if they are not embedded into $\bar{\mathbf{5}}$ and $\mathbf{10}$ of SU(5) at the high energy.

- In the fake GUT, the predictions of the nucleon decay rates and the branching ratios are different from those in the conventional GUT.

BACK UP

FAKE GUT

- $SU(5) \times H \xrightarrow{\text{Symmetry breaking}} SM$

- $SU(5) \supset (SU(3)_c , SU(2)_L , U(1)_Y)$

- Some of $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$ may be diagonal subgroup of $SU(5) \times H$

- Fermions

  Chiral fermion $\bar{\mathbf{5}}, \mathbf{10}$

  Vector-like fermion $\psi, \bar{\psi}$

FAKE GUT

- SU(5) × H —Symmetry breaking→ SM

- Fermions

Chiral fermion $\bar{\mathbf{5}}, \mathbf{10}$

Vector-like fermion $\psi, \bar{\psi}$

In the SU(5) GUT, all the SM fermions are contained in $\bar{\mathbf{5}}$ and $\mathbf{10}$.

FAKE GUT

- $SU(5) \times H$ —Symmetry breaking→ SM

- Fermions

Chiral fermion $\bar{5}, \mathbf{10}$

Vector-like fermion $\psi, \bar{\psi}$

In the fake GUT, the SM fermions can be contained in vector-like fermions.

$SU(5) \times U(2)_H$ model

Fermions ($SU(5)$, $SU(2)_H$,$U(1)_H$)

$\bar{5} : (\bar{5}, 1, 0)$ $\quad$ $10 : (10, 1, 0)$

$L_H : (1, 2, -1/2)$ $\quad$ $\bar{L}_H : (1, 2, 1/2)$

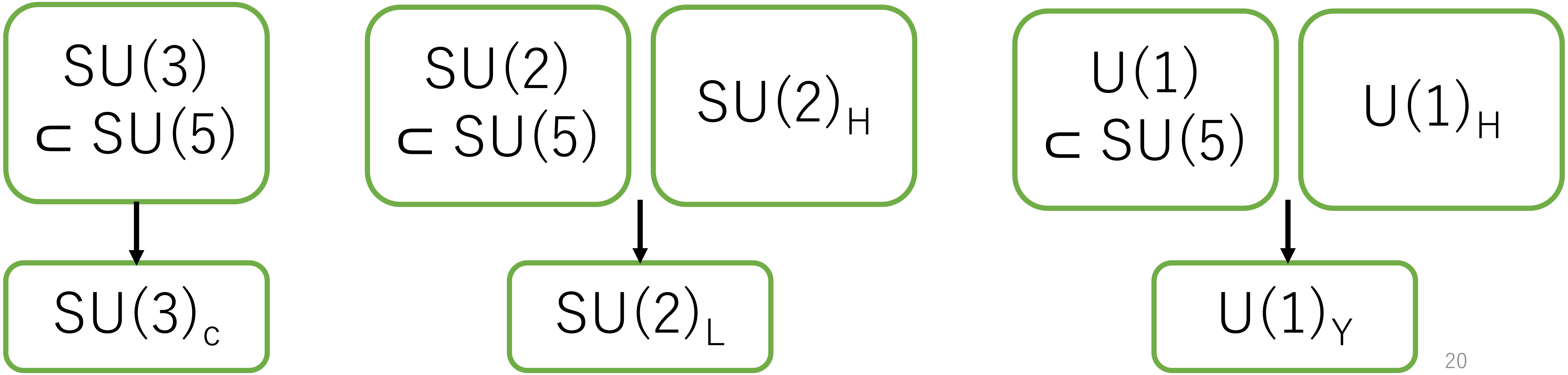
$E_H : (1, 1, -1)$ $\quad$ $\bar{E}_H : (1, 1, 1)$

- As we will show later, SM leptons are mostly contained in L_H and $\bar{E}_H$.
- SM quarks are all contained in $\bar{5}$ and 10.

$\mathrm{SU(5)} \times \mathrm{U(2)_H}$ model

Scalar ϕ_2 : $(5, 2, -1/2)$ of ($\mathrm{SU(5)}$, $\mathrm{SU(2)_H}$,$\mathrm{U(1)_H}$)

$$\langle \phi_2 \rangle = \left.\begin{pmatrix} \underbrace{\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix}}_{\mathrm{SU(3)}} & \underbrace{\begin{matrix} v_2 & 0 \\ 0 & v_2 \end{matrix}}_{\mathrm{SU(2)}} \end{pmatrix}\right\} \mathrm{SU(2)_H}$$



$SU(5) \times U(2)_H$ model

Lagrangian

$$\mathcal{L} = m_L L_H \bar{L}_H + \lambda_L \bar{5} \phi_2 \bar{L}_H + m_E E_H \bar{E}_H + \frac{\lambda_E}{\Lambda} E_H \phi_2^\dagger \phi_2^\dagger 10$$

(E_H and $\bar{E}_H$ are omitted) ↓

$$\bar{L}_H \left(\lambda_L \frac{v}{\sqrt{2}} \quad m_L \right) \begin{pmatrix} \bar{5}_L \\ L_H \end{pmatrix} \longrightarrow \bar{L}_H \left(M_L \quad 0 \right) \begin{pmatrix} L_M \\ L \end{pmatrix}$$

Only SM fermions remain massless at the low energy.

$\bar{5}_{\bar{d}}$ ⟶ quark

L_M —— $\bar{L}_H$ ($\langle \Phi \rangle, m_L$)

L ⟶ lepton

$SU(5) \times U(2)_H$ model

Mixing of lepton components

L : SM lepton $\qquad$ L_M : heavy lepton

$$\begin{pmatrix} \bar{5}_L \\ L_H \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} L_M \\ L \end{pmatrix} \qquad \tan\theta = \frac{m_L}{\lambda_L v}$$

$SU(5) \times U(2)_H$ model

Yukawa interactions

We consider a case one SM Higgs remains in the low energy.

Scalar containing the SM Higgs

$$H_5 : (5, 1, 0)$$

$$H_2 : (1, 2, 1/2)$$

$$H_5 = \begin{pmatrix} h_5^{color} \\ h_5^{SM} \end{pmatrix}$$

$$H_2 = h_2^{SM}$$

Higgs mixing term

$$\mathcal{L}_{52\,mix} = \mu_{mix} H_2 \phi_2 H_5^* + h.c.$$

$$h^{SM} = \cos\theta_h \, h_2^{SM} - \sin\theta_h \, h_5^{SM}$$

$SU(5) \times U(2)_H$ model

Yukawa interactions

$$\mathcal{L}_{YQ} = -(y_5)_{ij}\overline{5}_i\, 10_j H_5^* - (y_{10})_{ij} 10_i 10_j H_5 + h.c.$$

$$\mathcal{L}_{YL} = -(y_{LE})_{ij} L_{Hi} \bar{E}_{Hj} H_2^* + h.c.$$

$$(y_u^{SM})_{ij} = -\sin\theta_h\, (y_{10})_{ij}$$

$$\left(y_d^{SM}\right)_{ij} = -\sin\theta_h\, (y_5)_{ij}$$

$$(y_e^{SM})_{ij} = \cos\theta_h\, (y_{LE})_{ij} + \mathcal{O}(\theta_L\theta_E)\sin\theta_h\, (y_5)_{ij}$$

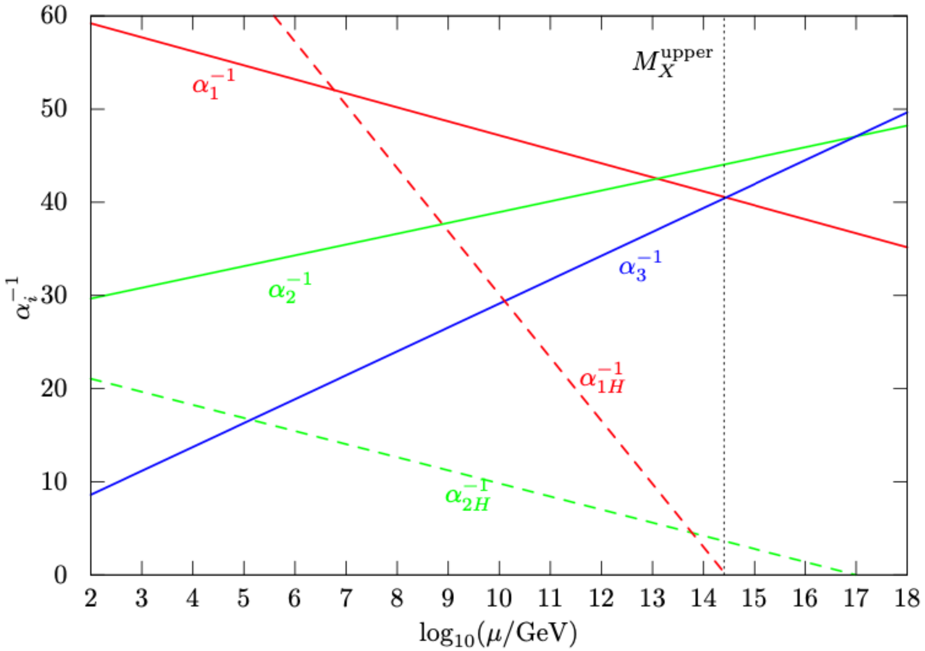
$SU(5) \times U(2)_H$ model

Gauge couplings

$$\alpha_1^{-1}(M_X) = \alpha_5^{-1}(M_X) + \frac{3}{5}\alpha_{1H}^{-1}(M_X)$$

$$\alpha_2^{-1}(M_X) = \alpha_5^{-1}(M_X) + \alpha_{2H}^{-1}(M_X)$$

$$\alpha_3^{-1}(M_X) = \alpha_5^{-1}(M_X)$$

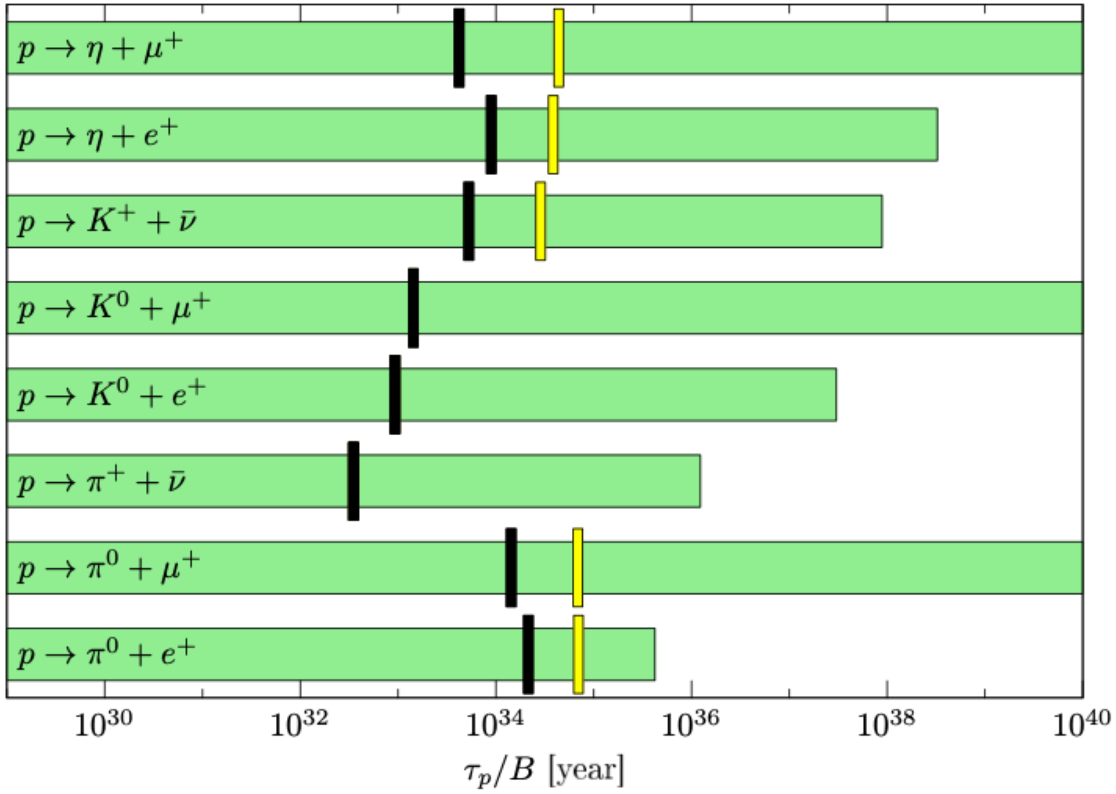


$$g_{1H}(M_2) \sim g_{2H}(M_2) \gg g_5(M_2)$$

$\longrightarrow$ $M_X \sim 10^{14-15}$ GeV

$SU(5) \times U(2)_H$ model

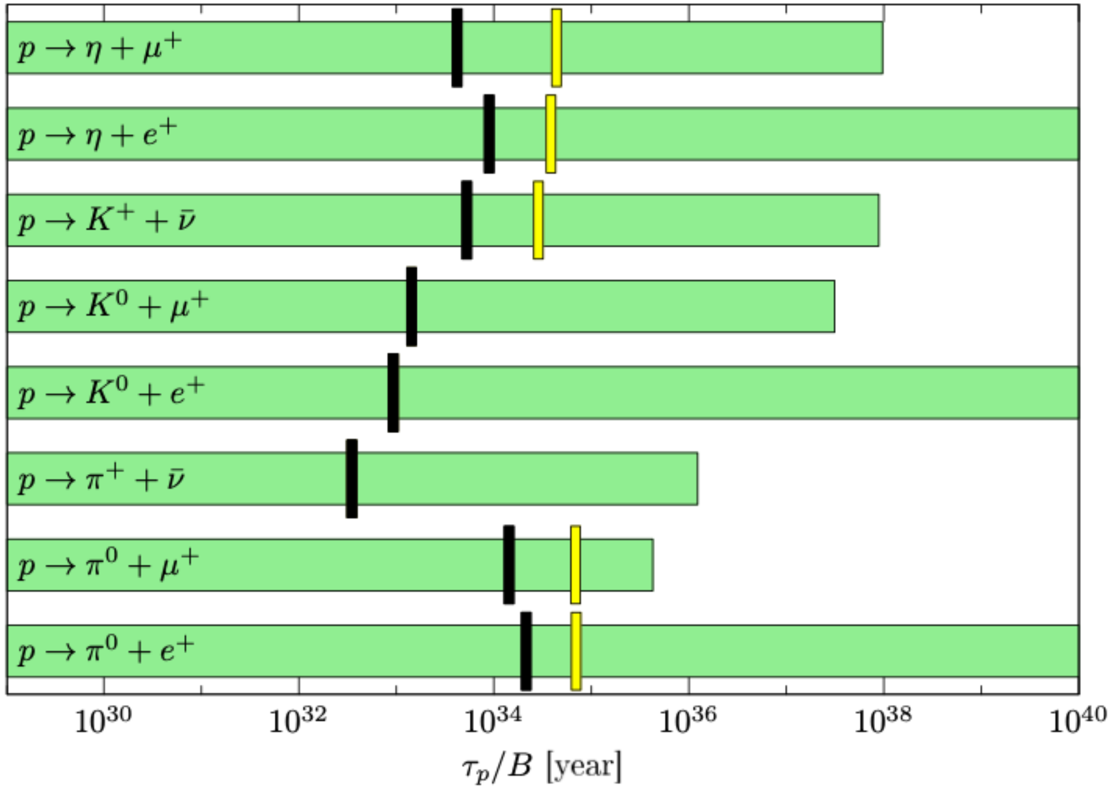
Proton lifetime



$$\mathcal{L}_{mass} = (\psi_{L1}\ \ \psi_{L2}) \begin{pmatrix} M & 0 & 10^{-4}M & 0 \\ 0 & M & 0 & 0 \end{pmatrix} (\bar{5}_{1L}\ \ \bar{5}_{2L}\ \ \bar{\psi}_{L1}\ \ \bar{\psi}_{L2})^T$$

$SU(5) \times U(2)_H$ model

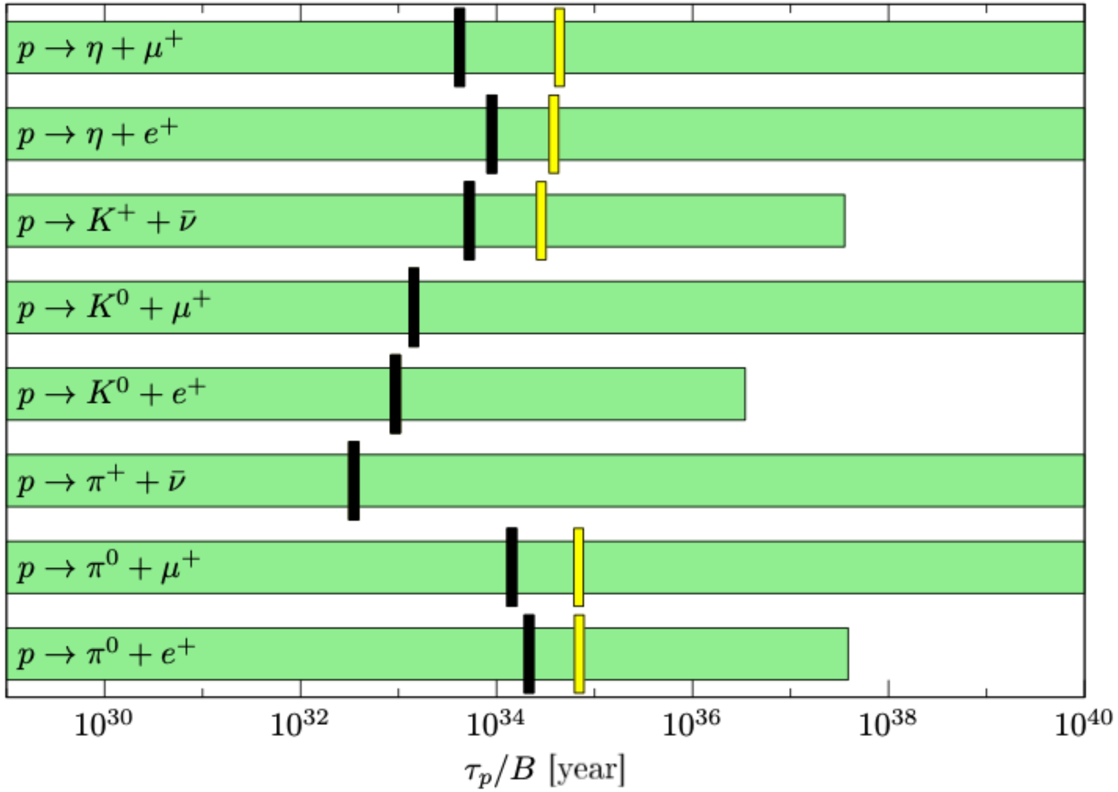
Proton lifetime



$$\mathcal{L}_{mass} = (\psi_{L1}\ \ \psi_{L2}) \begin{pmatrix} M & 0 & 0 & 10^{-4}M \\ 0 & M & 0 & 0 \end{pmatrix} (\bar{5}_{1L}\ \ \bar{5}_{2L}\ \ \bar{\psi}_{L1}\ \ \bar{\psi}_{L2})^T$$

$SU(5)\times U(2)_H$ model

Proton lifetime



$$\mathcal{L}_{mass} = (\psi_{L1}\ \ \psi_{L2})\begin{pmatrix} M & 0 & 0 & 0 \\ 0 & M & 10^{-4}M & 0 \end{pmatrix}(\bar{5}_{1L}\ \ \bar{5}_{2L}\ \ \bar{\psi}_{L1}\ \ \bar{\psi}_{L2})^T$$

$SU(5) \times SU(3)_H$ model

Fermions($SU(5)$, $SU(3)_H$)

$$\bar{5} : (\bar{5}, 1) \qquad 10 : (10, 1)$$

$$L_T : (1, 3) \qquad \bar{L}_T : (1, \bar{3})$$

$$L_T = (L_H \ \ \bar{E}_H) \qquad \bar{L}_T = (\bar{L}_H \ \ E_H)$$

- As we will show later, SM leptons are mostly contained in L_H and $\bar{E}_H$.
- SM quarks are all contained in $\bar{5}$ and 10.

$SU(5) \times SU(3)_H$ model

Scalars

$A : (1, 8)$, $SU(3)_H \longrightarrow SU(2)_H \times U(1)_H$

$\phi_3 : (5, \bar{3})$, $SU(5) \times SU(2)_H \times U(1)_H \longrightarrow SM$

$$\langle A \rangle = \begin{pmatrix} v_A & 0 & 0 \\ 0 & v_A & 0 \\ 0 & 0 & -2v_A \end{pmatrix} \quad \langle \phi_3 \rangle = \begin{pmatrix} 0 & 0 & 0 & v_3 & 0 \\ 0 & 0 & 0 & 0 & v_3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$SU(5) \times SU(3)_H$ model

Gauge couplings

$$SU(5) \times SU(3)_H \longrightarrow SU(5) \times SU(2)_H \times U(1)_H \longrightarrow SM$$

$$\frac{1}{3}\alpha_{1H}^{-1}(M_\Omega) = \alpha_{3H}^{-1}(M_\Omega)$$

$$\alpha_{2H}^{-1}(M_\Omega) = \alpha_{3H}^{-1}(M_\Omega)$$

$$M_{\mathrm{X\,max}} \leq 4 \times 10^{10}\ \mathrm{GeV}$$

$$\alpha_1^{-1}(M_X) = \alpha_5^{-1}(M_X) + \frac{3}{5}\alpha_{1H}^{-1}(M_X)$$

$$\alpha_2^{-1}(M_X) = \alpha_5^{-1}(M_X) + \alpha_{2H}^{-1}(M_X)$$

$$\alpha_3^{-1}(M_X) = \alpha_5^{-1}(M_X)$$

$SU(5)\times SU(3)_H$ model

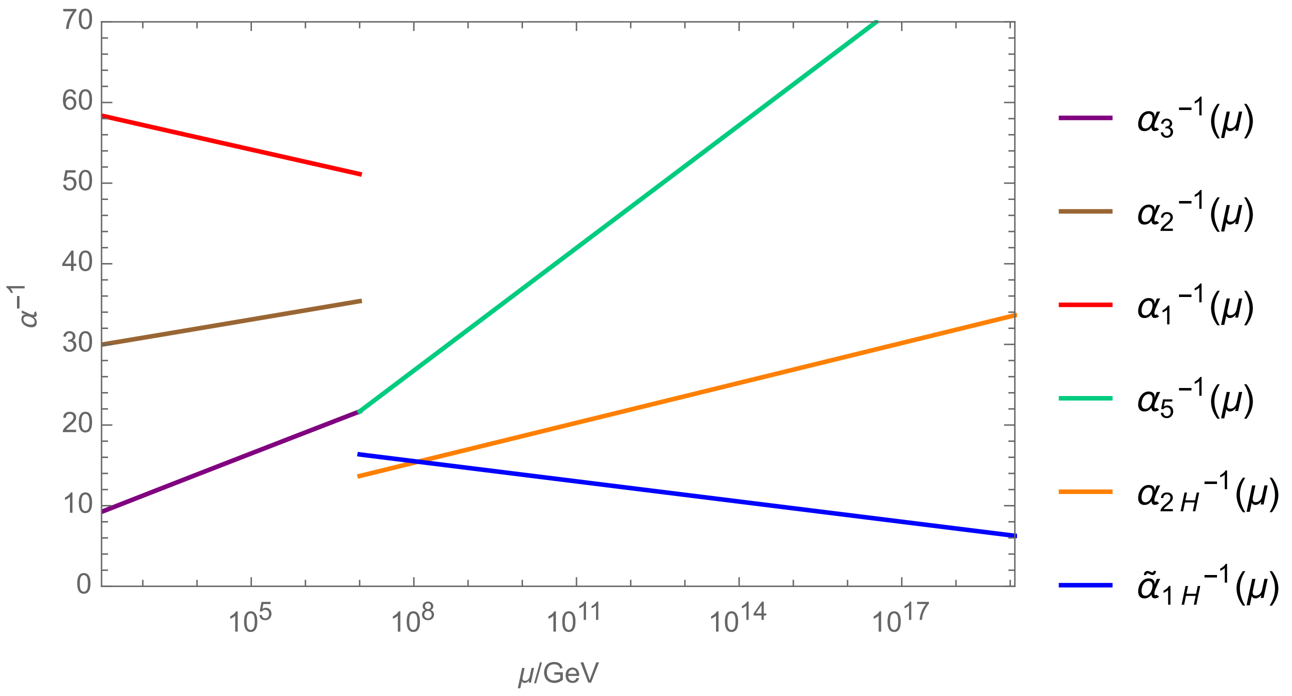
$$\alpha_1^{-1}(M_X) = \alpha_5^{-1}(M_X) + \frac{3}{5}\alpha_{1H}^{-1}(M_X)$$

$$\alpha_2^{-1}(M_X) = \alpha_5^{-1}(M_X) + \alpha_{2H}^{-1}(M_X)$$

$$\alpha_3^{-1}(M_X) = \alpha_5^{-1}(M_X)$$

$$\frac{1}{3}\alpha_{1H}^{-1}(M_\Omega) = \alpha_{3H}^{-1}(M_\Omega)$$

$$\alpha_{2H}^{-1}(M_\Omega) = \alpha_{3H}^{-1}(M_\Omega)$$



$$M_X = 1.0\times 10^7\ \text{GeV}$$

$$M_\Omega \cong 1.2\times 10^8\ \text{GeV}$$

$SU(5) \times SU(3)_H$ model

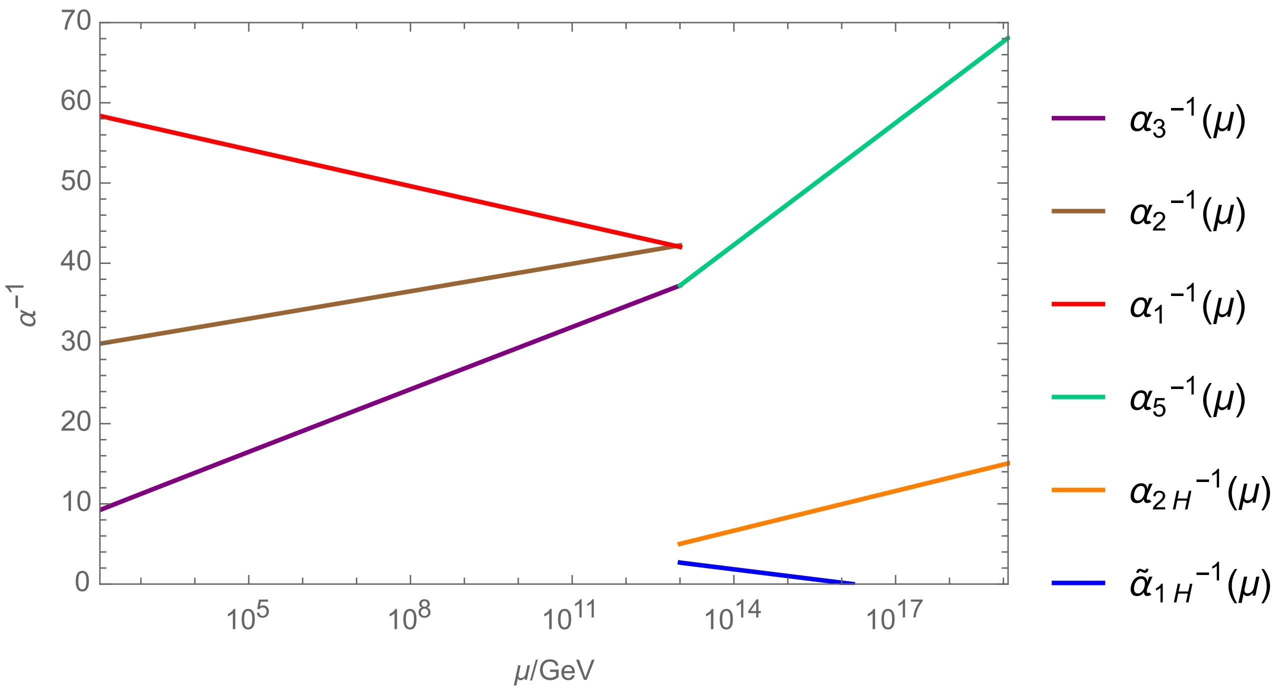
$$\alpha_1^{-1}(M_X) = \alpha_5^{-1}(M_X) + \frac{3}{5}\alpha_{1H}^{-1}(M_X)$$

$$\alpha_2^{-1}(M_X) = \alpha_5^{-1}(M_X) + \alpha_{2H}^{-1}(M_X)$$

$$\alpha_3^{-1}(M_X) = \alpha_5^{-1}(M_X)$$

$$\frac{1}{3}\alpha_{1H}^{-1}(M_\Omega) = \alpha_{3H}^{-1}(M_\Omega)$$

$$\alpha_{2H}^{-1}(M_\Omega) = \alpha_{3H}^{-1}(M_\Omega)$$



$$M_X = 1.0 \times 10^{13}\ \text{GeV}$$

$$M_\Omega \cong 1.1 \times 10^{12}\ \text{GeV}$$

$SU(5) \times SU(3)_H$ model

Proton lifetime

$$\tau(p \to \pi^0 e^+) \cong 10^{10} \, \frac{1}{\sin^2\theta} \left(\frac{M_X/g_5}{10^{10}\ \mathrm{GeV}} \right)^4 \ \mathrm{yrs}$$

$\longrightarrow$ $\sin\theta \lesssim 10^{-12}$ due to $\tau(p \to \pi^0 e^+) > 2.4 \times 10^{34}\ \mathrm{yrs}$

A. Takenaka et al. (SK collaboration) PRD102, 112011 (2020)

$SU(5) \times SU(3)_H$ model

Yukawa interactions

We consider a case one SM Higgs remains in the low energy.

Scalar containing the SM Higgs

$H_5 : (5, 1)$

$H_3 : (1, 3)$

$$H_5 = \begin{pmatrix} h_5^{color} \\ h_5^{SM} \end{pmatrix}$$

$$H_3 = \begin{pmatrix} h_3^{SM\dagger} \\ h_3^{singlet} \end{pmatrix}$$

Higgs mixing term

$$\mathcal{L}_{53\,mix} = \mu_{53} H_3^\dagger \phi_3 H_5^\dagger + h.c.$$

$\mathrm{SU}(5) \times \mathrm{SU}(3)_\mathrm{H}$ model

Yukawa interactions $\quad H_5\ (5,1) \quad H_3\ (1,3)$

quark $\quad \mathcal{L}_{YQ} = -(y_5)_{ij} \bar{5}_i\, 10_j H_5^* - (y_{10})_{ij} 10_i 10_j H_5 + h.c.$

lepton
$$\mathcal{L} = -(y_{LT})_{ij}\, \varepsilon_{abc} H_3^a L_{Ti}^b L_{Tj}^c = -(y_{LT})_{ij}\, \varepsilon_{abc} H_3^a L_{Tj}^c L_{Ti}^b$$
$$= (y_{LT})_{ij}\, \varepsilon_{acb} H_3^a L_{Tj}^c L_{Ti}^b = (y_{LT})_{ij}\, \varepsilon_{abc} H_3^a L_{Tj}^b L_{Ti}^c$$
$$= -(y_{LT})_{ji}\, \varepsilon_{abc} H_3^a L_{Ti}^b L_{Tj}^c$$

$\mathrm{SU}(5) \times \mathrm{SU}(3)_\mathrm{H}$ model

Yukawa interactions

$$(y_{LT})_{ij} = -(y_{LT})_{ji} \qquad y_{LT} = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$$

$$\rightarrow \quad \det y_{LT} = 0\,, \mathrm{Tr}\,[y_{LT}] = 0$$

→ massless electron

μ and τ lepton have the same mass

$\mathrm{SU}(5) \times \mathrm{SU}(3)_{\mathrm{H}}$ model

Yukawa interactions

$$\mathcal{L} = -\frac{(Y_{LT})_{ij}}{\Lambda_Y} L_{Ti}^{a} A_{a}^{b} L_{Tj}^{c} H_{3}^{d} \varepsilon_{bcd} + h.c.$$