Kramers-Wannier-like duality defects in (3+1)d gauge theories

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[2111.01141] JK, Kantaro Ohmori, Yunqin Zheng
See also [Koide, Nagoya, Yamaguchi ‘21; Choi, Córdova, Hsin, Lam, Shao ‘21]
Symmetries

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  – Higher-form symmetries [Gaiotto, Kapustin, Seiberg, Willett ‘14]
  
  – Vector and multipole symmetries [Pretko ‘18; Seiberg ‘19]

  – Subsystem symmetries [Lawler, Fradkin ‘04; Seiberg ‘19; Seiberg, Shao ‘20]

  – Non-invertible symmetries [Frölich, Fuchs, Runkel, Schweigert ‘09; Bhardwaj, Tachikawa ‘17;
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  – **Subsystem symmetries** [Lawler, Fradkin ‘04; Seiberg ‘19; Seiberg, Shao ‘20]

  – **Non-invertible symmetries** [Frölich, Fuchs, Runkel, Schweigert ‘09; Bhardwaj, Tachikawa ‘17; Chang, Lin, Shao, Wang, Yin ‘18]

• **Big conceptual breakthrough: symmetries = topological defects!**
  
  [Frölich, Fuchs, Runkel, Schweigert ‘09; Kapustin, Seiberg ‘14; Gaiotto, Kapustin, Seiberg, Willett ‘14; ...]

  – Why? Because the set of top. defects is a *robust/persistent* feature of the theory at any energy.
**Topological Defects**

- In a quantum theory, symmetry transformations can be implemented by codimension-1 operators $L_g$ acting on states.

  - This computes $L_g |\psi\rangle = |g\psi\rangle$ for $|\psi\rangle \in \mathcal{H}$.

  - For a usual group, $L_{g_1} L_{g_2} |\psi\rangle = |(g_1 \cdot g_2)\psi\rangle = L_{g_1 \cdot g_2} |\psi\rangle$.

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- Higher-form symmetries = higher-codimension topological operators.
  - A $p$-form symmetry is associated to a codimension-$(p + 1)$ defect.
  - Charged objects are $p$-dimensional.

- Today we will discuss only $p = 0, 1$. 
Topological Lines in 2d

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- There are also codimension-2 topological defects which act as junctions between two lines:
Topological Lines in 2d

• In two dimensions, codimension-1 defects are \textit{lines}.

• There are also codimension-2 topological defects which act as junctions between two lines:

• The lines are “objects” and the local operators are “morphisms” \Rightarrow \textit{category}.

• We can fuse defects together \Rightarrow \textit{fusion category}.

• If we want unitarity \Rightarrow \textit{unitary fusion category}.
Topological Lines in 2d

• Some subset of the defects may have grouplike fusion:

\[ L_{g_1} \times L_{g_2} = L_{g_1 \cdot g_2} \]
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• Others may not:

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L_{g_1} \times L_{g_2} = L_{g_3} + L_{g_4} + \ldots
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- For the grouplike symmetries, the category includes data about the 't Hooft anomalies of the group:
F-symbols and Pentagon Identity

- More generally, the category includes the “F-symbols”:

\[
\begin{array}{c}
L_x \\
L_g \\
\end{array}
\begin{array}{c}
L_h \\
L_k \\
\end{array}
\begin{array}{c}
L_\ell \\
\end{array}
= \sum_{L_y \in L_h \cdot L_k} (F^\ell_{g,h,k})^x_{L_g \cdot L_h \cdot L_k}
\begin{array}{c}
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= \sum_{L_y \in L_h \cdot L_k} (F^d_{g,h,k})^x_y \\
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\]

• The F-symbols must satisfy the pentagon identity:

• This is actually the full set of constraints on this data. The set of solutions is discrete ("Ocneanu rigidity"). [Etingof, Nikshych, Ostrik '05]
Classifying Fusion Categories

- The \textit{rank} \( r \) of the fusion category is the number of (simple) TDLs.
- Fusion categories of small rank \( r = 2, 3 \) have been classified \cite{Ostrik02, Ostrik13}.
Classifying Fusion Categories

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- Fusion categories of small rank $r = 2, 3$ have been classified [Ostrik ‘02; Ostrik ‘13]
- At rank 2, there are two simple TDLs $1$ and $X$. The general form of the fusion ring is $X^2 = 1 + aX$. It turns out only $a = 0, 1$ allow for solutions to the pentagon identity:
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  \[-a = 0: \text{Then } X^2 = 1, \text{ so } X \text{ is the generator of a global } \mathbb{Z}_2 \text{ symmetry.}
  \]

  To specify the full category, must also specify the F-symbols. There are two solutions to the pentagon equation, $F_{X,X,X}^X(1,1) = \pm 1$. The case $F_{X,X,X}^X(1,1) = -1$ corresponds to $\mathbb{Z}_2$ global symmetry with 't Hooft anomaly.
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  - $a = 0$: Then $X^2 = 1$, so $X$ is the generator of a global $\mathbb{Z}_2$ symmetry. To specify the full category, must also specify the F-symbols. There are two solutions to the pentagon equation, $F_{X,X,X}^X(1,1) = \pm 1$. The case $F_{X,X,X}^X(1,1) = -1$ corresponds to $\mathbb{Z}_2$ global symmetry with 't Hooft anomaly.
  - $a = 1$: Then $X^2 = 1 + X$, which is the Lee-Yang fusion algebra. From it we read off $\langle X \rangle = \frac{1 \pm \sqrt{5}}{2}$. The $-$ gives the “Lee-Yang category” which is realized by the $(2,5)$ Virasoro minimal model. The $+$ gives a category which is realized as a subset of tricritical Ising, three-state Potts, etc categories.
Classifying Fusion Categories

- At rank 3, there are three simple TDLs $1, X$, and $Y$. There are now five possible fusion algebras:

  - $X^2 = Y$, $Y^2 = X$, $XY = 1$  
    3 sols. to pent. eq.
  - $X^2 = 1$, $Y^2 = 1 + X$, $XY = Y$  
    2 sols. to pent. eq.
  - $X^2 = 1$, $Y^2 = 1 + X + Y$, $XY = Y$  
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  - $X^2 = 1$, $Y^2 = 1 + X + 2Y$, $XY = Y$  
    4 sols. to pent. eq.
Classifying Fusion Categories

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&- X^2 = 1, \ Y^2 = 1 + X + 2Y, \ XY = Y \quad \text{4 sols. to pent. eq.}
\end{align*}
\]

• Of particular interest are ones of the second type,

\[
\eta^2 = 1 \quad \mathcal{N}'^2 = 1 + \eta \quad \eta \mathcal{N} = \mathcal{N}
\]

- This is realized in the 2d Ising model. In that context \(\eta\) implements the \(\mathbb{Z}_2^{(0)}\) spin-flip symmetry, while \(\mathcal{N}\) implements Kramers-Wannier duality.
Non-invertible Symmetries in Higher Dimensions?

• To summarize: much is known about the general structure of symmetry in the case of $d = 2$.

• However, in higher dimensions the situation is far less well-understood.
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- In $d$-dim, we expect to have codim-$1$, -2, ..., -$d$ defects. The codim-2 ops. act as top. junctions between codim-1 ops, and so on. Hence our category has objects, morphisms, 2-morphisms, ..., $(d - 1)$-morphisms.

- The required structure is thus that of a fusion $(d - 1)$-category.
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• The required structure is thus that of a fusion $(d-1)$-category.

• Unfortunately, this is much more complicated than for the $d = 2$ case:
  
  – Upon fusing two codim-$1$ operators, we could get a sum of operators defined on higher codim.

  – Full set of pentagon-like identities unknown.
Non-invertible Symmetries in Higher Dimensions?

- **Goal**: Identify examples of non-inv. defects in 4d
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- **Strategies:**

  1) Find a theory which is self-dual under gauging a discrete group $G$. Then gauge $G$ in half of the space with Dirichlet boundary conditions. [Choi, Cordova, Hsin, Lam, Shao '21]

  2) Start with a theory with invertible symmetry $G$ and gauge a symmetry with mixed anomaly with $G$ to make $G$ noninvertible. Defects obtained in this way will again be related to self-dualities. [JK, Ohmori, Zheng '21]
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- **We will pursue the latter strategy today.**
Non-invertible Symmetries in Higher Dimensions

- Consider a 4d theory on $X_4$ with $\mathbb{Z}_2^{(0)}$ zero-form symmetry and $\mathbb{Z}_2^{(1)}$ one-form symmetry.
  - The $\mathbb{Z}_2^{(0)}$ has corresponding codim-1 defect $D(M_3, B^{(2)})$.
  - The $\mathbb{Z}_2^{(1)}$ has corresponding codim-2 defect $L(\Sigma_2)$. 
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- Say we have ’t Hooft anomaly of the form

  $$\pi \int_{X_5} A^{(1)} \cup \frac{\mathcal{P}(B^{(2)})}{2}, \quad \partial X_5 = X_4.$$ 

  - Here $\mathcal{P}(B^{(2)})$ is the Pontryagin square of $B^{(2)}$ – roughly speaking, it is like $B^{(2)} \cup B^{(2)}$ but mod 4.
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- This anomaly means that $D(M_3, B^{(2)})$ is not invariant under $\mathbb{Z}_2^{(1)}$ transfs, but
  \[ D(M_3, B^{(2)})e^{i\pi \int_{M_4} \frac{P(B^{(2)})}{2}}, \quad \partial M_4 = M_3 \]
  is invariant.
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- If we don’t gauge $\mathbb{Z}_2^{(1)}$, then $D(M_3, B^{(2)})$ is still a 3d invertible defect.
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- We can however couple $D(M_3, b^{(2)})$ to an appropriate 3d TQFT to absorb the bulk dependence.
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- Such a TQFT is not unique, but the minimal option is $U(1)_2$ CS theory.

[Hsin, Lam, Seiberg ‘18]
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  [Hsin, Lam, Seiberg ‘18]

- Claim: the resulting defect

\[ \mathcal{N}(M_3) \propto \int D a \ D(M_3, b^{(2)}) \ e^{\frac{i}{2\pi} \int_{M_3} a da - i \int_{M_3} a b^{(2)}} \]

is of Kramers-Wannier-type.
Kramers-Wannier-type Defects in 4d

To see this, note that

\[ \mathcal{N}(M_3) \times \mathcal{N}(M_3) \propto \int \mathcal{D}a \mathcal{D}a' \ e^{\frac{i}{2\pi} \int_{M_3} (a da - a' da') - i \int_{M_3} (a - a') b^{(2)}} \]
Kramers-Wannier-type Defects in 4d

To see this, note that

\[ \mathcal{N}(M_3) \times \mathcal{N}(M_3) \propto \int D\alpha D\alpha' e^{\frac{i}{\pi} \int_{M_3} (a\alpha - a'\alpha') - i \int_{M_3} (a - a') b^{(2)}} \]

= \int D\alpha D\hat{\alpha} e^{\frac{2i}{2\pi} \int a\hat{\alpha} - \frac{i}{2\pi} \int \hat{a}\hat{\alpha} - i \int \hat{a}b^{(2)}}
• To see this, note that

\[ N(M_3) \times N(M_3) \propto \int D\alpha D\alpha' e^{\frac{i}{2\pi} \int_{M_3} (\alpha \alpha' - \alpha' \alpha) - i \int_{M_3} (\alpha - \alpha') b^{(2)}} \]

\[ = \int D\alpha D\hat{\alpha} e^{\frac{2i}{2\pi} \int \alpha \hat{\alpha} - \frac{i}{2\pi} \int \hat{\alpha} \hat{\alpha} - i \int \hat{\alpha} b^{(2)}} \]

\[ = \sum_{\Sigma_2 \in H_2(M_3, \mathbb{Z}_2)} (-1)^{\#(\Sigma_2)} L(\Sigma_2) \]
Kramers-Wannier-type Defects in 4d

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\[ = \sum_{\Sigma_2 \in H_2(M_3,\mathbb{Z}_2)} (-1)^{\#(\Sigma_2)} L(\Sigma_2) \]

• This looks like a higher-categorical version of a KW defect \( \mathcal{N}^2 = \sum_g L_g \). The two main differences are:
  
  – There is a factor of \((-1)^{\#(\Sigma_2)}\). This can be removed by stacking the original theory with an invertible phase.
  
  – The defects on the right-hand side are of different codim than the defects on the left!
Kramers-Wannier-type Defects in 4d

- As in 2d, the KW-type defects imply presence of a self-duality.

  Rough argument: as in 2d, start by nucleating a bubble of $\mathcal{N}(M_3)$ which wraps all generators of $H_3(X_4, \mathbb{Z})$. Then use the fusion rules to get a fine mesh of the condensate:
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![Diagram showing the nucleation of a bubble and the resulting mesh]

- Summary so far: by gauging the 1-form symmetry in a theory with anomaly

$$\pi \int_{X_5} A^{(1)} \cup \frac{\mathcal{P}(B^{(2)})}{2}, \quad \partial X_5 = X_4$$

the codim-1 defect implementing the 0-form symmetry becomes non-invertible. In particular, it becomes a defect of KW-type, and implies existence of a self-duality.
Examples

- Begin by considering $SU(2)$ YM theory at $\theta = \pi$.
  
  - This theory has $\mathbb{Z}_2^{(1)}$ one-form symmetry, and time-reversal $T$.
  - These symmetries have a mixed anomaly given by
    \[
    \pi \int_{X_5} w_1(TX_5) \cup \frac{\mathcal{P}(B^{(2)})}{2}
    \]

- Thus via the statements on the previous slides, if we gauge $\mathbb{Z}_2^{(1)}$ to obtain $SO(3)$ YM, the codim-1 defect implementing $T$ becomes non-inv.
  
  - Since the non-invertible symmetries exist only for $\theta = \pm \pi$, one might expect a phase transition there. Indeed, such a phase transition is predicted by soft SUSY breaking [Aharony, Tachikawa, Seiberg ’13]:

![Diagram showing TQFTs and $\mathbb{Z}_2$ TQFT at different values of $\theta$.]
Examples

- Next consider $\mathcal{N} = 4$ $SO(3)$ SYM theory at $\tau = i$.
  - This theory has a $\mathbb{Z}_2^{(0)}$ zero-form symmetry generated by modular $S$.
  - It also has a $\mathbb{Z}_2^{(1)}$ one-form symmetry.
  - The two have a mixed anomaly,

$$Z_{SO(3)}[-1/\tau, B^{(2)}] = e^{i\frac{\pi}{2} \int X_4 P(B^{(2)})} Z_{SO(3)}[\tau, B^{(2)}]$$

- Thus upon gauging $\mathbb{Z}_2^{(1)}$, we obtain a non-invertible defect implementing self-duality of $\mathcal{N} = 4$ $SU(2)$ SYM theory at $\tau = i$.

- Hence $\mathcal{N} = 4$ $SU(2)$ SYM theory at $\tau = i$ has non-invertible defects and a notion of self-duality.
Summary

- The most general notion of symmetry in $d = 2$ is given by a fusion category. This contains both group-like (0-forms & higher-form) and non-invertible symmetries, together with their "'t Hooft anomalies."
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• Nevertheless, we have identified a general construction for one class of non-invertible defects in $d = 4$. Examples:

  1) $SO(3)$ YM at $\theta = \pi$

  2) $\mathcal{N} = 4$ $SU(2)$ SYM at $\tau = i$

Others include: $\mathcal{N} = 1$ $SO(3)$ SYM, Maxwell theory at $\tau = 2i$, $SO(8)$ YM, ...
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• There are also examples in $d = 3$, e.g. Chern-Simons matter theories. [JK, Ohmori, Zheng '21]
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- All examples I discussed today are duality defects. It would be interesting to identify non-invertible defects of other types as well.
The End (for now)

Thank you!