Quantum Computing and Quantum Error Correction

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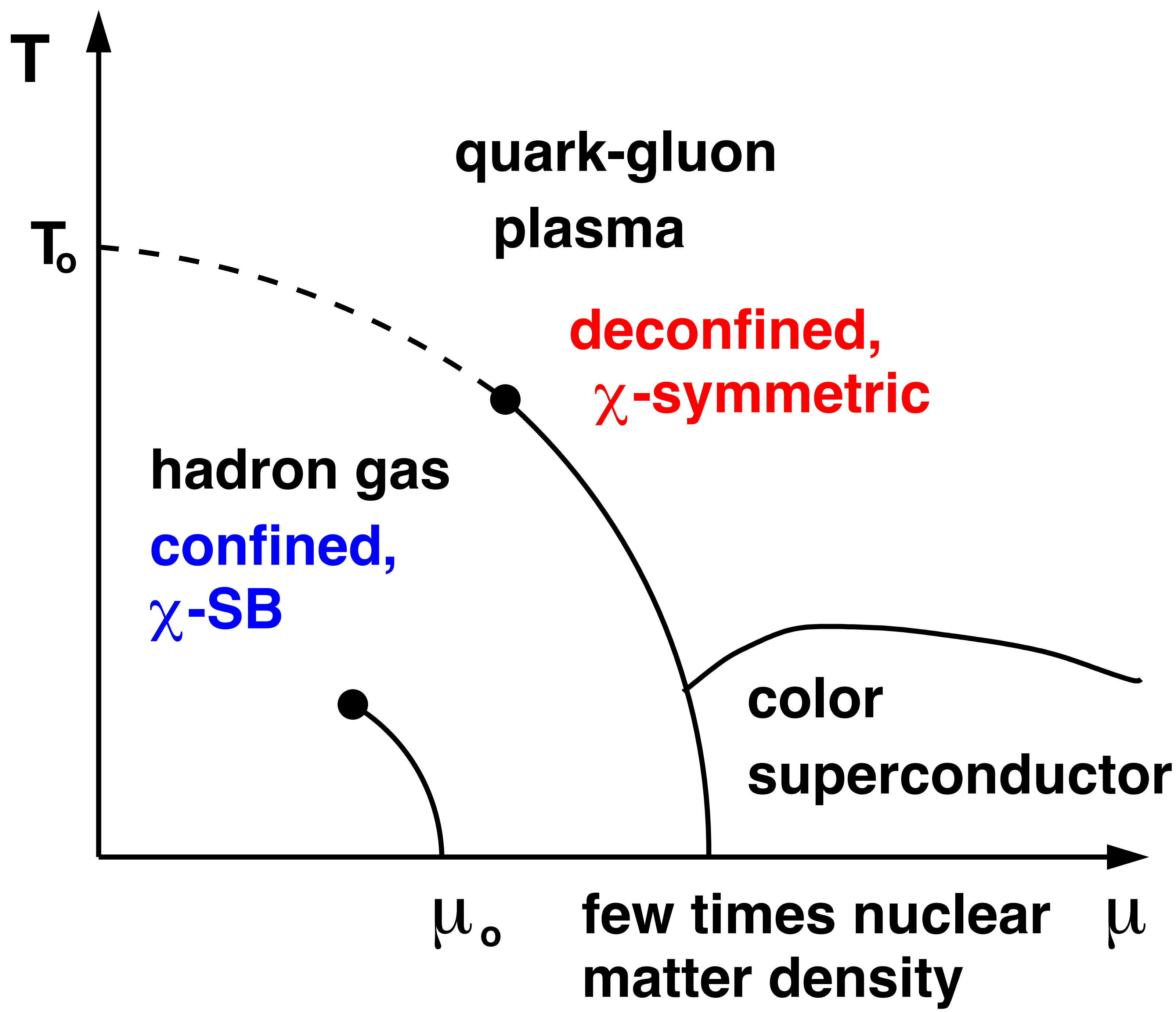
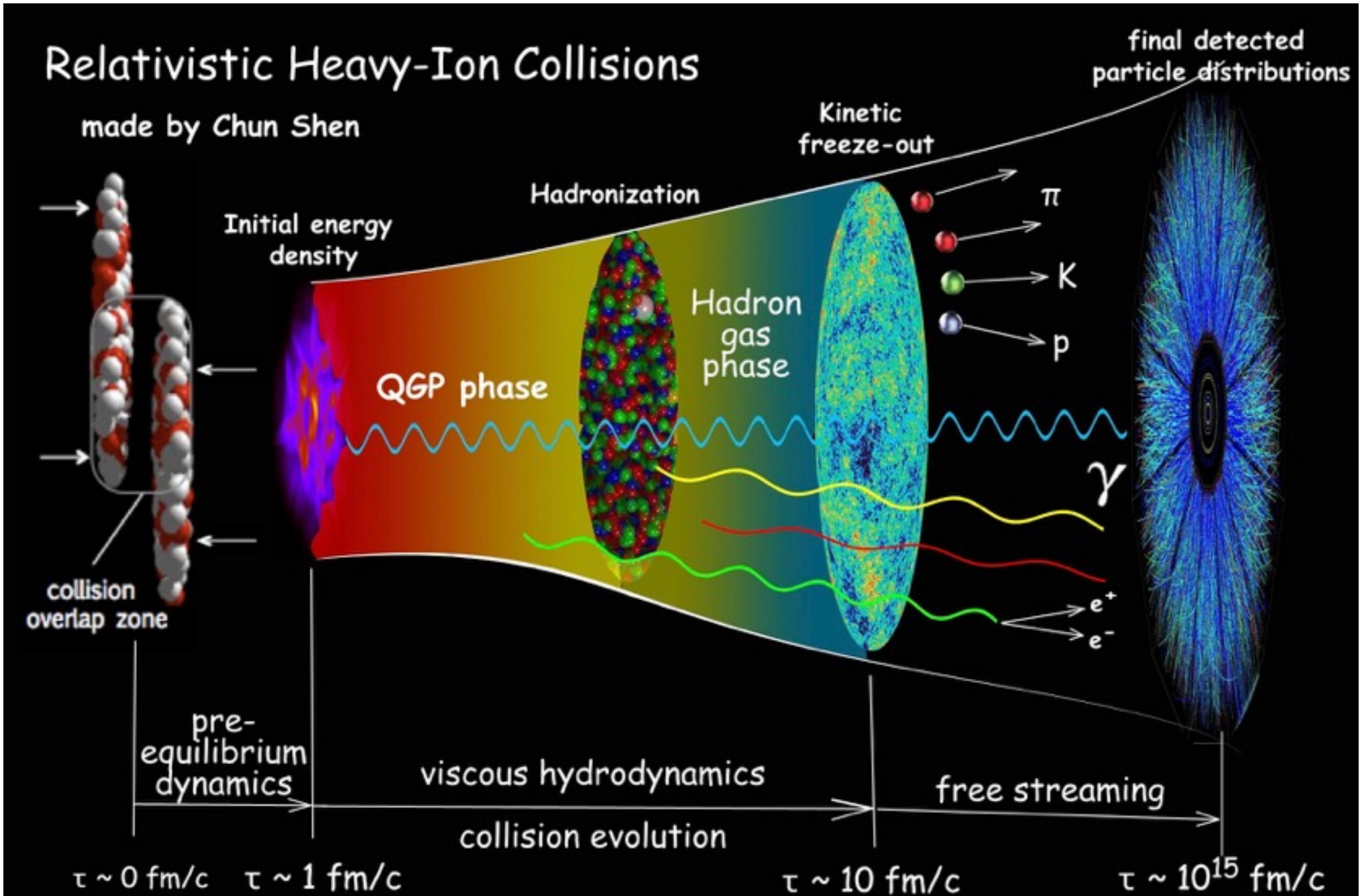
cf. D. Vodola, M. Rispler, SK, M. Mueller, "Fundamental thresholds of realistic quantum error correction circuits from classical spin models", arXiv:2021.04847 (now, Quantum 6, 618 (2022) !)

QCD phase diagram



Evolution of Nuclear Collision



Non-perturbative method: Lattice Gauge Theory

- A phase diagram is a property of a system in thermal/ chemical equilibrium
- Lattice QCD is suitable for studying a system in thermal equilibrium but not in chemical equilibrium (because the action becomes complex)
- We are studying QCD phase diagram by Relativistic Heavy Ion Collision (real-time, dynamical system)

Lattice Gauge Theory

- In quantum mechanics ((0+1)-dimensional quantum field theory), solve

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{x},t) = \hat{H}\psi(\mathbf{x},t)$$

and calculate

$$\langle\psi_f|e^{-\frac{i}{\hbar}\hat{H}(t_f-t_i)}|\psi_i\rangle$$

- interaction picture and perturbation theory

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}$$

Lattice Gauge Theory

- consider

$$
\begin{aligned}
& \langle \psi_f(t_f) | e^{-\frac{i}{\hbar}\hat{H}(t_f - t_i)} | \psi_i(t_i) \rangle \\
&= \langle \psi_f(t_f) | \int d^3\, x | \mathbf{x} \rangle \langle \mathbf{x} e^{-\frac{i}{\hbar}\hat{H}(t_f - t_i)} | \int d^3\, x' | \mathbf{x}' \rangle \langle \mathbf{x}' | \psi_i(t_i) \rangle \\
&= \int d^3\, x \int d^3\, x' \psi_f(\mathbf{x}, t_f)^\dagger \psi_i(\mathbf{x}', t_i) \langle \mathbf{x} | e^{-\frac{i}{\hbar}\hat{H}(t_f - t_i)} | \mathbf{x}' \rangle
\end{aligned}
$$

- and Green function method

$$G(\mathbf{x}, t_f | \mathbf{x}', t_i) = \langle \mathbf{x} | e^{-\frac{i}{\hbar}\hat{H}(t_f - t_i)} | \mathbf{x}' \rangle,$$

and

$$\psi_f(\mathbf{x}, t_f) = \int d^3 x' G(\mathbf{x}, t_f | \mathbf{x}', t_i) \psi_i(\mathbf{x}', t_i)$$

Lattice Gauge Theory

$$G(x, t_f | x', t_i) = \langle x | e^{-i\hat{H}T} | x' \rangle = \int \prod_{i=1}^{N} dx_i e^{-i\varepsilon \sum_i L(t_i)} = \int \mathcal{D}x(t) e^{-iS},$$

and

$$S = \int_{t_i}^{t_f} dt\, L[x(t)] = \int_{t_i}^{t_f} dt \left[\frac{1}{2} m \left(\frac{dx}{dt} \right)^2 - V(x(t)) \right]$$

Lattice Gauge Theory

- partition function

$$Z = \mathrm{Tr}\left(e^{-\frac{\hat{H}}{k_B T}}\right) = \sum_n e^{-\frac{E_n}{k_B T}} = \int d\,x \langle x | e^{-\frac{\hat{H}}{k_B T}} | x \rangle$$

With $k_B = 1$ and $\hbar = 1$ and $\tau_f - \tau_i = \frac{1}{T}$ and the periodic boundary condition,

$$Z = \int \mathcal{D}x(\tau) e^{-S},$$

and

$$S = \int_{\tau_i}^{\tau_f} d\,\tau \left[\frac{1}{2} m \left(\frac{dx}{d\tau}\right)^2 + V(x(\tau))\right],$$

- Euclidean space ($it \to \tau$) and statistical mechanics

Complex action problem

- lattice gauge theory with baryon density : grand canonical ensemble

$$\mathcal{L}_M = \overline{\psi}\gamma_\mu [i(\partial_\mu - iA_\mu) - m]\psi + \mu\psi^\dagger\psi \tag{1}$$

$$Z_G = \mathrm{Tr} e^{-\int d^4x \mathcal{L}_E} \tag{2}$$

- real-time physics: Schwinger-Keldysh formulation

Quantum Computer

- digital computer: bit
  - 0, 1
- quantum computer: qubit
  - |0>, |1>,   a |0> + b |1>

why quantum computer ?

- computing speed is measured in FLOPS (FLoating point OPeration per Second)
- computing speed is determined by (FLOP required for an algorithm) / FLOPS
- quantum computer uses less FLOP because of quantum algorithm

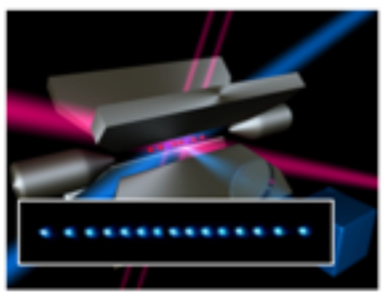
why quantum computer ?

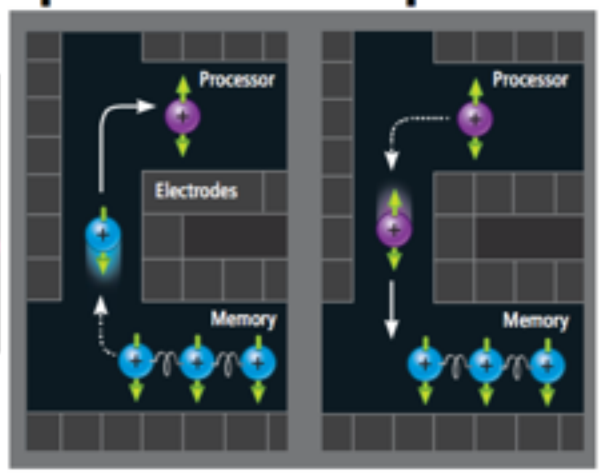
- the best FAST Fourier Transform (FFT) algorithm requires O (N log N) (N = 2^n) number of operations
- Quantum Fourier Transform algorithm requires n (n + 1)/2
- for n = 3 (N = 8), a classical computer needs ~ 8 log 8 operations and a quantum computer needs 3 operations

M. Mueller, talk in Korea-UK FP programme
(http://pyweb.swan.ac.uk/~aarts/ai-uk-korea-programme.html)

The ongoing race towards scalable quantum computers

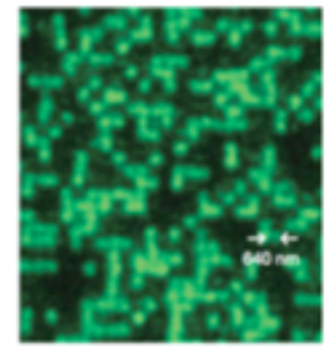
▶ 2D ion-trap quantum computers



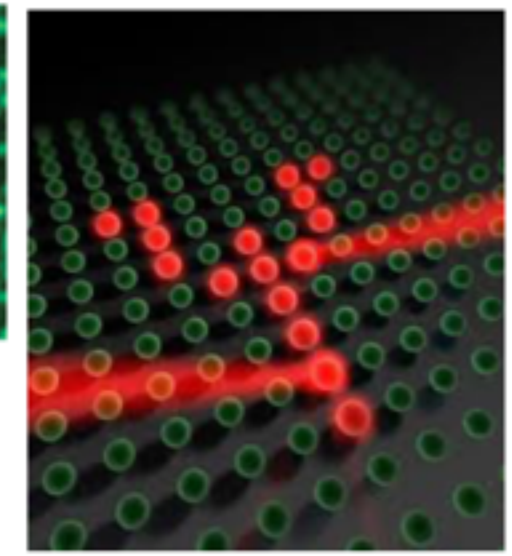


Innsbruck, Mainz, Maryland, Siegen, Zurich, AQT, IONQ …

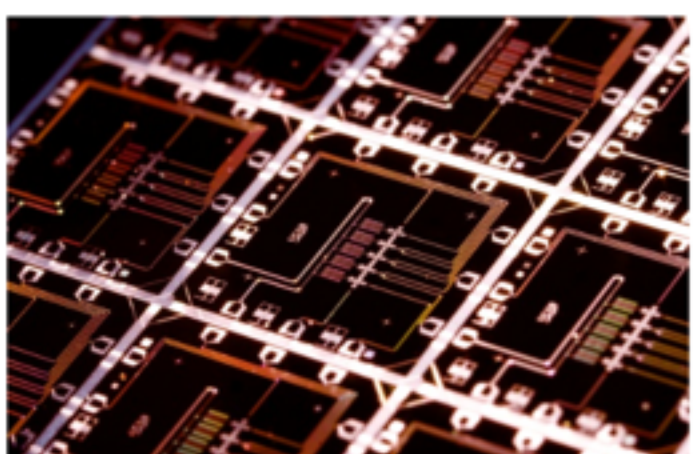
▶ Cold atoms in optical lattices



Munich,
Harvard
Strathclyde,
PASQAL
(Palaiseau)…

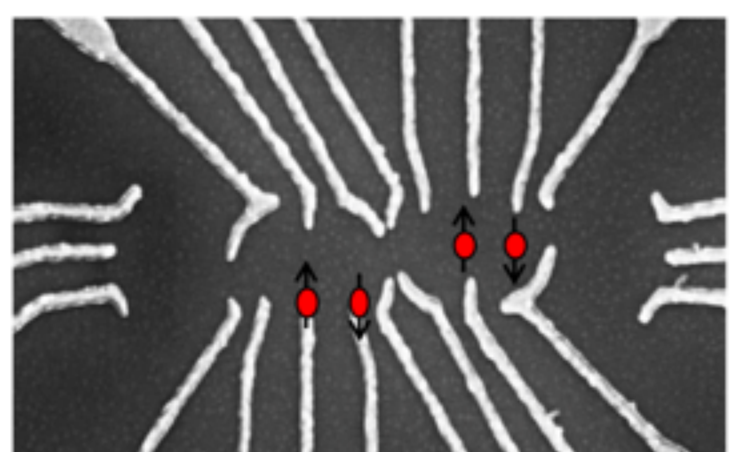


▶ **Super-conducting** qubits



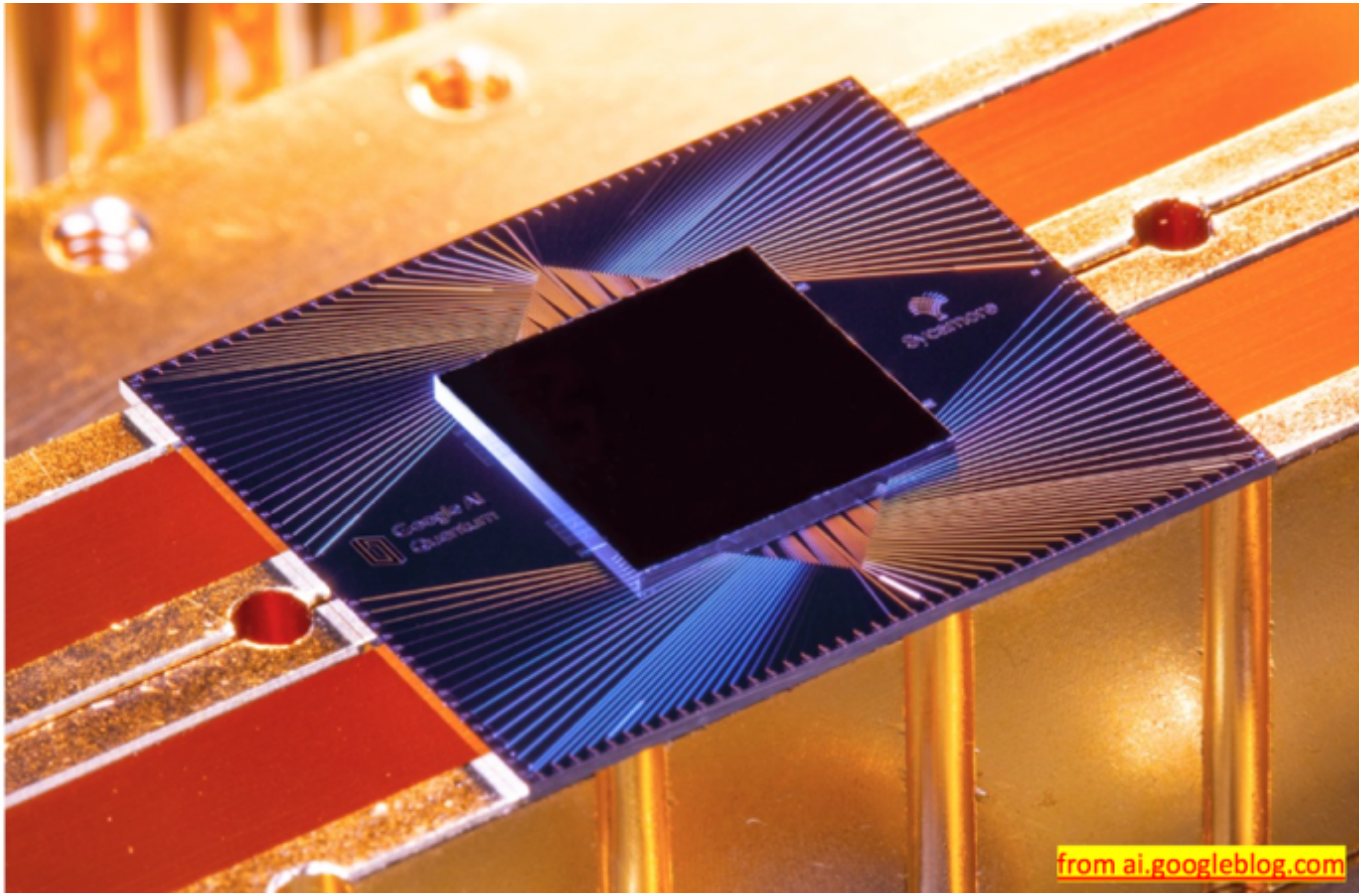
Jülich, Delft, Zurich, IBM, GOOGLE, …

▶ **Electron spin** qubits



Aachen, Jülich, Stuttgart, Cologne …

- **▶ Majorana qubits,**
- **▶ Photons,**
- **▶ NV centres, …**


from ai.googleblog.com

Quantum computers outperforming the best classical computers?

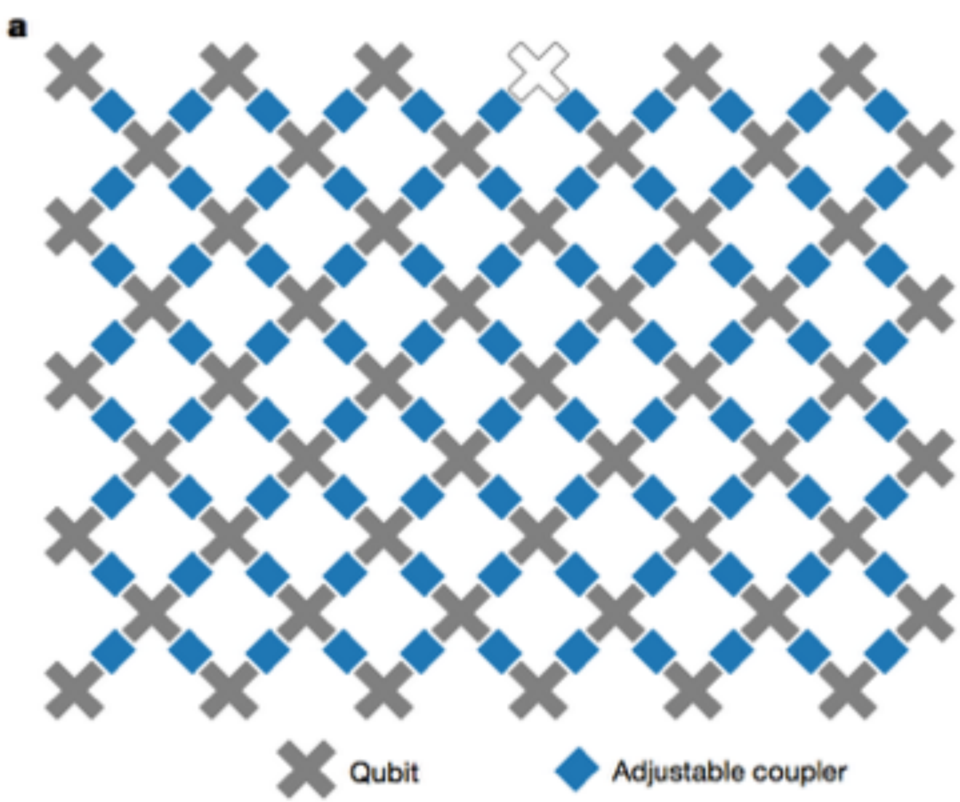
- First quantum computation a classical computer can't track
- … but for a completely useless problem

Article | Published: 23 October 2019

Quantum supremacy using a programmable superconducting processor

Frank Arute, Kunal Arya, [...] John M. Martinis

Nature **574**, 505–510(2019) | Cite this article



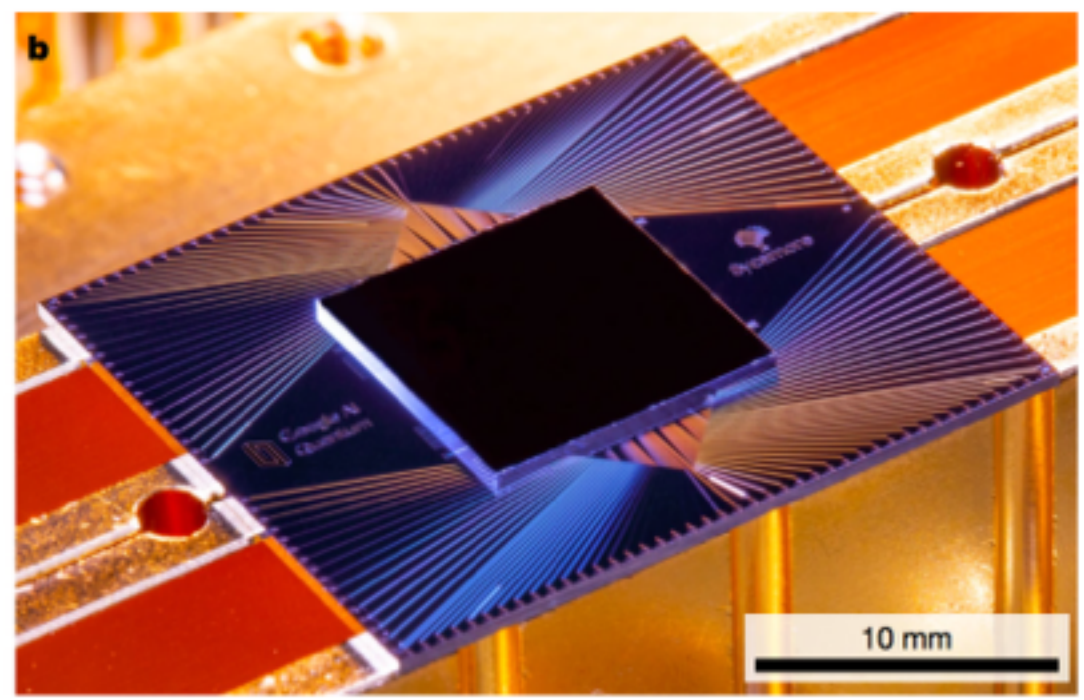


Fig. 1 | The Sycamore processor. a, Layout of processor, showing a rectangular array of 54 qubits (grey), each connected to its four nearest neighbours with couplers (blue). The inoperable qubit is outlined. **b**, Photograph of the Sycamore chip.


행사자료
양자기술 산·학·연·관 교류·소통의 場
제1차 K-퀀텀 스퀘어 미팅
2021년 12월 10일(금), 엘타워(서울)
주최 : 과학기술정보통신부
주관: 한국연구재단, 양자정보연구지원센터

D. Vodola, M. Rispler, SK, M. Mueller, arxiv:2104.04847

Fundamental thresholds of realistic quantum error correction circuits from classical spin models

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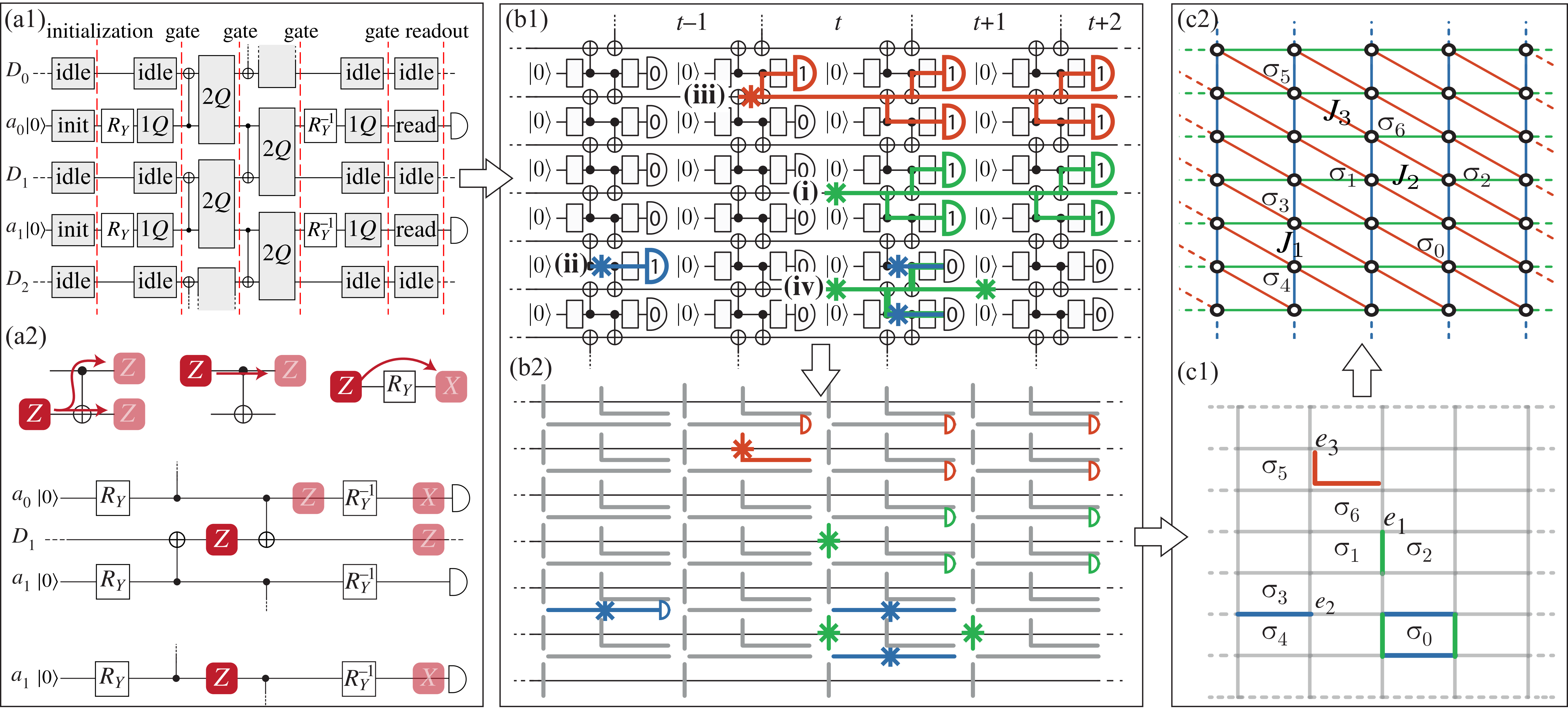
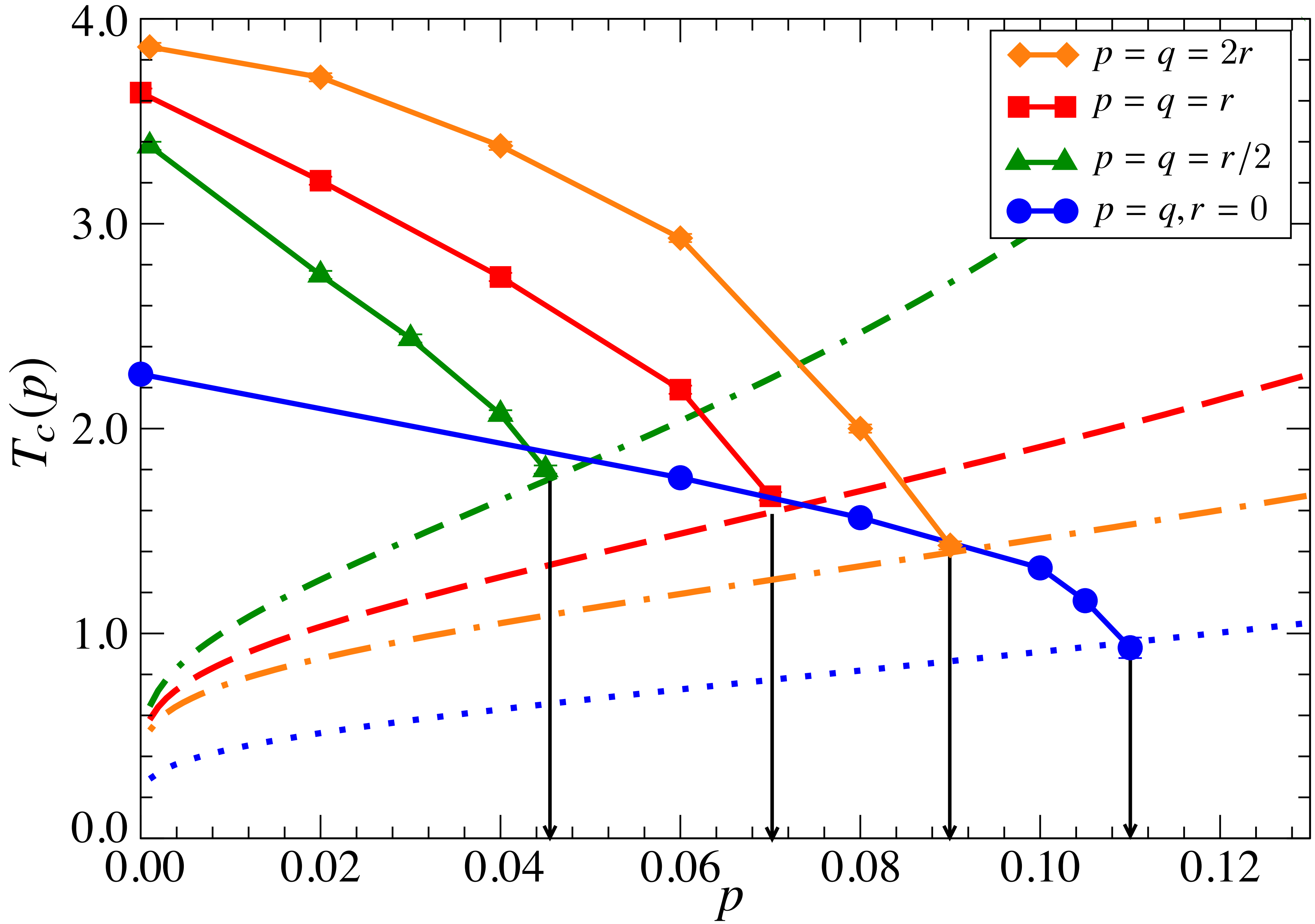
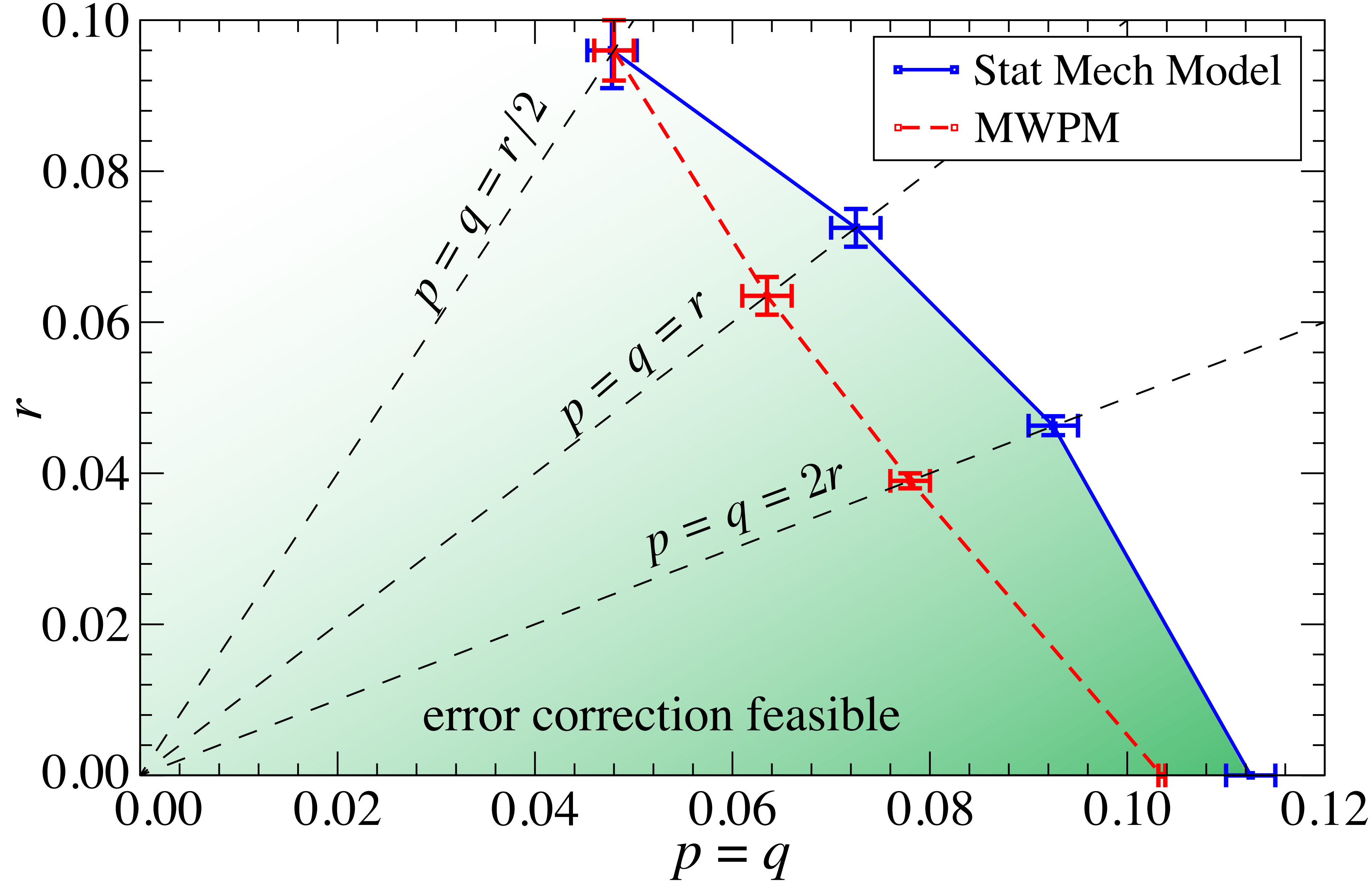
Mapping the decoding of quantum error correcting (QEC) codes to classical disordered statistical mechanics models allows one to determine critical error thresholds of QEC codes under phenomenological noise models. Here, we extend this mapping to admit realistic, multi-parameter noise models of faulty QEC circuits, derive the associated strongly correlated classical spin models, and illustrate this approach for a quantum repetition code with faulty stabilizer readout circuits. We use Monte-Carlo simulations to study the resulting phase diagram and benchmark our results against a minimum-weight perfect matching decoder. The presented method provides an avenue to assess

realistic quantum circuit for error correction and statistical mechanics model





4.0
3.0
2.0
1.0
0.0
0.00
0.02
0.04
0.06
0.08
0.10
0.12
p
$T_c(p)$
$p = q = 2r$
$p = q = r$
$p = q = r/2$
$p = q, r = 0$



0.10
0.08
0.06
0.04
0.02
0.00
r
0.00
0.02
0.04
0.06
0.08
0.10
0.12
p = q
Stat Mech Model
MWPM
p = q = r/2
p = q = r
p = q = 2r
error correction feasible

- Much more to do !