R-Matrix study of the β^{+} decay of ${}^{8}_{5}\mathrm{B}$ to the highly excited states of ${}^{8}_{4}\mathrm{Be}$

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Presented at CPAN-2022

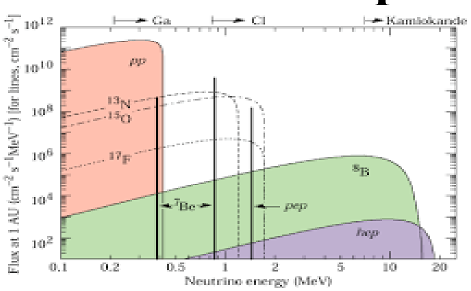
The β^+ decay of ${}^{8}_{5}$B is of interest for both astrophysics and nuclear structure

Astrophysics

Part of the stellar hydrogen-burning chain

Source of high-energy solar neutrinos above 2 MeV

Main contributor to what was known as the "solar neutrino problem"



JYFL08

O Kirsebom. Phys. Rev. C, 83(6):065802–065822, 2011.

Nuclear Structure

Through the β^+ decay of ^{8}B, we study the structure of ^{8}Be

The 16.6 and 16.9 MeV levels of ^{8}Be are <u>assumed</u> to form a <u>fully mixed</u> 2^+ isospin doublet ($^7Li \otimes p$; $7Be \otimes n$)

Only known case of Nuclear Chart

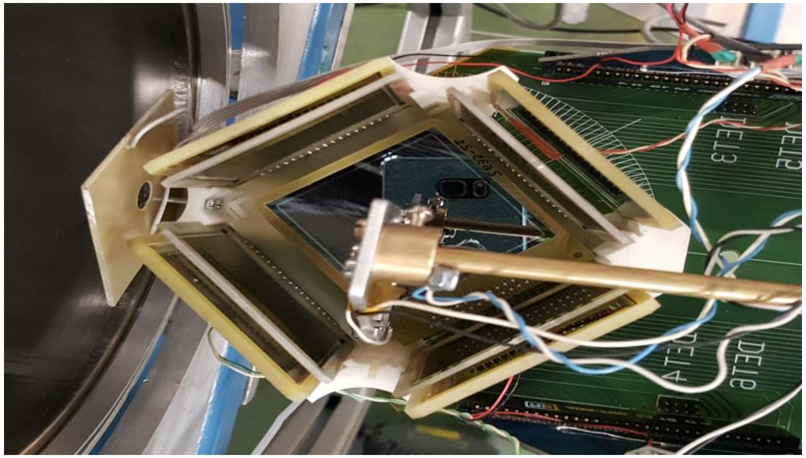
<u>We can experimentally check this assumption!</u>

IS633

S.Viñals, PhD Thesis (Complutense University of Madrid, Department of Physics, Sep. 2020).

Our objective is to determine the mixture coefficients of the 2^+ isospin doublet

Experiment IS633 was conducted at ISOLDE – CERN (May of 2017) to study the structure of ^{8}Be



- 4 Si △E-E **telescopes** DSSD + PAD
- **Coin** 60µm -- 60µm (U6 & U2)
    - (stop α: 1-10 MeV

  --- 40µm -- 40µm (U3 & U4)

  (low β-response)

Our experiment follows a four-step technique

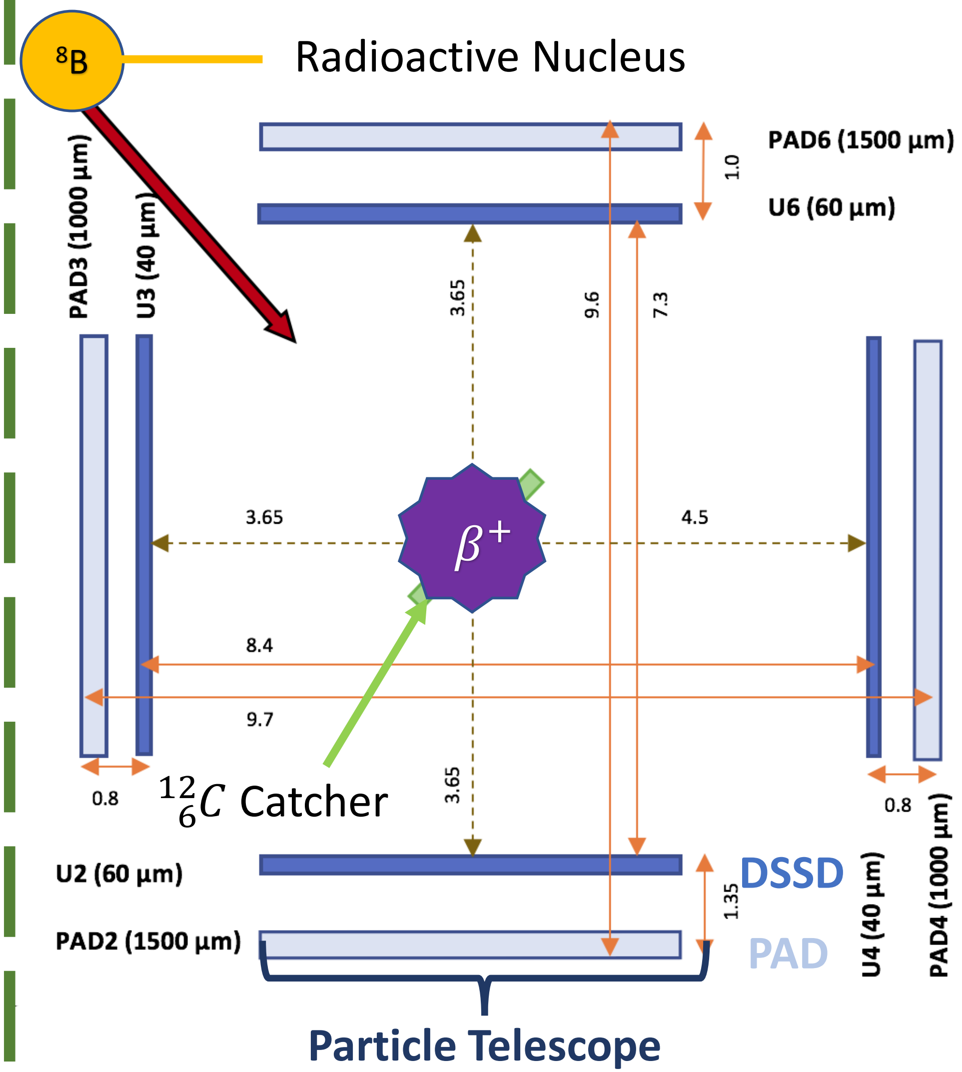
1- $^8_5$B nucleus is implanted in a $^{12}_6C$ foil

2- The β^+ decay of $^8_5$B ($T_{1/2} = 771.17(94)$ ms) populates states of $^8_4$Be

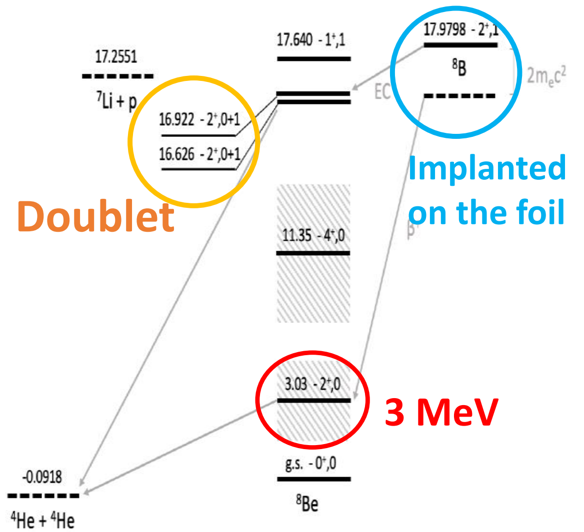
3- $^8_4$Be is unbound → $\alpha - \alpha$ break up

4- Reconstruction $\alpha - \alpha$ coincidence spectrum through a system of four telescopes

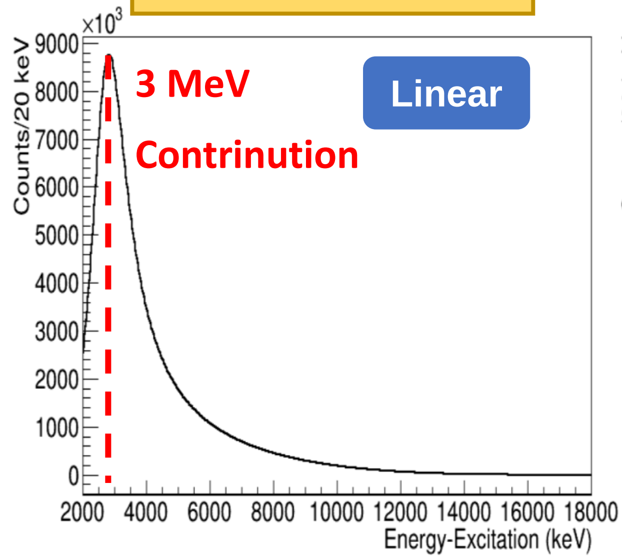
Set UP schematics

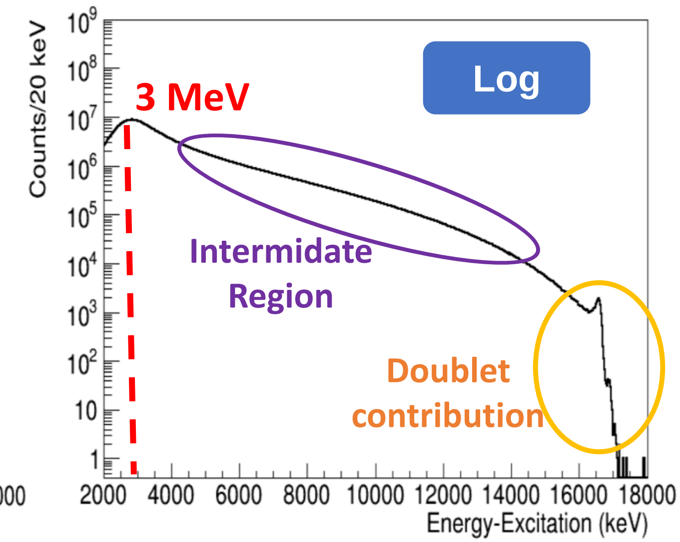


$$\Omega = 38\%\ 4\pi$$



Experimental data





How can we determine if the two states are mixed?

Theory

Each state in the doublet can be decomposed into pure isospín states

$|a\rangle = \alpha|T=0\rangle + \beta|T=1\rangle \quad \alpha^2+\beta^2=1$

$|b\rangle = \beta|T=0\rangle - \alpha|T=1\rangle$ **mixing coeficients**

If the states are completely mixed : $\alpha^2/\beta^2 = 1$

Method 1

$$\frac{\alpha^2}{\beta^2} = \frac{B_{16.6,GT}}{B_{16.9,GT}}$$

$$\frac{\alpha^2}{\beta^2} = \frac{B_{16.9,F}}{B_{16.6,F}}$$

Method 2

$$\alpha^2 = \frac{\Gamma_{16.6}}{\Gamma_0} = \frac{\Gamma_{16.6}}{\Gamma_{16.6}+\Gamma_{16.9}}$$

$$\beta^2 = \frac{\Gamma_{16.9}}{\Gamma_0} = \frac{\Gamma_{16.9}}{\Gamma_{16.6}+\Gamma_{16.9}}$$

Fitting the spectrum gives the relevant information about the levels (E, B_F, B_{GT}, Γ)

The β^+ decay feads levels to broad to be fitted with a simple function (Gauss, Landau, …)

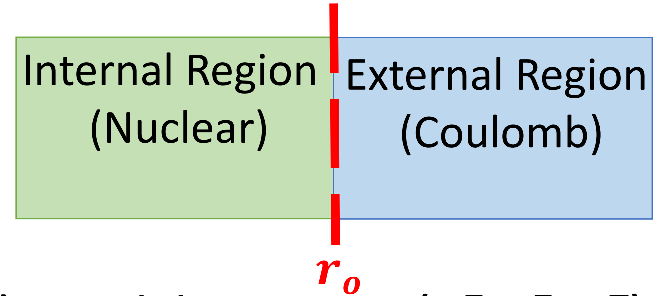
R-Matrix formalism → Nuclear resonances in reaction studies → β-decay followed by 2-body break up

A.M. Lane et al., Rev. Mod. Phys. 30(2):257-353, 1958 *F.C. Barker, Aus. Journ. Phys. 22(3):293-316, 1969*

R-Matrix Theory

- **The configuration space → 2 Regions (1- nuclear 2-columb)**
- **Log derivate of the w.f must be continuous in the boundary (r_o).**
- **Imposing continuity in r_o we obtain a Matrix relating both regions:**

$$R_{c'c} = \sum_{\lambda} \frac{\gamma_{\lambda c'} \gamma_{\lambda c}}{E_\lambda - E}$$



The R-Matrix is formed by individual nuclear resonances → Each with characteristics parameters(E,B_F, B_{GT},Γ)

If you feel confused remember: R-Matrix is just a parametrization in term of well-defined resonances

R-Matrix Praxis

¿How can we fit data with R-Matrix?

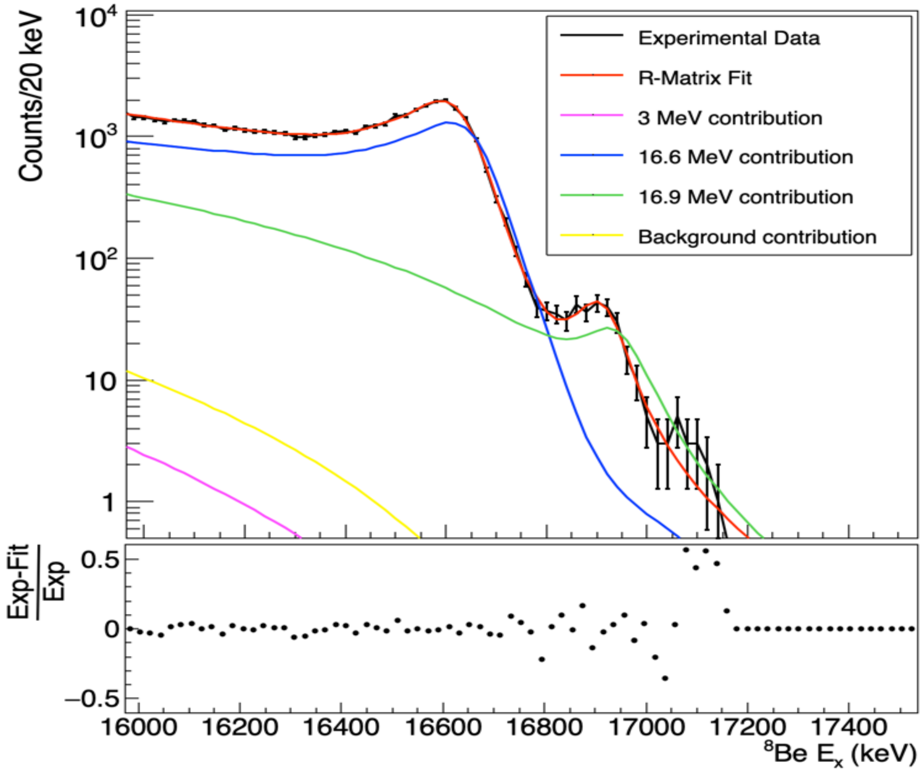
I. **Select the number of resonances with initial parameters (E, B_F, B_{GT}, Γ)**
II. **Liberate (allow to change) some of the parameters.**
III. **Modify the free parameters till the R-matrix spectrum fits the experimental data (Root-Minuit).**
IV. **Liberate other parameters and start again**
V. **Iterate until you get the best fit (χ^2 minimization)**

Our approach

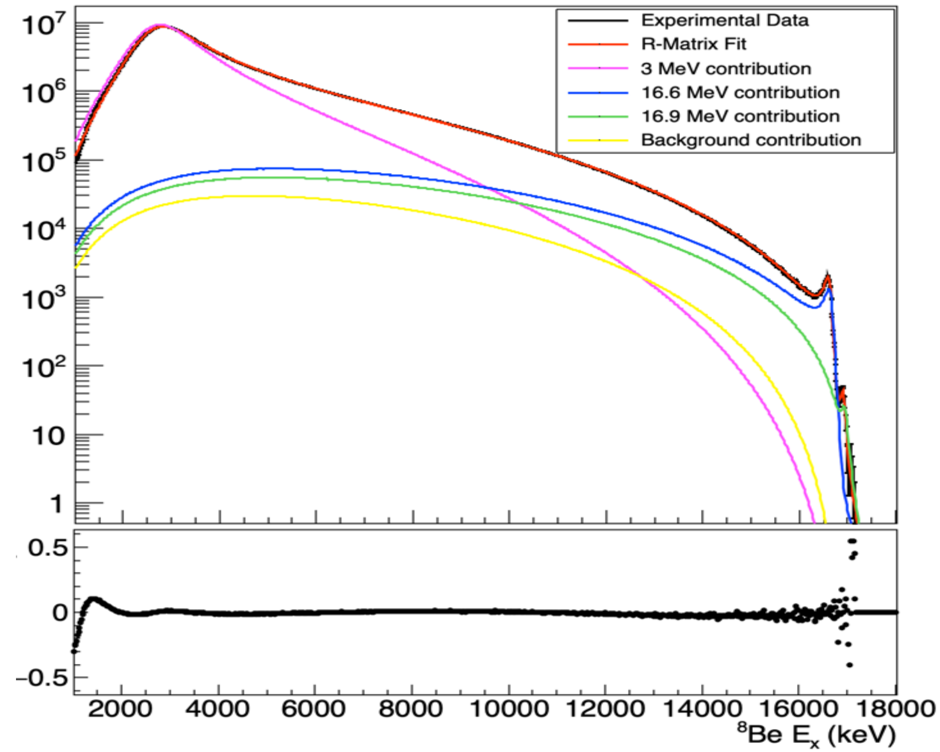
(4x R-Matrix resonances)

3 MeV →	**Dominant**
16,6 MeV / **16,9 MeV**	**Main decay**
BKG →	**Intermediate Region+ level tails**

Local 2^{+} doublet fit



Global fit



The residue function measures the quality of the fit

The contribution of the broad 3 MeV State does not influence the peaks of the 2^{+} doublet

The global fit includes the contributions of the 3 MeV and Intermediate región

So, that's all?

Not yet, we would like to know why there is a **discrepancy** between the **Decay Width ($\Gamma_{\alpha\alpha}$) obtained through our R-Matrix** and those of the **adopted published values [Tilley (2004)]**

We have performed **two types of cross-checks** to find the reason for this discrepancy

	Parameters	Tilley (2004)	Global Fit	Local 2^+ Fit
	r_0 (fm)	1.35	1.35	1.35
2^+_0	E (keV)	3030(10)	3052(37)	
	B_F		0	
	B_{GT}		0.011813(56)	
	$\Gamma_{\alpha\alpha}$(keV)	1513(15)	1957(15)	
2^+_1	E (keV)	16626(3)	16632(54)	16632(70)
	B_F		0.63(24)	0.32(81)
	B_{GT}		0.98(14)	1.17(35)
	$\Gamma_{\alpha\alpha}$(keV)	108.1(5)	129.47(28)	129.5(36)
2^+_2	E (keV)	16922	16921(20)	16919.5(90)
	B_F		1.08(24)	1.44(79)
	B_{GT}		0.57(14)	0.35(49)
	$\Gamma_{\alpha\alpha}$(keV)	74.0(4)	112.5(11)	108(13)
2^+_{Bkg}	E (keV)		21205	
	B_F		0	
	B_{GT}		1.3438	
	$\Gamma_{\alpha\alpha}$(keV)		119.11	

Consistency tests of our fits

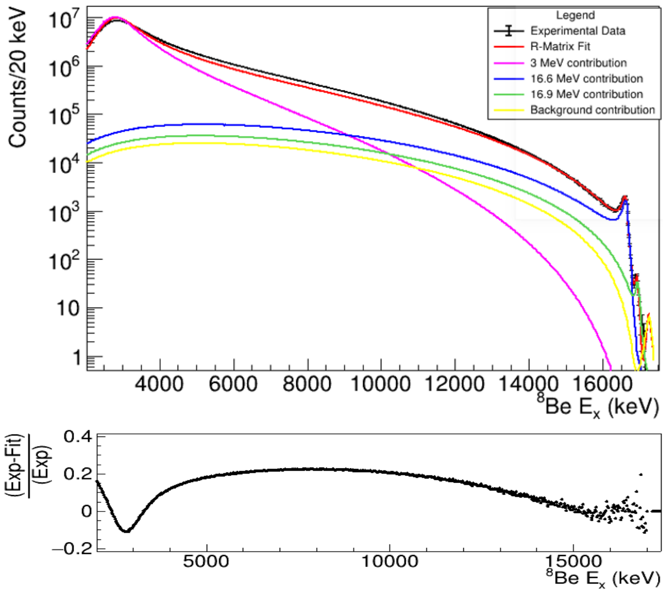
Repeat the fit under different initial parameters to ensure convergence

Comparison with previous results

To test if there is any systematic error in our data

Check I: Fix the decay widths to the literature values

Check I: $\Gamma_{\alpha\alpha}$ = Fixed



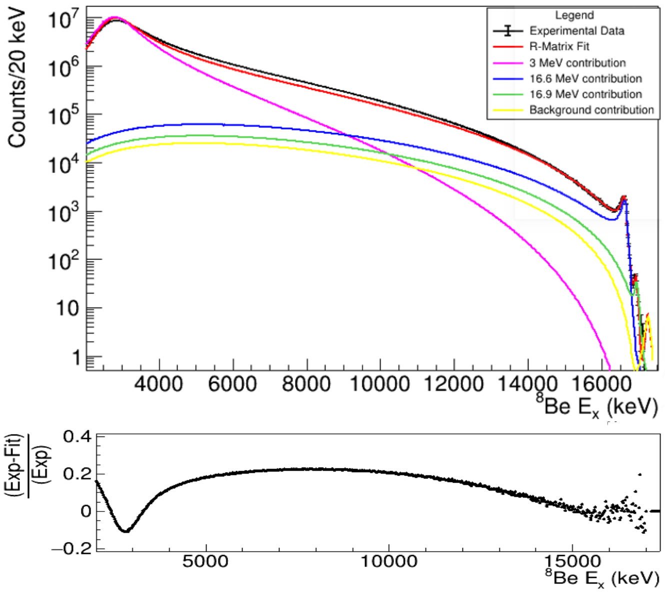
Levels	Parameters	Literature [Til04]	S.Viñals	Check I
	r_o(fm)	1.35	1.35	1.35
	χ^2(2-17.2 MeV)		14.4	3991
2_0^+	E (keV)	3030(10)	3058(31)	2959.3
	$\Gamma_{\alpha\alpha}$(keV)	1513(15)	1876(94)	1415.7
2_1^+	E (keV)	16626(3)	16632(83)	16616
	$\Gamma_{\alpha\alpha}$(keV)	108.1(5)	129.47(28)	180
2_2^+	E (keV)	16922	16921(85)	16919
	$\Gamma_{\alpha\alpha}$(keV)	74.0(4)	112.5	74.076
2_{Bkg}^+	E (keV)		21205	17238
	$\Gamma_{\alpha\alpha}$(keV)		119.11	104.26

Results

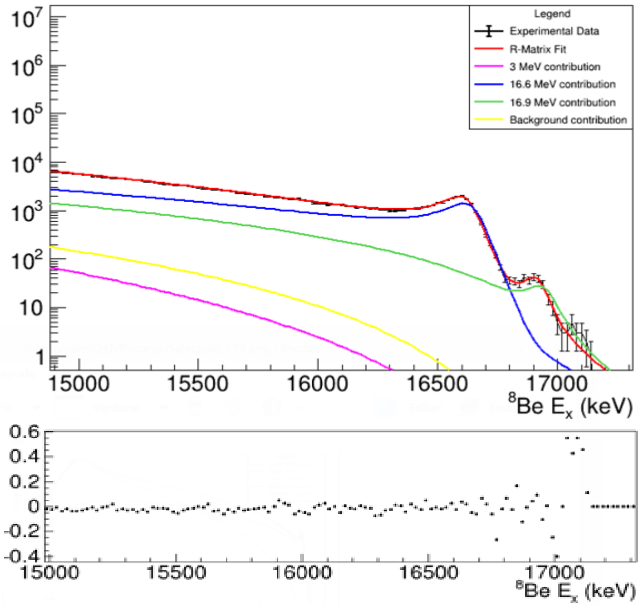
- **$\Gamma_{\alpha\alpha}$= Fixed to Literature:** does not generate good results.

Check II: Set the decay widths to the literature values, allowing them to change

Check I: $\Gamma_{\alpha\alpha}$ = Fixed



Check II: $\Gamma_{\alpha\alpha}$= Let Free



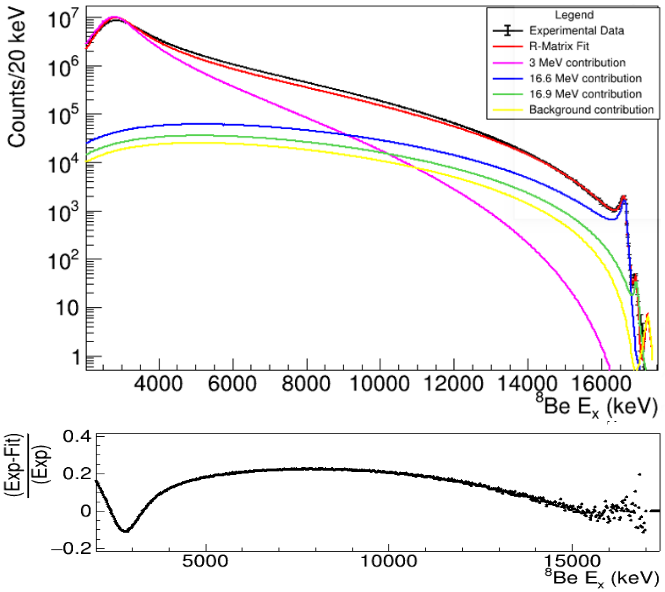
Levels	Parameters	Literature [Til04]	S.Viñals	Check I	Check II
	r_0(fm)	1.35	1.35	1.35	1.35
	χ^2(2-17.2 MeV)		14.4	3991	14.3
2^+_0	E (keV)	3030(10)	3058(31)	2959.3	3050.75
	$\Gamma_{\alpha\alpha}$(keV)	1513(15)	1876(94)	1415.7	1949.8
2^+_1	E (keV)	16626(3)	16632(83)	16616	16627
	$\Gamma_{\alpha\alpha}$(keV)	108.1(5)	129.47(28)	180	123.98
2^+_2	E (keV)	16922	16921(85)	16919	16917
	$\Gamma_{\alpha\alpha}$(keV)	74.0(4)	112.5	74.076	99.735
2^+_{Bkg}	E (keV)		21205	17238	23338
	$\Gamma_{\alpha\alpha}$(keV)		119.11	104.26	1331.4

Results

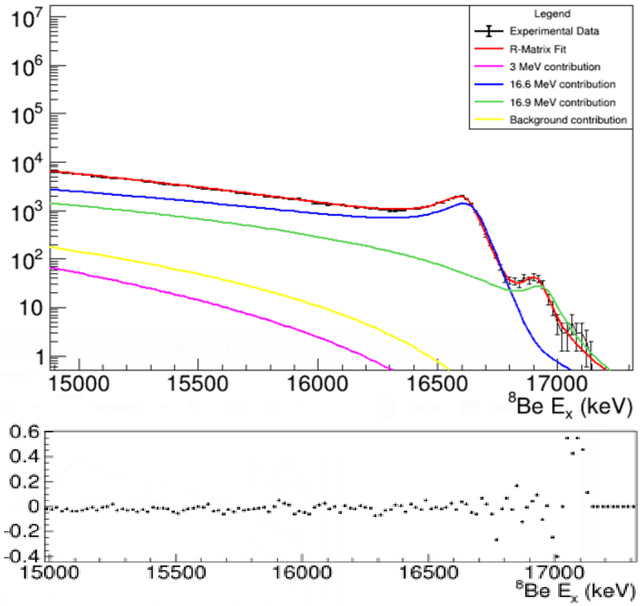
- **$\Gamma_{\alpha\alpha}$= Fixed to Literature:** does not generate good results.
- **$\Gamma_{\alpha\alpha}$= Let free :** improves the global fit

Check III: Modify the energy of the BKG level

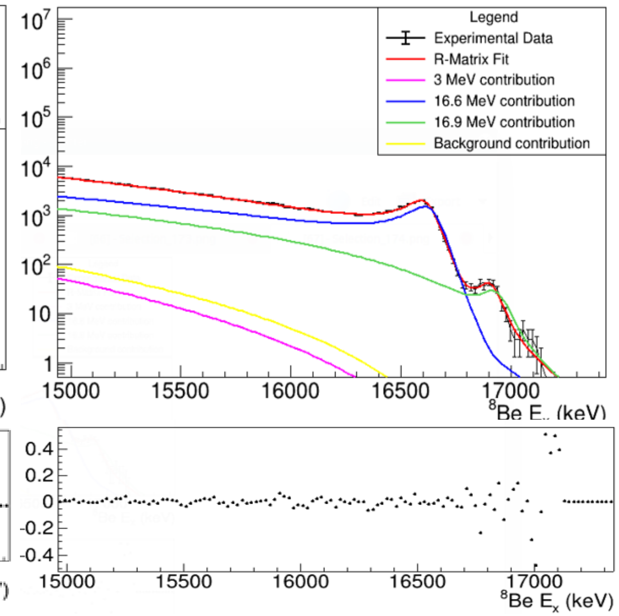
Check I: $\Gamma_{\alpha\alpha}$ = Fixed



Check II: $\Gamma_{\alpha\alpha}$= Let Free



Check III: E_{BKG} = 37 MeV



Levels	Parameters	Literature [Til04]	S.Viñals	Check I	Check II	Check III
	r_0(fm)	1.35	1.35	1.35	1.35	1.35
	χ^2(2-17.2 MeV)		14.4	3991	14.3	12.0
2_0^+	E (keV)	3030(10)	3058(31)	2959.3	3050.75	3036.9
	$\Gamma_{\alpha\alpha}$(keV)	1513(15)	1876(94)	1415.7	1949.8	1883.1
2_1^+	E (keV)	16626(3)	16632(83)	16616	16627	16623
	$\Gamma_{\alpha\alpha}$(keV)	108.1(5)	129.47(28)	180	123.98	114.67
2_2^+	E (keV)	16922	16921(85)	16919	16917	16913
	$\Gamma_{\alpha\alpha}$(keV)	74.0(4)	112.5	74.076	99.735	99.179
2_{Bkg}^+	E (keV)		21205	17238	23338	37000
	$\Gamma_{\alpha\alpha}$(keV)		119.11	104.26	1331.4	12116

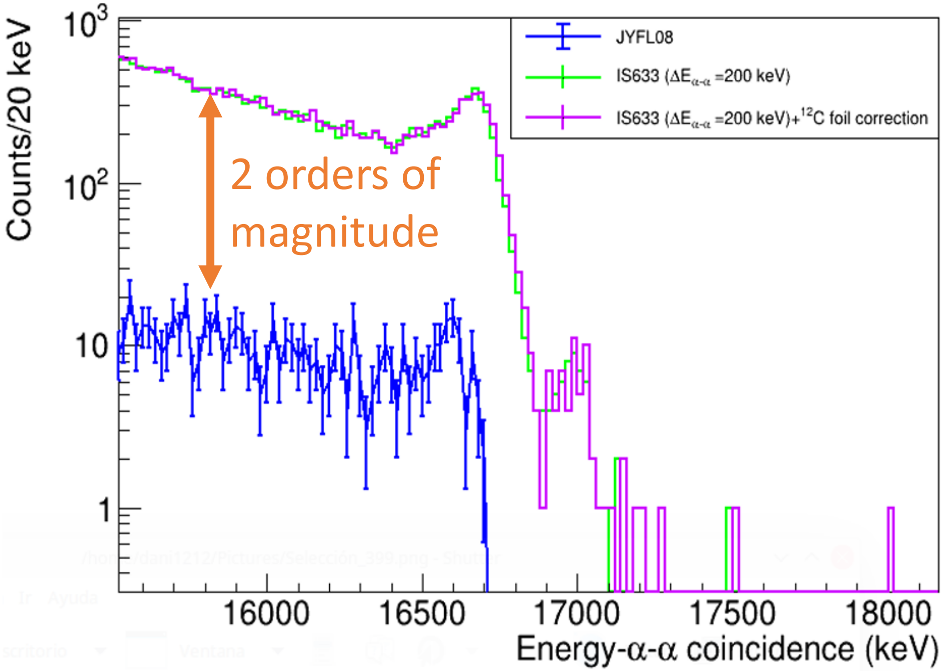
Results

- **$\Gamma_{\alpha\alpha}$= Fixed to Literature:** does not generate good results.
- **$\Gamma_{\alpha\alpha}$= Let free :** improves the global fit
- **E_{BKG} = 37 MeV:** Lowest χ^2 values; the larger value of the BKG.

JYFL08: experiment conducted in Jyväskylä studying the global shape of the spectrum

O Kirsebom. Phys. Rev. C, 83(6):065802–065822, 2011.

The R-Matrix fit produces a value of $\Gamma_{\alpha\alpha}^{3\,MeV}$ in accordance with the literature

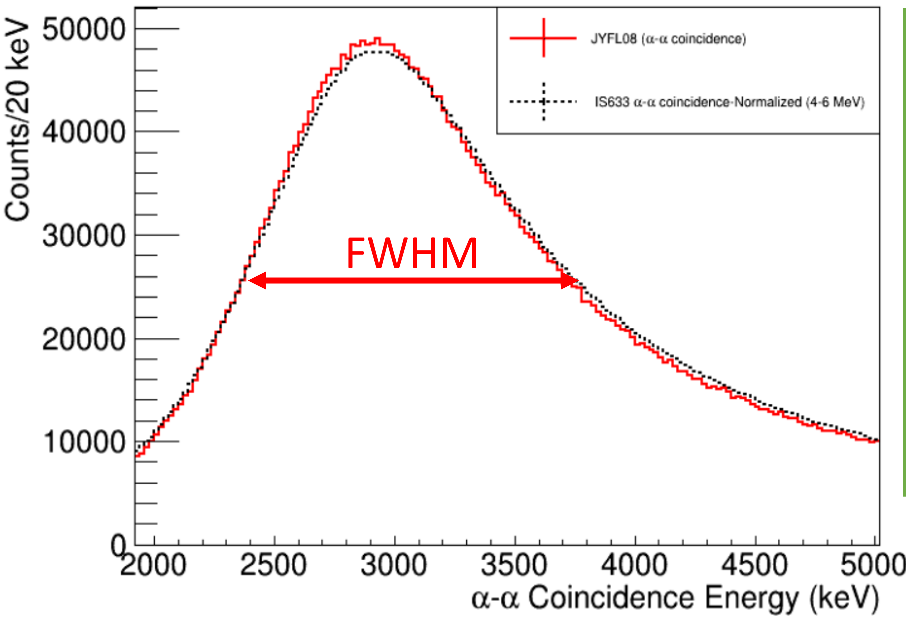


Levels	Parameters	Literature [Tea04]	JYFL08
	r_o(fm)	1.35	1.35
	χ^2(2-17.2 MeV)		0,97
2_0^+	E (keV)	3030(10)	3054
	B_{GT}	0.011813(56)	0.01020
	$\Gamma_{\alpha\alpha}$(keV)	1513(15)	1472
2_1^+	E (keV)	16626(3)	16544
	B_{GT}	-	-
	B_F	-	-
	$\Gamma_{\alpha\alpha}$(keV)	108.1(5)	355
2_2^+	E (keV)	16922	16887
	B_{GT}	-	-
	B_F	-	-
	$\Gamma_{\alpha\alpha}$(keV)	74.0(4)	120
2_{Bkg}^+	E (keV)		21000
	B_{GT}		0.032
	$\Gamma_{\alpha\alpha}$(keV)		176

- ❖ **JYFL08: production was not high enough to resolve the doublet**
- ❖ **linear fit to the most stable region (5-6 MeV) to determine a normalization factor.**

We will use the data of this experiment as a reference to compare with our data

Normalized IS633 data in agreement with JYFL08. → **Let´s analyze the 3 MeV peak**



- **Computing the FWHM maunually gives similar results in both data sets.**
- **Local fit to the 3 MeV level using 2 R-Matrix levels (3MeV+BKG).**
- **Fit to JYFL08 and IS633 data produce results in agreement with published values.**

Method Employed	Fitting Range (MeV)	Literature		JYFL08		IS633	
		E (keV)	$\Gamma_{\alpha\alpha}^{3\ MeV}$	E (keV)	$\Gamma_{\alpha\alpha}^{3\ MeV}$	E (keV)	$\Gamma_{\alpha\alpha}^{3\ MeV}$
Manual FWHM		3030	1513	2980	1510	2980	1525
R-Matrix Algorithm	2-4			3034	1488	2997	1470
	2-5			3036	1475	3006	1588
	2-6			3047	1516	3020	1655
	2-7			3060	1565	3030	1706

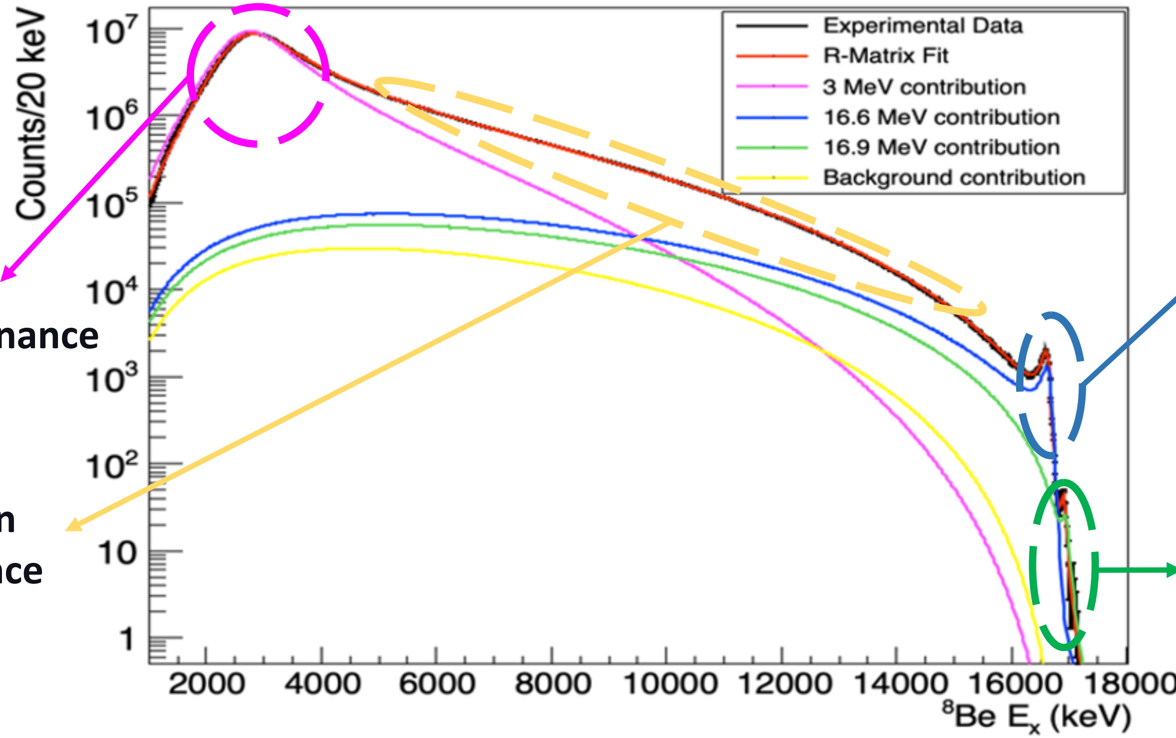
The local R-Matrix fit of the 3 MeV level, is in agreement, but starts to deviate when including the distribution > 6 MeV.

Local fits produce results in agreement with the literature → global fits don´t

The problem appears in the intermediate region (BKG level) → distortion of the 3 MeV resonance

3 MeV peak
3 MeV R-Matrix resonance

Intermediate Region
BKG (virtual) resonance



16,6 MeV peak
16,6 MeV R-Matrix resonance

16,9 MeV peak
16,9 MeV R-Matrix resonance

- **R-Matrix decomposes the spectrum in resonant levels.**
- **For an excitation to continuum, a virtual resonance must be used.**
- **This only works if the continuum is close to the resonant levels.**
- **But if that is not the case R-Matrix will not work.**

R-Matrix can not fit the whole spectrum due to the intermediate (not resonant) region.

Once finished with the R-Matrix discusión we obtain the mixture coefficients.

Method 1

$$\frac{\alpha^2}{\beta^2} = \frac{B_{16.9,F}}{B_{16.6,F}}$$

$$\frac{\alpha^2}{\beta^2} = \frac{B_{16.6,GT}}{B_{16.9,GT}}$$

Method 2

$$\alpha^2 = \frac{\Gamma_{16.6}}{\Gamma_0} = \frac{\Gamma_{16.6}}{\Gamma_{16.6} + \Gamma_{16.9}}$$

$$\beta^2 = \frac{\Gamma_{16.9}}{\Gamma_0} = \frac{\Gamma_{16.9}}{\Gamma_{16.6} + \Gamma_{16.9}}$$

	Isospin coefficient ratio $\left({\alpha^2}/{\beta^2}\right)$		
Method	Local 2^+ Fit	Global fit (S. Viñals)	Global fit (My best fit)
B_F	5 (12)	1.72(64)	1.71(48)
B_F	3.4 (5.0)	1.71(55)	1.73(42)
Γ	1.20 (15)	1.150(11)	1.22(13)

Low resolution in B_F and B_{GT}

Results in acordence with predicitons

The Isospin coefficient ratio obtained from the decay width is in accordance with theoretical predictions

First Experimental Confirmation!

- **IS633 is the first experiment that enables to study the 2^{+} doublet of ${}^{8}_{5}\mathrm{Be}$ by beta decay where Fermi and Gamow-Teller contributions could be separated.**
- **R-matrix formalism was employed to analyse the spectrum.**
- **The local fits to either low or high energy-region of the ^{8}Be excitation spectrum produces good results.**
- **The full spectrum fit produces E, Γ values for the 3 MeV state that differs from the ones adopted in the literature. It is important to indicate that we do global fits.**
- **We performed cross-checks to ensure that our results are consistent and do not suffer from systematic errors such as summing or piled-up.**
- **Comparison with JYFL08 assure that IS633 is consistent with previous results.**
- **Fitting including the intermediate "non-resonant" region → distorts the results**
- **The obtained result indicate the two doublet states are fully mixed.**

Thank you for your attention

Extra Slides

Residue Function (Discusion)

Re-visiting the LT Data

- There were some discussions concerning the residue function

  - Karsten proposes the following formula:

$$R = \frac{Set1 - Set2}{\sqrt{Set1 + Set2}}$$

- Which is strange since the formula is not adimensional

  - Maria Jose instead proposes this one

$$R = \frac{Set1 - Set2}{\sqrt{Set1^2 + Set2^2}}$$

- The difference between both formulas is significant

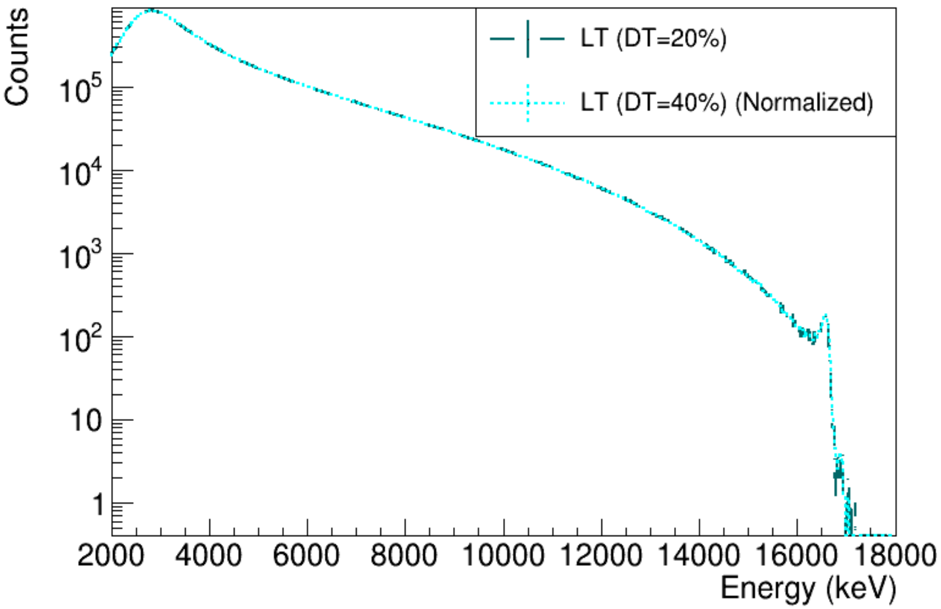
  - Lets see an example

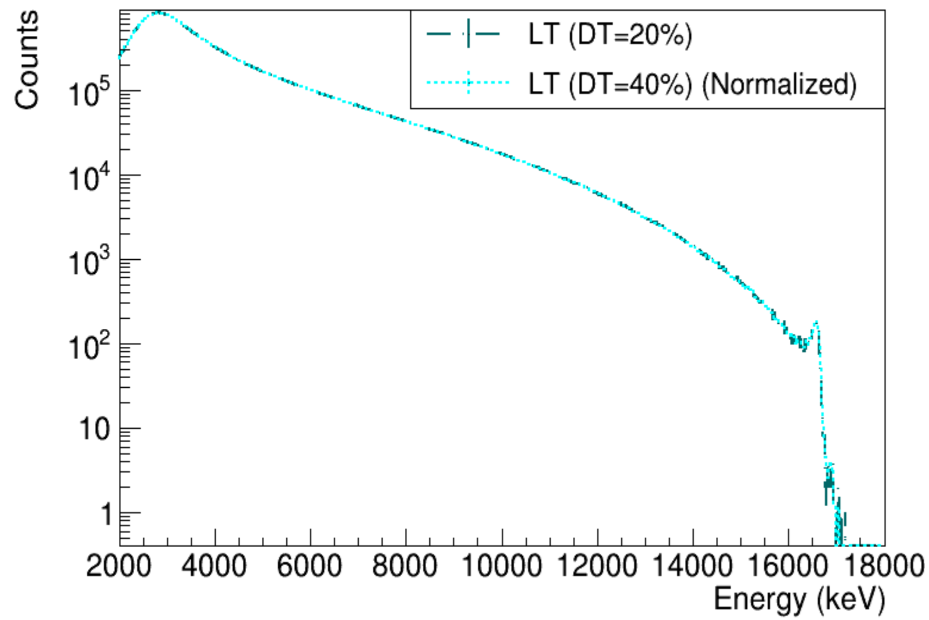
Comaprasion both formulas LT

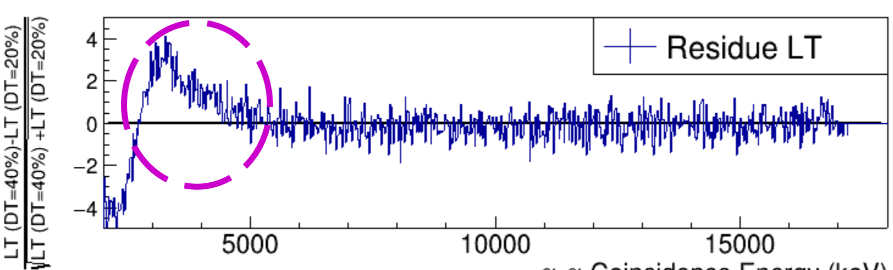
- We shall compare the behaviour of the Residue Function for both formulas using the LT data as an example

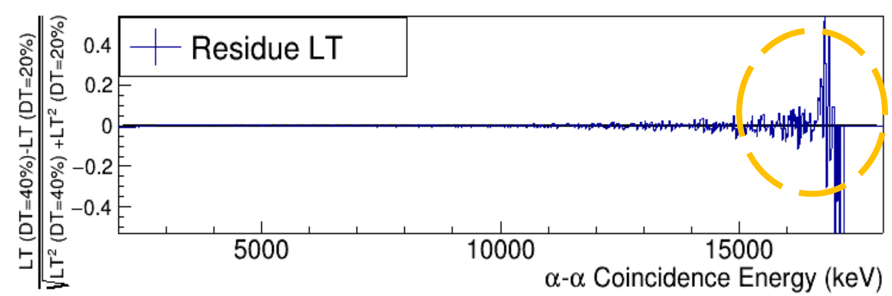
$$R = \frac{Set1 - Set2}{\sqrt{Set1 + Set2}}$$

$$R = \frac{Set1 - Set2}{\sqrt{Set1^2 + Set2^2}}$$









- For Karsten´s formula, the discrepancy is huge at 3 MeV.
- For Maria Jose´s the discrepancy is larger at the doulet.

Discusion of the Residuals

- The different behaviour makes sense if we examine the limits of both functions.
- For both LT Set 1 and LT Set 2 approaching infinity (according to Wolfram Alpha)

$$\lim_{Set1;Set2 \to \infty} \frac{Set1 - Set2}{\sqrt{Set1 + Set2}} \to \infty \qquad \lim_{Set1;Set2 \to \infty} \frac{Set1 - Set2}{\sqrt{Set1^2 + Set2^2}} \to 1$$

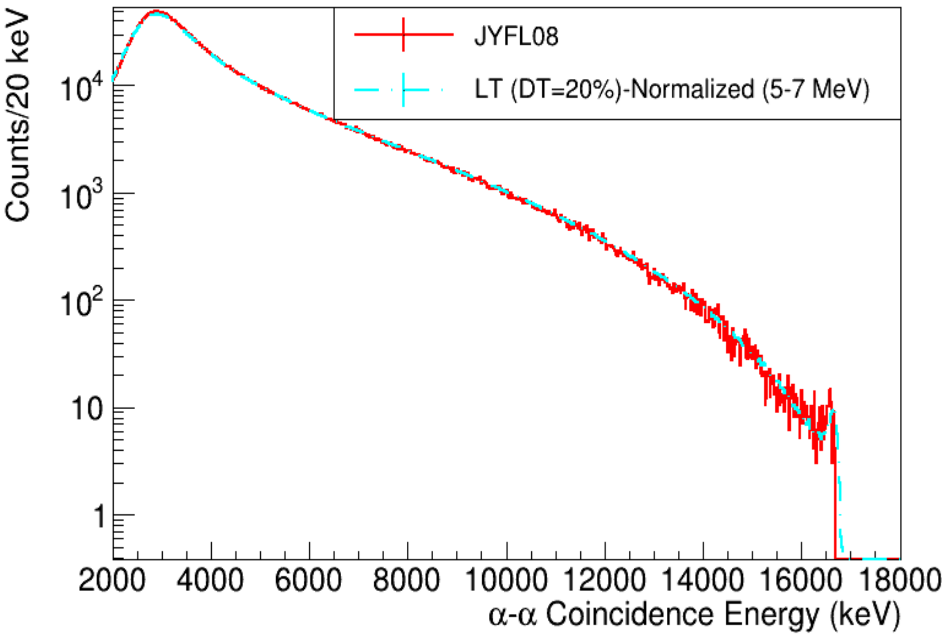
- This implies that when both LT sets exhibit a growing tendency the behaviour of the Residue Function can be different

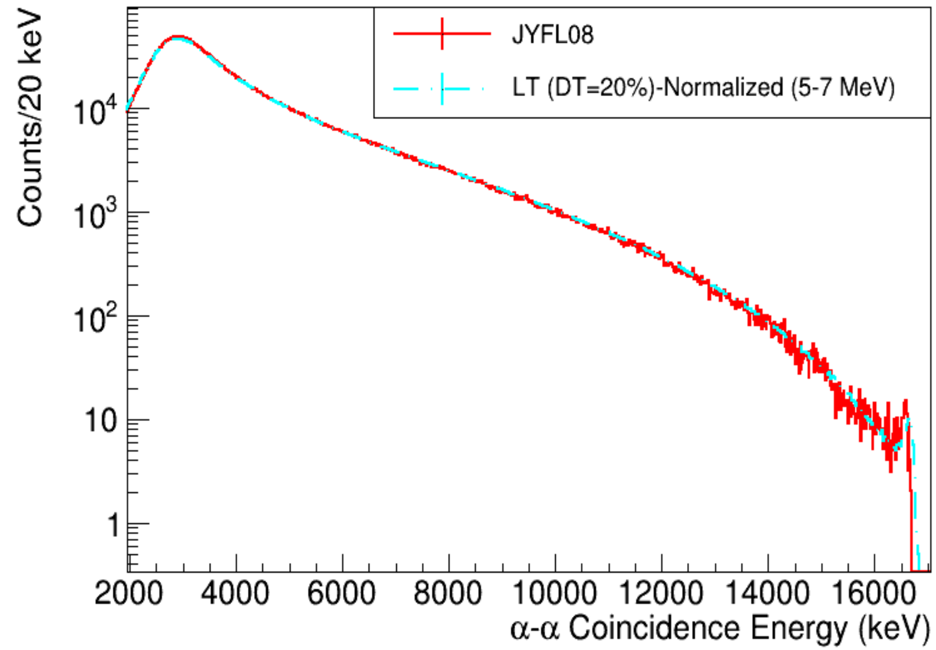
I will compare the behaviour of both functions when we compare JYFL098 and IS633.

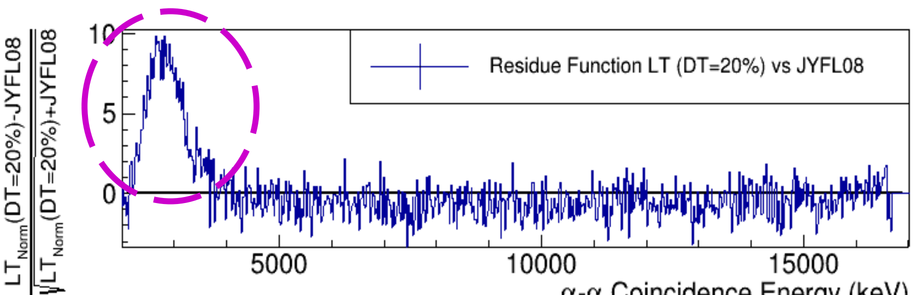
JYFL08 vs LT (IS633)

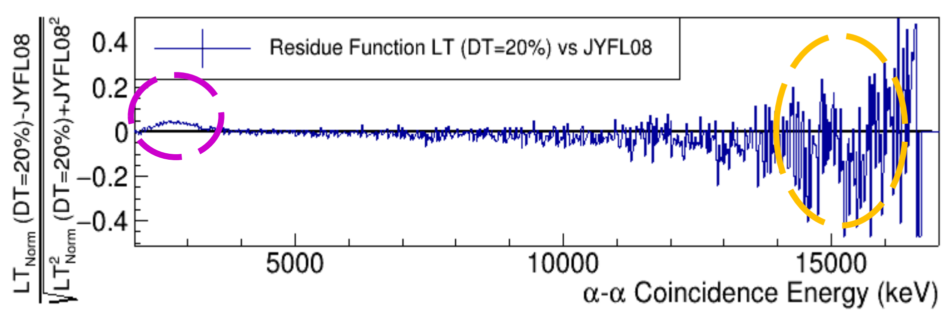
$$R = \frac{Set1 - Set2}{\sqrt{Set1 + Set2}}$$

$$R = \frac{Set1 - Set2}{\sqrt{Set1^2 + Set2^2}}$$









- As we can appreciate Karsten´s residue function indicates a huge discrepancy in the 3 MeV pea
- Maria Jose´s residue function indicates a larger discrepancy in the doublet

Results of the comparasion

- Karsten´s and Maria Joses´s definitions of the residue function give fundamentally different results:
  - Karsten: Greater difference in the 3 MeV peak
  - Maria Jose: Greater difference in the doublet.
- In my opinion the results of the second formula look more logical for the following reasons
  - Maria Jose´s formula is adimensional
  - If the discrepancy in the 3 MeV region is so pronounced, it should manifest in the FWHM (which is doesn't).

Even if there is such a discrepancy, ORM_FIT indicates that fitting to the 3 MeV region of IS633 and JYFL089 produces very similar results for all parameters. The intermediate region is the main problem

3 data set where recorded. Each of them with different electronic settings

Low Thresholds
(40% of dead time)

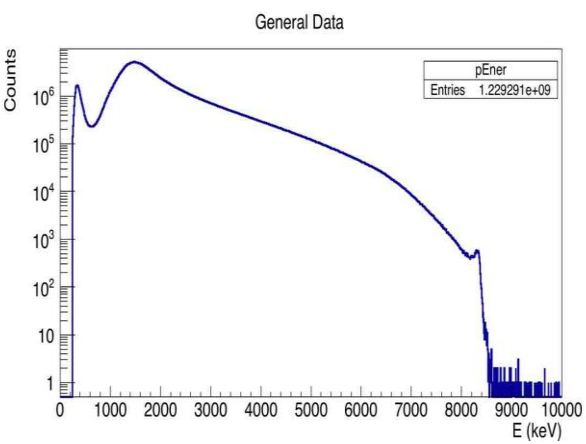
- **A = 6000 Bq**
- **Obtain general spectra**
- **60 GB**

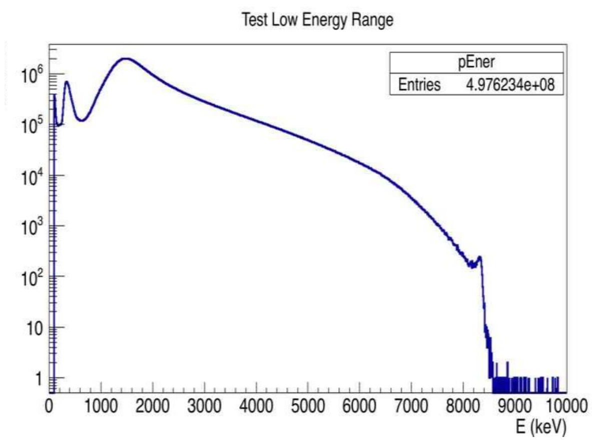
Low Thresholds
(20% of dead time)

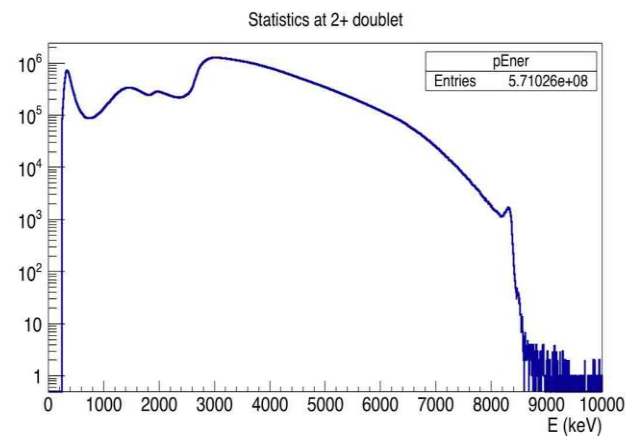
- **A = 5000 Bq**
- **Test sensitivity at low energy range**
- **22 GB**

High Thresholds
(15% of dead time)

- **A = 6000 Bq**
- **Statistics in 2+ doublet**
- **40 GB**
- **Distorted spectra at low energies**







Individual tests were employed to ensure the 3 MeV level is not distorted.

No significant difference was found in the 3 MeV peak

The R-Matrix fits allow us to study the Isospin mixing through two methods

Method 1

$$|a\rangle = \alpha|T=0\rangle + \beta|T=1\rangle$$
$$|b\rangle = \beta|T=0\rangle - \alpha|T=1\rangle$$
$$\alpha^2+\beta^2=1$$

$$M_{a,X} = \langle a|O_x|^8B\rangle \rightarrow \begin{cases} M_{a,F} = \sqrt{2}\beta \\ M_{a,GT} = \alpha M_{0,GT} + \beta M_{1,GT} \end{cases}$$

$$M_{b,X} = \langle b|O_x|^8B\rangle \rightarrow \begin{cases} M_{b,F} = -\sqrt{2}\alpha \\ M_{b,GT} = \beta M_{0,GT} - \alpha M_{1,GT} \end{cases}$$

$$B_{16.6,F} = 2C\beta^2$$
$$B_{16.9,F} = 2C\alpha^2$$

$$B_{16.6,GT} = C'(\alpha M_{0,GT} + \beta M_{1,GT})^2$$
$$B_{16.9,GT} = C'(\beta M_{0,GT} - \alpha M_{1,GT})^2$$

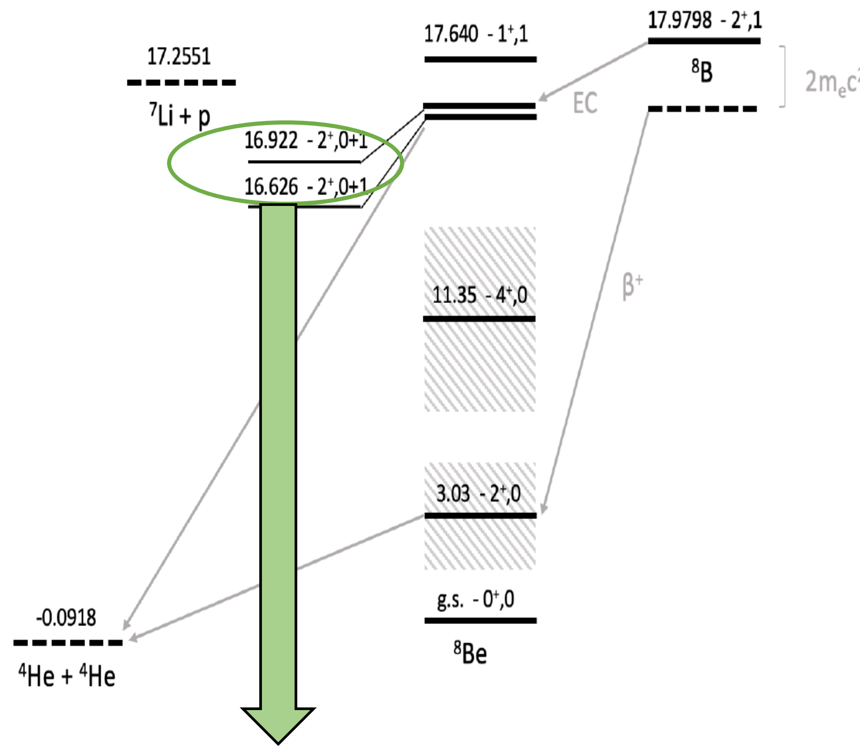
$$\frac{\alpha^2}{\beta^2} = \frac{B_{16.9,F}}{B_{16.6,F}}$$

$$\frac{\alpha^2}{\beta^2} = \frac{B_{16.6,GT}}{B_{16.9,GT}}$$

$$\cancel{\frac{\alpha^2}{\beta^2} = \frac{B_{16.9,GT}}{B_{16.6,GT}}}$$

Method 2

$$\alpha^2 = \frac{\Gamma_{16.6}}{\Gamma_0} = \frac{\Gamma_{16.6}}{\Gamma_{16.6}+\Gamma_{16.9}}$$
$$\beta^2 = \frac{\Gamma_{16.9}}{\Gamma_0} = \frac{\Gamma_{16.9}}{\Gamma_{16.6}+\Gamma_{16.9}}$$



Mix isospin coeficinets

	Isospin coefficient ratio $\left(\alpha^2/\beta^2\right)$	
Method	**Local 2^+ Fit**	**Global Fit**
B_F	5 (12)	1.72 (64)
B_{GT}	3.4 (5.0)	1.71 (55)
Γ	1.20 (15)	1.150 (11)