Phys.Rept. 658 (2016) 1

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Dispersive and analytic methods for light scalars
Where do we stand?

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JRP. Physics Reports 658-(2016)-1
JRP, A.Rodas, J. Ruiz de Elvira, Eur. Phys. J. Spec. Top. (2021) 230:1539
JRP & A.Rodas, Physics Reports 969-(2022)-1

EuNPC2022- Santiago de Compostela. 24-28th October 2022

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Motivation for dispersive/analytic studies of the scattering of pions, kaons and etas

- π,K,η are Goldstone Bosons of QCD → Test Chiral Symmetry Breaking

- π,K appear as final products of almost all hadronic processes: B,D, decays, CP violation…

- **SPECTROSCOPY**: main or relevant source for PDG parameters of many resonances.

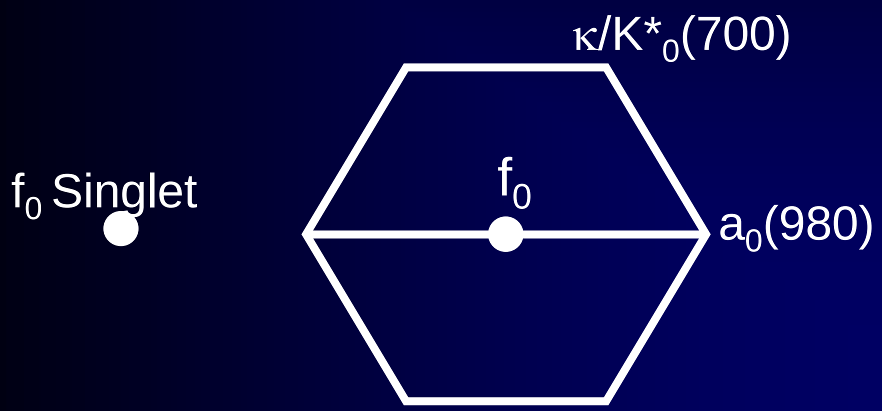
  - Relevant for glueball identification

  - **CRYPTOEXOTICS**: The controversial **light scalar resonances** appear here: **f_0(500)**, f_0(980), a_0(980) and strange **K^*_0(700)**.
    Strong indications for predominant non quark-antiquark nature of light scalars

Non-ordinary spectroscopic classification

- Lightest scalar SU(3) multiplets <2 GeV. Accepted picture at RPP

Light scalar nonet <1 GeV:

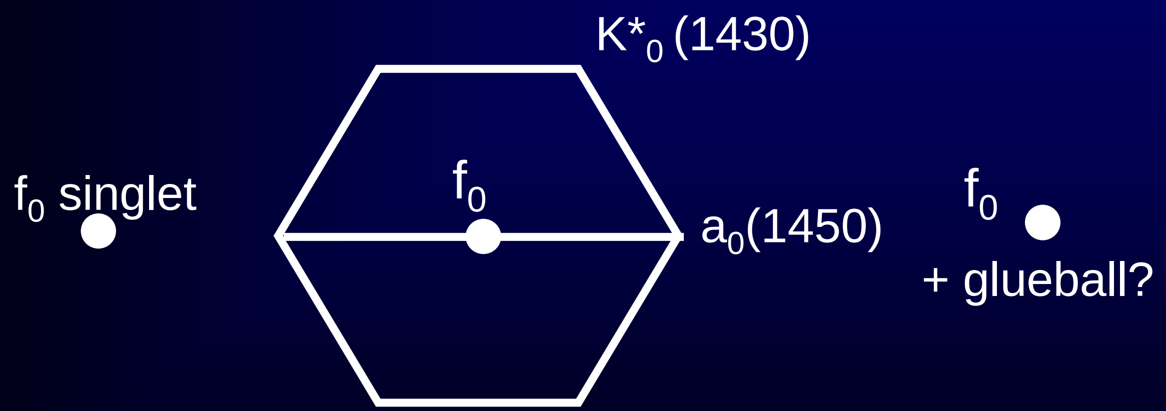


Non-strange heavier!!
Hugely Inverted $q\bar{q}$ hierarchy.
Cryptoexotics? (R.Jaffe 1976)

f_0(500) and f_0(980) octet/singlet mixtures
κ/K^*_0(700) only recently "well established at PDG"
Only in 2021 on-line update "Needs Confirmation"

Scalar nonet >1 GeV:

One extra state f_0(1370), f_0(1500), f_0(1710)



Also, not quite $q\bar{q}$ hierarchy

complicated mixtures
f_0(1370) worst determined and still contested because hard to see

Why cryptoexotics?

- Inverted hierarchy (tetraquarks, molecules) (Jaffe 76)

- Scalar quark antiquark, expected above 1 GeV in QM but once meson-meson interactions included, and additional companion pole appears at low energies (Van Beveren, Ribeiro, Rupp, Close, Tornqvist, Oller, Oset….)

- Contrary to vectors, do not saturate ChPT parameters, although being lighter (Gasser, Leutwyler, Ecker, De Rafael, Pich, Donoghue….)

- Dynamically generated from LO Unitarized ChPT whereas vectors need NLO (Oller, Oset, Pelaez,Nieves, Arriola…)

- Do not fit in linear Regge trajectories (J,M^2) (Anisovich, Sarantsev, Anisovich)

- Dispersively calculated Regge trajectories turn non-ordinary (Londergan, Nebreda, Peláez, Szczepaniak, Rodas)

- Non-ordinary 1/Nc leading behavior (Pelaez, Nebreda, Rios, Nieves, Pich, Oller, Ruiz de Elvira

DISCLAIMER: time is limited

Relatively old stuff, although now present in PDG review on “Spectroscopy of light mesons”

Here I concentrate on the existence and determination of parameters from **meson-meson scattering**

Two longstanding sources of trouble
In meson-meson scattering

DATA PROBLEM

MODEL-DEPENDENCE PROBLEM

THIS TALK

Overview of effort to discard inconsistent data and **eliminate or reduce model dependence:**
by using

DISPERSIVE/ANALYTICITY APPROACHES

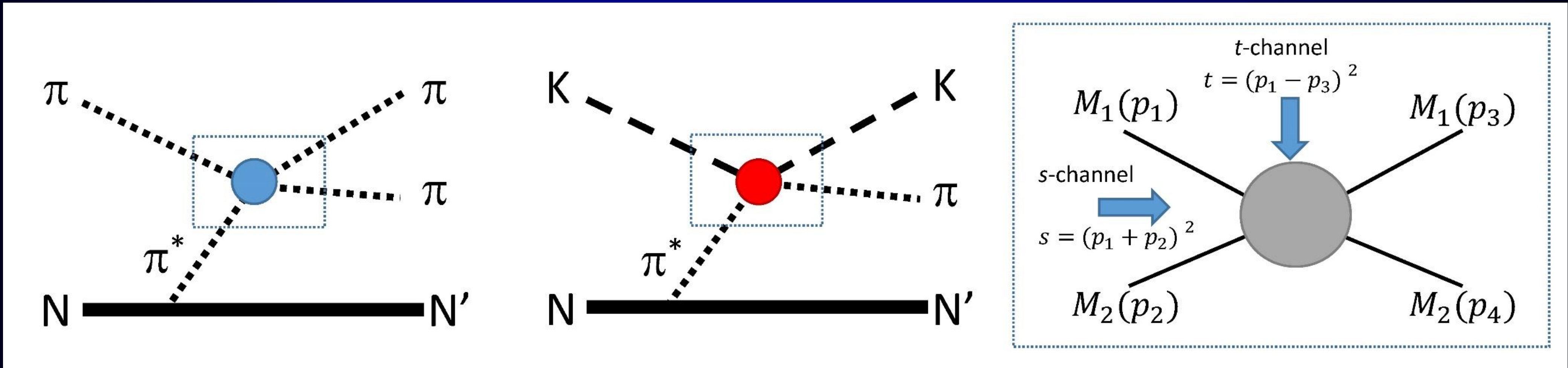
DATA PROBLEM: **Meson-meson SCATTERING data are poor**

π and K unstable. Beams NOT luminous enough for **ππ and πK** collisions:

<u>Indirect measurements</u>

<u>1) Very few good data from $K \to \pi\pi e\nu$</u>. But $E<M_K$. Geneva-Saclay (77), E865 (01), **NA48/2 (2010)**

<u>2) ALMOST ALL DATA from Meson-Nucleon scattering</u> (In the 70's and 80's)



CAVEATS: One-Pion-Exchange (OPE) Approximation

In initial state virtual pion not well defined, Chew-Low off-shell extrapolation

More contributions: absorption, A_2 exchange...

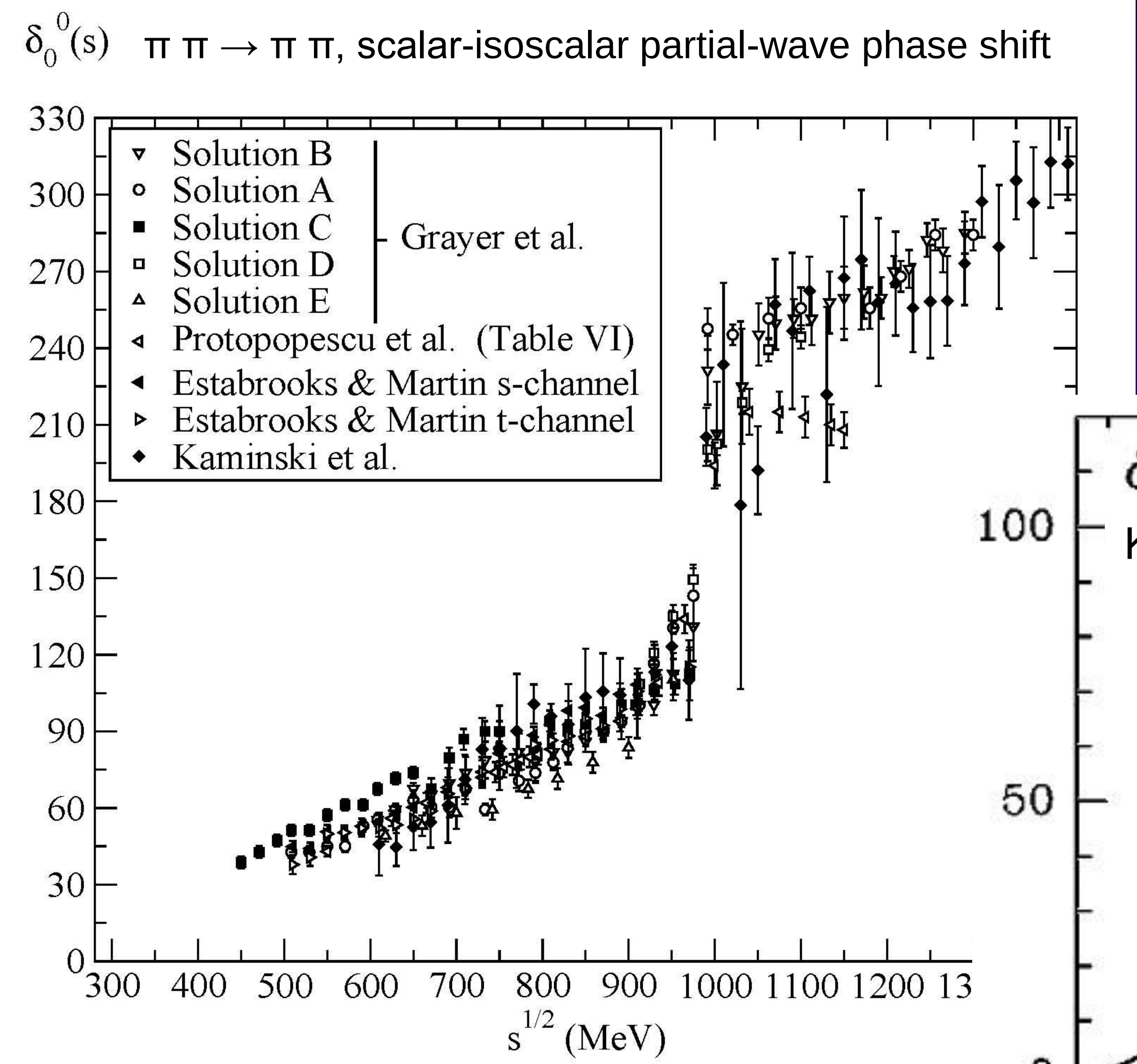
Needs Meson-N partial-wave extraction. Problems with phase shift ambiguities, etc...

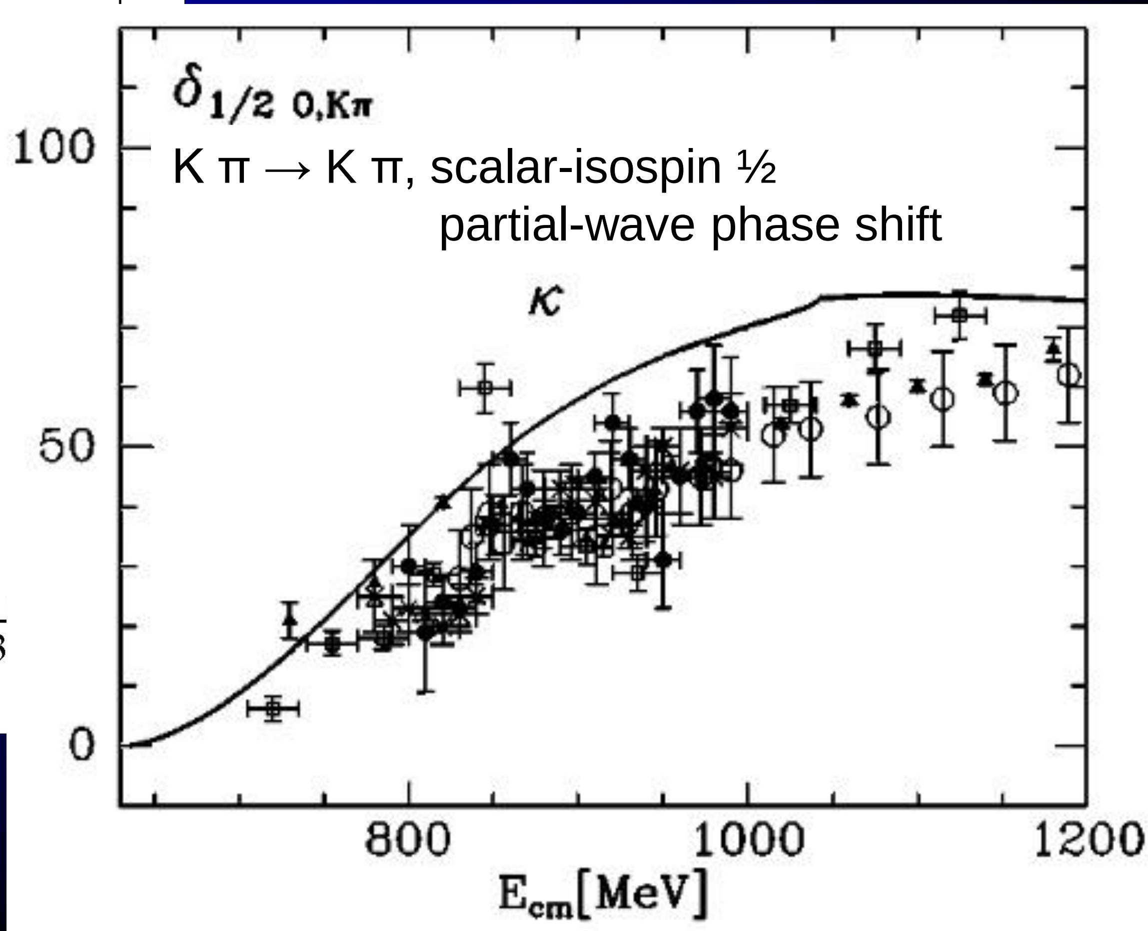
As a consequence... VERY LARGE SYSTEMATIC UNCERTAINTIES

DATA PROBLEM

ππ and πK SCATTERING data are often in conflict

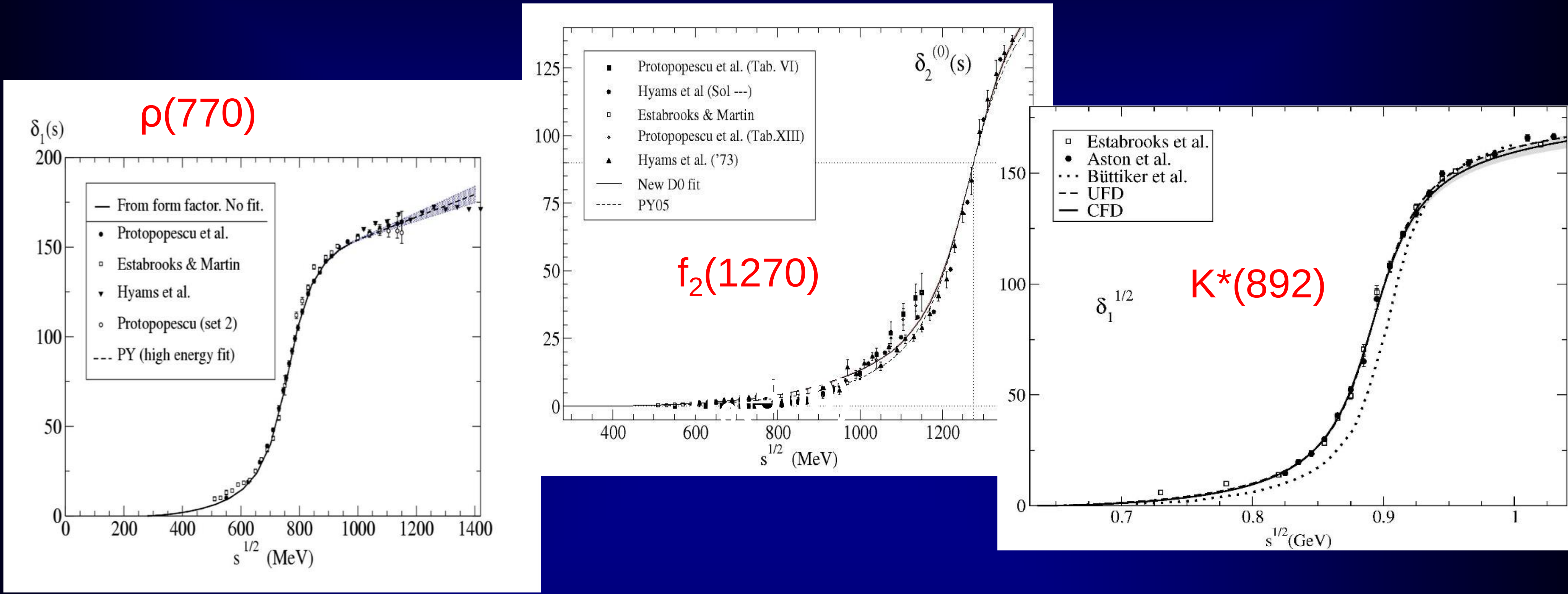
CONFLICTING DATA SETS & SYSTEMATIC uncertainties larger than STATISTICAL





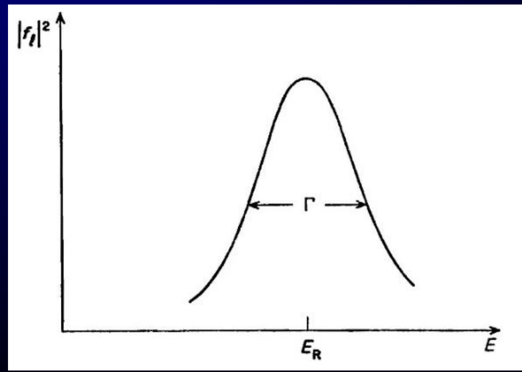
MODEL PROBLEM: many models used to fit data and extract resonances…

Narrow resonances when far from other resonances or singularities (thresholds, cuts, etc…) produce typical peaks and rapid 180º phase motions



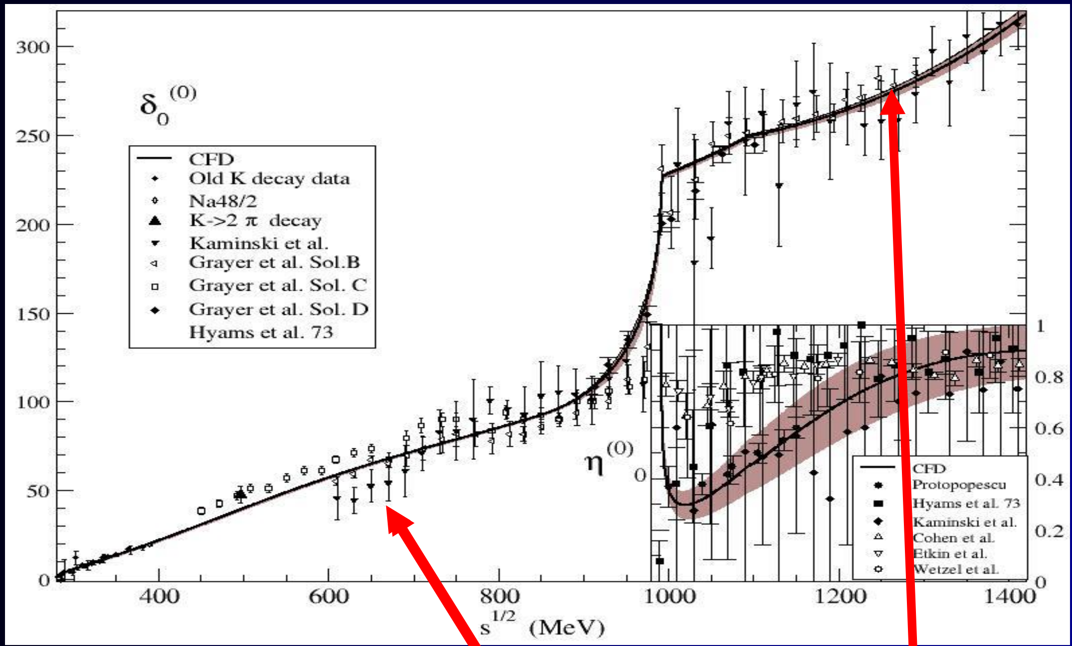
These were the “easy ones” and models are usually fine.
For instance, they may be reasonably well approximated by Breit-Wigner shapes

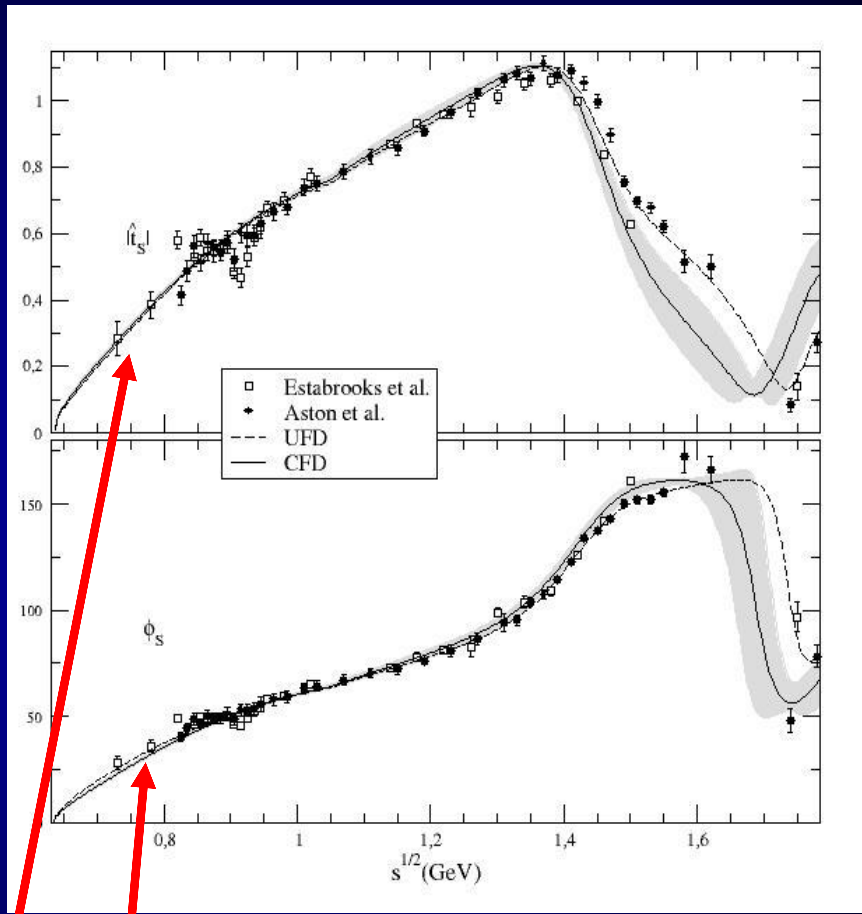
$$\sim \frac{M\,\Gamma(s)}{M^2 - s - iM\,\Gamma(s)}$$



MODEL PROBLEM: Resonances in meson-meson scattering

"Breit-Wigner" shapes are easily recognizable… but life is not that easy





Do you see resonances there?

Nevertheless there are resonances (poles) in these regions: the $\sigma/f_0(500)$, $f_0(1370)$ and $\kappa/K_0^*(700)$ light scalars

MODEL PROBLEM: Resonance shapes process dependent

The not so easy ones…

- Light scalars are wide, or even extremely wide and frequently overlap with one another or with thresholds like KK.

- Very often they do not produce clear peaks, nor rapid phase motions, and "peak searching" not valid anymore

- Moreover since they are not clear-cut peaks, **their shape**, apparent position, width… **can be different depending on the process where they are observed**.

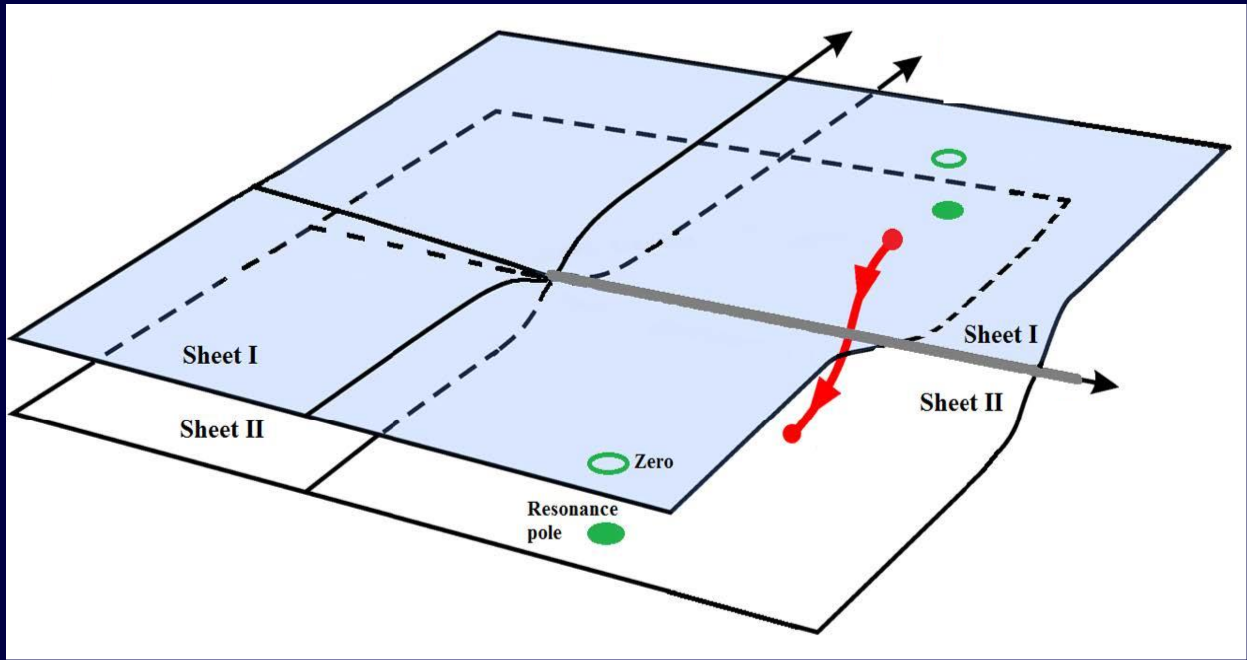
- Model fits to different process or partial data can yield different resonances

- Meson-meson scattering has the strongest theory constraints and is the most reliable theoretically to go to the complex plane. (non-linear unitarity condition)

The universal features of resonances are their pole positions and residues *

$\sqrt{s_{pole}} \approx M - i\,\Gamma/2$

*in the Riemann sheet obtained from an analytic continuation through the physical cut

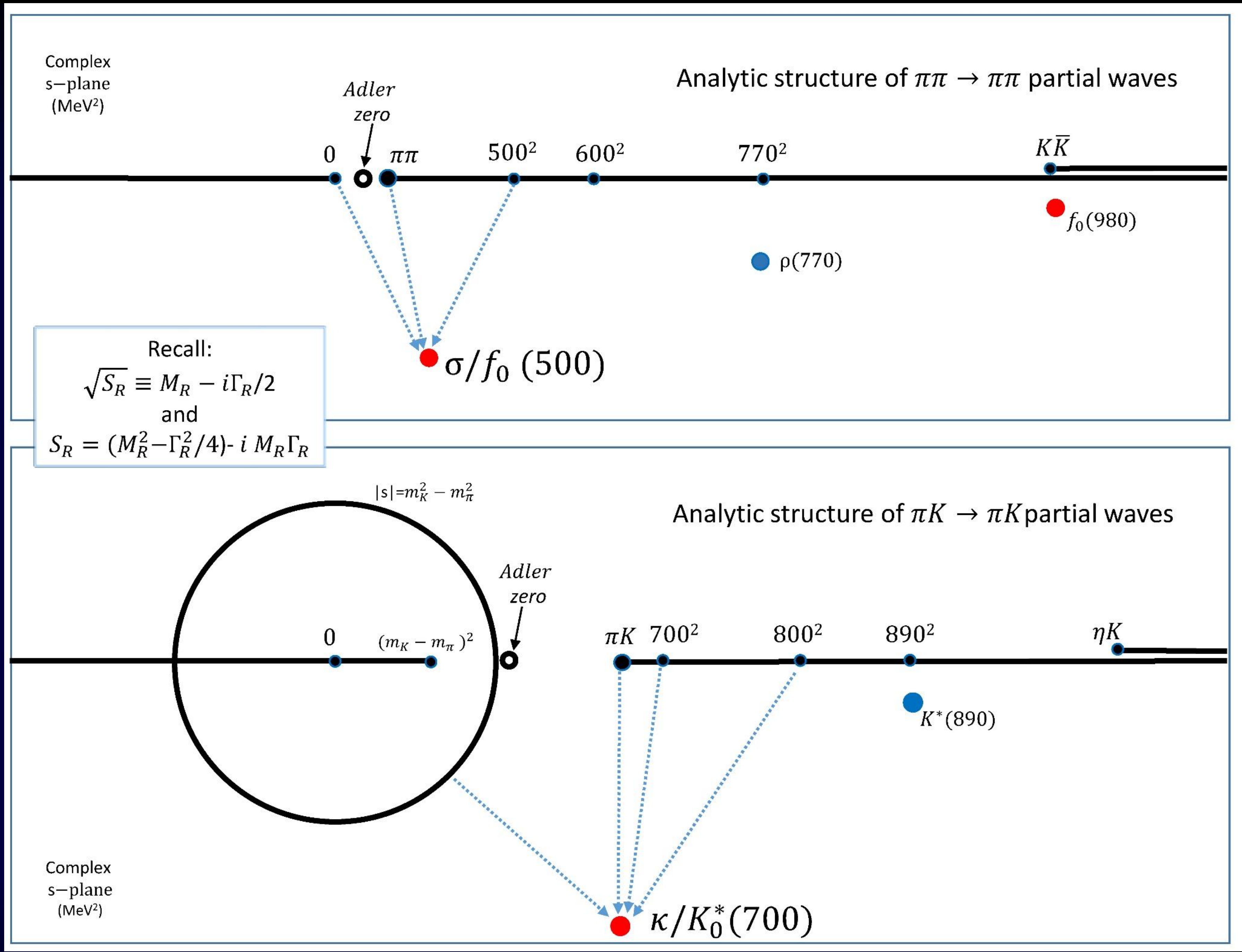


The Review of Particle Physics has been adding pole determinations for more and more resonances

unfortunately keeping also Breit-Wigner parameters even when not applicable

However, analytic continuations are a delicate mathematical problem and a good control of the analytic structure is needed. Many models fail at this.

Why worry about low-energy & ANALYTIC STRUCTURE in s-plane?

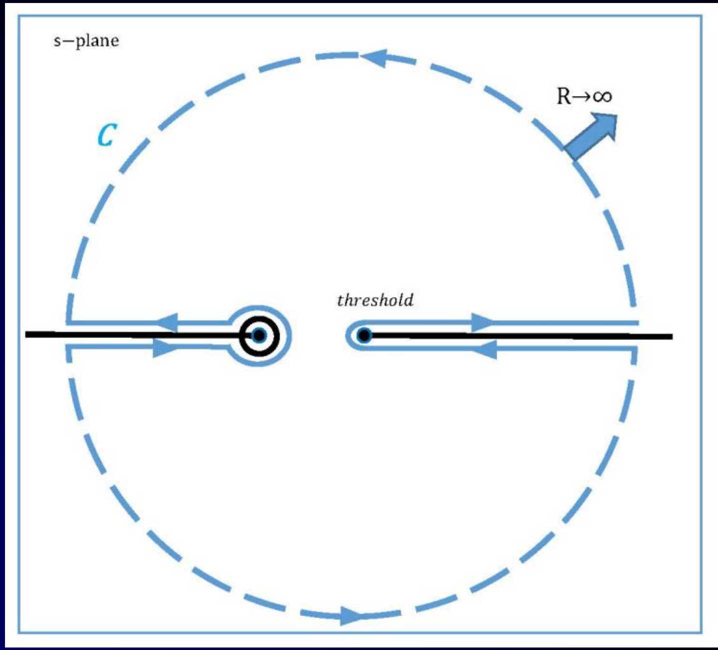


Important for
$\sigma/f_0(500)$
$\kappa/K_0{*}(700)$

- Threshold behavior
  Chiral symmetry
- Subthreshold behavior
  Chiral symmetry→Adler zeros
- Other cuts
  Left & circular

What is a dispersion relation.? Very Briefly and for π π

- CAUSALITY ⇒ Amplitudes t(s) are ANALYTIC in complex s plane with cuts due to thresholds (also in crossed channels)



- Cauchy Theorem:

If t(s)→0 fast enough at high s, curved part vanishes

$$t(s) = \frac{1}{\pi}\int_{th}^{\infty}\frac{Im\,t(s')}{s-s'}ds' + LC$$

Otherwise, determined up to polynomial (subtractions)

Good for:

1) Calculating t(s) as an integral where there is not data
2) Constraining data analysis: Input =output
3) ONLY MODEL INDEPENDENT extrapolation to complex s-plane

Last decades

Effort to **eliminate or reduce model dependence** by using **dispersive approaches** often combined with **Chiral Perturbation Theory (ChPT).**

We need to get rid of one variable to write CAUCHY THEOREM for the other

1) Fix one variable in terms of the other (fixed-t, hyperbolic relations…)

Most popular: $t_0=0$, Forward Dispersion Relations (FDRs).
(Kaminski, Pelaez , Yndurain, Garcia Martin, Ruiz de Elvira, Rodas)

PROS: One eq. per amplitude. Simple. High energy reliable. Applicable to all energies
Precision
CONS: No direct access to poles… until recently (see below)

2) Integrate one variable: Partial wave dispersion relations

- "Roy-like" equations. GKPY eqs, Roy Steiner Equation-

Crossing to rewrite Left/circular cuts with. crossing in terms of physical region.
CONS: Different partial waves or channels coupled. In practice, limited to a finite energy
PROS: Directly partial waves. Better to look for poles. Precision

Precision Dispersive studies

- Unitarized Amplitudes (IAM, N/D, Chew-Mandelstam…)

2-body unitarity exact on dispersion relation for inverse amplitude (single or coupled channels)
Ideally combined with ChPT for these approximations, but additional bare/preexisting resonances could be added, simple models for real part, use of Lagrangians, effective theories etc…
CONS: Unphysical cuts, higher energies, multibody, approximated
PROS: Directly partial waves. Better to look for poles. Connection with QCD through ChPT in UChPT

Precision studies. Two strategies on real axis:

- SOLVE equations: (Ananthanarayan, Colangelo, Gasser, Leutwyler, Caprini, Moussallam, Stern…)

  S and P wave solution for Roy or GKPY equations unique at low energy

  Needs input on other waves and high energy.

  NO scattering DATA used at low energies ($\sqrt{s} \leq 0.8 \sim 1\ GeV$)

  Good if interested in low energy scattering and do not trust data.

  Uses ChPT input for threshold parameters

- Impose Dispersion Relations on fits to data. (García-Martín, Kaminski,JRP, Ruiz de Elvira, Ynduráin)

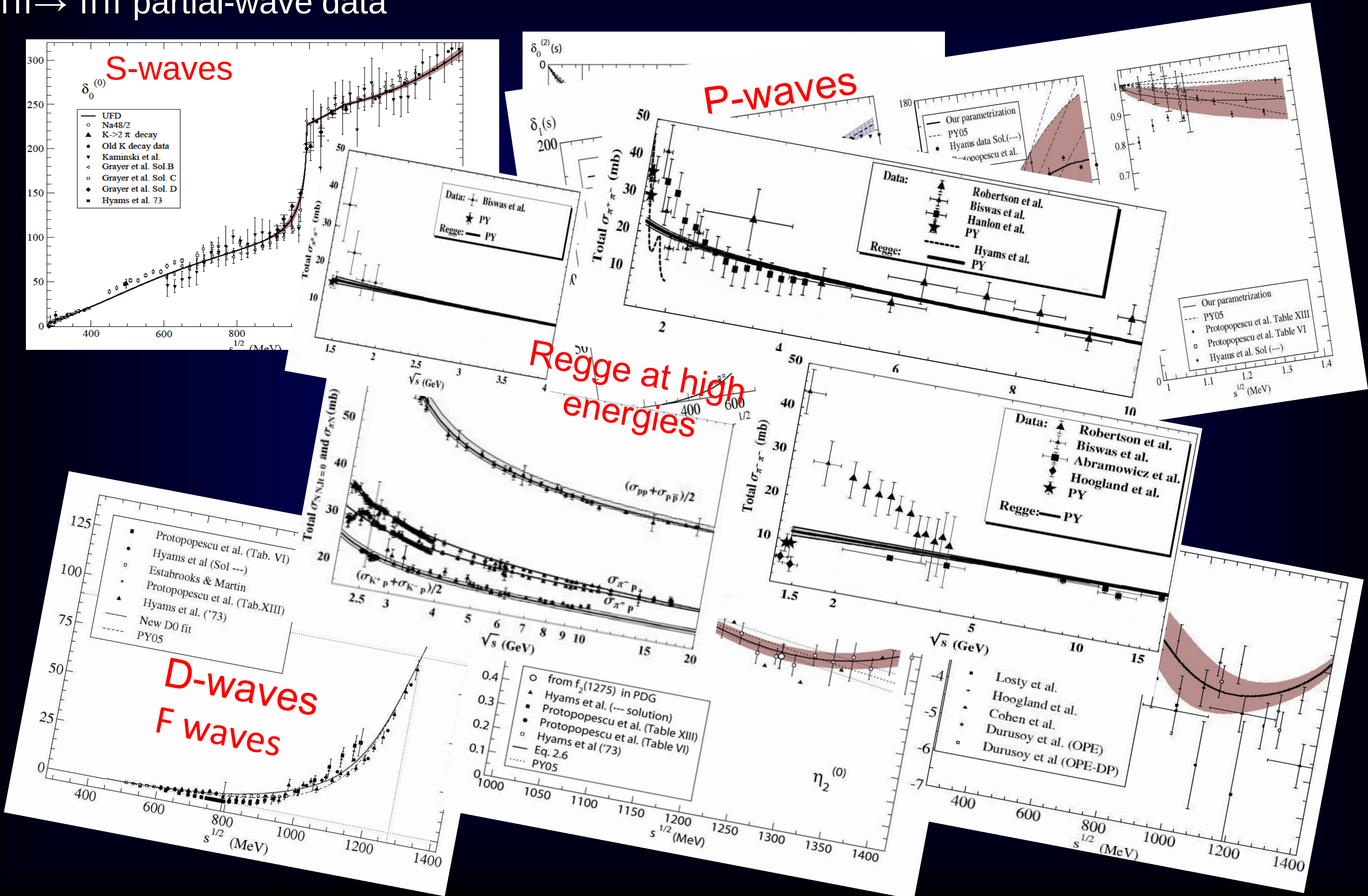
  Also known as "DATA driven dispersive analyses"

  Also needs input on other waves and high energy.

FIRST STEP: Simple Unconstrained Fits (UFD) to data

Estimation of statistical and SYSTEMATIC errors

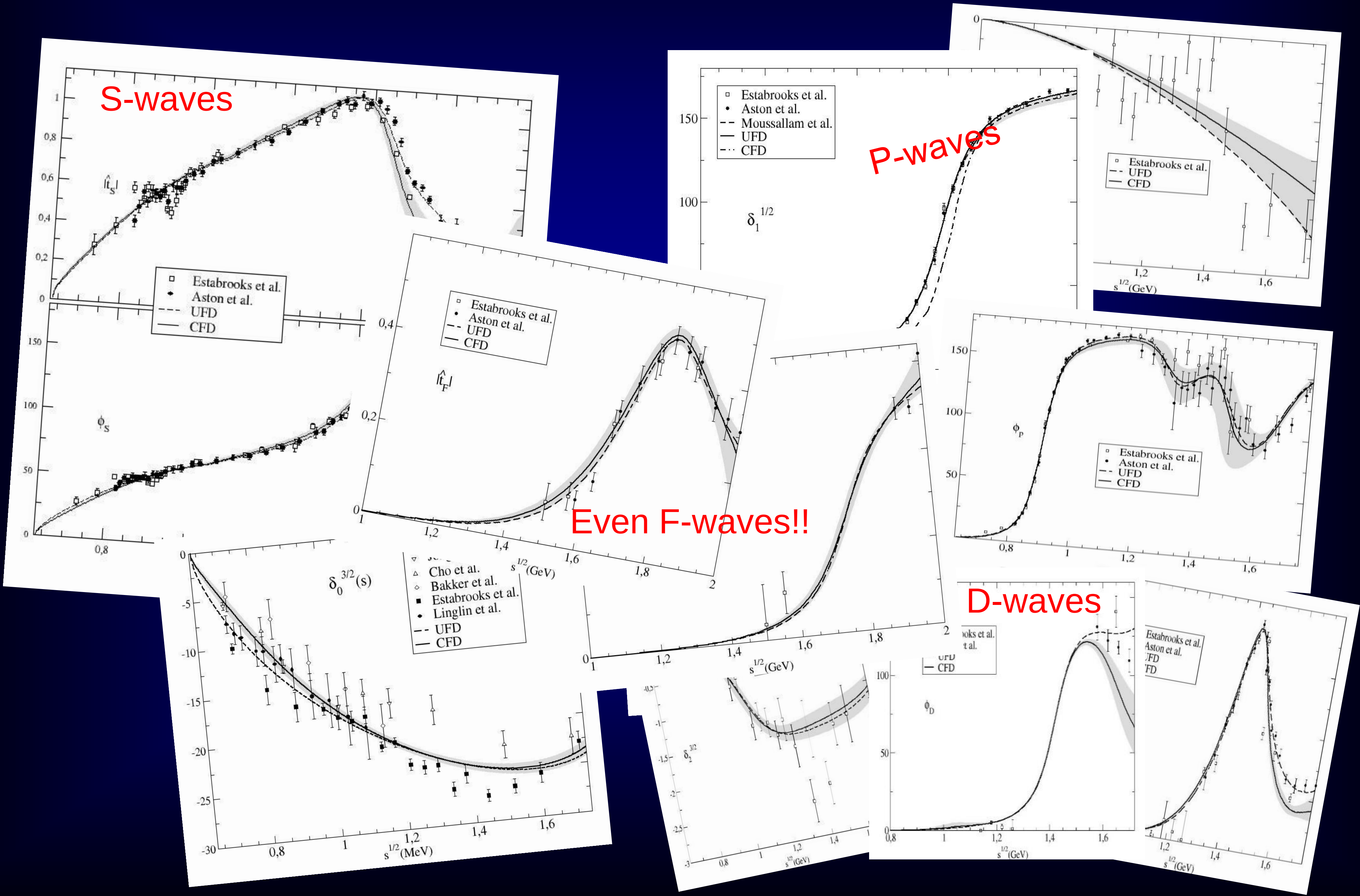
ππ→ ππ partial-wave data



FIRST STEP: Simple Unconstrained Fits (UFD) to data

Estimation of statistical and SYSTEMATIC errors

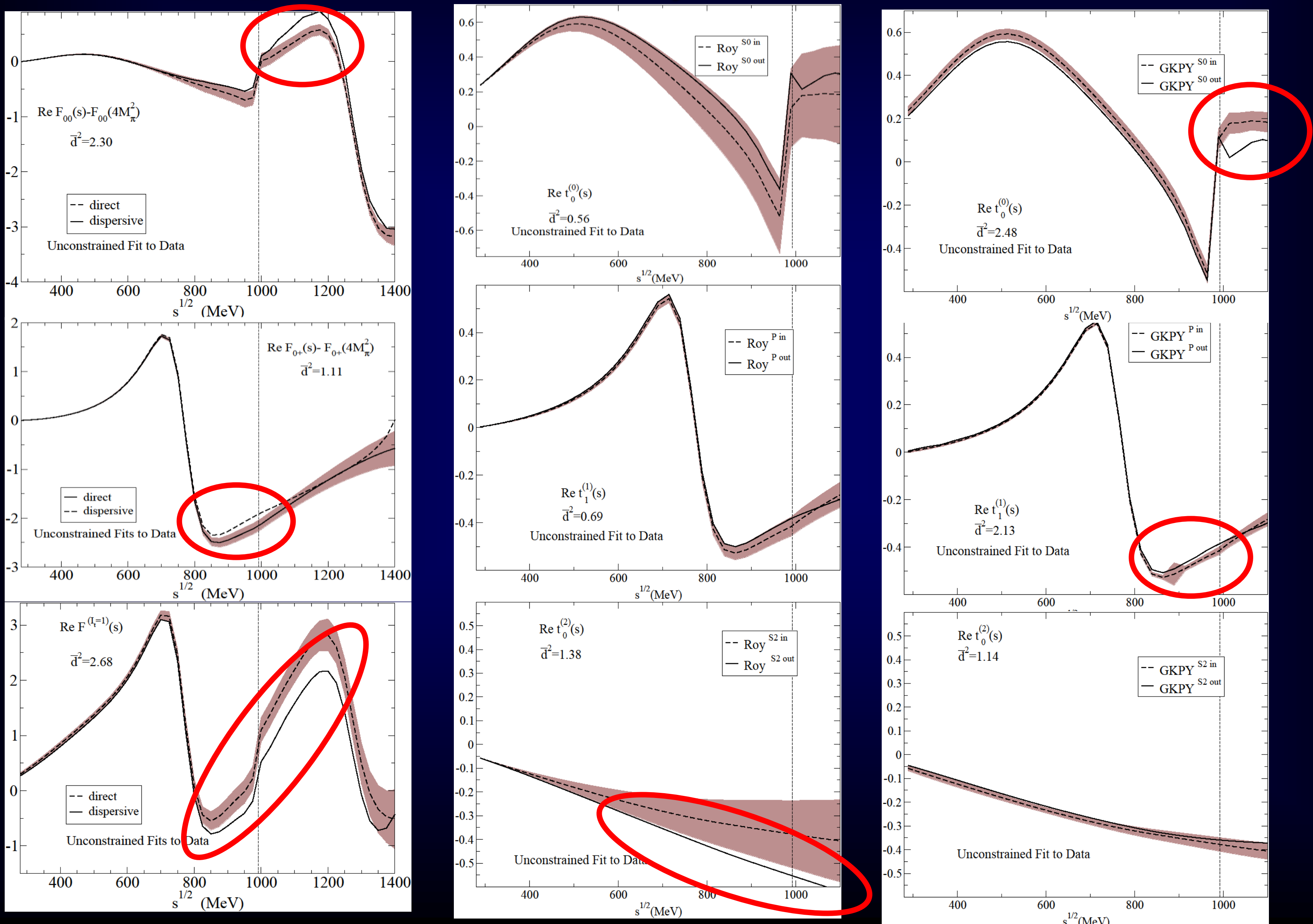
πK and ππ→KK partial-wave Data



SECOND STEP: Check dispersion relations: ππ→ππ

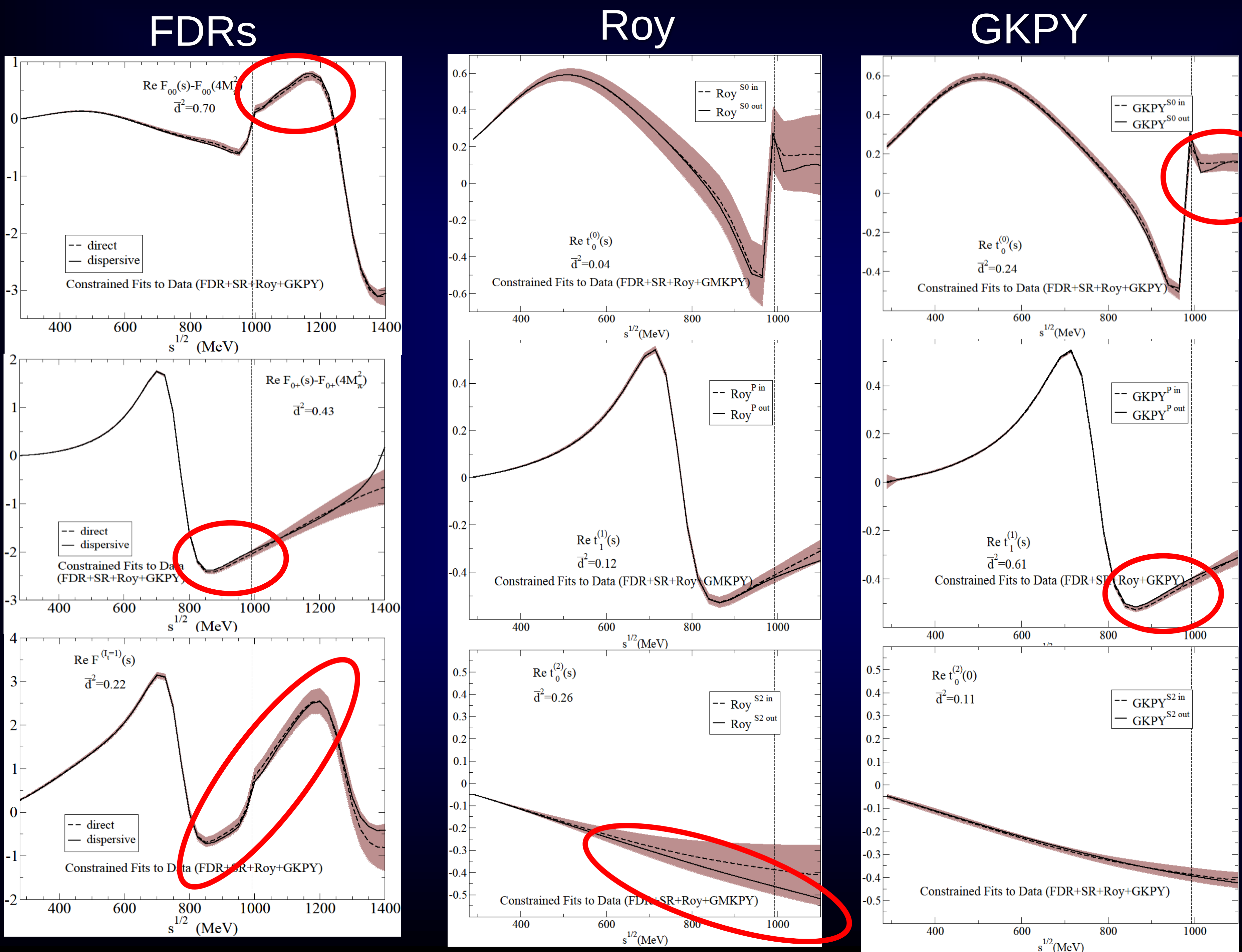
In general, data does not satisfy well DR. Sometimes very badly indeed

FDRs Roy GKPY



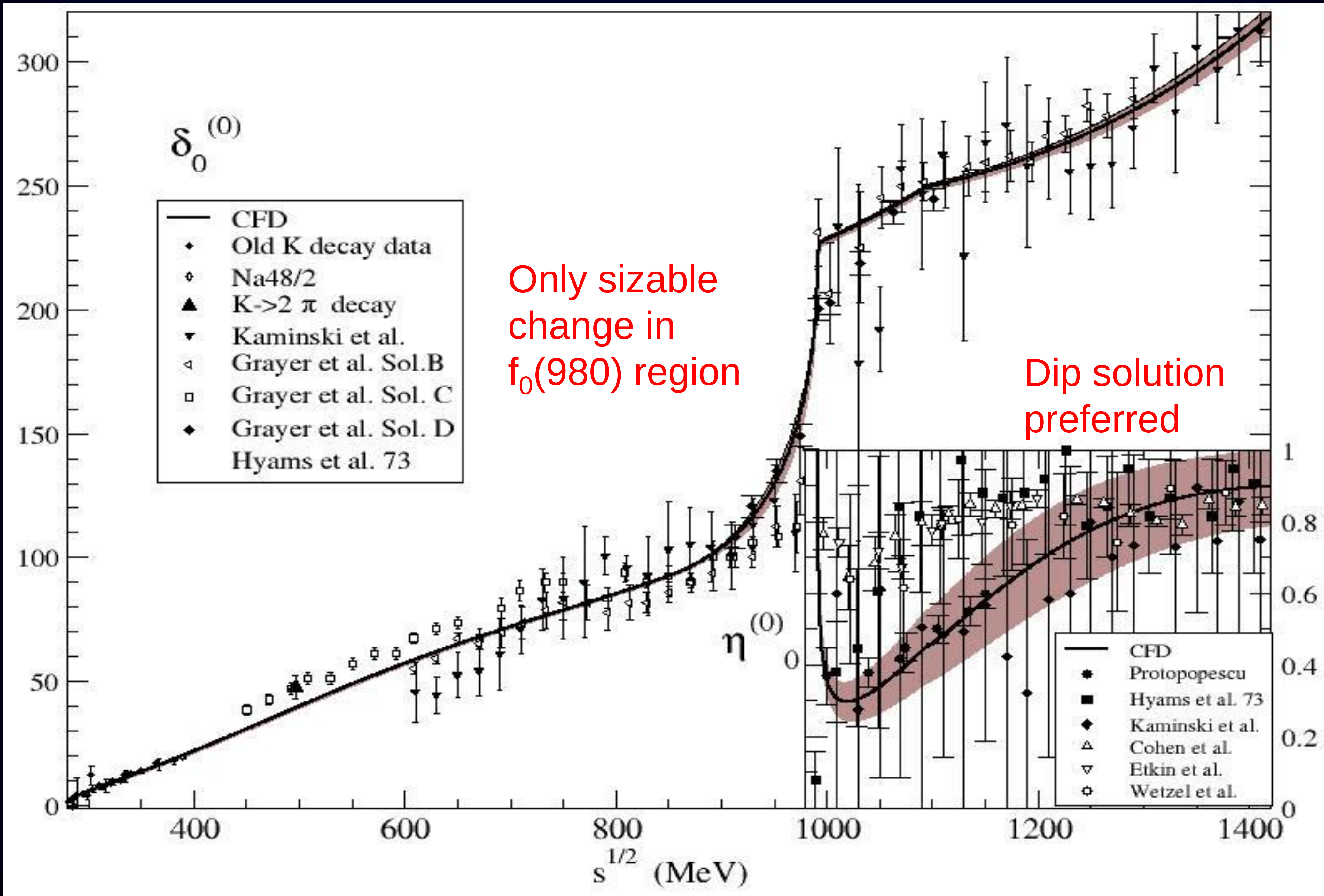
THIRD STEP: Use dispersion relations as constraints for the fits: ππ→ππ

Very good fulfillment: Constrained Fits to Data



S0 wave: from UFD to CFD

The changes from Unconstrained to Constrained fits are not too large, but relevant in some regions



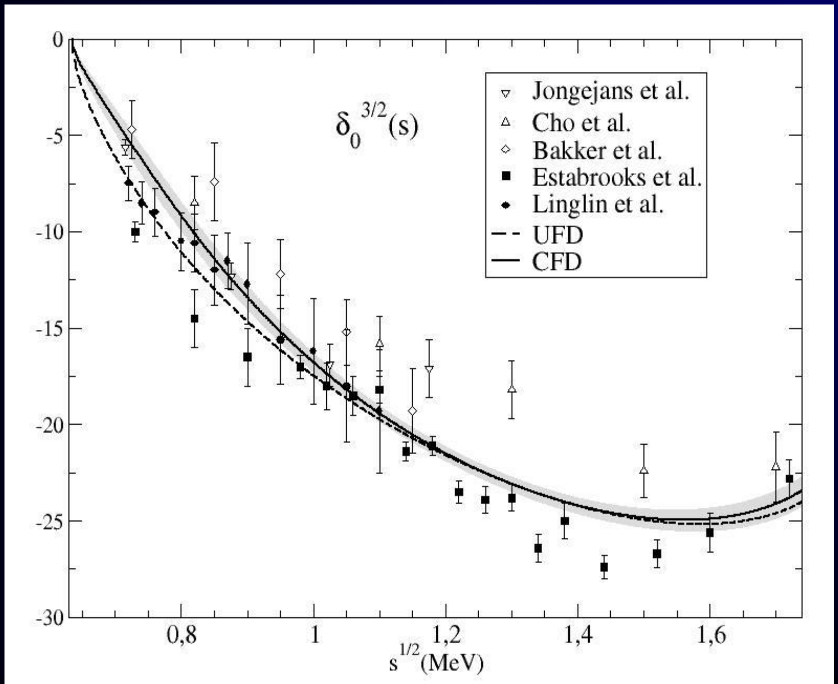
From Unconstrained (UFD) to Constrained Fits to data (CFD)

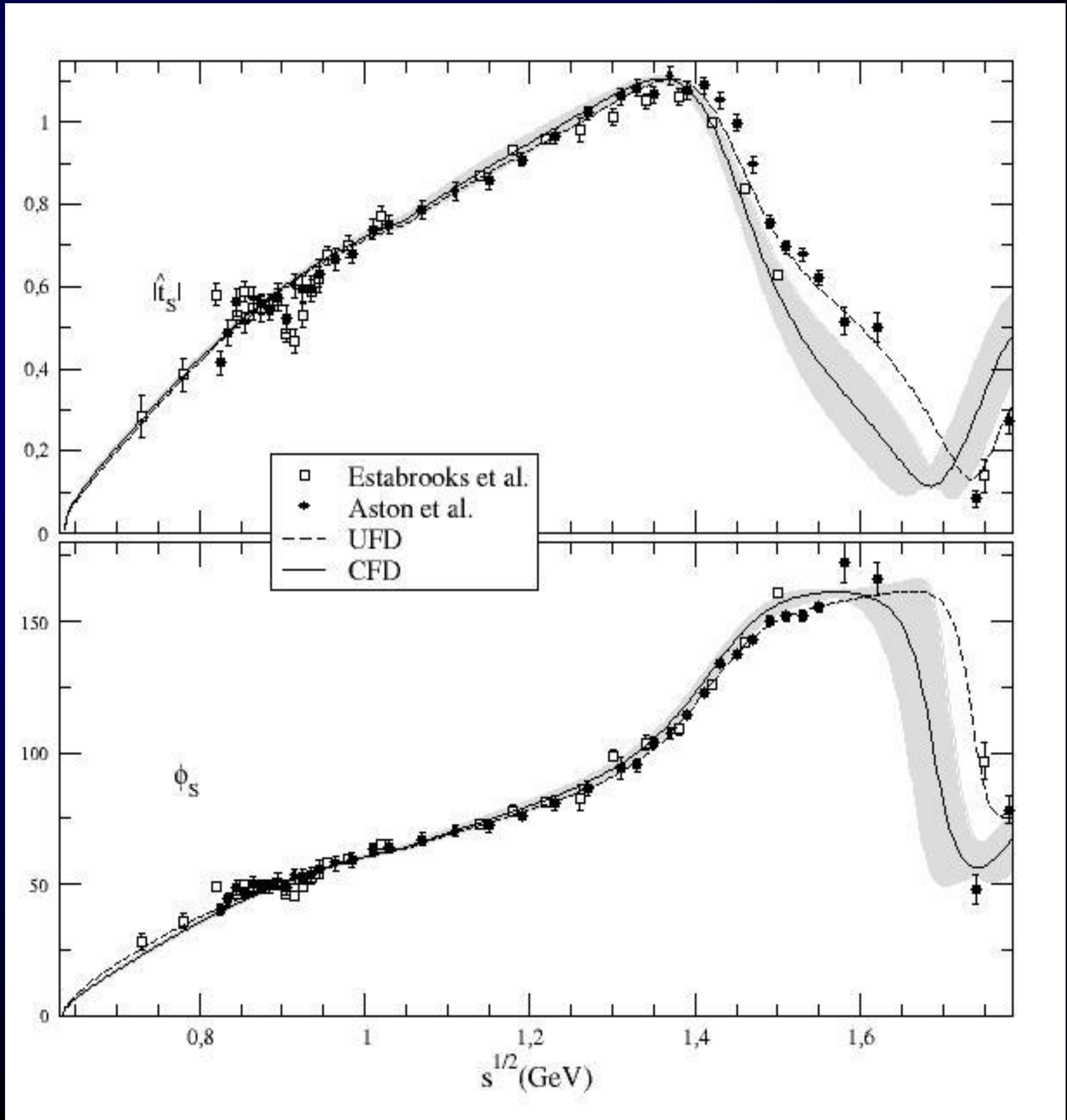
JRP, A.Rodas-PhysRevLett.124.172001-2020

S-waves. The most interesting for the K_0* resonances and the $K_0^*(700)$ in particular

Largest changes from UFD to CFD

at higher energies



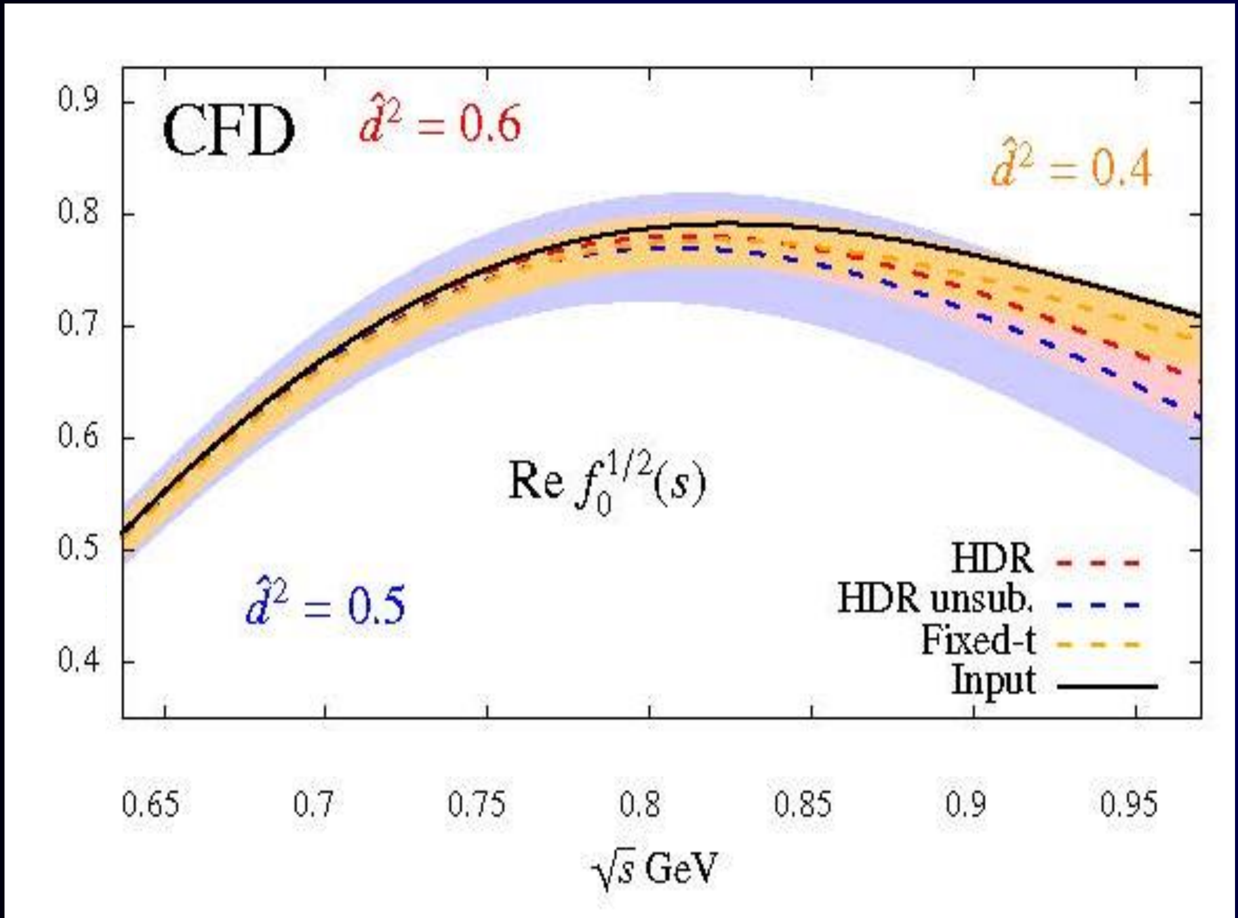


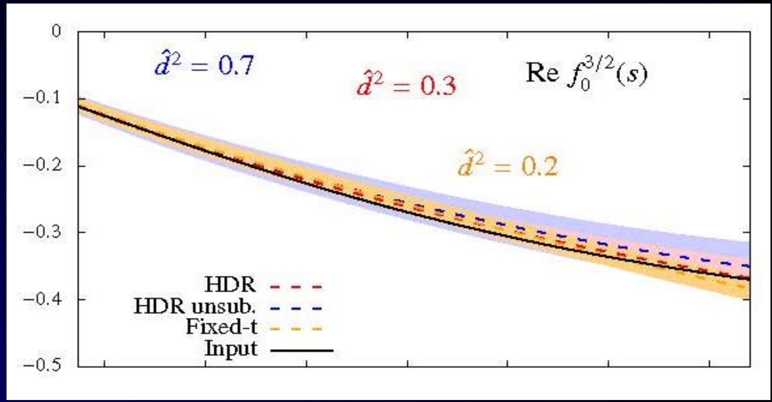
JRP, A.Rodas-PhysRevLett.124.172001-2020

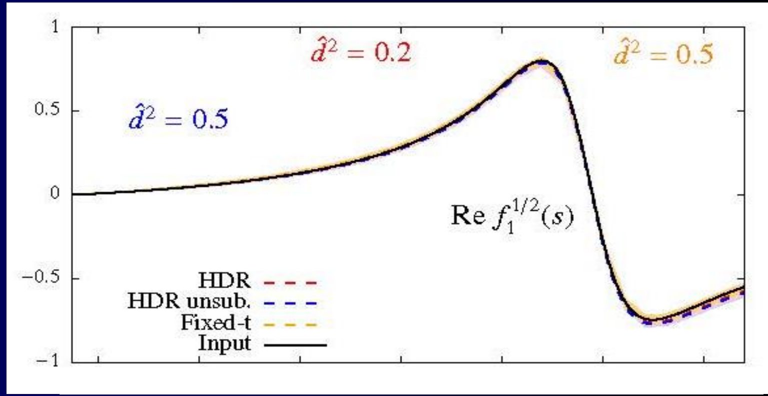
Unconstrained fits (UFD):
LARGE inconsistencies with 3 Roy-Steiner Eqs.

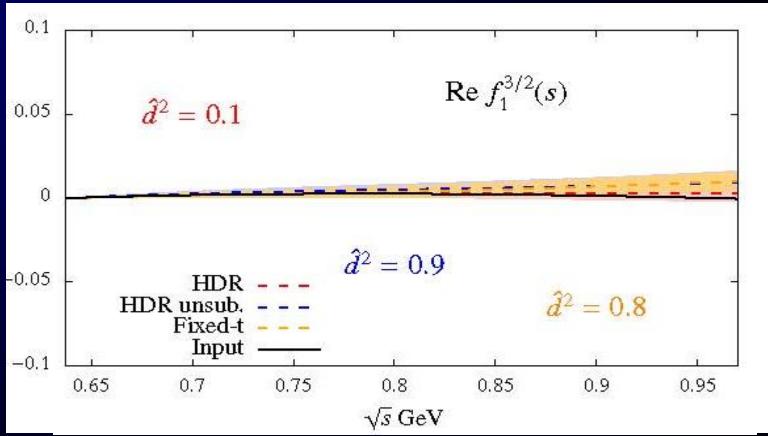
One or no subtraction for F- lie on opposite sides of input

The most relevant wave for the kappa resonance.









Constrained fits (UFD):
Consistent within uncertainties

Dispersion relations provide model-independent analytic continuation to **first** Riemann sheet, but the most relevant resonance poles live in the CONTIGUOUS sheet

- For elastic resonances, only second sheet, $S^{II}=1/S^{I}$

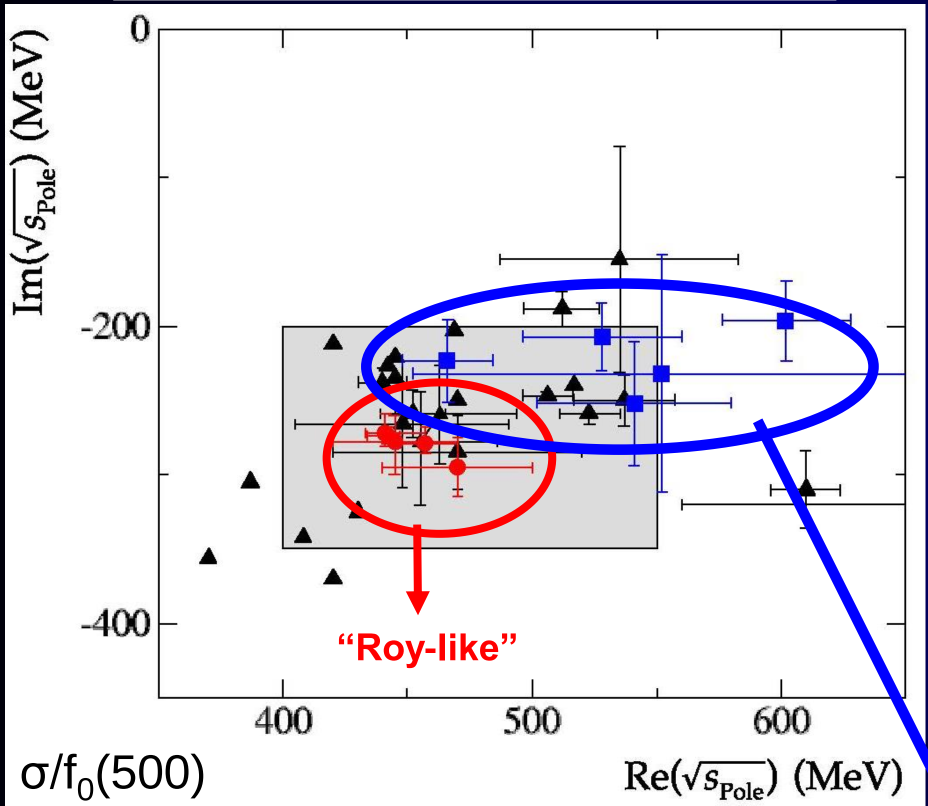
$\sigma/f_0(500)$, $\kappa/K_0^*(700)$.
Purely Dispersive Determination

WHERE DO WE STAND? Light Scalars at RPP 2021 (on-line update). "Note on Light Scalar Mesons below 1 GeV"

"Roy-like" and "Breit-Wigner" poles identified separately from the rest
Not all from meson-meson scattering

$\sigma/f_0(500)$ estimate

$(400\text{-}550)\text{-}i(200\text{-}350)$ MeV



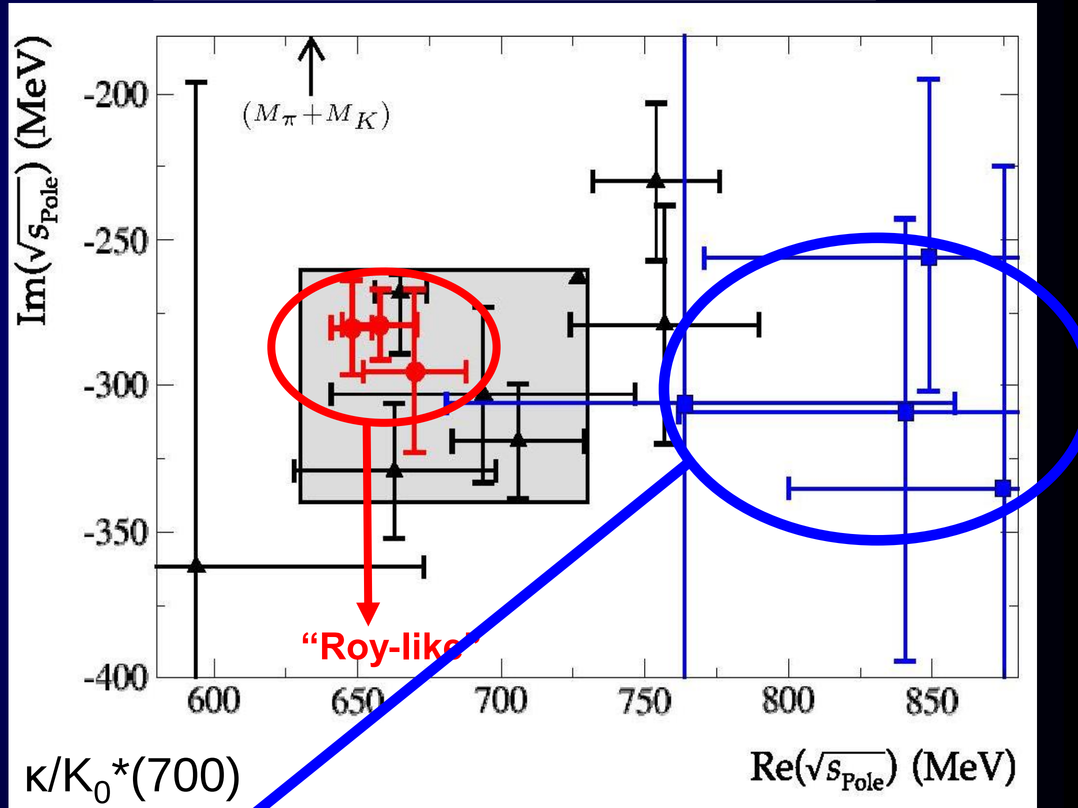
(My) Conservative Dispersive Estimate:

$(449^{+22}_{-16})\text{-}i(275\pm15)$ MeV

JRP, Physics Reports 658-2016-1
*Typo fixed in uncertainty from 12 to 15 MeV

$\kappa/K_0^*(700)$ estimate
2021 No longer "Needs Confirmation"

$(400\text{-}550)\text{-}i(200\text{-}350)$ MeV



From our data driven Roy-Steiner analysis:

No sub: $(648\pm6)\text{-}i(283\pm26)$ MeV
1 sub: $(648\pm7)\text{-}i(280\pm16)$ MeV

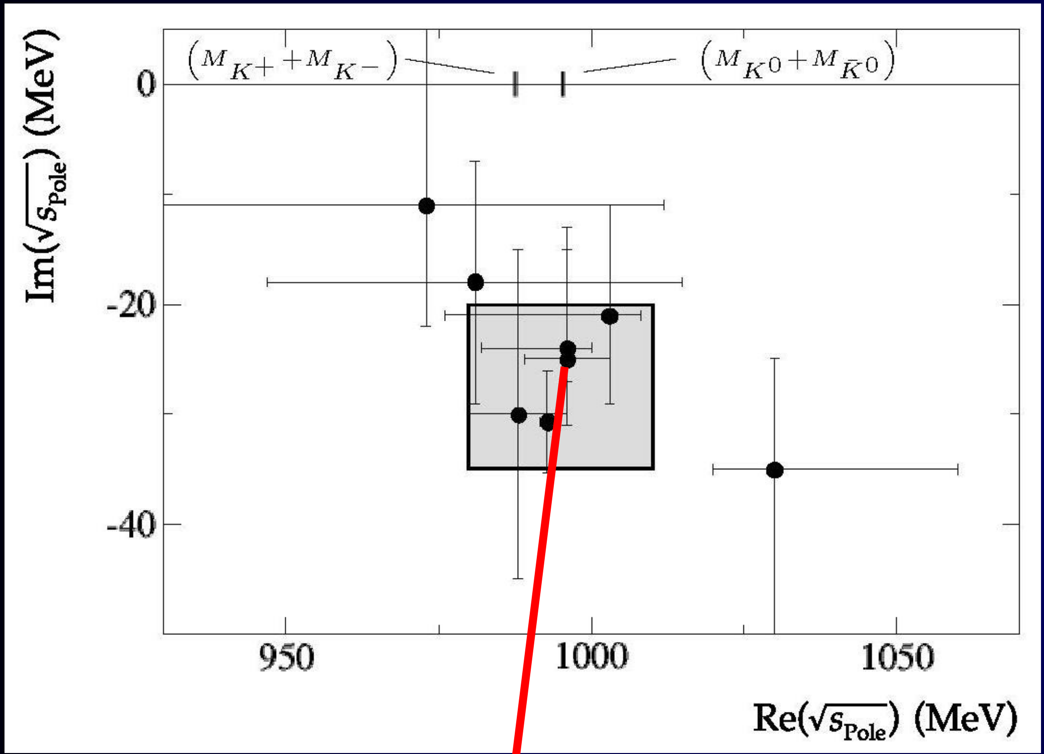
JRP, A.Rodas-PhysRevLett.124.172001-2020

☹ **But still Breit-Wigners!!**

$f_0(980)$ and $a_0(980)$ poles also provided. Less controversial, smaller uncertainties
Not all from meson-meson scattering

$f_0(980)$ estimate

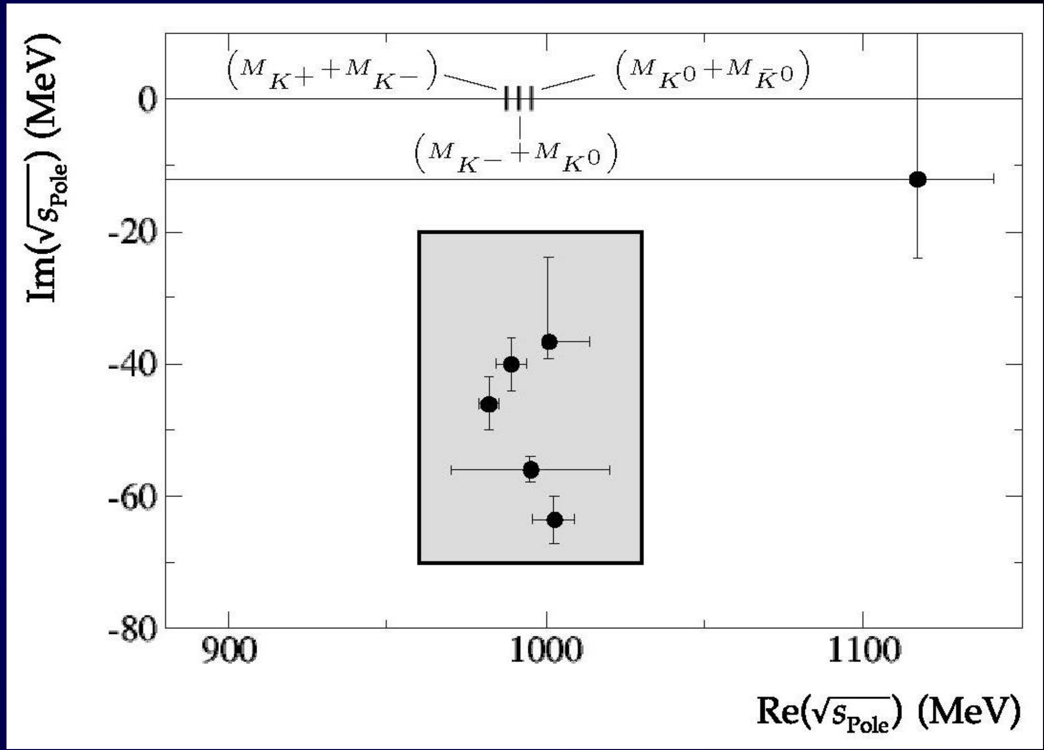
(980-1010)-i(20-35) MeV



From scattering “Roy-like”

$a_0(980)$ estimate

(980-1030)-i(20-70) MeV



Dispersion relations provide model-independent analytic continuation to **first** Riemann sheet, but the most relevant resonance poles live in the CONTIGUOUS sheet

- For elastic resonances (only second sheet), $S^{II}=1/S^{I}$

σ/f_0(500), κ/K_0*(700), f_0(980),
Purely Dispersive Determination
from meson-meson scattering

- To reach the contiguous sheet in the inelastic case, we need an analytic continuation to the second sheet by means of general analytic functions reproducing the Dispersion Relation in the real axis or the upper-half complex plane.

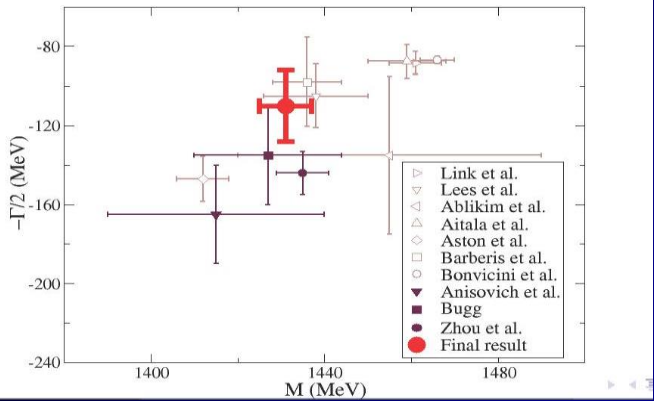
Several methods in the literature
- **Sequences of Padés**
- **Continued Fractions**
- Laurent-Pietarinen functions
- Conformal expansions…

These methods avoid specific parameterizations, reducing drastically the model-dependence
Tested then with the σ/f_0(500) and κ/K_0*(700). Compatible results.

The method can be used for inelastic resonances too. Provides resonance parameters WITHOUT ASSUMING SPECIFIC FUNCTIONAL FORM

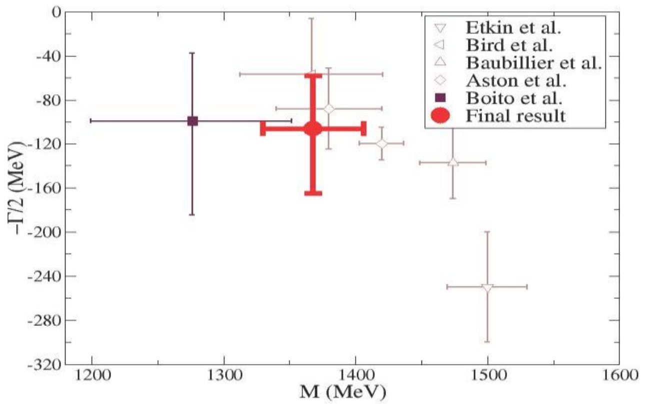
- For the $K_0^*(1430)$ we find

$$\sqrt{s_p} = (1431 \pm 6) - i(110 \pm 19)MeV$$
$$\sqrt{s_p} = (1425 \pm 50) - i(135 \pm 40)MeV(PDG)$$



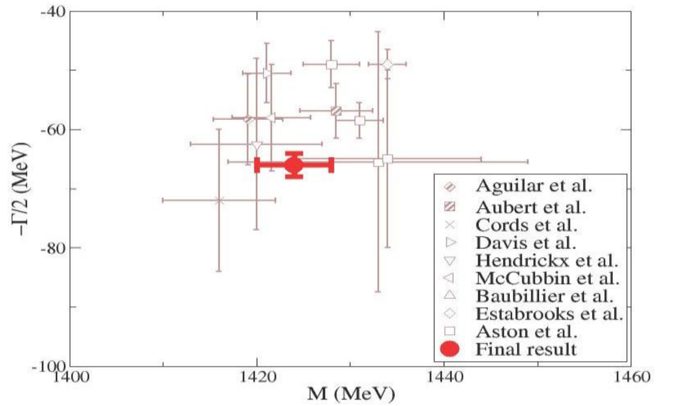
- For the $K_1^*(1410)$ we find

$$\sqrt{s_p} = (1368 \pm 38) - i(106^{+48}_{-59})MeV$$
$$\sqrt{s_p} = (1414 \pm 15) - i(116 \pm 10)MeV(PDG)$$



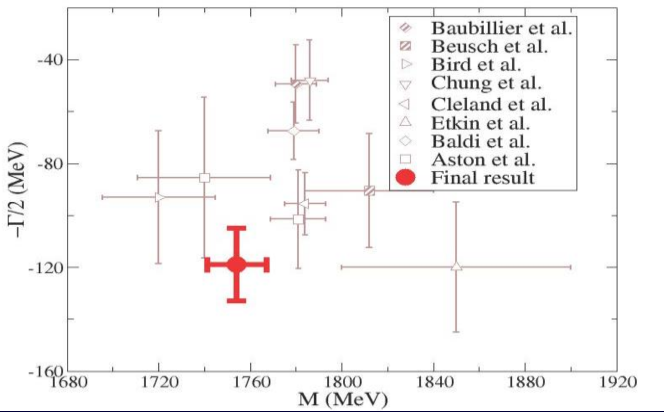
- For the $K_2^*(1430)$ we find

$$\sqrt{s_p} = (1424 \pm 4) - i(66 \pm 2)MeV$$
$$\sqrt{s_p} = (1432.4 \pm 1.3) - i(55 \pm 3)MeV(PDG)$$



- For the $K_3^*(1780)$ we find

$$\sqrt{s_p} = (1754 \pm 13) - i(119 \pm 14)MeV$$
$$\sqrt{s_p} = (1776 \pm 7) - i(80 \pm 11)MeV(PDG)$$



Using Padé Sequences, the kappa: (670±18)-i(295± 28) MeV Consistent with dispersive value

JRP, A.Rodas & J. Ruiz de Elvira. Eur. Phys. J. C (2017) 77:91

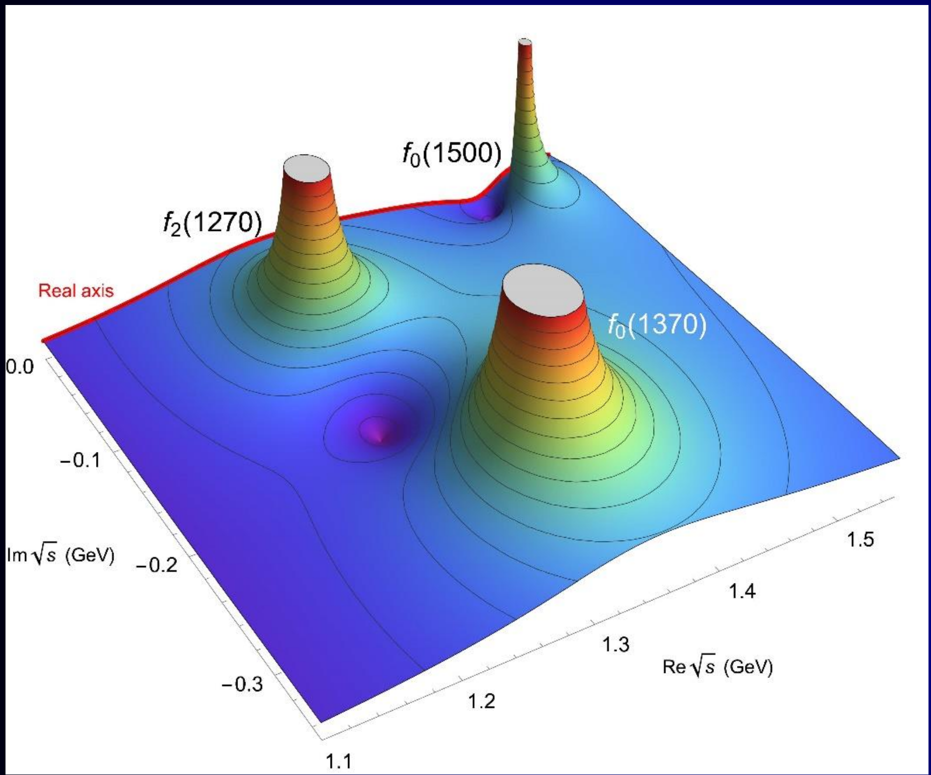
$f_0(1370)$ and $f_0(1500)$ poles from dispersive meson-meson + analytic continuation

JRP, A.Rodas & J. Ruiz de Elvira. e-Print: 2206.14822 [hep-ph]

Roy ππ→ππ equations strict applicability only up to 1.1 GeV.

NEW!

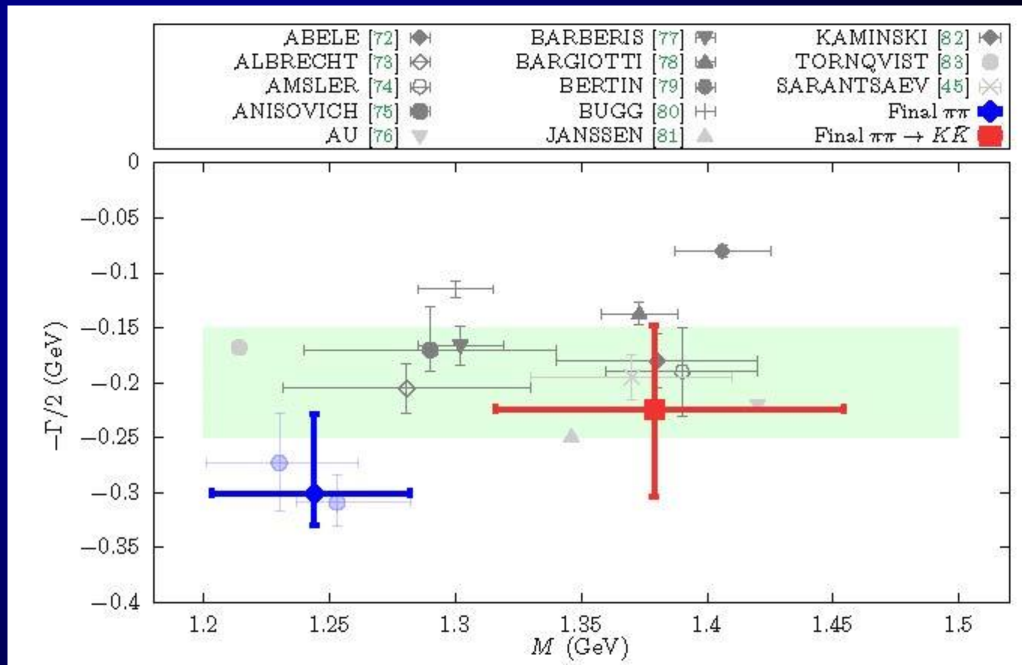
However, Forward Dispersion Relations applicability up to 1.42 GeV in our fits.
Complication, we see the isoscalar-wave with all spins



Continued fractions provide stable pole description absent in original experimental ππ→ππ data analyses.

Also in extrapolation of Roy-like equations

Roy-Steiner eqs. for ππ→KK are applicable there.
Analytically continued **also find f_0(1370)**
Larger uncertainty
But ~2σ tension on mass due to DATA



$$\mathrm{FDR}_{\pi\pi\to\pi\pi} + \mathrm{C_N} \quad (1245 \pm 40) - i\,(300^{+30}_{-70}) \quad 5.6^{+0.7}_{-1.2}$$

$$\mathrm{RS}_{\pi\pi\to K\bar{K}} + \mathrm{C_N} \quad (1380^{+70}_{-60}) - i\,(220^{+80}_{-70}) \quad 3.2^{+1.3}_{-1.1}$$

SUMMARY

- Over the last years, and as late as 2021, the use of techniques based on analyticity and dispersion relations applied to meson-meson scattering has finally settled the longstanding controversy about the existence of two light scalar nonets below 2 GeV

- It is of the uttermost importance to characterize resonances by their poles.
Some simple parameterizations may be useful to describe data, but using them to characterize resonant states only creates confusion.

- Models should be consistent with the dispersive data analyses and with its poles

- The present picture is that of a light scalar nonet below 1 GeV of a non-ordinary quark-antiquark nature (Criptoexotic). All its members are now identified

- Another nonet exists around 1.5 GeV, with one f_0 state too many. The $f_0(1370)$ is found in meson-meson scattering from dispersively constrained meson-meson scattering

WHERE DO WE STAND? Recent developments on Exotics from meson-meson scattering

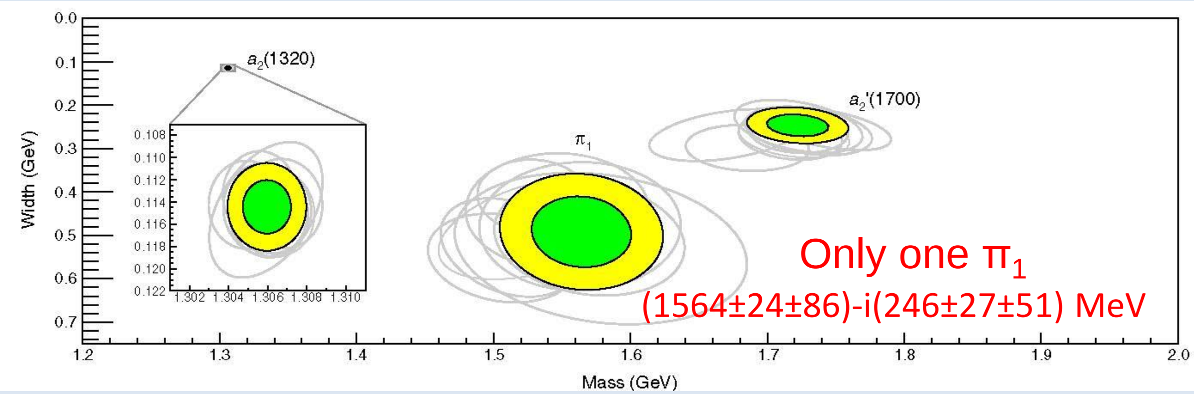
Candidates: $\pi_1(1400)$ and $\pi_1(1600)$ with $J=1^{-+}$ @Crystal Barrel, Brookhaven and COMPASS
Exotic P-wave in $\pi\eta\to\pi\eta$ and $\pi\eta'\to\pi\eta'$ scattering coupled to other channels as well.

But not real data on scattering of $\pi\eta$ and $\pi\eta'$. Just $\pi p \to \eta\pi p$, $\pi p \to \eta'\pi p$
Various analyses used BW.

Recent "Analytic" methods:

Rodas et al. (JPAC Collab) PRL122.042002 (2019)

N/D method for final state. Sampling of large # of models
Input: COMPASS data



Kopf et al. Eur. Phys. J. C (2021) 81:1056

K-matrix/Chew-Mandelstam
analytic parameterization
Input:Crystal Ball+COMPASS
Consistent with JPAC but slightly narrower

Lattice: HadSpec Collab.PRD103, 054502 (2021)
Actual scattering (caveats:
in a box, dicretization, large pion mass, extrapolation)