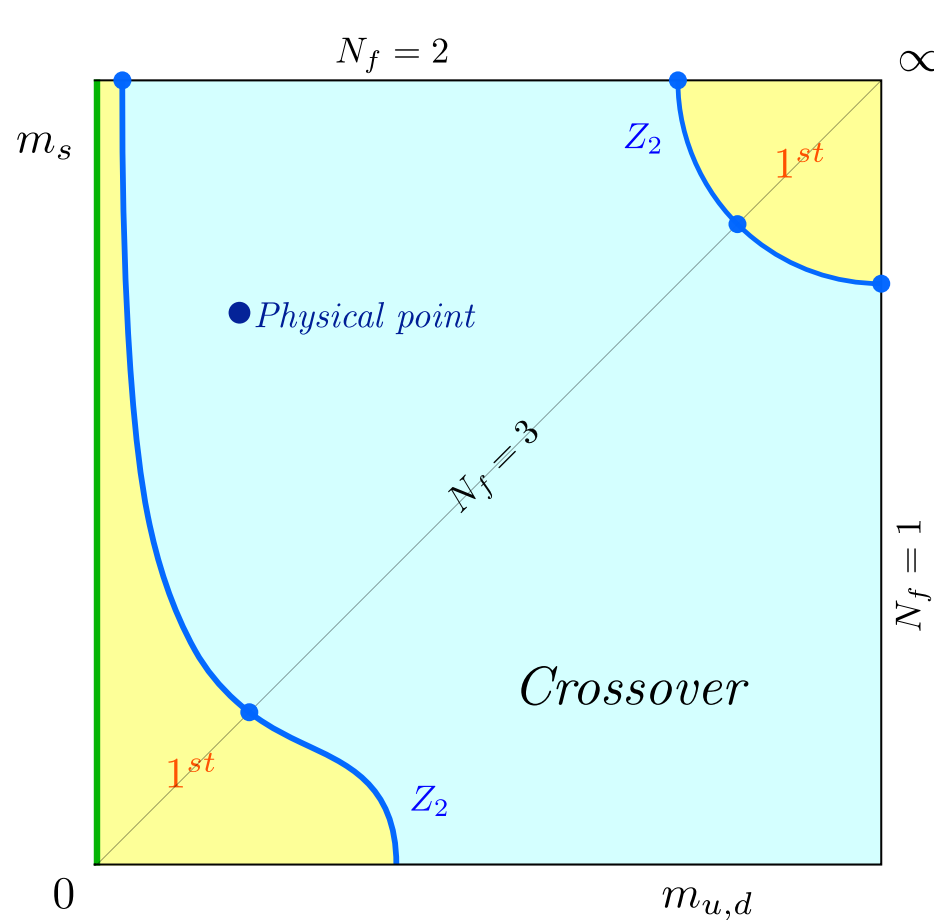


Constraining the QCD phase diagram

Owe Philipsen

- Chiral phase transition in massless limit, zero density
- Constraints on a possible critical endpoint
- A new emergent chiral spin symmetry

The nature of the QCD thermal transition, $\mu_B = 0$

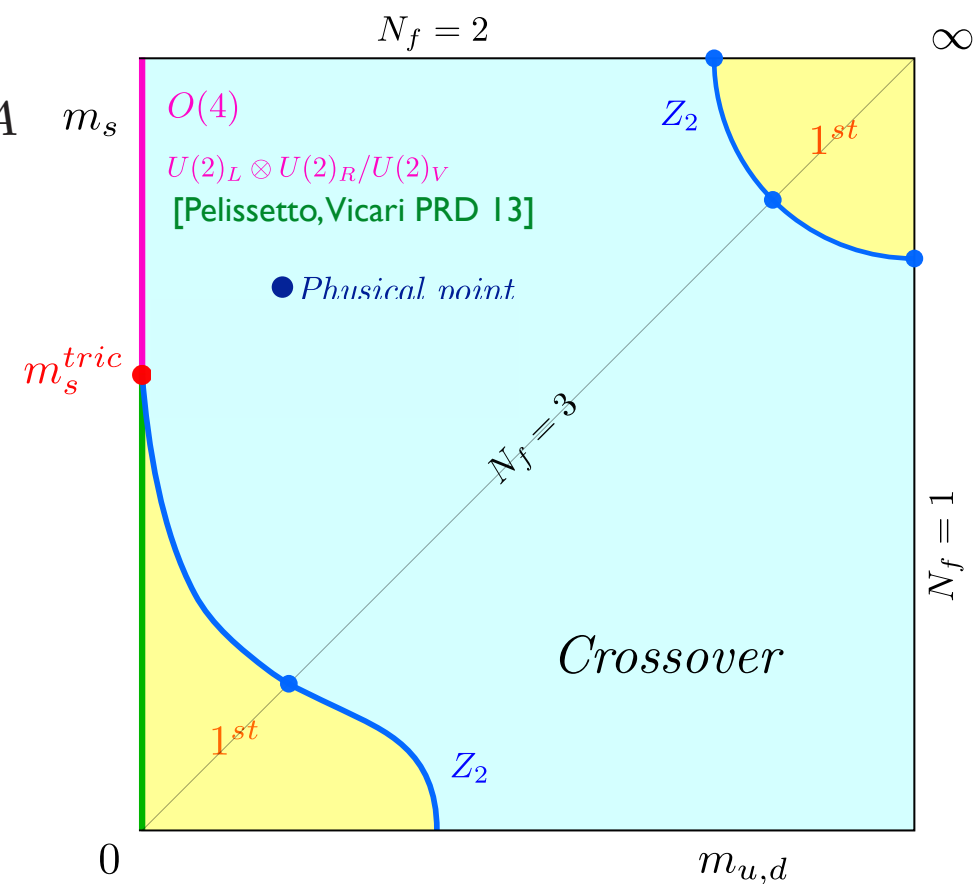


[Pisarski, Wilczek, PRD 84]:
 $N_f = 2$ depends on $U(1)_A$

restored

broken

$N_f \geq 3$ 1st order



chiral p.t.

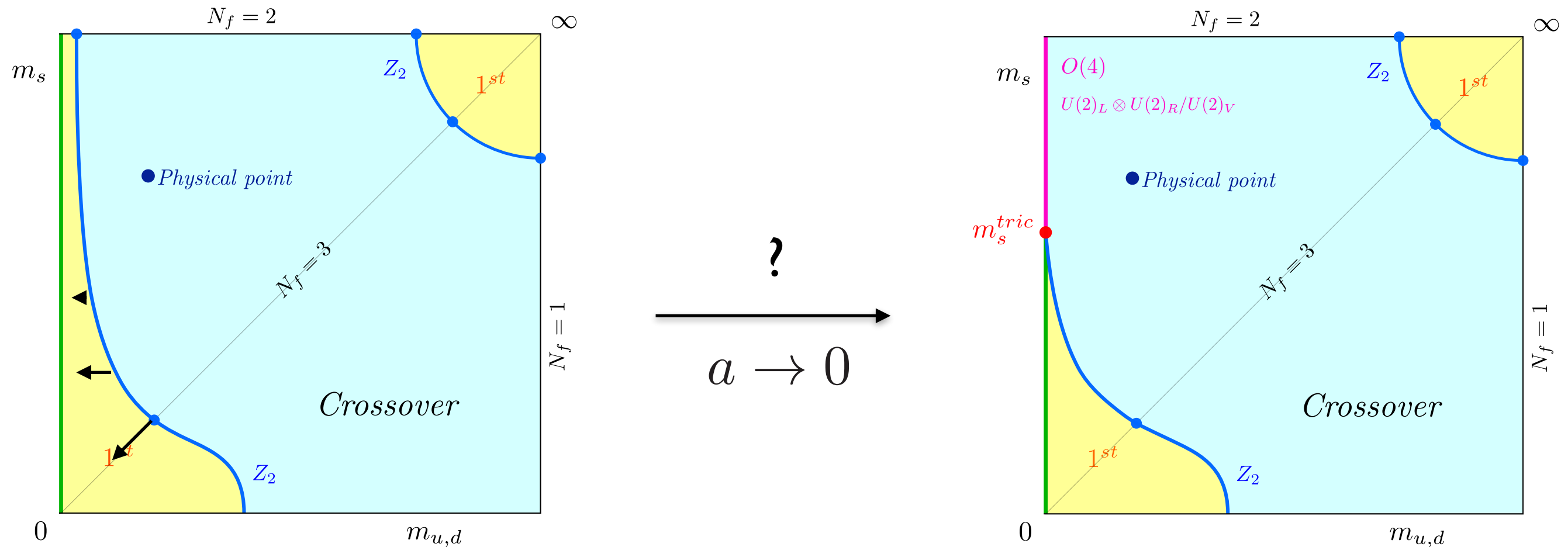
restoration of global symmetry in flavour space

$$SU(2)_L \times SU(2)_R \times U(1)_A$$

↑
 anomalous

The nature of the QCD chiral transition

...is elusive, massless limit **not simulable!**

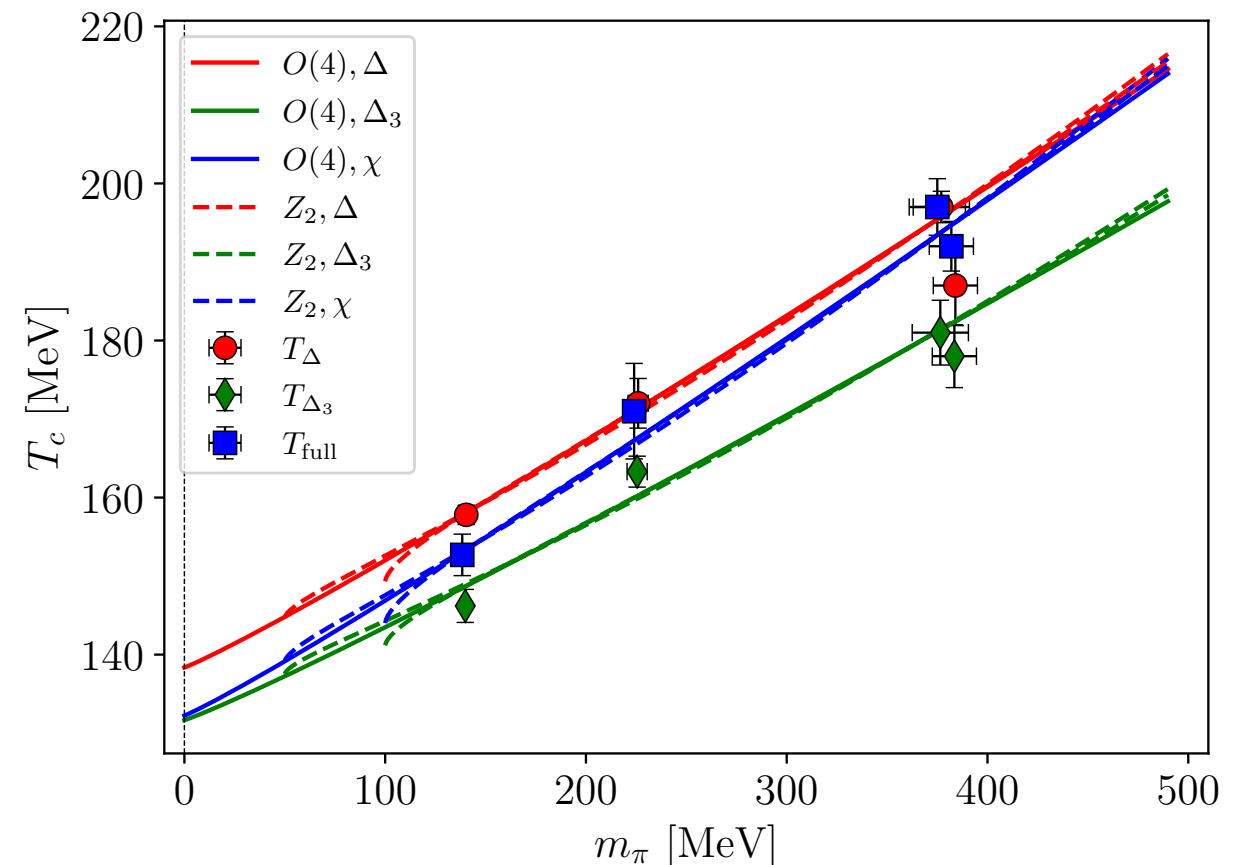
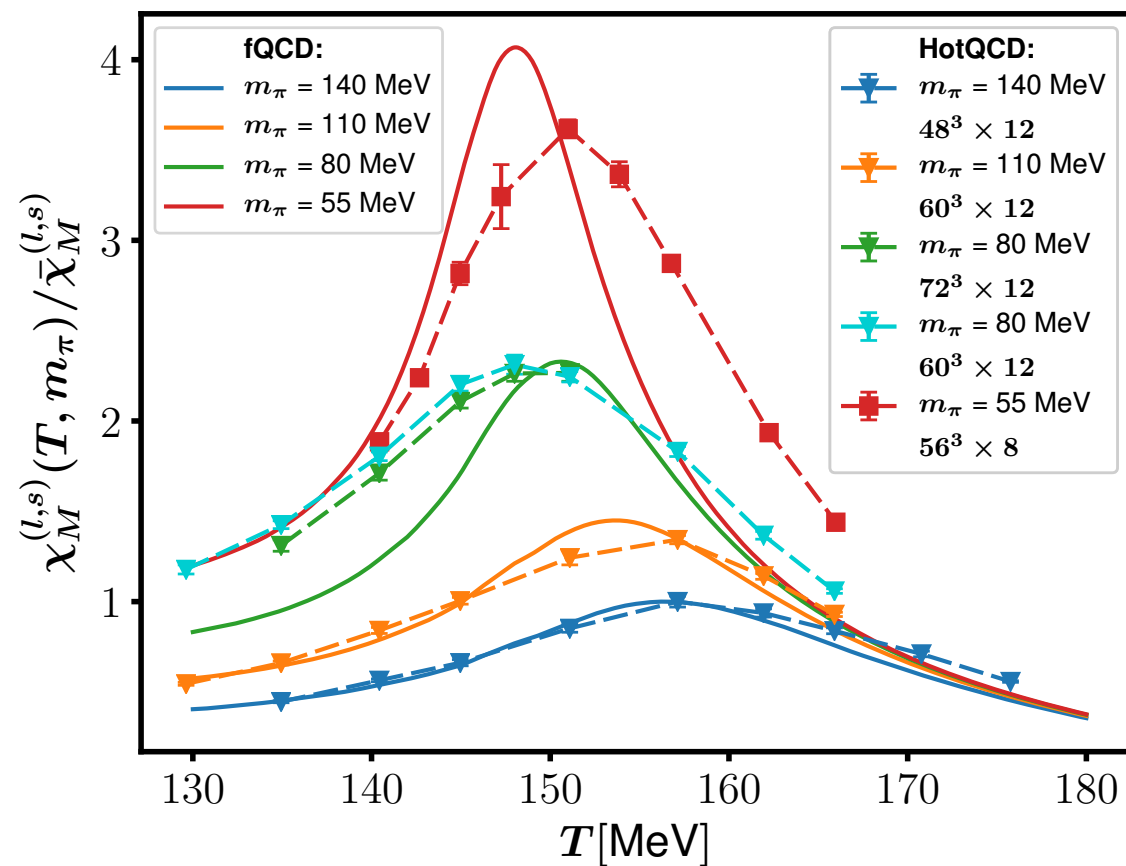


- Coarse lattices or unimproved actions: 1st order for $N_f = 2, 3$
- 1st order region shrinks rapidly as $a \rightarrow 0$
- Improved staggered actions: no 1st order region so far, even for $N_f = 3$ $m_{PS} > 45\text{MeV}$

Details and reference list: [\[O.P., Symmetry 13, 2021\]](#)

From the physical point towards the chiral limit

arXiv:2012.06231



[HotQCD, PRL 19] HISQ (staggered)

[Kotov, Lombardo, Trunin, PLB 21] Wilson twisted mass

$$T_c^0 = 132_{-6}^{+3} \text{ MeV}$$

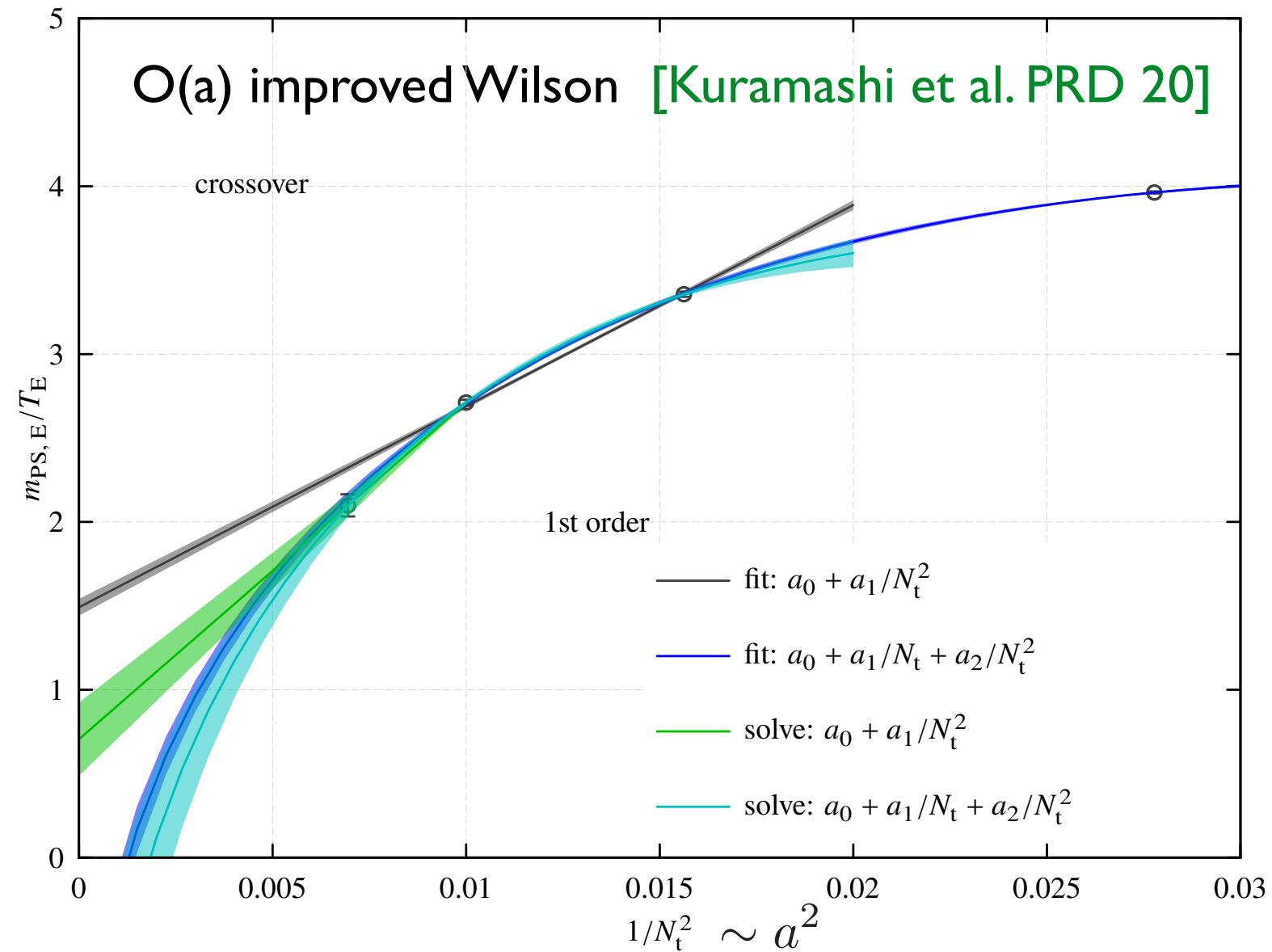
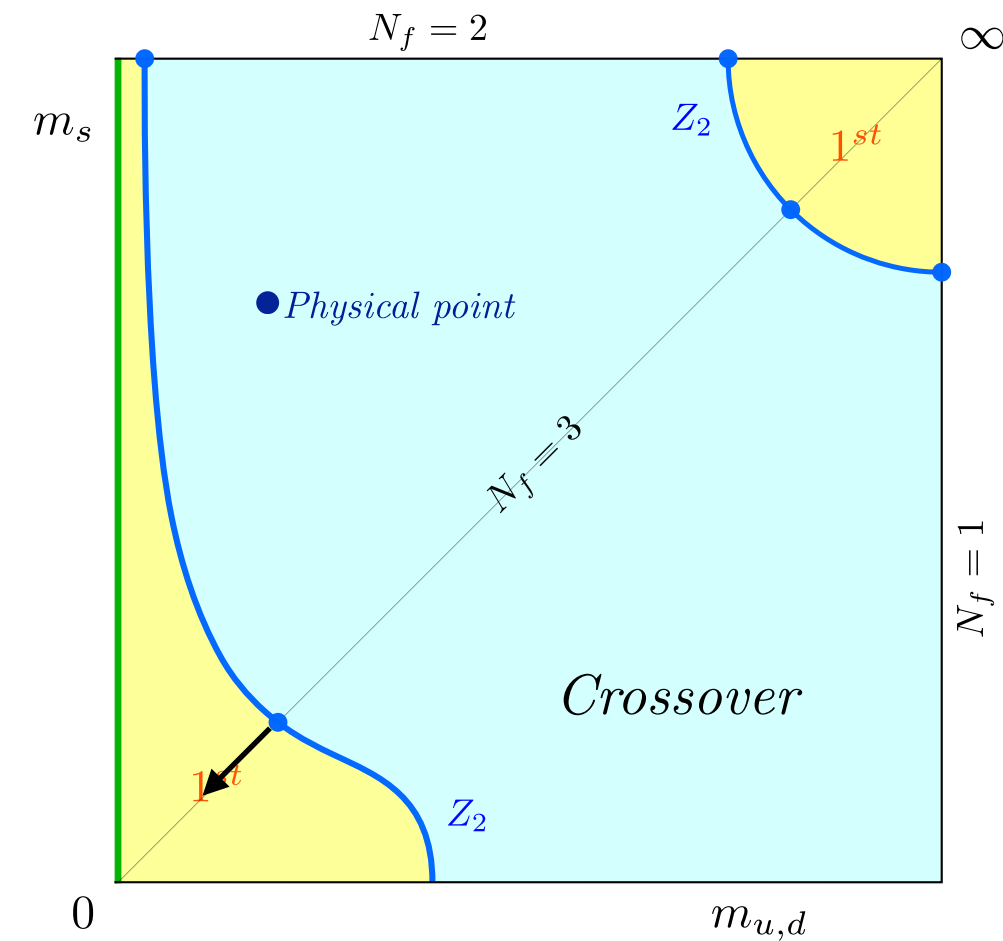
$$T_{pc}(m_l) = T_c^0 + K m_l^{1/\beta\delta}$$

$$T_c^0 = 134_{-4}^{+6} \text{ MeV}$$

- Keep strange quark mass fixed, crossover gets stronger as chiral limit approached
- Cannot distinguish between $Z(2)$ vs. $O(4)$ exponents, need exponential accuracy!
- Determination of chiral critical temperature possible, but not the order of the transition
- Comparison with fRG: $T_c^0 \approx 142 \text{ MeV}$, "most likely $O(4)$ " [Braun et al., PRD 20,21]

The nature of the QCD chiral transition, $N_f=3$

...has enormously large cut-off effects!

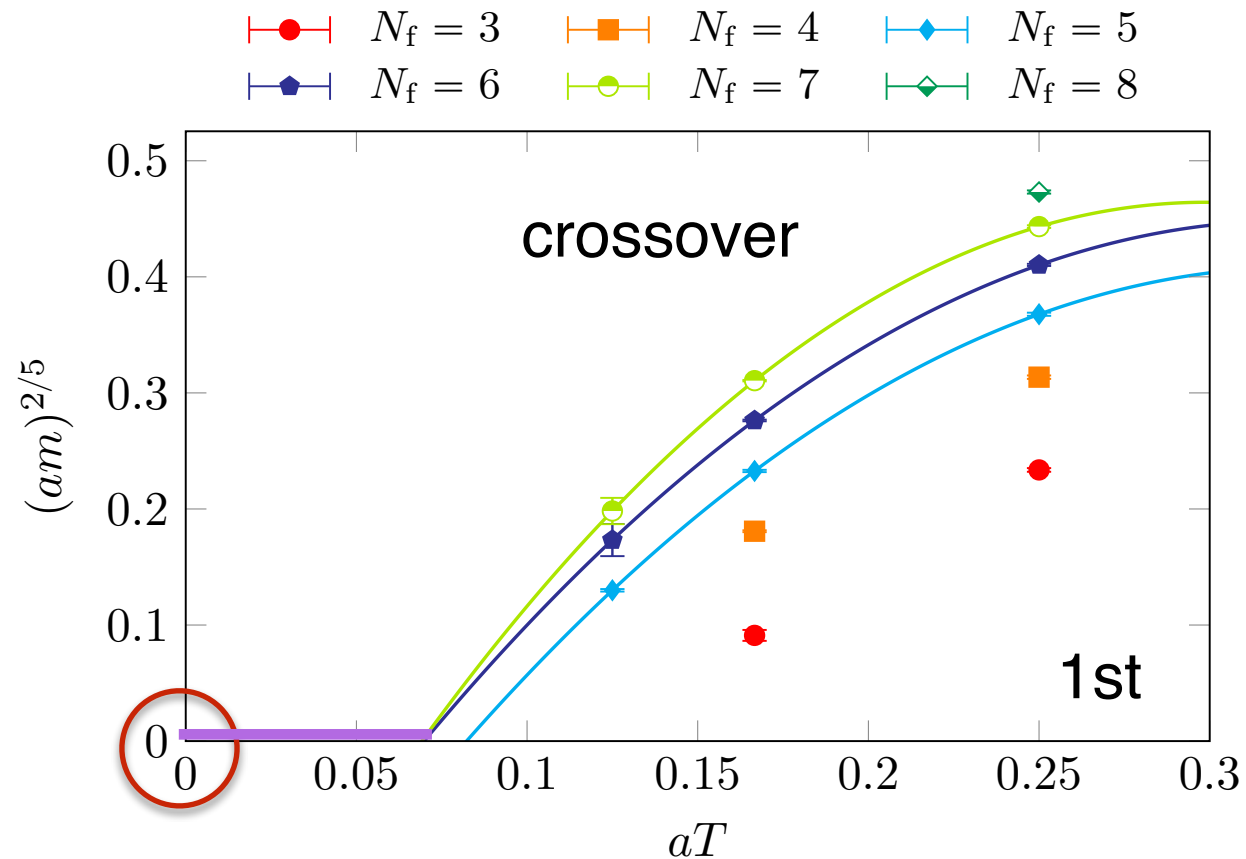


O(a)-improved Wilson:
1st order region shrinks for $a \rightarrow 0$

$$m_\pi^c \leq 110 \text{ MeV} \quad N_\tau = 4, 6, 8, 10, 12$$

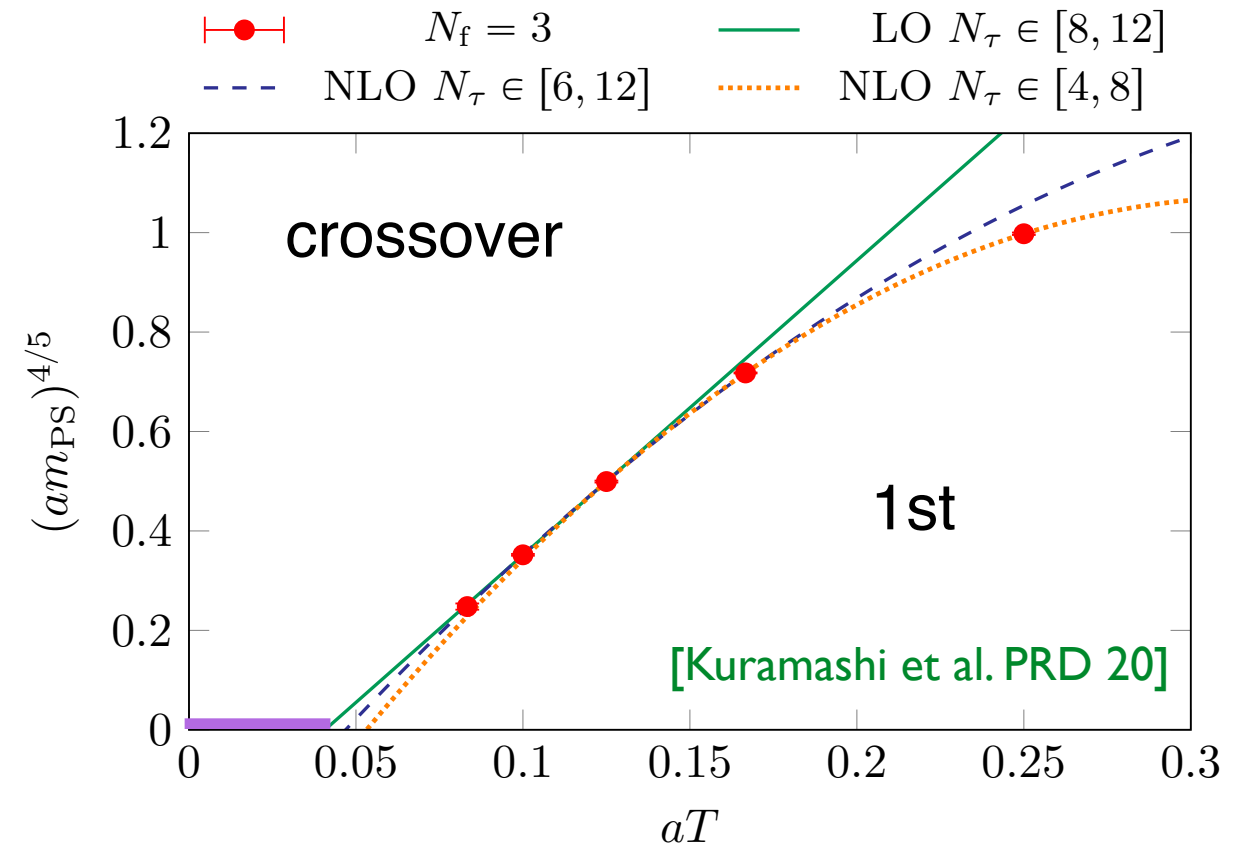
Resolution: scaling in lattice parameter space

[Cuteri, O.P., Sciarra JHEP 21] Staggered



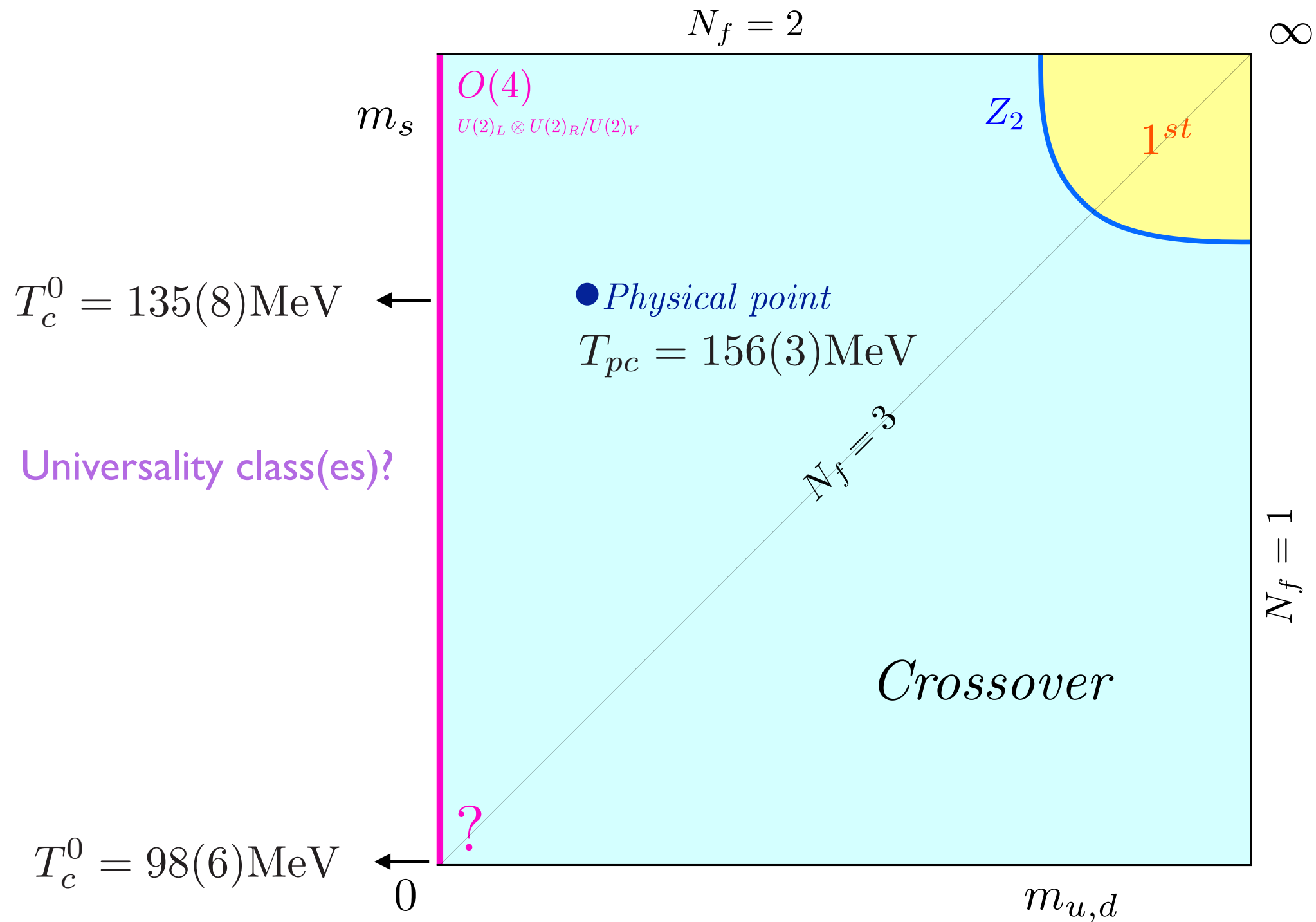
continuum limit

O(a) improved Wilson rescaled



- Tricritical scaling observed in lattice bare parameter space
- Allows extrapolation to lattice chiral limit, tricritical points $N_\tau^{\text{tric}}(N_f)$
- If tricritical point exists: region of 1st-order transitions not connected to continuum
- QCD chiral transition is second order for $N_f = 2 - 7$

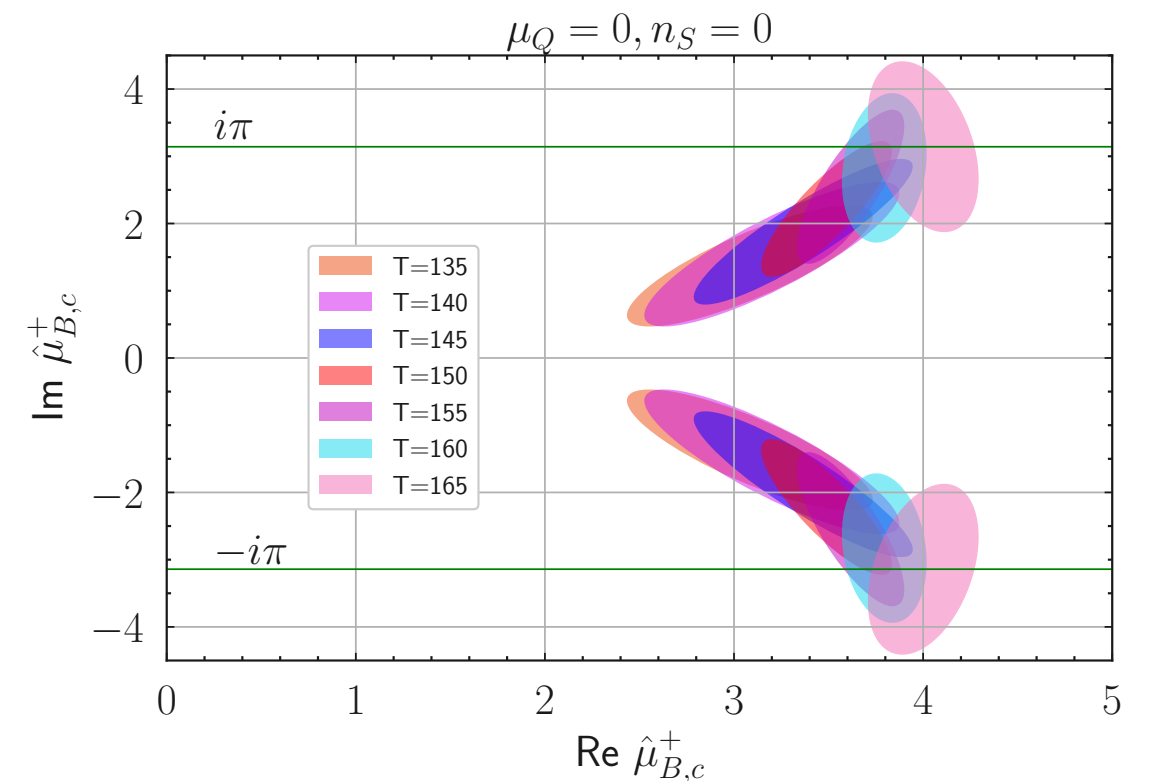
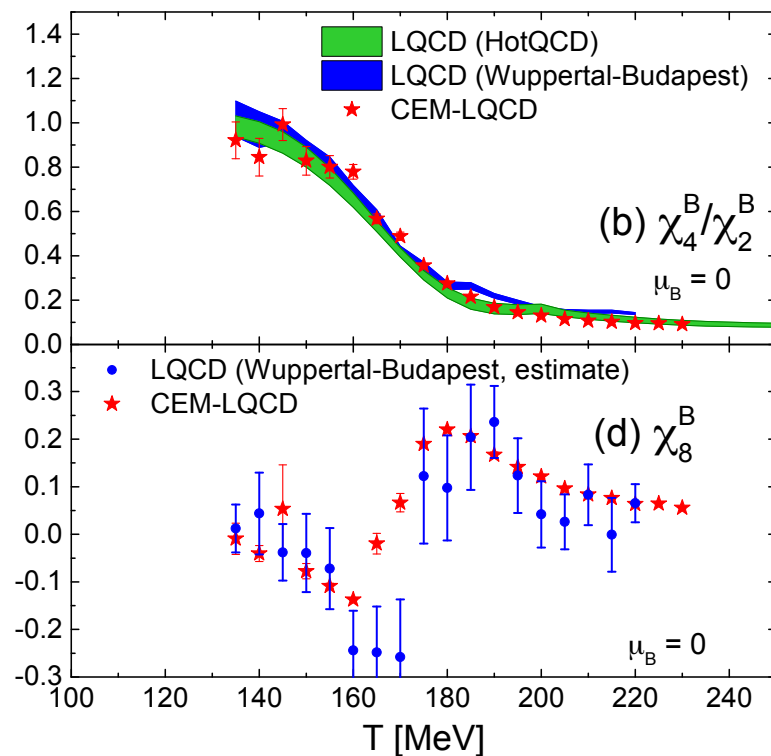
The Columbia plot in the continuum



Lattice results for fluctuations + resummations

$$\frac{p(T, \mu_B)}{T^4} = \frac{p(T, 0)}{T^4} + \sum_{n=1}^{\infty} \frac{1}{2n!} \chi_{2n}^B(T) \left(\frac{\mu_B}{T} \right)^{2n}$$

search for radius of convergence



Cluster expansion model, infinite order, describes all available lattice data

From lattice coeffs up to μ_B^8 , multi-point Pade resummation

$$\mu_B^{\text{cep}} > \pi T$$

[Vovchenko et al. PRD 18]

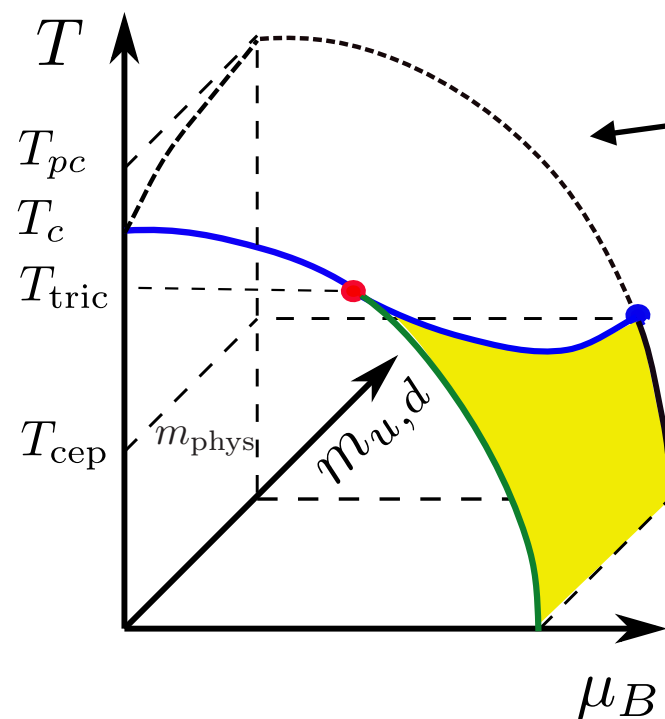
$$\mu_B^{\text{cep}} > 2.5T, T < 125 \text{ MeV}$$

[HotQCD PRD 22]

Constraints on the critical point

The “standard scenario”: [Halasz et al., PRD 98; Hatta, Ikeda, PRD 03...]

Importance of the chiral limit!



$$\frac{T_{pc}(\mu_B)}{T_{pc}(0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_{pc}(0)} \right)^2 + \dots$$

κ_2
0.0158(13)
0.0135(20)
0.0145(25)
0.016(5)

[Bellwied et al, PLB 15]

[Bonati et al, NPA 19]

[Bonati et al, PRD 18]

[HotQCD, PLB 19]

- ▶ Ordering of critical temperatures $\mu_B^{\text{cep}} > 3.1 T_{pc}(0) \approx 485 \text{ MeV}$ [O.P. Symmetry 21]
- ▶ Cluster expansion model of lattice fluctuations $\mu_B^{\text{cep}} > \pi T$ [Vovchenko et al. PRD 18]
- ▶ Singularities, Pade-approx. fluctuations $\mu_B^{\text{cep}} > 2.5T, T < 125 \text{ MeV}$ [Bollweg et al. PRD 21]
- ▶ Direct simulations with refined reweighting $\mu_B^{\text{cep}} > 2.5T$ [Wuppertal-Budapest collaboration, PRD 21]
- ▶ Consistent with DSE, fRG [Fischer PPNP 19; Fu, Pawłowski, Rennecke PRD 20; Gao, Pawłowski PRD 21]

A surprise: emergent chiral spin symmetry

Chiral spin transformation, $SU(2)_{CS}$: $\psi \rightarrow \psi' = \exp\left(i\frac{\varepsilon^n \Sigma^n}{2}\right) \psi$ $\Sigma^n = \{\gamma_k, -i\gamma_5 \gamma_k, \gamma_5\}$

$$SU(2)_{CS} \otimes SU(2)_V \simeq SU(4) \supset SU(2)_L \times SU(2)_R \times U(1)_A$$

QCD quark action, chiral limit: $\bar{\psi} \gamma^\mu D_\mu \psi = \bar{\psi} \gamma^0 D_0 \psi + \bar{\psi} \gamma^i D_i \psi$

$$[\Sigma^a, \gamma_0] = 0, [\Sigma^a, \gamma_i] \neq 0,$$

\uparrow
CS invariant

\uparrow
breaks CS

Necessary condition for approximate CS symmetry:

Quantum effective action **dynamically dominated by colour-electric interactions!**

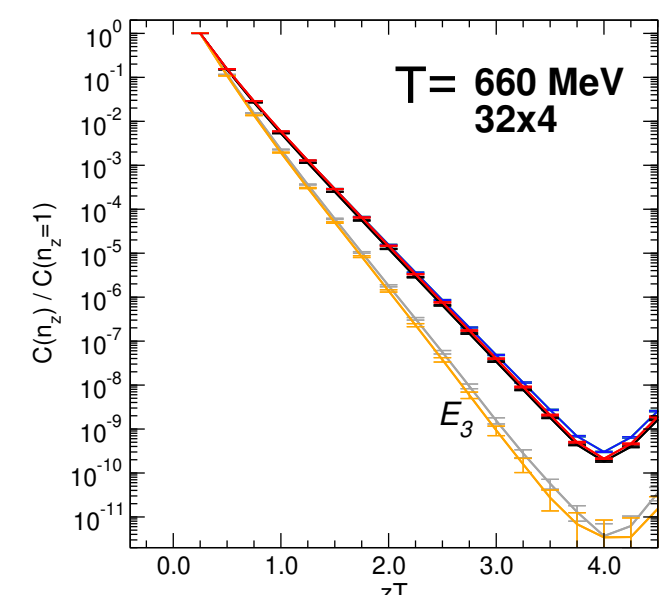
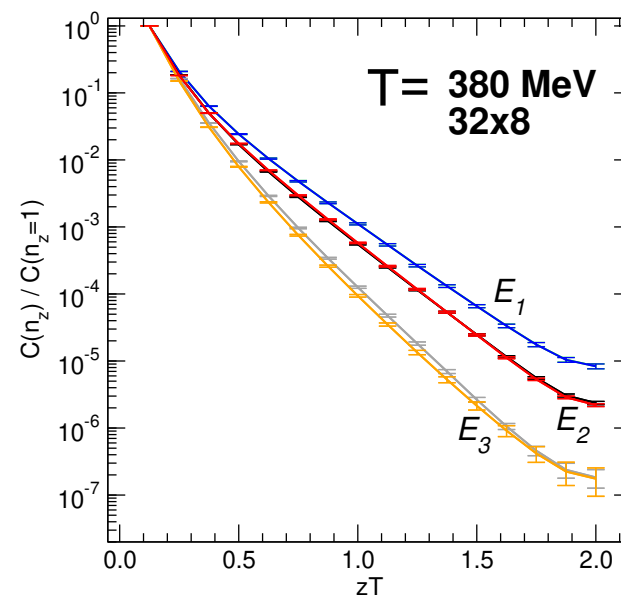
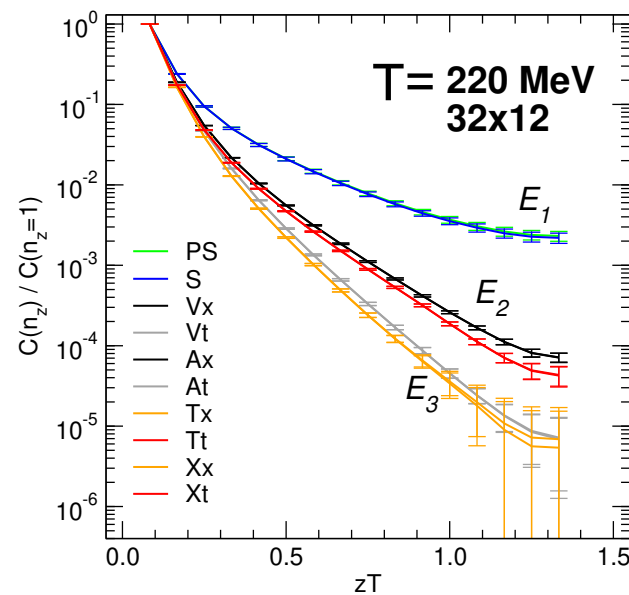
CS-symmetry observed in meson correlators

JLQCD domain wall fermions at phys. point

Spatial correlators:

[Rohrhofer et al., Phys. Rev. D 100 (2019)]

$$\begin{aligned}
 E_1 : & \quad PS \leftrightarrow S, & U(1)_A \\
 E_2 : & \quad V_x \leftrightarrow T_t \leftrightarrow X_t \leftrightarrow A_x, & SU(4) \\
 E_3 : & \quad V_t \leftrightarrow T_x \leftrightarrow X_x \leftrightarrow A_t, & SU(2)_L \times SU(2)_R \times U(1)_A
 \end{aligned}$$

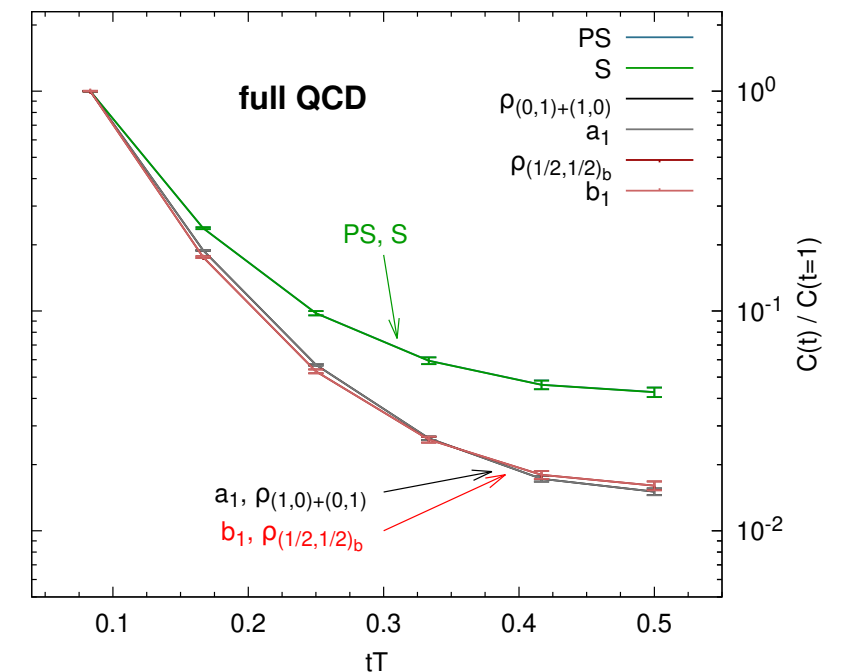
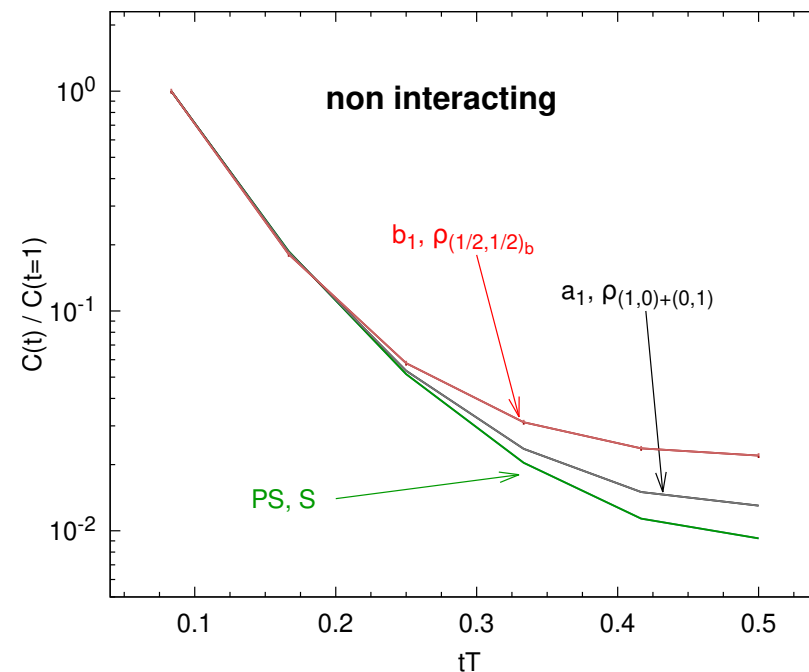


Temporal correlators:

[Rohrhofer et al., PLB 20]

$$T = 220 \text{ MeV} \quad (1.2T_c)$$

$$48^3 \times 12 \quad (a = 0.075 \text{ fm})$$

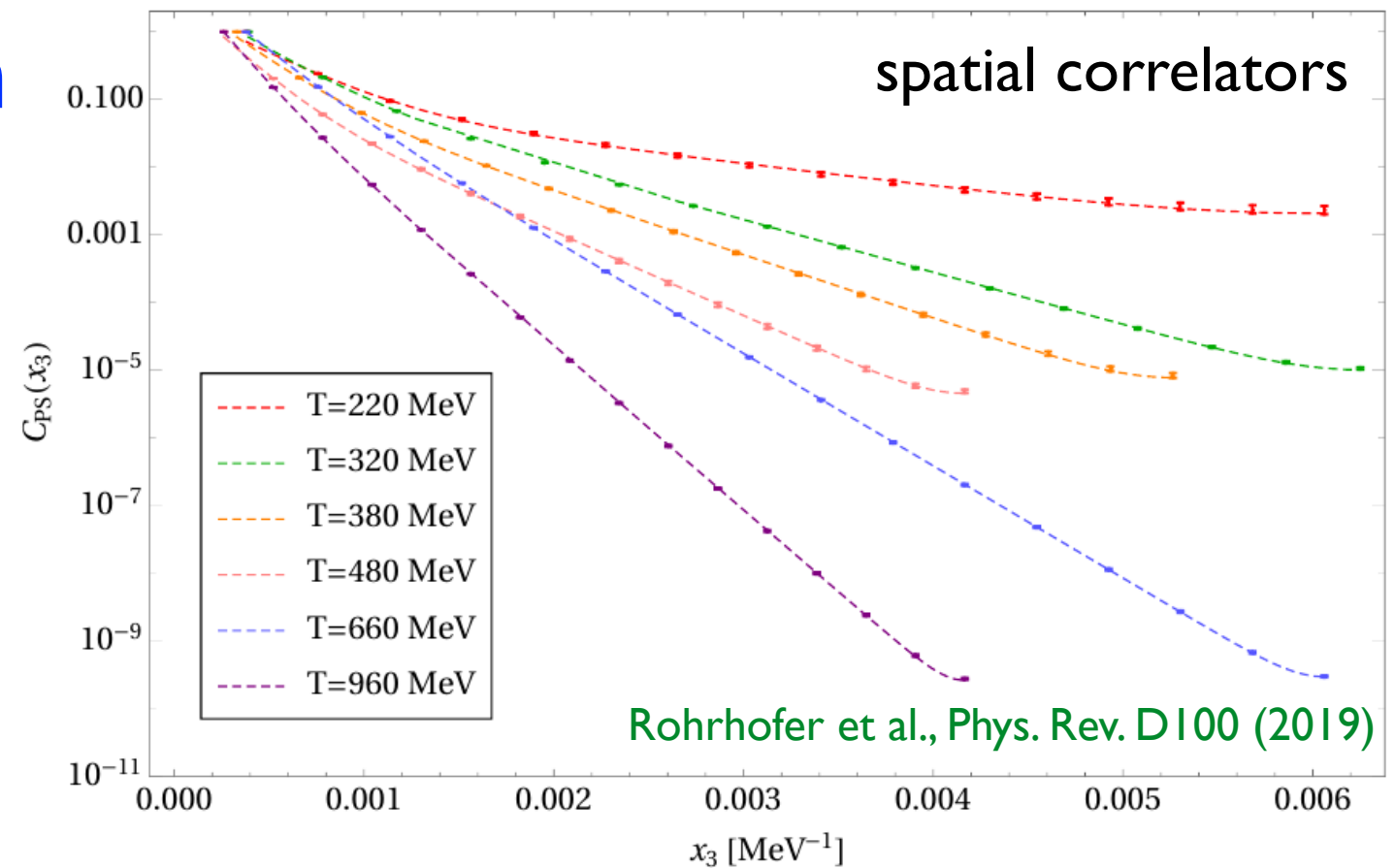
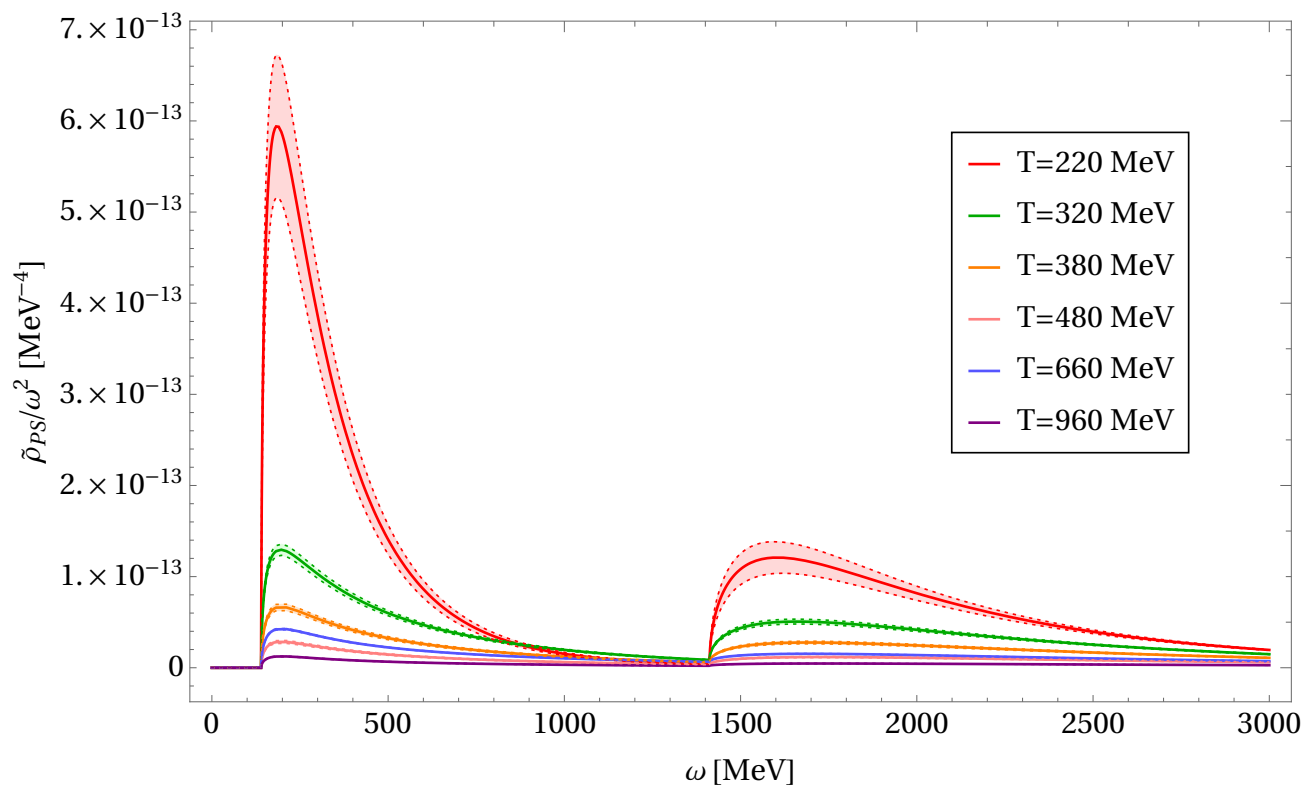


The pion spectral function

[Lowdon, O.P., arXiv:2207.14718]

damping factors 2-state fits π, π^*

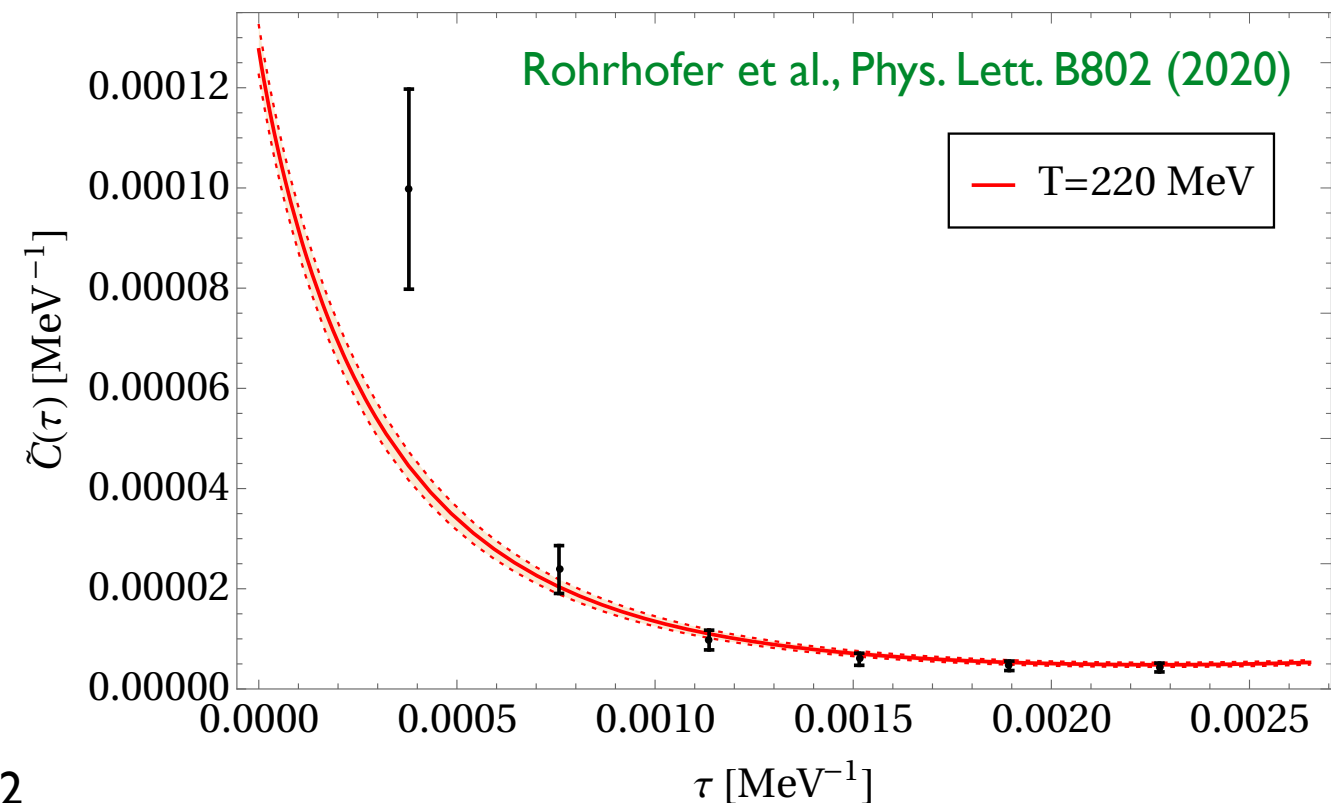
$$D_{m,\beta}$$



Rohrhofer et al., Phys. Rev. D 100 (2019)

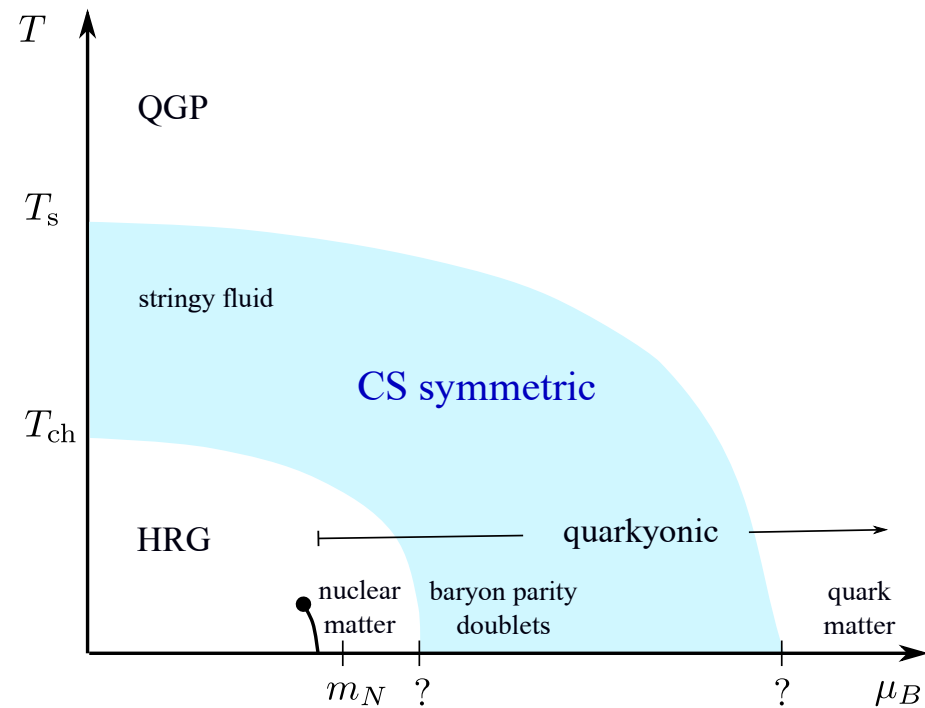
spectral functions

predict temporal correlators, compare with data

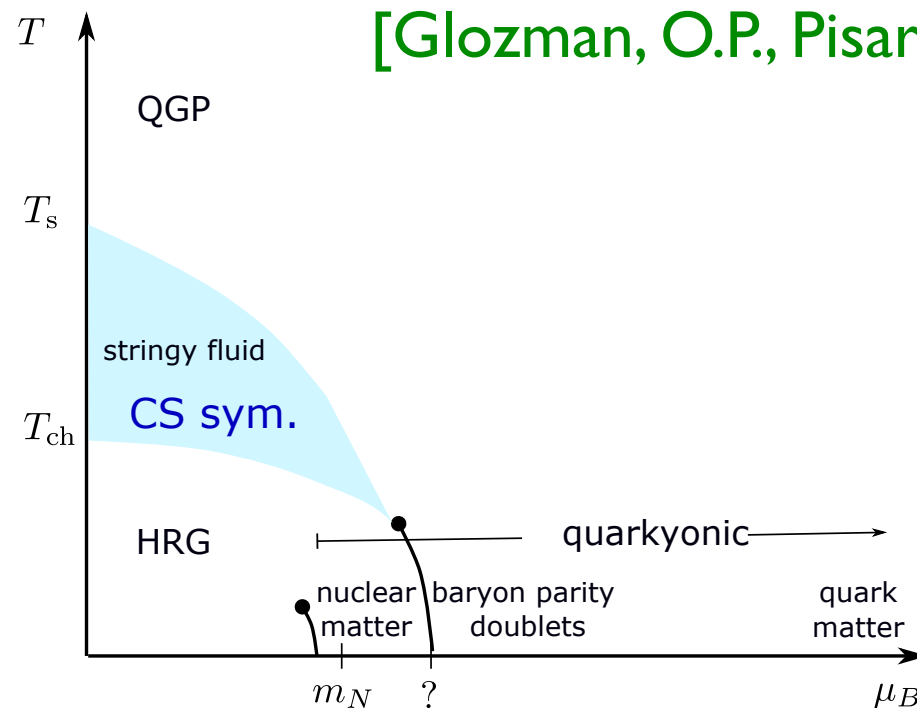


Rohrhofer et al., Phys. Lett. B 802 (2020)

CS symmetry at finite density

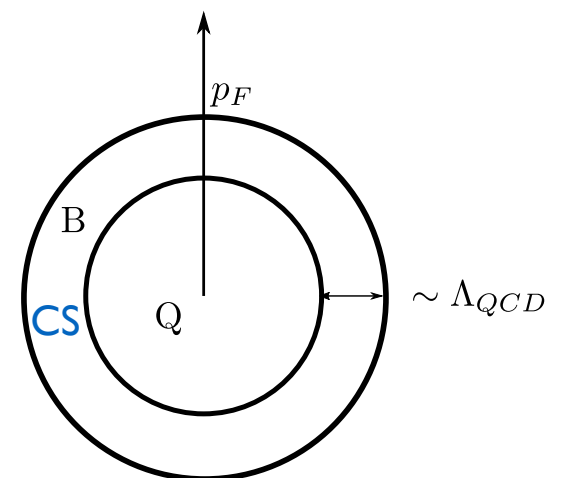


[Glozman, O.P., Pisarski, arXiv:2204.05083]



...

- Cold and dense candidate: baryon parity doublet models, **CS symmetric** [Glozman, Catillo PRD 18]
- Quarkyonic matter [McLerran, Pisarski, NPA 07; O.P., Scheunert JHEP 19]
 - contains chirally symmetric baryon matter
 - consistent with intermediate CS regime
- CS consistent with or without chiral phase transition



Conclusions

- Chiral transition is at zero density is second order for $N_f=2-7$
- Phenomenologically relevant constraints on critical point emerging
- Three regimes of QCD: chirally broken, chiral spin symmetric, chiral symmetric
- Intermediate temperature regime $T_{pc} < T < 3T_{pc}$
with CS symmetry and hadron-like states