Variations on Nuclear Shapes

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Outline

- The Quadrupole Interaction: Shapes
- Nuclear Shape; Meaning and Limitations
- Shape coexistence in ^{68}Ni and ^{78}Ni: K-plots
- Nuclear Shapes at the LHC ?

Nuclear Shapes

- To speak of the shape of a quantal system it is necessary to define an intrinsic reference frame, hence the rotational (and reflection) invariances must be broken. In addition, we usually rely on semiclassical models, liquid-drop like, to describe properties akin to the concept of shape.
- The surface of a drop can be expressed in the basis of the spherical harmonics $Y_{\lambda,\mu}(\theta,\phi)$. The coefficients of the development, $\alpha_{\lambda,\mu}$, are the shape parameters.

Nuclear Shapes

- **To characterise the quadrupole shapes in the intrinsic frame two parameters are used β and γ, and a large variety of recipes exist to relate them to the laboratory frame observables**

Quadrupole Invariants

- The only rigorous method to relate the intrinsic parameters to laboratory-frame observables is provided by the so-called quadrupole invariants Q^n of the second-rank quadrupole operator Q_2 introduced by Kumar.
- The calculation of β and γ requires the knowledge of the expectation values of the second- and third-order invariants defined, respectively, by $\hat{Q}^2 = \hat{Q} \cdot \hat{Q}$ and $\hat{Q}^3 = (\hat{Q} \times \hat{Q}) \cdot \hat{Q}$ (where $\hat{Q} \times \hat{Q}$ is the coupling of $\hat{Q}$ with itself to a second-rank operator).

Fluctuations

Indeed, it is not very meaningful to assign effective (average) values to β and γ without also studying their fluctuations. Our aim is to go beyond the extraction of effective values of these intrinsic parameters and obtain their variances.

With this goal, we calculate:

$$\sigma(\hat{Q}^2) = (\langle \hat{Q}^4 \rangle - \langle \hat{Q}^2 \rangle^2)^{1/2} \tag{1}$$

and

$$\sigma(\hat{Q}^3) = (\langle \hat{Q}^6 \rangle - \langle \hat{Q}^3 \rangle^2)^{1/2} . \tag{2}$$

Higher-order invariants

- **The choice of the fourth-order invariant $\hat{Q}^4$ is unique and we take it as $\hat{Q}^4 = (\hat{Q}^2)^2 = (\hat{Q}\cdot\hat{Q})^2$.**
- **The fifth-order invariant is also unique and we take it as $\hat{Q}^5 = \hat{Q}^2\ \hat{Q}^3 = (\hat{Q}\cdot\hat{Q})([\hat{Q}\times\hat{Q}]\cdot\hat{Q}])$.**
- **The sixth order invariant is not unique. There are two choices but the adequate one to use in Eq. (2) is $\hat{Q}^6 = (\hat{Q}^3)^2 = ([\hat{Q}\times\hat{Q}]\cdot\hat{Q}])^2$.**

We have been able to compute them using the Lanczos Projected Strength Function Method. See

A. Poves, F. Nowacki, and Y. Alhassid,
Phys. Rev. C 101, 054307 (2020),
for the details.

The intrinsic quadrupole moment Q_0 and the effective (average) values of the Bohr-Mottelson shape parameters β and γ can be calculated from the expectation values of the second- and third-order invariants using

$$Q_0 = \sqrt{\frac{16\pi}{5}} \langle \hat{Q}^2 \rangle^{1/2} , \tag{3}$$

$$\beta = \frac{4\pi}{3r_0^2} \frac{\langle \hat{Q}^2 \rangle^{1/2}}{A^{5/3}} , \tag{4}$$

with r_0=1.2 fm, and

$$\cos 3\gamma = -\sqrt{\frac{7}{2}} \frac{\langle \hat{Q}^3 \rangle}{\langle \hat{Q}^2 \rangle^{3/2}} \tag{5}$$

Fluctuations in β and γ

$$\frac{\Delta\beta}{\beta} = \frac{1}{2}\frac{\sigma\langle\hat{Q}^2\rangle}{\langle\hat{Q}^2\rangle} \ . \tag{6}$$

$$\frac{\sigma^2(\cos 3\gamma)}{(\overline{\cos 3\gamma})^2} = \frac{\sigma^2\langle\hat{Q}^3\rangle}{\langle\hat{Q}^3\rangle^2} + \frac{9}{4}\frac{\sigma^2\langle\hat{Q}^2\rangle}{\langle\hat{Q}^2\rangle^2} - 3\frac{\langle\hat{Q}^5\rangle - \langle\hat{Q}^3\rangle\langle\hat{Q}^2\rangle}{\langle\hat{Q}^3\rangle\langle\hat{Q}^2\rangle} \ . \tag{7}$$

Notice that the covariance term in (7) requires the knowledge of $\langle\hat{Q}^5\rangle$. The range of γ values at 1σ is given by

$$\cos^{-1}(\cos 3\gamma \pm \sigma(\cos 3\gamma)) \tag{8}$$

Miscellaneous results

	β	$\Delta\beta$	$\frac{\sigma\langle\hat{Q}^2\rangle}{\langle\hat{Q}^2\rangle}$	γ	γ range
^{20}Ne	0.62	0.07	0.24	3°	0° — 9°
^{24}Mg	0.60	0.07	0.25	18°	12° — 22°
^{48}Cr	0.31	0.06	0.41	13°	0° — 20°
^{34}Si	0.18	0.10	1.07	40°	0° — 60°
0_2^+	0.42	0.08	0.37	40°	30° — 60°
^{68}Ni	0.11	0.06	1.10	36°	0° – 60°
0_2^+	0.19	0.05	0.55	38°	23° – 60°
0_3^+	0.29	0.05	0.36	16°	0° – 24°
^{64}Cr	0.29	0.06	0.35	16°	0° – 24°

Do the intrinsic shape parameters β and γ survive in the laboratory frame?

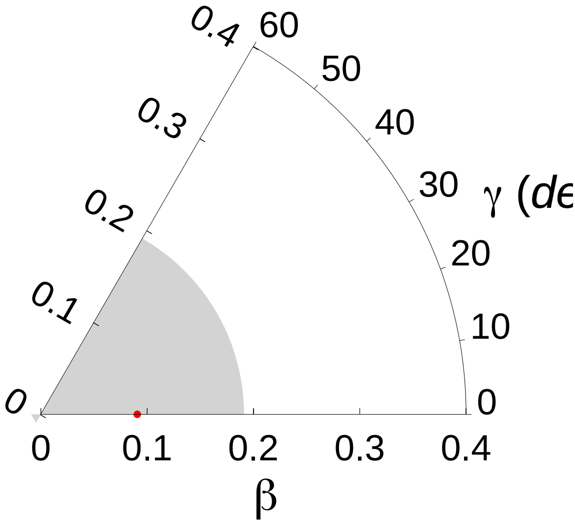
- β: **yes although nuclei are most often β-soft**
- γ: **rather not. The fluctuations in γ amount to 20°- 30°. In some cases the oblate or prolate character survives. In others, both sectors of the β-γ sextant are equally probable**
- β **and** γ **only have small fluctuations when the nucleus approaches the SU3 limit. And, probably, in heavier well deformed nuclei too.**

A bit of semantics: Are doubly magic nuclei spherical?

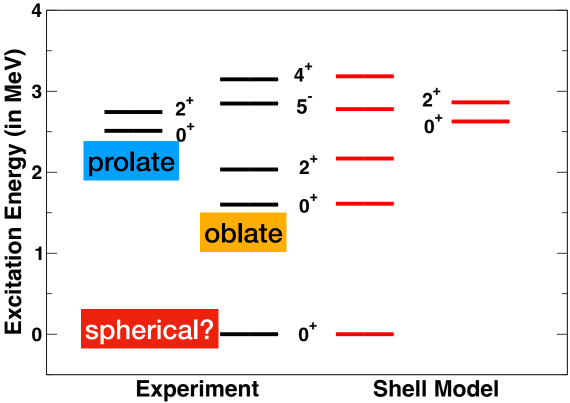
- Doubly magic nuclei are NOT spherical, they have NO shape, because $\frac{\sigma\langle\hat{Q}^2\rangle}{\langle\hat{Q}^2\rangle} \approx 1$, and the span of γ is close to 60°
- Hence, there are NO spherical nuclei at all as seen below
- ^{56}Ni β = 0.21$\pm$ 0.07 γ= 40.5° span 13° – 60°
- ^{48}Ca β = 0.15$\pm$ 0.05 γ= 33° span 0° – 60°
- Discomforting isn't it?

The K-plots are a representation in the (β, γ) sextant of the locus of their variances.

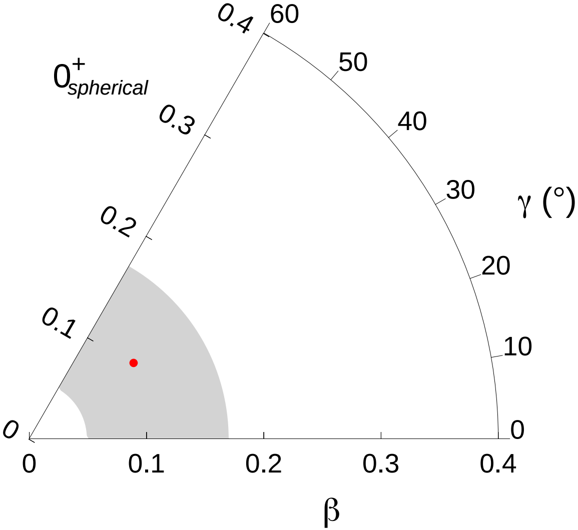
Doubly magic ^{40}Ca



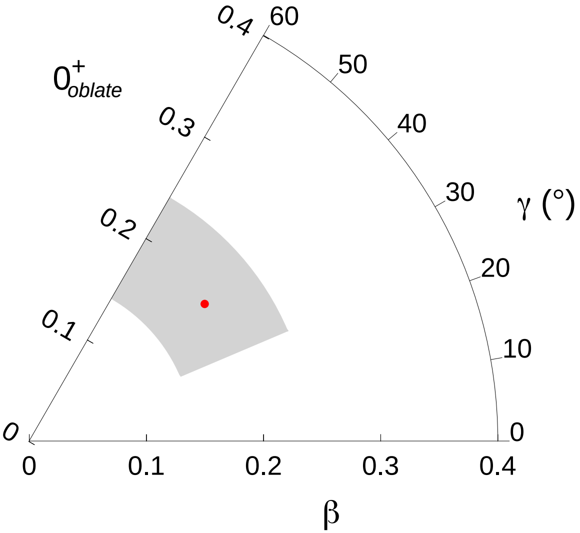
Shape Coexistence in ^{68}Ni


Excitation Energy (in MeV)
4
3
2
1
0
2+
0+
prolate
4+
5-
2+
0+
oblate
spherical?
0+
2+
0+
Experiment
Shell Model

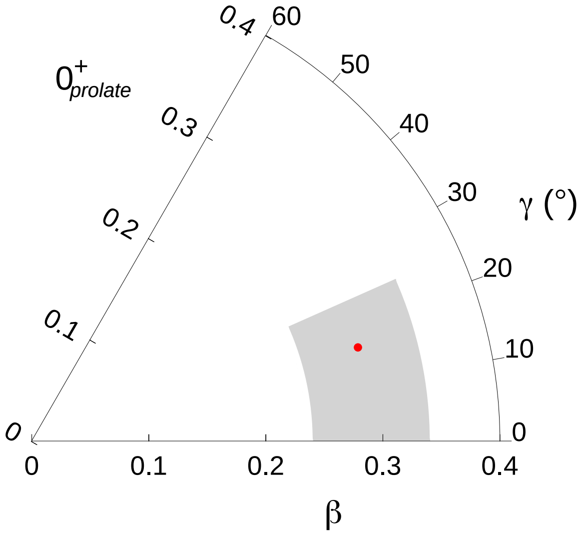
Is the closed-shell ground state spherical?



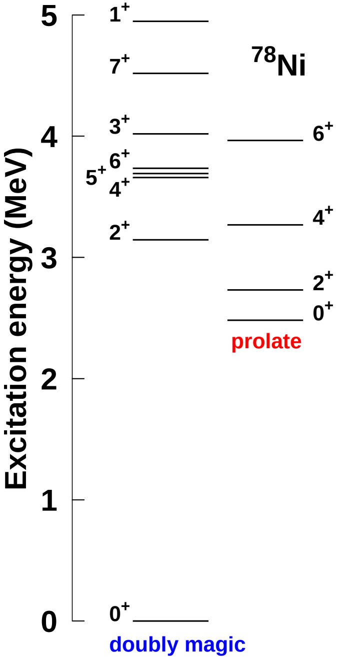
^{68}Ni



^{68}Ni

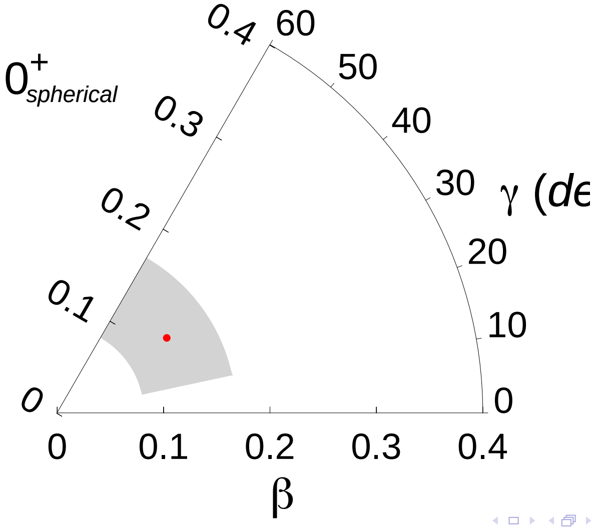


^{78}Ni

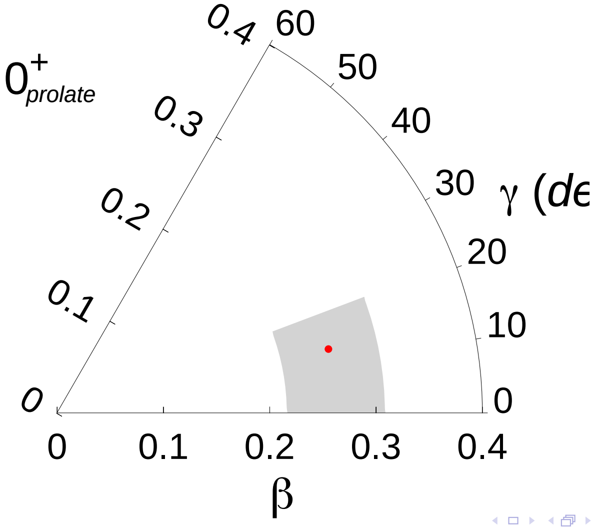


Excitation energy (MeV)
0
1
2
3
4
5
1+
7+
3+
6+
5+
4+
2+
0+
doubly magic
78Ni
6+
4+
2+
0+
prolate

^{78}Ni, the doubly magic ground state



^{78}Ni, the coexisting deformed 0^+



How affect these fluctuations our established ideas about shape evolution and shape coexistence?

- **Shape evolution: The shape of the nuclei changes along isotopic or isotonic chains.**
- **Shape coexistence: Different states of the same nucleus have different shapes.**
- **But, how to interpret the cases in which the different 1σ contours in the β-γ plane have large overlaps?**
- **Food for thought**

What can tell us the high energy heavy ion collisions, at RHIC or at LHC, about the nuclear shapes?

- **In several recent papers it is argued that these experiments can inform us abut the deformation and the triaxiality of the colliding nuclei.**
- **B. Bally talk. See also the Atlas collaboration paper, arXiv-2205.00039, where they compare the ^{208}Pb-^{208}Pb and the ^{129}Xe-^{129}Xe results.**
- **They submit that two of their experimentally extracted parameters are related to β and γ as:**

$$v_2^2 \approx a + b\,\beta^2 \text{ and } \rho^2 \approx \mathrm{a}' + \mathrm{b}'\cos(3\gamma)\beta^3$$

- **where a and a' are the values in the spherical case**

- The Atlas paper concludes that
- *Comparison of the model with the Pb+Pb and Xe+Xe data confirms that the ^{129}Xe nucleus is a highly deformed triaxial ellipsoid that has neither a prolate nor oblate shape. This provides strong evidence for a triaxial deformation of the ^{129}Xe nucleus from high-energy heavy-ion collisions.*
- We have computed the variances of β and γ in ^{130}Xe, which is an excellent proxy. We perform SM-CI calculations in a large valence space, with the GCN5082 effective interaction, which reproduce nicely the spectroscopy of the Xenon isotopes from A=128 to A=136, and its electromagnetic properties.

- In the NNDC database one finds:
- ^{128}Xe, B(E2) ($2^+ \rightarrow 0^+$) = 48(11) WU
- ^{130}Xe, B(E2) ($2^+ \rightarrow 0^+$) = 38(5) WU
- Corresponding to β_{ch}= 0.20$\pm$0.03
- and β_{ch}= 0.18$\pm$0.02, respectively
- using the standard BM recipe.

- Our calculation gives:
- ^{130}Xe, β_{ch}= 0.17 from the B(E2), with the BM prescription.
- Using the Kumar invariants, and for the mass deformation we get:
- β_m= 0.14$\pm$0.02
- γ=26°, with an interval at one σ (13° – 37°)
- from $cos3\gamma$=0.21 and $\sigma(cos3\gamma)$=0.57

- **Remember that the formula used by the Atlas coll, reads**

$$v_2^2 \approx a + b\,\beta^2 \text{ and } \rho^2 \approx \mathrm{a}' + \mathrm{b}'\cos(3\gamma)\beta^3$$

- **How to make sense of this given that $cos3\gamma$=0.21$\pm$0.57?**
- **By the way, β^2=0.020$\pm$0.006.**
- **and β^3=0.0027$\pm$0.0012**

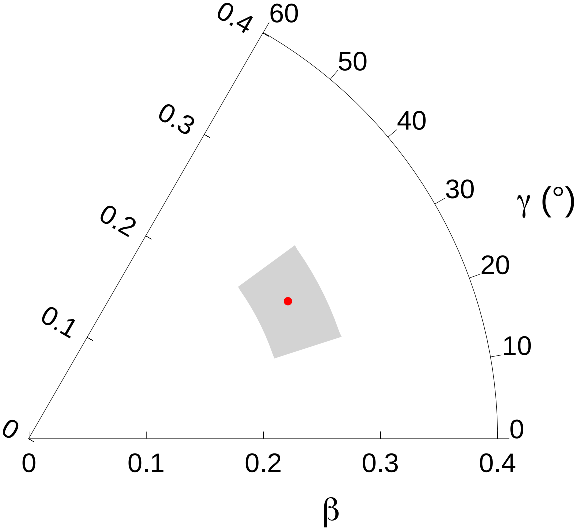
Whereof one cannot speak, thereof one must be silent

L. Wittgenstein,
Tractatus logico-philosophicus, Proposition 7,
Routledge and Kegan Paul eds., London (1922).

Rigid triaxiality in ^{76}Se and ^{76}Ge?

- **Shell Model Calculations in the r3g space with the jj44b interaction and standard effective charges**
- ^{76}**Ge:** $\beta = 0.17 \pm 0.02$ **and** γ **= (26$^{+9}_{-9}$)$^\circ$**
- ^{76}**Se:** $\beta = 0.20 \pm 0.03$ **and** γ **= (31$^{+17}_{-16}$)$^\circ$**
- **Shell Model Calculations in the LNPS space and standard effective charges**
- ^{76}**Ge:** $\beta = 0.25 \pm 0.03$ **and** γ **= (28$^{+8}_{-10}$)$^\circ$**

^{76}Ge



^{76}Se