From correlations to universal behavior in Few-Nucleon Systems

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EuNPC 2022

Santiago de Compostela, 24-29 October 2022

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Outline

- Appearance of universal behavior
- Equal long-range behavior but very different short-range behavior
- Weakly bound systems
- Definition of the unitary limit
- Definition of the universal window: Efimov physics
- Dynamics governed by a few parameters (control parameters)
- Continuous (or discrete) scale invariance
- Impact of Efimov physics in nuclear systems

Interplay of two aspects

- Weakly bound systems are strongly correlated
- In the universal regime details of the interaction are not important
- Effective interactions
- Are correlated systems and universal properties compatible?

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The universal window

Low energy quantities

- We consider a short-range interaction: $V(r > r_0) \to 0$

  In this case low energy means $E = k^2\hbar^2/m < \hbar^2/mr_0^2$

- In this regime the s-wave phase shift is well described by the effective range expansion up to second order

  $k \cot \delta_0 = -1/a + r_e k^2/2 + \ldots$

  with a the scattering length defined from the zero-energy Schrödinger equation, $H\phi_0 = 0$

  $$\phi_0(r \to \infty) \to u_0 = 1 - a/r$$

  and r_e the effective range

  $$r_e = \frac{2}{a^2} \int_0^\infty (\phi_0^2 - u_0^2) r^2 dr$$

The universal window

The presence of a shallow bound state

A bound state correspond to the S-matrix pole $k \cot \delta_0 - ik = 0$
In general all terms in the expansion of $k \cot \delta_0$ are needed.
However, when a shallow state appears (fine tuning), we can use the expansion up to second order ($i\kappa = k$)

$$\kappa = 1/a + r_e \kappa^2/2 + \ldots$$

which introduces a strict correlation between the low energy parameters ($E = \hbar^2 \kappa^2/m$).

Defining the lengths $a_B = 1/\kappa$ and $r_B = a - a_B$, they are related up to second order by

$$r_e a = 2 r_B a_B$$

The universal window is defined when $r_e/a \approx r_e/a_B \ << 1$

The universal window

Observables

Other observables are stricted correlated to the low-energy parameters (up to second order).
The mean square radius:

$$\langle r^2 \rangle = \frac{a^2}{8}\left[1 + \left(\frac{r_B}{a}\right)^2 + \cdots\right] = \frac{a_B^2}{8}\, e^{2r_B/a_B} = \frac{a_B^2}{8}\, f_{sc}$$

The asymptotic normalization constant

$$C_a^2 = \frac{2}{a_B}\frac{1}{1 - r_e/a_B} = \frac{2}{a_B}\, e^{2r_B/a_B} = \frac{2}{a_B}\, f_{sc}$$

the probability to be outside the interaction range

$$P_e = C_a^2 \int_{2r_B}^{\infty} e^{-2r/a_B} dr = e^{-2r_B/a_B} = \frac{1}{f_{sc}}$$

Universal behavior in few-body systems

Examples

- The helium dimer (as given by the LM2M2 potential):

  $a = 189.415$ a.u., $\rightarrow a_B = 182.221$ a.u.
  $r_e = 13.8447$ a.u., $\rightarrow r_B = 7.194$ a.u.
  $E_d = 1.303$ mk, $\rightarrow E(a, r_e) = 1.303$ mk

  $(r_e\, a)/(2 r_B a_B) = 1.0002$ $\rightarrow r_e/a = 0.073$

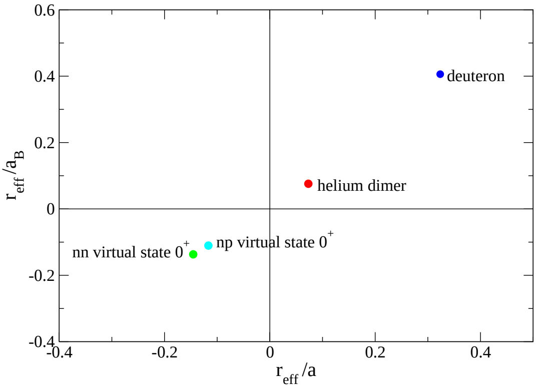
- The deuteron:

  $a^1 = 5.419 \pm 0.007$ fm, $\rightarrow a^1_B = 4.318$ fm
  $r^1_e = 1.753 \pm 0.008$ fm, $\rightarrow r^1_B = 1.101 \pm 0.007$ fm
  $E_d = 2.224575(9)$ MeV $\rightarrow E(a, r_e) = 2.223$ MeV
  $\langle r \rangle = 1.97535(85)$ fm $\rightarrow \langle r \rangle(a, r_e) = 1.970$ fm
  $C_a = 0.8781(44)$ fm$^{-1/2}$ $\rightarrow C_a(a, r_e) = 0.8782$ fm$^{-1/2}$

  $(r_e\, a)/(2 r_B a_B) = 0.9991$
  $(16/a_B^3)(\langle r^2 \rangle / C_a^2) = 1.005$ $\rightarrow r_e/a = 0.32$

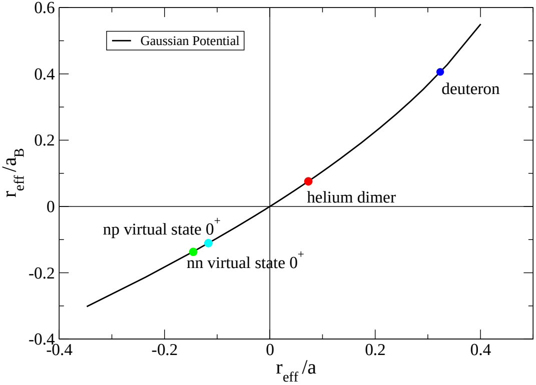
The universal window

Protagonists of the story:
$a \rightarrow$ scattering length
$r_{eff} \rightarrow$ effective range
$a_B \rightarrow E = \hbar^2/ma_B^2 \rightarrow$ energy length



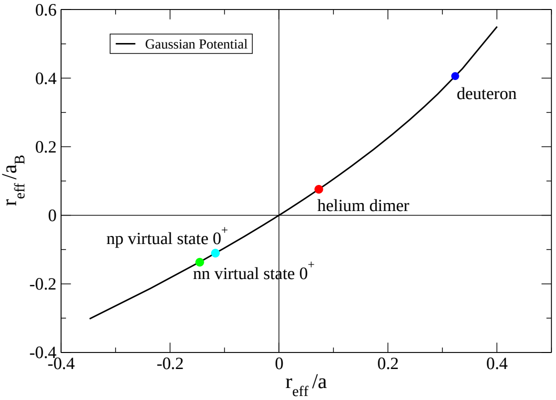
The Hero


G


Gaussian Potential
deuteron
helium dimer
np virtual state 0+
nn virtual state 0+
0.6
0.4
0.2
0
-0.2
-0.4
-0.4
-0.2
0
0.2
0.4
r_{eff}/a_B
r_{eff}/a

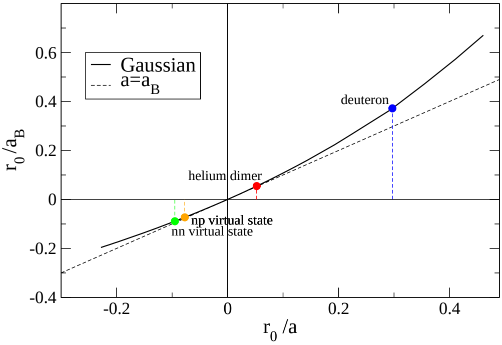
Universal behavior in few-body systems

- When a shallow state exists, the systems can be organized inside the universal window. The experimental points can be connected using a potential model (here a Gaussian potential).



Gaussian characterization of the universal window

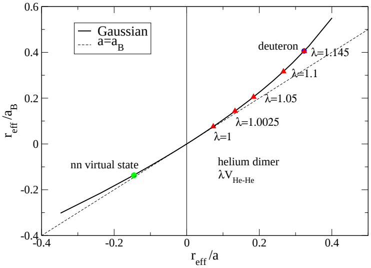
$V(1,2) = V_0 e^{-r^2/r_0^2}$, with the unitary limit at $V_0 = 2.684\hbar^2/mr_0^2$



At $r_0/a = 0.2877$ a Gaussian potential describes correctly the deuteron low-energy parameters a^1, r_e^1, a_B

Moving along the universal window

- By scaling the strength of the potential, systems can be (ideally) moved along the universal window
- The Gaussian charaterization can be used to continously connect systems inside the window



Continuous scale invariance

$a \to \alpha a$

$a_B \to \alpha a_B$

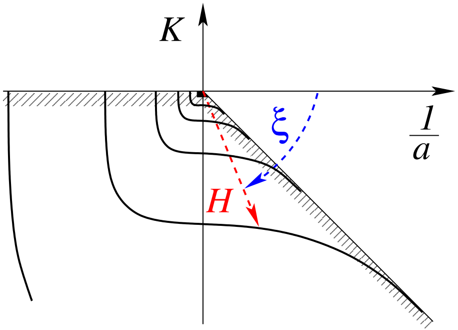
$r_B = a - a_B \to \alpha r_B$

$r_e = 2 r_B a_B / a \to \alpha r_e$

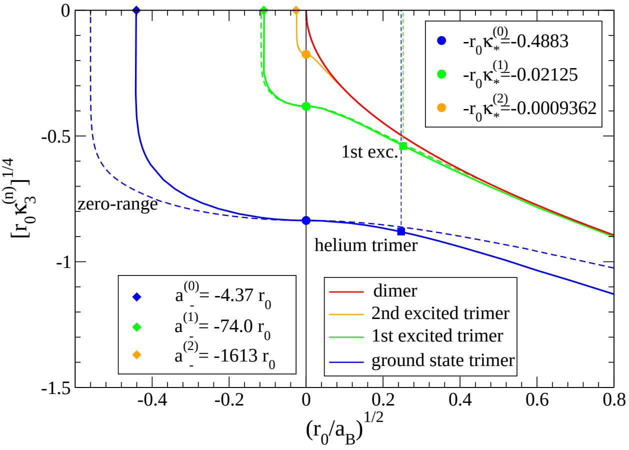
r_0 =constant

The three-body system inside the universal window

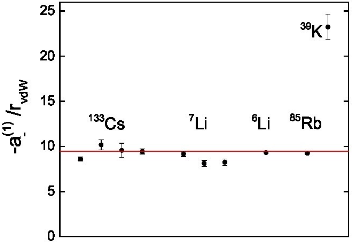
- The energy spectrum of three equal bosons shows a discrete scale invariance (DSI)
- As the range of the interaction goes to zero, ($r_0 \rightarrow 0$) $\rightarrow$ Thomas collapse
- At the unitary limit $1/a \rightarrow 0 \rightarrow$ Efimov effect



Gaussian characterization of the universal window for three bosons: $H = T + V_0 \sum_{i<j} e^{r_{ij}^2/r_0^2}$



Footprint of universality



where the van der Waals length is $\ell_{vdW} = \frac{1}{2}(2mC_6/\hbar^2)^{1/4}$
For the Gaussian characterization this is encoded in the (almost) model independet relation $\kappa_*^{(0)} a_-^{(0)} = -4.37 r_0 \frac{0.4883}{r_0} = -2.14$

And for van der Waals species $\kappa_*^{(0)} a_-^{(0)} \approx -2.2$

Gaussian characterization of the universal window for three 1/2 spin-isospin fermions

Two nucleon data

two nucleons	E_2 (MeV)	a^S (fm)	r_e^S (fm)
$np\ S=1$	2.2245	5.419	1.753
$np\ S=0$	0.0661	-23.740	2.77

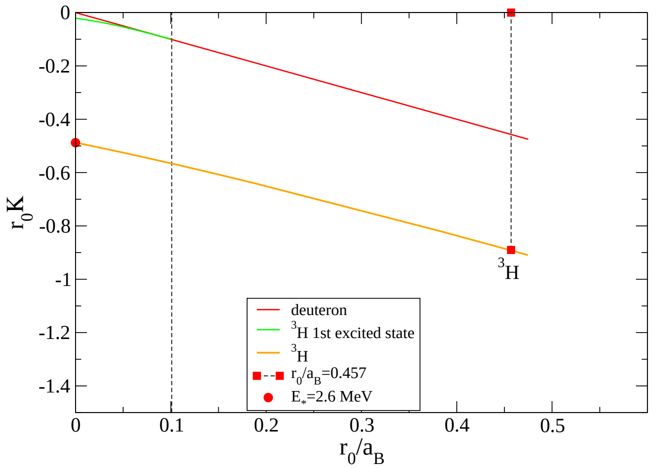
The Potential:

$$V(i,j) = V_0 e^{-r_{ij}^2/r_0^2}\mathcal{P}_{01} + V_1 e^{-r_{ij}^2/r_1^2}\mathcal{P}_{10}$$

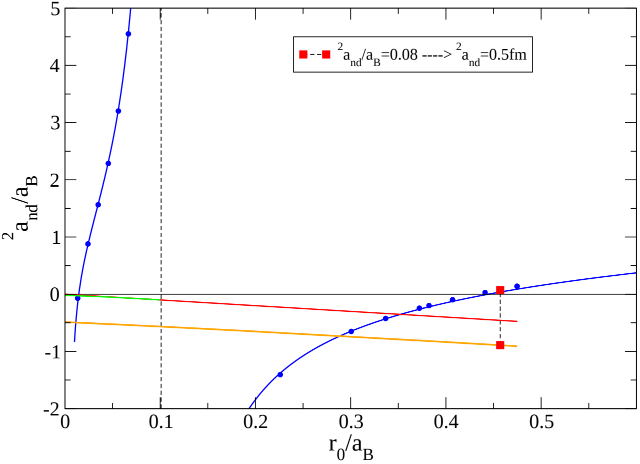
with $\mathcal{P}_{ST}$ the projector on the spin-isospin channels S, T

Walking along the nuclear cut:
Fixing $r_0 = r_1$, the strengths V_0 and V_1 are varied verifying $a^0/a^1 = -4.38$

Gaussian characterization of the universal window for three 1/2 spin-isospin fermions

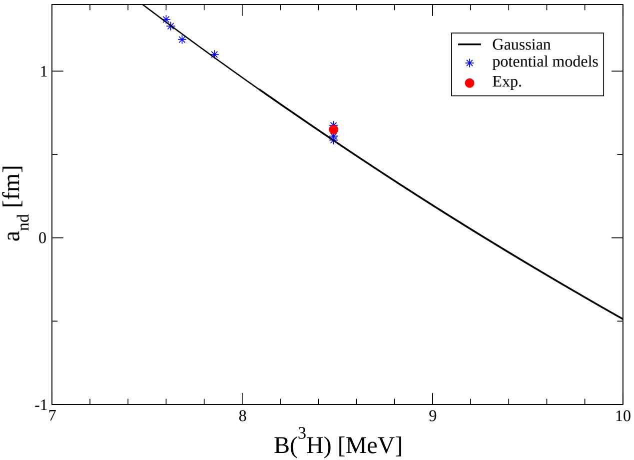


Correlations inside the universal window, the *nd* scattering length: ${}^2a_{nd} = 0.65 \pm 0.01$ fm

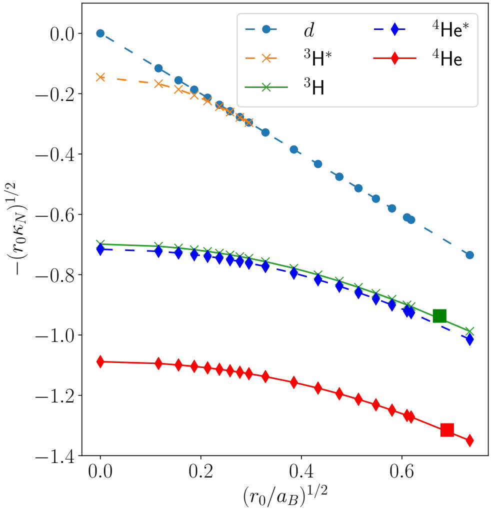


A. Deltuva, M. Gattobigio, A. K. and M. Viviani, PRC 102, 064001 (2020)

Correlations inside the universal window: the Phillips line



Gaussian characterization of the universal window for four 1/2 spin-isospin fermions



The ^{3}H and ^{4}He physical points

At the ^{3}H physical point $r_0/a_B = 0.457$ or $r_0 = 1.97\ \ \mathrm{fm}$
At the ^{4}He physical point $r_0/a_B = 0.483$ or $r_0 = 2.08\ \ \mathrm{fm}$
This implies that exist a potential

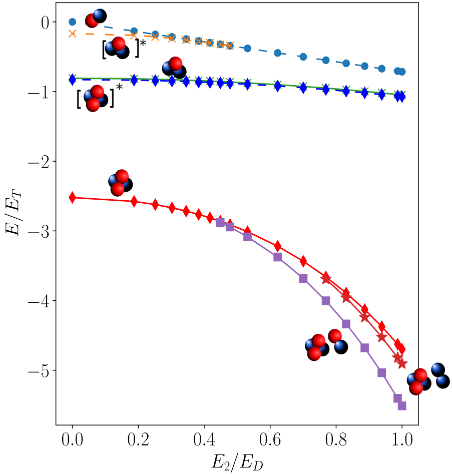
$$\sum_{i<j}\left(V_0 e^{-r_{ij}^2/r_0^2}\mathcal{P}_{01} + V_1 e^{-r_{ij}^2/r_0^2}\mathcal{P}_{10}\right)$$

that describes simultaneously ^{2}H and ^{3}H (for $r_0 = 1.97\ \ \mathrm{fm}$) and ^{2}H and ^{4}He (for $r_0 = 2.08\ \ \mathrm{fm}$).

At the unitary limit $E_*^3 = 2.6$ MeV and $E_*^4 = 13.5$ MeV

These potentials are a low-energy representation of the nuclear interaction. The different ranges in $N = 3$ and $N = 4$ reflect the fact that one ingredient is missing: the three-body force.

Gaussian characterization of the universal window for 1/2 spin-isospin fermions up to $A = 6$



Including the three-nucleon force

The potential is now

$$\sum_{i<j} V(i,j) + \sum_{i<j<k} W(i,j,k)$$

with

$$V(i,j) = \left(V_0 e^{-r_{ij}^2/r_0^2} \mathcal{P}_{01} + V_1 e^{-r_{ij}^2/r_0^2} \mathcal{P}_{10} \right)$$

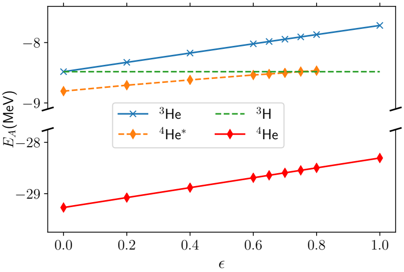
and

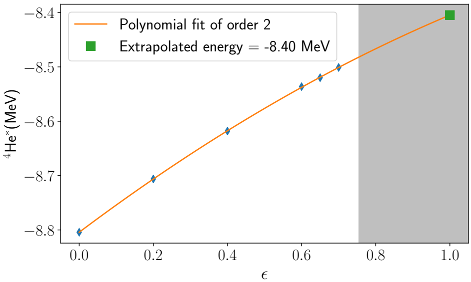
$$W(i,j) = W_0 e^{-r_{ijk}^2/\rho_0^2}$$

with $r_{ijk}^2 = r_{ij}^2 + r_{jk}^2 + r_{ki}^2$ and W_0 and ρ_0 fixed to describe simultaneously ^{3}H and ^{4}He.

The physical point turning on the Coulomb interaction

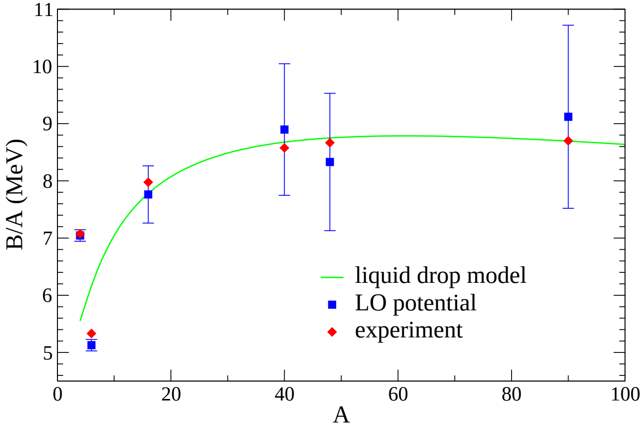
$$V_C(r) = \epsilon \frac{e^2}{r}$$





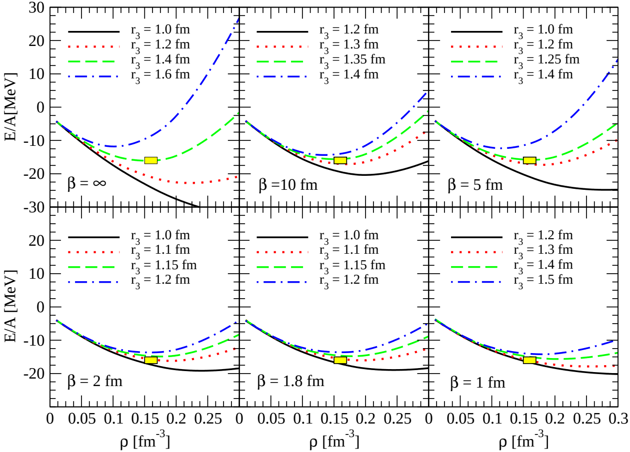
M. Gattobigio, A.K. and M. Viviani, PRC 100, 034004 (2019)

The nuclear chart with the two- plus three-body gaussian potential



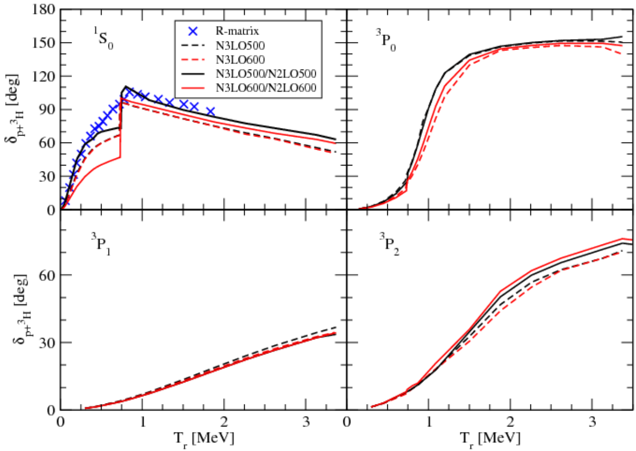
R. Schiavilla et al., PRC 103, 054003 (2021)

The saturation point of nuclear matter



A. Kievsky et al., PRL 121, 072701 (2018)

The 0^+ resonance of ^{4}He



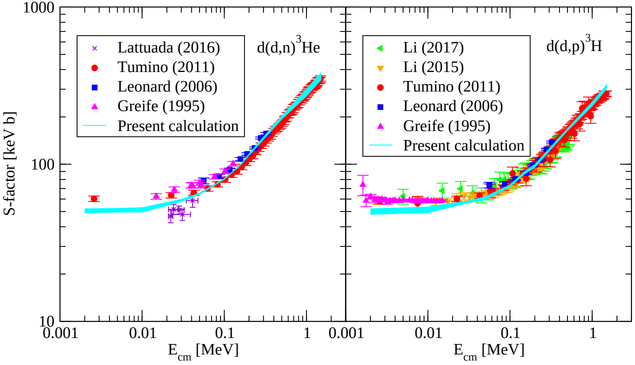
M. Viviani et al., PRC 102, 034007 (2020)

The 0^+ resonance of ^{4}He

Energy of the resonance and its width as extracted from the $p-{}^3$H phase-shifts using the HH method (M. Viviani et al, PRC 102, 034007 (2020)). The experimental values are extracted from the R-matrix analysis (D. R. Tilley, H. R. Weller, and G. M. Hale, Nucl. Phys. A541i, 1 (1992)).

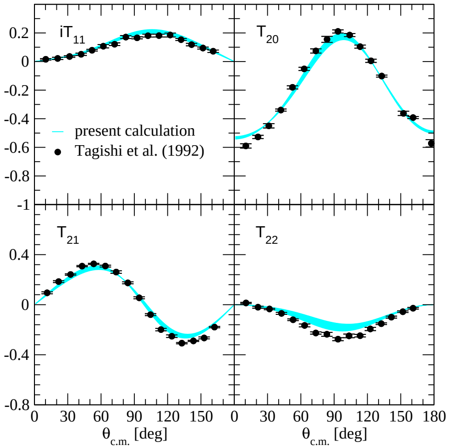
Interaction	E_R (MeV)	Γ (MeV)
N3LO500	0.126	0.556
N3LO600	0.134	0.588
N3LO500/N2LO500	0.118	0.484
N3LO600/N2LO600	0.130	0.989
N4LO450/N2LO450	0.126	0.400
N4LO500/N2LO500	0.118	0.490
N4LO550/N2LO550	0.130	0.740
Expt.	0.39	0.50

The $d(d,n)^3$He and $d(d,p)^3$H s-factor



M.Viviani, L.Girlanda, A.K., D.Logoteta, L.E.Marcucci, arXiv:nucl-th/2207.01433

The *dd* polarization observables



M.Viviani, L.Girlanda, A.K., D.Logoteta, L.E.Marcucci, arXiv:nucl-th/2207.01433

Conclusions

- Weakly bound systems can be located inside a window in which universal behavior emerges
- The universal behavior can be encoded in a two-parameter potential (as a Gaussian)
- Using this potential, trajectories (and correlations) can be studied along the universal window
- The physical point can be connected continuosly to the unitary point
- Nuclear levels emerge from the unitary point
- A two- plus a three-body soft interaction has been used to reproduce low energy observables in $A = 2, 3, 4$ and to predict energies along the nuclear chart
- The 0^+ stae of ^{4}He has been analyzed together with *dd* low energy observables