

Shape Coexistence in Sr isotopes

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FÍSICA
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What is shape coexistence?

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Shape coexistence: It appears in quantum systems where eigenstates with very different density distribution coexist.



Shape of the nucleus

(Implicit geometric interpretation)



- ▶ **Stabilizing effect:** closed shell
- ▶ **Deformed tendency:** pairing and quadrupole force

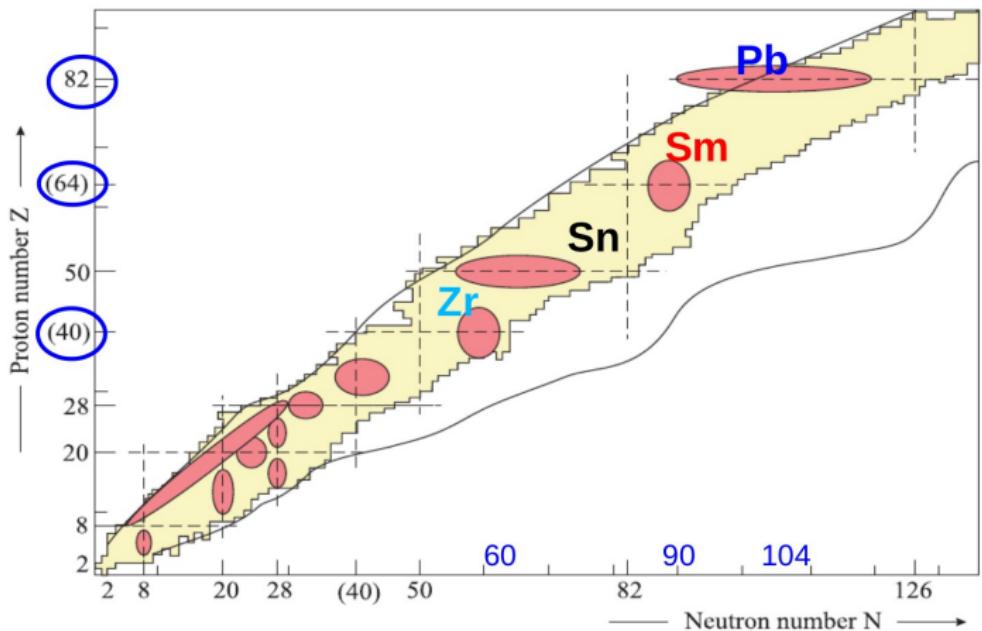


Regions around closed shells with **spherical shapes** and near mid-shell are **well deformed**

Regions of interest

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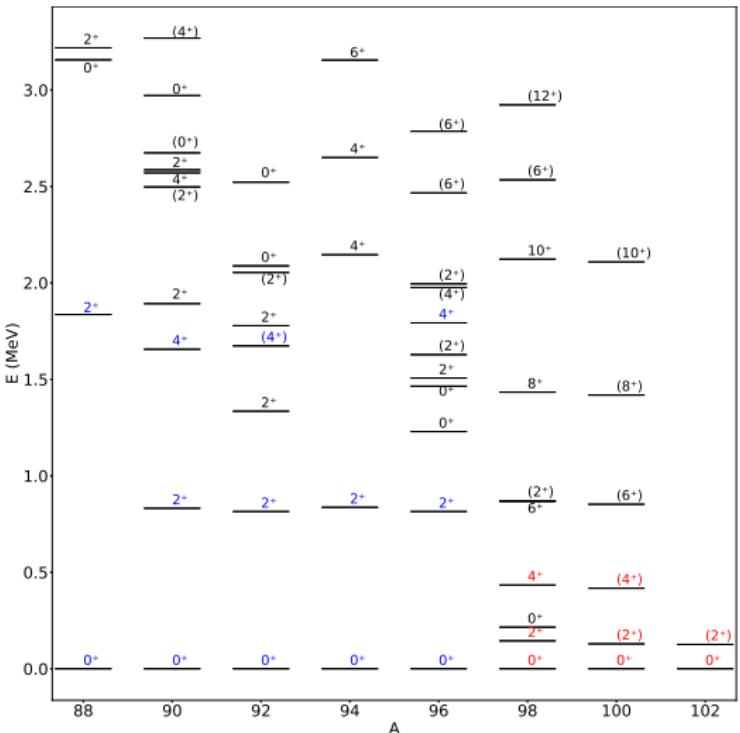
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Experimental data in the even-even nuclei



Blue labels are for spherical states while red labels correspond to the deformed ones.

Interacting boson Model. IBM

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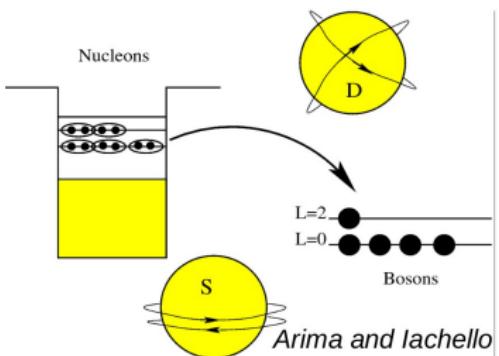
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Nucleons couple preferably in pairs with angular momentum either equal to 0 (S) or equal to 2 (D).

$$s^\dagger, d_m^\dagger(m = 0, \pm 1, \pm 2)$$

$$s, d_m(m = 0, \pm 1, \pm 2)$$

$$\hat{H}_{ECQF} = \epsilon \hat{n}_d + \kappa \hat{Q} \cdot \hat{Q} + \kappa' \hat{L} \cdot \hat{L}$$



- ▶ Model based on a **u(6) spectrum generator algebra**. It is especially suited for **medium and heavy-mass nuclei**.
- ▶ The number of bosons, **N**, corresponds the number of **nucleons pairs**, regardless its proton, neutron, particle or hole nature.

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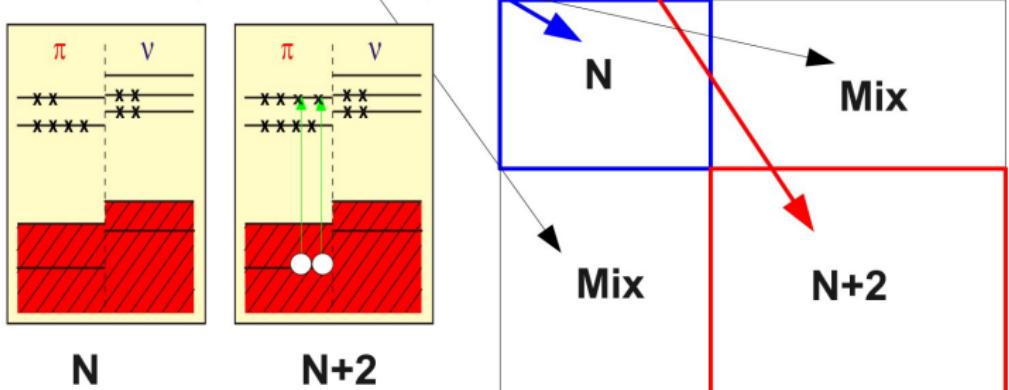
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IBM with configuration mixing

$$\hat{H} = \hat{P}_N^\dagger \hat{H}_{\text{ECQF}}^N \hat{P}_N + \hat{P}_{N+2}^\dagger (\hat{H}_{\text{ECQF}}^{N+2} + \Delta^{N+2}) \hat{P}_{N+2} + \hat{V}_{\text{mix}}^{N,N+2},$$



A different Hamiltonian, \hat{H}_{ECQF}^N and $\hat{H}_{\text{ECQF}}^{N+2}$, acts on the regular [N] and intruder [N+2] sectors, separately

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Least squares fit to the experimental data, including
excitation energies and absolute B(E2) transitions



$$\chi^2 = \frac{1}{N_{data} - N_{par}} \sum_{i=1}^{N_{data}} \frac{(X_i(data) - X_i(IBM))^2}{\sigma_i^2}$$

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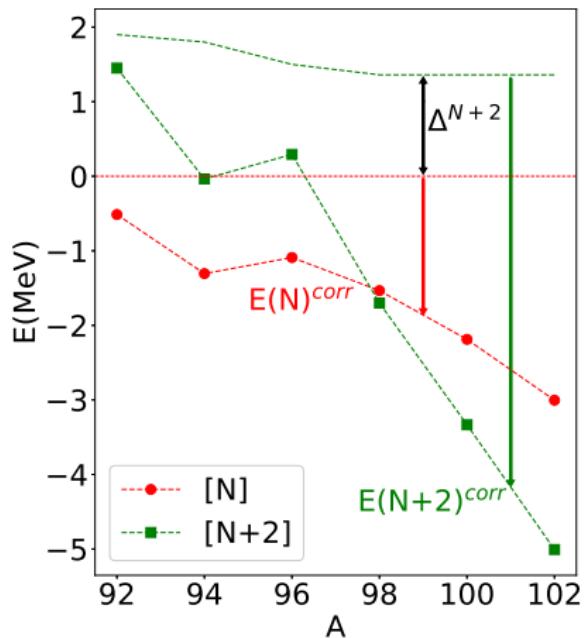
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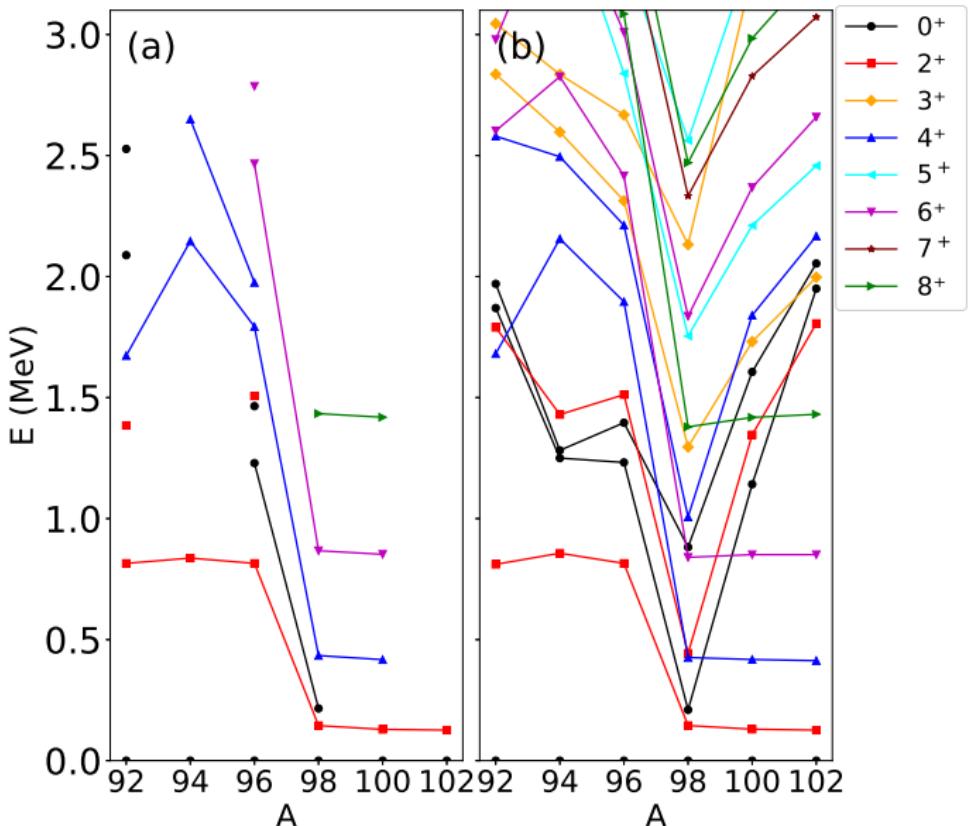
Absolute energy of the
lowest unperturbed regular
and intruder 0_1^+ states

$$\Downarrow$$

$V_{mix} = 0$

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Excitation energies



B(E2) transition probabilities-Yrast Band

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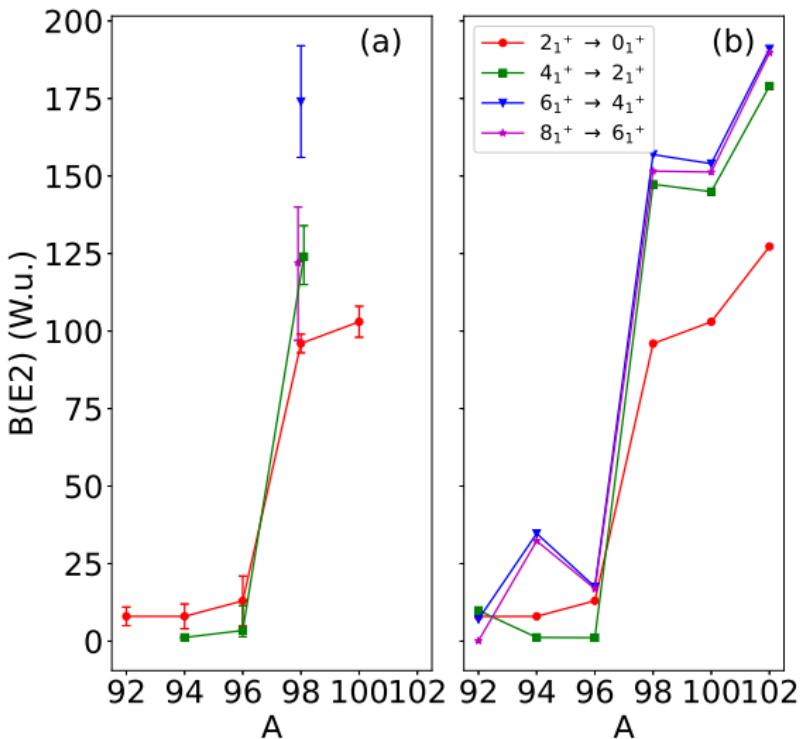
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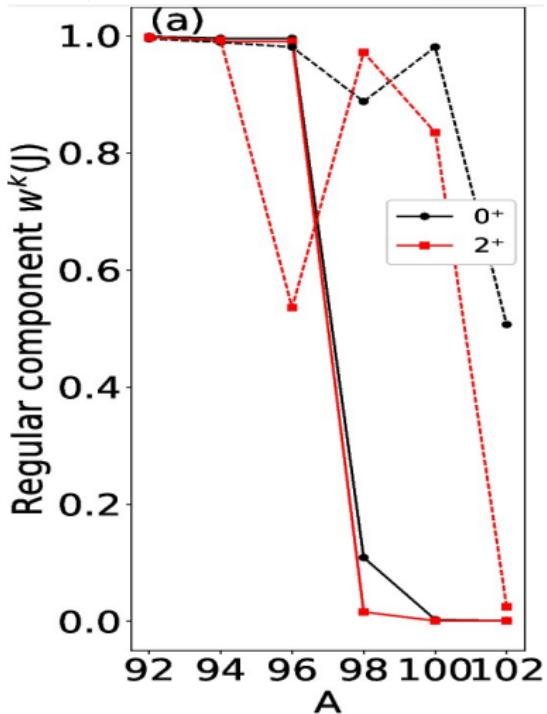
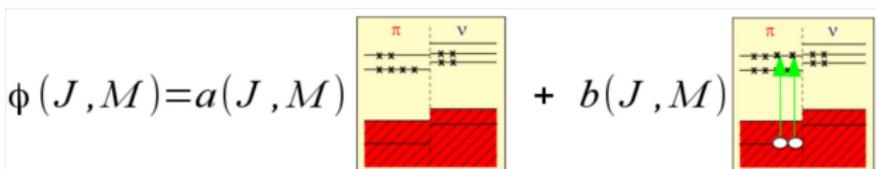
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Wave function: Regular component



Wave function within the
regular sector



$$w^k(J) \equiv \sum_i |a_i^k(J)|^2$$

Radii

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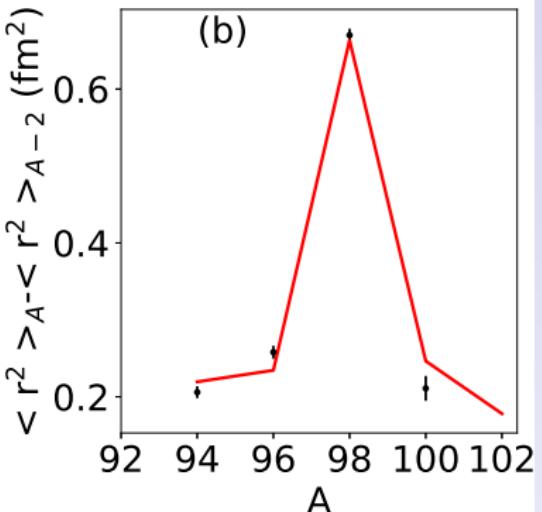
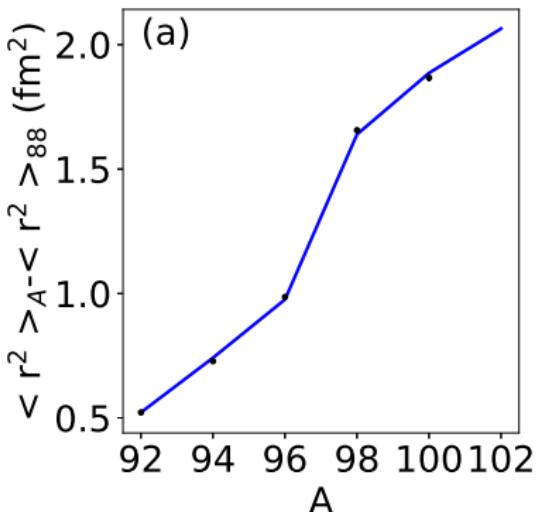
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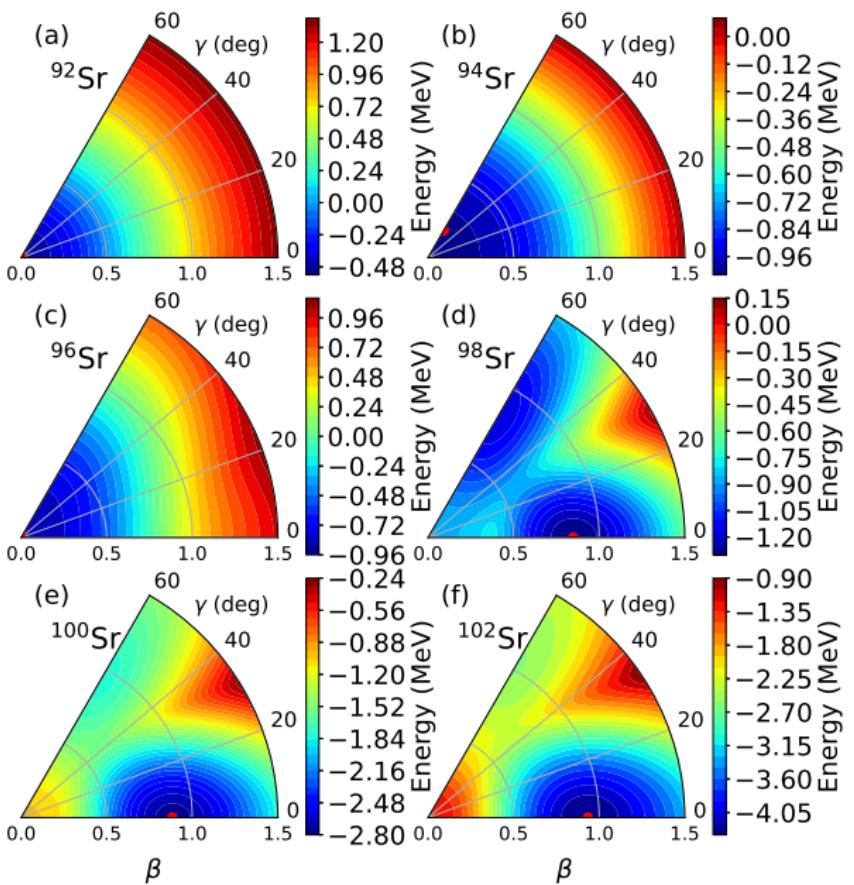
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$$r^2 = r_c^2 + \hat{P}_N^\dagger (\gamma_N \hat{N} + \beta_N \hat{n}_d) \hat{P}_N + \hat{P}_{N+2}^\dagger (\gamma_{N+2} \hat{N} + \beta_{N+2} \hat{n}_d) \hat{P}_{N+2}$$



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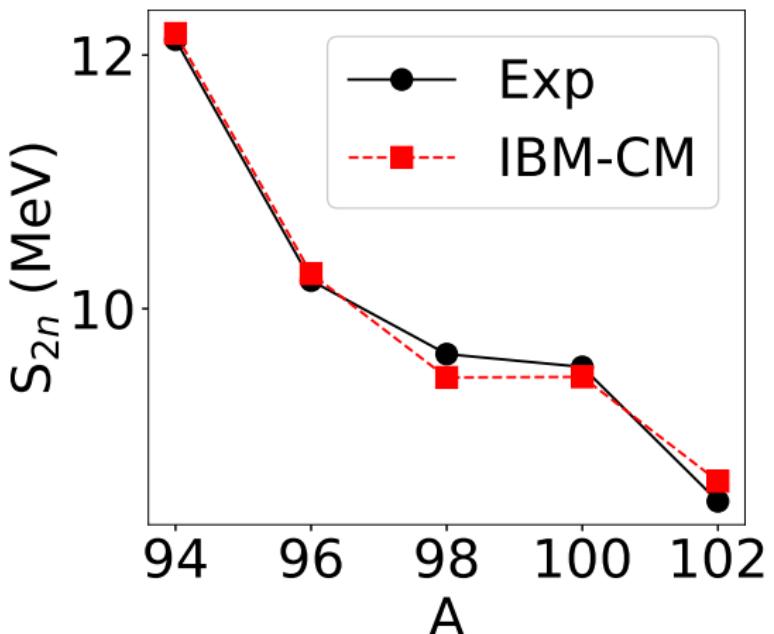
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Quantum Phase Transitions in Sr isotopes

Some hints points towards a Quantum Phase Transition in the Sr region

- ▶ Two neutron separation energies



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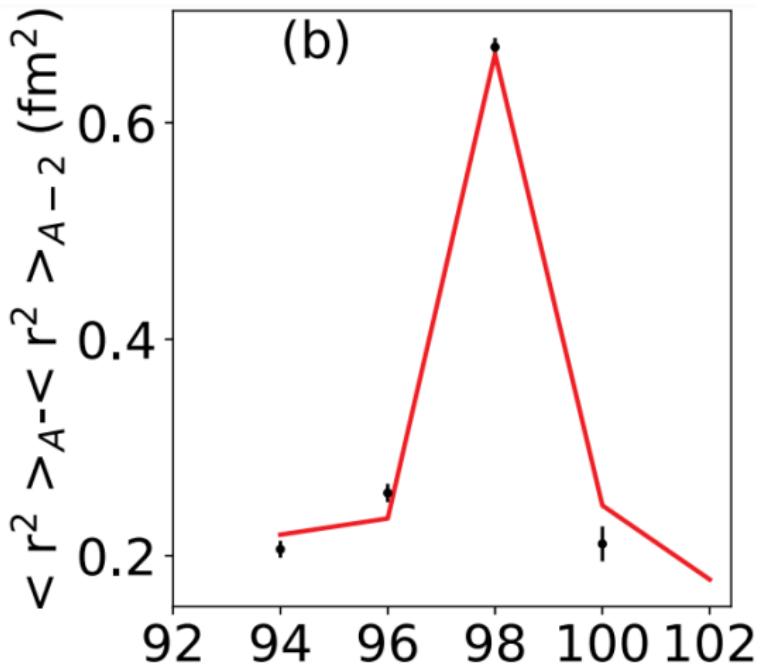
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Quantum Phase Transitions in Sr isotopes

Some hints points towards a Quantum Phase Transition in the Sr region

- ▶ Isotopic shift



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- ▶ In the Sr isotopes studied we have noticed a clear change in structure, with an evolution from spherical shapes to more deformed ones, with a clear change in ^{98}Sr .
- ▶ We have developed a phenomenological study in order to obtain the spectrum from experimental energy spectra and $B(E2)$.
- ▶ The IBM-CM provides an accurate description of the observed changes and of the different shapes in the spectrum.
- ▶ We have described the theoretical S_{2n} and radii in comparison with the experimental results, as a method to the reliability of our calculations and as possible hints pointing to a QPT.

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THANK YOU

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Nucleus	N	ε_N	κ_N	χ_N	κ'_N	ε_{N+2}	κ_{N+2}	χ_{N+2}	κ'_{N+2}	ω	Δ^{N+2}	e_N	e_{N+2}
⁹² Sr	3	838	-32.01	0.00	-7.84	347.2	-15.00	-0.88	0.00	15	1900	1.53	1.53 ^a
⁹⁴ Sr	4	365	-50.00	0.00	75.01	451.7	-41.81	0.00	0.00	15	1800	1.16	1.53
⁹⁶ Sr	5	620	-35.00	0.64	53.43	242.7	-20.00	-0.79	9.84	15	1500	1.33	0.86
⁹⁸ Sr	6	526	-28.19	1.88	18.59	279.1	-34.96	-0.72	0.23	15	1360	0.78	2.22
¹⁰⁰ Sr	7	526 ^b	-28.19 ^b	1.88 ^b	18.59 ^b	387.3	-43.16	-0.77	-2.99	15	1360	0.78 ^b	1.93
¹⁰² Sr	8	526 ^b	-28.19 ^b	1.88 ^b	18.59 ^b	387.3	-46.41	-0.77	-2.99	15	1360	0.78 ^b	1.93 ^c

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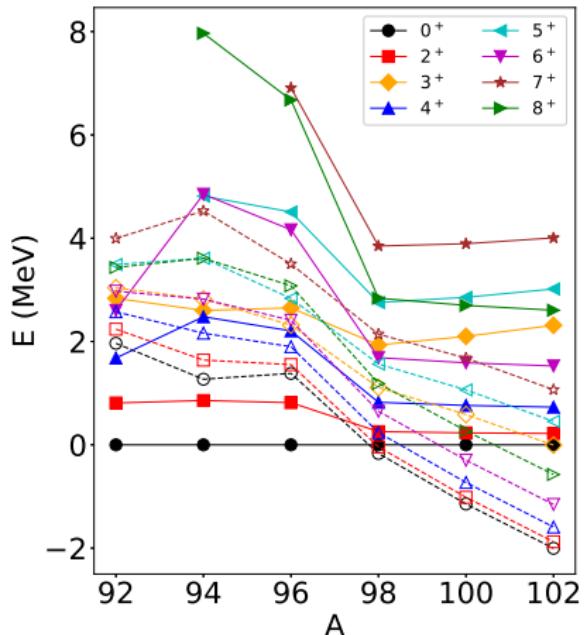
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Energy spectra for the
IBM-CM Hamiltonian
obtained



$$V_{mix} = 0$$

B(E2) transition probabilities-Non Yrast Band

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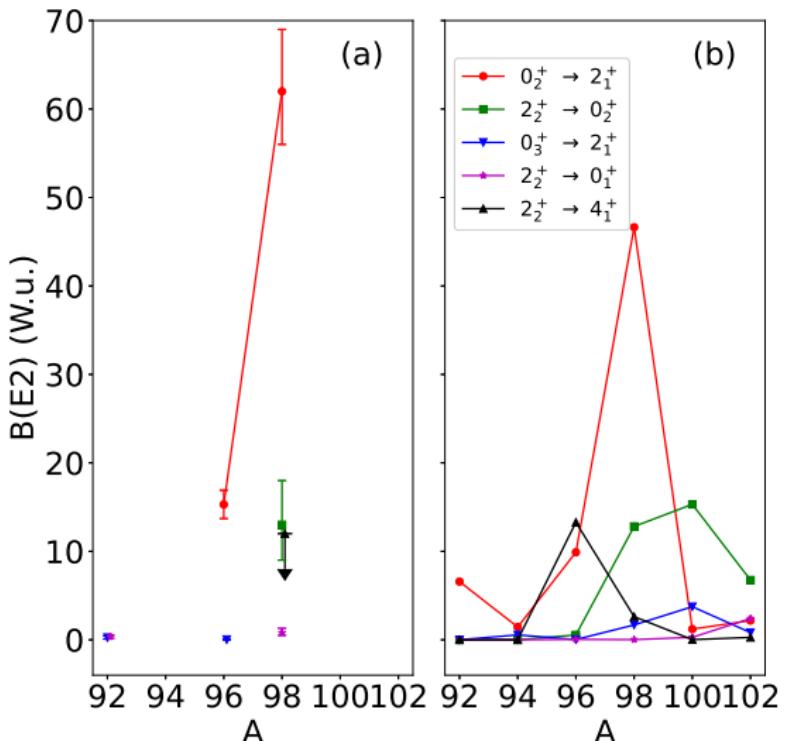
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Wave function: energy systematics

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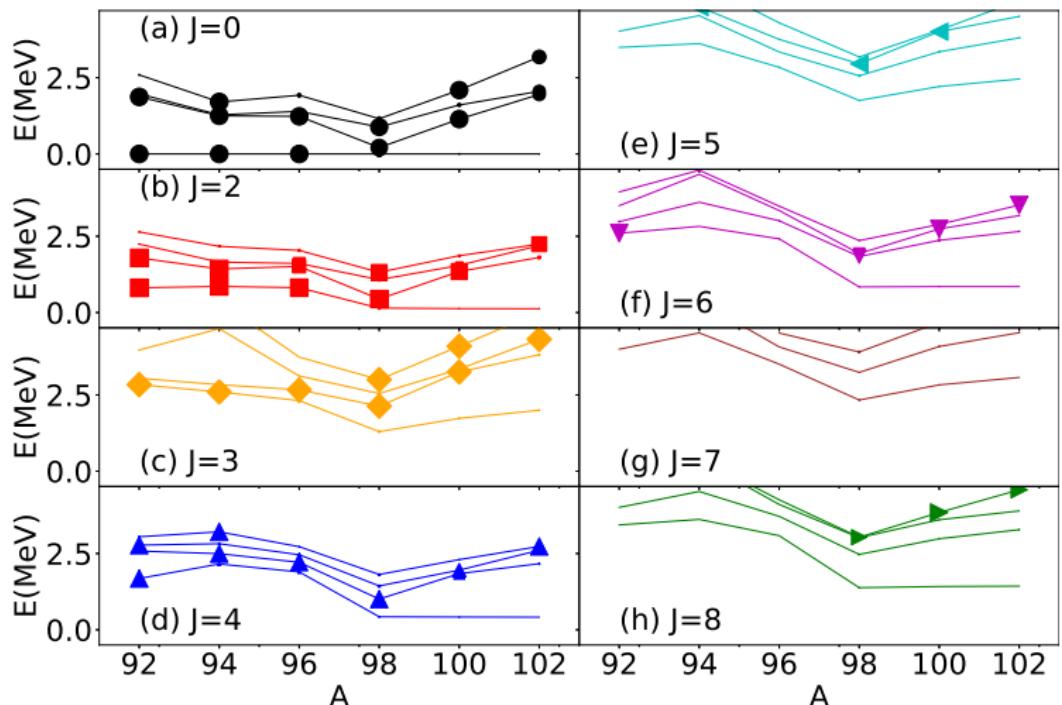
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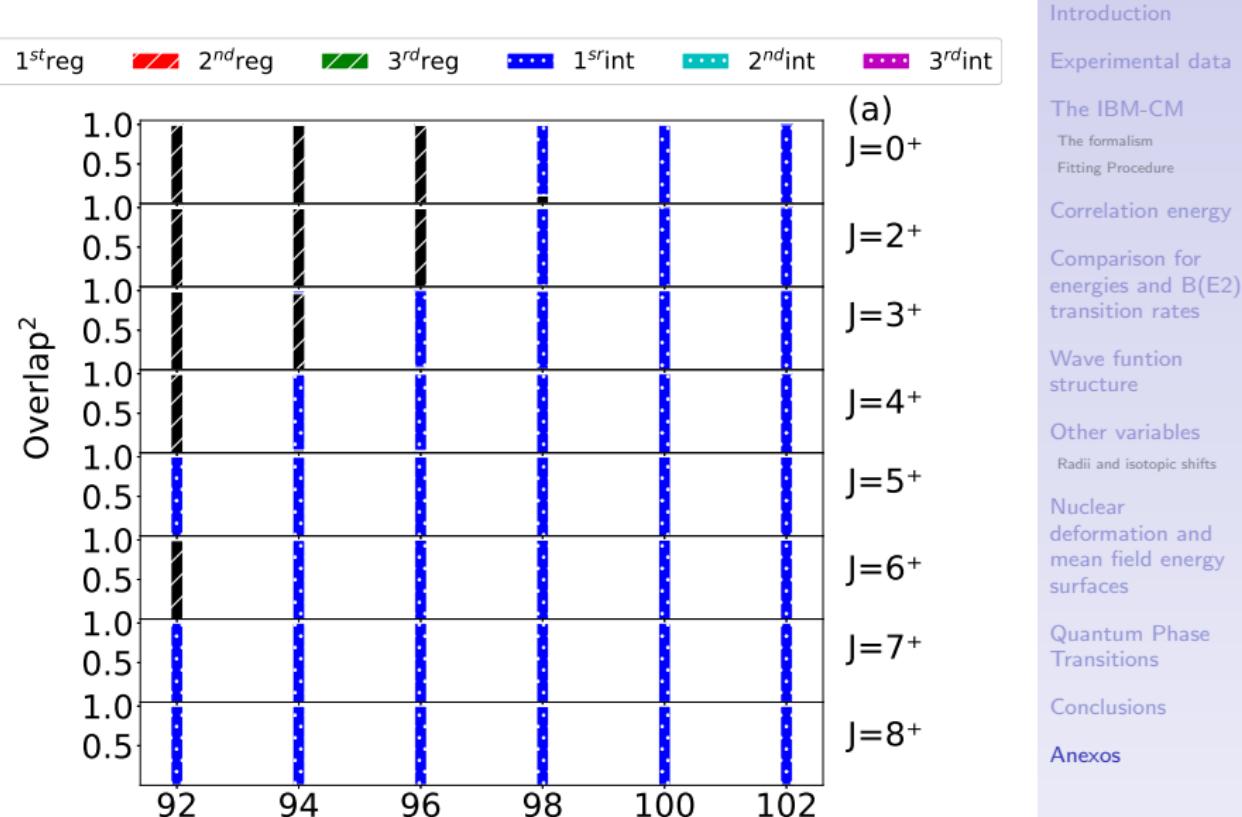
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Overlap of the wave funtion

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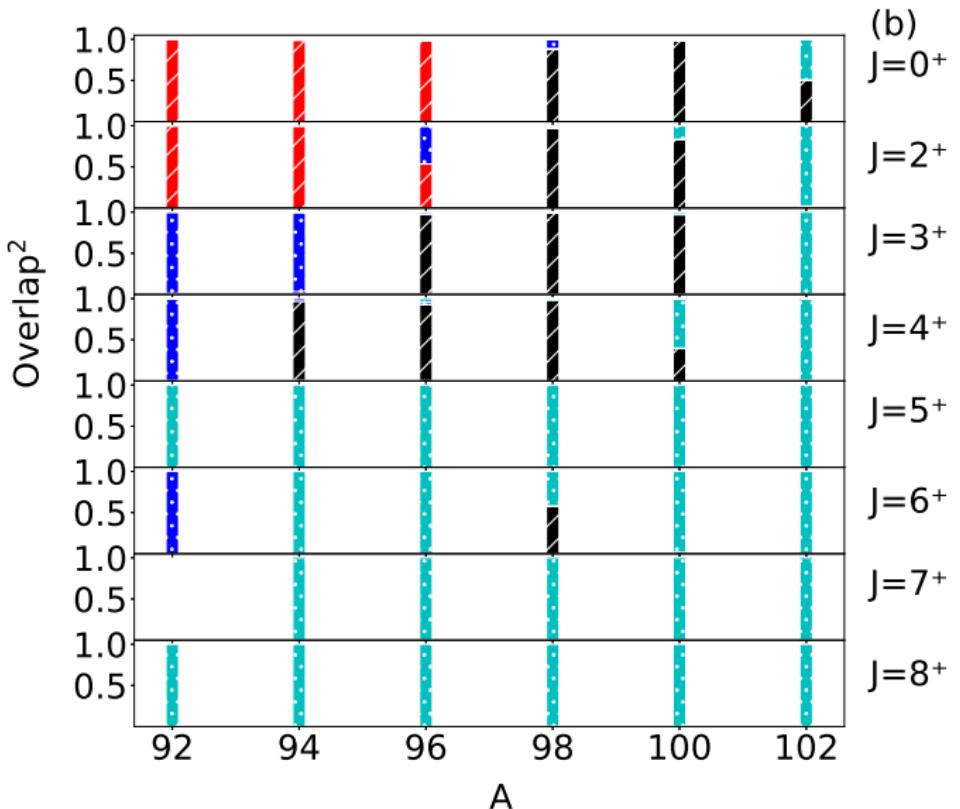
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Two-neutron separation energies

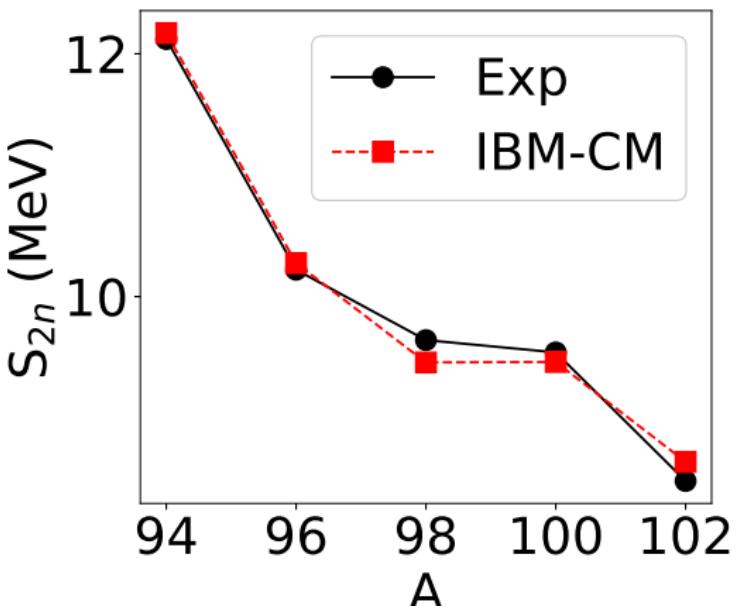
$$S_{2n}(A) = \mathcal{A} + \mathcal{B}A + BE^{lo}(A) - BE^{lo}(A-2)$$

↓

Because of the influence of intruder states

↓

$$S_{2n}(A) = \mathcal{A} + \mathcal{B}(A + 2(1 - w)) + BE^{lo}(A) - BE^{lo}(A-1)$$



β from IBM

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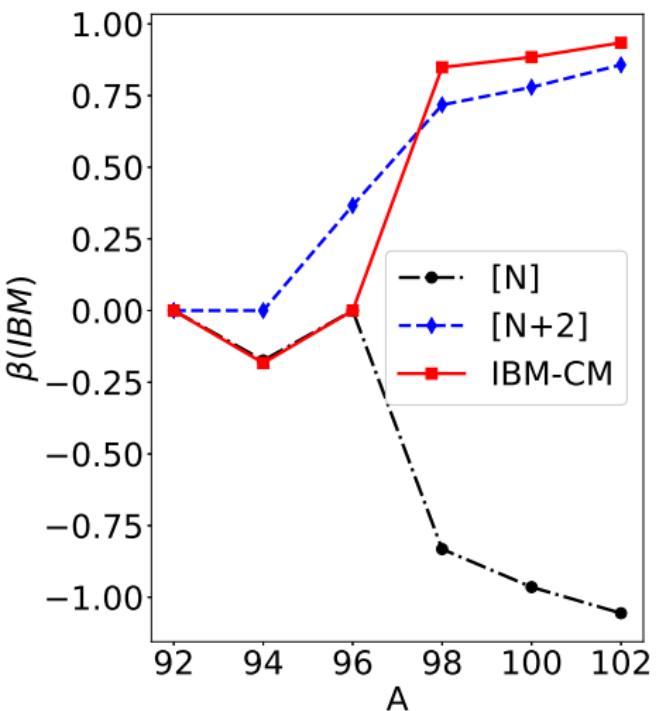
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β from the quadrupole shape invariants

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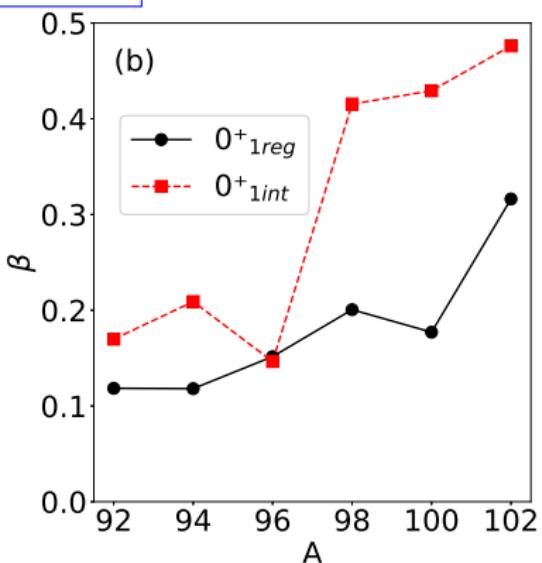
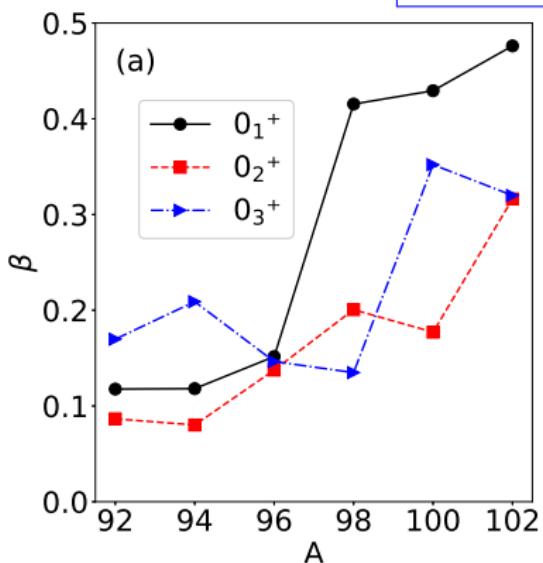
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$$q_{2,i} = \sqrt{5} \langle 0_i^+ | [\hat{Q} \times \hat{Q}]^{(0)} | 0_i^+ \rangle$$



$$\beta = \frac{4\pi\sqrt{q_2}}{3Zer_0^2 A^{2/3}}$$



Interacting Boson Model

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$$\hat{H}_{\text{ecqf}}^i = \varepsilon_i \hat{n}_d + \kappa'_i \hat{L} \cdot \hat{L} + \kappa_i \hat{Q}(\chi_i) \cdot \hat{Q}(\chi_i)$$

$$\hat{L}_\mu = [d^\dagger \times \tilde{d}]_\mu^{(1)}$$

$$\hat{Q}_\mu(\chi_i) = [s^\dagger \times \tilde{d} + d^\dagger \times s]_\mu^{(2)} + \chi_i [d^\dagger \times \tilde{d}]_\mu^{(2)}$$

$$\hat{V}_{\text{mix}}^{N,N+2} = \omega_0^{N,N+2} (s^\dagger \times s^\dagger + s \times s) + \omega_2^{N,N+2} (d^\dagger \times d^\dagger + \tilde{d} \times \tilde{d})^{(0)}$$

$$\hat{T}(E2)_\mu = \sum_{i=N,N+2} e_i \hat{P}_i^\dagger \hat{Q}_\mu(\chi_i) \hat{P}_i$$

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We have considered the coherent state:

$$|N, \alpha_m\rangle = \left(s^\dagger + \sum_m \alpha_m d_m^\dagger \right)^N |0\rangle$$

Where the relation with the collective parameters:

$$\alpha_0 = \beta \cos \gamma, \quad \alpha_{\pm 1} = 0, \quad \alpha_{\pm 2} = \frac{\beta}{\sqrt{2}} \cos \gamma$$

$$|N; \beta, \gamma\rangle = \left\{ s^\dagger + \beta \left[\cos \gamma d_0^\dagger + 1/\sqrt{2} \sin \gamma (d_{+2}^\dagger + d_{-2}^\dagger) \right] \right\}^N |0\rangle$$

$$E(N; \beta, \gamma) = \frac{\langle N; \beta, \gamma | H | N; \beta, \gamma \rangle}{\langle N; \beta, \gamma | N; \beta, \gamma \rangle}$$

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Weisskopf units

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One Weisskopf unit of $B(E\lambda)$ is equal to

$$B(E\lambda) = \frac{(1.2)^{2\lambda}}{4\pi} \left(\frac{3}{\lambda + 2} \right)^2 A^{2\lambda/3} \quad \text{in unit of } e^2(fm)^\lambda$$

Transition probability

$$T(E2) = 1.223 \times 10^9 E_\gamma^5 B(E2)[1/\text{sec}]$$

E_γ is in MeV.

$B(E2)$ in $e^2(fm)^4$