A Scintillating Fiber-based Ion Beam Profile Monitor*PART I

HighRR Bi-Weekly Seminar Talk

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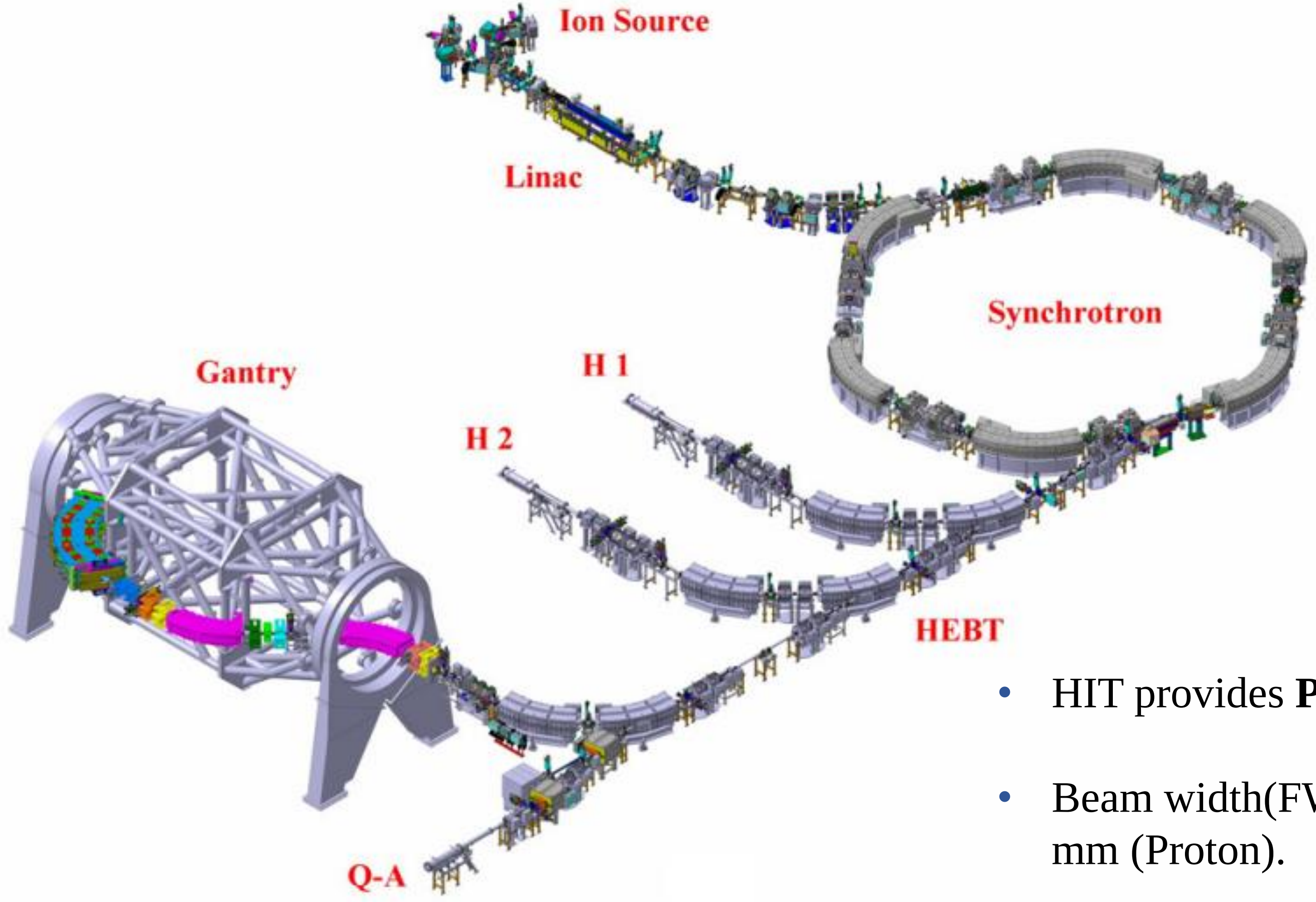
Overview

- Heidelberg Ion Beam Center
- Scintillating Fiber-based Beam Profile Monitor
- Linear Regression for a Gaussian-like Profile
- Resolution Calculation
- Calibration and Radiation Damage
- Outlook

Heidelberg Ion Therapy Center (HIT)

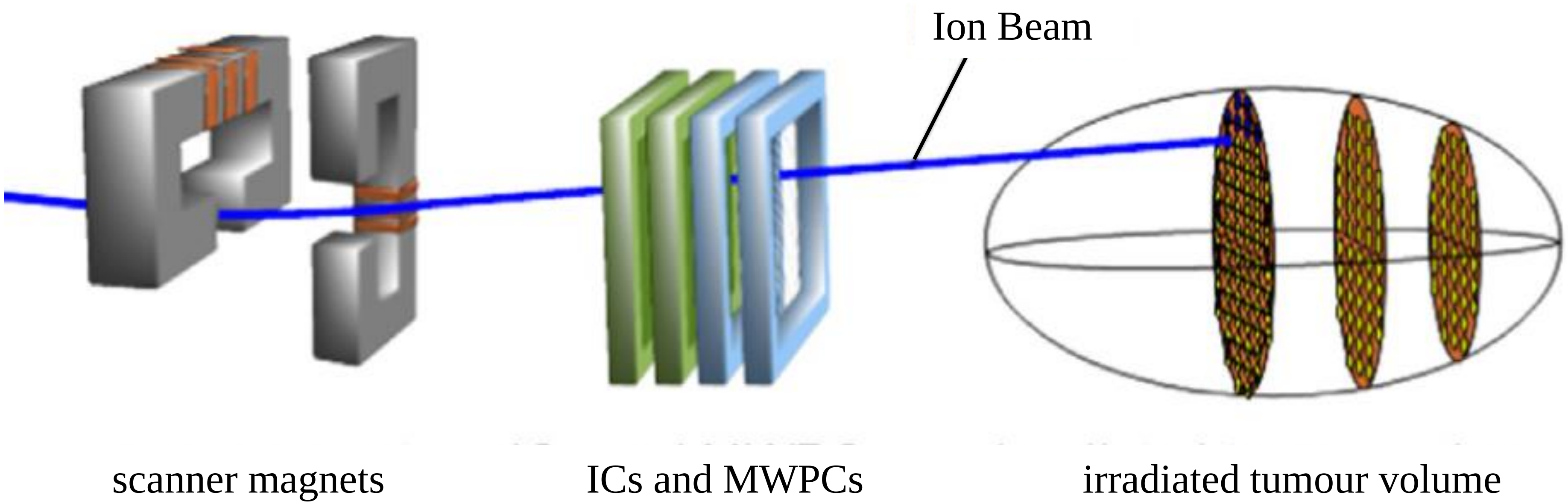


QA: quality assurance. H1, H2 and Gantry rooms are for patients treatment. QA room is for QA and experiments.

- HIT provides **Proton, Carbon, Helium, and Oxygen** beams.
- Beam width(FWHM, Focus) from 3.4 mm (carbon) to 32.9 mm (Proton).
- Scan area 20×20 cm^2 .

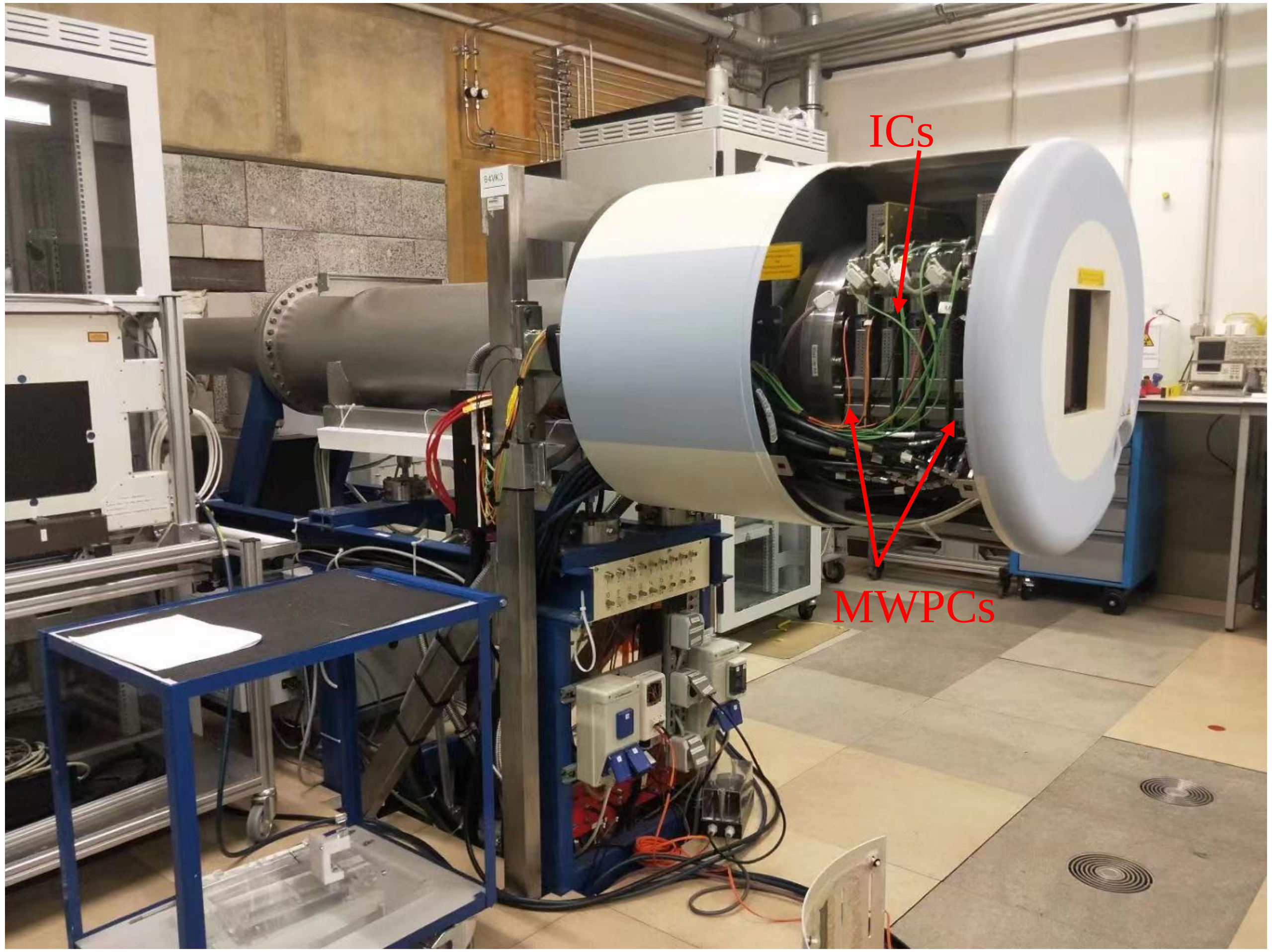
Figure from: David Ondreka, Udo Weinrich, The Heidelberg ion therapy (HIT) acceleratorcoming into operation, in: Proceedings of EPAC08, Genoa, Italy, 2008.

HIT Beam Application and Monitor System

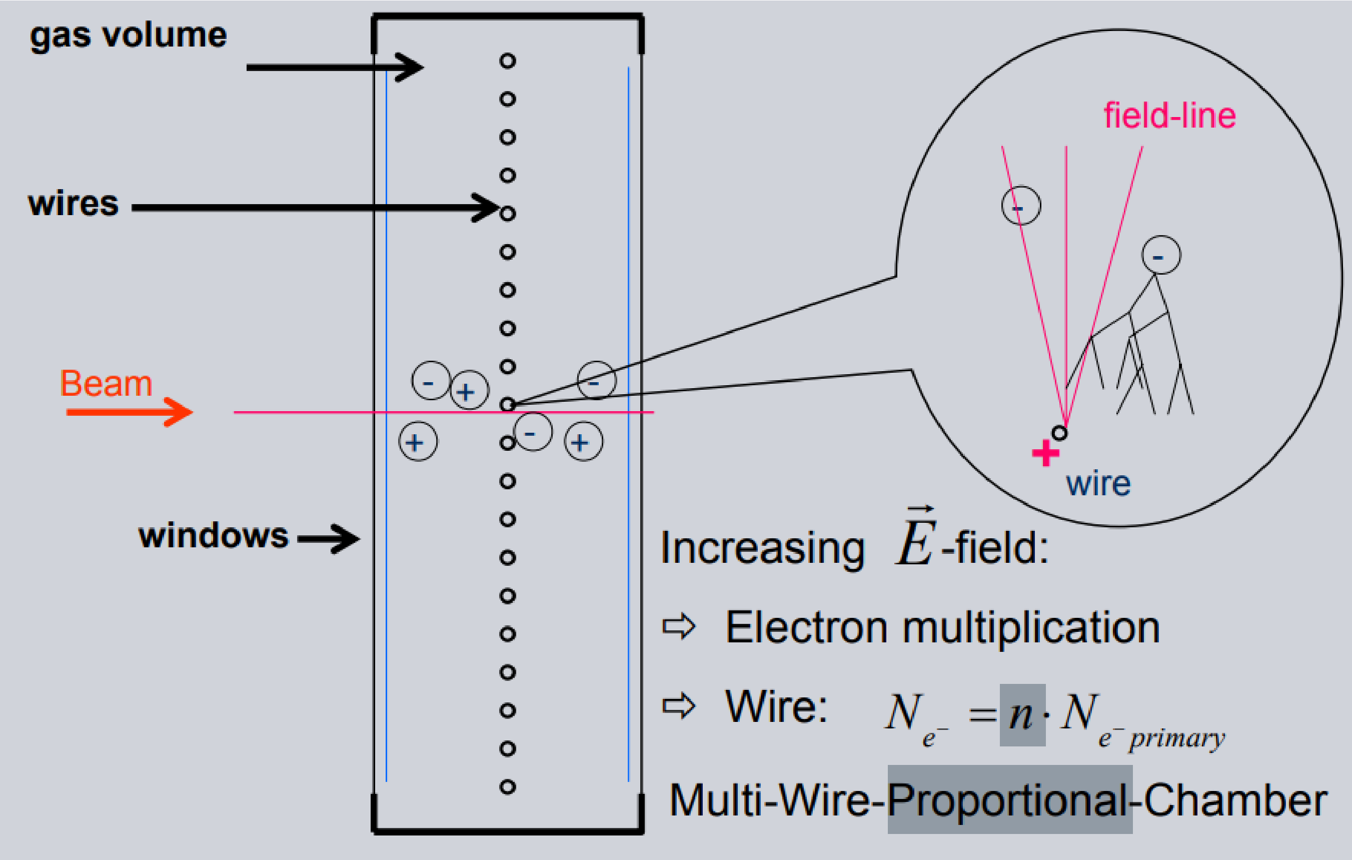


MWPCs: Multi-wire Proportional Chambers. ICs: Ionization Chambers. MWPCs provide beam position and width information.

HIT Beam Application and Monitor System



MWPCs



Benefits of MWPCs:

- Very light in material(gas detector)
- Geometric acceptance(easy to produce huge size)
- Standard technology

(Simons will not supply the boards in 10 years??)

Drawbacks of MWPCs:

- Space charge limits intensity;
- Drifting time of ions(dead time) limits the readout rate;
- Deposit on anode wires of thin insulating layers

(needs 2 days to be cleaned every two years? Hard to replace?)

Figure from:The intensity feedback system at Heidelberg Ion-Beam Therapy Centre,
Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment

Requirements for new Beam Profile Monitor

Requirements for detector properties

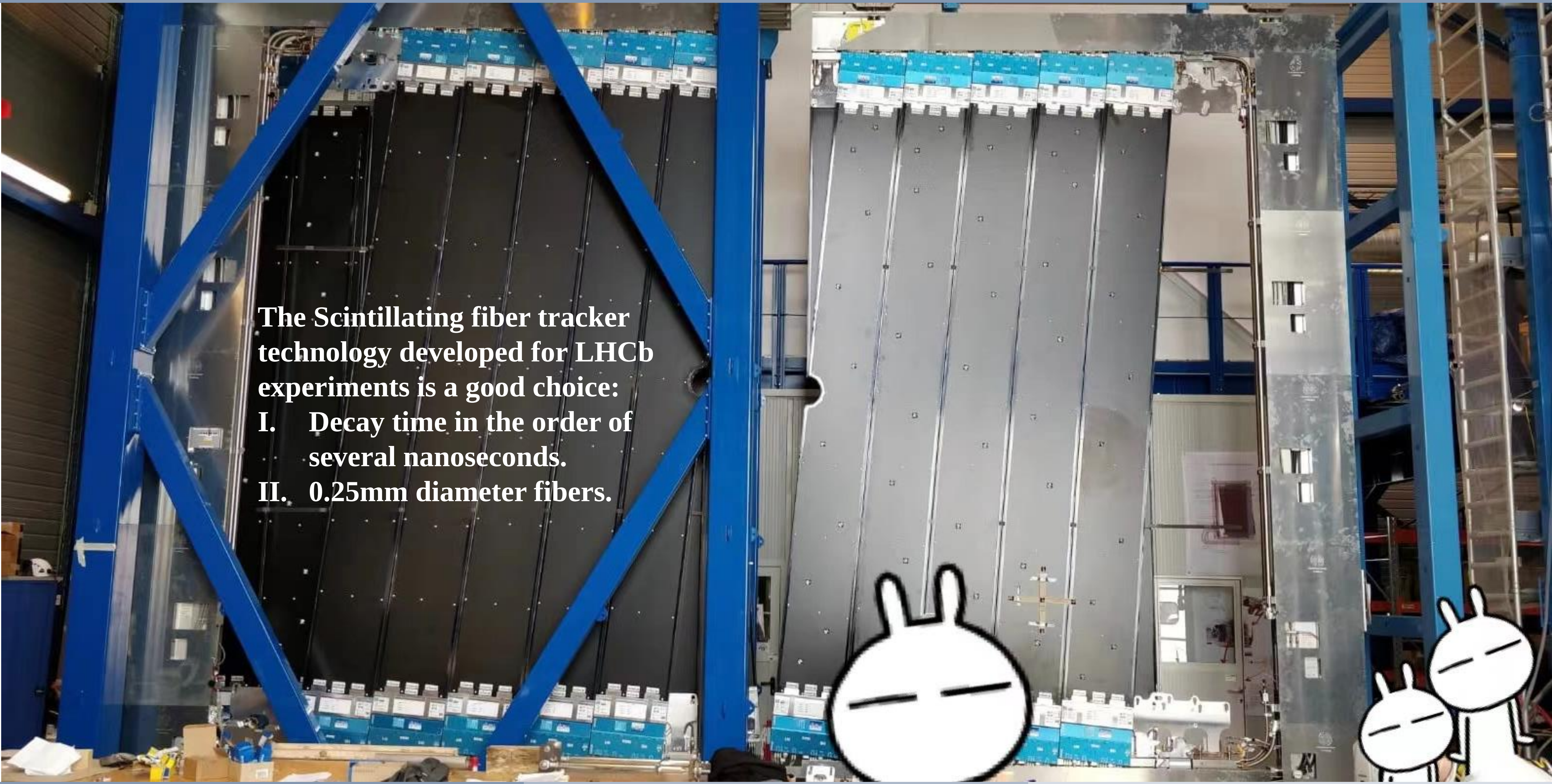
Requirement	**Value**
Beam Spot Size (FWHM)	1 - 33 mm
Beam Position Resolution	< 0.2 mm
Beam Width Resolution	< 0.4 mm
Readout Rate	> 4 – 8 kHz
Material in Active Area	< 0.35 mm Water Equiv./ plane

The energy and intensities available for the four ions types at Heidelberg Ion Therapy Center
Requirements from beam settings

	Protons	**Helium**	**Carbon**	**Oxygen**
Energy (MeV/u)	48 – 221	51 – 221	89 – 430	104 – 515
Intensity (s^{-1})	$8\cdot10^7$ – $3.2\cdot10^9$	$2\cdot10^7$ – $8\cdot10^8$	$2\cdot10^6$ – $8\cdot10^7$	$1\cdot10^6$ – $4\cdot10^7$

Tables from: Blake Leverington, A scintillating fibre based beam profile monitor for ion therapy beams. DFG form 53.01-03/18

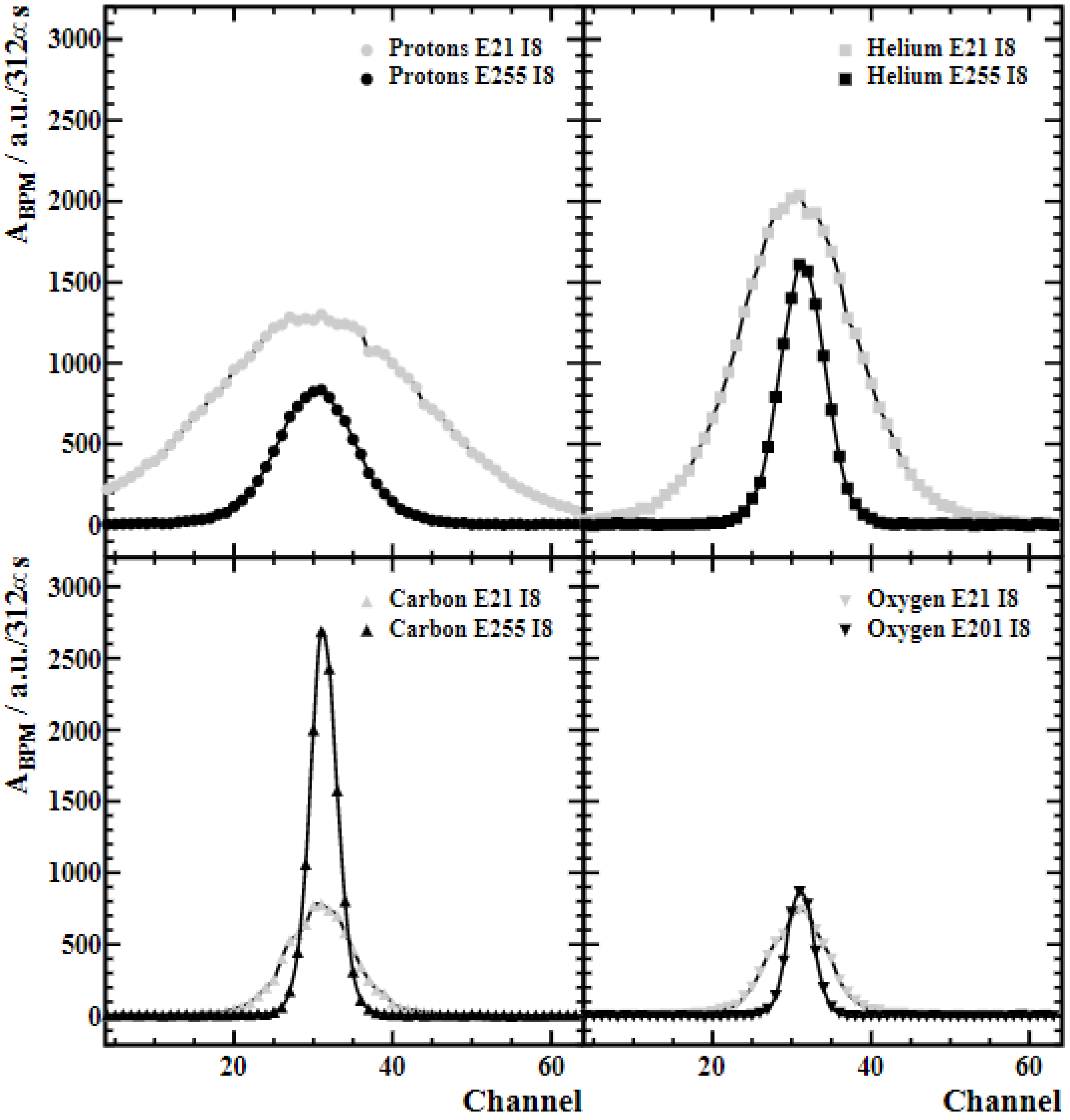
Scintillating Fiber Tracker at LHCb



The Scintillating fiber tracker technology developed for LHCb experiments is a good choice:

I. **Decay time in the order of several nanoseconds.**
II. **0.25mm diameter fibers.**

Beam Shape measured by 5-layer Fiber Prototype

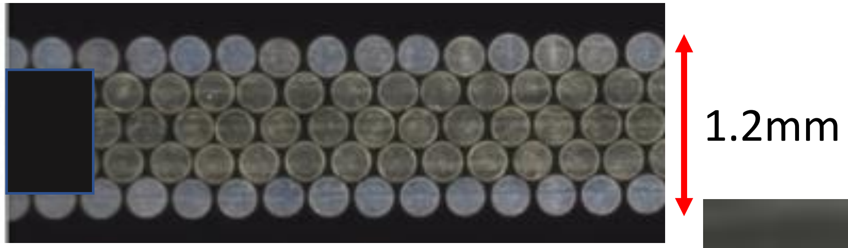


Proton	
E21	70.03 MeV/u
E255	221.06 MeV/u
I8	$1.2 \cdot 10^9$/s

Helium	
E21	71.73 MeV/u
E255	220.51 MeV/u
I8	$3 \cdot 10^8$/s

Carbon	
E21	129.79 MeV/u
E255	430.10 MeV/u
I8	$3 \cdot 10^7$/s

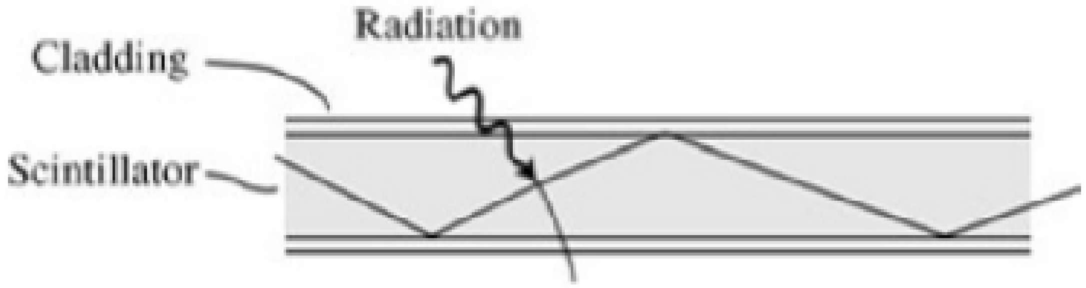
Oxygen	
E21	152.42 MeV/u
E255	423.23 MeV/u
I8	$1.5 \cdot 10^7$/s



Cross Section of 5 layer Scintillating Fiber Plane

Plot from: B.D. Leverington and M. Dziewiecki and L. Renner and R. Runze. A prototype scintillating fiber beam profile monitor for Ion Therapy beams

Scintillating Fiber



Cross section of a typical fiber scintillator. Some fraction of the emitted light is trapped by total internal reflection at core-cladding interface.

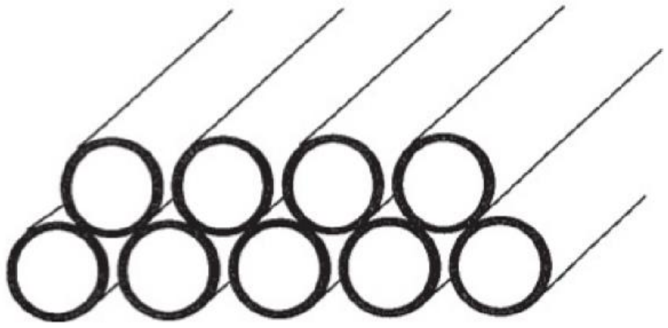
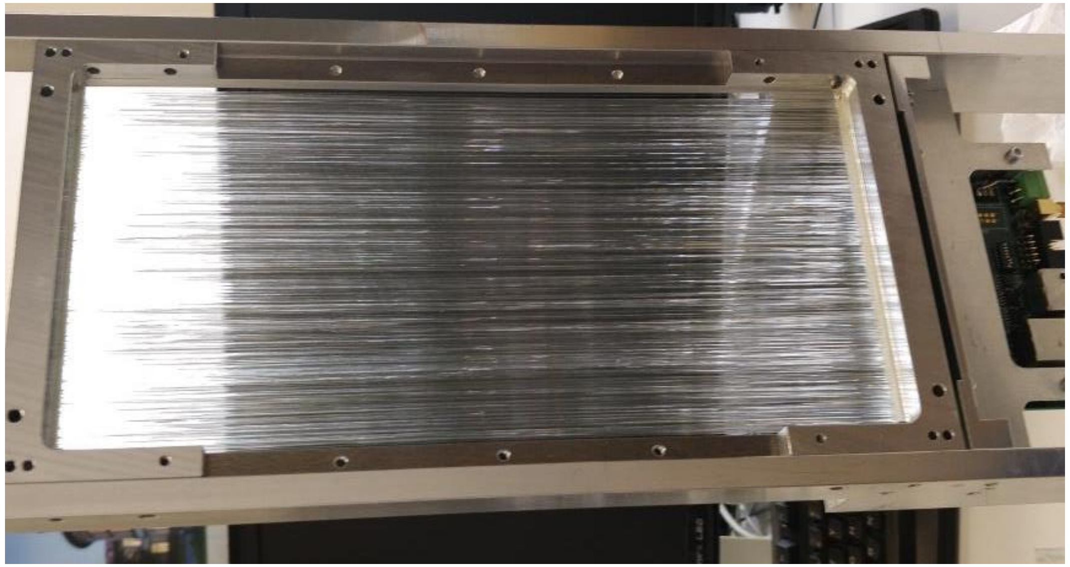


Diagram for Scintillating Fiber Plane



Scintillating Fiber Plane

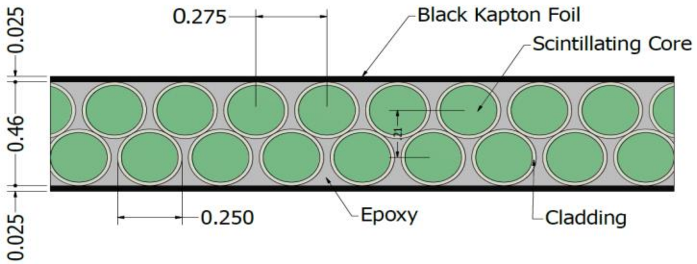
Advantages:

- Decay time in the order of several nanoseconds.
- Geometric acceptance
- Easy to replace

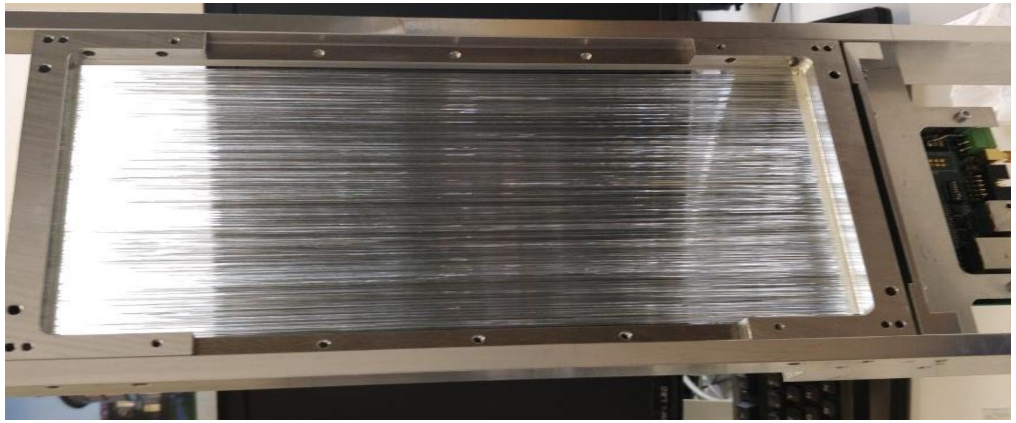
Drawbacks:

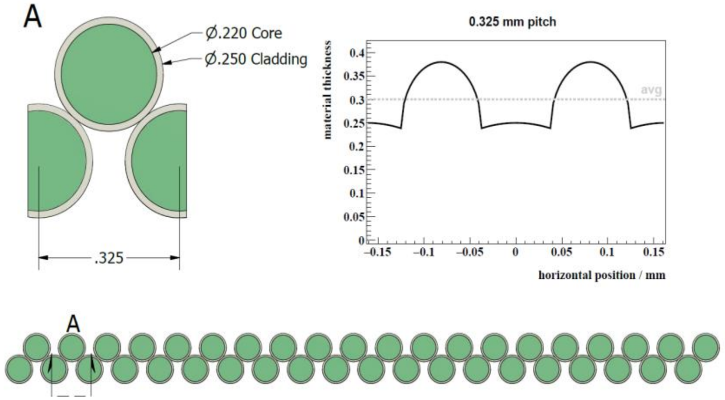
- Not light in material

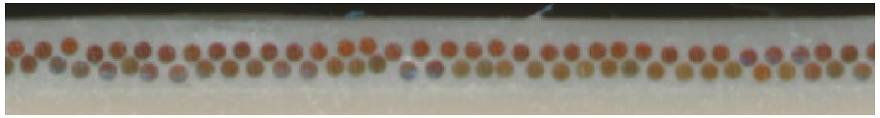
Scintillating fiber-based Beam Profile Monitor ---- Fiber Mat



2-layer fiber with glue between fiber and black Kapton foil on the front and back, fixed by epotek glue.

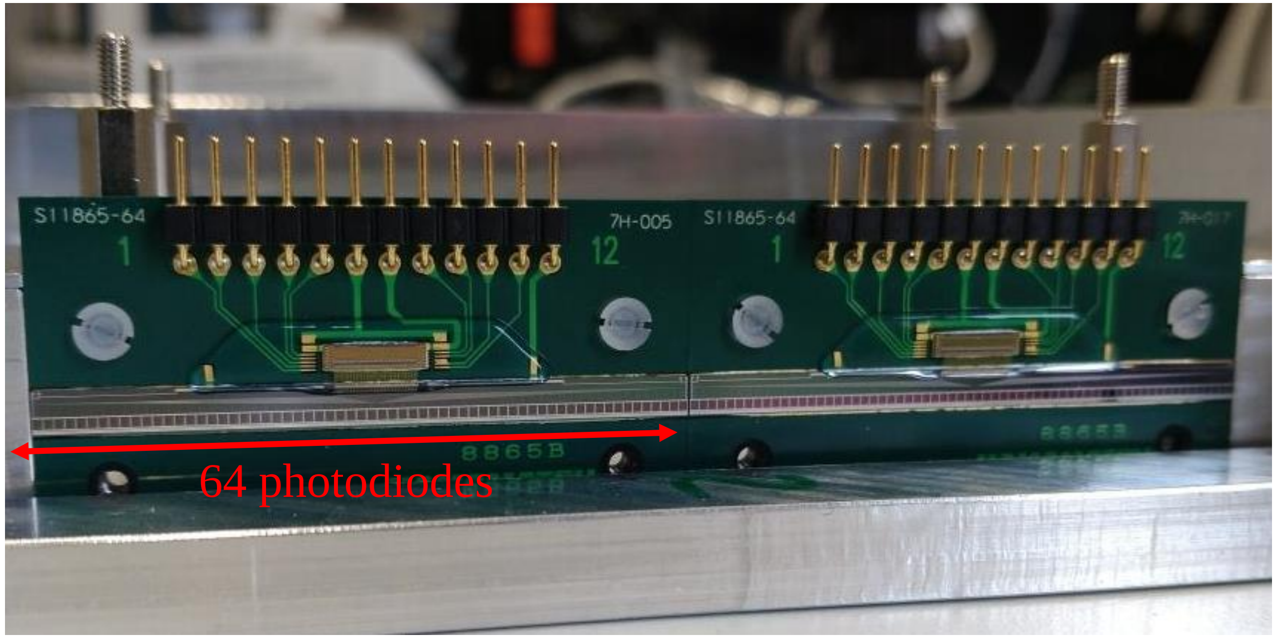






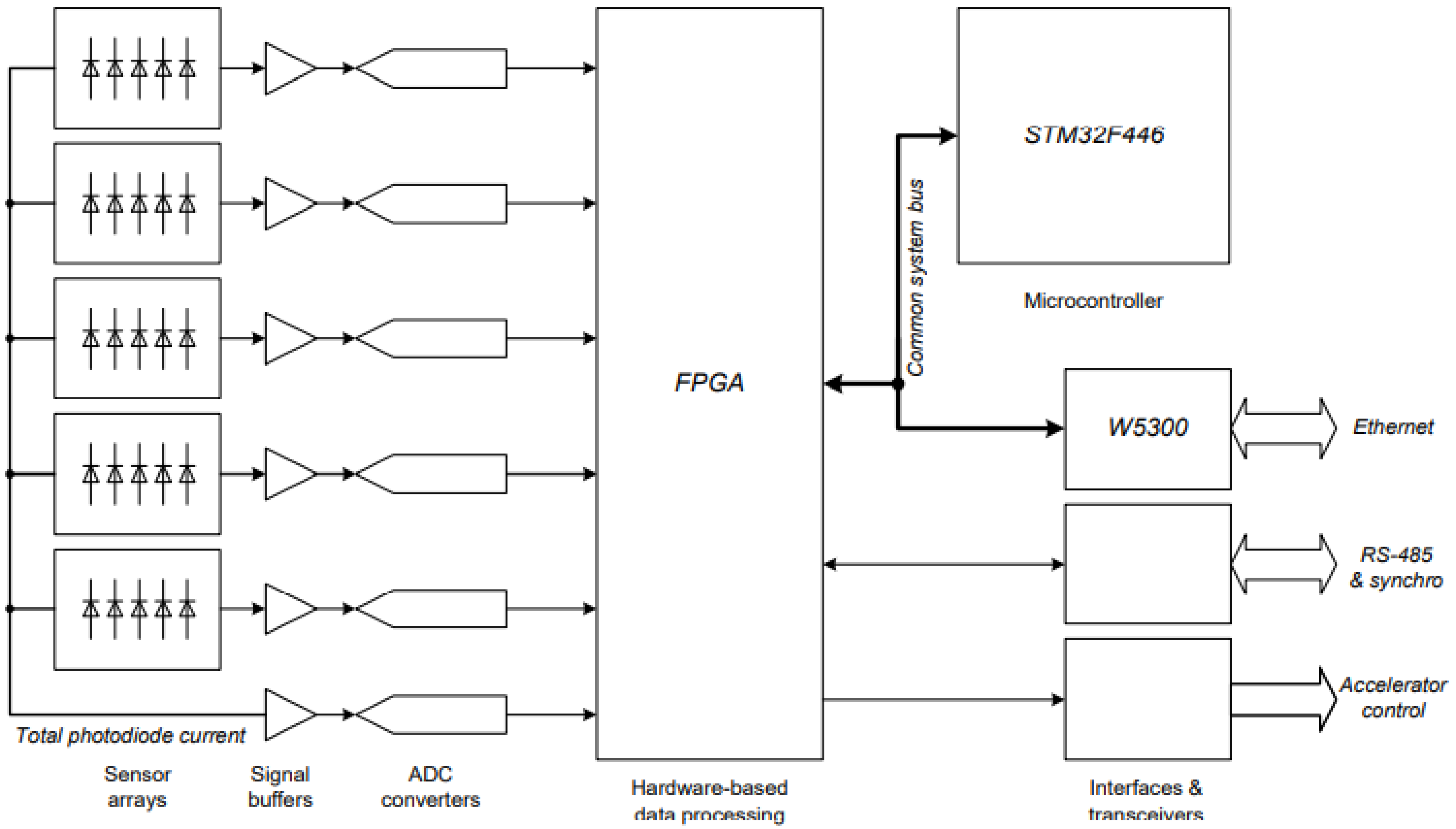
The first 2-layer glueless fiber, in order to keep the material budget to the requirements, fixed by araldite glue.

Scintillating fiber-based Beam Profile Monitor ---- Photodiodes



Hamamatsu S11865-64, an array of 64 photodiodes with an integration circuit. This circuit allows simultaneous integration of photodiode currents for some period and sending it out as a series of voltage pulses over a single wire.

Features:
- Data rate: 1 MHz max.
- Large element pitch: 0.8 mm pitch × 64 ch
- Pitch size: 0.7 × 0.8 mm
- Simultaneous integration by using a charge amplifier array
- Sequential readout with a shift register



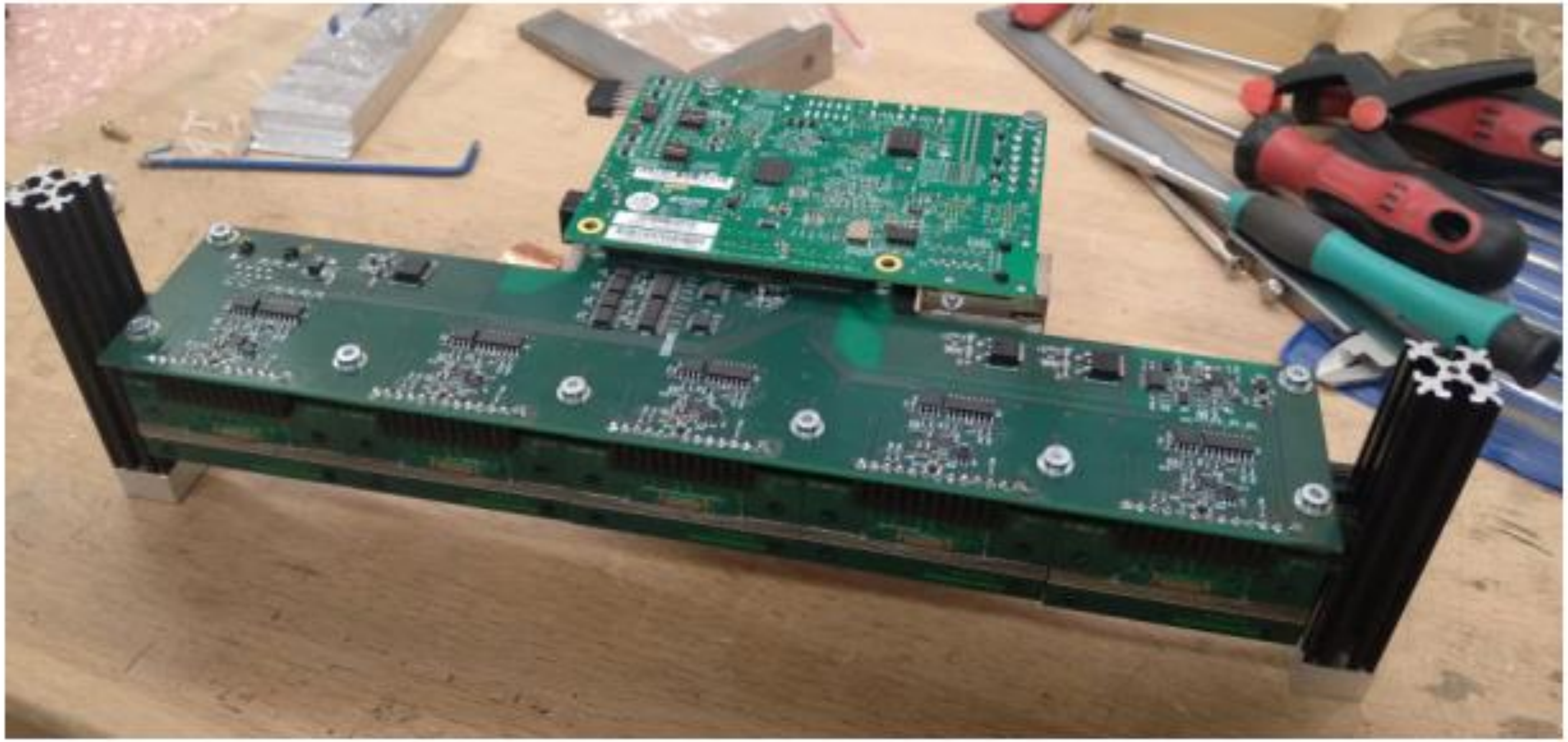
A simplified block diagram of readout board for 5 sensors.

Feature:

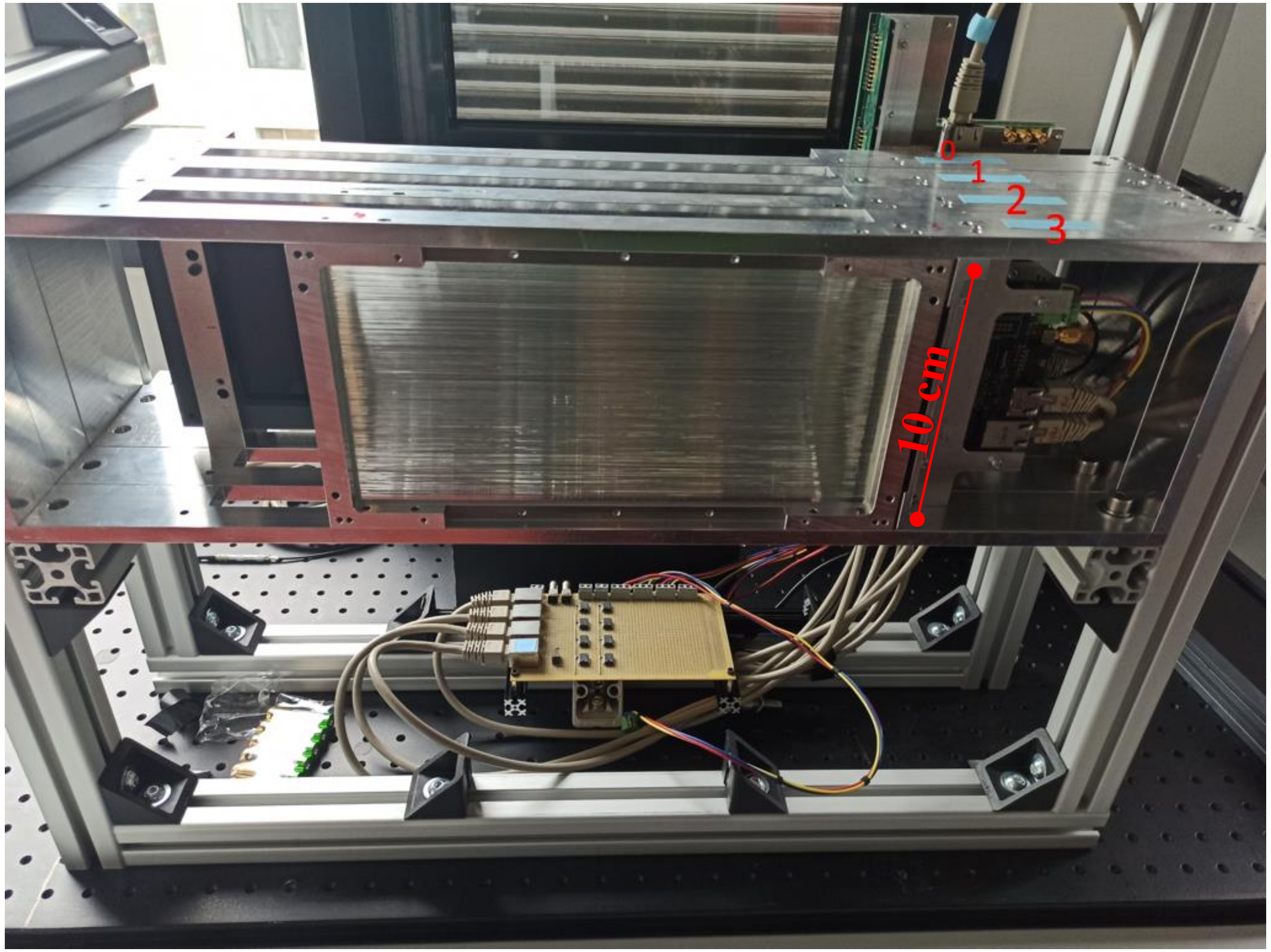
- Two PCBs: frontend PCB, FPGAs PCB
- MAX10 development kit
- Adjustable integration time, ~100 µs by default.
- Maximum 10kHz readout rate

Currently, the readout collect signal from sensors and send to PC.
In future, Beam reconstruction algorithm will be implemented on FPGAs.

Scintillating fiber-based Beam Profile Monitor ---- Readout



Scintillating fiber-based Beam Profile Monitor ---- 4 Detection Planes



Beam profile monitor with 4 scintillating fiber detection planes.

Features:

- Collect integrated signal and send to PC every 100 μs => one Frame
- 4 detection planes
- Plane 0 and 1 are with glue and foil
- Plane 2 and 3 are glueless

Synchronization for multiple boards:

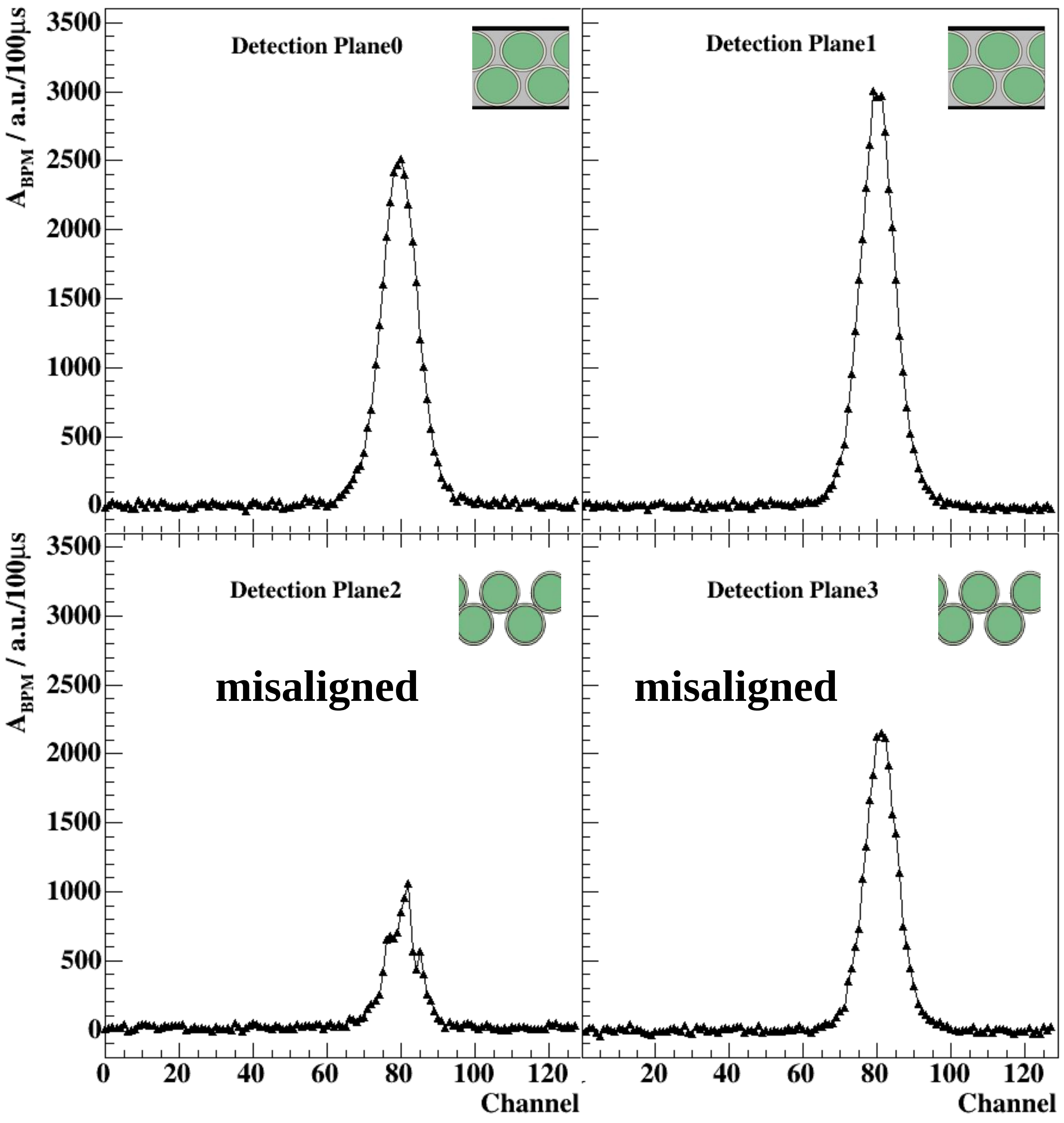
- Frame-level synchronization. Master board sends a number over the synchronization link. It's a unique identifier
- Fine synchronization. Use the same clock signal to trigger. Match integration window for all planes within some dozens of nanoseconds.

Synchronization for external signal:

A 8-bit parallel, digital input (time-stamper signal) is latched and transmitted with every frame for collecting external signal at same time.

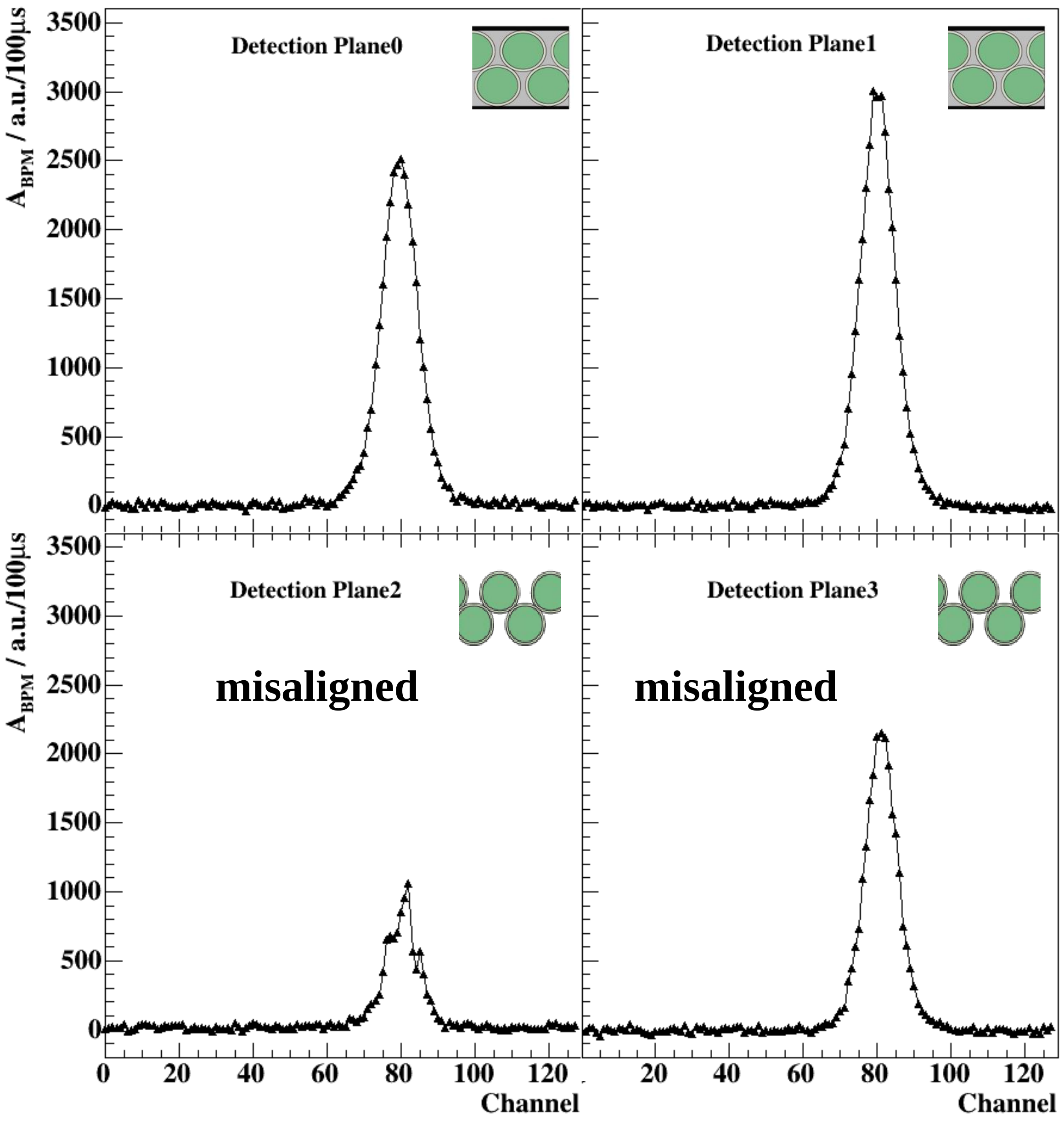
Experiment Setup

Signal collected in 100 µs by 2-layer Fiber Sensor



Proton	
E225	221.06 MeV/u
W1	8.1 mm
I9	$2 \cdot 10^9$/s

Signal collected in 100 μs by 2-layer Fiber Sensor

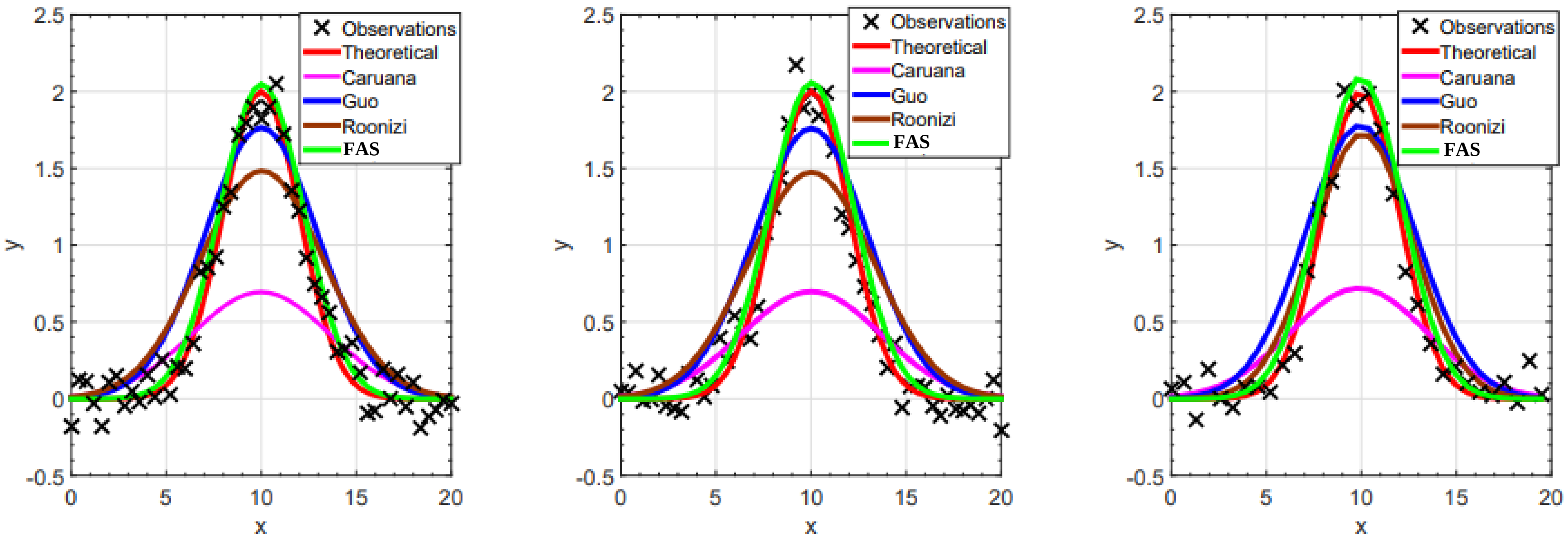


Proton	
E225	221.06 MeV/u
W1	8.1 mm
I9	$2 \cdot 10^9$/s

An algorithm to reconstruct beam position and width is needed, which can be implemented on FPGAs.
It should be fast, robust, precise and be able to work at all beam settings.

Linear Regression for a Gaussian-like Profile

FAS： A computationally efficient, accurate, and separable linear regression algorithm for Gaussian Function. The basic idea is using the area and Maximum of observed data for the FWHM, and then using the weighted least-squares method for Amplitude and Mean.



(a) $N = 50$ (b) $N = 40$ (c) $N = 30$

Figure 2: Results of different algorithms for fitting the GF with $A = 2$, $\sigma = 2$ and $\mu = 10$ in the presence of observation noise with $\sigma_w = 0.1$

Arxiv:1907.07241v1

Linear Regression for a Gaussian-like Profile: Linearization

$$y(x) = Ae^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Algorithms for Gauss linear regression are basically in two categories according to the way of linearization the parameters, by taking the logarithm or integration by parts.

Taking the logarithm:

$$\ln(y) = \ln(A) + \frac{-(x-\mu)^2}{2\sigma^2} = \ln(A) - \frac{\mu^2}{2\sigma^2} + \frac{2\mu x}{2\sigma^2} - \frac{x^2}{2\sigma^2} = a + bx + cx^2$$

Integrating by parts:

$$y(x) = \beta_1\phi_1(x) + \beta_2\phi_2(x) + y(x_0)$$

Where $\beta_1 = -1/\sigma^2$, $\beta_2 = \mu/\sigma^2$, and

$$\phi_1(x) = \int_{x_0}^{x} uy(u)du \text{ , and } \phi_2(x) = \int_{x_0}^{x} y(u)du$$

Linear Regression for a Gaussian-like Profile: Error Function

Define an error function:

$$\varepsilon = \ln(\hat{y}) - \ln(y) = \ln(\hat{y}) - (a + bx + cx^2)$$

$$\delta = \hat{y}\varepsilon = \hat{y}[\ln(\hat{y}) - (a + bx + cx^2)]$$

$$\varepsilon = \hat{y} - y = \hat{y} - [\beta_1\phi_1(x) + \beta_2\phi_2(x) + y(x_0)]$$

$$\delta = \hat{y}\varepsilon = \hat{y}\{\hat{y} - [\beta_1\phi_1(x) + \beta_2\phi_2(x) + y(x_0)]\}$$

Minimize the sum of the squares of the errors:

For example, $$\begin{bmatrix} N & \Sigma x_n & \Sigma x_n^2 \\ \Sigma x_n & \Sigma x_n^2 & \Sigma x_n^3 \\ \Sigma x_n^2 & \Sigma x_n^3 & \Sigma x_n^4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \Sigma\ln(\hat{y}_n) \\ \Sigma x_n \ln(\hat{y}_n) \\ \Sigma x_n^2 \ln(\hat{y}_n) \end{bmatrix}$$

Linear Regression for a Gaussian-like Profile: FAS

The area under Gaussian function:

$$\Lambda = \int_{-\infty}^{\infty} Ae^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = A\sigma\sqrt{2\pi} \approx \sum_{n=1}^{N} \Delta x_n \hat{y}_n$$

Use the area under Gaussian and the maximum value of observed data estimating width :

$$\hat{\sigma} = \frac{\sum_{n=1}^{N} \Delta x_n \hat{y}_n}{\sqrt{2\pi}\hat{y}_{max}}$$

Taking the logarithm and weighted error function:

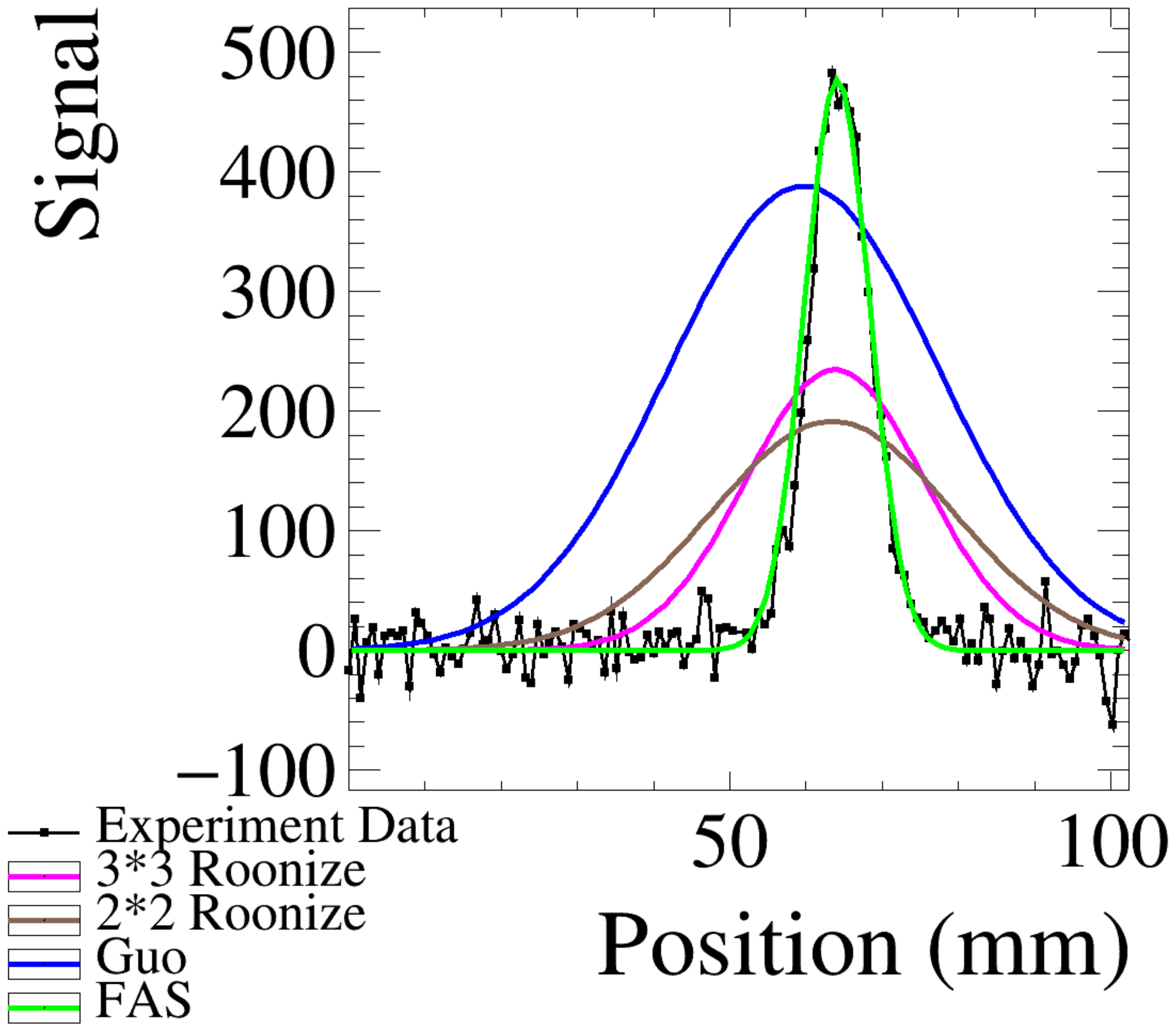
$$\ln(y) = \ln(A) + \frac{-(x-\mu)^2}{2\sigma^2} = \ln(A) - \frac{\mu^2}{2\sigma^2} + \frac{2\mu x}{2\sigma^2} - \frac{x^2}{2\sigma^2} = a + bx + cx^2$$

$$\delta = \hat{y}\varepsilon = \hat{y}[\ln(\hat{y}) - (a + bx + cx^2)]$$

Minimize the sum of the squares of the errors:

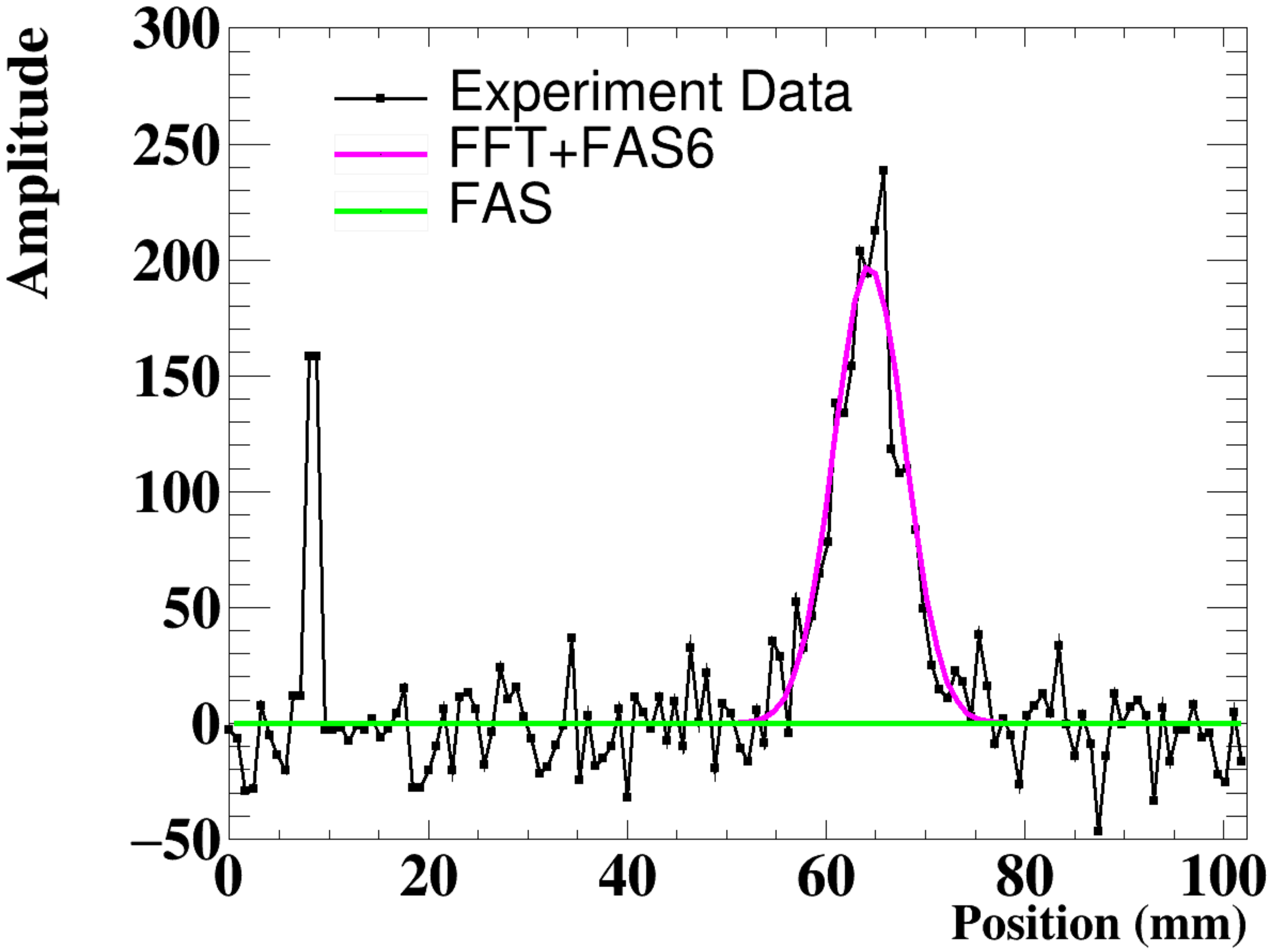
$$\begin{bmatrix} \Sigma\hat{y}_n^2 & \Sigma x_n\hat{y}_n^2 \\ \Sigma x_n\hat{y}_n^2 & \Sigma x_n^2\hat{y}_n^2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \Sigma\hat{y}_n^2 \ln(\hat{y}_n) - c\Sigma x_n^2\hat{y}_n^2 \\ \Sigma x_n\hat{y}_n^2 \ln(\hat{y}_n) - c\Sigma x_n^3\hat{y}_n^2 \end{bmatrix}$$

Linear Regression Algorithm : FAS



Proton	
E225	221.06 MeV/u
W1	8.1 mm
I7	$8 \cdot 10^8$/s

FFT+FAS6



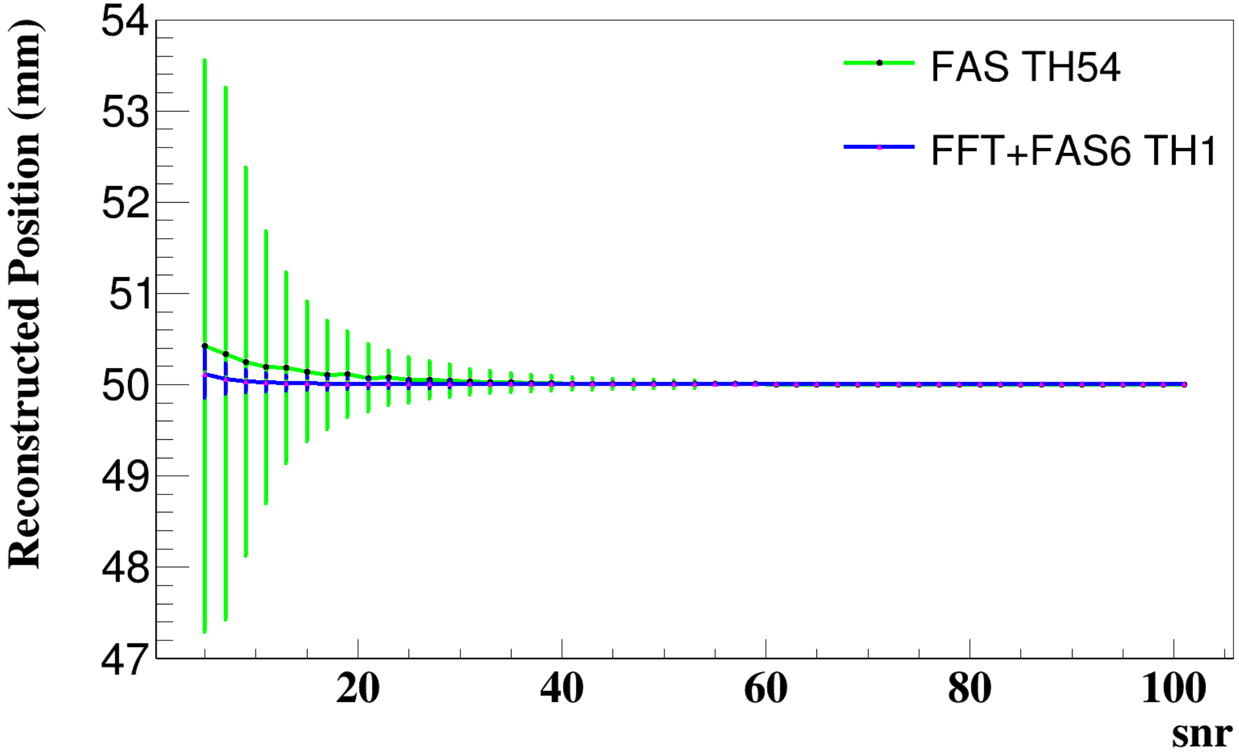
Proton	
E225	221.06 MeV/u
W1	8.1 mm
I3	$2 \cdot 10^8$/s

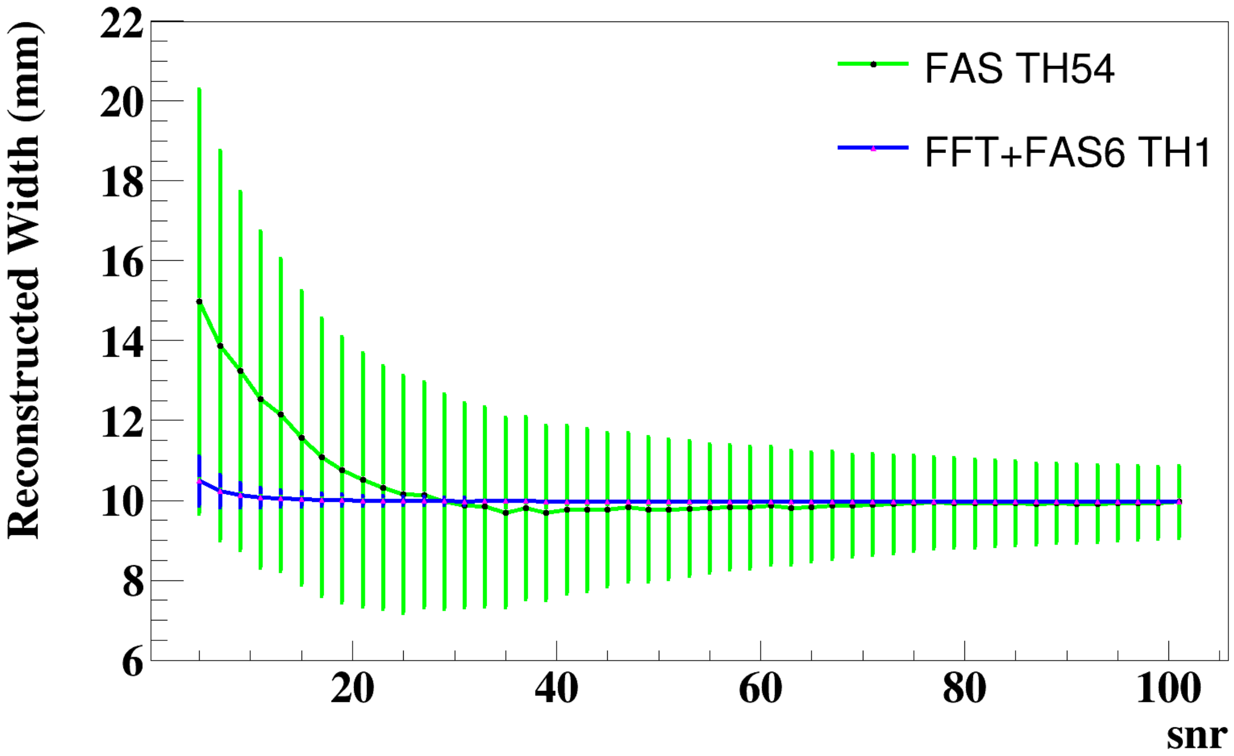
When there is outlier or SNR is low, FAS could fail.
Add FFT to filter the noise and outlier.
Iterate FAS to get better accuracy.

When there is outlier or SNR is low, FAS could fail.
Add FFT to filter the noise and outlier.
Iterate FAS to get better accuracy.

Simulation Test

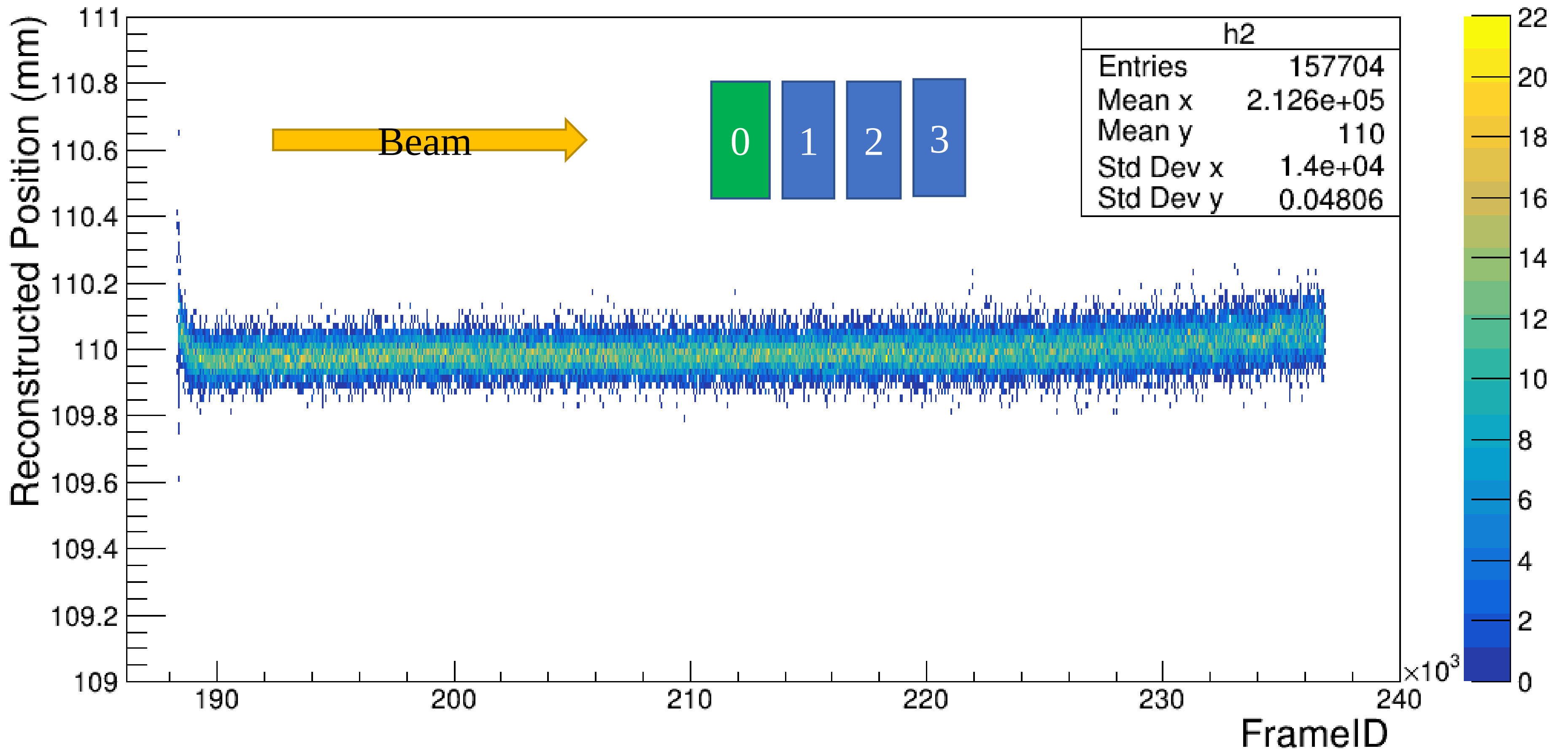
Test the accuracy of the reconstruction algorithm.





The reconstructed position and width and their standard deviations with signal to noise ratio between 5 and 100, position = 50 mm, width = 10 mm.

Resolution Calculation



Reconstructed Position for detection plane 0 for one spill of synchrotron : Proton 221.06 MeV 8.1 mm $2 \cdot 10^9$/s

The fluctuation of Position contains the resolution of detector and the jitter and drift of beam.

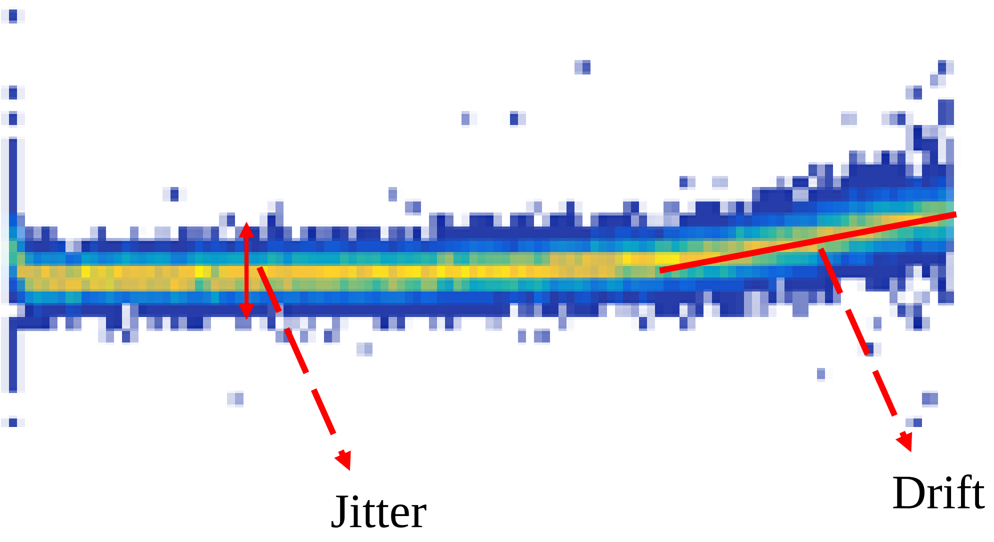
Resolution Calculation

For Single Board:

$$\sigma_{B_i}^2 = \sigma_{\det\_B_i}^2 + \sigma_{beam}^2$$

$$\sigma_{B_j}^2 = \sigma_{\det\_B_j}^2 + \sigma_{beam}^2$$

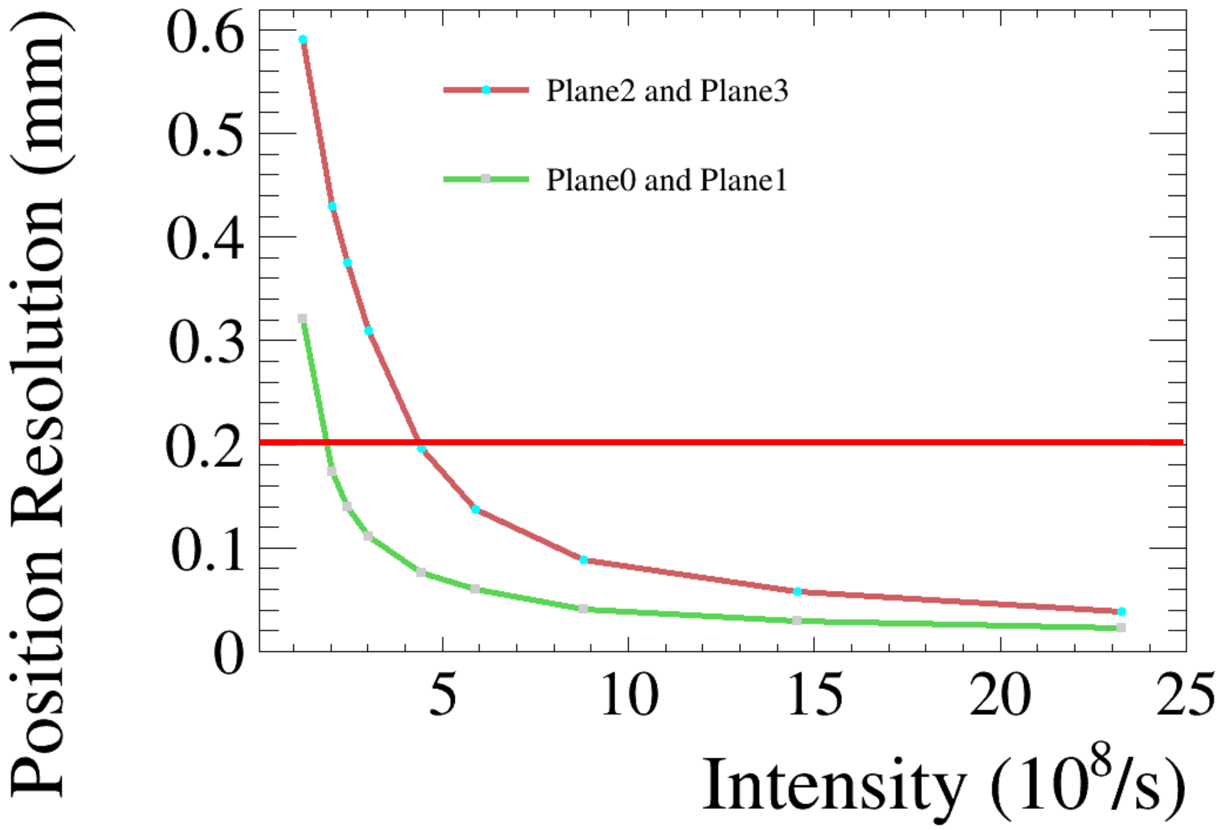
$$\sigma_{beam}^2 = \sigma_{beam-jitter}^2 + \sigma_{beam-drift}^2$$

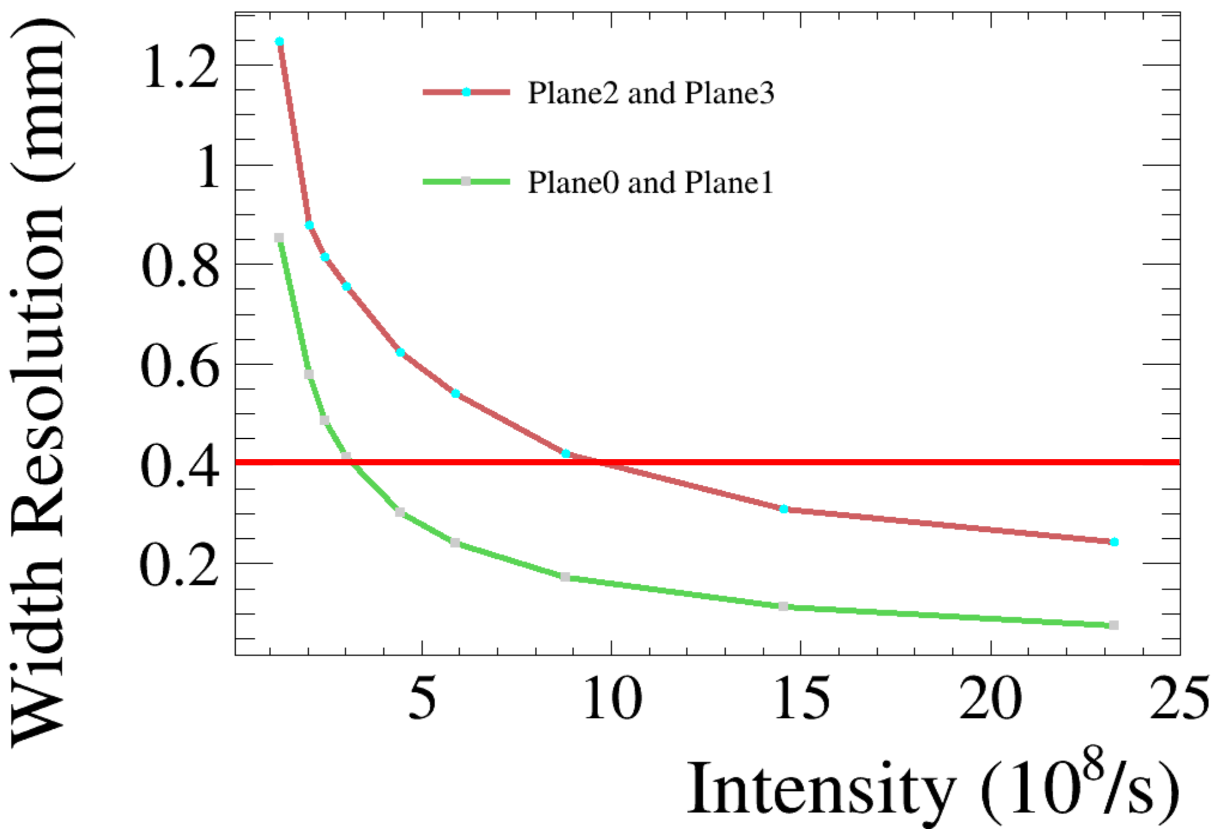


By Boardi – Boardj:

$$\sigma_{B_j-B_i}^2 = \sigma_{\det\_B_i}^2 + \sigma_{\det\_B_j}^2 \Rightarrow \sigma = \frac{\sigma_{B_j-B_i}}{\sqrt{2}}$$

Detector Properties: Proton

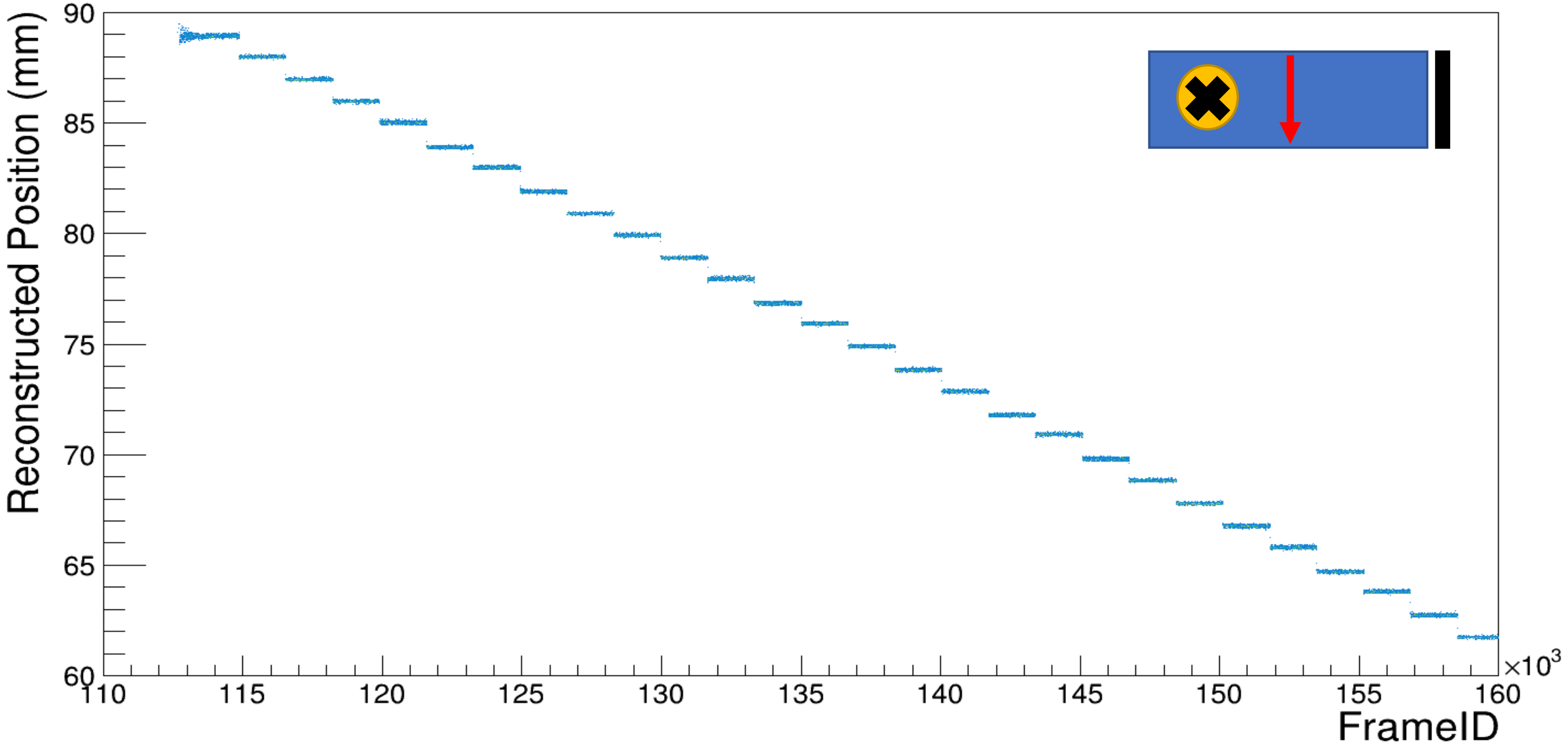




The position and width resolution of detection planes. Beam setting: Proton 221.06 MeV 8.1 mm.

Calibration

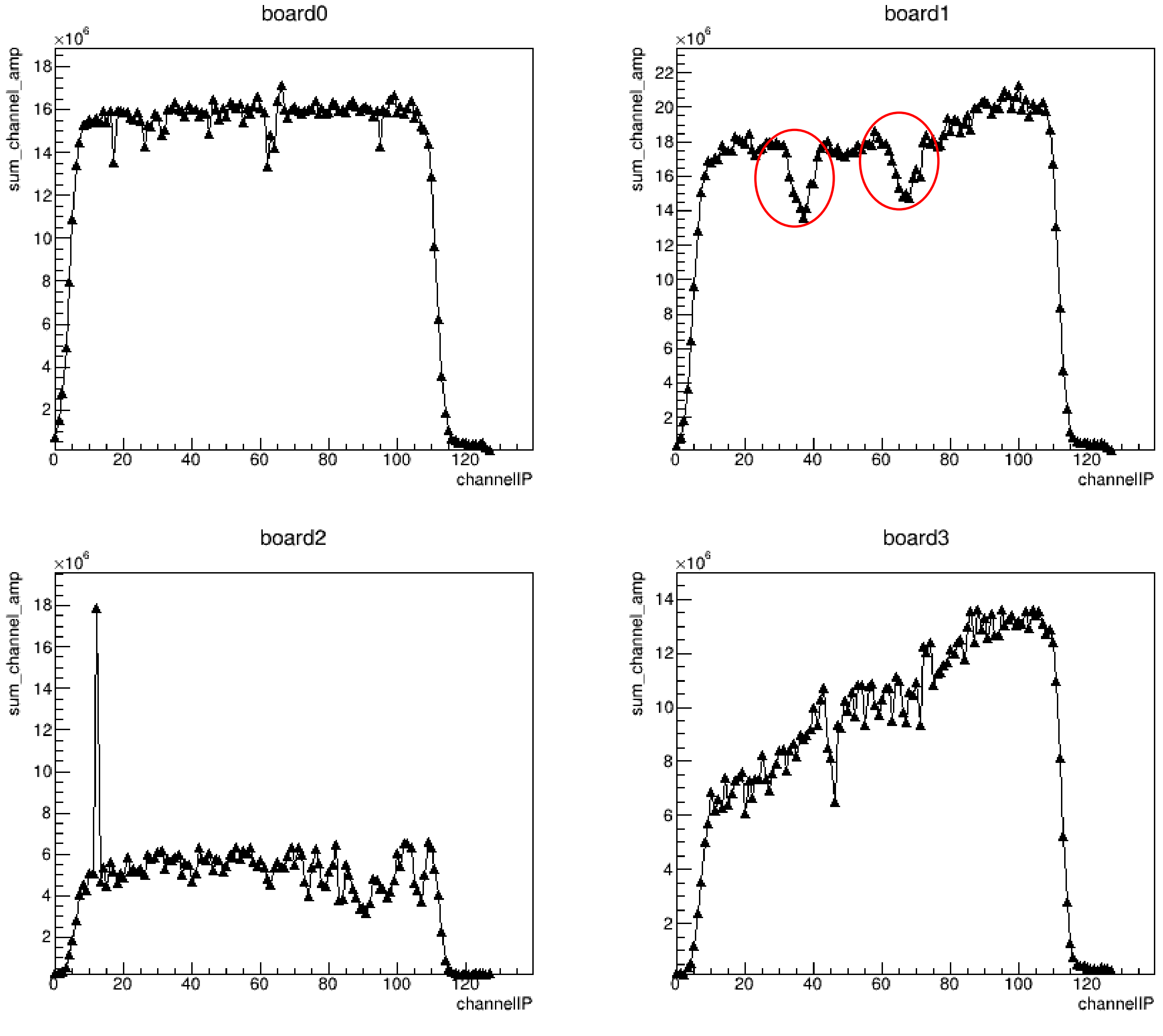
Due to the difference between channels, calibration is needed.



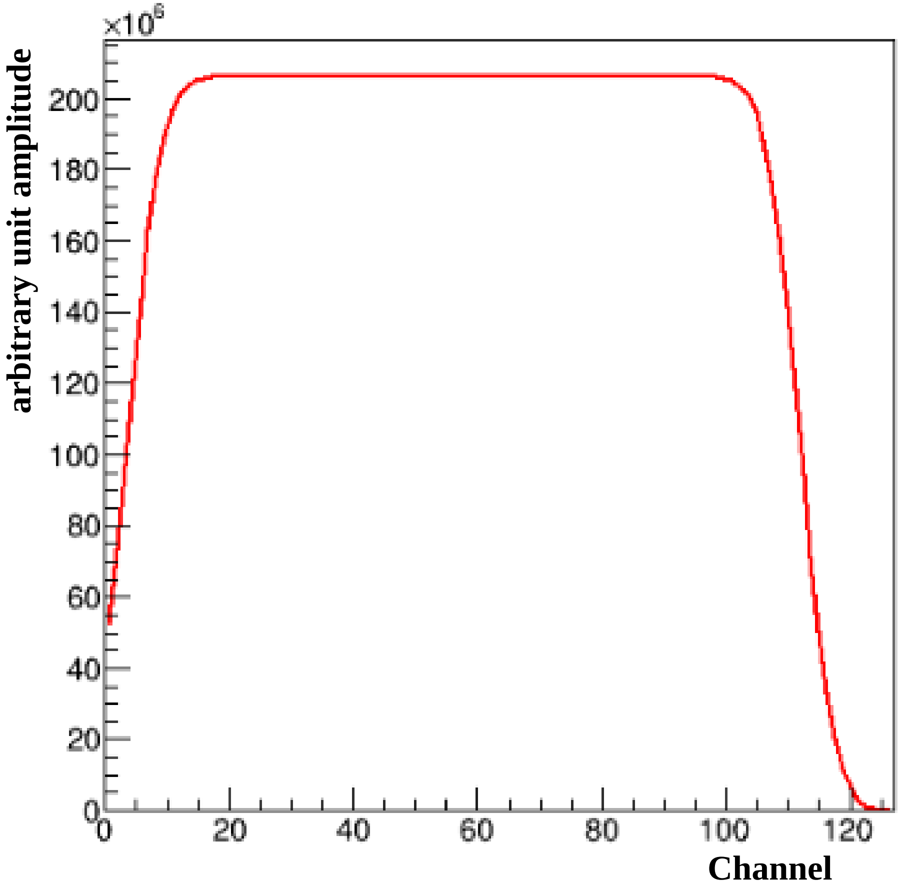
Scan along the detection plane. reconstructed Position for detection plane 1 : Carbon 430 MeV/u 3.4 mm 5· 10^7/s

Distribute the same amount of particle with fixed energy and width 1mm/step along 128 channels.

Calibration

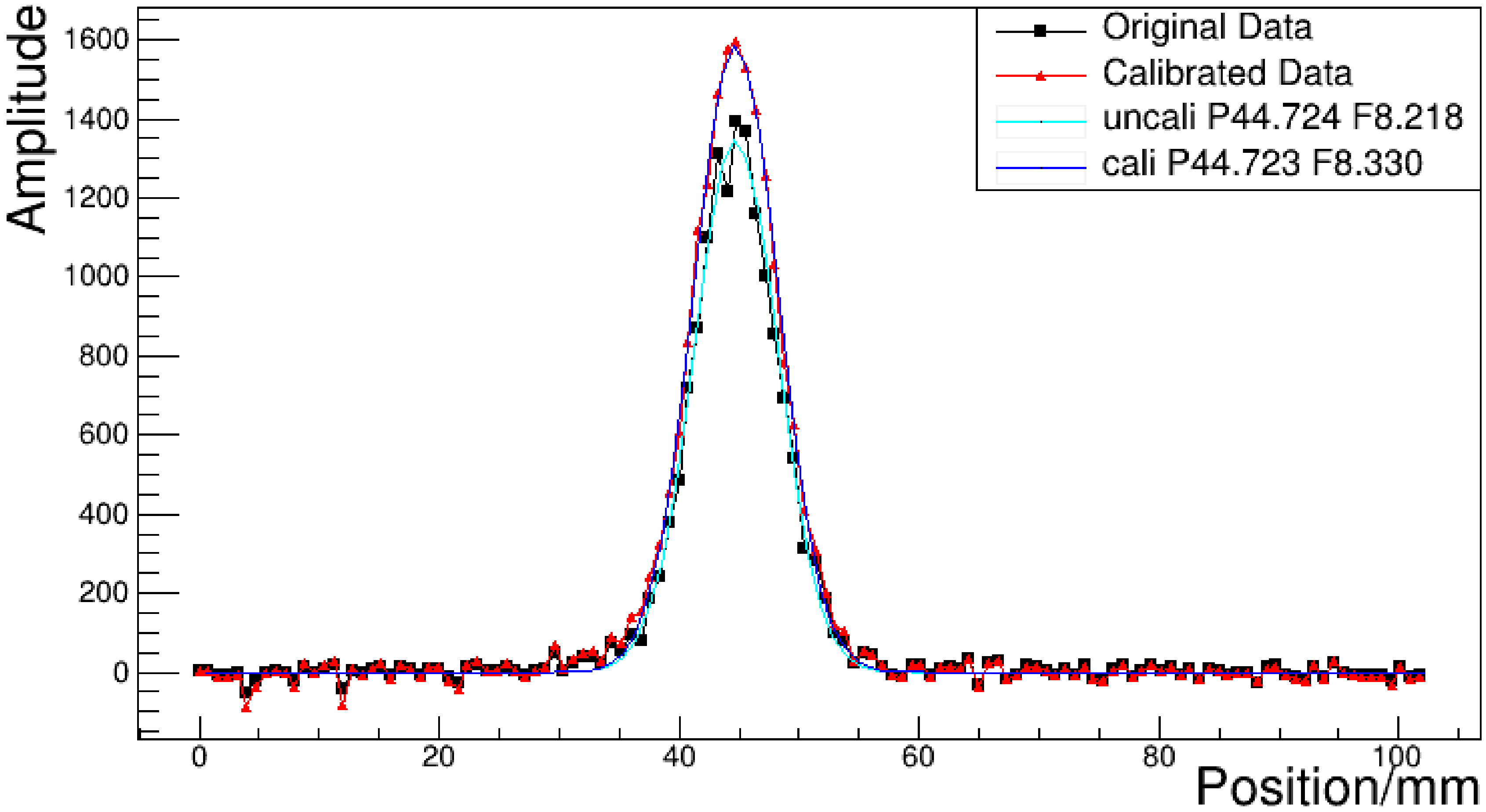


Integrated Signal Amplitude for each channel. Particles are uniformly distributed.



Ideal Integrated Signal Amplitude.

Calibration



Original signal vs. calibrated signal for detection plane 3. Proton 221.06 MeV 8.1 mm $2 \cdot 10^9$/s

Calibration make the beam shape closer to Gaussian shape.

Calibration and Radiation Damage

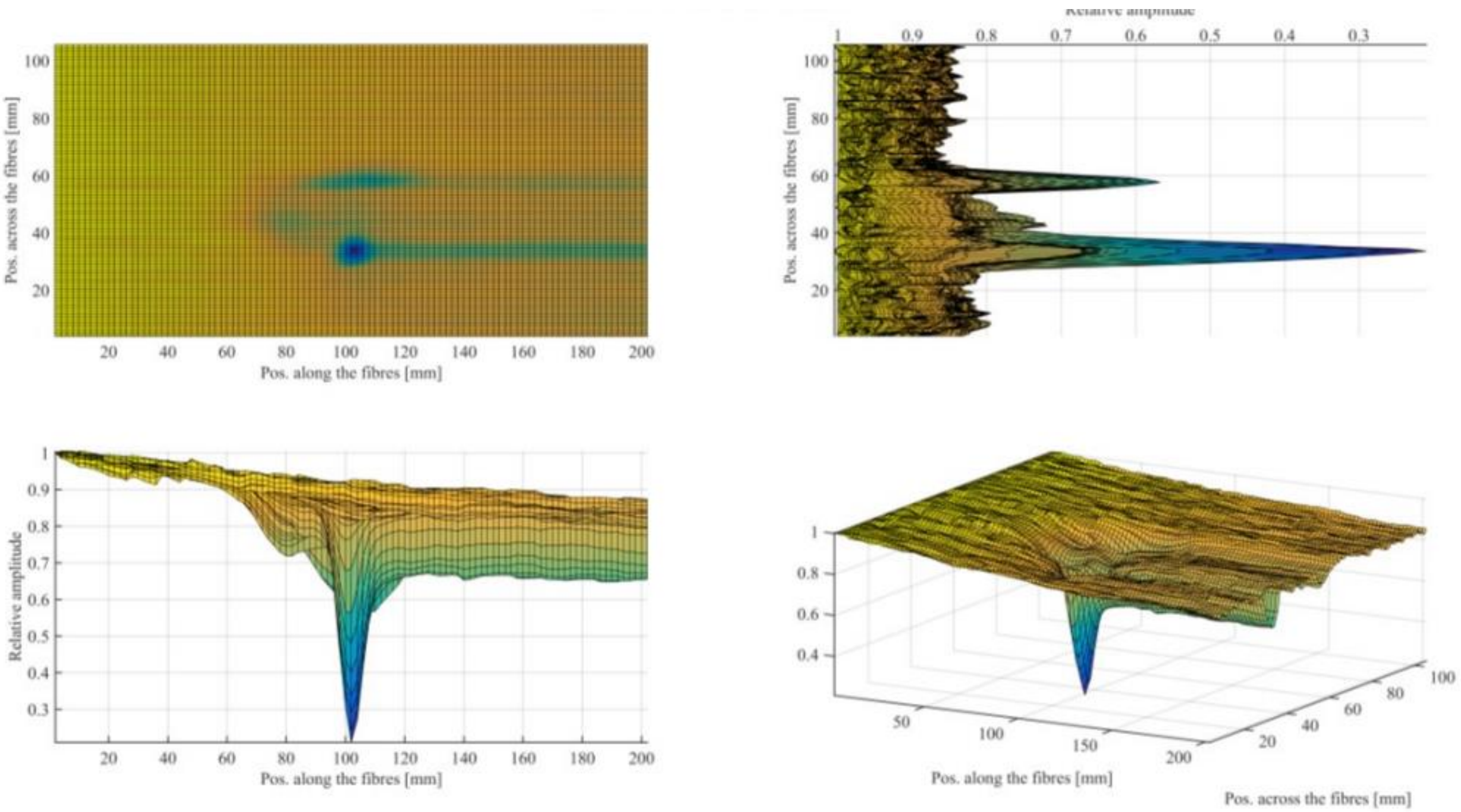
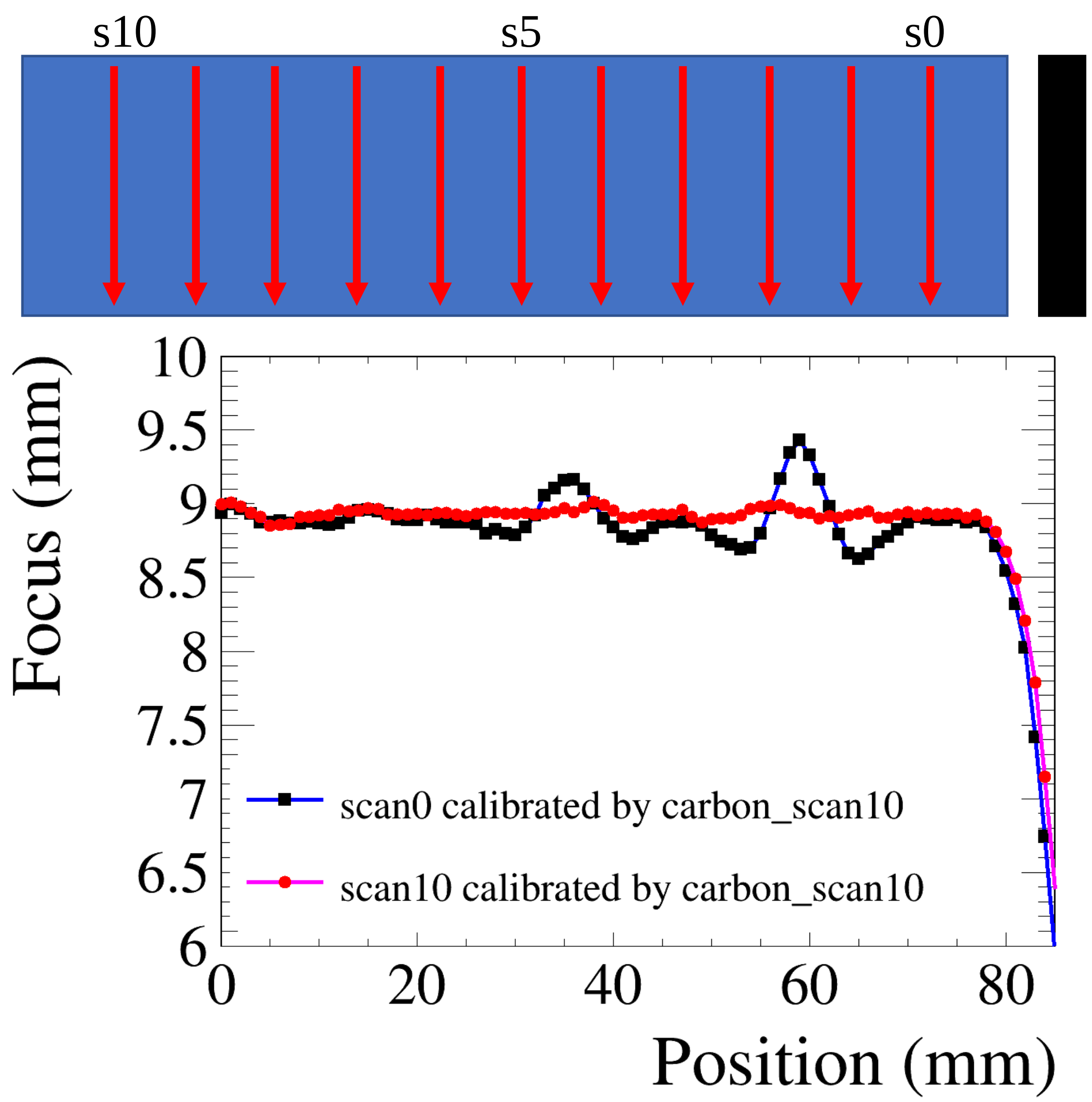
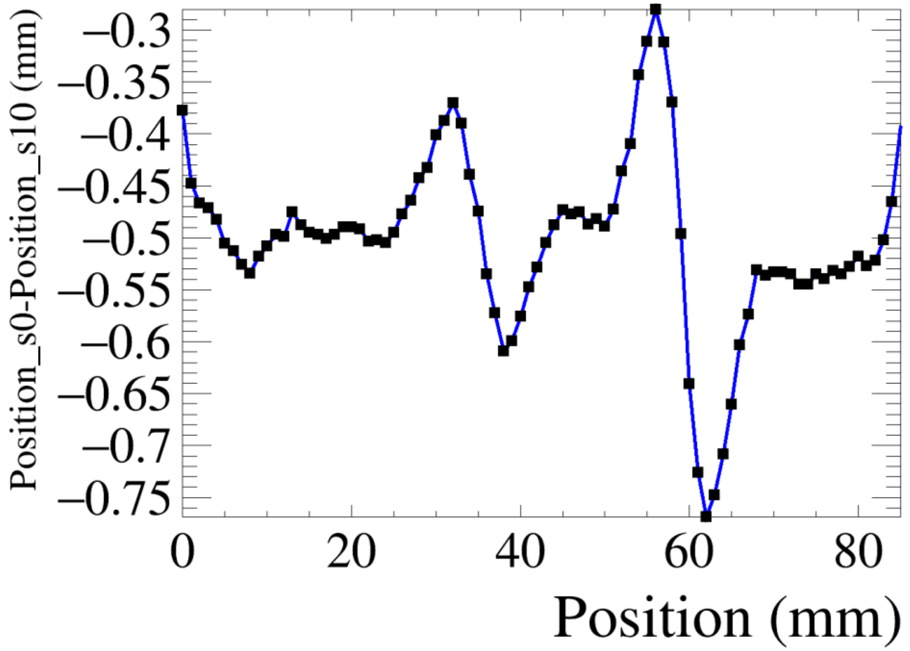


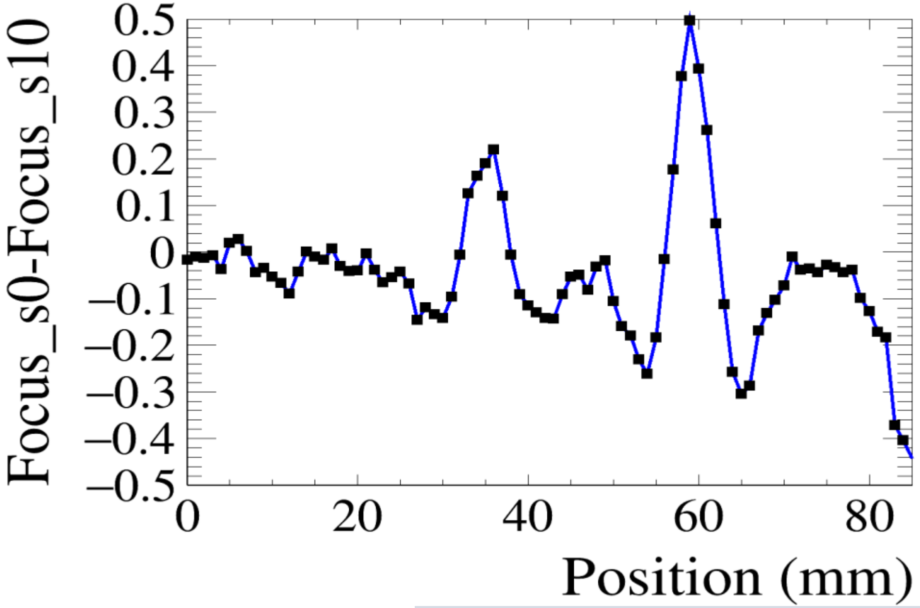
Figure 11. The signal observed in preliminary data from a fibre mat with SCSF-3HF 'green' fibres using a 4mm FWHM carbon ion beam scanning over an area of 10x20 cm^2 with 2 mm steps. The signal amplitudes are normalised to the data at 2 mm. The damage is from approximately 0.6 of a year of equivalent ionising dose. A second spot is visible due to a misalignment of the beam for a short period.

Calibration and Radiation Damage



Average reconstructed Focus at each step by Proton beam scan.
Proton 221.06 MeV 8.1 mm $2 \cdot 10^9$/s





Summary

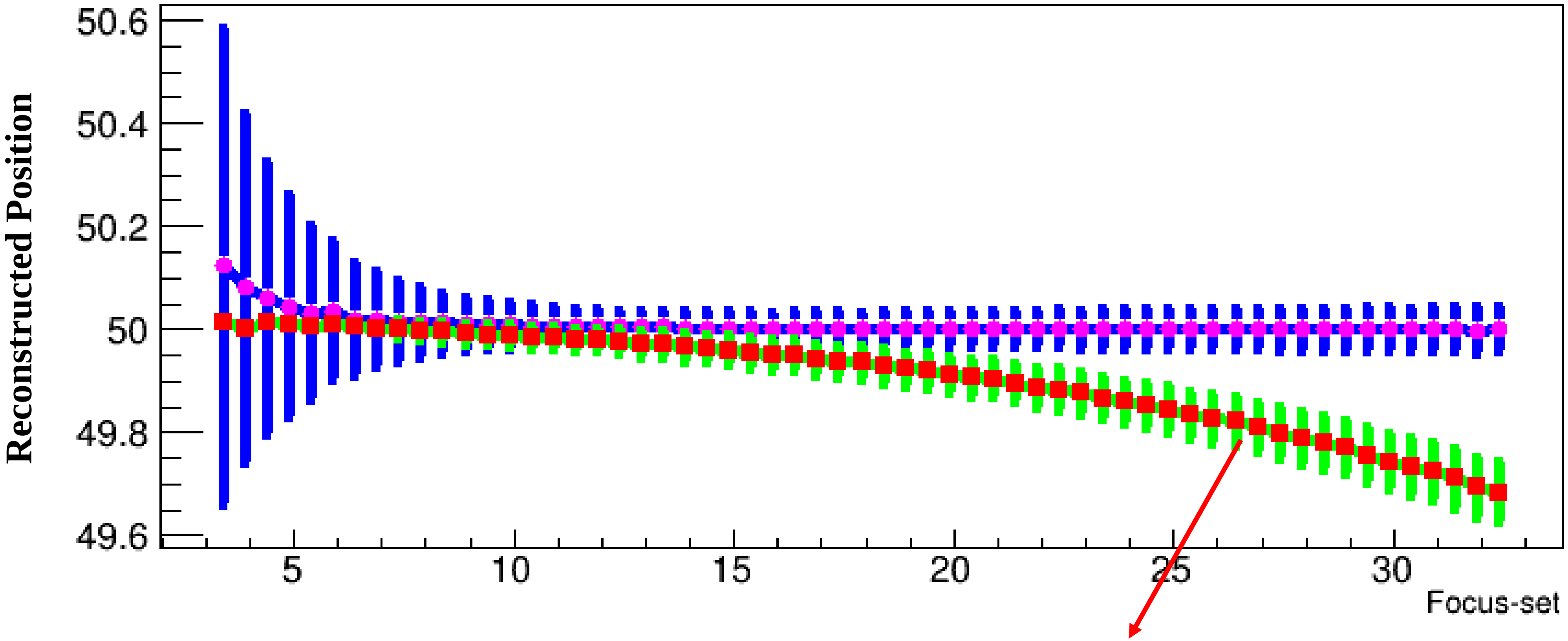
We found a FAS algorithm for beam reconstruction.

The detector satisfies the requirement at high intensity.

Calibration needs to be done in 2 dimensions.

Outlook

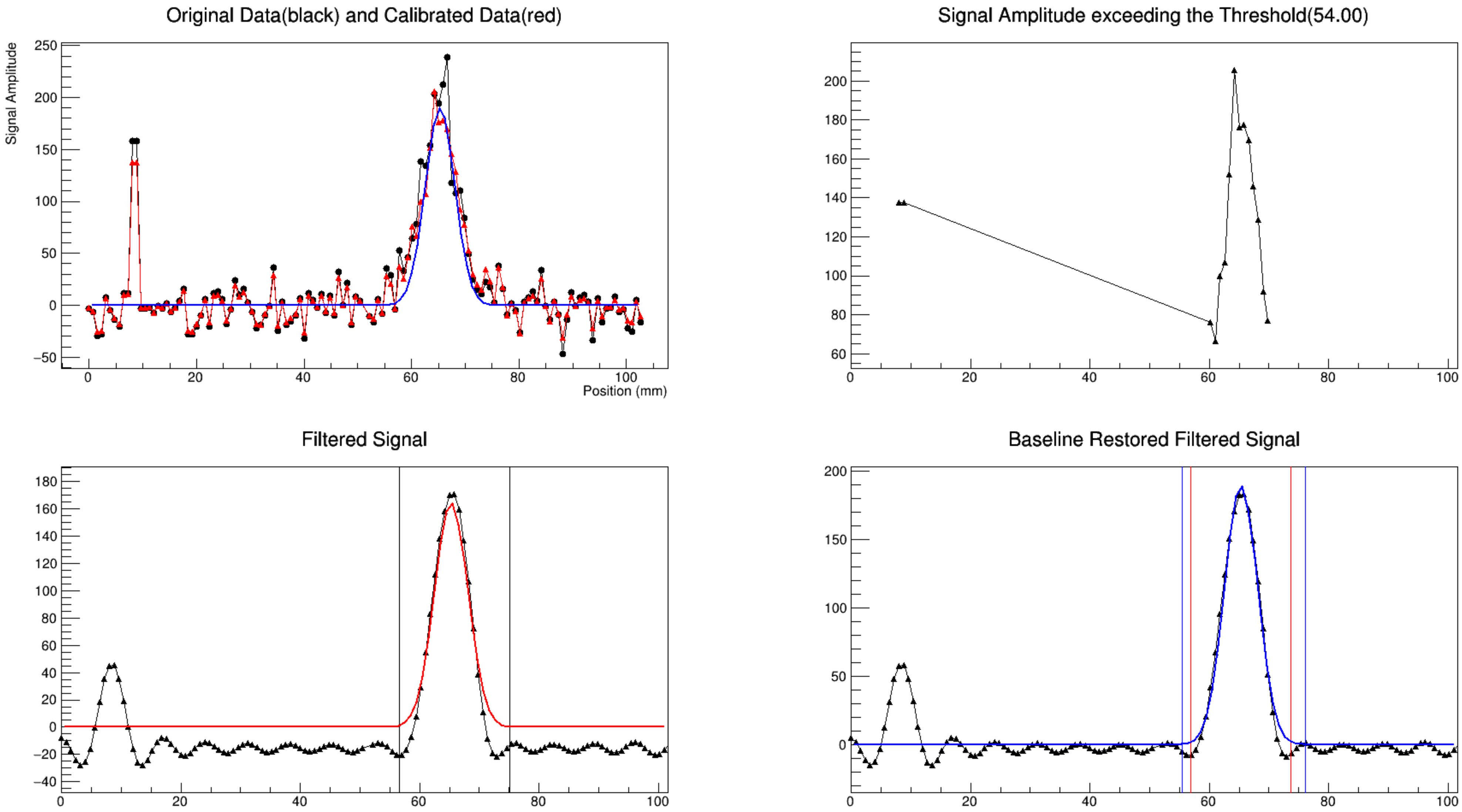
Simplify the FAS6+FFT algorithm.



Caused by a 0.2 mm gap around 50.5 mm and over-iteration.

Outlook

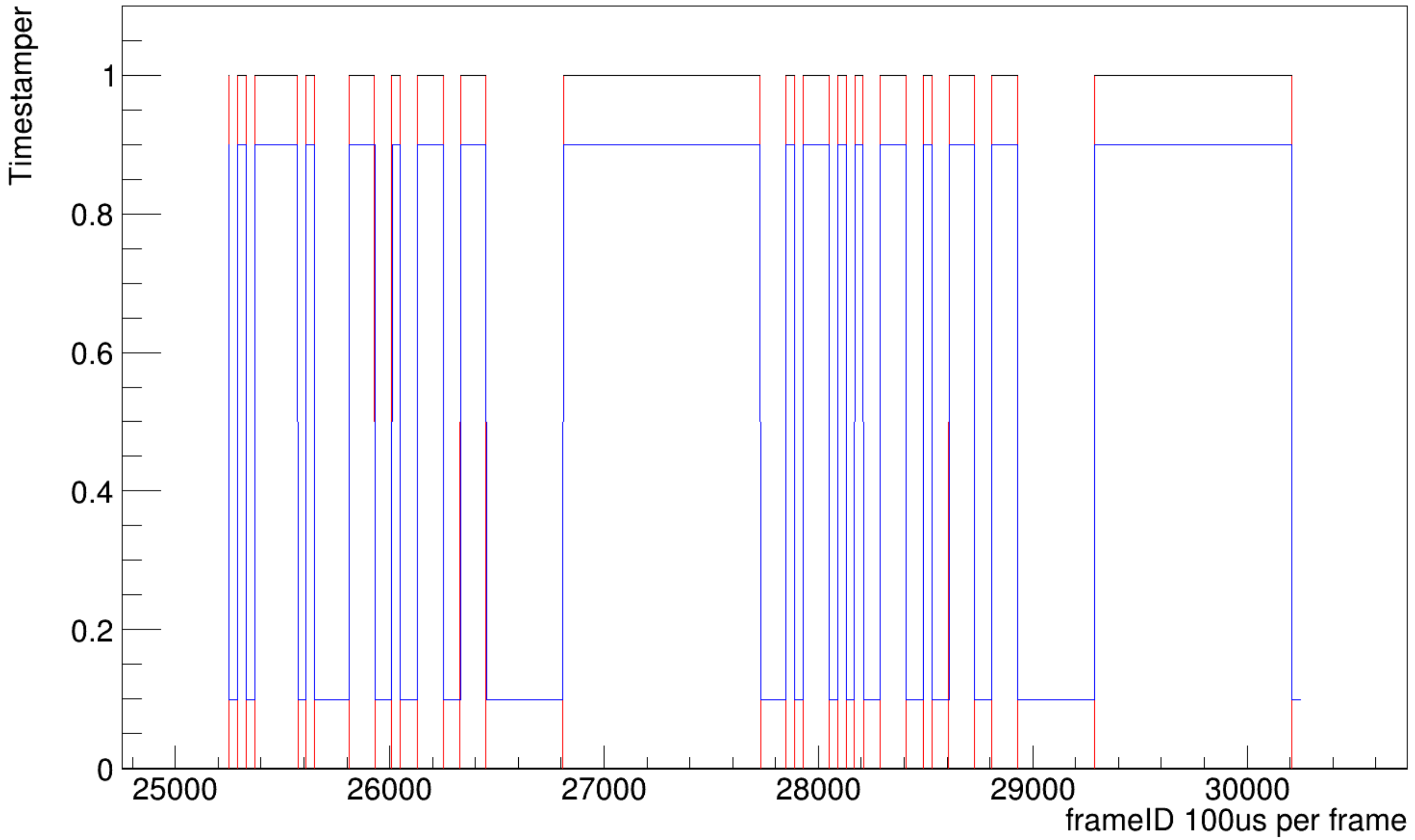
Apply the data processing process on FPGAs.



Outlook

Nr.	Name	Sample Period
1	IC1	50us
2	IC2	50us
3	MW1_FOCUSX	100us
4	MW1_FOCUSY	100us
5	MW2_FOCUSX	100us
6	MW2_FOCUSY	100us
7	DEBUG-X	50us
8	DEBUG-Y	50us
9	ACS_SIGNALS	50us
10	MW1_POSITIONX	100us
11	MW1_POSITIONY	100us
12	MW2_POSITIONX	100us
13	MW2_POSITIONY	100us
14	ENERGY	10ms
15	FOCUS	10ms
16	INTENSITY	10ms
17	IONSORT	10ms
18	Analog_IN2	50us

Comparing with HIT detectors; Calibration in 2 dimensions.

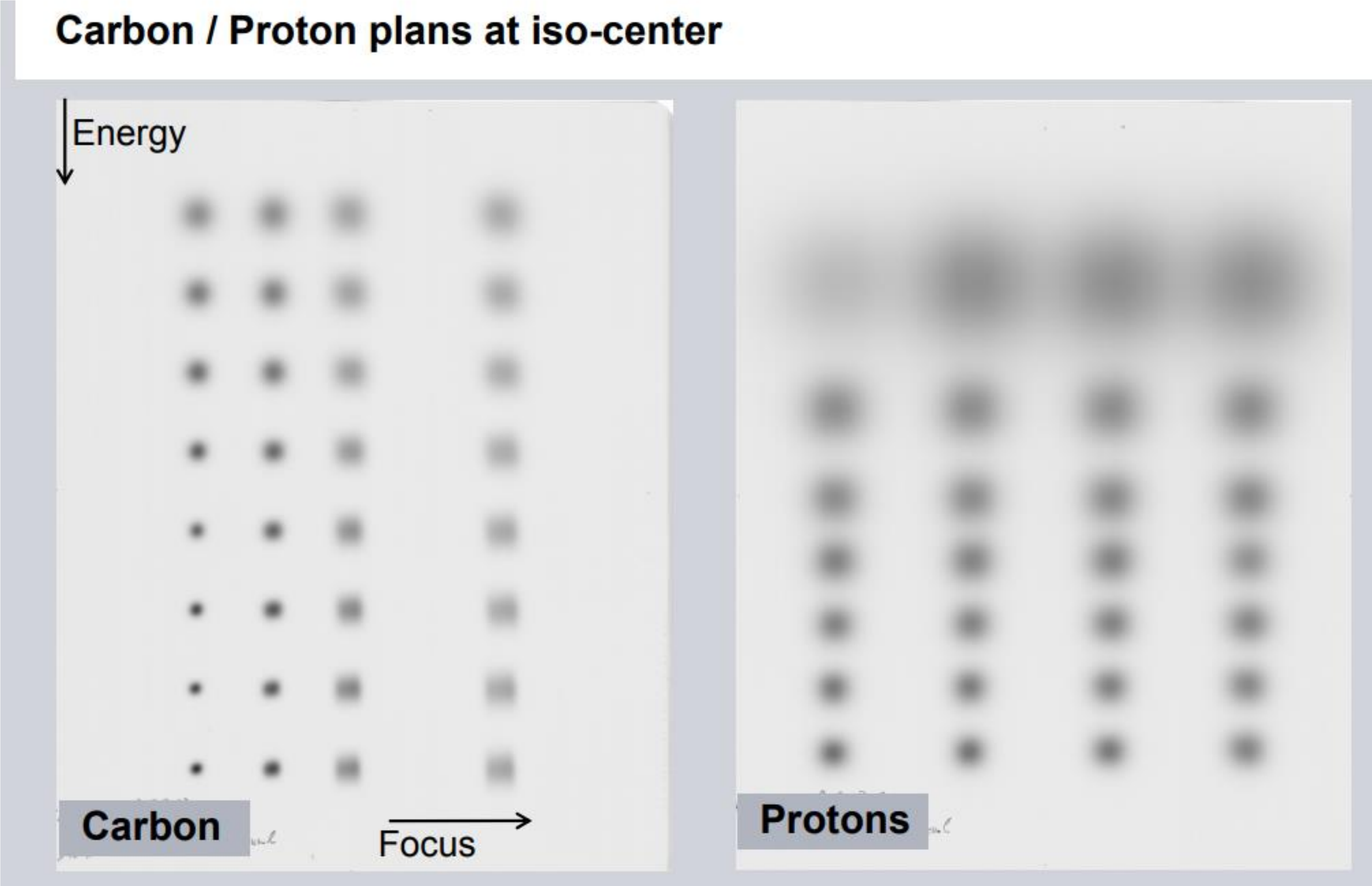


Aligned timestamper signal to hit detector(red) and scintillating fiber BPM(blue)

Thank you for you attention

BACK UP

Focus


Carbon / Proton plans at iso-center
Energy
Carbon
Focus
Protons

How to get Position and Focus from Data?

Original Data

↓

Threshold

↓

Calibration

↓

FFT

↓ Set the 1st window

FAS regression

Set the 2nd window →

Baseline Restore

↑ Set the 3rd window

FAS regression

Data Feature:

1. Noise is troubling
2. Outliers happens
3. Baseline is not stable

Data Structure

A structure of a single frame is shown below.

	Word offset	Symbol	Description
Header	0	N	Number of sensors
	1	C1	Board 1: number of channels
	(boards 2 to N-1)		
	N	CN	Board N: number of channels
Data	N+1	S1	Board 1: sync frame (8 words = 16 bytes)
	...		
	N+8		
	N+9	D1	Board 1: first sensor channel
	...		*(other sensor channels)*
	N+9+C1		Board 1: last sensor channel
	(boards 2 to N-1)		
	(X)+1	SN	Board N: sync frame
	...		
	(X)+8		
	(X)+9	DN	Board N: first sensor channel
	...		*(other sensor channels)*
	(X)+9+CN		Board N: last sensor channel

One Single Frame stored by PC contain both the signal information and synchronize information.

Version 2 readout

The new readout:
In order to calculate beam parameters on boards.

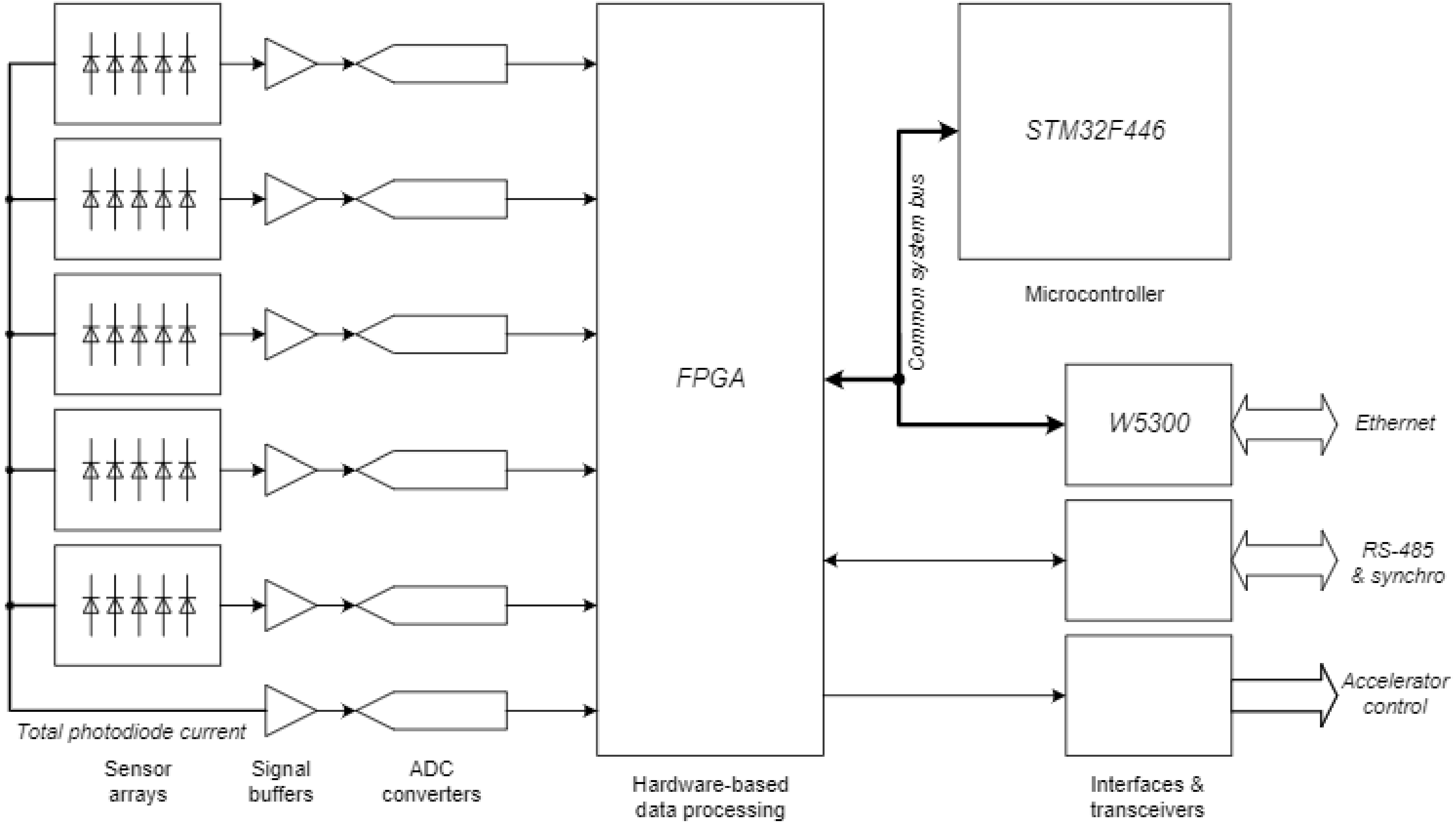
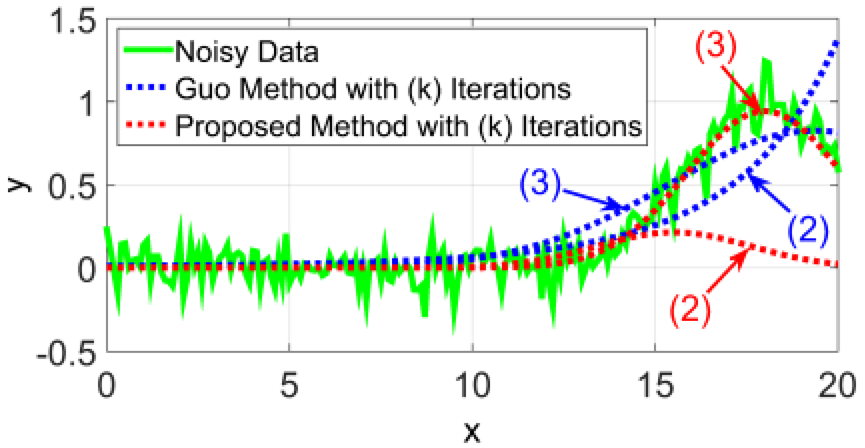
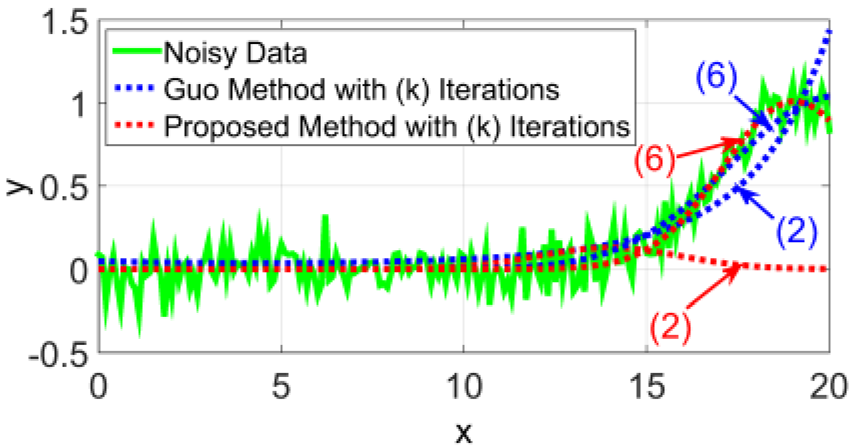


Figure 12. A simplified block diagram of the Phase-2 readout board for 5 sensors.

Iteration needs by different algorithm



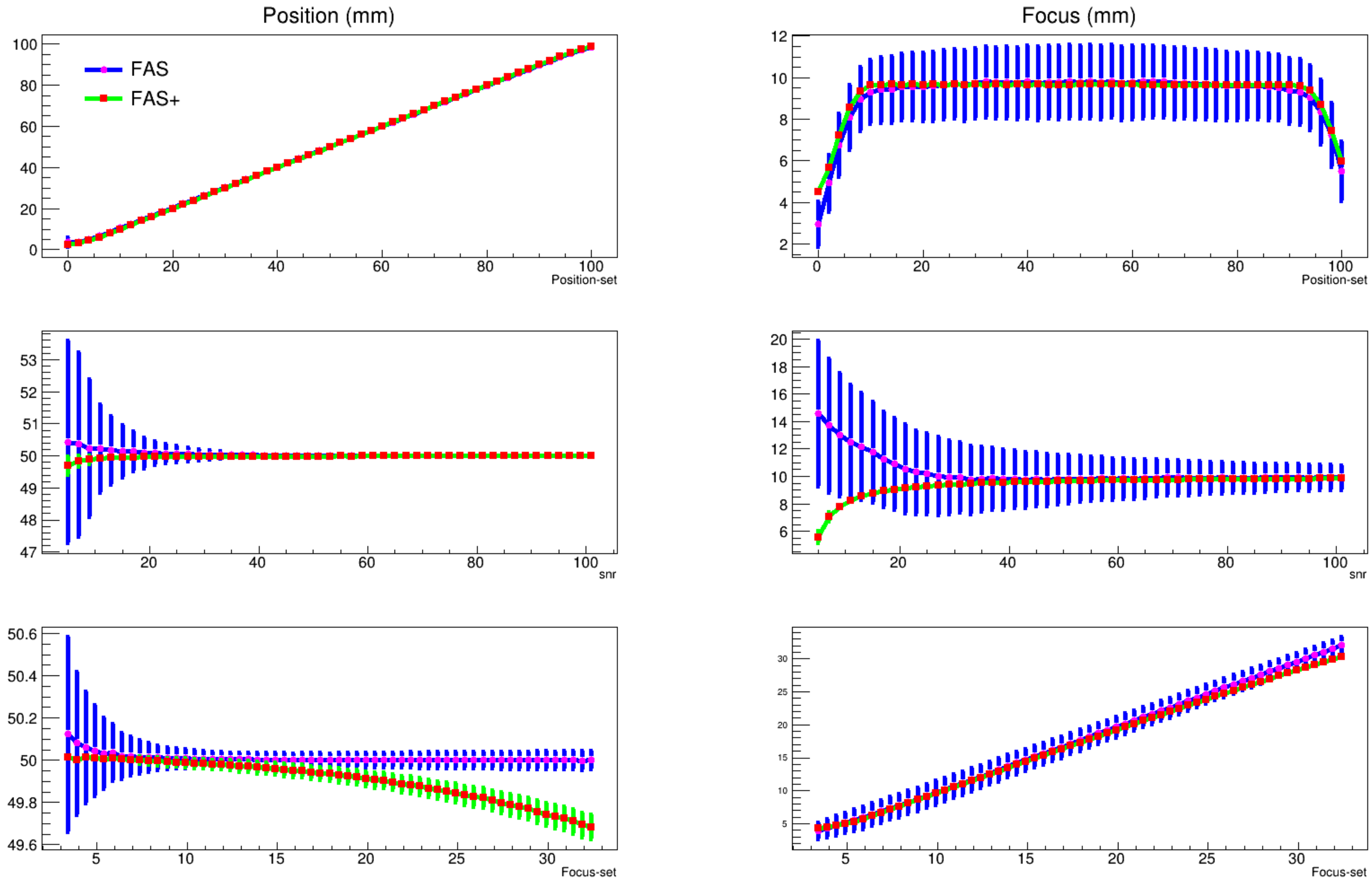
(a) $\mu = 18$



(b) $\mu = 19$

Figure 3: Results of the proposed FAS iterative algorithm in comparison with Guo's algorithm for fitting the GF of $N = 200$, $A = 1$, $\sigma = 2$ and $\sigma_w = 0.1$ (i.e., snr = 10).

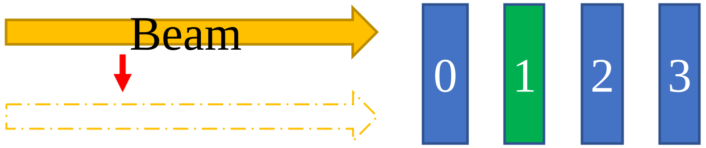
Simulation test

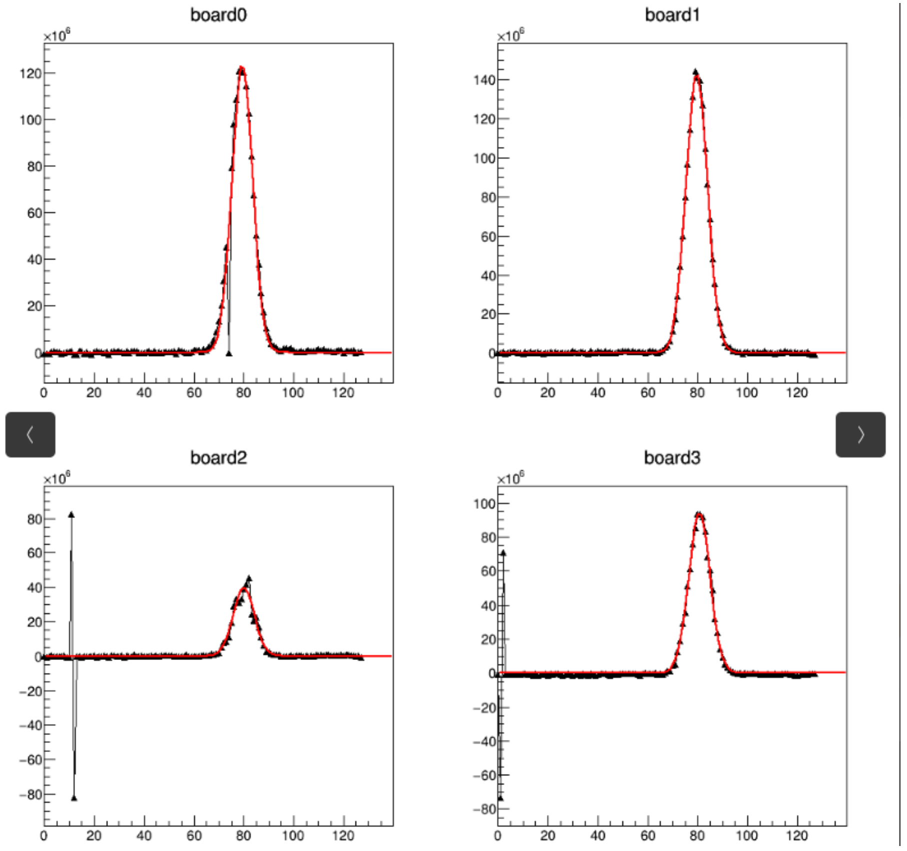


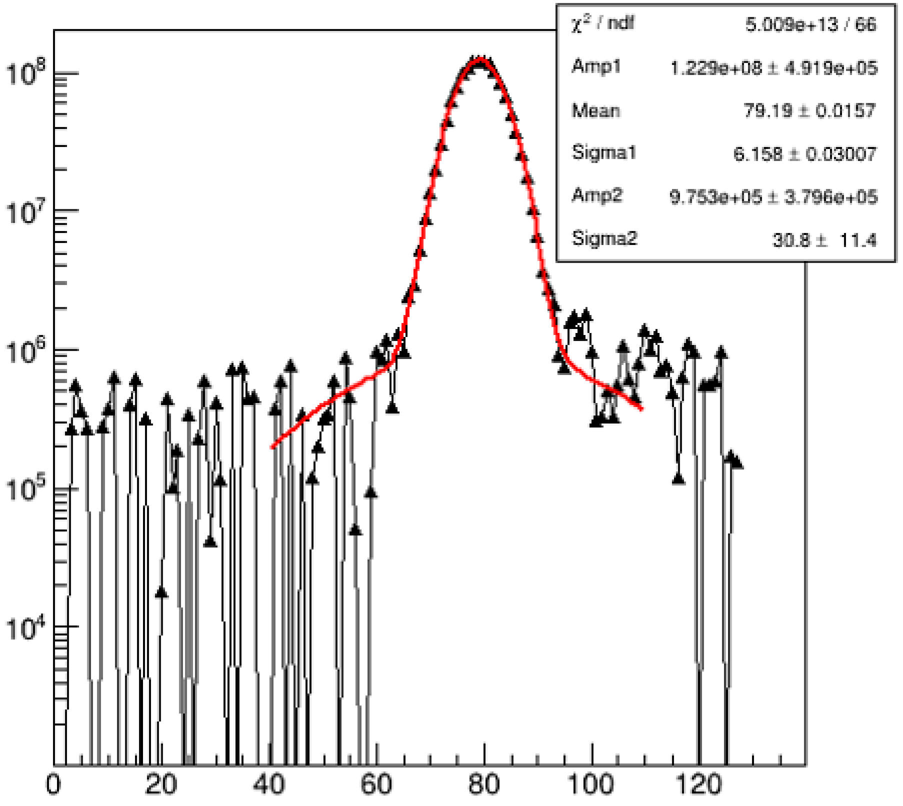
~~Snr=50;~~
~~Focus=10 mm;~~
~~Position=0~100 mm~~

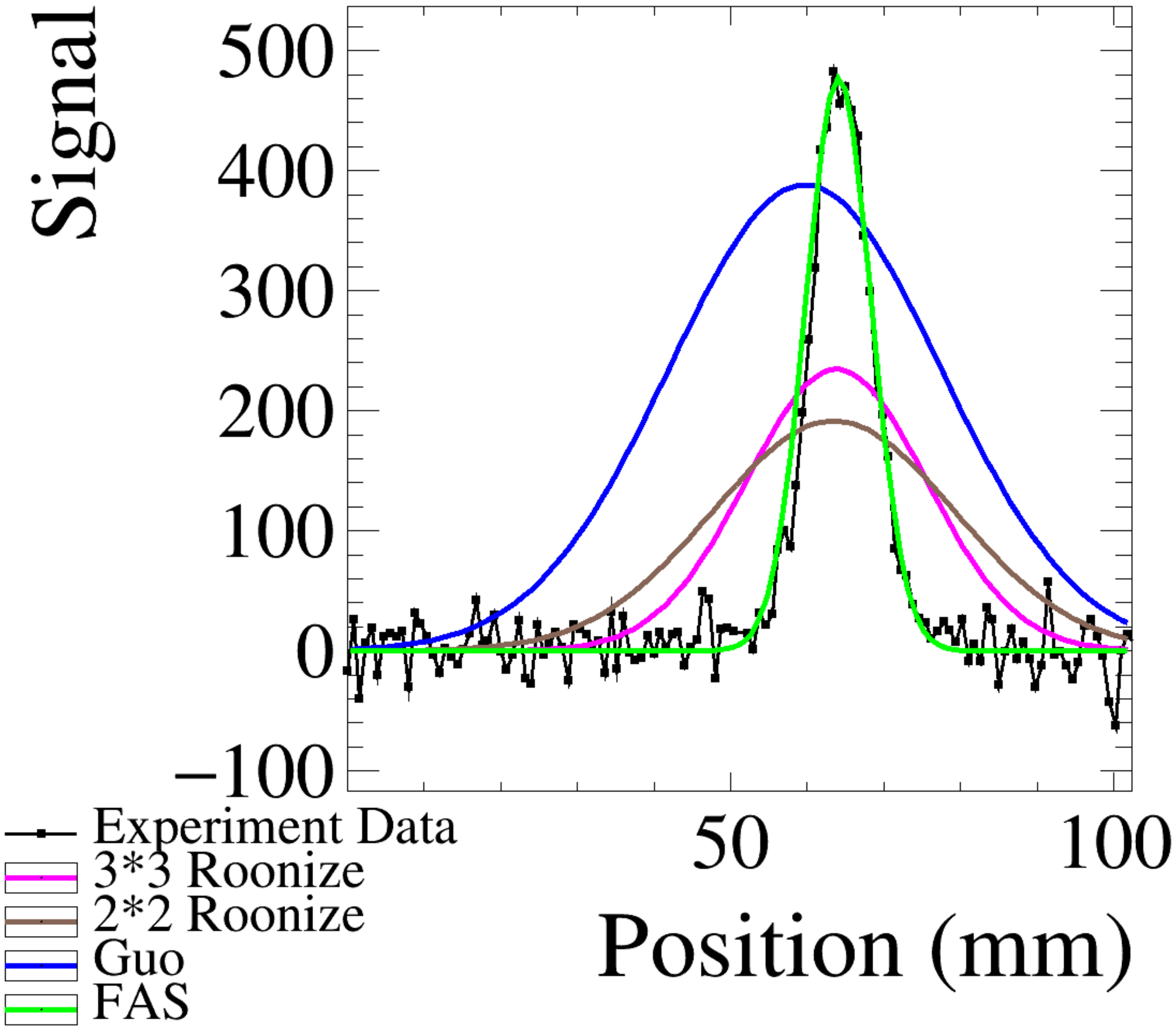
Snr=5~100;
Focus=10 mm;
Position=50 mm

Snr=50;
Focus=3.4~32.4 mm;
Position=50 mm

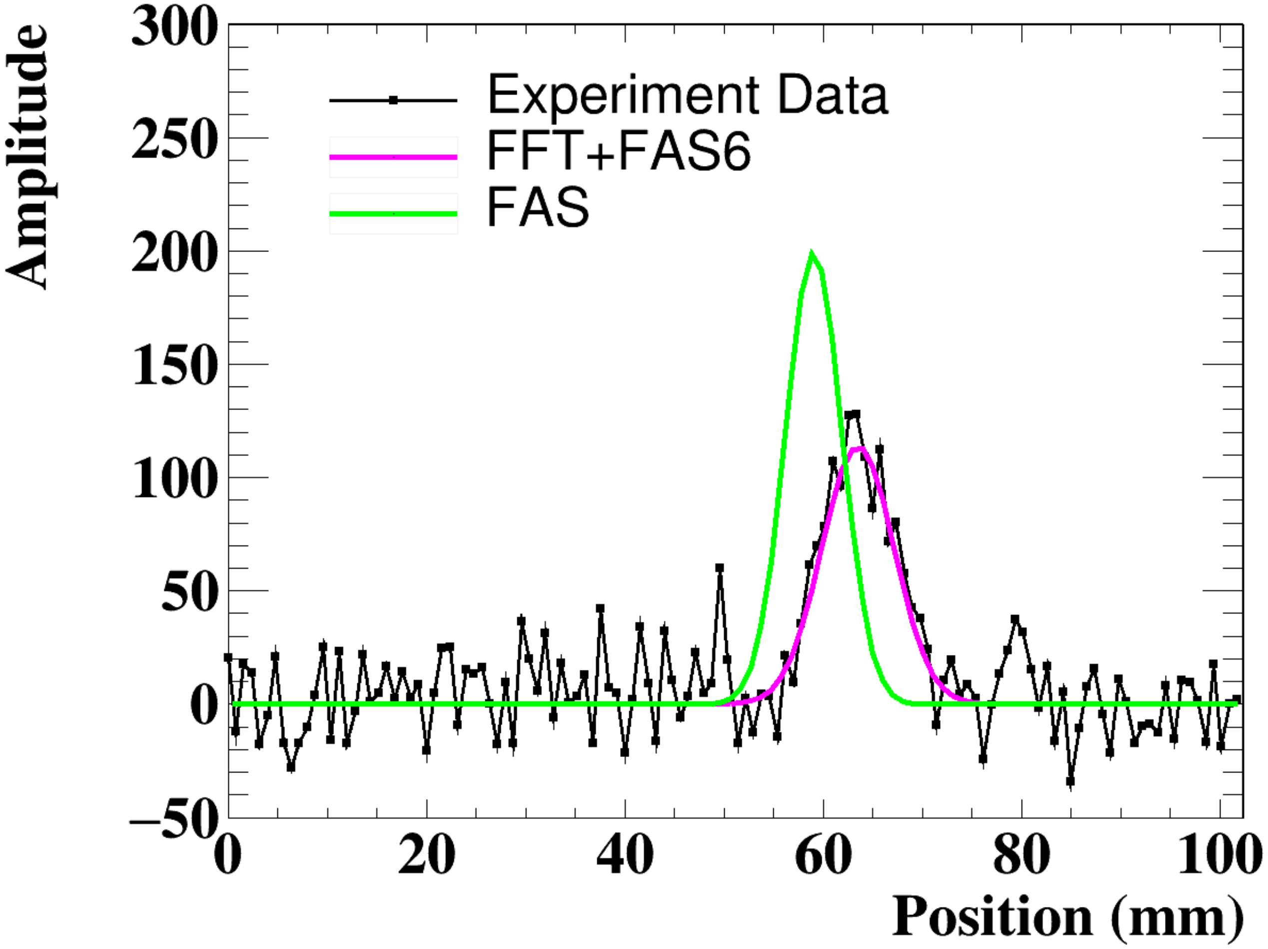

Beam
0
1
2
3


board0
board1
board2
board3


χ^2 / ndf 5.009e+13 / 66
Amp1 1.229e+08 ± 4.919e+05
Mean 79.19 ± 0.0157
Sigma1 6.158 ± 0.03007
Amp2 9.753e+05 ± 3.796e+05
Sigma2 30.8 ± 11.4

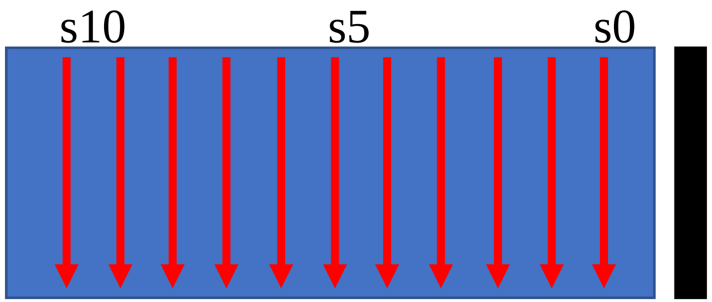


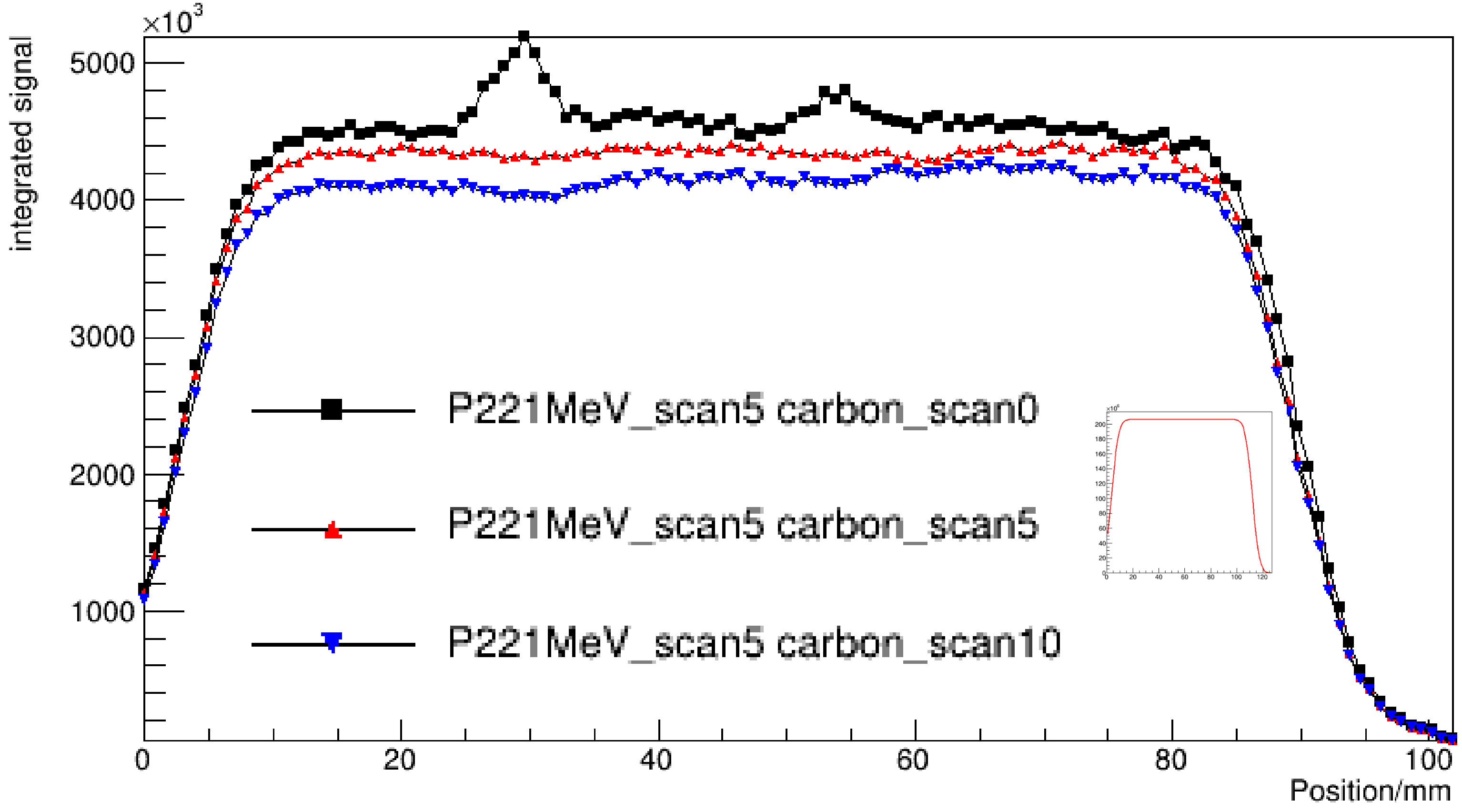
Signal
500
400
300
200
100
0
−100
50
100
Position (mm)
Experiment Data
3*3 Roonize
2*2 Roonize
Guo
FAS



Amplitude
300
250
200
150
100
50
0
−50
0
20
40
60
80
100
Position (mm)
Experiment Data
FFT+FAS6
FAS

Calibration and Radiation Damage





Integrated signal amplitude of proton scan5 calibrated by different carbon scan.

FAS algorithms

$$\ln(y) = \ln(A) + \frac{-(x-\mu)^2}{2\sigma^2} = \ln(A) - \frac{\mu^2}{2\sigma^2} + \frac{2\mu x}{2\sigma^2} - \frac{x^2}{2\sigma^2} = a + bx + cx^2,$$

$$A = e^{a-\frac{b^2}{4c}}, \quad \mu = \frac{-b}{2c}, \quad \sigma = \sqrt{\frac{-1}{2c}}.$$

$$\Lambda = \int_{-\infty}^{\infty} Ae^{-\frac{(x-\mu)^2}{2\sigma^2}}dx = A\sigma\sqrt{2\pi}.$$

$$\hat{\sigma} = \frac{\sum_{n=1}^{N} \Delta x_n \hat{y}_n}{\sqrt{2\pi}\hat{y}_{\max}}.$$

Error Function:

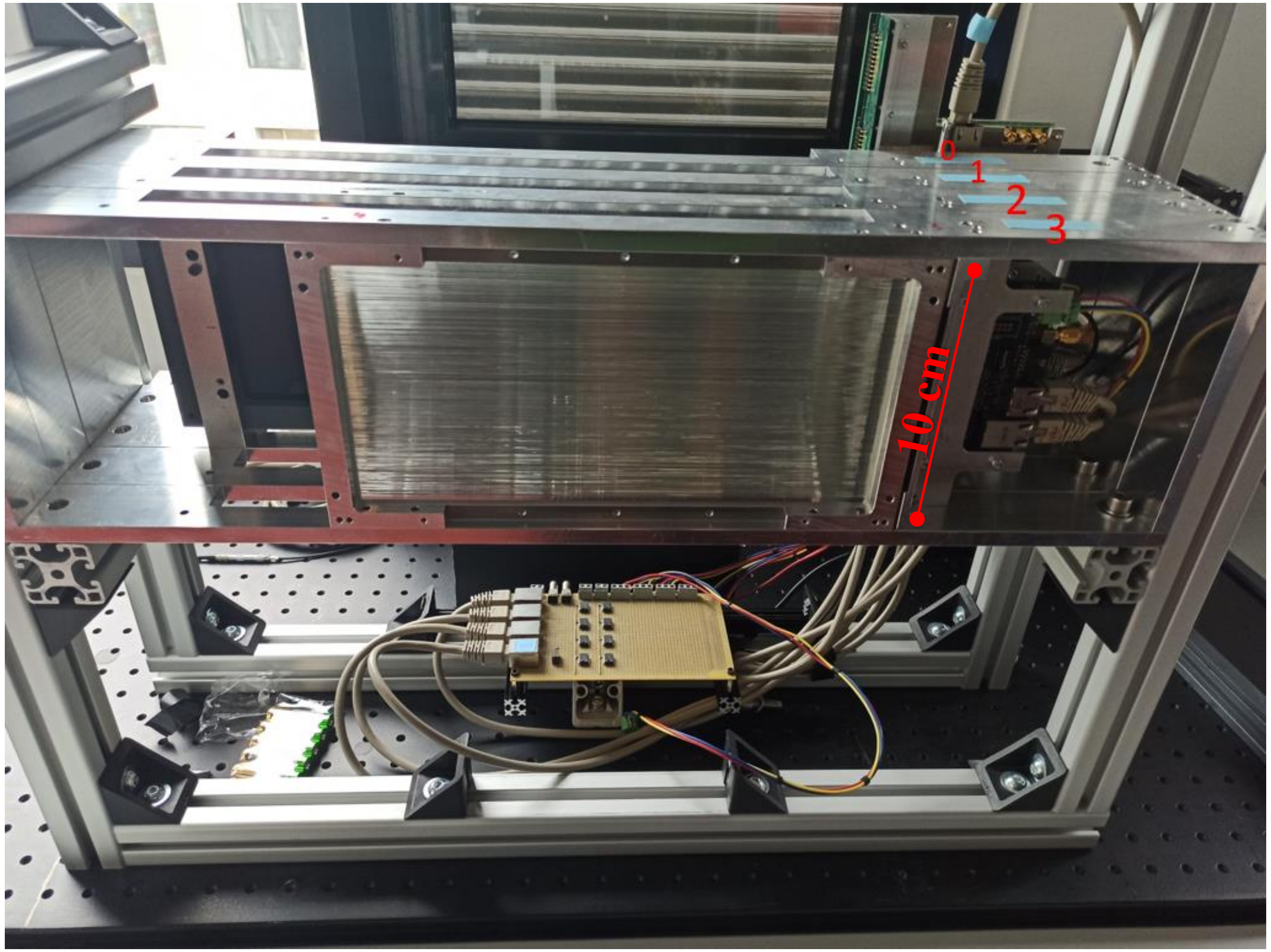
$$\delta = \hat{y}\varepsilon = \hat{y}[\ln(\hat{y}) - (a + bx + cx^2)]$$

$$\begin{bmatrix} \sum \hat{y}_n^2 & \sum x_n\hat{y}_n^2 \\ \sum x_n\hat{y}_n^2 & \sum x_n^2\hat{y}_n^2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum \hat{y}_n^2\ln(\hat{y}_n) - c\sum x_n^2\hat{y}_n^2 \\ \sum x_n\hat{y}_n^2\ln(\hat{y}_n) - c\sum x_n^3\hat{y}_n^2 \end{bmatrix}$$

Iterative form: $\hat{y}_{n,(k)} = e^{a_{(k)}+b_{(k)}x_n+cx_n^2}$

$$\begin{bmatrix} \sum \hat{y}_{n,(k-1)}^2 & \sum x_n\hat{y}_{n,(k-1)}^2 \\ \sum x_n\hat{y}_{n,(k-1)}^2 & \sum x_n^2\hat{y}_{n,(k-1)}^2 \end{bmatrix} \begin{bmatrix} a_{(k)} \\ b_{(k)} \end{bmatrix} = \begin{bmatrix} \sum \hat{y}_{n,(k-1)}^2\ln(\hat{y}_n) - c\sum x_n^2\hat{y}_{n,(k-1)}^2 \\ \sum x_n\hat{y}_{n,(k-1)}^2\ln(\hat{y}_n) - c\sum x_n^3\hat{y}_{n,(k-1)}^2 \end{bmatrix}$$

Scintillating fiber-based Beam Profile Monitor



Beam profile monitor with 4 scintillating fiber detection planes.

2 layers-fiber plane cross section diagram.

Detector:

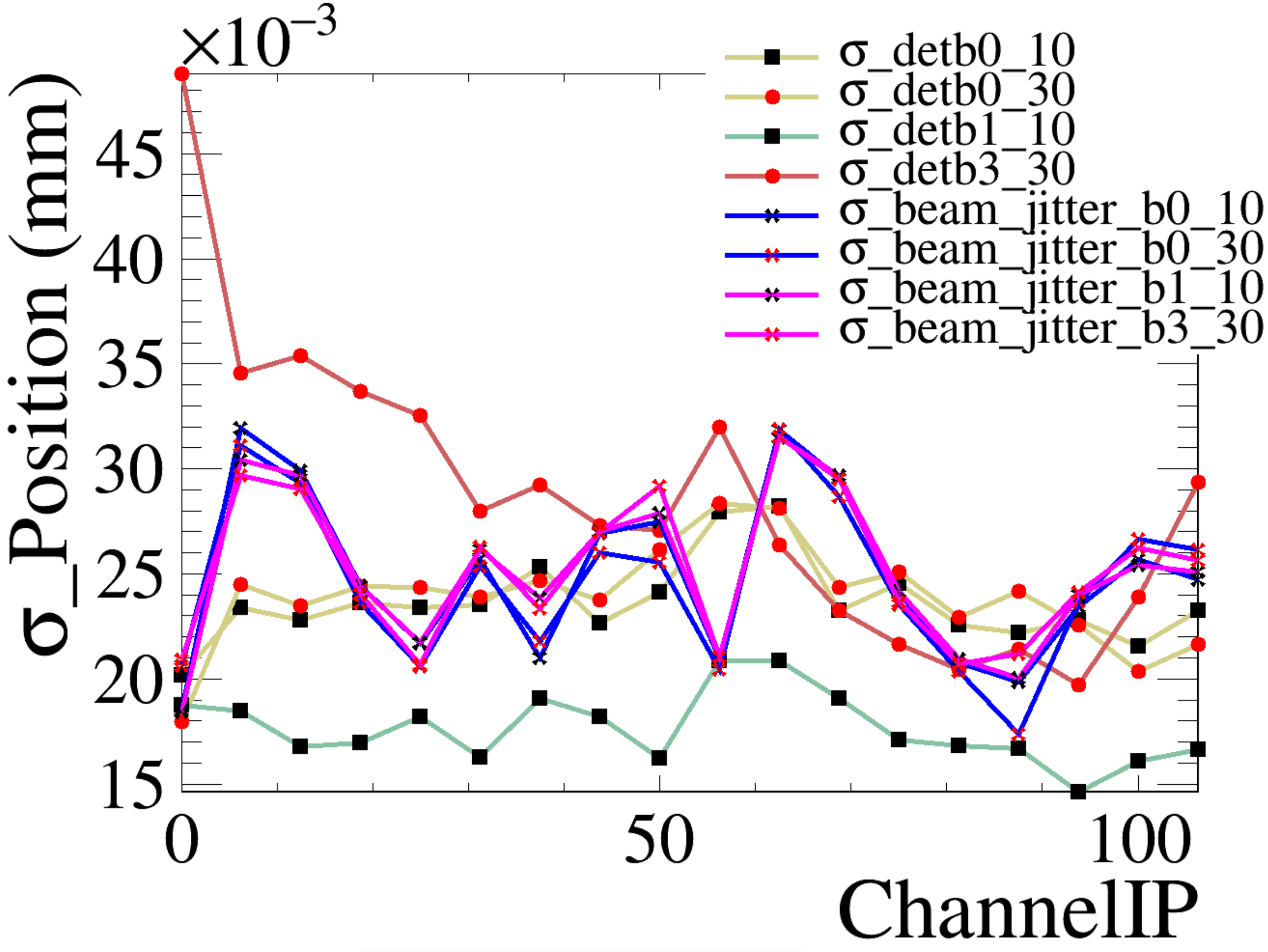
- Scintillating fiber SCSF-3HF from Kuraray.
- 0.250mm diameter; 0.325mm pitch.
- 2×64-channel photodiode array for each detection plane.

Readout:

- Adjustable integration time, ~100 μs by default.
- Maximum 10kHz readout rate.

Beam reconstruction algorithm will be implemented on FPGAs.

Detector Properties: Proton



Position Resolution for detection plane 0, 1 and 3. Proton 221.06 MeV 8.1 mm $2 \cdot 10^9$/s

Resolution Calculation

For Single Board:

$$\sigma_{B_i}^2 = \sigma_{\det\_B_i}^2 + \sigma_{beam}^2$$

$$\sigma_{B_j}^2 = \sigma_{\det\_B_j}^2 + \sigma_{beam}^2$$

By Boardi – Boardj:

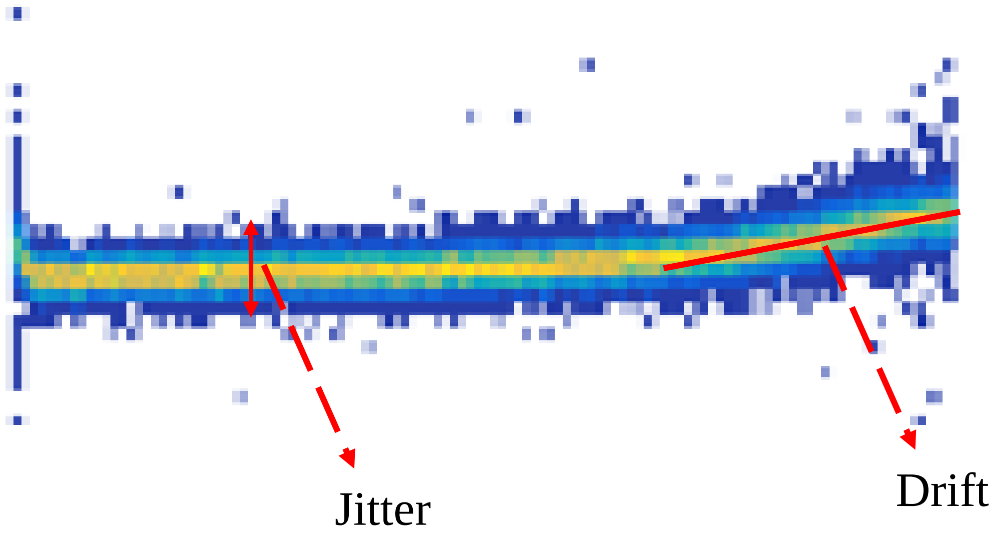
$$\sigma_{B_j - B_i}^2 = \sigma_{\det\_B_i}^2 + \sigma_{\det\_B_j}^2$$

For Beam: $\sigma_{beam}^2 = \sigma_{beam-jitter}^2 + \sigma_{beam-drift}^2$

By Framek– Framek-1:

$$\sigma_{Bi}{}_{F_k - F_{k-i}}^2 = 2\sigma_{\det\_B_i}^2 + 2\sigma_{beam-jitter}^2$$

$$\sigma_{Bj}{}_{F_k - F_{k-i}}^2 = 2\sigma_{\det\_B_j}^2 + 2\sigma_{beam-jitter}^2$$



Resolution Calculation

For Single Board:

$$\sigma_{B_i}^2 = \sigma_{\det\_B_i}^2 + \sigma_{beam}^2$$

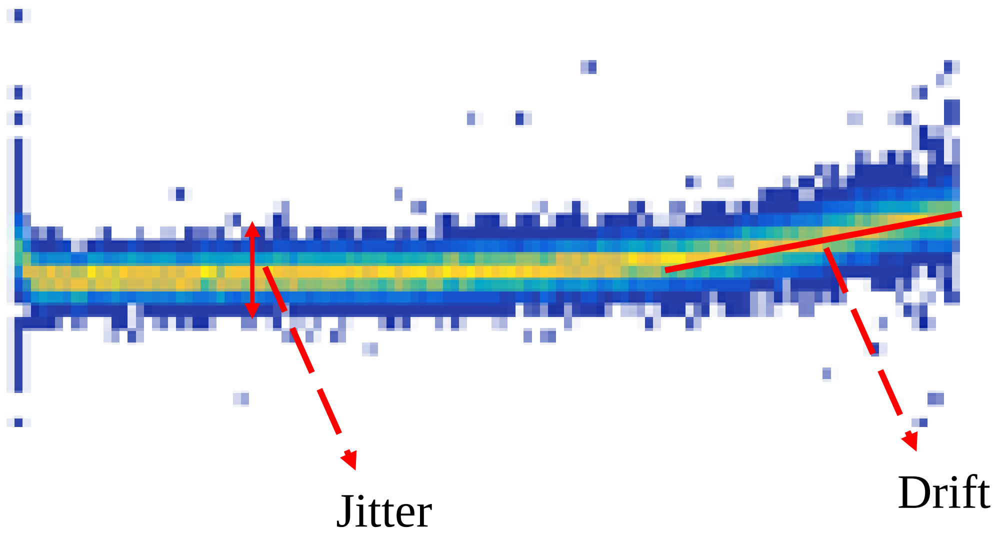
$$\sigma_{B_j}^2 = \sigma_{\det\_B_j}^2 + \sigma_{beam}^2$$

Provided the beam fluctuation for these two boards are the same.
Should be careful when the energy is low.
Simulation work is required.

By Boardi – Boardj:

$$\sigma_{B_j-B_i}^2 = \sigma_{\det\_B_i}^2 + \sigma_{\det\_B_j}^2 \Rightarrow \sigma = \frac{\sigma_{B_j-B_i}}{\sqrt{2}}$$

For Beam: $\sigma_{beam}^2 = \sigma_{beam-jitter}^2 + \sigma_{beam-drift}^2$



By Framek– Framek-1:

$$\sigma_{Bi}{}_{F_k-F_{k-i}}^2 = 2\sigma_{\det\_B_i}^2 + 2\sigma_{beam-jitter}^2$$

$$\sigma_{Bj}{}_{F_k-F_{k-i}}^2 = 2\sigma_{\det\_B_j}^2 + 2\sigma_{beam-jitter}^2$$

Linear Regression for a Gaussian-like Profile: FAS

Algorithms for Gauss Regression are basically in two categories according to the form of Gauss used.

$$y(x) = Ae^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

1 $$\ln(y) = \ln(A) + \frac{-(x-\mu)^2}{2\sigma^2} = \ln(A) - \frac{\mu^2}{2\sigma^2} + \frac{2\mu x}{2\sigma^2} - \frac{x^2}{2\sigma^2} = a + bx + cx^2,$$

2 $$y(x) = \beta_1\phi_1(x) + \beta_2\phi_2(x)$$

where $\beta_1 = -1/\sigma^2$, $\beta_2 = \mu/\sigma^2$, and

$$\phi_1(x) = \int_{-\infty}^{x} u\,y(u)\,du, \quad \phi_2(x) = \int_{-\infty}^{x} y(u)\,du.$$

DO: $$\Lambda = \int_{-\infty}^{\infty} Ae^{-\frac{(x-\mu)^2}{2\sigma^2}}dx = A\sigma\sqrt{2\pi}.$$

Error Function:

a $$\varepsilon = \ln(\hat{y}) - \ln(y) = \ln(\hat{y}) - (a + bx + cx^2).$$ Or $\varepsilon = \hat{y} - y$

b $$\delta = \hat{y}\varepsilon = \hat{y}[\ln(\hat{y}) - (a + bx + cx^2)]$$

ROONIZI: 2 a
Guo: 1 b
FAS: DO 1 b